Subject – Math (Standard Level)
Topic - Vector
Year - Nov 2011 – Nov 2017
Paper- 1

Question 2

(a) (i) evidence of approach *(M1)*

e.g. $\vec{PO}+\vec{OQ}$, $P-Q$

$\vec{PQ}=\begin{pmatrix}2\\1\\-4\end{pmatrix}$ ***A1*** ***N2***

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+s\boldsymbol{b}$ (accept any parameter for s)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\4\\8\end{pmatrix}$ or $\begin{pmatrix}4\\5\\4\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}2\\1\\-4\end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r}=\begin{pmatrix}2\\4\\8\end{pmatrix}+s\begin{pmatrix}2\\1\\-4\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}4+2s\\5+1s\\4-4s\end{pmatrix}$, $\boldsymbol{r}=2\boldsymbol{i}+4\boldsymbol{j}+8\boldsymbol{k}+s(2\boldsymbol{i}+1\boldsymbol{j}-4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+s\boldsymbol{b}$, ***A1*** for $\boldsymbol{L}=\boldsymbol{a}+s\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+s\boldsymbol{a}$.

[4 marks]

(b) (i) choosing correct direction vectors for L_1 and L_2 *(A1)(A1)*

e.g. $\begin{pmatrix}2\\1\\-4\end{pmatrix}$, $\begin{pmatrix}3p\\2p\\4\end{pmatrix}$

evidence of equating scalar product to 0 *(M1)*

correct calculation of scalar product ***A1***

e.g. $2\times3p+1\times2p+(-4)\times4$, $8p-16=0$

$p=2$ ***A1*** ***N3***

(ii) **any** correct expression in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}10\\6\\-40\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\4\\4\end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r}=\begin{pmatrix}10\\6\\-40\end{pmatrix}+t\begin{pmatrix}6\\4\\4\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}10+6s\\6+4s\\-40+4s\end{pmatrix}$, $\boldsymbol{r}=10\boldsymbol{i}+6\boldsymbol{j}-40\boldsymbol{k}+s(6\boldsymbol{i}+4\boldsymbol{j}+4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$ (unless they have been penalised for $\boldsymbol{L}=\boldsymbol{a}+s\boldsymbol{b}$ in part (a)), ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[7 marks]

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(c) appropriate approach *(M1)*

e.g. $\begin{pmatrix}2\\4\\8\end{pmatrix}+s\begin{pmatrix}2\\1\\-4\end{pmatrix}=\begin{pmatrix}10\\6\\-40\end{pmatrix}+t\begin{pmatrix}6\\4\\4\end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***

e.g. $2+2s=10+6t$, $4+s=6+4t$, $8-4s=-40+4t$

attempt to solve simultaneous equations ***(M1)***

correct working ***(A1)***

e.g. $-6=-2-2t$, $4=2t$, $-4+5s=46$, $5s=50$

one correct parameter $s=10$, $t=2$ ***A1***

$x=22$ $\left(\text{accept } (22, 14, -32)\right)$ ***A1*** ***N4***

[7 marks]

Total [18 marks]

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Question 2

(a) (i) evidence of correct approach *A1*

e.g. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$, $Q - P$

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$ *AG* *N0*

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ *A2* *N2*

where $\boldsymbol{a}$ is either $\overrightarrow{OP}$ or $\overrightarrow{OQ}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{PQ}$

e.g. $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} t \\ 4-2t \\ 1+2t \end{pmatrix}$, $\boldsymbol{r} = 4\boldsymbol{j} + \boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

[3 marks]

(b) choosing a correct direction vector for L_2 *(A1)*

e.g. $\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$

finding scalar products and magnitudes *(A1)(A1)(A1)*

scalar product $= 1(3) - 2(0) + 2(-4) \quad (= -5)$

magnitudes $= \sqrt{1^2 + (-2)^2 + 2^2} \quad (= 3)$, $\sqrt{3^2 + 0^2 + (-4)^2} \quad (= 5)$

substitution into formula *M1*

e.g. $\cos\theta = \dfrac{-5}{\sqrt{9} \times \sqrt{25}}$

$\cos\theta = -\dfrac{1}{3}$ *A2* *N5*

[7 marks]

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QQ

(c) evidence of valid approach *(M1)*

e.g. equating lines, $L_1 = L_2$

EITHER

one correct equation in one variable *A2*

e.g. $6 - 2t = 2$

OR

two correct equations in two variables *A1A1*

e.g. $2t + 4s = 0$, $t - 3s = 5$

THEN

attempt to solve *(M1)*

one correct parameter *A1*

e.g. $t = 2$, $s = -1$

correct substitution of either parameter *(A1)*

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + (-1)\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + (2)\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

coordinates $\mathrm{R}(1, 2, 3)$ *A1* *N3*

[7 marks]

Total [17 marks]

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Question 3

(a) **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$, $\boldsymbol{r} = 5\boldsymbol{i} - 4\boldsymbol{j} + 10\boldsymbol{k} + t(-8\boldsymbol{i} + 4\boldsymbol{j} - 10\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

(b) recognizing that $y = 0$ or $z = 0$ at x-intercept (seen anywhere) ***(R1)***

attempt to set up equation for x-intercept (must suggest $x \neq 0$) ***(M1)***

e.g. $L = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$, $5 + 4t = x$, $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

one correct equation in one variable ***(A1)***

e.g. $-4 - 2t = 0$, $10 + 5t = 0$

finding $t = -2$ ***A1***

correct working ***(A1)***

e.g. $x = 5 + (-2)(4)$

$x = -3$ (accept $(-3, 0, 0)$) ***A1*** ***N3***

[6 marks]

Total [8 marks]

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Question 4

(a) correct approach *A1*

e.g. $\vec{AO}+\vec{OB}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}-\begin{pmatrix}5\\2\\1\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}1\\-2\\2\end{pmatrix}$ *AG* *N0*

[1 mark]

(b) recognizing $\vec{AD}$ is perpendicular to $\vec{AB}$ (may be seen in sketch) ***(R1)***

e.g. adjacent sides of rectangle are perpendicular

recognizing dot product must be zero ***(R1)***

e.g. $\vec{AD}\bullet\vec{AB}=0$

correct substitution ***(A1)***

e.g. $(1\times 4)+(-2\times p)+(2\times 1)$, $4-2p+2=0$

equation which clearly leads to $p=3$ ***A1***

e.g. $6-2p=0$, $2p=6$

$p=3$ ***AG*** ***N0***

[4 marks]

(c) correct approach (seen anywhere including sketch) ***(A1)***

e.g. $\vec{OC}=\vec{OB}+\vec{BC}$, $\vec{OD}+\vec{DC}$

recognizing opposite sides are equal vectors (may be seen in sketch) ***(R1)***

e.g. $\vec{BC}=\vec{AD}$, $\vec{DC}=\vec{AB}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}+\begin{pmatrix}4\\3\\1\end{pmatrix}$, $\begin{pmatrix}9\\5\\2\end{pmatrix}+\begin{pmatrix}1\\-2\\2\end{pmatrix}$

coordinates of point C are $(10, 3, 4)$ $\left(\text{accept } \begin{pmatrix}10\\3\\4\end{pmatrix}\right)$ ***A2*** ***N4***

Note: Award ***A1*** for two correct values.

[4 marks]

(d) attempt to find one side of the rectangle *(M1)*
e.g. substituting into magnitude formula

two correct magnitudes ***A1A1***
e.g. $\sqrt{(1)^2+(-2)^2+2^2}$, 3; $\sqrt{16+9+1}$, $\sqrt{26}$

multiplying magnitudes *(M1)*
e.g. $\sqrt{26}\times\sqrt{9}$

area $=\sqrt{234}\left(=3\sqrt{26}\right)$ $\left(\text{accept } 3\times\sqrt{26}\right)$ ***A1*** ***N3***

[5 marks]

Total [14 marks]

Question 5

(a) (i) $2\boldsymbol{a}=\begin{pmatrix}4\\-6\end{pmatrix}$ *(A1)*

correct expression for $2\boldsymbol{a}+\boldsymbol{b}$ ***A1*** ***N2***

eg $\begin{pmatrix}5\\-2\end{pmatrix}$, $(5,-2)$, $5\boldsymbol{i}-2\boldsymbol{j}$

(ii) correct substitution into length formula *(A1)*

eg $\sqrt{5^2+2^2}$, $\sqrt{5^2+-2^2}$

$|2\boldsymbol{a}+\boldsymbol{b}|=\sqrt{29}$ ***A1*** ***N2***

[4 marks]

(b) valid approach *(M1)*

eg $\boldsymbol{c}=-(2\boldsymbol{a}+\boldsymbol{b})$, $5+x=0, -2+y=0$

$\boldsymbol{c}=\begin{pmatrix}-5\\2\end{pmatrix}$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question 6

(a) (i) valid approach *(M1)*

eg $\begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $A - B$, $\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ *A1* *N2*

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\vec{AB}$ *A2* *N2*

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $(x, y, z) = (1, -2, 3) + t(3, -1, 2)$, $\boldsymbol{r} = \begin{pmatrix} 1+6t \\ -2-2t \\ 3+4t \end{pmatrix}$

Note: Award *A1* for $\boldsymbol{a} + t\boldsymbol{b}$, *A1* for $L_1 = \boldsymbol{a} + t\boldsymbol{b}$, *A0* for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

(b) recognizing that scalar product $= 0$ (seen anywhere) *R1*

correct calculation of scalar product *(A1)*

eg $6(3) - 2(-3) + 4p$, $18 + 6 + 4p$

correct working *A1*

eg $24 + 4p = 0$, $4p = -24$

$p = -6$ *AG* *N0*

[3 marks]

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(c) setting lines equal *(M1)*

eg $L_1 = L_2$, $\begin{pmatrix}1\\-2\\3\end{pmatrix}+t\begin{pmatrix}6\\-2\\4\end{pmatrix}=\begin{pmatrix}-1\\2\\15\end{pmatrix}+s\begin{pmatrix}3\\-3\\-6\end{pmatrix}$

any two correct equations with **different** parameters *A1A1*

eg $1+6t=-1+3s$, $-2-2t=2-3s$, $3+4t=15-6s$

attempt to solve **their** simultaneous equations *(M1)*

one correct parameter *A1*

eg $t=\frac{1}{2}$, $s=\frac{5}{3}$

attempt to substitute parameter into vector equation *(M1)*

eg $\begin{pmatrix}1\\-2\\3\end{pmatrix}+\frac{1}{2}\begin{pmatrix}6\\-2\\4\end{pmatrix}$, $1+\frac{1}{2}\times 6$

$x=4$ (accept $(4, -3, 5)$, ignore incorrect values for y and z) *A1* *N3*

[7 marks]

Total [14 marks]

Question 7

(a) appropriate approach *(M1)*

eg $\vec{QP}=\vec{QO}+\vec{OP}$, $P-Q$

$\vec{QP}=\boldsymbol{p}-\boldsymbol{q}$ *A1* *N2*

[2 marks]

(b) recognizing correct vector for $\vec{QT}$ or $\vec{PT}$ *(A1)*

eg $\vec{QT}=\frac{1}{2}(\boldsymbol{p}-\boldsymbol{q})$, $\vec{PT}=\frac{1}{2}(\boldsymbol{q}-\boldsymbol{p})$

appropriate approach *(M1)*

eg $\vec{OT}=\vec{OP}+\vec{PT}$, $\vec{OQ}+\vec{QT}$, $\vec{OP}+\frac{1}{2}\vec{PQ}$

$\vec{OT}=\frac{1}{2}(\boldsymbol{p}+\boldsymbol{q})$ $\left(\text{accept } \frac{\boldsymbol{p}+\boldsymbol{q}}{2}\right)$ *A1* *N2*

[3 marks]

[Total 5 marks]

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Question 8

(a) correct approach *A1*

eg $\begin{pmatrix}1\\1\\5\end{pmatrix}-\begin{pmatrix}2\\1\\4\end{pmatrix}$, $\text{AO}+\text{OB}$, $\boldsymbol{b}-\boldsymbol{a}$

$\vec{\text{AB}}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *AG* *N0*

[1 mark]

(b) (i) correct vector (or any multiple) *A1* *N1*

eg $\boldsymbol{d}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for *t*)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\1\\4\end{pmatrix}$ or $\begin{pmatrix}1\\1\\5\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *A2* *N2*

eg $\boldsymbol{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+t\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2-s\\1\\4+s\end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $L_1=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[3 marks]

(c) valid approach *(M1)*

eg $r_1 = r_2$, $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

one correct equation in one parameter ***A1***

eg $2 - t = 4, 1 = 7 - s, 1 - t = 4$

attempt to solve *(M1)*

eg $2 - 4 = t, s = 7 - 1, t = 1 - 4$

one correct parameter ***A1***

eg $t = -2, s = 6, t = -3,$

attempt to substitute **their** parameter into vector equation *(M1)*

eg $\begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + 6\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

P(4, 1, 2) (accept position vector) ***A1*** ***N2***

[6 marks]

(d) (i) correct direction vector for L_2 ***A1*** ***N1***

eg $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

(ii) correct scalar product and magnitudes for **their** direction vectors *(A1)(A1)(A1)*

scalar product $= 0 \times -1 + -1 \times 0 + 1 \times 1\ (= 1)$

magnitudes $= \sqrt{0^2 + (-1)^2 + 1^2}, \sqrt{-1^2 + 0^2 + 1^2}\ \left(\sqrt{2}, \sqrt{2}\right)$

attempt to substitute **their** values into formula ***M1***

eg $\dfrac{0 + 0 + 1}{\left(\sqrt{0^2 + (-1)^2 + 1^2}\right) \times \left(\sqrt{-1^2 + 0^2 + 1^2}\right)}, \dfrac{1}{\sqrt{2} \times \sqrt{2}}$

correct value for cosine, $\dfrac{1}{2}$ ***A1***

angle is $\dfrac{\pi}{3}\ (= 60°)$ ***A1*** ***N1***

[7 marks]

Total [17 marks]

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Question 9

(a) attempt to find gradient *(M1)*

eg reference to change in x is 3 and/or y is 2, $\frac{3}{2}$

gradient $=\frac{2}{3}$ *A1* *N2*

[2 marks]

(b) attempt to substitute coordinates and/or gradient into Cartesian equation for a line *(M1)*

eg $y-4=m(x-9)$, $y=\frac{2}{3}x+b$, $9=a(4)+c$

correct substitution *(A1)*

eg $4=\frac{2}{3}(9)+c$, $y-4=\frac{2}{3}(x-9)$

$y=\frac{2}{3}x-2$ $\left(\text{accept } a=\frac{2}{3}, b=-2\right)$ *A1* *N2*

[3 marks]

(c) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t), where $\boldsymbol{a}$ indicates position *eg* $\begin{pmatrix}9\\4\end{pmatrix}$ or $\begin{pmatrix}0\\-2\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}3\\2\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}9\\4\end{pmatrix}+t\begin{pmatrix}3\\2\end{pmatrix}$, $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3t+9\\2t+4\end{pmatrix}$, $\boldsymbol{r}=0i-2j+s(3i+2j)$ *A2* *N2*

Note: Award ***A1*** for $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $L=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[2 marks]

Total [7 marks]

Question 10

(a) valid approach *(M1)*

eg magnitude of direction vector

correct working *(A1)*

eg $\sqrt{(-4)^2+2^2+4^2}$, $\sqrt{-4^2+2^2+4^2}$

6 (ms^{-1}) ***A1*** ***N2***

[3 marks]

(b) substituting 2 for t *(A1)*

eg $0+2(4)$, $\boldsymbol{r}=\begin{pmatrix}5\\6\\0\end{pmatrix}+2\begin{pmatrix}-4\\2\\4\end{pmatrix}$, $\begin{pmatrix}-3\\10\\8\end{pmatrix}$, $y=10$

8 (metres) ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

choosing correct direction vectors $\begin{pmatrix}-4\\2\\4\end{pmatrix}$ and $\begin{pmatrix}4\\-6\\7\end{pmatrix}$ *(A1)(A1)*

evidence of scalar product ***M1***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$

correct substitution into scalar product *(A1)*

eg $(-4\times4)+(2\times-6)+(4\times7)$

evidence of correct calculation of the scalar product as 0 ***A1***

eg $-16-12+28=0$

directions are perpendicular ***AG*** ***N0***

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METHOD 2

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

attempt to find angle between vectors ***M1***

correct substitution into numerator ***A1***

eg $\cos\theta = \dfrac{-16-12+28}{|a||b|}$, $\cos\theta = 0$

$\theta = 90^\circ$ ***A1***

directions are perpendicular ***AG*** ***N0***

[5 marks]

(d) **METHOD 1**

one correct equation for Ryan's airplane ***(A1)***

eg $5-4t=-23,\ 6+2t=20,\ 0+4t=28$

$t=7$ ***A1***

one correct equation for Jack's airplane ***(A1)***

eg $-39+4s=-23,\ 44-6s=20,\ 0+7s=28$

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

METHOD 2

valid approach ***(M1)***

eg $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$, one correct equation

two correct equations ***(A1)***

eg $5-4t=-39+4s,\ 6+2t=44-6s,\ 4t=7s$

$t=7$ ***A1***

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

[5 marks]

Total [15 marks]

Question 11

attempt to find $\cos C\hat{A}B$ (seen anywhere) ***(M1)***

eg $\cos\theta = \dfrac{\vec{AB}\bullet\vec{AC}}{|\vec{AB}||\vec{AC}|}$

$\cos C\hat{A}B = \dfrac{-5\sqrt{3}}{10}$ $\left(= -\dfrac{\sqrt{3}}{2}\right)$ ***A1***

valid attempt to find $\sin C\hat{A}B$ ***(M1)***

eg triangle, Pythagorean identity, $C\hat{A}B = \dfrac{5\pi}{6}, 150^{\circ}$

$\sin C\hat{A}B = \dfrac{1}{2}$ ***(A1)***

correct substitution into formula for area ***(A1)***

eg $\dfrac{1}{2}\times 10\times\dfrac{1}{2}, \dfrac{1}{2}\times 10\times\sin\dfrac{\pi}{6}$

area $= \dfrac{10}{4}$ $\left(= \dfrac{5}{2}\right)$ ***A1*** ***N3***

[6 marks]

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Question 12

(a) attempt to substitute $x=1$ *(M1)*

eg $\boldsymbol{r}=\begin{pmatrix}1\\ \frac{2}{1}\end{pmatrix}+t\begin{pmatrix}1^2\\ -2\end{pmatrix}$, $L_1=\begin{pmatrix}1\\ 2\end{pmatrix}+t\begin{pmatrix}1\\ -2\end{pmatrix}$

correct equation (vector or Cartesian, but do not accept "$L_1=$")

eg $\boldsymbol{r}=\begin{pmatrix}1\\ 2\end{pmatrix}+t\begin{pmatrix}1\\ -2\end{pmatrix}$, $y=-2x+4$ (must be an equation) ***A1*** ***N2***

[2 marks]

(b) appropriate approach *(M1)*

eg $\begin{pmatrix}0\\ y\end{pmatrix}=\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}+t\begin{pmatrix}a^2\\ -2\end{pmatrix}$

correct equation for x-coordinate ***A1***

eg $0=a+ta^2$

$t=\frac{-1}{a}$ ***A1***

substituting **their** parameter to find y *(M1)*

eg $y=\frac{2}{a}-2\left(\frac{-1}{a}\right)$, $\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}-\frac{1}{a}\begin{pmatrix}a^2\\ -2\end{pmatrix}$

correct working ***A1***

eg $y=\frac{2}{a}+\frac{2}{a}$, $\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}-\begin{pmatrix}a\\ -\frac{2}{a}\end{pmatrix}$

finding correct expression for y ***A1***

eg $y=\frac{4}{a}$, $\begin{pmatrix}0\\ \frac{4}{a}\end{pmatrix}$

$\mathrm{P}\left(0, \frac{4}{a}\right)$ ***AG*** ***N0***

[6 marks]

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(c) valid approach *M1*

eg distance formula, Pythagorean Theorem, $\overrightarrow{PQ} = \begin{pmatrix} 2a \\ -\frac{4}{a} \end{pmatrix}$

correct simplification *A1*

eg $(2a)^2 + \left(\frac{4}{a}\right)^2$

$d = 4a^2 + \frac{16}{a^2}$ *AG* *N0*

[2 marks]

(d) recognizing need to find derivative *(M1)*

eg d', $d'(a)$

correct derivative *A2*

eg $8a - \frac{32}{a^3}$, $8x - \frac{32}{x^3}$

setting **their** derivative equal to 0 *(M1)*

eg $8a - \frac{32}{a^3} = 0$

correct working *(A1)*

eg $8a = \frac{32}{a^3}$, $8a^4 - 32 = 0$

working towards solution *(A1)*

eg $a^4 = 4$, $a^2 = 2$, $a = \pm\sqrt{2}$

$a = \sqrt[4]{4}$ $\left(a = \sqrt{2}\right)$ (do not accept $\pm\sqrt{2}$) *A1* *N3*

[7 marks]

Total [17 marks]

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Question 13

(e) **METHOD 1** (area $= 0.5 \times$ height $\times$ base)

$\left|\vec{OC}\right| = \sqrt{0+2^2+(-1)^2}$ $\left(=\sqrt{5}\right)$ (seen anywhere) ***A1***

valid approach ***(M1)***

eg $\frac{1}{2} \times \left|\vec{AB}\right| \times \left|\vec{OC}\right|$, $\left|\vec{OC}\right|$ is height of triangle

correct substitution ***A1***

eg $\frac{1}{2} \times \sqrt{6} \times \sqrt{0+(2)^2+(-1)^2}$, $\frac{1}{2} \times \sqrt{6} \times \sqrt{5}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

METHOD 2 (difference of two areas)

one correct magnitude (seen anywhere) ***A1***

eg $\left|\vec{OC}\right| = \sqrt{2^2+(-1)^2}$ $\left(=\sqrt{5}\right)$, $\left|\vec{AC}\right| = \sqrt{4+4+16}$ $\left(=\sqrt{24}\right)$, $\left|\vec{BC}\right| = \sqrt{6}$

valid approach ***(M1)***

eg $\Delta OAC - \Delta OBC$

correct substitution ***A1***

eg $\frac{1}{2} \times \sqrt{24} \times \sqrt{5} - \frac{1}{2} \times \sqrt{5} \times \sqrt{6}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

METHOD 3 (area $= \frac{1}{2}ab\sin C$ for ΔOAB)

one correct magnitude of $\vec{OA}$ or $\vec{OB}$ (seen anywhere) ***A1***

eg $\left|\vec{OA}\right| = \sqrt{(-2)^2+4^2+3^2}$ $\left(=\sqrt{29}\right)$, $\left|\vec{OB}\right| = \sqrt{1+9+1}$ $\left(=\sqrt{11}\right)$

valid attempt to find $\cos\theta$ ***or*** $\sin\theta$ ***(M1)***

eg $\cos C = \frac{-1-3-2}{\sqrt{6} \times \sqrt{11}} \left(= \frac{-6}{\sqrt{66}}\right)$, $29 = 6+11-2\sqrt{6}\sqrt{11}\cos\theta$, $\frac{\sin\theta}{\sqrt{5}} = \frac{\sin 90}{\sqrt{29}}$

correct substitution into $\frac{1}{2}ab\sin C$ ***A1***

eg $\frac{1}{2} \times \sqrt{6} \times \sqrt{11} \times \sqrt{1-\frac{36}{66}}$, $0.5 \times \sqrt{6} \times \sqrt{29} \times \frac{\sqrt{5}}{\sqrt{29}}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

[4 marks]

Total [16 marks]

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Question 14

(c) **METHOD 1**

substituting **their** x to find y-value *(M1)*

eg $f(-1)$, $-4(-1+3)(-1-1)$

correct calculation *(A1)*

eg $-4(2)(-2)$

largest value is 16 **A1** **N2**

METHOD 2

valid attempt to complete the square *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

correct vertex form *(A1)*

eg $-4(x+1)^2+16$

largest value is 16 **A1** **N2**

METHOD 3

valid approach (may be seen in (b)) *(M1)*

eg $f'(x)=0$, $-8x-8=0$

substituting $x=-1$ into $f(x)$ *(A1)*

eg $-4(-1)^2-8(-1)+12$

largest value is 16 **A1** **N2**

[3 marks]

(d) **METHOD 1**

recognizing coordinates of vertex *(M1)*

eg $(-1, 16)$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ **A1A1** **N3**

METHOD 2

valid attempt to complete the square (may be seen in (c)) *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ **A1A1** **N3**

[3 marks]

Total [15 marks]

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Question 15

(a) correct approach *(A1)*

eg $\vec{CB} = \vec{CA} + \vec{AB}$, $\vec{AB} - \vec{AC}$, $\vec{AC} + \vec{CB} = \vec{AB}$

$\vec{CB} = -\boldsymbol{q} + \boldsymbol{p}$ ***A1*** ***N2***

[2 marks]

(b) correct approach *(A1)*

eg $\vec{CD} = \vec{BA}$

$\vec{CD} = -\boldsymbol{p}$ ***A1*** ***N2***

[2 marks]

(c) correct approach *(A1)*

eg $\vec{DB} = \vec{DC} + \vec{CB}$, $\vec{DA} + \vec{AB}$

correct working *(A1)*

eg $\vec{DB} = \boldsymbol{p} - (\boldsymbol{q} - \boldsymbol{p})$, $\boldsymbol{p} + \boldsymbol{p} - \boldsymbol{q}$

$\vec{DB} = 2\boldsymbol{p} - \boldsymbol{q}$ ***A1*** ***N2***

[3 marks]

[Total 7 marks]

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Question 16

(a) (i) correct approach **A1**

eg $\text{OB}-\text{OA}$, $\begin{pmatrix}-2\\5\\3\end{pmatrix}-\begin{pmatrix}0\\-3\\1\end{pmatrix}$

$\vec{\text{AB}}=\begin{pmatrix}-2\\8\\2\end{pmatrix}$ **AG** **N0**

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}0\\-3\\1\end{pmatrix}$ or $\begin{pmatrix}-2\\5\\3\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}-2\\8\\2\end{pmatrix}$ **A2** **N2**

eg $\boldsymbol{r}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+t\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}-2-2s\\5+8s\\3+2s\end{pmatrix}$, $\boldsymbol{r}=-2\boldsymbol{i}+5\boldsymbol{j}+3\boldsymbol{k}+t(-2\boldsymbol{i}+8\boldsymbol{j}+2\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $L=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[3 marks]

(b) valid approach ***(M1)***

eg equating lines, $L_1=L_2$

one correct equation in one variable ***A1***

eg $-2t=-1$, $-2-2t=-1$

valid attempt to solve ***(M1)***

eg $2t=1$, $-2t=1$

one correct parameter ***A1***

eg $t=\frac{1}{2}$, $t=-\frac{1}{2}$, $s=-6$

correct substitution of either parameter ***A1***

eg $r=\begin{pmatrix}0\\-3\\1\end{pmatrix}+\frac{1}{2}\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $r=\begin{pmatrix}-2\\5\\3\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $r=\begin{pmatrix}-1\\7\\-4\end{pmatrix}-6\begin{pmatrix}0\\1\\-1\end{pmatrix}$

the coordinates of C are $(-1,1,2)$, or position vector of C is $\begin{pmatrix}-1\\1\\2\end{pmatrix}$ ***AG*** ***N0***

Note: If candidate uses the same parameter in both vector equations and working shown, award ***M1A1M1A0A0***.

[5 marks]

(c) valid approach **(M1)**

eg attempt to find $\overrightarrow{CA}$, $\cos A\hat{C}D = \dfrac{\overrightarrow{CA} \bullet \overrightarrow{CD}}{\left|\overrightarrow{CA}\right|\left|\overrightarrow{CD}\right|}$, $A\hat{C}D$ formed by $\overrightarrow{CA}$ and $\overrightarrow{CD}$

$\overrightarrow{CA} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$ **(A1)**

finding $\left|\overrightarrow{CA}\right|$ (may be seen in cosine formula) **A1**

eg $\sqrt{1^2 + (-4)^2 + (-1)^2}$, $\sqrt{18}$

correct substitution into cosine formula **(A1)**

eg $\dfrac{-9}{\sqrt{18}\sqrt{18}}$

finding $\cos A\hat{C}D = -\dfrac{1}{2}$ **(A1)**

$A\hat{C}D = \dfrac{2\pi}{3}$ (120°) **A2** **N2**

Notes: Award ***A1*** if additional answers are given.

[7 marks]

Total [15 marks]

Question 17

correct scalar product **(A1)**
eg $m + n$

setting up their scalar product equal to 0 (seen anywhere) **(M1)**

eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0$, $-3(0) + 1(m) + 1(n) = 0$, $m = -n$

correct interpretation of unit vector **(A1)**

eg $\sqrt{0^2 + m^2 + n^2} = 1$, $m^2 + n^2 = 1$

valid attempt to solve their equations (must be in one variable) **M1**

eg $(-n)^2 + n^2 = 1$, $\sqrt{1 - n^2} + n = 0$, $m^2 + (-m)^2 = 1$, $m - \sqrt{1 - m^2} = 0$

correct working **A1**

eg $2n^2 = 1$, $2m^2 = 1$, $\sqrt{2} = \dfrac{1}{n}$, $m = \pm\dfrac{1}{\sqrt{2}}$

both correct pairs **A2** **N3**

eg $m = \dfrac{1}{\sqrt{2}}$ and $n = -\dfrac{1}{\sqrt{2}}$, $m = -\dfrac{1}{\sqrt{2}}$ and $n = \dfrac{1}{\sqrt{2}}$,

$m = (0.5)^{\frac{1}{2}}$ and $n = -(0.5)^{\frac{1}{2}}$, $m = -\sqrt{\dfrac{1}{2}}$ and $n = \sqrt{\dfrac{1}{2}}$

Note: Award ***A0*** for $m = \pm\dfrac{1}{\sqrt{2}}$, $n = \pm\dfrac{1}{\sqrt{2}}$, or any other answer that does not clearly indicate the correct pairs.

[7 marks]

Question 18

(a) valid attempt to find direction vector **(M1)**

eg $\overrightarrow{PQ}, \overrightarrow{QP}$

correct direction vector (or multiple of) **(A1)**

eg $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (any parameter for t) **A2** **N3**

where $\boldsymbol{a}$ is $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ or $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

eg $\boldsymbol{r} = 7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k} + t(6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k})$, $\boldsymbol{r} = \begin{pmatrix} 1+6s \\ 2+1s \\ -1-3s \end{pmatrix}$, $r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$

Notes: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L} = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

(b) correct expression for scalar product **(A1)**

eg $6 \times 2 + 1 \times 0 + (-3) \times n$, $-3n + 12$

setting scalar product equal to zero (seen anywhere) **(M1)**

eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0$, $-3n + 12 = 0$

$n = 4$ **A1** **N2**

[3 marks]

[Total 7 marks]

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Question 19

(a) (i) valid approach to find $\vec{AB}$ *(M1)*

eg $\vec{OB}-\vec{OA}$, $\begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\vec{AB}=\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1*** ***N2***

(ii) valid approach to find $\left|\vec{AB}\right|$ *(M1)*

eg $\sqrt{(5)^2+(1)^2+(-1)^2}$

$\left|\vec{AB}\right|=\sqrt{27}$ ***A1*** ***N2***

[4 marks]

(b) correct approach ***A1***

eg $\vec{OC}=\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}+\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

C has coordinates $(-2, 1, 3)$ ***AG*** ***N0***

[1 mark]

(c) (i) $A\hat{D}B=\pi-\theta$, $\hat{D}=180-\theta$ ***A1*** ***N1***

(ii) any correct expression for the area involving θ ***A1*** ***N1***

eg $\text{area}=\frac{1}{2}\times AD\times BD\times\sin(180-\theta)$, $\frac{1}{2}ab\sin\theta$, $\frac{1}{2}\left|\vec{DA}\right|\left|\vec{DB}\right|\sin(\pi-\theta)$

[2 marks]

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(d) **METHOD 1** (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\text{AD}\times\text{DC}\times\sin\theta$

correct equation involving areas ***A1***

eg $\dfrac{\frac{1}{2}\text{AD}\times\text{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\text{AD}\times\text{DC}\times\sin\theta}=3$

recognizing that $\sin(\pi-\theta)=\sin\theta$ (seen anywhere) ***(A1)***

$\frac{\text{BD}}{\text{DC}}=3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\text{DC}}+\overrightarrow{\text{DC}}=\overrightarrow{\text{BC}}$, $\overrightarrow{\text{BC}}=4\overrightarrow{\text{DC}}$

correct ratio $\frac{\text{BD}}{\text{BC}}=\frac{3}{4}$ ***AG*** ***N0***

METHOD 2 (Geometric approach)

recognising ΔABD and ΔACD have same height ***(A1)***

eg use of h for both triangles, $\dfrac{\frac{1}{2}\text{BD}\times h}{\frac{1}{2}\text{CD}\times h}=3$

correct approach ***A2***

eg $\text{BD}=3x$ and $\text{DC}=x$, $\frac{\text{BD}}{\text{DC}}=3$

correct working ***A2***

eg $\text{BC}=4x$, $\text{BD}+\text{DC}=4\text{DC}$, $\frac{\text{BD}}{\text{BC}}=\frac{3x}{4x}$, $\frac{\text{BD}}{\text{BC}}=\frac{3\text{DC}}{4\text{DC}}$

$\frac{\text{BD}}{\text{BC}}=\frac{3}{4}$ ***AG*** ***N0***

[5 marks]

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(e) correct working (seen anywhere) ***(A1)***

eg $\vec{BD} = \frac{3}{4}\vec{BC}$, $\vec{OD} = \vec{OB} + \frac{3}{4}\begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix}$, $\vec{CD} = \frac{1}{4}\vec{CB}$

valid approach (seen anywhere) ***(M1)***

eg $\vec{OD} = \vec{OB} + \vec{BD}$, $\overrightarrow{BC} = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix}$

correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix}$, $x = 4 + \frac{3}{4}(-6)$, $-2 + \frac{1}{4}(6)$

D is $\left(-\frac{1}{2}, 1, 3\right)$ ***A1*** ***N3***

[4 marks]

[Total 16 marks]

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Question 20

(a) (i) valid approach **(M1)**

eg $A - B,\ -\begin{pmatrix}0\\1\\8\end{pmatrix}+\begin{pmatrix}3\\5\\2\end{pmatrix}$

$\vec{AB} = \begin{pmatrix}3\\4\\-6\end{pmatrix}$ **A1** **N2**

(ii) **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (any parameter for t) **A2** **N2**

where $\boldsymbol{a}$ is $\begin{pmatrix}0\\1\\8\end{pmatrix}$ or $\begin{pmatrix}3\\5\\2\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}3\\4\\-6\end{pmatrix}$

eg $\boldsymbol{r} = \begin{pmatrix}0\\1\\8\end{pmatrix} + t\begin{pmatrix}3\\4\\-6\end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix}3+3t\\5+4t\\2-6t\end{pmatrix}$, $\boldsymbol{r} = \boldsymbol{j} + 8\boldsymbol{k} + t(3\boldsymbol{i} + 4\boldsymbol{j} - 6\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L} = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

(b) valid approach **(M1)**

eg $a \cdot b = 0$

choosing correct direction vectors (may be seen in scalar product) **A1**

eg $\begin{pmatrix}3\\4\\-6\end{pmatrix}$ and $\begin{pmatrix}p\\0\\1\end{pmatrix}$, $\begin{pmatrix}3\\4\\-6\end{pmatrix} \cdot \begin{pmatrix}p\\0\\1\end{pmatrix} = 0$

correct working/equation **A1**

eg $3p - 6 = 0$

$p = 2$ **AG** **N0**

[3 marks]

(c) valid approach **(M1)**

eg $L_1 = \begin{pmatrix} 9 \\ 13 \\ z \end{pmatrix}$, $L_1 = L_2$

one correct equation (must be different parameters if both lines used) **(A1)**

eg $3t = 9$, $1 + 2s = 9$, $5 + 4t = 13$, $3t = 1 + 2s$

one correct value **A1**

eg $t = 3$, $s = 4$, $t = 2$

valid approach to substitute their t or s value **(M1)**

eg $8 + 3(-6)$, $-14 + 4(1)$

$z = -10$ **A1** **N3**

[5 marks]

(d) (i) $\left|\vec{d}\right| = \sqrt{2^2 + 1}$ $\left(= \sqrt{5}\right)$ **(A1)**

$\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\left(\text{accept } \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{0}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}\right)$ **A1** **N2**

(ii) **METHOD 1 (using unit vector)**

valid approach **(M1)**

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} \pm \sqrt{5}\,\hat{d}$

correct working **(A1)**

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

one correct point **A1** **N2**

eg $(11, 13, -9)$, $(7, 13, -11)$

METHOD 2 (distance between points)

attempt to use distance between $(1 + 2s, 13, -14 + s)$ and $(9, 13, -10)$ **(M1)**

eg $(2s - 8)^2 + 0^2 + (s - 4)^2 = 5$

solving $5s^2 - 40s + 75 = 0$ leading to $s = 5$ or $s = 3$ **(A1)**

one correct point **A1** **N2**

eg $(11, 13, -9)$, $(7, 13, -11)$

[5 marks]

Total [17 marks]

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Question 21

(a) evidence of scalar product **M1**
eg $\boldsymbol{a} \cdot \boldsymbol{b}$, $4(k+3)+2k$

recognizing scalar product must be zero ***(M1)***
eg $\boldsymbol{a} \cdot \boldsymbol{b} = 0$, $4k+12+2k=0$

correct working (must involve combining terms) ***(A1)***
eg $6k+12$, $6k=-12$

$k=-2$ ***A1*** ***N2***

[4 marks]

(b) attempt to substitute **their** value of k (seen anywhere) ***(M1)***
eg $\boldsymbol{b} = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}$, $2\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

correct working ***(A1)***
eg $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix}$

$\boldsymbol{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ ***A1*** ***N2***

[3 marks]

[Total 7 marks]

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Question 22

(a) recognizing $t=0$ at A **(M1)**

A is $(4, -1, 3)$ ***A1*** ***N2***

[2 marks]

(b) (i) **METHOD 1**

valid approach ***(M1)***

eg $\begin{pmatrix}4\\-1\\3\end{pmatrix}+2\begin{pmatrix}1\\2\\-2\end{pmatrix}$, $(6, 3, -1)$

correct approach to find $\overrightarrow{AB}$ ***(A1)***

eg $AO+OB$, $B-A$, $\begin{pmatrix}6\\3\\-1\end{pmatrix}-\begin{pmatrix}4\\-1\\3\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ ***A1*** ***N2***

METHOD 2

recognizing $\overrightarrow{AB}$ is two times the direction vector ***(M1)***

correct working ***(A1)***

eg $\overrightarrow{AB}=2\begin{pmatrix}1\\2\\-2\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ ***A1*** ***N2***

(ii) correct substitution ***(A1)***

eg $\left|\overrightarrow{AB}\right|=\sqrt{2^2+4^2+4^2}$, $\sqrt{4+16+16}$, $\sqrt{36}$

$\left|\overrightarrow{AB}\right|=6$ ***A1*** ***N2***

[5 marks]

(c) **METHOD 1 (vector approach)**

valid approach involving $\vec{AB}$ and $\vec{AC}$ ***(M1)***

eg $\vec{AB}\bullet\vec{AC}$, $\frac{\vec{BA}\bullet\vec{AC}}{AB\times AC}$

finding scalar product and $\left|\vec{AC}\right|$ ***(A1)(A1)***

scalar product $2(3)+4(0)-4(4)$ $(=-10)$

$\left|\vec{AC}\right|=\sqrt{3^2+0^2+4^2}$ $(=5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos B\hat{A}C=\frac{6+0-16}{6\sqrt{3^2+4^2}}$

$\cos B\hat{A}C=-\frac{10}{30}$ $\left(=-\frac{1}{3}\right)$ ***A1*** ***N2***

METHOD 2 (triangle approach)

valid approach involving cosine rule ***(M1)***

eg $\cos B\hat{A}C=\frac{AB^2+AC^2-BC^2}{2\times AB\times AC}$

finding lengths AC and BC ***(A1)(A1)***

$AC=5$, $BC=9$

substitution of **their** lengths into cosine formula ***(M1)***

eg $\cos B\hat{A}C=\frac{5^2+6^2-9^2}{2\times5\times6}$

$\cos B\hat{A}C=-\frac{20}{60}$ $\left(=-\frac{1}{3}\right)$ ***A1*** ***N2***

[5 marks]

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(d) **Note:** Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

METHOD 1 (using cosine rule)

recognizing need to find BC *(M1)*

choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS *A1*

eg $\text{BC}^2 = (6)^2 + (5)^2 - 2(6)(5)\left(-\frac{1}{3}\right)$, $36 + 25 + 20$

distance is 9 *A1* *N2*

METHOD 2 (finding magnitude of $\overrightarrow{\text{BC}}$)

recognizing need to find BC *(M1)*

valid approach *(M1)*

eg attempt to find $\overrightarrow{\text{OB}}$ or $\overrightarrow{\text{OC}}$, $\overrightarrow{\text{OB}} = \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ or $\overrightarrow{\text{OC}} = \begin{pmatrix} 7 \\ -1 \\ 7 \end{pmatrix}$, $\overrightarrow{\text{BA}} + \overrightarrow{\text{AC}}$

correct working *A1*

eg $\overrightarrow{\text{BC}} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$, $\overrightarrow{\text{CB}} = \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 *A1* *N2*

METHOD 3 (finding coordinates and using distance formula)

recognizing need to find BC *(M1)*

valid approach *(M1)*

eg attempt to find coordinates of B or C, $\text{B}(6, 3, -1)$ or $\text{C}(7, -1, 7)$

correct substitution into distance formula *A1*

eg $\text{BC} = \sqrt{(6-7)^2 + (3-(-1))^2 + (-1-7)^2}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 *A1* *N2*

[4 marks]

[Total 16 marks]

Question 23

(a) (i) correct approach **A1**

eg $\begin{pmatrix}-1\\3\\3\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}$, $\begin{pmatrix}3\\-4\\-2\end{pmatrix}+\begin{pmatrix}-1\\3\\3\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **AG** **N0**

(ii) any correct equation in the form $r=a+tb$ (any parameter for t)

where a is $\begin{pmatrix}-3\\4\\2\end{pmatrix}$ or $\begin{pmatrix}-1\\3\\3\end{pmatrix}$ and b is a scalar multiple of $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **A2** **N2**

eg $r=\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}$, $(x, y, z)=(-1, 3, 3)+s(-2, 1, -1)$, $r=\begin{pmatrix}-3+2t\\4-t\\2+t\end{pmatrix}$

> **Note:** Award ***A1*** for the form $a+tb$, ***A1*** for the form $L=a+tb$, ***A0*** for the form $r=b+ta$.

[3 marks]

(b) **METHOD 1 – finding value of parameter**

valid approach ***(M1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1, 3, 3)+s(-2, 1, -1)=(3, 1, p)$

one correct equation (not involving p) ***(A1)***

eg $-3+2t=3$, $-1-2s=3$, $4-t=1$, $3+s=1$

correct parameter from their equation (may be seen in substitution) **A1**

eg $t=3$, $s=-2$

correct substitution ***(A1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+3\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $3-(-2)$

$p=5$ $\left(\text{accept }\begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ **A1** **N2**

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METHOD 2 – eliminating parameter

valid approach ***(M1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1,\,3,\,3)+s(-2,\,1,\,-1)=(3,\,1,\,p)$

one correct equation (not involving p) ***(A1)***

eg $-3+2t=3$, $-1-2s=3$, $4-t=1$, $3+s=1$

correct equation (with p) ***A1***

eg $2+t=p$, $3-s=p$

correct working to solve for p ***(A1)***

eg $7=2p-3$, $6=1+p$

$p=5$ $\left(\text{accept } \begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ ***A1*** ***N2***

[5 marks]

(c) valid approach to find $\overrightarrow{DC}$ or $\overrightarrow{CD}$ ***(M1)***

eg $\begin{pmatrix}3\\1\\5\end{pmatrix}-\begin{pmatrix}q^2\\0\\q\end{pmatrix}$, $\begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\5\end{pmatrix}$, $\begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\p\end{pmatrix}$

correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$ (may be seen in scalar product) ***A1***

eg $\begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}$, $\begin{pmatrix}q^2-3\\-1\\q-5\end{pmatrix}$, $\begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}$

recognizing scalar product of $\overrightarrow{DC}$ or $\overrightarrow{CD}$ with direction vector of L is zero (seen anywhere) ***(M1)***

eg $\begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$, $\overrightarrow{DC}\bullet\overrightarrow{AC}=0$, $\begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$

correct scalar product in terms of only q ***A1***

eg $6-2q^2-1+5-q$, $2q^2+q-10=0$, $2\left(3-q^2\right)-1+5-q$

correct working to solve quadratic ***(A1)***

eg $(2q+5)(q-2)$, $\dfrac{-1\pm\sqrt{1-4(2)(-10)}}{2(2)}$

$q=-\dfrac{5}{2},\ 2$ ***A1A1*** ***N3***

[7 marks]

Total [15 marks]

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