A-level

Topic :Complex Number

May 2013-May 2025

Answers

Question 1

(a)	State or imply $3a+3b\mathrm{i}+2\mathrm{i}(a-b\mathrm{i})=17+8\mathrm{i}$		B1	
	Consider real and imaginary parts to obtain two linear equations in *a* and *b*		M1*	
	Solve two simultaneous linear equations for *a* or *b*		M1 (dep*)	
	Obtain $7-2\mathrm{i}$		A1	[4]
(b)	Either	Show or imply a triangle with side 2	B1	
		State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$	B1	
		State or imply argument is $\frac{1}{4}\pi$	B1	
		Use sine rule or equivalent to find *r*	M1	
		Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$	A1	
	Or	State $y = x$.	B1	
		State $y=\frac{1}{\sqrt{3}}x+2$ or $\frac{\sqrt{3}}{2}=\frac{x}{\sqrt{x^2+(y-2)^2}}$ or $\frac{1}{2}=\frac{y-2}{\sqrt{x^2+(y-2)^2}}$	B1	
		State or imply argument is $\frac{\pi}{4}$	B1	
		Solve for *x* or *y*.	M1	
		Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	[5]

Question 2

(a)	Substitute $w = x + \mathrm{i}y$ and state a correct equation in *x* and *y*	B1	
	Use $\mathrm{i}^2 = -1$ and equate real parts	M1	
	Obtain $y = -2$	A1	
	Equate imaginary parts and solve for *x*	M1	
	Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
(b)	Show a circle with centre 2i	B1	
	Show a circle with radius 2	B1	
	Show half line from –2 at $\frac{1}{4}\pi$ to real axis	B1	
	Shade the correct region	B1	
	Carry out a complete method for calculating the greatest value of $\|z\|$	M1	
	Obtain answer 3.70	A1	[6]

Question 3

(i)	Show that $a^2 + b^2 = (a + \mathrm{i}b)(a - \mathrm{i}b)$	B1	
	Show that $(a + \mathrm{i}b - k\mathrm{i})^* = a - \mathrm{i}b + k\mathrm{i}$	B1	[2]
(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
	Obtain a correct equation in any form, e.g. $(z - 10\mathrm{i})(z^* + 10\mathrm{i}) = 4(z - 4\mathrm{i})(z^* + 4\mathrm{i})$	A1	
	Obtain the given equation	A1	
	Either express $\lvert z - 2i \rvert = 4$ in terms of z and z^* or reduce the given equation to the form $\lvert z - u \rvert = r$	M1	
	Obtain the given answer correctly	A1	[5]
(iii)	State that the locus is a circle with centre 2i and radius 5	B1	[1]

Question 4

(a)	*EITHER:*	Solve for u or for v	M1	
		Obtain $u = \dfrac{2\mathrm{i} - 6}{1 - 2\mathrm{i}}$ or $v = \dfrac{5}{1 - 2\mathrm{i}}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent		
		Or: Set u or v equal to x + iy, obtain two equations by equating real and imaginary parts and solve for x or for y	M1	
	OR:	Using a + ib and c +id for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d	M1	
		Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent	A1	
		Solve for one unknown	M1	
	Obtain final answer $u = -2 - 2\mathrm{i}$, or equivalent		A1	
	Obtain final answer $v = 1 + 2\mathrm{i}$, or equivalent		A1	**[5]**
(b)	Show a circle with centre –i		B1	
	Show a circle with radius 1		B1	
	Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis		B1	
	Use a correct method for finding the least value of the modulus		M1	
	Obtain final answer $\dfrac{3}{\sqrt{2}} - 1$, or equivalent, e.g. 1.12 (allow 1.1)		A1	**[5]**

Question 5

(a)	Solve using formula, including simplification under square root sign	M1*	
	Obtain $\dfrac{-2 \pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents	A1	
	Multiply by $\dfrac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case	M1(d*M)	
	Obtain final answer $-\dfrac{4}{5}+\dfrac{3}{5}\mathrm{i}$	A1	
	Obtain final answer $-\mathrm{i}$	A1	[5]
(b)	Show *w* in first quadrant with modulus and argument relatively correct	B1	
	Show w^3 in second quadrant with modulus and argument relatively correct	B1	
	Show *w** in fourth quadrant with modulus and argument relatively correct	B1	
	Use correct method for area of triangle	M1	
	Obtain 10 by calculation	A1	[5]

Question 6

(i)	Either	Multiply numerator and denominator by $\sqrt{3}+\mathrm{i}$ and use $\mathrm{i}^2=-1$	M1	
		Obtain correct numerator $18+18\sqrt{3}\mathrm{i}$ or correct denominator 4	B1	
		Obtain $\dfrac{9}{2}+\dfrac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18+18\sqrt{3}\mathrm{i}\right)/4$	A1	
		Obtain modulus or argument	M1	
		Obtain $9\mathrm{e}^{\frac{1}{3}\pi \mathrm{i}}$	A1	[5]
	OR	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
		Obtain moduli and argument 18 and $\dfrac{1}{6}\pi$ or 2 and $-\dfrac{1}{6}\pi$ or moduli 18 and 2 or arguments $\dfrac{1}{6}\pi$ and $-\dfrac{1}{6}\pi$ (allow degrees)	B1	
		Obtain $18\mathrm{e}^{\frac{1}{6}\pi \mathrm{i}} \div 2\mathrm{e}^{-\frac{1}{6}\pi \mathrm{i}}$ or equivalent	A1	
		Divide moduli and subtract arguments	M1	
		Obtain $9\mathrm{e}^{\frac{1}{3}\pi \mathrm{i}}$	A1	[5]
(ii)		State $3\mathrm{e}^{\frac{1}{6}\pi \mathrm{i}}$, following through their answer to part (i)	B1√	
		State $3\mathrm{e}^{\frac{1}{6}\pi \mathrm{i} \pm \frac{1}{2}\pi \mathrm{i}}$, following through their answer to part (i)	B1√	
		Obtain $3\mathrm{e}^{-\frac{5}{6}\pi \mathrm{i}}$	B1	[3]

Question 7

(a)	*EITHER*:	Substitute and expand $(-1+\sqrt{5}\,\mathrm{i})^3$ completely	M1	
		Use $\mathrm{i}^2 = -1$ correctly at least once	M1	
		Obtain $a = -12$	A1	
		State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
	OR1:	State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
		State the quadratic factor $z^2 + 2z + 6$	B1	
		Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a	M1	
		Obtain $a = -12$	A1	
(b)	*EITHER:*	Substitute $w = \cos 2\theta + \mathrm{i}\sin 2\theta$ in the given expression	B1	
		Use double angle formulae throughout	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain given answer correctly	A1	
	OR:	Substitute $w = \mathrm{e}^{2\mathrm{i}\theta}$ in the given expression	B1	
		Divide numerator and denominator by $\mathrm{e}^{\mathrm{i}\theta}$, or equivalent	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain the given answer correctly	A1	**4**

Question 8

(a)	*EITHER*:	Multiply numerator and denominator by 1 – 4i, or equivalent, and use $\mathrm{i}^2 = -1$	M1	
		Simplify numerator to $-17-17\mathrm{i}$, or denominator to 17	A1	
		Obtain final answer $-1-\mathrm{i}$	A1	
	OR:	Using $\mathrm{i}^2 = -1$, obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = -1$ or $y = -1$, or equivalent	A1	
		Obtain final answer $-1-\mathrm{i}$	A1	**3**
(b)	**(i)**	Show a point representing 2 + i in relatively correct position	B1	
		Show a circle with centre 2 + i and radius 1	B1√	
		Show the perpendicular bisector of the line segment joining i and 2	B1	
		Shade the correct region	B1	**4**
	(ii)	State or imply that the angle between the tangents from the origin to the circle is required	M1	
		Obtain answer 0.927 radians (or 53.1°)	A1	**2**

Question 9

(i)	Substitute $z = 1 + \mathrm{i}$ and obtain $w = \dfrac{1+2\mathrm{i}}{1+\mathrm{i}}$	B1	
	EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
	Simplify numerator to $3 + \mathrm{i}$ or denominator to 2	A1	
	Obtain final answer $\dfrac{3}{2}+\dfrac{1}{2}\mathrm{i}$, or equivalent	A1	
	OR: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain $x = \dfrac{3}{2}$ or $y = \dfrac{1}{2}$, or equivalent	A1	
	Obtain final answer $\dfrac{3}{2}+\dfrac{1}{2}\mathrm{i}$, or equivalent	A1	**[4]**
(ii)	*EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$	B1	
	Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y	M1	
	OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts	B1	
	Solve for x and y	M1	
	Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$	A1	
	Obtain final answer $-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}\mathrm{i}$	A1	**[4]**

Question 10

(i)	State or imply $\mathrm{i}w = -3 + 5\mathrm{i}$	B1	
	Carry out multiplication by $\dfrac{4-\mathrm{i}}{4-\mathrm{i}}$	M1	
	Obtain final answer $-\dfrac{7}{17}+\dfrac{23}{17}\mathrm{i}$ or equivalent	A1	[3]
(ii)	Multiply w by z to obtain $17 + 17\mathrm{i}$	B1	
	State $\arg w = \tan^{-1}\dfrac{3}{5}$ or $\arg z = \tan^{-1}\dfrac{1}{4}$	B1	
	State $\arg wz = \arg w + \arg z$	M1	
	Confirm given result $\tan^{-1}\dfrac{3}{5} + \tan^{-1}\dfrac{1}{4} = \dfrac{1}{4}\pi$ legitimately	A1	[4]

Question 11

(i)	Either	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Multiply by $\frac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	
	Or	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Obtain two equations in x and y and solve for x or y	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	[3]
(ii)	Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$		B1	
	Use appropriate method to find both critical values of p		M1	
	State $-6 \leqslant p \leqslant 2$		A1	[3]
(iii)	Identify equation as of form $\lvert z-a\rvert = a$ or equivalent		M1	
	Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$		A1	
	State $\lvert z-5\rvert = 5$		A1	[3]

Question 12

(i)	Square $x+\mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$	M1	
	Obtain $x^2-y^2=-1$ and $2xy=4\sqrt{3}$	A1	
	Eliminate one unknown and find an equation in the other	M1	
	Obtain $x^4+x^2-12=0$ or $y^4-y^2-12=0$, or three term equivalent	A1	
	Obtain answers $\pm(\sqrt{3}+2\mathrm{i})$	A1	**[5]**
	[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]		
(ii)	Show a circle with centre $-1+4\sqrt{3}$ in a relatively correct position	B1	
	Show a circle with radius 1 and centre not at the origin	B1	
	Carry out a complete method for calculating the greatest value of arg z	M1	
	Obtain answer 1.86 or 106.4°	A1	**[4]**

Question 13

(i)	*EITHER*:	Substitute for u in $\frac{i}{u}$ and multiply numerator and denominator by $1 + i$	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}i$, or equivalent	A1	
	OR:	Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}i$, or equivalent	A1	**2**
(ii)		Show a point representing u in a relatively correct position	B1	
		Show the bisector of the line segment joining u to the origin	B1	
		Show a circle with centre at the point representing i	B1	
		Show a circle with radius 2	B1	**4**
(iii)		State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270°	B1	
		State or imply the intersection in the first quadrant represents $2 + i$	B1	
		State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625)	B1	**3**

Question 14

(i)	Show u in a relatively correct position	**B1**	
	Show u^* in a relatively correct position	**B1**	
	Show $u^* - u$ in a relatively correct position	**B1**	
	State or imply that $OABC$ is a parallelogram	**B1**	[4]
(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by $3 + i$, or equivalent	**M1**	
	Simplify the numerator to $8 + 6i$ or the denominator to 10	**A1**	
	Obtain final answer $\frac{4}{5}+\frac{3}{5}i$, or equivalent	**A1**	
	OR: Substitute for u, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain $x=\frac{4}{5}$ or $y=\frac{3}{5}$, or equivalent	**A1**	
	Obtain final answer $\frac{4}{5}+\frac{3}{5}i$, or equivalent	**A1**	[3]
iii)	State or imply $\arg(u^*/u)=\tan^{-1}(\frac{3}{4})$	**B1**	
	Substitute exact arguments in $\arg(u^*/u)=\arg u^* - \arg u$	**M1**	
	Fully justify the given statement using exact values	**A1**	[3]

Question 15

(a)	Either	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
		Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
		Square $x+\mathrm{i}y$ form to find w^2	**M1**	
		Obtain $w^2=\frac{48-14\mathrm{i}}{25}$ and confirm modulus is 2	**A1**	
		Use correct process for finding argument of w^2	**M1**	
		Obtain –0.284 radians or $-16.3°$	**A1**	
	Or 1	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
		Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
		Find modulus of w and hence of w^2	**M1**	
		Confirm modulus is 2	**A1**	
		Find argument of w and hence of w^2	**M1**	
		Obtain –0.284 radians or $-16.3°$	**A1**	
(b)		Draw circle with centre the origin and radius 5	**B1**	
		Draw straight line parallel to imaginary axis in correct position	**B1**	
		Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
		Obtain $5e^{\pm\frac{1}{3}\pi\mathrm{i}}$ or equivalents in required form	**A1**	[4]

Question 16

(a)		Substitute and obtain a correct equation in x and y	**B1**	
		Use $\mathrm{i}^2=-1$ and equate real and imaginary parts	**M1**	
		Obtain two correct equations, e.g. $x+2y+1=0$ and $y+2x=0$	**A1**	
		Solve for x or for y	**M1**	
		Obtain answer $z=\frac{1}{3}-\frac{2}{3}\mathrm{i}$	**A1**	[5]
(b)	**(i)**	Show a circle with centre $-1+3\mathrm{i}$	**B1**	
		Show a circle with radius 1	**B1**	
		Show the line $\mathrm{Im}\, z=3$	**B1**	
		Shade the correct region	**B1**	[4]
	(ii)	Carry out a complete method to calculate the relevant angle	**M1**	
		Obtain answer 0.588 radians (accept 33.7°)	**A1**	[2]

Question 17

(a)	Square x+ iy and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively	**M1**
	Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$	**A1**
	Eliminate one variable and find an equation in the other	**M1**
	Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent	**A1**
	Obtain answers $\pm(3 - i\sqrt{2})$	**A1**
		[5]
(b) (i)	Show point representing 1 + 2i	**B1**
	Show circle with radius 1 and centre 1 + 2i	**B1**√
	Show a half line from the point representing 1	**B1**
	Show line making the correct angle with the real axis	**B1**
		[4]
(ii)	State or imply the relevance of the perpendicular from 1 + 2i to the line	**M1**
	Obtain answer $\sqrt{2} - 1$ (or 0.414)	**A1**
		[2]

Question 18

(a)	*EITHER*: Use quadratic formula to solve for z	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain a correct answer in any form, simplified as far as $(-2 \pm i\sqrt{8})/2i$	**A1**	
	Multiply numerator and denominator by i, or equivalent	**M1**	
	Obtain final answers $\sqrt{2}$ + i and $-\sqrt{2}$ + i	**A1**	
	OR: Substitute x + iy and equate real and imaginary parts to zero	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent	**A1**	
	Solve for x and y	**M1**	
	Obtain final answers $\sqrt{2} + i$ and $-\sqrt{2} + i$	**A1**	[5]
(b) (i)	*EITHER*: Show the point representing 4 + 3i in relatively correct position	**B1**	
	Show the perpendicular bisector of the line segment joining this point to the origin	**B1**√	[2]
	OR: Obtain correct Cartesian equation of the locus in any form, e.g. $8x + 6y = 25$	**B1**	
	Show this line	**B1**√	
	[This f.t. is dependent on using a correct method to determine the equation.]		
(ii)	State or imply the relevant point is represented by 2 + 1.5i or is at (2, 1.5)	**B1**	
	Obtain modulus 2.5	**B1**√	
	Obtain argument 0.64 (or 36.9°) (allow decimals in [0.64, 0.65] or [36.8, 36.9])	**B1**√	[3]

Question 19

(i)	*EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by 2 + i, or equivalent	**M1**
	Simplify the numerator to –5 +5i or denominator to 5	**A1**
	Obtain final answer –1 + I	**A1**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = -1$ or $y = 1$	**A1**
	Obtain final answer –1 + I	**A1)**
		[3]
(ii)	Obtain $u + v = 1 + 2i$	**B1**
	In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively	**B1**√
	State that OB and AC are parallel	**B1**
	State that $OB = AC$	**B1**
		[4]
(iii)	Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u/v)$	**M1**
	Show sufficient working to justify the given answer $\frac{3}{4}\pi$	**A1**
		[2]

Question 20

(a)	EITHER:	Use quadratic formula to solve for w	**M1**	
		Use $i^2 = -1$	**M1**	
		Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$	**A1**	
		Multiply numerator and denominator of an answer by –2i + 1, or equivalent	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	
	OR1:	Multiply the equation by 1 – 2i	**M1**	
		Use $i^2 = -1$	**M1**	
		Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	**A1**	
		Use quadratic formula or factorise to solve for w	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	
	OR2:	Substitute $w = x + iy$ and form equations for real and imaginary parts	**M1**	
		Use $i^2 = -1$	**M1**	
		Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	**A1**	
		Form equation in x only or y only and solve	**M1**	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	[5]
(b)	Show a circle with centre 1 + i		**B1**	
	Show a circle with radius 2		**B1**	
	Show half-line $\arg z = \frac{1}{4}\pi$		**B1**	
	Show half-line $\arg z = -\frac{1}{4}\pi$		**B1**	
	Shade the correct region		**B1**	[5]

Question 21

(i)	State modulus $2\sqrt{2}$, or equivalent State argument $-\frac{1}{3}\pi$ (or $-60°$)	**B1** **B1**	[2]
(ii) (a)	State answer $3\sqrt{2}+\sqrt{6}\,\mathrm{i}$	**B1**	
(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of iz Simplify the numerator to $4\sqrt{3}+4\,\mathrm{i}$ or the denominator to 8 Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$ *OR*: Substitute for z, obtain two equations in x and y and solve for x or for y Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$ Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	**M1** **A1** **A1** **M1** **A1** **A1**	[4]
(iii)	Show points A and B in relatively correct positions Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\frac{z^*}{\mathrm{i}z}$ Obtain the given answer	**B1** **M1** **A1**	[3]

Question 22

(i)	Substitute $z=-1+\mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**
	Use $\mathrm{i}^2=-1$ at least once	**M1**
	Complete the verification correctly	**A1**
	Total:	**3**
(ii)	State second root $z=-1-\mathrm{i}$	**B1**
	Carry out a complete method for finding a quadratic factor with zeros $-1+\mathrm{i}$ and $-1-\mathrm{i}$	**M1**
	Obtain z^2+2z+2, or equivalent	**A1**
	Attempt division of p(z) by z^2+2z+2 and reach a partial quotient z^2+kz	**M1**
	Obtain quadratic factor z^2-2z+5	**A1**
	Solve 3-term quadratic and use $\mathrm{i}^2=-1$	**M1**
	Obtain roots 1 + 2i and 1 – 2i	**A1**
	Total:	**7**

Question 23

(i)	State that $u - 2w = -7 - i$	B1
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by $3 - 4i$, or equivalent	(M1
	Simplify the numerator to $25 + 25i$ or denominator to 25	A1
	Obtain final answer $1 + i$	A1)
	OR: Obtain two equations in x and y and solve for x or for y	(M1
	Obtain $x = 1$ or $y = 1$	A1
	Obtain final answer $1 + i$	A1)
	Total:	4
(ii)	Find the argument of $\frac{u}{w}$	M1
	Obtain the given answer	A1
	Total:	2
(iii)	State that OB and CA are parallel	B1
	State that $CA = 2OB$, or equivalent	B1
	Total:	2

Question 24

(i)	*EITHER:* Substitute $x = 2 - \mathrm{i}$ (or $x = 2 + \mathrm{i}$) in the equation and attempt expansions of x^2 and x^3	(M1
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*1: Substitute $x = 2 - \mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	(M1
	Substitute $x = 2 + \mathrm{i}$ in the equation and add/subtract the two equations	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)

(ii)	Show a circle with centre $2 - \mathrm{i}$ in a relatively correct position	B1
	Show a circle with radius 1 and centre not at the origin	B1
	Show the perpendicular bisector of the line segment joining 0 to $-\mathrm{i}$	B1
	Shade the correct region	B1
	Total:	**4**

Question 25

(a)	Solve for z or for w	M1
	Use $\mathrm{i}^2 = -1$	M1
	Obtain $w = \dfrac{\mathrm{i}}{2-\mathrm{i}}$ or $z = \dfrac{2+\mathrm{i}}{2-\mathrm{i}}$	A1
	Multiply numerator and denominator by the conjugate of the denominator	M1
	Obtain $w = -\frac{1}{5} + \frac{2}{5}\mathrm{i}$	A1
	Obtain $z = \frac{3}{5} + \frac{4}{5}\mathrm{i}$	A1
	Total:	**6**

(b)	*EITHER:* Find $\pm\left[2+\left(2-2\sqrt{3}\right)\mathrm{i}\right]$	**(B1**
	Multiply by 2i (or –2i)	**M1***
	Add result to v	**DM1**
	Obtain answer $4\sqrt{3}-1+6\mathrm{i}$	**A1)**
	OR: State $\frac{z-v}{v-u}=k\mathrm{i}$, or equivalent	**(M1**
	State $k=2$	**A1**
	Substitute and solve for z even if i omitted	**M1**
	Obtain answer $4\sqrt{3}-1+6\mathrm{i}$	**A1)**
	Total:	**4**

Question 26

(a)	Square $x+\mathrm{i}y$ and equate real and imaginary parts to 8 and –15	**M1**
	Obtain $x^2-y^2=8$ and $2xy=-15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4-32x^2-225=0$ or $4y^4+32y^2-225=0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5-3\mathrm{i})$ or equivalent	**A1**
		5

(b)	Show a circle with centre $2+\mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

Question 27

(i)	State modulus 2	B1
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or $300°$)	B1
		2
(ii)	*EITHER:* Expand $(1-(\sqrt{3})\text{i})^3$ completely and process i^2 and i^3	(M1
	Verify that the given relation is satisfied	A1)
	OR: $u^3 = 2^3\left(\cos(-\pi)+i\sin(-\pi)\right)$ or equivalent: follow their answers to **(i)**	(M1
	Verify that the given relation is satisfied	A1)
		2
(iii)	Show a circle with centre $1-(\sqrt{3})\text{i}$ in a relatively correct position	B1
	Show a circle with radius 2 passing through the origin	B1
	Show the line Re $z = 2$	B1
	Shade the correct region	B1
		4

Question 28

(i)(a)	Substitute $x = 1 + 2i$ in the equation and attempt expansions of x^2 and x^3	M1
	Use $i^2 = -1$ correctly at least once and solve for k	M1
	Obtain answer $k = 15$	A1
		3
(i)(b)	State answer $1 - 2i$	B1
	Carry out a complete method for finding a quadratic factor with zeros $1 + 2i$ and $1 - 2i$	M1
	Obtain $x^2 - 2x + 5$	A1
	Obtain root $-\frac{3}{2}$, or equivalent, *via* division or inspection	A1
		4

(ii)	Show a circle with centre $1 + 2i$	B1
	Show a circle with radius 1	B1
	Carry out a complete method for calculating the least value of arg z	M1
	Obtain answer 0.64	A1
		4

Question 29

(i)	Use quadratic formula, or completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
		3
(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
(iii)	State or imply correct value of a relevant length or angle, e.g. *OA*, *OB*, *AB*, angle between *OA* or *OB* and the real axis	**B1ft**
	Carry out a complete method for finding angle *OAB*	**M1**
	Obtain $AOB = 60°$ correctly	**A1**
		3
(iv)	Give a complete justification of the given statement	**B1**
		1

Question 30

(i)	Substitute in *uv*, expand the product and use $i^2 = -1$		**M1**	
	Obtain answer $uv = -11 - 5\sqrt{3}i$		**A1**	
	EITHER:	Substitute in *u/v* and multiply numerator and denominator by the conjugate of *v*, or equivalent	**M1**	
		Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	**A1**	
		Obtain final answer $-1 + \sqrt{3}i$	**A1**	
	OR:	Substitute in *u/v*, equate to $x + iy$ and solve for x or for y	**M1**	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$		**A1**	
	Obtain final answer $-1 + \sqrt{3}\,i$		**A1**	
			5	

(ii)	Show the points A and B representing u and v in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB, e.g. calculate arg(u/v) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	M1	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan(a-b) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	A1	Given answer so check working carefully
		3	

Question 31

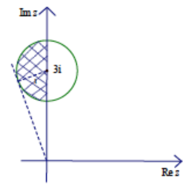
(a)	Substitute and obtain a correct equation in x and y	B1
	Use $i^2 = -1$ and equate real and imaginary parts	M1
	Obtain two correct equations in x and y, e.g. $3x - y = 1$ and $3y - x = 5$	A1
	Solve and obtain answer $z = 1 + $ **2(i)**	A1
	Total:	**4**
(b)	Show a circle with radius 3	B1
	Show the line $y = 2$ extending in both quadrants	B1
	Shade the correct region	B1
	Carry out a complete method for finding the greatest value of arg z	M1
	Obtain answer 2.41	A1
	Total:	**5**

Question 32

(i)	*EITHER*: Multiply numerator and denominator by $1 + 2i$, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	M1	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}i$, or equivalent	A1	
	Use correct method to find either r or θ	M1	
	Obtain $r = 1.61$	A1	
	Obtain $\theta = 2.09$	A1	
	OR: Find modulus or argument of $2 + 3i$ or of $1 - 2i$	B1	
	Use correct method to find r	M1	
	Obtain $r = 1.61$	A1	
	Use correct method to find θ	M1	
	Obtain $\theta = 2.09$	A1	
		5	
(ii)	Show a circle with centre $3 - 2i$	B1	
	Show a circle with radius 1	B1ft	Centre not at the origin
	Carry out a correct method for finding the least value of $\lvert z \rvert$	M1	
	Obtain answer $\sqrt{13} - 1$	A1	
		4	

Question 33

(a)(i)	Multiply numerator and denominator by $1 + 2i$, or equivalent	M1	Requires at least one of $2+10i+12i^2$ and $1-4i^2$ together with use of $i^2 = -1$. Can be implied by $\frac{-10+10i}{5}$
	Obtain quotient $-2 + 2i$	A1	
	Alternative		
	Equate to $x + iy$, obtain two equations in x and y and solve for x or for y	M1	$x+2y=2, \quad y-2x=6$
	Obtain quotient $-2 + 2i$	A1	
		2	
(a)(ii)	Use correct method to find either r or θ	M1	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r = 2\sqrt{2}$, or exact equivalent	A1ft	ft their $x + iy$
	Obtain $\theta = \frac{3}{4}\pi$ from exact work	A1ft	ft on $k(-1 + i)$ for $k > 0$ Do not ISW
		3	

(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and shade the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z = \frac{\pi}{2} + \sin^{-1}\frac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

Question 34

(a)	Use quadratic formula to solve for z	**M1**
	Use $i^2 = -1$ throughout	**M1**
	Obtain correct answer in any form	**A1**
	Multiply numerator and denominator by $1 - i$, or equivalent	**M1**
	Obtain final answer, e.g. $1 - i$	**A1**
	Obtain second final answer, e.g. $\frac{5}{2} + \frac{1}{2}i$	**A1**
		6
(b)	Show the point representing u in relatively correct position	**B1**
	Show the horizontal line through $z = i$	**B1**
	Show correct half-lines from u, one of gradient 1 and the other vertical	**B1ft**
	Shade the correct region	**B1**
		4

Question 35

(i)	State or imply $r = 2$	B1
	State or imply $\theta = \frac{1}{6}\pi$	B1
	Use a correct method for finding the modulus or the argument of u^4	M1
	Obtain modulus 16	A1
	Obtain argument $\frac{2}{3}\pi$	A1
		5
(ii)	Substitute u and carry out a correct method for finding u^3	M1
	Verify u is a root of the given equation	A1
	State that the other root is $\sqrt{3} - \mathrm{i}$	B1
	Alternative method	
	State that the other root is $\sqrt{3} - \mathrm{i}$	B1
	Form quadratic factor and divide cubic by quadratic	M1
	Verify that remainder is zero and hence that u is a root of the given equation	A1
		3

(iii)	Show the point representing u in a relatively correct position	B1
	Show a circle with centre u and radius 2	B1
	Show the line $y = 2$	B1
	Shade the correct region	B1
	Show that the line and circle intersect on $x = 0$	B1
		5

Question 36

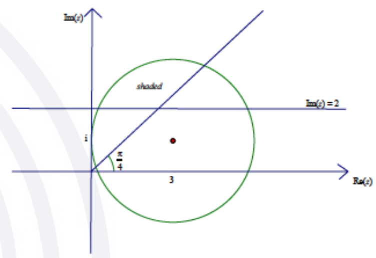
(i)	State answer $-1-\sqrt{3}\mathrm{i}$	B1
		1

(ii)	Substitute $x = -1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1
	Obtain $k = 2$	A1
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	M1
	Obtain $x^2 + 2x + 4$	A1
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	A1

Question 37

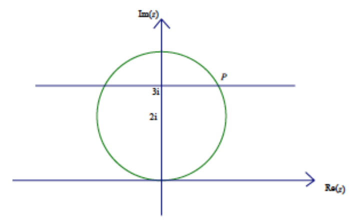
(i)	Multiply numerator and denominator by $1+\sqrt{3}\mathrm{i}$, or equivalent	M1
	$4\mathrm{i} - 4\sqrt{3}$ and $3+1$	A1
	Obtain final answer $-\sqrt{3}+\mathrm{i}$	A1
		3
(ii)	State that the modulus of u is 2	B1
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	B1
		2
(iii)	Show a circle with centre the origin and radius 2	B1
	Show u in a relatively correct position	B1
	Show the perpendicular bisector of the line joining u and the origin	B1
	Shade the correct region	B1
		4

Question 38

(a)	Square $a + ib$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2} - \sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
(b)	Show point representing 3 + i in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	
		5	

Question 39

(a)	Substitute and obtain a correct horizontal equation in x and y in any form	B1
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero OE	*M1
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	A1
	Solve for x or for y	DM1
	Obtain answer $\frac{6}{5} - \frac{3}{5}\mathrm{i}$ and no other	A1
		5

(b)(i)	Show a circle with centre 2i and radius 2	B1	
	Show horizontal line $y = 3$ – in first and second quadrant	B1	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	M1	$\left(z = \sqrt{3} + 3\text{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or 60°)	A1	**SC:** Allow B2 for 60° with no working
		2	

Question 40

(i)	Obtain answer $w = \frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}$	**B1**
		1
(ii)	Show point representing u	**B1**
	Show point representing v in relatively correct position	**B1**
		2
(iii)	Explain why the moduli are equal	**B1**
	Explain why the arguments are equal	**B1**
	Use $\text{i}^2 = -1$ and obtain $2uw$ in the given form	**M1**
	Obtain answer $1 - 2\sqrt{3} + \left(2 + \sqrt{3}\right)\text{i}$	**A1**
		4

Question 41

(a)	Solve for v or w	M1
	Use $i^2 = -1$	M1
	Obtain $v = -\frac{2i}{1+i}$ or $w = \frac{5+7i}{-1+i}$	A1
	Multiply numerator and denominator by the conjugate of the denominator	M1
	Obtain $v = -1 - i$	A1
	Obtain $w = 1 - 6i$	A1
		6
(b)(i)	Show a circle with centre $2 + 3i$	B1
	Show a circle with radius 1 and centre not at the origin	B1
		2
(b)(ii)	Carry out a complete method for finding the least value of arg z	M1
	Obtain answer $40.2°$ or 0.702 radians	A1
		2

Question 42

(a)(i)	Multiply numerator and denominator by $a - 2i$, or equivalent	M1
	Use $i^2 = -1$ at least once	A1
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	A1
		3
(a)(ii)	Either state that arg $u = -\frac{1}{3}\pi$ or express u^* in terms of a (FT on u)	B1
	Use correct method to form an equation in a	M1
	Obtain answer $a = -2\sqrt{3}$	A1
		3

b)(i)	Show the perpendicular bisector of points representing $2i$ and $1+i$	**B1**
	Show the point representing $2+i$	**B1**
	Show a circle with radius 2 and centre $2+i$ (FT on the position of the point for $2+i$)	**B1FT**
	Shade the correct region	**B1**
		4
b)(ii)	State or imply the critical point $2+3i$	**B1**
	Obtain answer 56.3° or 0.983 radians	**B1**
		2

Question 43

(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2=-1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $x-y=3$ and $3x+y=5$	**A1**
	Solve and obtain answer $z=2-i$	**A1**
		4

(b)(i)	Show a point representing $2+2i$	**B1**
	Show a circle with radius 1 and centre not at the origin (**FT** is on the point representing the centre)	**B1 FT**
	Show the correct half line from $4i$	**B1**
	Shade the correct region	**B1**
		4
b)(ii)	Carry out a complete method for finding the least value of Im z	**M1**
	Obtain answer $2-\frac{1}{2}\sqrt{2}$, or exact equivalent	**A1**
		2

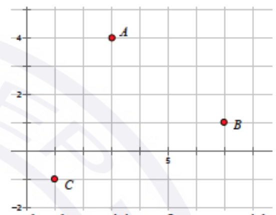
Question 44

(a)	Eliminate u or w and obtain an equation w or u	**M1**
	Obtain a quadratic in u or w, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for u or for w	**M1**
	Obtain answer $u = \sqrt{5} + \mathrm{i}$, $w = \sqrt{5} - \mathrm{i}$	**A1**
	Obtain answer $u = -\sqrt{5} + \mathrm{i}$, $w = -\sqrt{5} - \mathrm{i}$	**A1**
		5
(b)	Show the point representing $2 + 2\mathrm{i}$	**B1**
	Show a circle with centre $2 + 2\mathrm{i}$ and radius 2 (**FT** is on the position of $2 + 2\mathrm{i}$)	**B1 FT**
	Show half-line from origin at 45° to the positive x-axis	**B1**
	Show line for $\mathrm{Re}\, z = 3$	**B1**
	Shade the correct region	**B1**
		5

Question 45

'(a)	Substitute $-1 + \sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1 - 2\sqrt{5}\mathrm{i} - 5$ or $1 - \sqrt{5}\mathrm{i} - \sqrt{5}\mathrm{i} - 5$ and $4 - 2\sqrt{5}\mathrm{i} + 10$ or $4 - 4\sqrt{5}\mathrm{i} + 2\sqrt{5}\mathrm{i} + 10$
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	$1 - 5$ or $4 + 10$ seen
	Complete the verification correctly	**A1**	$2\left(14 - 2\sqrt{5}\mathrm{i}\right) + \left(-4 - 2\sqrt{5}\mathrm{i}\right) + 6\left(-1 + \sqrt{5}\mathrm{i}\right) - 18 = 0$
		3	
'(b)	State second root $-1 - \sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1 + \sqrt{5}\mathrm{i}$ and $-1 - \sqrt{5}\mathrm{i}$	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1 - \sqrt{5}\mathrm{i}$	**B1**	
	$(x + 1 - \sqrt{5}\mathrm{i})(x + 1 + \sqrt{5}\mathrm{i})(2x + a) = 2x^3 + x^2 + 6x - 18$	**M1**	
	$(1 - \sqrt{5}\,\mathrm{i})(1 + \sqrt{5}\,\mathrm{i})\,a = -18$	**A1**	
	$6a = -18$ $a = -3$ leading to $x = \frac{3}{2}$	**A1**	OE

Question 46

(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy) = 7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
		3	
(b)	Show the point A representing u in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points B and C in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of A as a guide to 'scale' if axes not marked
		2	
(c)	State or imply $\arg(1-i) = -\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in $\arg(7+i) - \arg(1-i) = \arg u$	**M1**	Must see a statement about the relationship between the Args e.g. $\text{Arg}A = \text{Arg}B - \text{Arg}C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u = k(3+4i)$
		3	

Question 47

(a)	Substitute $-1+\sqrt{5}\,i$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}i-5$ or $1-\sqrt{5}i-\sqrt{5}i-5$ and $4-2\sqrt{5}i+10$ or $4-4\sqrt{5}i+2\sqrt{5}i+10$
	Use $i^2 = -1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}i\right)+\left(-4-2\sqrt{5}i\right)+6\left(-1+\sqrt{5}i\right)-18=0$
		3	
(b)	State second root $-1-\sqrt{5}i$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}i$ and $-1-\sqrt{5}i$	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE

Question 48

(a)	Multiply numerator and denominator by 3 – i	**M1**
	Obtain numerator – 10 + 10i or denominator 10	**A1**
	Obtain final answer – 1 + i	**A1**
		3
(b)	State or imply $r = \sqrt{2}$	**B1 FT**
	State or imply that $\theta = \frac{3}{4}\pi$	**B1 FT**
		2
(c)	State that *OA* and *BC* are parallel	**B1**
	State that $BC = 2OA$	**B1**
		2
(d)	Use angle $AOB = \arg u - \arg v = \arg\frac{u}{v}$	**M1**
	Obtain the given answer	**A1**
	Alternative method for question 8(d)	
	Obtain tan *AOB* from gradients of *OA* and *OB* and the $\tan(A \pm B)$ formula	**M1**
	Obtain the given answer	**A1**
	Alternative method for question 8(d)	
	Obtain $\cos AOB$ by using the cosine rule or a scalar product	**M1**
	Obtain the given answer	**A1**
		2

Question 49

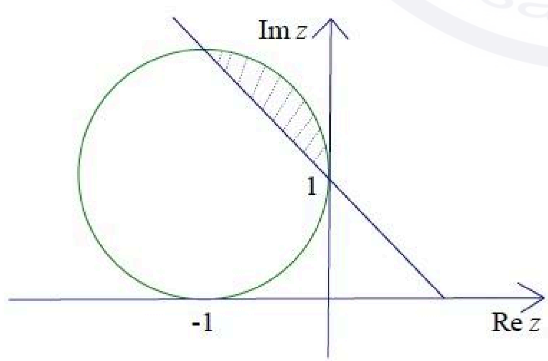
(a)	Substitute $-1+\sqrt{2}\mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**
	Use $\mathrm{i}^2=-1$ at least once	**M1**
	Complete the verification correctly	**A1**
		3

(b)	State second root $-1-\sqrt{2}\mathrm{i}$	**A1**
	Carry out a method to find a quadratic factor with zeros $-1\pm\sqrt{2}\mathrm{i}$	**M1**
	Obtain z^2+2z+3	**A1**
	Commence division and reach partial quotient z^2+kz	**M1**
	Obtain second quadratic factor z^2-2z+4	**A1**
	Solve a 3-term quadratic and use $\mathrm{i}^2=-1$	**M1**
	Obtain roots $1+\sqrt{3}\mathrm{i}$ and $1-\sqrt{3}\mathrm{i}$	**A1**
		7

Question 50

Square $a + \mathrm{i}b$, use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to 10 and $-4\sqrt{6}$ respectively	**M1**
Obtain $a^2 - b^2 = 10$ and $2ab = -4\sqrt{6}$	**A1**
Eliminate one unknown and find an equation in the other	**M1**
Obtain $a^4 - 10a^2 - 24[=0]$, or $b^4 + 10b^2 - 24[=0]$, or 3-term equivalent	**A1**
Obtain final answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	**A1**

Question 51

Show a circle with centre – 1 + i.	**B1**
Show a circle with radius 1 and centre not at the origin (or relevant part thereof).	**B1**
Show correct half line from 1(or relevant part thereof) .	**B1**
Shade the correct region on a correct diagram.	**B1**
	**4**

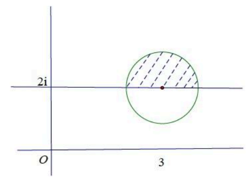
Question 52

(a)	Use quadratic formula and $i^2 = -1$	**M1**
	Obtain answers $pi + \sqrt{q - p^2}$ and $pi - \sqrt{q - p^2}$	**A1**
		2
(b)	State or imply that the discriminant must be negative	**M1**
	State condition $q < p^2$	**A1**
		2
(c)	Carry out a correct method for finding a relation, e.g. use the fact that the argument of one of the roots is $(\pm)60°$	**M1**
	State a correct relation in any form, e.g. $\dfrac{p}{\sqrt{q-p^2}} = (\pm)\sqrt{3}$	**A1**
	Simplify to $q = \dfrac{4}{3}p^2$	**A1**

Question 53

(a)	State or imply $r = 2$	**B1**	
	State or imply $\theta = \dfrac{5}{6}\pi$	**B1**	
		2	
(b)	Use a correct method for finding the modulus or argument of u^6	**M1**	
	Show correctly that u^6 is real and has value -64	**A1**	
		2	
(c)(i)	Show half lines from the point representing $-\sqrt{3} + i$	**B1**	
	Show correct half lines	**B1**	
	Show the line $x = 2$ in the first quadrant	**B1**	
	Shade the correct region	**B1**	
		4	
(c)(ii)	Carry out a correct method to find the greatest value of $\lvert z \rvert$	**M1**	
	Obtain answer 5.14	**A1**	
		2	

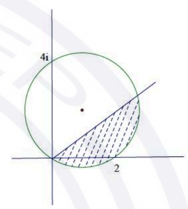
Question 54

(a)	Show circle with centre 3 + 2i	**B1**	
	Show circle with radius 1. Must match *their* scales: if scales not identical should have an ellipse.	**B1**	
	Show line $y = 2$ in at least the diameter of a circle in the first quadrant	**B1**	
	Shade the correct region in a correct diagram	**B1**	
		4	
(b)	Identify the correct point	**B1**	
	Carry out a correct method for finding the argument	**M1**	e.g. $\arg x = \tan^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{\sqrt{13}}$ Exact working required.
	Obtain answer $49.8°$	**A1**	Or better. 0.869 radians scores **B1M1A0**.
		3	**Special Case 1: B1M0** for $45°$ if they have shaded the wrong half of the circle. **Special Case 2:** 3 out of 3 available if they identify the correct point on the correct circle and it is consistent with *their* shading.

Question 55

(a)	Substitute for u and w and state correct conjugate of one side	**B1**	
	Express the other side without conjugates and confirm $(u+w)^* = u^* + w^*$	**B1**	**Given answer.** Needs explicit reference to conjugate of both sides.
		2	
(b)	Substitute and remove conjugates to obtain a correct equation in x and y	**B1**	e.g. $x + 2 - (y+1)\mathrm{i} + (2+\mathrm{i})(x+\mathrm{i}y) = 0$
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero	**M1**	
	Obtain two correct equations in x and y	**A1**	e.g. $3x - y + 2 = 0$ and $x + y - 1 = 0$. Allow $x\mathrm{i} + y\mathrm{i} - \mathrm{i} = 0$.
	Solve and obtain answer $z = -\frac{1}{4} + \frac{5}{4}\mathrm{i}$	**A1**	Allow for real and imaginary parts stated separately.
		4	

Question 56

(a)	Substitute 1 + 2i in the polynomial and attempt expansions of x^2 and x^3	**M1**	$u^2 = -3+4\text{i},\ u^3 = -11-2\text{i}$ Full substitution but need not simplify.
	Equate real and/or imaginary parts to zero	**M1**	$-18-3a+b=0,\ 4+4a=0$
	Obtain $a = -1$	**A1**	
	Obtain $b = 15$	**A1**	
		4	
(b)	State second root 1 – 2i	**B1**	
		1	
(c)	State the quadratic factor x^2-2x+5	**B1**	
	State the linear factor $2x+3$	**B1**	
		2	
(d)(i)	Show a circle with centre 1 + 2i	**B1**	
	Show circle passing through the origin	**B1**	
	Show the half line $y = x$ in the first quadrant (accept chord of circle)	**B1**	
	Shade the correct region on a correct diagram	**B1**	
		4	
(d)(ii)	State answer $2-\sqrt{5}$	**B1**	
		1	

Question 57

Substitute and obtain a correct equation in x and y	**B1**	$(x+\text{i}y)^2 + 2\text{i}(x-\text{i}y) = 1$
Use $\text{i}^2 = -1$ at least once and equate real and imaginary parts	**M1**	
Obtain two correct equations, e.g. $x^2-y^2+2y=1$ and $2xy+2x=0$	**A1**	
Solve for x or for y	**M1**	
Using $y = -1$, obtain answer $w = -2-\text{i}$ only	**A1**	A0 if $w = 2-\text{i}$ as well
Using $x = 0$, obtain answer $w = \text{i}$	**A1**	
	6	

Question 58

Show a circle with centre – 2 + 3i	**B1**	Must see (– 2, 3) or appropriate marks on axes
Show a circle of radius 2 and centre not at the origin.	**B1**	
Show correct **half line** from the origin	**B1**	$\frac{3\pi}{4}$ or $\frac{\pi}{4}$ seen, or half line that approximately bisects angle $\frac{\pi}{2}$.
Shade the correct region.	**B1**	
	4	N.B. Maximum 3 out of 4 if any errors seen.

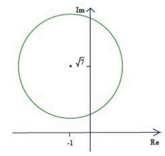
Question 59

(a)	Show u and u^* in relatively correct positions. Must have sense of scale on axes	**B1**	$u = 3 - \text{i}$, $u^* = 3 + \text{i}$ Ignore labels.
	Show $u^* - u$ in a relatively correct position. Must have sense of scale on axes	**B1**	2i. Scale only on Imaginary axis is sufficient for this mark.
	State that *OABC* is a parallelogram [independent of previous marks]	**B1**	Ignore 'quadrilateral'. Allow 'trapezium' from correct work.
		3	
(b)	Multiply *their* numerator and the given denominator by 3 + i and attempt to evaluate either	**M1**	Can have missing term and arithmetic errors but need $\text{i}^2 = -1$ once, seen or implied.
	Obtain numerator 8 + 6i or denominator 10	**A1**	
	State final answer $\frac{4}{5}+\frac{3}{5}\text{i}$ or $\frac{8}{10}+\frac{6}{10}\text{i}$ or 0.8 + 0.6i	**A1**	Correct answer with no working scores 0/3.
	Alternative method for question 5(b)		
	Obtain two equations in x and y, and attempt to solve for x or for y	**M1**	$3 = 3x + y$ and $1 = -x + 3y$
	Obtain $x = \frac{4}{5}$ or $\frac{8}{10}$ or 0.8 $y = \frac{3}{5}$ or $\frac{6}{10}$ or 0.6	**A1**	
	State final answer $\frac{4}{5}+\frac{3}{5}\text{i}$ or $\frac{8}{10}+\frac{6}{10}\text{i}$ or 0.8 + 0.6i	**A1**	Correct answer with no working scores 0/3.
		3	

(c)	State or imply $\arg \frac{u^*}{u} = \arg u^* - \arg u$ or $2\arg u^*$	M1	
	Justify the given statement correctly	A1	AG $\arg \frac{u^*}{u} = \tan^{-1}\frac{3}{4}$, $\arg u^* = \tan^{-1}\frac{1}{3}$ and $\arg u = \tan^{-1} - \frac{1}{3}$ (or $\arg u = -\tan^{-1}\frac{1}{3}$), needed if use first expression in M1; or $\arg \frac{u^*}{u} = \tan^{-1}\frac{3}{4}$ and $\arg u^* = \tan^{-1}\frac{1}{3}$, needed if use second expression in M1.
	Alternative method for question 5(c)		
	Use $\tan 2A$ formula with $\tan A = \frac{1}{3}$	M1	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, $\tan A = \frac{1}{3}$, hence $\tan 2A = \frac{3}{4}$.
	Justify the given statement correctly	A1	AG So $2A = \tan^{-1}\frac{3}{4} = \arg\frac{u^*}{u}$ and $A = \tan^{-1}\frac{1}{3} = \arg u^*$ hence $\arg\frac{u^*}{u} = 2\arg u^*$.
		2	

Question 60

(a)	Substitute $x = -1+\sqrt{7}\text{i}$ in the equation and attempt expansions of x^2 and x^3	*M1	
	Use $\text{i}^2 = -1$ correctly at least once and solve for k	DM1	$2\left(20-4\sqrt{7}\text{i}\right)+3\left(-6-2\sqrt{7}\text{i}\right)+14\left(-1+\sqrt{7}\text{i}\right)+k=0$
	Obtain answer $k = -8$	A1	
			SC B1 only for those who show no working for the cube and square and obtain answer $k = -8$.
(b)	State answer $-1-\sqrt{7}\text{i}$	B1	Can be seen simply stated on its own, or in a list of roots. Allow if stated clearly in part **10(a)**.
	Carry out a method for finding a quadratic factor with zeros $-1+\sqrt{7}\text{i}$ and $-1-\sqrt{7}\text{i}$	M1	Or state $\left(x-\left(-1+\sqrt{7}\text{i}\right)\right)\left(x-\left(-1-\sqrt{7}\text{i}\right)\right)(2x-p)$
	Obtain x^2+2x+8	A1	Or obtain $\left(-1+\sqrt{7}\text{i}\right)\left(-1-\sqrt{7}\text{i}\right)p=-8$ Or obtain $\left(-1+\sqrt{7}\text{i}\right)+\left(-1-\sqrt{7}\text{i}\right)+\frac{p}{2}=-\frac{3}{2}$
	Obtain root $x = \frac{1}{2}$, or equivalent, via division or inspection	A1	Needs to follow from the working.
		4	

(c)	Show a circle with centre $-1+\sqrt{7}\mathrm{i}$	**B1**	
	Show circle with radius 2 and centre not at the origin There needs to be some evidence of scale e.g. radius marked or a scale on the axes	**B1**	If the scales are very different from each other then B1 for centre in the correct position and B1 for an ellipse. If there is more than one circle the max score is B1.
		2	
(d)	Carry out a complete method for calculating the maximum value of arg z for correct circle	**M1**	e.g. $\frac{\pi}{2}+\tan^{-1}\frac{1}{\sqrt{7}}+\frac{\pi}{4}$ Can be implied by $155.7°$.
	Obtain answer 2.72 radians	**A1**	CAO. The question requires radians.
		2	

Question 61

(a)	Multiply numerator **and** denominator by 1 – 2i, or equivalent	**M1**	At least one multiplication completed.
	Obtain correct numerator $(1-2a)\sqrt{2}-(2+a)\sqrt{2}\mathrm{i}$	**A1**	OE
	Obtain final answer $\frac{1-2a}{5}\sqrt{2}-\frac{2+a}{5}\sqrt{2}\mathrm{i}$	**A1**	OE
(b)	Obtain $r=2$	**B1 FT**	
	Obtain $\theta=-\frac{3}{4}\pi$	**B1**	
		2	
(c)	Use correct method to find r or θ	**M1**	
	State answer $\sqrt{2}e^{-\frac{3}{8}\pi\mathrm{i}}$	**A1 FT**	
	State answer $\sqrt{2}e^{\frac{5}{8}\pi\mathrm{i}}$	**A1 FT**	
		3	

Question 62

(a)	Show a circle with centre –2	**B1**	
	Show a circle with radius 2 and centre not the origin	**B1**	
	Show the line $y = 1$	**B1**	
	Shade the correct region	**B1**	
		4	
(b)	Identify the correct point and carry out a correct method to find the argument	**M1**	
	Obtain answer $\frac{11}{12}\pi$	**A1**	2.88 radians or 165°.
		2	

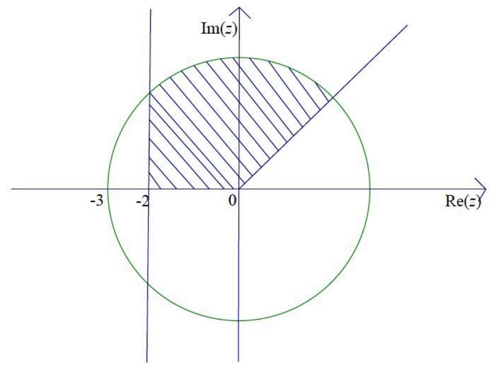
Question 63

Use quadratic formula to solve for z	**M1**	**SC M1**: For substitution of $x + iy$ and multiplying out.
Use $i^2 = -1$ throughout	**M1**	**SC M1**: Use $i^2 = -1$ throughout.
Obtain correct answer in any form	**A1**	**SC A1**: For two correct equations $x^2 - y^2 + 6xy - 2x + y = 0$ and $-3(x^2 - y^2) + 2xy - x - 2y + 1 = 0$.
Multiply numerator and denominator by (1 + 3i), or equivalent	**M1**	
Obtain final answer, e.g. $-\frac{1}{2} + \frac{1}{2}i$	**A1**	
Obtain second final answer, e.g. $\frac{2}{5} + \frac{1}{5}i$	**A1**	
	6	

Question 64

(a)	Use quadratic formula, or completing the square $\left((z-3i)^2 - 3 = 0\right)$ and use $i^2 = -1$ to find a root	**M1**	
	Obtain a root, e.g. $\sqrt{3} + 3i$	**A1**	Or exact 2 term equivalent e.g. $\frac{6i}{2} + \frac{\sqrt{12}}{2}$ ISW.
	Obtain the other root, e.g. $-\sqrt{3} + 3i$	**A1**	Or exact 2 term equivalent ISW.
		3	
(b)	Show points representing the roots correctly	**B1 FT**	2 roots consistent with *their* **(a)** and with no errors seen on the diagram. B0 if they only have one root or more than 2 roots Must match their scale and $1 < \sqrt{3} < 2$ Linear scales seen or implied. Need some indication of scale (numbers or dashes). Scales along an axis must be approximately consistent but scales may be different on the 2 axes.
		1	
(c)	State modulus of either root is $2\sqrt{3}$, or simplified exact equivalent	**B1 FT**	ISW if converted to decimal . Ignore modulus of second root if seen. Follow their root(s) not on either axis (from **(a)** or **(b)**).
	Find the argument of one of their roots – get as far as $\tan^{-1}(...)$	**M1**	SOI but must be correct for their root.
	Obtain correct arguments $\frac{1}{3}\pi$ and $\frac{2}{3}\pi$, or simplified exact equivalents	**A1**	Must obtain values. Allow degrees.
		3	
(d)	Give a **complete justification** that the **correct triangle** is equilateral	**B1**	Check *their* diagram in **(b)**. Possible justifications: 3 equal sides, or all angles equal to $\frac{\pi}{3}$, or isosceles and an angle of $\frac{\pi}{3}$.
		1	

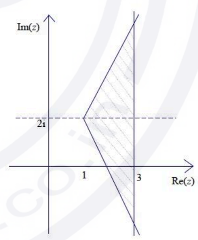
Question 65

Show a circle with radius 3 and centre the origin	**B1**	
Show the line $x = -2$	**B1**	
Show the correct half line for $\frac{\pi}{4}$	**B1**	
Shade the correct region	**B1**	For the vertical line and the circle, allow the B1 marks if all you see is the relevant part.
	4	

Question 66

(a)	State or imply $u^2 = 4e^{\frac{1}{2}\pi i}$	**B1**	
	Obtain answer $v = \frac{4}{3}e^{\frac{1}{6}\pi i}$	**B1 + B1**	For the modulus and the argument.
		3	
(b)	State $n = 6$	**B1**	
		1	

Question 67

(a)	Show correct half-lines from 1 + 2i, symmetrical about $y = 2i$ (drawn between $\frac{\pi}{4}$ and $\frac{5\pi}{12}$).	**B1**	
	Show the line $x = 3$ extending in both quadrants.	**B1**	
	Shade the correct region. Allow dashes on axes as scale. FT If only error is one of following: FULL lines or $x \neq 3$ or one sign error in 1 + 2i or angle outside tolerance or scale missing on one axis.	**B1 FT**	
			SC No scale on either axis allow **B1 FT** for otherwise correct figure in correct position.
		3	
(b)	Carry out a complete method for finding the least value of arg z	**M1**	e.g. $-\tan^{-1}\frac{(2\sqrt{3}-2)}{3}$ or $\tan^{-1}\frac{(-2\sqrt{3}+2)}{3}$.
	Obtain answer –0.454 3dp	**A1**	**SC B1** 0.454 .
		2	

Question 68

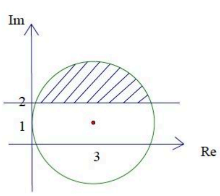
Substitute $z = x + iy$ and $z^* = x - iy$ to obtain a correct equation, horizontal or with $(1-2i)/(1-2i)$ seen, in x and y	**B1**	$5(x + iy) - (x + iy)(x - iy)(1 + 2i) + (30 + 10i)(1 + 2i) = 0$ $5(x + iy)(1 - 2i)/[(1 + 2i)(1 - 2i)] - (x + iy)(x - iy) + (30 + 10i) = 0$ $x - 2ix + iy + 2y - x^2 - y^2 + 30 + 10i = 0$.
Use $i^2 = -1$ at least once and equate real and imaginary parts to zero	***M1**	OE For their horizontal equation.
Obtain two correct equations e.g. $x + 2y - x^2 - y^2 + 30 = 0$ and $-2x + y + 10 = 0$	**A1**	$5x - (x^2 + y^2) + 10 = 0$ $5y - 2(x^2 + y^2) + 70 = 0$ $5y - 10x + 50 = 0$ $x + 2y - (x^2 + y^2) + 30 = 0$ Allow $-2ix + iy + 10i = 0$.
Solve quadratic equation for x or for y	**DM1**	$x^2 - 9x + 18 = (x - 3)(x - 6) = 0$ $y^2 + 2y - 8 = (y + 4)(y - 2) = 0$ **DM0** If x or y imaginary.
Obtain answers $3 - 4i$ and $6 + 2i$	**A1**	
	5	

Question 69

(a)	Multiply numerator and denominator by $(3 - ai)$	**M1**	Must perform complete multiplications but need not simplify i^2. Can have errors but no term duplicated or missing. $\dfrac{(5a-2i)(3-ai)}{9-a^2} = \dfrac{13a - i(5a^2+6)}{9-a^2}$ M0 M1 A0 No working so unsure if denominator multiplied by $3 - ai$ M1 M1 A0
	Use $i^2 = -1$ at least once and separate real and imaginary parts	**M1**	
	Obtain $\dfrac{13a - i(5a^2+6)}{9+a^2}$ or $\dfrac{13a - 5a^2 i - 6i}{9+a^2}$	**A1**	OE If $15a - 2a = 13a$ seen later award this A1.
	Use $\arg z$ to form equation in a $-\dfrac{5a^2+6}{13a} = \pm\tan\left(\pm\dfrac{\pi}{4}\right)$ or $-\dfrac{13a}{5a^2+6} = \pm\tan\left(\pm\dfrac{\pi}{4}\right)$ or $\tan^{-1}\left(-\dfrac{5a^2+6}{13a}\right) = \pm\dfrac{\pi}{4}$ or $\tan^{-1}\left(-\dfrac{13a}{5a^2+6}\right) = \pm\dfrac{\pi}{4}$	**M1**	Allow expression given in answer column or $5a^2 + 6 = \pm 13a$ or use $-(x \pm xi) = (13a - i(5a^2 + 6))/(9 + a^2)$ and eliminate x so $5a^2 + 6 = \pm 13a$ M1.
	Obtain $a = 2$	**A1**	Need to reject $a = \dfrac{3}{5}$ or ignore it in future work. May not see second root, but if present, must be $\dfrac{3}{5}$.
	Obtain $z = 2 - 2i$ only	**A1**	Allow $z = -2i + 2$.

(b)	State $\arg(z^3) = -\frac{3}{4}\pi$ or evaluate from $z = b - b\text{i}$ or from $-2b^3(1 + \text{i})$	**B1**	If 2 different values given award B0. Do not ISW.
	Complete method to obtain r from *their* z	**M1**	$\lvert z^3 \rvert = \left(\sqrt{x^2 + y^2}\right)^3$. If z correct, may see $\lvert z^3 \rvert = \left(\sqrt{2^2 + (-2)^2}\right)^3$ or $\lvert z^3 \rvert = \sqrt{(-16)^2 + (-16)^2}$.
	$r = 16\sqrt{2}$	**A1**	CAO A1 if $z = 2 - 2\text{i}$ obtained correctly. or $z =$ used with $a = 2$ found correctly, otherwise A0XP. May see arg and r given in a final answer i.e. $16\sqrt{2}e^{-\frac{3}{4}\pi\text{i}}$. Allow this form for arg and r to collect full marks, even if i missing. Ignore answers outside the given interval. If 2 different values given award A0.
		3	

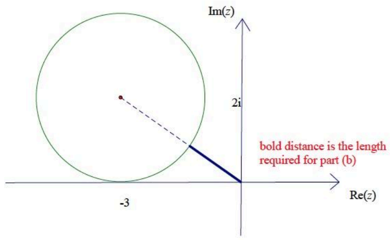
Question 70

Show a circle with centre $3 + \text{i}$	**B1**	Must be some evidence of scale on both axes or centre stated as $3 + \text{i}$ or $(3, 1)$.
Show a circle with radius 3 and centre not at the origin	**B1**	Must be some evidence that radius = 3 or stated $r = 3$
Show the line $y = 2$	**B1**	Line $y = 2$ can be represented by 2 or correct dashes.
Shade the correct region	**B1**	Line and circle must be correct.
	**4**	Scales may be replaced by dashes on axes for all marks. Correct figure, with no scale on either axis then allow 1/3 and the B1 for correct shaded region Max 2/4. If B0 above for line but relatively correct position then B1 for correct shaded region Max 3/4. Re and Im axes interchanged but clearly labelled, allow **SCB1** for centre and radius of circle correct and **SCB1** for line and shading correct Max 2/4.

Question 71

(a)	Substitute $2 + y\text{i}$ in $a^3 - a^2 - 2a$ and attempt expansions of a^2 and a^3	**M1**	$a^2 = 4 + 4y\text{i} - y^2 \quad a^3 = 8 + 12y\text{i} - 6y^2 - y^3\text{i}$. If using $a(a^2 - a - 2)$ must then expand fully. Must see working.
	Use $\text{i}^2 = -1$	**M1**	Seen at least once (e.g. in squaring).
	Obtain **final** answer $-5y^2 + (6y - y^3)\text{i}$	**A1**	Or simplified equivalent e.g. $6y\text{i} - 5y^2 - y^3\text{i}$. Do not ISW.
		3	No evidence of working for the square or the cube can score SC B1 for the correct answer.
(b)	Equate *their* $-5y^2$ to -20 and solve for y	**M1**	Need to obtain a value for y. Available even if *their* y is not real.
	Obtain $y = -2$	**A1**	From correct work. Allow after incorrect f(a) if the real part was correct. Condone ± 2 with positive not rejected.
	Obtain **final** answer $\arg a = -\frac{\pi}{4}$	**A1**	Correct only (must have rejected y positive). OE e.g. $-\frac{\pi}{4} \pm 2n\pi$. Accept -0.785, 5.50. Allow after incorrect f(a) if the real part was correct. Accept degrees. Do not ISW.
		3	

Question 72

(a)		**B1**	Show a circle with centre –3 + 2i. Allow for a curved figure with 'centre' in roughly the correct position. Accept marks or numbers on axes, coordinates of centre shown. B0B1 available for axes the wrong way round (and M1 A1 in part **(b)**).
	Show a circle with radius 2	**B1 FT**	FT centre not at the origin. Allow 'near miss' on x axis. Different scales on axes require an ellipse for B1 B1. Scales on the axes and any label of the radius must be consistent for B1 B1. Correct circle shaded scores B1 B0.
		2	
(b)	Carry out a correct method for finding the least value of $\lvert z \rvert$	**M1**	e.g. distance of centre from origin – radius or find point of intersection of circle and $3y = -2x$ and use Pythagoras. If they subtract the wrong way round M0. If their diagram is a reflection or a rotation of the correct diagram, M1 A1 is available (requires equivalent work). Any other circle M0.
	Obtain answer $\sqrt{13} - 2$ or $\sqrt{17 - 4\sqrt{13}}$	**A1**	Or exact equivalent e.g. $\sqrt{17 - \frac{26}{3}\sqrt{\frac{36}{13}}}$. Correct solution only. Allow A1 if exact answer seen and then decimal given.
		2	

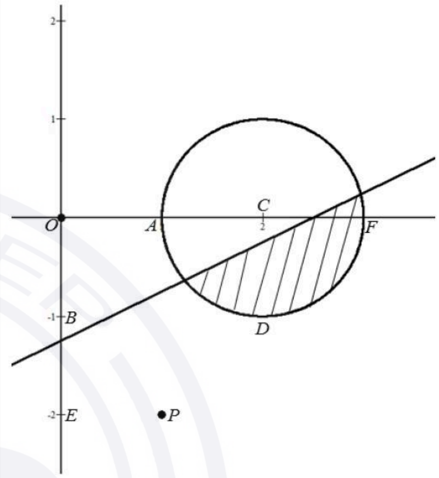
Question 73

(a)	Substitute $x=-3$ to obtain value of $p(-3)$	**M1**	
	Obtain $p(-3)=0$ and hence given result	**A1**	
	Alternative method for Question 10(a)		
	Divide $p(x)$ by $(x+3)$ to obtain quotient $x^2 \pm 2x + ...$	**M1**	
	Obtain quotient $x^2+2x+25$, with zero remainder and hence given result	**A1**	
		2	
(b)	Substitute $z=-1+2\sqrt{6}i$ and attempt expansions of z^2 and z^3	**M1**	$z^2=-23-4\sqrt{6}i$, $z^3=-1+6\sqrt{6}i+72-48\sqrt{6}i$.
	Use $i^2=-1$	**M1**	Seen at least once.
	Obtain $p(z)=0$ and hence given result	**A1**	**SC B1** if there is no evidence of working for the square or the cube. Total 1/3.
	Alternative Method 1		
	Use roots $z=-1+2\sqrt{6}i$ to form quadratic factor	**M1**	$z^2+2z+25$.
	Divide $p(z)$ by *their* quadratic factor	**M1**	
	Obtain zero remainder and hence given result.	**A1**	
	Alternative Method 2		
	Set *their* quadratic factor from **(a)** equal to zero	**M1**	
	Solve for z	**M1**	Need to see method here as answer is given.
	Obtain $z=-1+2\sqrt{6}i$ (and $z=-1-2\sqrt{6}i$)	**A1**	
	Alternative Method 3		
	Substitute $z=-1+2\sqrt{6}i$ into *their* quadratic factor and attempt expansion of z^2	**M1**	
	Use $i^2=-1$	**M1**	
	Obtain 0 and hence given result	**A1**	
		3	
(c)	State $z_1=\sqrt{3}i$ and $z_2=-\sqrt{3}i$	**B1**	
	Expand $(x+iy)^2=-1+2\sqrt{6}i$ and compare real and imaginary parts	**M1**	Allow for use of $z^2=-1-2\sqrt{6}i$.
	Obtain $x^2-y^2=-1$ and $xy=\sqrt{6}$	**A1**	
	Solve to obtain x and y	**M1**	
	Obtain $z_3=\sqrt{2}+\sqrt{3}i$ and $z_4=-\sqrt{2}-\sqrt{3}i$	**A1**	
	Use $z^2=-1-2\sqrt{6}i$ to obtain z_5 and z_6	**M1**	Allow for use of $z^2=-1+2\sqrt{6}i$.
	Obtain $z_5=\sqrt{2}-\sqrt{3}i$ and $z_6=-\sqrt{2}+\sqrt{3}i$	**A1**	
		7	

Question 74

Obtain final answers $\frac{3}{5}+\frac{4}{5}\text{i}$ and $-\text{i}$ Accept $0.6 + 0.8\text{i}$ and $0 - \text{i}$	**A1**	**SC** Both correct final answers from $10w^2 - 2(3 - \text{i})w + (3 - \text{i})^2 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}, y = \frac{4}{5}$ or $x = 0, y = -1$. A0 for $\frac{3+4\text{i}}{5}$.
Alternative method for Question 4		
Substitute $w = x + \text{i}y$ and form equations for real and imaginary parts	**M1**	
Use $\text{i}^2 = -1$ in $(x + \text{i}y)^2$	**M1**	
Obtain $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$	**A1**	OE
Form quartic equation in x only or y only using the correct substitution and solve for x or y	**M1**	Use correct $y = \frac{(3-x)}{10x-3}$ to attempt to form and solve $50x^4 - 60x^3 + 63x^2 - 27x = 0$ $x(5x - 3)(10x^2 - 6x + 9) = 0$. Use correct $x = \frac{3(1+y)}{10y+1}$ to attempt to form and solve $100y^4 + 40y^3 - 66y^2 - 14y - 8 = 0$ $(y + 1)(5y - 4)(20y^2 + 4y + 2) = 0$.
Obtain final answers $\frac{3}{5}+\frac{4}{5}\text{i}$ and $-\text{i}$ Accept $0.6 + 0.8\text{i}$ and $0 - \text{i}$	**A1**	**SC** Both correct final answers from $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}, y = \frac{4}{5}$ or $x = 0, y = -1$. A0 for $\frac{3+4\text{i}}{5}$.
	5	

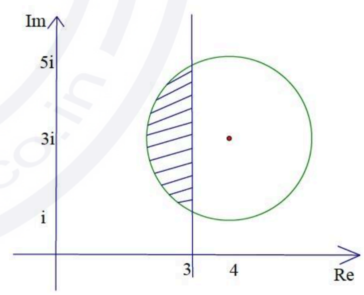
Question 75

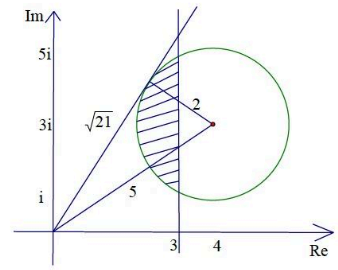
Show a circle centre (2, 0)	**B1**	
Show the relevant part of a circle with radius 1	**B1 FT**	FT centre not at the origin even if centre at 1 – 2i. Must clearly go through (1, 0) or (3, 0) (oe for FT mark).
Show the point representing 1 – 2i	**B1**	Can be implied by correct perpendicular bisector
Show the perpendicular bisector of the line joining 1 – 2i and the origin. Perpendicular to *OP* by eye and at midpoint of *OP* by eye sufficient. Must reach midpoint of *OP* and if extended will cut *BE*.	**B1 FT**	FT on the position of 1 – 2i.
Shade the correct region. Dependent on all previous marks, except in case 3 below, and the perpendicular must cut axes between *CF* and *BE*, but not actually through *C* or *F* and not through *B* or *E* Scale can be implied by dashes 1 Scale only on *y*-axis and 2*OA* = *OC* B1, B1FT, B1, B1FT, B1 2 Scale only on *x*-axis and 2*OB* = *OE* B1, B1FT, B1, B1FT, B1 3 No scale on either axis, but 2*OA* = *OC* B0, B1FT, B0, B1FT, B1 then 2*OB* = *OE*	**B1**	
	5	

Question 76

(a)	Multiply both sides by $a+2\mathrm{i}$ and attempt expansion of right-hand side	***M1**	
	Use of $\mathrm{i}^2=-1$ seen at least once (or implied)	**DM1**	e.g. $2+3a\mathrm{i}=\lambda(2a+2)+\lambda\mathrm{i}(-a+4)$
	Compare real and imaginary parts to obtain an equation in a only $\left[2=\lambda(2a+2),\quad 3a=\lambda(-a+4)\right]$	**M1**	e.g. $\frac{3a}{2}=\frac{-a+4}{2a+2}$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	**A1**	AG
	Alternative method for question 8(a)		
	Multiply top and bottom of the left-hand side by $a-2\mathrm{i}$ and attempt both expansions	***M1**	Do not need the right-hand side at this stage.
	Use of $\mathrm{i}^2=-1$ seen at least once or implied	**DM1**	e.g. $\left[\lambda(2-\mathrm{i})=\right]\frac{8a+\mathrm{i}(3a^2-4)}{a^2+4}$.
	Compare real and imaginary parts to obtain an equation in a only	**M1**	e.g. $8a=-2(3a^2-4)$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	**A1**	AG
		4	
(b)	Solve given quadratic to obtain a value of a and use this to form an equation in λ only (based on an equation seen in *their* working in **(a)** or **(b)**)	**M1**	Can be implied by relevant working seen or a correct value for λ seen.
	Obtain $a=-2,\ \lambda=-1$ or $a=\frac{2}{3},\ \lambda=\frac{3}{5}$	**A1**	Allow $\frac{6}{10}$ and 0.6.
	Obtain second correct pair of values	**A1**	
		3	

Question 77

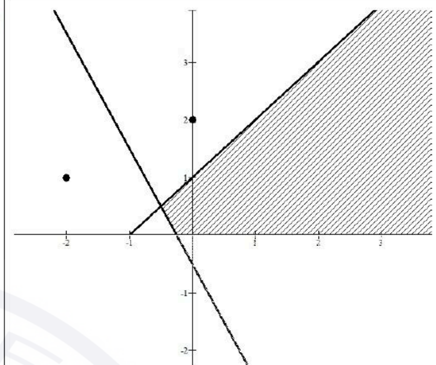
(a)	Show a circle with centre $4+3\mathrm{i}$. Accept a curved shape with correct point roughly in the middle.	**B1**	
	Show a circle with radius 2 and centre not at the origin. The shape should be consistent with their scales	**B1**	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	**B1**	
	Shade the correct region on a correct diagram Any other shading must be accompanied by words to explain which region is required	**B1**	Need some indication of scale e.g. label the centre, mark key points on the axes or dashes on the axes. Condone dotted lines in place of solid lines Condone correct shaded shape but not an entire circle
		4	

(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\tan^{-1}\frac{3}{4}+\sin^{-1}\frac{2}{5}$ $(0.6435+0.4115)$	**M1**	
	Obtain answer 1.06 (accept 1.055 or 1.056) radians or $60.45°$ (accept $60.4°$ or $60.5°$)	**A1**	
	Alternative method for question 4(b)		
	Tangent to circle passing through origin has equation $y=mx$. The equation $(x-4)^2+(y-3)^2=4$ will have one root. Hence $(1+m^2)x^2-(8+6m)x+21=0$, discriminant = 0 = $48m^2-96m+20$ and $m=\frac{6\pm\sqrt{21}}{6}$ with the larger value needed to give greatest arg(z). Required angle is $\tan^{-1}m$.	**M1**	Complete method for finding the greatest value of arg(z).
	Obtain answer 1.06 radians or $60.45°$	**A1**	Accept 1.055 or 1.056 radians. Accept $60.4°$ or $60.5°$.
		2	

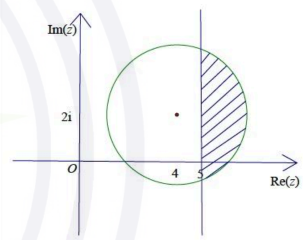
Question 78

(a)	Multiply numerator and denominator by $a+5\mathrm{i}$	**M1**	OE
	Use $\mathrm{i}^2=-1$	**M1**	At least once.
	Obtain answer $\frac{3a-10}{a^2+25}+\frac{2a+15}{a^2+25}\mathrm{i}$	**A1**	
	Alternative Method for Question 4(a)		
	Multiply $x+\mathrm{i}y$ by $a-5\mathrm{i}$ and use $\mathrm{i}^2=-1$	**M1**	
	Compare real and imaginary parts	**M1**	$3=ax+5y$, $2=ay-5x$.
	Obtain answer $\frac{3a-10}{a^2+25}+\frac{2a+15}{a^2+25}\mathrm{i}$	**A1**	
		3	
(b)	State or imply Im(**a**) ÷ Re(**a**) = 1	**M1**	Or Im(**a**) = Re(**a**) or equivalent for *their u*.
	Obtain answer $a=25$	**A1**	
		2	

Question 79

Show points representing 2i and –2 + i	**B1**	Can be implied if the correct perpendicular is drawn.
Show perpendicular bisector of *their* (2i and –2 + i)	**B1FT**	
Show correct half–line of gradient 1 from point (–1, 0)	**B1**	Should pass through (0, 1).
Correct loci and shade correct region	**B1**	
	4	

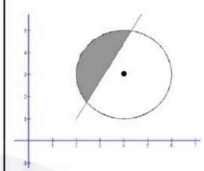
Question 80

(a)	Show a circle with centre $4+2\text{i}$	**B1**	
	Show a circle with radius 3 and centre not at the origin	**B1**	
	Show the straight line $\text{Re}(z)=5$	**B1**	
	Shade the correct region Allow even if radius 3 mark not gained or shown incorrectly	**B1**	
		4	If 4 and 6 seen on diagram and line is at mid point, but 5 not marked, allow final two B1 marks.
(b)	Carry out a complete method for finding the greatest value of arg z	**M1**	e.g. $\tan^{-1}\dfrac{2+2\sqrt{2}}{5}$. Allow $2\sqrt{2}$ as $\sqrt{(3^2-1^2)}$.
	Obtain answer 0.768 radians or 44.0°	**A1**	
		2	**SC B1** $\tan^{-1}(2/4)+\sin^{-1}(3/\sqrt{(4^2+2^2)})=26.565°+42.130°=68.695°$ 68.7° or [1.19896] 1.20 radians.

Question 81

(a)	Obtain $r=4$	**B1**	$\lvert z\rvert=\sqrt{\left(\left(-\sqrt{3}\right)^2+1^2\right)}$ so $r=\lvert z\rvert^2=\left(-\sqrt{3}\right)^2+1^2$.
	Correct method for the argument	**M1**	$\theta=2\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$ or $2\times\frac{5\pi}{6}$.
	Obtain $\theta=-\frac{\pi}{3}$	**A1**	Arg with no working **B1** instead of **M1 A1**. **A0** if decimals. Allow separate mod and arg to gain full marks
	Alternative solution for Question 3(a)		
	$z^2=2-2\sqrt{3}\,\mathrm{i}$ so $r=\sqrt{2^2+\left(-2\sqrt{3}\right)^2}=4$	**B1**	
	Correct method for the argument	**M1**	$\arg z^2=\tan^{-1}\frac{-2\sqrt{3}}{2}$
	Obtain $\theta=-\frac{\pi}{3}$	**A1**	Arg with no working **B1** instead of **M1 A1**. **A0** if decimals. Allow separate mod and arg to gain full marks
		3	
(b)	Use of $\alpha+\mathit{their}\,\theta=0$ or $\alpha+\mathit{their}\,\theta=-\pi$ or $\alpha+\mathit{their}\,\theta=\pi$	**M1**	Seen or implied. Using their θ or new value calculated in **(b)**.
	Use of $R=\frac{\mathit{their}\ \mathrm{r}}{12}$	**M1**	Seen or implied.
	Obtain $\frac{1}{3}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}$ and $\frac{1}{3}\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$	**A1**	
		3	

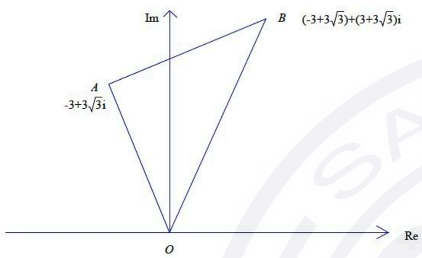
Question 82

(a)	Show a circle centre (4, 3) Allow dashes for coordinates on axes	**B1**	Note full circle is not required but must show centre and include relevant arc.
	Show a circle with radius 2. Can be implied by at least two of the points (2, 3), (6, 3), (4, 1) and (4, 5) being correct	**B1FT**	FT centre not at the origin.
	Point representing (2, 1)	**B1**	Half-line or 'correct' full line extending into the third quadrant implies point (2, 1).
	Show a half-line at their (2, 1) at an angle of $\frac{1}{3}\pi$, cutting top of circle between $x = 3$ and $x = 5$	**B1FT**	FT the point (±2, ±1) or (±1, ±2).
	Shade the correct region Needs correct half-line or "correct" full line extending into the third quadrant **AND** correct circle	**B1**	
		5	
(b)	Carry out a correct method for finding the greatest value of arg z in the correct region in (a)	**M1**	E.g. $\sin^{-1}(2/\sqrt{(25)}) + \tan^{-1}(3/4)$ or $\sin^{-1}(2/\sqrt{(25)}) + \sin^{-1}(3/5)$. Or, e.g., substitute $y = kx$ in circle equation, solve when discriminant = 0, to get $\tan^{-1}\left(\frac{6+\sqrt{21}}{6}\right)$.
	Obtain answer 1.06, or 1.05 or 1.055 or 1.056 or 60.4° or 60.5°	**A1**	The marks in **(b)** are available even if errors in **(a)**. No working seen scores 0/2 marks.
		2	

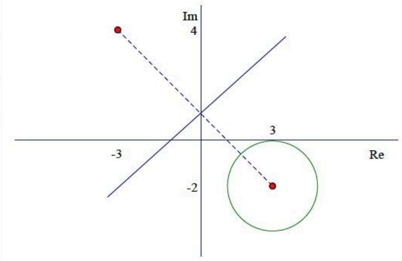
Question 83

Square $x + iy$ obtaining three terms when simplified and equate real and imaginary parts to 24 and −7 respectively	**M1**	Having used $i^2 = -1$.
Obtain equations $x^2 - y^2 = 24$ and $2xy = -7$	**A1**	Allow $2xyi = -7i$.
Eliminate one variable by correct method and find a horizontal equation in the other	**M1**	All powers of x or y are positive and are in the numerator.
Obtain $4x^4 - 96x^2 - 49 = 0$ or $4y^4 + 96y^2 - 49 = 0$ or 3-term equivalents	**A1**	
Obtain answers $\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ and $-\frac{7\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ or exact equivalents and no others	**A1**	E.g. $\pm\left(\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$, but not $\pm\left(\frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i\right)$ or $\left(\pm\frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i\right)$. Allow coordinates or $x = \ldots$, $y = \ldots$ paired correctly. ISW converting to different form. Must simplify $\sqrt{49}$.
	5	

Question 84

(a)	State $z\omega = \left(-3+3\sqrt{3}\right)+\left(3+3\sqrt{3}\right)\mathrm{i}$	**B1**	Or exact equivalent with real and imaginary parts collected. Need brackets around the coefficient of i. Allow for $a =$, $b =$ stated correctly.
		1	
(b)	Obtain $\lvert z\rvert = \sqrt{2}$	**B1**	
	Obtain $\arg z = -\frac{\pi}{4}$ final answer	**B1**	
	Obtain $\lvert\omega\rvert = 6$	**B1**	
	Obtain $\arg\omega = \frac{2\pi}{3}$ final answer	**B1**	
		4	
(c)			Note: The question does not require the diagram. If they use $\frac{5\pi}{12}$ they need to demonstrate where it comes from. Complex number equivalent to AB is $3\sqrt{3}+3\mathrm{i}$.
	Show $\lvert OA\rvert = \lvert AB\rvert = 6,$ hence isosceles	**B1**	One mark for 'isosceles' and one mark for 'right angle'. There will be alternatives e.g. use of Pythagoras (ratio of lengths is $1:1:\sqrt{2}$), expressing each number in "vector" form and using scalar product or explaining the effect of multiplying by 1 – i.
	$\angle AOB = \arg\omega - \arg z\omega = -\arg z = \frac{\pi}{4}$ hence third angle is a right angle	**B1**	
		2	
(d)	$\arg z\omega = \arg z + \arg\omega \left(= \frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}\right)$	**M1**	For showing correct use of their angles from part **(b)**. Must demonstrate where $\frac{5\pi}{12}$ comes from.
	$\arg z\omega = \tan^{-1}\dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$	**M1**	Correct method for *their* $z\omega$ from part **(a)**. Must link to point B on diagram or to $\arg z\omega$. Need to see $\tan^{-1}\dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ or $\tan\theta = \dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ and not just $\tan^{-1}\dfrac{1+\sqrt{3}}{-1+\sqrt{3}}$.
	$\Rightarrow \tan\left(\frac{5}{12}\pi\right) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$	**A1**	Obtain given answer from full and correct working.
		3	

Question 85

(a)	Show a circle centre (3, –2)	**B1**	
	Show a circle with radius 2 FT centre not at the origin	**B1FT**	
	Show the point representing (–3, 4) or the midpoint (0, 1)	**B1**	
	Show the perpendicular bisector of the line joining (–3, 4) and centre of the circle FT is on the position of (–3, 4) and centre of the circle	**B1FT**	
		4	
(b)	Carry out a correct method for finding the least value of $\|z - w\|$	**M1**	(Distance (3, –2) to (0, 1)) – 2.
	Obtain answer $\sqrt{18} - 2$ or $3\sqrt{2} - 2$	**A1**	
		2	

Question 86

(a)	State or imply $r = 2$	**B1**	
	State or imply $\theta = -\frac{2}{3}\pi$	**B1**	
		2	
(b)	State or imply $r = \frac{5}{2}$	**B1FT**	FT $\dfrac{5}{\textit{their } 2}$.
	State or imply $\theta = \frac{5}{6}\pi$	**B1FT**	FT $\frac{1}{6}\pi - \textit{their } -\frac{2}{3}\pi$.
		2	

Question 87

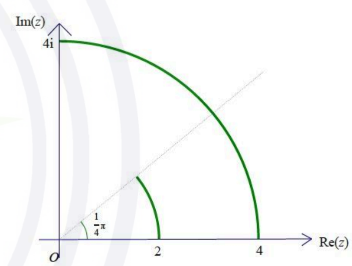
Substitute $z = x + \mathrm{i}y$ and obtain a horizontal equation Do not allow if this would lead to an equation containing xy terms which do not cancel	***M1**	E.g. $5(x + (y-3)\mathrm{i}) = (2 - 9\mathrm{i})(x + (y+3)\mathrm{i})$
Use $\mathrm{i}^2 = -1$ anywhere	**M1**	
Obtain e.g. $5x + 5(y-3)\mathrm{i} = (2x + 9y + 27) + \mathrm{i}(2y + 6 - 9x)$ or e.g. $3x^2 + 3y^2 - 12y - 63 + (9x^2 + 9y^2 - 30x + 54y + 81)\mathrm{i} = 0$	**A1**	Or equivalent expression free of products of complex numbers. Terms can be in any order.
Obtain simultaneous equations by equating real and imaginary parts	**DM1**	E.g. $3x - 9y = 27$ and $3y + 9x = 21$ $3x^2 + 3y^2 - 12y - 63 = 0$ and $9x^2 + 9y^2 - 30x + 54y + 81 = 0$
Obtain $[z =]\ 3 - 2\mathrm{i}$ only	**A1**	

Alternative Method for Question 4:

Obtain a horizontal equation in z Do not allow if it would lead to an equation containing z^2 where the xy terms do not cancel	*M1	E.g. $5z - 15i = 2z + 6i - 27i^2 - 9iz$. Allow errors, but no brackets.
Use $i^2 = -1$ anywhere	M1	
Obtain $z = \dfrac{9+7i}{1+3i}$	A1	OE (might have an uncancelled factor of 3)
Multiply top and bottom by $1-3i$ or equivalent for *their* z	DM1	Must see working for numerator or denominator, e.g. $9-27i+21+7i$ or $1 + 9$ or 10. If $\dfrac{9+7i}{1+3i} = 3-2i$ M0A0. If $\dfrac{9+7i}{1+3i} \times \dfrac{1-3i}{1-3i} = 3-2i$ M0A0 **SC B1**. If $\dfrac{9+7i}{1+3i} \times \dfrac{1-3i}{1-3i}$ and working in numerator or denominator and $3-2i$ M1A1.
Obtain $[z =]\ 3-2i$ only	A1	
	5	

Question 88

(a)	For all 4 marks, scales must be approximately equal, dashes can replace numbers. Arcs don't have to be perfectly circular, mark intention.	
	Show an arc of a circle, centre the origin and radius 2. Only need 2 on Re(z) or 2i on Im(z) or r = 2 to show correct radius	B1
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$ with any radius	B1
	Max B1 if sector shaded	
		2
(b)	Show an arc of a circle, centre the origin and radius 4. Only need 4 on Re(z) or 4i on Im(z) or r = 4 to show correct radius	B1
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{2}\pi$ with any radius	B1
	Max B1 if sector shaded	
		2



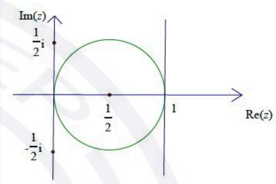
Question 89

(a)	$\frac{4\pi}{7}$ and/or $-\frac{\pi}{7}$	M1	SOI Allow $\frac{4}{7}\pi-\left(-\frac{1}{7}\pi\right)$ or $\frac{4}{7}\pi-\frac{1}{7}\pi$ Note: Many multiply top and bottom by the conjugate, which is fine, but to score the M1 they need to state or imply the argument of a complex number.
	Obtain arg $u = \frac{5}{7}\pi$	A1	Do not accept degrees.
		2	
(b)	Reflection (in the) real axis	B1	Correct non-contradictory statement. Condone x–axis or horizontal axis. Need 'reflection'. Not 'mirror', 'flip'.
	arg $u^* = -\frac{5}{7}\pi$	B1FT	FT *their* exact **(a).** Accept $2\pi-$ *their* exact **(a)**. Accept an 'exact' expression in place of an exact value. Need to see a value or an expression. Do not accept $\arg u^* = -\arg u$ without a value seen.
		2	

Question 90

Square $x + iy$ and equate real and imaginary parts to 6 and −8 respectively	*M1	Condone +8 in place of -8 and/or $i^2 = 1$.
Obtain equations $x^2 - y^2 = 6$ and $2xy = -8$	A1	OE
Eliminate one variable and find an equation in the other (from 2 equations each in 2 unknowns)	DM1	Condone a slip but not seriously incorrect algebra, e.g. use of $x = -4y$ is M0.
Obtain $x^4 - 6x^2 - 16 = 0$ or $y^4 + 6y^2 - 16 = 0$	A1	Accept 3-term equivalents e.g. $x^4 = 6x^2 + 16$. Condone missing '= 0' if implied by subsequent work.
Obtain answers $\pm\left(2\sqrt{2} - \sqrt{2}i\right)$ or exact equivalents	A1	Allow if values of x and y stated separately but the pairing is clear. Ignore additional correct solutions for x and y not real, but A0 if any additional incorrect answers.
	5	

Question 91

(a)	Multiply numerator and denominator by $1-y\text{i}$	**M1**	OE
	Obtain $\frac{1}{1+y^2}+\frac{-y}{1+y^2}\text{i}$	**A1**	OE
		2	
(b)	Express $\left(a-\frac{1}{2}\right)^2+b^2$ in terms of y and expand the bracket	**M1**	$\left(\frac{1}{1+y^2}-\frac{1}{2}\right)^2+\left(\frac{(-)y}{1+y^2}\right)^2$
	Obtain $\left(\frac{1}{\left(1+y^2\right)^2}-\frac{2}{2\left(1+y^2\right)}+\frac{1}{4}\right)+\frac{y^2}{\left(1+y^2\right)^2}$	**A1FT**	Follow *their* answer from **(a)** provided it gives an expression in y.
	Obtain $\frac{1}{4}$ from full and correct working	**A1**	AG
		3	
(c)	Show a vertical straight line through $1+0\text{i}$	**B1**	
	Show a circle centre $\frac{1}{2}+0\text{i}$	**B1**	
	Show a circle with radius $\frac{1}{2}$ and centre not at the origin	**B1**	
		3	
(d)	circle centre $\frac{1}{2}+0\text{i}$ with radius $\frac{1}{2}$	**B1**	OE Condone inclusion of the origin.
		1	

Question 92

Square $x + \text{i}y$ and equate real and imaginary parts to -4 and $6\sqrt{5}$ respectively	**M1**	
Obtain equations $x^2 - y^2 = -4$ and $2xy = 6\sqrt{5}$ or $\sqrt{x^2+y^2}=\sqrt{\sqrt{(-4)^2+\left(6\sqrt{5}\right)^2}}$	**A1**	Or $x^2 + y^2 = 14$.
Eliminate one variable and find a horizontal equation in the other	**M1**	Allow slips in e.g. signs, powers etc.
Obtain $x^4 + 4x^2 - 45 = 0$ or $y^4 - 4y^2 - 45 = 0$ or three-term equivalents, or $2x^2 = 10$ or $2y^2 = 18$	**A1**	May be implied by further work.
Obtain answers $\pm\left(\sqrt{5}+3\text{i}\right)$	**A1**	Accept e.g. $x=\sqrt{5},\ y=3$ and $x=-\sqrt{5}$, $y=-3$ or $\pm\left(\sqrt{5},3\right)$, but must be clearly paired. Can be implied by (e.g.) column working.
	5	

Question 93

(a)	Obtain $\text{Im}(z) \leqslant -1$	B1	Condone strict inequalities throughout **(a)**.
	Obtain answer of the form $\|z-a\| \leqslant b$	M1	Accept equation or any inequality sign. $a = \pm 2 \pm \text{i}$ and $b = 3$, e.g. $\|z+2-\text{i}\| = 3$ or $\|z+2-\text{i}\| > 3$.
	Obtain answer $\|z+2-\text{i}\| \leq 3$	A1	Accept $\|z-(-2+\text{i})\| \leqslant 3$ as final answer. Do not ISW.
		3	
(b)	Identify the coordinates of correct point	M1*	$(-2-\sqrt{5}, -1)$, if correct. From solving $(x \pm 2)^2 + (y \pm 1)^2 = 3^2$ (or = 3) with $y = -1$, or attempt to get $2+\sqrt{5}$ using a right-angled triangle.
	Carry out a correct method for finding the greatest value of $\|z\|$	DM1	
	Obtain answer 4.35 or $\sqrt{10+4\sqrt{5}}$	A1	AWRT 4.35, e.g. 4.3525…
		3	

Question 94

$\dfrac{(x+4)+\text{i}y}{x+\text{i}(y+4)} \times \dfrac{x-\text{i}(y+4)}{x-\text{i}(y+4)}$	*M1	Multiply numerator (and denominator) by the conjugate of the denominator.
Equate imaginary part of numerator to zero	DM1	Numerator $= x(x+4)+y(y+4)+\text{i}[xy-(y+4)(x+4)]$
Obtain $x+y=-4$	A1	OE
Correct use of modulus and solve for x or y	DM1	$x^2+(-x-4)^2=10 \Rightarrow x^2+4x+3=0$ or $(-y-4)^2+y^2=10 \Rightarrow y^2+4y+3=0$
$\Rightarrow z=-3-\text{i}$	A1	One correct solution A1. **SC A1 A0** for both pairs of x and y correct but not in given form.
or $z=-1-3\text{i}$	A1	Two solutions only.

Alternative Method for Question 6

$z+4=c(z+4i)$, $c \in \mathbb{R}$ or $x+iy+4=c(x+iy+4i)$	*M1	Equate to a real constant
$x+4=cx$ and $y=cy+4c$	DM1	Equate real and imaginary parts
$x=\dfrac{4}{c-1}$, $y=\dfrac{4c}{1-c}$ or $\dfrac{y}{x+4}=\dfrac{y+4}{x}$	A1	
Correct use of modulus and solve for a value of c, or x or y	DM1	$6c^2+20c+6=0$ OE $x^2+(-x-4)^2=10 \Rightarrow x^2+4x+3=0$ or $(-y-4)^2+y^2=10 \Rightarrow y^2+4y+3=0$.
$c=-\frac{1}{3}$ leading to $z=-3-\text{i}$	A1	One correct solution A1. **SC A1 A0** for both pairs of x and y correct but not in given form.
$c=-3$ leading to $z=-1-3\text{i}$	A1	Two solutions only.

Alternative Method 2 for Question 6

$\arg(z+4)=\arg(z+4i)$	*M1	Use of $\arg\left(\frac{z+4}{z+4i}\right)=0$.
$\frac{y}{x+4}=\frac{y+4}{x}$	DM1	
Obtain $x+y=-4$	A1	
Correct use of modulus and solve for x or y	DM1	$x^2+(-x-4)^2=10 \Rightarrow x^2+4x+3=0$ or $(-y-4)^2+y^2=10 \Rightarrow y^2+4y+3=0$.
$\Rightarrow z=-3-\text{i}$	A1	One correct solution A1. **SC A1 A0** for both pairs of x and y correct but not in given form.
or $z=-1-3\text{i}$	A1	Two solutions only.
	6	

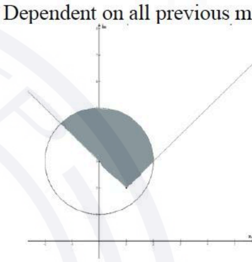
Question 95

(a)	State $z_1z_2=r_1r_2\text{e}^{\text{i}(\theta_1+\theta_2)}$	B1	Allow equivalent forms.
	State correct conjugate of z_1z_2, z_1 or z_2	B1	Allow equivalent forms.
	Obtain given result from correct working	B1	Clear demonstration that the product of the conjugates is identical to the conjugate of the product. Need to see a conclusion.
		3	
(b)	State $(z^*=)\ 3\text{e}^{-\frac{1}{4}\pi\text{i}}$	B1	Allow equivalent forms. Allow $3\text{e}^{\frac{7}{4}\pi\text{i}}$.
	Complete method to find both b and c	M1	E.g. find the product and sum of the roots, or expand $\left(z-3\text{e}^{-\frac{1}{4}\pi\text{i}}\right)\left(z-3\text{e}^{\frac{1}{4}\pi\text{i}}\right)$.
	Obtain $b=-3\sqrt{2},\ c=9$	A1	OE Allow $z^2-3\sqrt{2}z+9=0$.
		3	

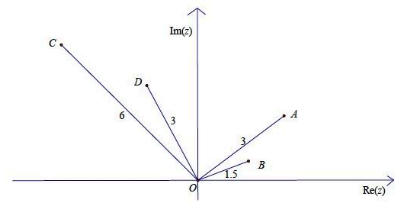
Question 96

Square $x+\text{i}y$ and equate real and imaginary parts to -1 and $-4\sqrt{5}$ respectively	*M1	
Obtain equations $x^2-y^2=-1$ and $2xy=-4\sqrt{5}$ from *their* expansion	A1	
Working from two equations in two unknowns, eliminate one variable and find an equation in the other	DM1	Do not condone incorrect algebra, e.g. $x=-2\sqrt{5}y$.
Obtain $x^4+x^2-20=0$ or $y^4-y^2-20=0$	A1	Accept 3-term equivalents. Condone missing "= 0" if implied by subsequent working.
Obtain answers $\pm\left(2-\sqrt{5}\text{i}\right)$ and no others	A1	Must state the square roots, not x and y separately, and not coordinates. Do not allow $\sqrt{4}$ in place of 2. A0 if there are additional incorrect solutions. No working seen scores 0/5.
	5	

Question 97

Show a circle centre (0, 3)	**B1**	Allow for a circle in the correct place even if their scales require an ellipse. '3' marked on the axis, centre implied by vertical intercepts or dashes to indicate a scale.
Show a circle with radius 2	**B1FT**	FT centre not at the origin. Consistent with *their* scale if they have unequal scales on the axes.
Show the point representing (1, 2)	**B1**	Position can be implied provided there is some indication of scale, e.g. numbers or marks on the axes.
Show correct half-lines from (1, 2), one at an angle of $\frac{1}{4}\pi$ and the other at an angle of $\frac{3}{4}\pi$	**B1FT**	FT from *their* (1, 2) not at the origin. Correct lines can imply previous B1. Half-lines must be intended to be 'symmetrical'. Angles consistent with *their* scale: correct lines need to pass very close to (0, 3) and (2, 3).
Shade the correct region	**B1**	Dependent on all previous marks. 
	5	

Question 98

(a)	Obtain $\omega z_1 = 6\mathrm{e}^{\mathrm{i}\frac{3}{4}\pi}$	**B1**	
	Obtain $\omega z_2 = 3\mathrm{e}^{\mathrm{i}\frac{2}{3}\pi}$	**B1**	
			SC B1 for both moduli correct or both arguments correct.
		2	
(b)	*A* and *B* plotted correctly	**B1**	 Follow *their* answers to part (a)
	C and *D* plotted with angles relatively correct and the same stretch implied.	**B1FT**	*C* and *D* correct, or FT *their A* and *B*.
		2	
(c)	Rotation $\frac{\pi}{2}$ radians (anticlockwise), or rotation $\frac{3\pi}{2}$ radians clockwise	**B1**	Accept 90°. Not required to state the centre of the rotation, but B0 if an incorrect statement seen.
	Enlargement (scale) factor 2	**B1**	Not required to state the centre of the enlargement, but B0 if an incorrect statement seen. Allow 'expansion' or 'stretch'.
		2	

Question 99

$\dfrac{x+\mathrm{i}(y+5)}{(x-5)+\mathrm{i}y}\times\dfrac{(x-5)-\mathrm{i}y}{(x-5)-\mathrm{i}y}$	***M1**	Multiply numerator (and denominator) by the conjugate of the denominator.
Equate imaginary part of numerator to zero	**DM1**	Numerator $=x(x-5)+y(y+5)+\mathrm{i}((y+5)(x-5)-xy)$.
Obtain $x-y=5$	**A1**	OE
Correct use of modulus and solve for x and y	**M1**	$x^2+(x-5)^2=17 \Rightarrow x^2-5x+4=0$ Or $x-y=5$ and $xy=-4$.
$\Rightarrow z=4-\mathrm{i}$	**A1**	
$\Rightarrow z=1-4\mathrm{i}$	**A1**	
Alternative Method for the first two marks of Question 3		
Quotient real $\Rightarrow \dfrac{x}{x+5}=\dfrac{y+5}{y}$	**M1**	
Rearrange to linear form and simplify	**M1**	
	6	