A-Level

Calculus

2013-2018

Question 1

(i)	Use product rule	M1	
	Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$	A1	
	Equate derivative to zero and use a double angle formula	M1*	
	Reduce equation to one in a single trig function	M1(dep*)	
	Obtain a correct equation in any form, e.g. $10\cos^3 x = 6\cos x$, $4 = 6\tan^2 x$ or $4 = 10\sin^2 x$	A1	
	Solve and obtain $x = 0.685$	A1	[6]
(ii)	Using $du = \pm\cos x\, dx$, or equivalent, express integral in terms of u and du	M1	
	Obtain $\int 4u^2(1-u^2)du$, or equivalent	A1	
	Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	M1	
	Obtain answer $\frac{8}{15}$ (or 0.533)	A1	[4]

Question 2

(i)	State $R = 2$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
(ii)	Substitute denominator of integrand and state integral $k\tan(x-\alpha)$	M1*	
	State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
	Substitute limits	M1 (dep*)	
	Obtain the given answer correctly	A1	[4]

Question 3

(i)	Use correct quotient or chain rule to differentiate $\sec x$	M1	
	Obtain given derivative, $\sec x \tan x$, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
(ii)	Using $dx\sqrt{3}\sec^2\theta\, d\theta$, or equivalent, express integral in terms of θ and $d\theta$	M1	
	Obtain $\int \sec\theta\, d\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]

Question 4

State $2ay\dfrac{dy}{dx}$ as derivative of ay^2	B1	
State $y^2 + 2xy\dfrac{dy}{dx}$ as derivative of xy^2	B1	
Equate derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero	M1	
Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent	A1	
Eliminate y and obtain an equation in x	M1	
Solve for x and obtain answer $x = \sqrt{3}a$	A1	

Question 5

(a)	Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{x} x^2 dx$	M1*	
	Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2\, dx$	A1	
	Obtain $2x^2 \ln x - x^2$	A1	
	Use limits, having integrated twice	M1 (dep*)	
	Confirm given result $56 \ln 2 - 12$	A1	[5]
(b)	State or imply $\frac{du}{dx} = 4\cos 4x$	B1	
	Carry out complete substitution except limits	M1	
	Obtain $\int (\frac{1}{4} - \frac{1}{4}u^2)\, du$ or equivalent	A1	
	Integrate to obtain form $k_1 u + k_2 u^3$ with non-zero constants k_1, k_2	M1	
	Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]

Question 6

(i)	Use correct quotient rule or equivalent	M1	
	Obtain $\dfrac{(1+e^{2x})2x - (1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = 0$ and obtain $-\dfrac{1}{2}$ or equivalent	A1	[3]
(ii)	Differentiate y^3 and obtain $3y^2\dfrac{dy}{dx}$	B1	
	Differentiate $5xy$ and obtain $5y + 5x\dfrac{dy}{dx}$	B1	
	Obtain $6x^2 + 5y + 5x\dfrac{dy}{dx} + 3y^2\dfrac{dy}{dx} = 0$	B1	
	Substitute $x = 0, y = 2$ to obtain $-\dfrac{5}{6}$ or equivalent following correct work	B1	[4]

Question 7

Differentiate y^3 to obtain $3y^2\frac{dy}{dx}$	B1	
Use correct product rule at least once	*M1	
Obtain $6e^{2x}y + 3e^{2x}\frac{dy}{dx} + e^x y^3 + 3e^x y^2\frac{dy}{dx}$ as derivative of LHS	A1	
Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{dy}{dx}$	M1(d*M)	
Obtain $-\frac{4}{3}$ or equivalent as **final answer**	A1	[5]

Question 8

Carry out complete substitution including the use of $\frac{du}{dx} = 3$	M1	
Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$	A1	
Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$	M1	
Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and $+c$	A1	[4]

Question 9

Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u} - m\int e^{\frac{1}{2}u}du$	M1*
Obtain $2ue^{\frac{1}{2}u} - 2\int e^{\frac{1}{2}u}du$, or equivalent	A1
Integrate again and obtain $2ue^{\frac{1}{2}u} - 4e^{\frac{1}{2}u}$, or equivalent	A1
Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)
Obtain answer $4\ln 4 - 4$, or exact equivalent	A1

Question 10

Use correct product or quotient rule at least once	M1*	
Obtain $\frac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\frac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
EITHER: Express $\frac{dy}{dx}$ in terms of tan t only	M1(dep*)	
Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
Show expression is identical to $\frac{dy}{dx}$	A1	**[6]**

Question 11

(i)	Use Pythagoras	M1	
	Use the sin2A formula	M1	
	Obtain the given result	A1	**[3]**
(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
	Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
	Substitute limits correctly	M1(dep)*	
	Obtain the given answer correctly having shown appropriate working	A1	**[4]**

Question 12

(i)	Substitute for x and dx throughout using $u = \sin x$ and du = cos x dx, or equivalent	M1	
	Obtain integrand e^{2u}	A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
	Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**
(ii)	Use chain rule or product rule	M1	
	Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
	Equate derivative to zero and obtain a quadratic equation in sin x	M1	
	Solve a 3-term quadratic and obtain a value of x	M1	
	Obtain answer 0.896	A1	**6**

Question 13

Obtain correct derivative of RHS in any form	B1	
Obtain correct derivative of LHS in any form	B1	
Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
Obtain $y = \frac{1}{2}$	A1	**7**

Question 14

(i)	Use product rule	M1	
	Obtain derivative in any correct form	A1	
	Differentiate first derivative using the product rule	M1	
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x-\frac{1}{4}x\cos\frac{1}{2}x-\frac{1}{2}\sin\frac{1}{2}x$	A1	
	Verify the given statement	A1	**5**
(ii)	Integrate and reach $kx\sin\frac{1}{2}x+l\int\sin\frac{1}{2}x\,\mathrm{d}x$	M1*	
	Obtain $2x\sin\frac{1}{2}x-2\int\sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent	A1	
	Obtain indefinite integral $2x\sin\frac{1}{2}x+4\cos\frac{1}{2}x$	A1	
	Use correct limits $x=0$, $x=\pi$ correctly	M1(dep*)	
	Obtain answer $2\pi-4$, or exact equivalent	A1	**5**

Question 15

(i)	State $\frac{\mathrm{d}x}{\mathrm{d}t}=1-\sec^2 t$, or equivalent	B1	
	Use chain rule	M1	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{\sin t}{\cos t}$, or equivalent	A1	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
	Obtain the given answer correctly.	A1	**5**
(ii)	State or imply $t=\tan^{-1}(\frac{1}{2})$	B1	
	Obtain answer $x=-0.0364$	B1	**2**

Question 16

(i)	Use of product or quotient rule	M1	
	Obtain $-5e^{-\frac{1}{2}x}\sin 4x+40e^{-\frac{1}{2}x}\cos 4x$	A1	
	Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and obtain $\tan 4z=k$ or $R\cos(4x\pm\alpha)$	M1	
	Obtain $\tan 4x=8$ or $\sqrt{65}\cos\left(4x\pm\tan^{-1}\frac{1}{8}\right)$	A1	
	Obtain 0.362 or 20.7°	A1	
	Obtain 1.147 or 65.7°	A1	[6]
(ii)	State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45°	B1	
	Attempt solution of inequality (or equation) of form $x_1+(n-1)k\pi\,.\,25$	M1	
	Obtain $n>\frac{4}{\pi}(25-0.362)+1$, following through on their value of x_1	A1√	
	n = 33	A1	[4]

Question 17

Obtain $\frac{2}{2t+3}$ for derivative of x	B1	
Use quotient of product rule, or equivalent, for derivative of y	M1	
Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent	A1	
Obtain $t=-1$	B1	
Use $\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$ in algebraic or numerical form	M1	
Obtain gradient $\frac{5}{2}$	A1	[6]

Question 18

State $\frac{du}{dx}=3\sec^2 x$ or equivalent	B1	
Express integral in terms of u and du (accept unsimplified and without limits)	M1	
Obtain $\int\frac{1}{3}u^{\frac{1}{2}}\,du$	A1	
Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$	M1	
Obtain $\frac{14}{9}$	A1	[5]

Question 19

State or imply $\dfrac{du}{dx} = e^x$	B1	
Substitute throughout for x and dx	M1	
Obtain $\displaystyle\int \frac{u}{u^2+3u+2}\,du$ or equivalent (ignoring limits so far)	A1	
State or imply partial fractions of form $\dfrac{A}{u+2}+\dfrac{B}{u+1}$, following their integrand	B1	
Carry out a correct process to find at least one constant for their integrand	M1	
Obtain correct $\dfrac{2}{u+2}-\dfrac{1}{u+1}$	A1	
Integrate to obtain $a\ln(u+2)+b\ln(u+1)$	M1	
Obtain $2\ln(u+2)-\ln(u+1)$ or equivalent, follow their A and B	A1√	
Apply appropriate limits and use at least one logarithm property correctly	M1	
Obtain given answer $\ln\dfrac{8}{5}$ legitimately	A1	[10]

SR for integrand $\dfrac{u^2}{u(u+1)(u+2)}$

State or imply partial fractions of form $\dfrac{A}{u}+\dfrac{B}{u+1}+\dfrac{C}{u+2}$	(B1)	
Carry out a correct process to find at least one constant	(M1)	
Obtain correct $\dfrac{2}{u+2}-\dfrac{1}{u+1}$	(A1)	
…complete as above.		

Question 20

Use correct product rule or correct chain rule to differentiate y	M1	
Use $\dfrac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
Obtain $\dfrac{-4\cos\theta\sin^2\theta+2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
Express $\dfrac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
Confirm given answer $6\cos^5\theta-4\cos^3\theta$ legitimately	A1	[5]

Question 21

(i)	State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
	Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
	Obtain 0.255 with no errors seen	A1	[3]
(ii)	Obtain or imply $a = -6$	B1	
	Obtain x^4 term including correct attempt at coefficient	M1	
	Obtain or imply $b = 27$	A1	
	<u>Either</u> Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1√	
	Obtain 0.259	B1	
	<u>Or</u> Use correct trapezium rule with at least 3 ordinates	M1	
	Obtain 0.259 (from 4)	A1	[5]

Question 22

(i)	Use chain rule correctly at least once	M1	
	Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain the given answer	A1	**[4]**
(ii)	State a correct equation for the tangent in any form	B1	
	Use Pythagoras	M1	
	Obtain the given answer	A1	**[3]**

Question 23

(i)	Integrate and reach $bx\ln 2x - c\int x.\frac{1}{x}\,dx$, or equivalent	M1*	
	Obtain $x\ln 2x - \int x.\frac{1}{x}\,dx$, or equivalent	A1	
	Obtain integral $x\ln 2x - x$, or equivalent	A1	
	Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
	Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
	Obtain the given answer	A1	**[6]**

Question 24

(i)	State or imply $f(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = 2, B = -1, C = 3$	A1	
	Obtain the remaining values A1 +	A1	**5**

[Apply an analogous scheme to the form $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1, E = 1$.]

(ii)	Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)-\frac{3}{x+2}$	B1√ + B1√ + B1√	
	Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
	Obtain the given answer following full and exact working	A1	**5**
	[The t marks are dependent on *A*, *B*, *C* etc.]		
	[SR: If *B*, *C* or *E* omitted, give B1M1 in part (**i**) and B1√B1√M1 in part (**ii**).]		
	[NB: Candidates who follow the *A*, *D*, *E* scheme in part (**i**) and then integrate $\frac{-x+1}{(x+2)^2}$ by parts should obtain $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)+\frac{x-1}{x+2}$ (the third term is equivalent to $-\frac{3}{x+2}+1$).]		

Question 25

Use correct quotient or product rule	M1	
Obtain correct derivative in any form	A1	
Equate derivative to zero and obtain a horizontal equation	M1	
Carry out complete method for solving an equation of the form $ae^{3x}=b$, or $ae^{5x}=be^{2x}$	M1	
Obtain $x=\ln 2$, or exact equivalent	A1	
Obtain $y=\frac{1}{3}$, or exact equivalent	A1	**6**

Question 26

(i)	State $\frac{dx}{dt}=-4a\cos^3 t\sin t$, or $\frac{dy}{dt}=4a\sin^3 t\cos t$	B1	
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1	
	Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
(ii)	Form the equation of the tangent	M1	
	Obtain a correct equation in any form	A1	
	Obtain the given answer	A1	**3**
(iii)	State the *x*-coordinate of *P* or the *y*-coordinate of *Q* in any form	B1	
	Obtain the given result correctly	B1	**2**

Question 27

(i)	State or imply $du = -\frac{1}{2\sqrt{x}}dx$, or equivalent	B1	
	Substitute for x and dx throughout	M1	
	Obtain integrand $\frac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu$, and cu^2	M1*	
	Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain the given answer correctly having shown sufficient working	A1	**[4]**

Question 29

EITHER:	Use correct product rule	M1	
	Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x\sin 2x$	A1	
	Use the correct double angle formulae to express derivative in $\cos x$ and $\sin x$, or $\cos 2x$ and $\sin x$	M1	
*OR*1:	Use correct double angle formula to express y in terms of $\cos x$ and attempt differentiation	M1	
	Use chain rule correctly	M1	
	Obtain correct derivative in any form, e.g. $-6\cos^2 x\sin x + \sin x$	A1	
*OR*2:	Use correct factor formula and attempt differentiation	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
	Use correct trig formulae to express derivative in terms of $\cos x$ and $\sin x$, or $\sin x$	M1	
Equate derivative to zero and obtain an equation in one trig function		M1	
Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$		A1	
Obtain answer $x = 1.15$ (or $65.9°$) and no other in the given interval		A1	**[6]**

[Ignore answers outside the given interval.]

[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:

Equate derivative to zero and obtain an equation in $\tan 2x$ and $\tan x$ — M1

Use correct double angle formula to obtain an equation in $\tan x$ — M1]

Question 30

(i)	Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2+2)(t+2)$	A1	
	Identify value of t at the origin as -1	B1	
	Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5

Question 31

(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
	Obtain integral of form $ax + b\tan 2x$	M1	
	Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of $+ c$	A1	[3]
(b)	State $\sin x \cos \frac{1}{2}\pi + \cos x \sin \frac{1}{6}\pi$	B1	
	Simplify integrand to $\cos \frac{1}{6}\pi + \frac{\cos x \sin \frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
	Integrate to obtain at least term of form $a\ln(\sin x)$	*M1	
	Apply limits and simplify to obtain two terms	M1 dep *M	
	Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln\left(\frac{1}{\sqrt{2}}\right)$ or equivalent	A1	[5]

Question 32

Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
Obtain correct $-6\sin 2x + 7\cos x$	A1	
Use identity $\sin 2x = 2\sin x \cos x$	B1	
Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
Obtain 0.623	A1	
Obtain 2.52	A1	
Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
Treat answers in degrees as MR – 1 situation		[7]

Question 33

State $du = 3\sin x\, dx$ or equivalent	**B1**	
Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
Carry out complete substitution, for x and dx	**M1**	
Obtain $\int \frac{8-2u}{\sqrt{u}}\, du$, or equivalent	**A1**	
Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
Apply correct limits correctly	**dep M1***	
Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]

Question 34

Use correct quotient rule or equivalent to find first derivative	**M1***	
Obtain $\dfrac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
Substitute $x = \frac{1}{4}\pi$ to find gradient	**dep M1***	
Obtain $-\frac{3}{2}$	**A1**	
Form equation of tangent at $x = \frac{1}{4}\pi$	**M1**	
Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent	**A1**	[6]

Question 35

(i)	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	**M1**	
	Obtain answer $p = 3.40$	**A1**	[2]

Question 36

(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	**B1**	
	Use product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Use Pythagoras	**M1**	
	Justify the given form	**A1**	[5]
(ii)	Fully justify the given statement	**B1**	[1]
(iii)	State answer $x = \frac{1}{4}\pi$	**B1**	[1]

Question 37

(i) *EITHER*:	State correct derivative of $\sin y$ with respect to x	**B1**	
	Use product rule to differentiate the LHS	**M1**	
	Obtain correct derivative of the LHS	**A1**	
	Obtain a complete and correct derived equation in any form	**A1**	
	Obtain a correct expression for $\frac{dy}{dx}$ in any form	**A1**	
OR:	State correct derivative of $\sin y$ with respect to x	**B1**	
	Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides	**B1**	
	Use quotient or product rule to differentiate the RHS	**M1**	
	Obtain correct derivative of the RHS	**A1**	
	Obtain a correct expression for $\frac{dy}{dx}$ in any form	**A1**	[5]

(ii)	Equate $\frac{dy}{dx}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$	**M1**	
	Solve for $\ln x$	**M1**	
	Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent	**A1**	[3]

Question 38

(i)	State or imply $dx = \sqrt{3}\sec^2\theta \, d\theta$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Obtain the given answer correctly	**A1**	[3]

(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$	**B1**	
	Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$	**B1**√	
	Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$	**M1**	
	Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent	**A1**	[4]
	[The f.t. is on integrands of the form $a\cos 2\theta + b$, where $ab \neq 0$.]		

Question 3

(i)	State or imply $du = 2x\,dx$, or equivalent	**B1**	
	Substitute for x and dx throughout	**M1**	
	Reduce to the given form and justify the change in limits	**A1**	
		[3]	

(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**	
	Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent	**A1 + A1**	
	(deduct A1 for each error or omission)		
	Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**	
	Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**	
		[5]	

Question 40

(i)	State $\frac{dx}{dt} = 1 - \sin t$	**B1**
	Use chain rule to find the derivative of y	**M1**
	Obtain $\frac{dy}{dt} = \frac{\cos t}{1 + \sin t}$, or equivalent	**A1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**
	Obtain the given answer correctly	**A1**
		[5]
(ii)	State or imply $t = \cos^{-1}(\frac{1}{3})$	**B1**
	Obtain answers $x = 1.56$ and $x = -0.898$	**B1 + B1**
		[3]

Question 41

State or imply derivative of $(\ln x)^2$ is $\frac{2\ln x}{x}$	**B1**
Use correct quotient or product rule	**M1**
Obtain correct derivative in any form, e.g. $\frac{2\ln x}{x^2} - \frac{(\ln x)^2}{x^2}$	**A1**
Equate derivative (or its numerator) to zero and solve for $\ln x$	**M1**
Obtain the point (1, 0) with no errors seen	**A1**
Obtain the point $(e^2,\ 4e^{-2})$	**A1** [6]

Question 42

Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x\ dx$	**M1***
Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**
Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**
Use limits correctly having integrated twice	**DM1***
Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1** [5]

Question 43

(i)	State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$	**B1**
	State $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		[4]
(ii)	Equate numerator to zero	**M1***
	Obtain $x = 2y$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1***
	Obtain the point $(-2, -1)$	**A1**
	State the point $(0, 1.44)$	**B1**
		[5]

Question 44

Use product rule	**M1**
Obtain correct derivative in any form, e.g. $\cos x\cos 2x - 2\sin x\sin 2x$	**A1**
Equate derivative to zero and use double angle formulae	**M1**
Remove factor of $\cos x$ and reduce equation to one in a single trig function	**M1**
Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**
Solve and obtain $x = 0.421$	**A1**
	[6]

Question 45

Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**
Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent	**A1**
Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice	**M1**
Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent	**A1**
	[5]

Question 46

(i)	State or imply $du = \dfrac{1}{2\sqrt{x}}dx$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Justify the change in limits and obtain the given answer	**A1**	[3]
(ii)	Convert integrand into the form $A + \dfrac{B}{u+1}$	**M1***	
	Obtain integrand $A = 1$, $B = -2$	**A1**	
	Integrate and obtain $u - 2\ln(u+1)$	**A1**√ + **A1**√	
	Substitute limits correctly in an integral containing terms au and $b\ln(u + 1)$, where $ab \neq 0$	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[6]
	[The f.t. is on A and B.]		

Question 47

Use correct quotient or product rule	**M1**	
Obtain correct derivative in any form	**A1**	
Use Pythagoras to simplify the derivative to $\dfrac{1}{1+\cos x}$, or equivalent	**A1**	
Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	**A1**	[4]

Question 48

(i)	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x}+\frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x=\sqrt{5}-1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x}+b\int(2-2x)e^{\frac{1}{2}x}\,dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2x-x^2)-2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x=0$, $x=2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer $24-8e$, or exact simplified equivalent	**A1**	[5]

Question 49

(i)	*EITHER*:	Use tan $2A$ formula to express LHS in terms of $\tan\theta$	**M1**	
		Express as a single fraction in any correct form	**A1**	
		Use Pythagoras or cos $2A$ formula	**M1**	
		Obtain the given result correctly	**A1**	
	OR:	Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin\theta$ and $\cos\theta$	**M1**	
		Express as a single fraction in any correct form	**A1**	
		Use Pythagoras or cos $2A$ formula or $\sin(A-B)$ formula	**M1**	
		Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)		**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta)+\ln(\cos\theta)$, or equivalent		**A1**	
	Substitute limits correctly (expect to see use of both limits)		**DM1**	
	Obtain the given answer following full and correct working		**A1**	[4]

Question 50

EITHER:	*EITHER*:	State $2xy + x^2\dfrac{dy}{dx}$, or equivalent, as derivative of x^2y	**B1**	
		State $6y^2 + 12xy\dfrac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**	
	OR:	Differentiating LHS using correct product rule, state term $xy(1-6\dfrac{dy}{dx})$, or equivalent	**B1**	
		State term $(y + x\dfrac{dy}{dx})(x-6y)$, or equivalent	**B1**	
		Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero	**M1***	
		Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**	
		Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	**A1**	
		Obtain an equation in x or y	**DM1**	
		Obtain answer $(-3a, -a)$	**A1**	
OR:	Rearrange to $y = \dfrac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\dfrac{9a^3}{x^2(x-6y)^2}\times .....$		**B1**	
	State term $(x-6y)+x(1-6y')$, or equivalent		**B1**	
	Justify division by $x(x-6y)$		**B1**	
	Set $\dfrac{dy}{dx}$ equal to zero		**M1***	
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)		**A1**	
	Obtain an equation in x or y		**DM1**	
	Obtain answer $(-3a, -a)$		**A1**	[7]

Question 51

(i)	State or imply derivative is $2\frac{\ln x}{x}$	**B1**
	State or imply gradient of the normal at $x = \mathrm{e}$ is $-\frac{1}{2}\mathrm{e}$, or equivalent	**B1**
	Carry out a complete method for finding the x-coordinate of Q	**M1**
	Obtain answer $x = \mathrm{e} + \frac{2}{\mathrm{e}}$, or exact equivalent	**A1**
	Total:	**4**
(ii)	Justify the given statement by integration or by differentiation	**B1**
	Total:	**1**
(iii)	Integrate by parts and reach $ax(\ln x)^2 + b\int x.\frac{\ln x}{x}\,\mathrm{d}x$	**M1***
	Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent	**A1**
	Use limits $x = 1$ and $x = \mathrm{e}$ correctly, having integrated twice	**DM1**
	Obtain exact value $\mathrm{e} - 2$	**A1**
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e} - 2 + \frac{1}{\mathrm{e}}$	**B1**$\sqrt{}$
	Total:	**5**

Question 52

Use product rule	**M1**
Obtain correct derivative in any form	**A1**
Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	**M1**
Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent	**A1**
Use the condition for a quadratic to have only one root	**M1**
Obtain answer $a = 2$	**A1**
Obtain answer $x = \frac{1}{4}\pi$	**A1**
Total:	**7**

Question 53

(i)	Use correct quotient rule or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = 2$	**A1**
	Total:	**4**

Question 54

5(i)	Use the chain rule	**M1**
	Obtain correct derivative in any form	**A1**
	Use correct trigonometry to express derivative in terms of $\tan x$	**M1**
	Obtain $\frac{dy}{dx} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	**A1**
	Total:	**4**
(ii)	Equate derivative to –1 and solve a 3–term quadratic for $\tan x$	**M1**
	Obtain answer x=1.11 and no other in the given interval	**A1**
	Total:	**2**

Question 55

Rearrange as $3u^2 + 4u - 4 = 0$, or $3e^{2x} + 4e^x - 4 = 0$, or equivalent	**B1**
Solve a 3-term quadratic for e^x or for u	**M1**
Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	**A1**
Obtain answer $x = -0.405$ and no other	**A1**
Total:	**4**

Question 56

Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ d\theta$	*M1
Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
Substitute limits correctly, having integrated twice	DM1
Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
Total:	**4**

Question 57

(i)	Use quotient or chain rule	M1
	Obtain given answer correctly	A1
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	(M1
	Use Pythagoras and express LHS in terms of sec θ and tan θ	M1
	Complete the proof	A1)
	OR1: Express RHS in terms of cos θ and sin θ	(M1
	Use Pythagoras and express RHS in terms of sin θ	M1
	Complete the proof	A1)
	OR2: Express LHS in terms of secθ and tanθ	(M1
	Multiply numerator and denominator by $\sec\theta + \tan\theta$ and use Pythagoras	M1
	Complete the proof	A1)
	Total:	**3**
(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	B2
	Use correct limits correctly in an integral containing terms a tanθ and b secθ	M1
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	A1
	Total:	**4**

Question 58

(i)	State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**
	Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$	**A1**
	Total:	**3**
(ii)	Use correct method to find the gradient of the normal at $t = 1$	**M1**
	Use a correct method to form an equation for the normal at $t = 1$	**M1**
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	**A1**
	Total:	**3**

Question 59

(i)	State or imply $du = -\sin x \, dx$	**B1**
	Using correct double angle formula, express the integral in terms of u and du	**M1**
	Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2 \, du$ with no errors seen	**A1**
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

Question 60

(i)	Use chain rule to differentiate x $\left(\frac{dx}{d\theta} = -\frac{\sin\theta}{\cos\theta}\right)$	M1
	State $\frac{dy}{d\theta} = 3 - \sec^2\theta$	B1
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$	A1
	Obtain $\frac{dy}{dx} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	A1
	Total:	**5**
(ii)	Equate gradient to −1 and obtain an equation in $\tan\theta$	M1
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	M1
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	A1
	Total:	**3**

Question 61

(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	B1
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	B1
	Total:	**2**
(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	*M1
	State correct integral $-\ln(1+e^{-x})$	A1
	Use limits correctly	DM1
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	A1
	Total:	**4**

Question 62

(i)	Use correct product or quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a 3 term quadratic equation in x	M1
	Obtain answers $x = 2 \pm \sqrt{3}$	A1
		4
(ii)	Integrate by parts and reach $k(1+x^2)\mathrm{e}^{-\frac{1}{2}x} + l\int x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	*M1
	Obtain $-2(1+x^2)\mathrm{e}^{-\frac{1}{2}x} + 4\int x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$, or equivalent	A1
	Complete the integration and obtain $(-18-8x-2x^2)\mathrm{e}^{-\frac{1}{2}x}$, or equivalent	A1
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	DM1
	Obtain the given answer	A1
		5

Question 63

5(i)	State or imply $y^3 + 3xy^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of xy^3	B1
	State or imply $4y^3\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^4	B1
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	Obtain the given answer	A1
		4
(ii)	Equate numerator to zero	*M1
	Obtain $y = -2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	A1
		4

Question 64

(i)	State or imply $3x^2y + x^3\frac{dy}{dx}$ as derivative of x^3y	B1
	State or imply $9xy^2\frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer AG	A1
		4
(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	M1
	Obtain answer $x = a$ and $y = -a$	A1
	Obtain answer $x = -a$ and $y = a$	A1
	Consider and reject $y = 0$ and $x = y$ as possibilities	B1
		4

Question 65

Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	M1
Obtain correct derivative in any form	A1
Equate the derivative to zero and solve for x	M1
Obtain $x = \frac{1}{6}\pi$	A1
Obtain $y = \sqrt{3}$	A1
	5
Carry out an appropriate method for determining the nature of a stationary point	M1
Show the point is a minimum point with no errors seen	A1
	2

Question 66

(i)	Use correct product or quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a 3 term quadratic equation in x	M1
	Obtain answers $x = 2 \pm \sqrt{3}$	A1
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\ dx$	*M1
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\ dx$, or equivalent	A1
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	A1
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	DM1
	Obtain the given answer	A1
		5

Question 67

(i)	State or imply $y^3 + 3xy^2\frac{dy}{dx}$ as derivative of xy^3	B1
	State or imply $4y^3\frac{dy}{dx}$ as derivative of y^4	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
		4
(ii)	Equate numerator to zero	*M1
	Obtain $y = -2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain final answer $x = -1, y = 2$ and $x = 1, y = -2$	A1
		4

Question 68

(i)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1
	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
	Total:	**4**
(ii)	Equate denominator to zero and solve for y	M1*
	Obtain $y = 0$ and $x = a$	A1
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	M1(dep*)
	Obtain $x = -a$	A1
	Obtain $y = 2a$	A1
	Total:	**5**

Question 69

(i)	State answer $R = \sqrt{5}$	B1
	Use trig formulae to find $\tan\alpha$	M1
	Obtain $\tan\alpha = 2$	A1
	Total:	**3**
(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	B1FT
	State correct indefinite integral $3\tan(\theta - \alpha)$	B1FT
	Substitute limits correctly	M1
	Use $\tan(A \pm B)$ formula	M1
	Obtain the given exact answer correctly	A1
	Total:	**5**

Question 70

(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
(iii)	Use iterative formula $x_{n+1} = \frac{3+x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

Question 71

Integrate by parts and reach $ax\sin 3x + b\int \sin 3x\,dx$	**M1***
Obtain $\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$, or equivalent	**A1**
Complete the integration and obtain $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$, or equivalent	**A1**
Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
Obtain answer $\frac{1}{18}(\pi - 2)$ OE	**A1**
Total:	**5**

Question 72

3(i)	Use correct product or quotient rule	M1	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
3(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use correct limits $x = -1$ and $x = 0$ in the correct order, having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	A1	
		5	

Question 73

(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y) + x^2\left(1 + 3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	B1	$3x^2 + 6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	Given answer so check working carefully
	Obtain the given answer	A1	
		4	
(ii)	Equate derivative to -1 and solve for y	M1*	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	M1(dep*)	
	Obtain answer $(1, -2)$	A1	
	Obtain answer $(\sqrt[3]{3}, 0)$	B1	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question 74

(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	M1	$\frac{2\sin x - 2\sin x\cos x}{1-(2\cos^2 x - 1)}$
	Obtain a correct expression	A1	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	M1	
	Obtain the given RHS correctly *OR (working R to L)*:	A1	
	$\frac{\sin x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}=\frac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\frac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\frac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
(ii)	State integral of the form $a\ln(1+\cos x)$	M1*	If they use the substitution $u=1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	A1	
	Substitute correct limits in correct order	M1(dep)*	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	A1	
		4	

Question 75

5(i)	State or imply $dx=-2\cos\theta\sin\theta\,d\theta$, or equivalent	B1
	Substitute for x and dx, and use Pythagoras	M1
	Obtain integrand $\pm 2\cos^2\theta$	A1
	Justify change of limits and obtain given answer correctly	A1
		4
(ii)	Obtain indefinite integral of the form $a\theta+b\sin 2\theta$	M1*
	Obtain $\theta+\frac{1}{2}\sin 2\theta$	A1
	Use correct limits correctly	M1(dep*)
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	A1
		4

Question 76

Use quotient or product rule	**M1**
Obtain correct derivative in any form	**A1**
Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a\sin x = b$	**M1***
Solve for x	**M1(dep*)**
Obtain answer 0.340	**A1**
Obtain second answer 2.802 and no other in the given interval	**A1**
	6

Question 77

(i)	State correct derivative of x or y with respect to t		**B1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		**M1**
	Obtain $\frac{dy}{dx} = \frac{4\sin 2t}{2 + 2\cos 2t}$, or equivalent		**A1**
	Use double angle formulae throughout		**M1**
	Obtain the given answer correctly	**AG**	**A1**
			5
(ii)	State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$		**B1**
	Obtain answer $x = -0.961$		**B1**
			2

Question 78

(i)	State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x + \cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
(ii)	Obtain integral $\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3