Subject – Math(Standard Level)
Topic - Algebra
Year - Nov 2011 – Nov 2019
Paper -2

Question 1

(a) 10 terms *A1* *N1*

[1 mark]

(b) evidence of binomial expansion *(M1)*

e.g. $a^9b^0+\binom{9}{1}a^8b+\binom{9}{2}a^7b^2+\ldots,\ \binom{9}{r}(a)^{n-r}(b)^r$, Pascal's triangle

evidence of correct term *(A1)*

e.g. 8^{th} term, $r=7$, $\binom{9}{7}$, $(3x^2)^2 2^7$

correct expression of complete term *(A1)*

e.g. $\binom{9}{7}(3x^2)^2(2)^7$, ${}^9_2C(3x^2)^2(2)^7$, $36\times9\times128$

$41472\,x^4$ (accept $41500\,x^4$) *A1* *N2*

[4 marks]

Total [5 marks]

Question 2

(a) (i) correct approach *(A1)*

e.g. $u_4 = (40)\frac{1}{2}^{(4-1)}$, listing terms

$u_4 = 5$ ***A1*** ***N2***

(ii) correct substitution into formula for infinite sum *(A1)*

e.g. $S_\infty = \frac{40}{1-0.5}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80$ ***A1*** ***N2***

[4 marks]

(b) (i) attempt to set up expression for u_8 *(M1)*

e.g. $-36+(8-1)d$

correct working ***A1***

e.g. $-8=-36+(8-1)d$, $\frac{-8-(-36)}{7}$

$d=4$ ***A1*** ***N2***

(ii) correct substitution into formula for sum *(A1)*

e.g. $S_n = \frac{n}{2}(2(-36)+(n-1)4)$

correct working ***A1***

e.g. $S_n = \frac{n}{2}(4n-76)$, $-36n+2n^2-2n$

$S_n = 2n^2 - 38n$ ***AG*** ***N0***

[5 marks]

(c) multiplying S_n (AP) by 2 or dividing S (infinite GP) by 2 *(M1)*

e.g. $2S_n$, $\frac{S_\infty}{2}$, 40

evidence of substituting into $2S_n = S_\infty$ ***A1***

e.g. $2n^2 - 38n = 40$, $4n^2 - 76n - 80\ (=0)$

attempt to solve **their** quadratic (equation) *(M1)*

e.g. intersection of graphs, formula

$n = 20$ ***A2*** ***N3***

[5 marks]

Total [14 marks]

Question 3

(a) (i) $d = 4$ **A1** **N1**

(ii) evidence of valid approach **(M1)**

e.g. $u_8 = 36 + 7(4)$, repeated addition of d from 36

$u_8 = 64$ **A1** **N2**

[3 marks]

(b) (i) correct substitution into sum formula **A1**

e.g. $S_n = \frac{n}{2}\{2(36)+(n-1)(4)\}$, $\frac{n}{2}\{72+4n-4\}$

evidence of simplifying

e.g. $\frac{n}{2}\{4n+68\}$ **A1**

$S_n = 2n^2 + 34n$ **AG** **N0**

(ii) 868 **A1** **N1**

[3 marks]

Total [6 marks]

Question 4

(a) Valid attempt to find term in x^{20} **(M1)**

e.g. $\binom{8}{1}(2^7)(b)$, $(2x^3)^7\left(\frac{b}{x}\right) = 3072$

correct equation **A1**

e.g. $\binom{8}{1}(2^7)(b) = 3072$

$b = 3$ **A1** **N2**

[3 marks]

(b) evidence of choosing correct term **(M1)**

e.g. 7^{th} term, $r = 6$

correct expression **A1**

e.g. $\binom{8}{6}(2x^3)^2\left(\frac{3}{x}\right)^6$

$k = 81648$ (accept 81600) **A1** **N2**

[3 marks]

Total [6 marks]

Question 5

(a) correct substitution into sum of a geometric sequence *(A1)*

e.g. $200\left(\frac{1-r^4}{1-r}\right)$, $200+200r+200r^2+200r^3$

attempt to set up an equation involving a sum and 324.8 ***M1***

e.g. $200\left(\frac{1-r^4}{1-r}\right)=324.8$, $200+200r+200r^2+200r^3=324.8$

$r=0.4$ (exact) ***A2*** ***N3***

[4 marks]

(b) correct substitution into formula ***A1***

e.g. $u_{10}=200\times 0.4^9$

$u_{10}=0.0524288$ (exact), 0.0524 ***A1*** ***N1***

[2 marks]

Total [6 marks]

Question 6

(a) valid method *(M1)*

e.g. subtracting terms, using sequence formula

$d=1.7$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into term formula *(A1)*

e.g. $5+27(1.7)$

28th term is 50.9 (exact) ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum formula *(A1)*

e.g. $S_{28}=\frac{28}{2}(2(5)+27(1.7))$, $\frac{28}{2}(5+50.9)$

$S_{28}=782.6$ (exact) [782, 783] ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question 7

attempt to expand binomial **(M1)**

e.g. $(2x)^6 p^0 + \binom{6}{1}(2x)^5 (p)^1 + \dots$, $\binom{n}{r}(2x)^r (p)^{n-r}$

one correct calculation for term in x^4 in the expansion for power 6 **(A1)**

e.g. $15,\ 16x^4$

correct expression for term in x^4 **(A1)**

e.g. $\binom{6}{2}(2x)^4 (p)^2$, $15.2^4 p^2$

> **Notes:** Accept sloppy notation *e.g.* omission of brackets around $2x$.
> Accept absence of x in middle factor.

correct term **(A1)**

e.g. $240p^2x^4$ (accept absence of x^4)

setting up equation with **their** coefficient equal to 60 **M1**

e.g. $\binom{6}{2}(2)^4 (p)^2 = 60$, $240p^2x^4 = 60x^4$, $p^2 = \frac{60}{240}$

$p = \pm\frac{1}{2} (p = \pm 0.5)$ **A1A1** **N3**

[7 marks]

Question 8

(a) $d = 3$ **A1** **N1**

[1 mark]

(b) (i) correct substitution into term formula **(A1)**

eg $u_{100} = 5 + 3(99)$, $5 + 3(100-1)$

$u_{100} = 302$ **A1** **N2**

(ii) correct substitution into sum formula **(A1)**

eg $S_{100} = \frac{100}{2}(2(5) + 99(3))$, $S_{100} = \frac{100}{2}(5 + 302)$

$S_{100} = 15350$ **A1** **N2**

[4 marks]

(c) correct substitution into term formula **(A1)**

eg $1502 = 5 + 3(n-1)$, $1502 = 3n + 2$

$n = 500$ **A1** **N2**

[2 marks]

Total [7 marks]

Question 9

(a) $p = 5$, $q = 7$, $r = 7$ (accept $r = 5$) *A1A1A1* *N3*

[3 marks]

(b) correct working *(A1)*

eg $\binom{12}{7} \times (3x)^5 \times (-2)^7$, 792, 243, -2^7, 24634368

coefficient of term in x^5 is -24634368 *A1* *N2*

Note: Do not award the final ***A1*** for an answer that contains x.

[2 marks]

Total [5 marks]

Question 10

correct substitution into sum of a geometric sequence *A1*

eg $62.755 = u_1\left(\frac{1-r^3}{1-r}\right)$, $u_1 + u_1r + u_1r^2 = 62.755$

correct substitution into sum to infinity *A1*

eg $\frac{u_1}{1-r} = 440$

attempt to eliminate one variable *(M1)*

eg substituting $u_1 = 440(1-r)$

correct equation in one variable *(A1)*

eg $62.755 = 440(1-r)\left(\frac{1-r^3}{1-r}\right)$, $440(1-r)(1+r+r^2) = 62.755$

evidence of attempting to solve the equation in a single variable *(M1)*

eg sketch, setting equation equal to zero, $62.755 = 440\left(1-r^3\right)$

$r = 0.95 = \frac{19}{20}$ *A1* *N4*

[6 marks]

Question 11

evidence of binomial expansion *eg* selecting correct term, $\left(\frac{x}{a}\right)^6\left(\frac{a^2}{x}\right)^0+\binom{6}{1}\left(\frac{x}{a}\right)^5\left(\frac{a^2}{x}\right)^1+\ldots$	*(M1)*	
evidence of identifying constant term in expansion for power 6 *eg* $r=3$, 4^{th} term	*(A1)*	
evidence of correct term (may be seen in equation) *eg* $20\frac{a^6}{a^3}$, $\binom{6}{3}\left(\frac{x}{a}\right)^3\left(\frac{a^2}{x}\right)^3$	*A2*	
attempt to set up **their** equation *eg* $\binom{6}{3}\left(\frac{x}{a}\right)^3\left(\frac{a^2}{x}\right)^3=1280$, $a^3=1280$	*(M1)*	
correct equation in one variable *a* *eg* $20a^3=1280$, $a^3=64$	*(A1)*	
$a=4$	*A1*	*N4* ***[7 marks]***

Question 12

(a) 11 terms	*A1*	*N1* ***[1 mark]***
(b) evidence of binomial expansion *eg* $\binom{n}{r}a^{n-r}b^r$, attempt to expand	*(M1)*	
evidence of choosing correct term *eg* 8^{th} term, $r=7$, $\binom{10}{7}$, $(x)^3(3)^7$	*(A1)*	
correct working *eg* $\binom{10}{7}(x)^3(3)^7$, $\binom{10}{3}(x)^3(3)^7$,	*(A1)*	
$262440x^3$ (accept $262000x^3$)	*A1*	*N3* ***[4 marks]***

Total [5 marks]

Question 13

valid approach **(M1)**

eg $\binom{8}{r}(3x^2)^{8-r}\left(\frac{k}{x}\right)^r$,

$(3x^2)^8+\binom{8}{1}(3x^2)^7\left(\frac{k}{x}\right)+\binom{8}{2}(3x^2)^6\left(\frac{k}{x}\right)^2+\ldots$, Pascal's triangle to 9th line

attempt to find value of r which gives term in x^0 **(M1)**

eg exponent in binomial must give x^{-2}, $x^2(x^2)^{8-r}\left(\frac{k}{x}\right)^r=x^0$

correct working **(A1)**

eg $2(8-r)-r=-2$, $18-3r=0$, $2r+(-8+r)=-2$

evidence of correct term **(A1)**

eg $\binom{8}{2}$, $\binom{8}{6}(3x^2)^2\left(\frac{k}{x}\right)^6$, $r=6$, $r=2$

equating **their** term and 16128 to solve for k **M1**

eg $x^2\binom{8}{6}(3x^2)^2\left(\frac{k}{x}\right)^6=16128$, $k^6=\frac{16128}{28(9)}$

$k=\pm2$ **A1A1** **N2**

Note: If no working shown, award ***N0*** for $k=2$.

Total [7 marks]

Question 14

valid approach to find the required term *(M1)*

eg $\binom{8}{r}\left(\frac{x^3}{2}\right)^{8-r}\left(\frac{p}{x}\right)^r$, $\left(\frac{x^3}{2}\right)^8\left(\frac{p}{x}\right)^0+\binom{8}{1}\left(\frac{x^3}{2}\right)^7\left(\frac{p}{x}\right)^1+\ldots$, Pascal's triangle to required value

identifying constant term (may be indicated in expansion) *(A1)*

eg 7^{th} term, $r=6$, $\left(\frac{1}{2}\right)^2$, $\binom{8}{6}$, $\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6$

correct calculation (may be seen in expansion) *(A1)*

eg $\binom{8}{6}\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6$, $\frac{8\times 7}{2}\times\frac{p^6}{2^2}$

setting up equation with **their** constant term equal to 5103 ***M1***

eg $\binom{8}{6}\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6=5103$, $p^6=\frac{5103}{7}$

$p=\pm 3$ ***A1A1*** ***N3***

[6 marks]

Question 15

(a) (i) valid approach *(M1)*

eg $r = \frac{u_2}{u_1}$, $\frac{4}{4.2}$

$r = 1.05$ (exact) *A1* *N2*

(ii) attempt to substitute into formula, with **their** r *(M1)*

eg 4×1.05^n, $4 \times 1.05 \times 1.05 \ldots$

correct substitution *(A1)*

eg 4×1.05^4, $4 \times 1.05 \times 1.05 \times 1.05 \times 1.05$

$u_5 = 4.862025(\text{exact})$, 4.86 $[4.86, 4.87]$ *A1* *N2*

[5 marks]

(b) (i) attempt to substitute $n = 1$ *(M1)*

eg $0.05 = a \times 1^k$

$a = 0.05$ *A1* *N2*

(ii) correct substitution of $n = 2$ into v_2 *A1*

eg $0.25 = a \times 2^k$

correct work *(A1)*

eg finding intersection point, $k = \log_2 \left(\frac{0.25}{0.05} \right)$, $\frac{\log 5}{\log 2}$

2.32192

$k = \log_2 5$ (exact), 2.32 [2.32, 2.33] *A1* *N2*

[5 marks]

(c) correct expression for u_n *(A1)*

eg $4 \times 1.05^{n-1}$

EITHER

correct substitution into inequality (accept equation) *(A1)*

eg $0.05 \times n^k > 4 \times 1.05^{n-1}$

valid approach to solve inequality (accept equation) *(M1)*

eg finding point of intersection, $n = 7.57994$ (7.59508 from 2.32)

$n = 8$ (must be an integer) *A1* *N2*

OR

table of values

when $n = 7$, $u_7 = 5.3604$, $v_7 = 4.5836$ *A1*

when $n = 8$, $u_8 = 5.6284$, $v_8 = 6.2496$ *A1*

$n = 8$ (must be an integer) *A1* *N2*

[4 marks]

Total [14 marks]

Question 16

(a) 9 terms **A1** **N1**

[1 mark]

(b) valid approach to find the required term **(M1)**

eg $\binom{8}{r}(2x)^{8-r}(3)^r$, $(2x)^8(3)^0+(2x)^7(3)^1+\ldots$, Pascal's triangle to 8th row

identifying correct term (may be indicated in expansion) **(A1)**

eg 6th term, $r=5$, $\binom{8}{5}$, $(2x)^3(3)^5$

correct working (may be seen in expansion) **(A1)**

eg $\binom{8}{5}(2x)^3(3)^5$, $56\times 2^3\times 3^5$

$108864x^3$ (accept $109000x^3$) **A1** **N3**

[4 marks]

tes: Do not award any marks if there is clear evidence of adding instead of multiplying.
Do not award final ***A1*** for a final answer of 108864, even if $108864x^3$ is seen previously.
If no working shown award ***N2*** for 108864.

Total [5 marks]

Question 17

(a) $d = -1.5$ *A1* *N1*
[1 mark]

(b) **METHOD 1**

valid approach *(M1)*

eg $u_{10} = u_1 + 9d$, $8 = u_1 - 9(-1.5)$

correct working *(A1)*

eg $8 = u_1 + 9d$, $6.5 = u_1 + 10d$, $u_1 = 8 - 9(-1.5)$

$u_1 = 21.5$ *A1* *N2*

METHOD 2

attempt to list 3 or more terms in either direction *(M1)*

eg 9.5, 11, 12.5,…; 5, 3.5, 2,… …

correct list of 4 or more terms in **correct** direction *(A1)*

eg 9.5 , 11 , 12.5 , 14

$u_1 = 21.5$ *A1* *N2*
[3 marks]

(c) correct expression *(A1)*

eg $\frac{50}{2}(2(21.5) + 49(-1.5))$, $\frac{50}{2}(21.5 - 52)$, $\sum_{k=1}^{50} 21.5 + (k-1)(-1.5)$

sum $= -762.5$ (exact) *A1* *N2*
[2 marks]

Total [6 marks]

Question 18

valid approach to find the required term *(M1)*

eg $\binom{8}{r} x^{8-r} k^r$, Pascal's triangle to 8th row, $x^8 + 8x^7k + 28x^6k^2 + \ldots$

identifying correct term (may be indicated in expansion) *(A1)*

eg $\binom{8}{2} x^6k^2$, $\binom{8}{6} x^6k^2$, $r = 2$

setting up equation in k with **their** coefficient/term *(M1)*

eg $28k^2x^6 = 63x^6$, $\binom{8}{6} k^2 = 63$

$k = \pm 1.5$ (exact) *A1A1* *N3*
[5 marks]

Question 19

METHOD 1

recognize that the distance walked each minute is a geometric sequence ***(M1)***

eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***

eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence ***(A1)***

eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

any correct equation with sum of a geometric sequence ***(A1)***

eg $80\left(\frac{0.9^n-1}{0.9-1}\right) = 660$, $1-0.9^n = \frac{66}{80}$

attempt to solve **their** equation involving the sum of a GP ***(M1)***

eg graph, algebraic approach

$n = 16.54290788$ ***A1***

since $n > 15$ ***R1***

he will be late ***AG*** ***N0***

Continue….

METHOD 2

recognize that the distance walked each minute is a geometric sequence **(M1)**
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence **(M1)**

eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence **(A1)**

eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

attempt to substitute $n = 15$ into sum of a geometric sequence **(M1)**
eg S_{15}

correct substitution **(A1)**

eg $80\left(\frac{0.9^{15}-1}{0.9-1}\right)$

$S_{15} = 635.287$ **A1**

since $S < 660$ **R1**
he will not be there on time **AG** **N0**

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

METHOD 3
recognize that the distance walked each minute is a geometric sequence **(M1)**
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence **(M1)**

eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

listing at least 5 correct terms of the GP **(M1)**

15 correct terms **A1**
80, 72, 64.8, 58.32, 52.488, 47.2392, 42.5152, 38.2637, 34.4373, 30.9936, 27.8942, 25.1048, 22.59436, 20.3349, 18.3014

attempt to find the sum of the terms **(M1)**

eg S_{15}, $80 + 72 + 64.8 + 58.32 + 52.488 + ... + 18.301433$

$S_{15} = 635.287$ **A1**

since $S < 660$ **R1**
he will not be there on time **AG** **N0**

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

[7 marks]

Question 20

(a) valid approach *(M1)*

eg $\frac{u_1}{u_2}$, $\frac{4}{1.6}$, $1.6 = r(0.64)$

$r = 2.5 \left(= \frac{5}{2}\right)$ **A1** **N2**

[2 marks]

(b) correct substitution into S_6 *(A1)*

eg $\frac{0.64(2.5^6 - 1)}{2.5 - 1}$

$S_6 = 103.74$ (exact), 104 **A1** **N2**

[2 marks]

(c) **METHOD 1 (analytic)**

valid approach *(M1)*

eg $\frac{0.64(2.5^n - 1)}{2.5 - 1} > 75000$, $\frac{0.64(2.5^n - 1)}{2.5 - 1} = 75000$

correct inequality (accept equation) *(A1)*

eg $n > 13.1803$, $n = 13.2$

$n = 14$ **A1** **N1**

METHOD 2 (table of values)

both crossover values **A2**

eg $S_{13} = 63577.8$, $S_{14} = 158945$

$n = 14$ **A1** **N1**

[3 marks]

Total [7 marks]

Question 21

(a) valid approach to find the required term **(M1)**

eg $\binom{9}{r}(x)^{9-r}(2)^r$, $x^9+9x^8(2)+\binom{9}{2}x^7(2)^2+\ldots$, Pascal's triangle to the 9th row

identifying correct term (may be indicated in expansion) **(A1)**

eg 4th term, $r=6$, $\binom{9}{3}$, $(x)^6(2)^3$

correct calculation (may be seen in expansion) **(A1)**

eg $\binom{9}{3}(x)^6(2)^3$, 84×2^3

$672x^6$ **A1 N3**

[4 marks]

(b) valid approach **(M1)**

eg recognizing x^7 is found when multiplying $5x\times 672x^6$

$3360x^7$ **A1 N2**

[2 marks]

Total [6 marks]

Question 22

correct equation to find r **(A1)**

eg $u_1r^3=8u_1$, $r^3=8$

$r=2$ (seen anywhere) **(A1)**

correct equation to find u_1 **A1**

eg $u_1\left(2^{10}-1\right)=2557.5$, $u_1=\frac{2557.5}{r^{10}-1}(r-1)$

$u_1=2.5$ **(A1)**

$u_{10}=2.5(2)^9$ **(M1)**

1280 **A1 N4**

[6 marks]

Question 23

(a) valid approach **(M1)**

eg $1.5-0.3, 1.5-2.7, 2.7=0.3+2d$

$d=1.2$ **A1 N2**

[2 marks]

(b) correct substitution into term formula **(A1)**

eg $0.3+1.2(30-1)$, $u_{30}=0.3+29(1.2)$

$u_{30}=35.1$ **A1 N2**

[2 marks]

(c) correct substitution into sum formula **(A1)**

eg $S_{30}=\frac{30}{2}(0.3+35.1)$, $\frac{30}{2}\left(2(0.3)+29(1.2)\right)$

$S_{30}=531$ **A1 N2**

[2 marks]

Total [6 marks]

Question 24

(a) 11 terms **A1 N1**

[1 mark]

(b) valid approach **(M1)**

eg $\binom{10}{r}\left(x^2\right)^{10-r}\left(\frac{2}{x}\right)^r$, $a^{10}b^0+\binom{10}{1}a^9b^1+\binom{10}{2}a^8b^2+\ldots$

Pascal's triangle to 11^{th} row

valid attempt to find value of r which gives term in x^8 **(M1)**

eg $\left(x^2\right)^{10-r}\left(\frac{1}{x^r}\right)=x^8$, $x^{2r}\left(\frac{2}{x}\right)^{10-r}=x^8$

identifying required term (may be indicated in expansion) **(A1)**

eg $r=6$, 5th term, 7th term

correct working (may be seen in expansion) **(A1)**

eg $\binom{10}{6}\left(x^2\right)^6\left(\frac{2}{x}\right)^4$, 210×16

3360 **A1 N3**

[5 marks]

Total [6 marks]

Question 25

attempt to find r	*(M1)*	
eg $\frac{576}{768}, \frac{768}{576}, 0.75$		
correct expression for u_n	*(A1)*	
eg $768(0.75)^{n-1}$		
EITHER (solving inequality)		
valid approach (accept equation)	*(M1)*	
eg $u_n < 7$		
valid approach to find n	***M1***	
eg $768(0.75)^{n-1} = 7$, $n - 1 > \log_{0.75}\left(\frac{7}{768}\right)$, sketch		
correct value		
eg $n = 17.3301$	*(A1)*	
$n = 18$ (must be an integer)	***A1***	***N2***
OR (table of values)		
valid approach	*(M1)*	
eg $u_n < 7$,one correct crossover value		
both crossover values, $u_{17} = 7.69735$ **and** $u_{18} = 5.77301$	***A2***	
$n = 18$ (must be an integer)	***A1***	***N2***
OR (sketch of functions)		
valid approach	***M1***	
eg sketch of appropriate functions		
valid approach	*(M1)*	
eg finding intersections or roots (depending on function sketched)		
correct value		
eg $n = 17.3301$	*(A1)*	
$n = 18$ (must be an integer)	***A1***	***N2***
		[6 marks]

Question 26

valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for r) **(M1)**

eg $\binom{11}{r}(2)^{11-r}ax^r$, $\binom{11}{3}(2)^8(ax)^3$, $2^{11}+\binom{11}{1}(2)^{10}(ax)^1+\binom{11}{2}(2)^9(ax)^2+\ldots$

recognizing need to find term in x^2 in binomial expansion **(A1)**

eg $r=2$, $(ax)^2$

correct term or coefficient in binomial expansion (may be seen in equation) **(A1)**

eg $\binom{11}{2}(ax)^2(2)^9$, $55(a^2x^2)(512)$, $28160a^2$

setting up equation in x^5 with **their** coefficient/term (do not accept other powers of x) **(M1)**

eg $ax^3\binom{11}{2}(ax)^2(2)^9=11880x^5$

correct equation **(A1)**

eg $28160a^3=11880$

$a=\frac{3}{4}$ **A1** **N3**

[6 marks]

Question 27

(a) valid approach to find maxima **(M1)**

eg one correct value of x_k, sketch of f

any two correct consecutive values of x_k **(A1)(A1)**

eg $x_1=1$, $x_2=5$

$a=4$ **A1** **N3**

[4 marks]

(b) recognizing the sequence $x_1, x_2, x_3, \ldots, x_n$ is arithmetic **(M1)**

eg $d=4$

correct expression for sum **(A1)**

eg $\frac{n}{2}(2(1)+4(n-1))$

valid attempt to solve for n **(M1)**

eg graph, $2n^2-n-861=0$

$n=21$ **A1** **N2**

[4 marks]

Total [8 marks]

Question 28

(a) valid approach **(M1)**

eg one correct value

-0.453620, 6.14210

$a = -0.454$, $b = 6.14$ **A1A1** **N3**

[3 marks]

(b) correct substitution **(A1)**

eg $-0.454 \ln 3.57 + 6.14$

correct working **(A1)**

eg $\ln y = 5.56484$

261.083 (260.409 from 3 sf)

$y = 261$, ($y = 260$ from 3sf) **A1** **N3**

Note: If no working shown, award ***N1*** for 5.56484.
If no working shown, award ***N2*** for $\ln y = 5.56484$.

[3 marks]

(c) **METHOD 1**

valid approach for expressing $\ln y$ in terms of $\ln x$ **(M1)**

eg $\ln y = \ln(kx^n)$, $\ln(kx^n) = a \ln x + b$

correct application of addition rule for logs **(A1)**

eg $\ln k + \ln(x^n)$

correct application of exponent rule for logs **A1**

eg $\ln k + n \ln x$

comparing one term with regression equation (check ***FT***) **(M1)**

eg $n = a$, $b = \ln k$

correct working for k **(A1)**

eg $\ln k = 6.14210$, $k = e^{6.14210}$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) **A1A1** **N2N2**

METHOD 2

valid approach **(M1)**

eg $e^{\ln y} = e^{a \ln x + b}$

correct use of exponent laws for $e^{a \ln x + b}$ **(A1)**

eg $e^{a \ln x} \times e^{b}$

correct application of exponent rule for $a \ln x$ **(A1)**

eg $\ln x^{a}$

correct equation in *y* **A1**

eg $y = x^{a} \times e^{b}$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = e^{b}$, $n = a$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) **A1A1** **N2N2**

METHOD 3

valid approach for expressing $\ln y$ in terms of $\ln x$ (seen anywhere) **(M1)**

eg $\ln y = \ln(kx^{n})$, $\ln(kx^{n}) = a \ln x + b$

correct application of exponent rule for logs (seen anywhere) **(A1)**

eg $\ln(x^{a}) + b$

correct working for b (seen anywhere) **(A1)**

eg $b = \ln(e^{b})$

correct application of addition rule for logs **A1**

eg $\ln(e^{b} x^{a})$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = e^{b}$, $n = a$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) **A1A1** **N2N2**

[7 marks]

Total [13 marks]

Question 29

(a) correct substitution into infinite sum **(A1)**

eg $200 = \frac{4}{1-r}$

$r = 0.98$ (exact) **A1** **N2**

[2 marks]

(b) correct substitution **(A1)**

$\frac{4(1-0.98^8)}{1-0.98}$

29.8473

29.8 **A1** **N2**

[2 marks]

(c) attempt to set up inequality (accept equation) **(M1)**

eg $\frac{4(1-0.98^n)}{1-0.98} > 163$, $\frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values **(A1)**

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ **A1** **N1**

[3 marks]

[Total: 7 marks]

Question 30

valid approach to find one of the required terms (must have correct substitution for parameters but accept " r " or an incorrect value for r) ***(M1)***

eg $\binom{9}{r}(2x)^{9-r}\left(\frac{k}{x}\right)^r$, $\binom{9}{6}(2x)^6\left(\frac{k}{x}\right)^3$, $\binom{9}{0}(2x)^0\left(\frac{k}{x}\right)^9+\binom{9}{1}(2x)^1\left(\frac{k}{x}\right)^8+\ldots$, Pascal's triangle to 9th row

te: Award ***M0*** if there is clear evidence of adding instead of multiplying.

identifying correct terms (must be clearly indicated if only seen in expansion) ***(A1)(A1)***

eg for x^3 term: $r=3$, $r=6$, 7th term, $\binom{9}{6}$, $\binom{9}{3}$, $(2x)^6\left(\frac{k}{x}\right)^3$, $5376k^3$

for x^5 term: $r=2$, $r=7$, 8th term, $\binom{9}{7}$, $\binom{9}{2}$, $(2x)^7\left(\frac{k}{x}\right)^2$, $4608k^2$

correct equation (may include powers of x) ***A1***

eg $\binom{9}{3}(2x)^6\left(\frac{k}{x}\right)^3=\binom{9}{2}(2x)^7\left(\frac{k}{x}\right)^2$

valid attempt to solve their equation in terms of k only ***(M1)***

eg sketch, $84\times 64k^3-36\times 128k^2=0$, $5376k-4608=0$, $\binom{9}{3}2^6k^3=\binom{9}{2}2^7k^2$

0.857142

$k=\frac{4608}{5376}\left(=\frac{6}{7}\right)$ (exact), 0.857 ***A1*** ***N4***

[6 marks]

Question 31

correct substitution into formula for infinite geometric series **(A1)**

eg $33.25 = \frac{u_1}{1-r}$

correct substitution into formula for u_n (seen anywhere) **(A1)**

eg $7.98 = u_1 r$

attempt to express u_1 in terms of r (or vice-versa) **(M1)**

eg $u_1 = \frac{7.98}{r}$, $u_1 = 33.25(1-r)$, $r = \frac{7.98}{u_1}$, $r = \frac{33.25 - u_1}{33.25}$

correct working **(A1)**

eg $\frac{\left(\frac{7.98}{r}\right)}{1-r} = 33.25$, $33.25(1-r) = \frac{7.98}{r}$, $(0.4, 19.95)$, $(0.6, 13.3)$, $\frac{u_1}{1-\frac{7.98}{u_1}} = 33.25$

$r = 0.4 \ \left(=\frac{2}{5}\right)$, $r = 0.6 \ \left(=\frac{3}{5}\right)$ **A1A1** **N3**

Total [6 marks]

Question 32

valid approach for expanding binomial **(M1)**

eg $\binom{12}{r}(2x^4)^{12-r}\left(\frac{x^2}{k}\right)^r$, $(2x^4)^{12} + \binom{12}{1}(2x^4)^{11}\left(\frac{x^2}{k}\right)^1 + \binom{12}{2}(2x^4)^{10}\left(\frac{x^2}{k}\right)^2 + \ldots$

valid attempt to find r for x^{40} or x^{38} **(M1)**

eg $(x^4)^{12-r}(x^2)^r = (x)^{40}$, $(x^4)^r(x^2)^{12-r} = (x)^{40}$,

$\binom{12}{r}(2^r)\left(\frac{1}{k}\right)^{12-r}(x^4)^r(x^2)^{12-r} = \binom{12}{r}(2^r)\left(\frac{1}{k}\right)^{12-r}x^{38}$

correct equation for finding one value of r **(A1)**

eg $48 - 2r = 40$, $48 - 2r = 38$, $24 + 2r = 40$, $2r + 24 = 38$

correct values for r (seen anywhere) **(A1)(A1)**

eg $r = 4$, $r = 5$ **OR** $r = 7$, $r = 8$

correct equation to solve for k **A1**

eg $\binom{12}{4}(2^8)\left(\frac{1}{k}\right)^4 = 5\binom{12}{5}(2^7)\left(\frac{1}{k}\right)^5$, $\frac{126\,720}{k^4} = 5 \times \frac{792(128)}{k^5}$, $990k = 3960$

$k = 4$ **A1** **N2**

Total [7 marks]

Question 33

(a) attempt to find d *(M1)*

eg $1.4-1.3$, u_1-u_2, $1.4=1.3+(2-1)d$

$d=0.1$ (may be seen in expression for u_n) *(A1)*

correct equation *(A1)*

eg $1.3+(k-1)\times 0.1=31.2$, $0.1k=30$

$k=300$ **A1** **N3**

[4 marks]

(b) correct substitution *(A1)*

eg $\frac{300}{2}(1.3+31.2)$, $\frac{300}{2}\left[2(1.3)+(300-1)(0.1)\right]$, $\frac{300}{2}\left[2.6+299(0.1)\right]$

$S_k=4875$ **A1** **N2**

[2 marks]

(c) recognizing need to find the sequence of multiples of 3 (seen anywhere) *(M1)*

eg first term is u_3 $(=1.5)$ (accept notation $u_1=1.5$), $d=0.1\times 3$ $(=0.3)$, 100 terms (accept $n=100$), last term is 31.2 (accept notation $u_{100}=31.2$), $u_3+u_6+u_9+\dots$ (accept $F=u_3+u_6+u_9+\dots$)

correct working for sum of sequence where n is a multiple of 3 **A2**

$\frac{100}{2}(1.5+31.2)$, $50(2\times 1.5+99\times 0.3)$, 1635

valid approach (seen anywhere) *(M1)*

eg $S_k-(u_3+u_6+\dots)$, $S_k-\frac{100}{2}(1.5+31.2)$, S_k-(their sum for $(u_3+u_6+\dots)$)

correct working (seen anywhere) **A1**

eg S_k-1635, $4875-1635$

$F=3240$ **AG** **N0**

[5 marks]

(d) attempt to find r *(M1)*

eg dividing consecutive terms

correct value of r (seen anywhere, including in formula)

eg $\frac{1}{\sqrt{2}}$, $0.707106\ldots$, $\frac{a}{0.293\ldots}$ **A1**

correct working (accept equation) *(A1)*

eg $\frac{a}{1-\frac{1}{\sqrt{2}}} < 3240$

correct working **A1**

METHOD 1 (analytical)

eg $3240 \times \left(1-\frac{1}{\sqrt{2}}\right)$, $a < 948.974$, 948.974

METHOD 2 (using table, must find both S_∞ values)

eg when $a = 948$, $S_\infty = 3236.67\ldots$ **AND** when $a = 949$, $S_\infty = 3240.08\ldots$

$a = 948$ **A1** **N2**

[5 marks]

Total [16 marks]

Question 35

valid approach for expanding binomial (must have correct substitution for parameters, but accept "r" or an incorrect value for r) **(M1)**

eg $\binom{15}{r}\left(\frac{1}{2x}\right)^{(15-r)}\left(x^2\right)^r,\ \left(\frac{1}{2x}\right)^{15}\left(x^2\right)^0+15\left(\frac{1}{2x}\right)^{14}\left(x^2\right)^1+\binom{15}{2}\left(\frac{1}{2x}\right)^{13}\left(x^2\right)^2+\ldots$

recognizing need to find the term containing x^{-3} in the expansion of $\left(\frac{1}{2x}+x^2\right)^{15}$ **(M1)**

correct equation **(A1)**

eg $\left(x^{-1}\right)^{15-r}\left(x^2\right)^r=x^{-3},\ \left(x^{-1}\right)^r\left(x^2\right)^{15-r}=x^{-3},\ -15+r+2r=-3$

identifying the correct term (seen anywhere) **A1**

eg $r=4,\ r=11,\ n-r=4$

correct working **(A1)(A1)**

eg $\binom{15}{4}\left(\frac{1}{2x}\right)^{15-4},\ 1365\times\frac{1}{2^{11}}$

e: Award ***A1*** for each factor.

$\frac{1365}{2048}$ **A1** **N2**

[7 marks]

Question 35

(a) attempt to add corresponding terms **(M1)**

eg $2+2,\ 6+(-6),\ 2(3)^{n-1}+2(-3)^{n-1}$

correct value for w_5 **(A1)**

eg 324

$4, 36, 324$ (accept $4+36+324$) **A1** **N3**

[3 marks]

(b) (i) valid approach **(M1)**

eg $4\times r^1=36$, $4\times 9^{n-1}$

$r=9$ (accept $\sum_{k=0}^{m}4\times 9^k$; m may be incorrect) **A1** **N2**

(ii) recognition that 225 terms of w_n consists of 113 non-zero terms **(M1)**

eg $\sum_{1}^{113}$, $\sum_{0}^{112}$, 113

$m=112$ (accept $\sum_{k=0}^{112}4\times r^k$; r may be incorrect) **A1** **N2**

[4 marks]

Total [7 marks]

Question 36

valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for r) ***(M1)***

eg $\binom{n}{r}\left(x^2\right)^{n-r}(1.2)^r$, $\binom{n}{0}\left(x^2\right)^n+\binom{n}{1}\left(x^2\right)^{n-1}(1.2)+\binom{n}{2}\left(x^2\right)^{n-2}(1.2)^2+\ldots$

attempt to identify correct term ***(M1)***

eg $2r=6$, $n-r=3$, $\binom{n}{3}$, $\binom{n}{n-3}$

correct expression ***(A1)***

eg $\binom{n}{n-3}\times 1.2^{n-3}x^6$, $\binom{n}{n-3}\times 1.2^{n-3}$

EITHER (solving inequality)

attempt to set up inequality in terms of n (accept equation) ***M1***

eg $\binom{n}{3}\times 1.2^{n-3}>200000$, $1.2^{n-3}\binom{n}{3}x^6=200000$

correct working for binomial coefficient (may be seen in equation) ***(A1)***

eg $\frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!}$, $\frac{n(n-1)(n-2)}{6}\times 1.2^{n-3}=200000$

$n>26.4959$ (accept 26.4959 or $n=26.4959$) ***A1***

$n=27$ ***A1*** ***N2***

Note: If no working shown, award ***N1*** for 26.4959.

OR (using table)

valid approach ***(M1)***

eg $\binom{n}{3}\times 1.2^{n-3}>200000$, one correct coefficient of x^6 for a value of n

correct crossover values for $n=26$ and $n=27$ ***A1A1***

eg 172243, 232528

$n=27$ ***A1*** ***N2***

[7 marks]

Question 37

(a) vertex is $(-10, 15)$ **A1A1** **N1N1**

[2 marks]

(b) valid approach **(M1)**

eg $f(0) = -20$, $-20 = a(0+10)^2 + 15$

$a = -0.35$ (exact) **A1** **N2**

[2 marks]

(c) valid approach **(M1)**

eg $f(8)$, $-0.35(8+10)^2 + 15$

$b = -98.4$ (exact) (accept $f(8) = -98.4$) **A1** **N2**

[2 marks]

Total [6 marks]

Question 38

(a) valid approach **(M1)**

eg $\frac{u_1}{u_2}, \frac{2.226}{2.1}, 2.226 = 2.1r$

$r = 1.06$ (exact) **A1** **N2**

[2 marks]

(b) correct substitution **(A1)**

eg 2.1×1.06^9

3.54790 **A1** **N2**

$u_{10} = 3.55$

[2 marks]

(c) correct substitution into S_n formula **(A1)**

eg $\frac{2.1(1.06^n - 1)}{1.06 - 1}$, $\frac{2.1(1.06^n - 1)}{1.06 - 1} > 5543$, $2.1(1.06^n - 1) = 332.58$,

sketch of S_n and $y = 5543$

correct inequality for n **or** crossover values **A1**

eg $n > 87.0316$, $S_{87} = 5532.73$ **and** $S_{88} = 5866.79$

$n = 88$ **A1** **N2**

[3 marks]

Total [7 marks]