A-level

Topic :Differential Equation

May 2013-May 2025

Answers

Question 1

(i)	State $\frac{dV}{dt} = 80 - kV$	B1	
	Correctly separate variables and attempt integration of one side	M1	
	Obtain $a\ln(80 - kV) = t$ or equivalent	M1*	
	Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent	A1	
	Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
	Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent	A1	
	Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly	A1	[7]
(ii)	Use iterative formula correctly at least once	M1	
	Obtain final answer 0.14	A1	
	Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
(iii)	State a value between 530 and 540 cm^3 inclusive	B1	
	State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive)	B1	[2]

Question 2

(i)	Use any relevant method to determine a constant	M1	
	Obtain one of the values $A = 1$, $B = -2$, $C = 4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[4]
	[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]		
(ii)	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
	Obtain $\ln y = -\frac{1}{2} - 2\ln(2x+1) + c$, or equivalent	A3√	

Question 3

(i) Separate variables correctly and integrate at least one side — M1

Obtain term ln t, or equivalent — B1

Obtain term of the form $a \ln(k - x^3)$ — M1

Obtain term $-\frac{2}{3}\ln(k - x^3)$, or equivalent — A1

EITHER: Evaluate a constant or use limits $t = 1$, $x = 1$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ — M1*

Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k - x^3) + \frac{2}{3}\ln(k - 1)$ — A1

Use limits $t = 4$, $x = 2$, and solve for k — M1(dep*)

Obtain $k = 9$ — A1

OR: Using limits $t = 1$, $x = 1$ and $t = 4$, $x = 2$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ obtain an equation in k — M1*

Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k - 8) + \frac{2}{3}\ln(k - 1)$ — A1

Solve for k — M1(dep*)

Obtain $k = 9$ — A1

Substitute $k = 9$ and obtain $x = (9 - 8t^{-\frac{3}{2}})^{\frac{1}{3}}$ — A1 [9]

(ii) State that x approaches $9^{\frac{1}{3}}$, or equivalent — B1√ [1]

Question 4

(i) State or imply $V = \pi h^3$ — B1

State or imply $\frac{dV}{dt} = -k\sqrt{h}$ — B1

Use $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$, or equivalent — M1

Obtain the given equation — A1 **[4]**

[The M1 is only available if $\frac{dV}{dh}$ is in terms of h and has been obtained by a correct method.]

[Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.]

(ii) Separate variables and integrate at least one side — M1

Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent — A1

Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ — M1

Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$ — M1

Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$ — A1

(ii) Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent — A1 **[6]**

(iii) Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$ — B1 **[1]**

Question 5

Use $2\cos^2 x = 1 + \cos 2x$ or equivalent	B1
Separate variables and integrate at least one side	M1
Obtain $\ln\left(y^3 + 1\right) = \ldots$ or equivalent	A1
Obtain $\ldots = 2x + \sin 2x$ or equivalent	A1
Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3 + 1\right)$, bx or $c \sin 2x$	M1*
Obtain $\ln\left(y^3 + 1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant	A1
Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point	B1
Use correct process to find y-coordinate for at least one x-coordinate	M1(d*M)
Obtain 5.9	A1
Obtain 48.1	A1 [10]

Question 6

Separate variables correctly and recognisable attempt at integration of at least one side	M1
Obtain $\ln y$, or equivalent	B1
Obtain $k\ln\left(2 + e^{3x}\right)$	B1
Use $y(0) = 36$ to find constant in $y = A\left(2 + e^{3x}\right)^k$ or $\ln y = k\ln\left(2 + e^{3x}\right) + c$ or equivalent	M1*
Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2 + e^{3x}\right)^k\right)$	*M1
Obtain $y = 4\left(2 + e^{3x}\right)^2$	A1 [6]

Question 7

(i)	State or imply $\frac{dN}{dt} = kN(1-0.01N)$ and obtain the given answer $k = 0.02$	B1	**1**
(ii)	Separate variables and attempt integration of at least one side	M1	
	Integrate and obtain term $0.02t$, or equivalent	A1	
	Carry out a relevant method to obtain A or B such that $\frac{1}{N(1-0.01N)} \equiv \frac{A}{N} + \frac{B}{1-0.01N}$, or equivalent	M1*	
	Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
	Integrate and obtain terms $\ln N - \ln(1-0.01N)$, or equivalent	A1√	
	Evaluate a constant or use limits $t = 0$, $N = 20$ in a solution with terms $a\ln N$ and $b\ln(1-0.01N)$, $ab \neq 0$	M1(dep*)	
	Obtain correct answer in any form, e.g. $\ln N - \ln(1-0.01N) = 0.02t + \ln 25$	A1	
	Rearrange and obtain $t = 50\ln(4N/(100-N))$, or equivalent	A1	**8**
(iii)	Substitute $N = 40$ and obtain $t = 49.0$	B1	**1**

Question 8

Separate variables correctly and attempt integration of at least one side	B1	
Obtain term in the form $a\sqrt{(2x+1)}$	M1	
Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
Obtain term of the form $k\tan\theta$	M1	
Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k\tan\theta$, $ak \neq 0$	M1	
Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2}\tan\theta + \frac{1}{2}$	A1	
Rearrange and obtain $x = \frac{1}{8}(\tan\theta+1)^2 - \frac{1}{2}$, or equivalent	A1	**7**

Question 9

(i)	Separate variables correctly and attempt to integrate at least one side	B1	
	Obtain term ln$R$	B1	
	Obtain $\ln x - 0.57x$	B1	
	Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
	Obtain correct solution in any form	A1	
	Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$	A1	**[6]**
(ii)	Equate $\frac{dR}{dx}$ to zero and solve for x	M1	
	State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
	Obtain $R = 28.8$ (allow 28.9)	A1	**[3]**

Question 10

(i)	Sensibly separate variables and attempt integration of at least one side	M1	
	Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
	Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\,dx$	M1	
	Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\,dx$ or equivalent	A1	
	Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
	Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
	Substitute 25 and calculate value of y	DM*1	
	Obtain 203	A1	[3]

Question 11

Separate variables and factorise to obtain $\frac{\mathrm{d}y}{(3y+1)(y+3)} = 4x\,\mathrm{d}x$ or equivalent	B1	
State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B	M1	
Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
Integrate to obtain form $k_1 \ln(3y+1) + k_2 \ln(y+3)$	M1	
Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y=3) = 2x^2$ or equivalent	A1	
Substitute $x = 0$ and $y = 1$ in equation of form $k_1 \ln(3y+1) + k_2 \ln(y+3) = k_3 x^2 + c$ to find a value of c	M1	
Obtain $c = 0$	A1	
Use correct process to obtain equation without natural logarithm present	M1	
Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]

Question 12

(i)	Separate variables correctly and attempt integration of one side	B1	
	Obtain term $\ln x$	B1	
	Obtain term of the form $a\ln(k + \mathrm{e}^{-t})$	M1	
	Obtain term $-\ln(k + \mathrm{e}^{-t})$	A1	
	Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k + \mathrm{e}^{-t})$ and $b\ln x$	M1*	
	Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k + \mathrm{e}^{-t}) + \ln(k+1)$	A1	**[6]**
(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
	Obtain the given answer	A1	**[2]**
(iii)	Using $\mathrm{e}^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
	Justify the given answer	A1	**[2]**

Question 13

(i)	Separate variables correctly and integrate one side	B1	
	Obtain term $2\sqrt{M}$, or equivalent	B1	
	Obtain term $50k\sin(0.02t)$, or equivalent	B1	
	Evaluate a constant of integration, or use limits $M = 100$, $t = 0$ in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
	Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	**5**
(ii)	Use values $M = 196$, $t = 50$ and calculate k	M1(dep*)	
	Obtain answer $k = 0.190$	A1	**2**
(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
	State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	**2**

Question 14

Separate variables and integrate one side	**B1**	
Obtain term $\ln(x + 2)$	**B1**	
Use cos 2A formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	**M1**	
Obtain correct form $(1 - \cos 4\theta)/2$, or equivalent	**A1**	
Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	**A1**√	
Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x + 2), d\sin(4\theta), e\theta$	**M1**	
Obtain correct solution in any form, e.g. $\ln(x + 2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	**A1**	
Use correct method for solving an equation of the form $\ln(x + 2) = f$	**M1**	
Obtain answer $x = 0.962$	**A1**	[9]

Question 15

(i)	State $\frac{dN}{dt} = k(N - 150)$	**B1**	[1]
(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
	Obtain $k = 0.08$	**A1**	
	Separate variables and obtain general solution involving $\ln(N - 150)$	**M1***	
	Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$	**A1**√	
	Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
	Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent	**A1**	
	Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
(iii)	<u>Either</u> Substitute $t = 15$ to find N <u>or</u> solve for t with $N = 2000$	**M1**	
	Obtain <u>Either</u> $N = 1810$ <u>or</u> $t = 16.4$ and conclude target not met	**A1**	[2]

Question 16

(i)	Separate variables and attempt integration of one side	**M1**	
	Obtain term $-e^{-y}$	**A1**	
	Integrate xe^x by parts reaching $xe^x \pm \int e^x \, dx$	**M1**	
	Obtain integral $xe^x - e^x$	**A1**	
	Evaluate a constant, or use limits $x = 0, y = 0$	**M1**	
	Obtain correct solution in any form	**A1**	
	Obtain final answer $y = -\ln(e^x(1-x))$, or equivalent	**A1**	[7]
(ii)	Justify the given statement	**B1**	[1]

Question 17

Separate variables and attempt integration of at least one side	**M1***	
Obtain term $\ln y$	**A1**	
Obtain terms $\ln x - x^2$	**A1**	
Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1***	
Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$	**A1**	
Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1-x^2)$	**A1**	
		[6]

Question 18

(i)	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term $\ln x$	**B1**	
	Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	**M1**	
	Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	**A1**	
	Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$, where $ab \neq 0$	**M1**	
	State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta) + \frac{3}{2}\ln 3$	**A1**	
	Rearrange in a correct form, e.g. $x = \sqrt{\left(\frac{27}{3+\cos 2\theta}\right)}$	**A1**	[7]
(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	**B1**	[1]

Question 19

Separate variables and make reasonable attempt at integration of either integral	**M1**
Obtain term $\frac{1}{2}e^{2y}$	**B1**
Use Pythagoras	**M1**
Obtain terms $\tan x - x$	**A1**
Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, $(ab \neq 0)$	**M1**
Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**
Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**
Obtain answer $y = 0.179$	**A1**
	[8]

Question 20

(i)	Separate variables correctly and integrate at least one side	**M1**	
	Integrate and obtain term kt, or equivalent	**A1**	
	Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	**M1***	
	Obtain $A = B = \frac{1}{4}$, or equivalent	**A1**	
	Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	**A1**√	
	EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	**A1**	
	Use a second pair of limits and determine k	**DM1**	
	Obtain the given exact answer correctly	**A1**	
	OR: Use both pairs of limits in a definite integral	**M1***	
	Obtain the given exact answer correctly	**A1**	
	Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
(ii)	Substitute $x = 3.6$ and solve for t	**M1**	
	Obtain answer $t = 4$	**A1**	[2]

Question 21

(i)	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	**B1**	[1]
(ii)	Separate variables correctly and attempts to integrate one side of equation	**M1**	
	Obtain terms of the form $a\ln y$ and bx^2	**A1**	
	Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing a ln y or bx^2	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$	**A1**	
	Obtain correct expression for y, e.g. $y = 2e^{\frac{1}{4}x^2}$	**A1**	[5]
(iii)	Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient.	**B1**	[1]

Question 22

(i)	State or imply $\frac{dV}{dt} = 2\frac{dh}{dt}$	**B1**
	State or imply $\frac{dV}{dt} = 1 - 0.2\sqrt{h}$	**B1**
	Obtain the given answer correctly	**B1**
	Total:	**3**
(ii)	State or imply $du = -\frac{1}{2\sqrt{h}}\ dh$, or equivalent	**B1**
	Substitute for h and dh throughout	**M1**
	Obtain $T = \int_3^5 \frac{20(5-u)}{u}\ du$, or equivalent	**A1**
	Integrate and obtain terms $100\ln u - 20u$, or equivalent	**A1**
	Substitute limits $u = 3$ and $u = 5$ correctly	**M1**
	Obtain answer 11.1, with no errors seen	**A1**
	Total:	**6**

Question 23

(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	M1
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	A1
	Total:	**2**
(ii)	Separate variables and integrate one side	B1
	Obtain term $\ln y$	B1
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	B2 FT
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	A1
	Obtain answer $y = 1.29$ (3s.f. only)	A1
	Total:	**7**

Question 24

(i)	State $\frac{dy}{dt} = -\frac{2y}{(1+t)^2}$, or equivalent	B1
	Separate variables correctly and attempt integration of one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $\frac{2}{(1+t)}$, or equivalent	A1
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a\ln y$ and $\frac{b}{1+t}$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	A1
	Total:	**6**

(ii)	State that the mass of B approaches $\frac{100}{e^2}$, or **exact** equivalent	**B1**
	State or imply that the mass of A tends to zero	**B1**
	Total:	**2**

Question 25

(i)	Justify the given differential equation	**B1**
	Total:	**1**
(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x=0$, $t=0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x)=kt-\ln 50$, or equivalent	**A1**
	Use $x=25$, $t=10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50-\ln(50-x)=\frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x=50(1-\exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

Question 26

Separate variables correctly and attempt integration of one side	**B1**
Obtain term $\tan y$, or equivalent	**B1**
Obtain term of the form $k\ln\cos x$, or equivalent	**M1**
Obtain term $-4\ln\cos x$, or equivalent	**A1**
Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a\tan y$ and $b\ln\cos x$ to evaluate a constant, or as limits	**M1**
Obtain correct solution in any form, e.g. $\tan y = 4\ln\sec x + 1$	**A1**
Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a\tan y$ and $b\ln\cos x$, and use correct method to find x	**M1**
Obtain answer $x = 0.587$	**A1**
	8

Question 27

Separate variables and obtain $\int \frac{1}{y}\mathrm{d}y = \int \frac{x+2}{x+1}\mathrm{d}x$	**B1**
Obtain term $\ln y$	**B1**
Use an appropriate method to integrate $(x+2)/(x+1)$	***M1**
Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	**A1**
Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1**
Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	**A1**
Obtain answer $y = (x+1)\mathrm{e}^{x-1}$	**A1**
	7

Question 28

(i)	Show sufficient working to justify the given statement **AG**	**B1**
		1
(ii)	Separate variables correctly and attempt integration of at least one side	**B1**
	Obtain term $\frac{1}{2}x^2$	**B1**
	Obtain terms $\tan^2\theta + \tan\theta$, or $\sec^2\theta + \tan\theta$	**B1 + B1**
	Evaluate a constant, or use limits $x = 1$, $\theta = \frac{1}{4}\pi$, in a solution with two terms of the form ax^2 and $b\tan\theta$, where $ab \neq 0$	**M1**
	State correct answer in any form, e.g. $\frac{1}{2}x^2 = \tan^2\theta + \tan\theta - \frac{3}{2}$	**A1**
	Substitute $\theta = \frac{1}{3}\pi$ and obtain $x = 2.54$	**A1**
		7

Question 29

(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln x$	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms $a \ln x$ and $b\, t\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

Question 30

(i)	Fully justify the given statement	B1	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
(ii)	Separate variables and attempt integration of at least one side Obtain terms $\ln y$ and $\frac{1}{2}x$	B1 B1	Must be working from $\int \frac{1}{y}\,dy = \int k\,dx$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	A1	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y \rvert = \ldots$ scores A0
		5	

Question 31

(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	**M1**
	Obtain $A = B = \frac{1}{4}$	**A1**
	Total:	**2**
(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2 + y)$ or $c \ln (2 - y)$	**M1**
	Obtain term $\ln x$	**B1**
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	**A1FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2 + y)$ and $c \ln (2 - y)$	**M1**
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	**A1**
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	**A1**
	Total:	**6**

Question 32

Separate variables correctly and integrate at least one side	**B1**
Obtain term ln y	**B1**
Obtain terms $2\ln x - \dfrac{1}{2}x^2$	**B1+B1**
Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**
Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \dfrac{1}{2}x^2 + \dfrac{1}{2}$	**A1**
Rearrange as $y = x^2 \exp\left(\dfrac{1}{2} - \dfrac{1}{2}x^2\right)$, or equivalent	**A1**
	7

Question 33

State equation $\dfrac{dy}{dx} = k\dfrac{y^2}{x}$, or equivalent	**B1**	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
Separate variables correctly and integrate at least one side	**B1**	$\int \dfrac{k}{x}dx = \int \dfrac{1}{y^2}dy$ Allow with incorrect value substituted for k
Obtain terms $-\dfrac{1}{y}$ and $k\ln x$	**B1 + B1**	Incorrect k used scores max. B1B0
Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**	SC: If an incorrect method is used to find k, M1 is allowable for a correct method to find C
Obtain $k = \dfrac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**	$\dfrac{1}{2}\ln x = 1 - \dfrac{1}{y}$ A0 for fortuitous answers.
Obtain answer $y = \dfrac{2}{2 - \ln x}$, or equivalent, and ISW	**A1**	$y = \dfrac{-1}{-1 + \ln\sqrt{x}}$
		SC: MR of the fraction. $\dfrac{dy}{dx} = k\dfrac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\dfrac{-1}{y} = \dfrac{-k}{x}(+C)$ B1+B1 Substitute to find k and/or c M1 $k = \dfrac{e}{2(e-1)}$, $c = \dfrac{2-e}{2(e-1)}$ A1+A1 Answer A0
	8	

Question 34

Separate variables correctly and attempt integration of at least one side	**B1**
Obtain term $-\frac{1}{2y^2}$, or equivalent	**B1**
Obtain term $-k\,e^{-x}$	**B1**
Use a pair of limits, e.g. $x=0$, $y=1$ to obtain an equation in k and an arbitrary constant c	**M1**
Use a second pair of limits, e.g. $x=1$, $y=\sqrt{e}$, to obtain a second equation and solve for k or for c	**M1**
Obtain $k=\frac{1}{2}$ and $c=0$	**A1**
Obtain final answer $y=e^{\frac{1}{2}x}$, or equivalent	**A1**
	7

Question 35

(i)	Use chain rule	**M1**	$k\cos\theta\sin^{-3}\theta\left(=-k\operatorname{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	**A1**	e.g. $-2\operatorname{cosec}^2\theta\cot\theta$, $\frac{-2\cos\theta}{\sin^3\theta}$ Accept $\frac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int x\,dx=\int -\operatorname{cosec}^2\theta\cot\theta\,d\theta$
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain term of the form $\frac{k}{\sin^2\theta}$	**M1***	or equivalent
	Obtain term $\frac{1}{2\sin^2\theta}$	**A1**	or equivalent
	Use $x=4$, $\theta=\frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\frac{b}{\sin^2\theta}$, where $ab\neq 0$	**DM1**	Dependent on the preceding M1
	Obtain solution $x=\sqrt{\left(\operatorname{cosec}^2\theta+12\right)}$	**A1**	or equivalent
		6	

Question 36

(i)	Separate variables correctly and attempt integration of at least one side	B1	$\int e^{-y}\,dy = \int xe^{x}\,dx$
	Obtain term $-e^{-y}$	B1	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\,dx$	M1	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	A1	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	M1	Must see this step
	Obtain correct solution in any form	A1	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	A1	or equivalent e.g. $y = \ln\frac{1}{e^{x}(1-x)}$ ISW
		7	
(ii)	Justify the given statement	B1	e.g. require $1 - x > 0$ for the ln term to exist, hence $x < 1$ Must be considering the range of values of x, and must be relevant to *their* y involving $\ln(1-x)$
		1	

Question 37

Separate variables correctly and integrate at least one side	B1
Obtain term $\ln(x+1)$	B1
Obtain term of the form $a\ln(y^2+5)$	M1
Obtain term $\frac{1}{2}\ln(y^2+5)$	A1
Use $y = 2$, $x = 0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2+5)$ and $b\ln(x+1)$, where $ab \neq 0$	M1
Obtain correct solution in any form	A1
Obtain final answer $y^2 = 9(x+1)^2 - 5$	A1
	7

Question 38

(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	B1	OE $(-10 = k \times 1 \times 1000)$
		1	
(ii)	Separate variables correctly and integrate at least one side	B1	$\int \frac{1}{N} dN = \int -0.01e^{-0.02t} dt$
	Obtain term $\ln N$	B1	OE
	Obtain term $0.5e^{-0.02t}$	B1	OE
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a \ln N$ and $be^{-0.02t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$	A1	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N = 800$ and obtain $t = 29.6$	A1	
		6	
(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	B1	Accept 606 or 607 or 606.5
		1	

Question 39

Separate variables correctly to obtain $\int \frac{1}{x+2} dx = \int \cot \frac{1}{2}\theta \, d\theta$	B1	Or equivalent integrands. Integral signs SOI
Obtain term $\ln(x+2)$	B1	Modulus signs not needed.
Obtain term of the form $k \ln \sin \frac{1}{2}\theta$	M1	
Obtain term $2 \ln \sin \frac{1}{2}\theta$	A1	
Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p \ln(x+2)$ and $q \ln\left(\sin \frac{1}{2}\theta\right)$	M1	Reach $C =$ an expression or a decimal value
Obtain correct solution in any form e.g. $\ln(x+2) = 2 \ln \sin \frac{1}{2}\theta + \ln 12$	A1	$\ln 12 = 2.4849$….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln \frac{1}{2}$
Remove logarithms and use correct double angle formula	M1	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
Obtain answer $x = 4 - 6\cos\theta$	A1	
	8	

Question 40

(i)	Separate variables correctly and integrate one side	**B1**
	Obtain term $0.2t$, or equivalent	**B1**
	Carry out a relevant method to obtain A and B such that $\frac{1}{(20-x)(40-x)} \equiv \frac{A}{20-x} + \frac{B}{40-x}$	***M1**
	Obtain $A = \frac{1}{20}$ and $B = -\frac{1}{20}$	**A1**
	Integrate and obtain terms $-\frac{1}{20}\ln(20-x) + \frac{1}{20}\ln(40-x)$ OE	**A1FT +A1FT**
	Use $x = 10$, $t = 0$ to evaluate a constant, or as limits	**DM1**
	Obtain correct answer in any form	**A1**
	Obtain final answer $x = \frac{60e^{4t} - 40}{3e^{4t} - 1}$	**A1**
		9
(ii)	State that x approaches 20	**B1**
		1

Question 41

(a)	Separate variables correctly and attempt integration of at least one side	B1
	Obtain term of the form $a\tan^{-1}(2y)$	M1
	Obtain term $\frac{1}{2}\tan^{-1}(2y)$	A1
	Obtain term $-e^{-x}$	B1
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan^{-1}(by)$ and $ce^{\pm x}$	M1
	Obtain correct answer in any form	A1
	Obtain final answer $y = \frac{1}{2}\tan\left(2e^{-1} - 2e^{-x}\right)$, or equivalent	A1
		7
(b)	State that y approaches $\frac{1}{2}\tan\left(2e^{-1}\right)$, or equivalent	B1FT
		1

Question 42

(a)	State $\frac{dy}{dx} = k\frac{y}{x\sqrt{x}}$, or equivalent	B1
	Separate variables correctly and attempt integration of at least one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $-2k\frac{1}{\sqrt{x}}$, or equivalent	A1
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a \ln y$ and $\frac{b}{\sqrt{x}}$, where $ab \neq 0$	M1
	Obtain $k = 1$ and $c = 2$	A1 + A1
	Obtain final answer $y = \exp\left(-\frac{2}{\sqrt{x}} + 2\right)$, or equivalent	A1
		8
(b)	State that y approaches e^2 (FT *their* c in part **(a)** of the correct form)	B1FT
		1

Question 43

Separate variables correctly and integrate at least one side	**B1**
Obtain term $\ln(y-1)$	**B1**
Carry out a relevant method to determine A and B such that $\frac{1}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$	**M1**
Obtain $A = \frac{1}{2}$ and $B = -\frac{1}{2}$	**A1**
Integrate and obtain terms $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$ $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$, or equivalent **(FT** is on A and B**)**	**A1 FT** **+ A1 FT**
Use $x=0$, $y=2$ to evaluate a constant, or as limits in a solution containing terms of the form $a\ln(y-1)$, $b\ln(x+1)$ and $c\ln(x+3)$, where $abc \neq 0$	**M1**
Obtain correct answer in any form	**A1**
Obtain final answer $y = 1 + \sqrt{\left(\frac{3x+3}{x+3}\right)}$, or equivalent	**A1**
	9

Question 44

(a)	State or imply $\frac{dV}{dt} = -k\sqrt{h}$	**B1**
	State or imply $\frac{dV}{dh} = 2\pi rh - \pi h^2$, or equivalent	**B1**
	Use $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$	**M1**
	Obtain the **given answer** correctly	**A1**
		4
(b)	Separate variables and attempt integration of at least one side	**M1**
	Obtain terms $\frac{4}{3}rh^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	**A3, 2, 1, 0**
	Use $t=0$, $h=r$ to find a constant of integration c	**M1**
	Use $t=14$, $h=0$ to find B	**M1**
	Obtain correct c and B, e.g. $c = \frac{14}{15}r^{\frac{5}{2}}$, $B = \frac{1}{15}r^{\frac{5}{2}}$	**A1**
	Obtain final answer $t = 14 - 20\left(\frac{h}{r}\right)^{\frac{3}{2}} + 6\left(\frac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	**A1**
		8

Question 45

Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
Obtain term $\ln y$	**B1**	
Obtain terms $\ln x - x^2$	**B1**	
Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
Obtain correct solution in any form	**A1**	
Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
	6	

Question 46

(a)	Correct separation of variables	**B1**	$\int \sec^2 2x\,\mathrm{d}x = \int \mathrm{e}^{-3t}\mathrm{d}t$ Needs correct structure
	Obtain term $-\frac{1}{3}\mathrm{e}^{-3t}$	**B1**	
	Obtain term of the form $k\tan 2x$	**M1**	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	**A1**	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a\tan 2x$ and be^{-3t}, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}\mathrm{e}^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1-\mathrm{e}^{-3t}\right)\right)$	**A1**	
		7	
(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	**B1 FT**	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $\mathrm{e}^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

Question 47

Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
Obtain term $\ln y$	**B1**	
Obtain terms $\ln x - x^2$	**B1**	
Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
Obtain correct solution in any form	**A1**	
Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
	6	

Question 48

(a)	Separate variables correctly and attempt integration of at least one side	**M1**
	Obtain term $\ln y$	**A1**
	Obtain term of the form $\pm\ln(1-\cos x)$	**M1**
	Obtain term $\ln(1-\cos x)$	**A1**
	Use $x=\pi$, $y=4$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln y$ and $b\ln(1-\cos x)$	**M1**
	Obtain final answer $y=2(1-\cos x)$	**A1**
		6
(b)	Show a correct graph for $0<x<2\pi$ with the maximum at $x=\pi$	**B1 FT**
		1

Question 49

(a)(i)	Justify the given statement $\frac{MN}{y}=\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**
		1
(a)(ii)	Express the area of PMN in terms of y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ and equate to $\tan x$	**M1**
	Obtain the given equation correctly	**A1**
		2

(b)	Separate variables and integrate at least one side	**M1**
	Obtain term $\frac{1}{6}y^3$	**A1**
	Obtain term of the form $\pm\ln\cos x$	**M1**
	Evaluate a constant or use $x = 0$ and $y = 1$ in a solution containing terms ay^3 and $\pm\ln\cos x$, or equivalent	**M1**
	Obtain correct answer in any form, e.g. $\frac{1}{6}y^3 = -\ln\cos x + \frac{1}{6}$	**A1**
	Obtain final answer $y = \sqrt[3]{(1-6\ln\cos x)}$	**A1**
		6

Question 50

State equation $\frac{dy}{dx} = k\frac{y}{\sqrt{x+1}}$	**B1**
Separate variables correctly for *their* differential equation and integrate at least one side	***M1**
Obtain $\ln y$	**A1**
Obtain $2[k]\sqrt{x+1}$	**A1**
Use (0, 1) **and** (3, e) in an expression containing $\ln y$ and $\sqrt{x+1}$ and a constant of integration to determine k and/or a constant of integration c (or use (0, 1), (3, e) and (x, y) as limits on definite integrals)	**DM1**
Obtain $k = \frac{1}{2}$ and $c = -1$	**A1**
Obtain $y = \exp\left(\sqrt{x+1} - 1\right)$	**A1**
	7

Question 51

State a suitable form of partial fractions for $\dfrac{1}{x^2(1+2x)}$	**B1**
Use a relevant method to determine a constant	**M1**
Obtain one of $A = -2$, $B = 1$ and $C = 4$	**A1**
Obtain a second value	**A1**
Obtain the third value	**A1**
Separate variables correctly and integrate at least one term	**M1**
Obtain terms $-2\ln x - \dfrac{1}{x} + 2\ln(1+2x)$ and t	**B3 FT**
Evaluate a constant, or use limits $x = 1$, $t = 0$ in a solution containing terms t, $a\ln x$ and $b\ln(1+2x)$, where $ab \neq 0$	**M1**
Obtain a correct expression for t in any form, e.g. $t = -\dfrac{1}{x} + 2\ln\left(\dfrac{1+2x}{3x}\right) + 1$	**A1**
	11

Question 52

(a)	State or imply equation of the form $\frac{dx}{dt} = k\frac{x}{20-x}$	M1	
	Obtain $k = 19$	A1	AG
		2	
(b)	Separate variables and integrate at least one side	M1	
	Obtain terms $20\ln x - x$ and $19t$, or equivalent	A1 A1	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a\ln x$ and bt	M1	
	Substitute $t = 1$ and rearrange the equation in the given form	A1	AG
		5	
(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	M1	
	Obtain final answer $x = 2.83$	A1	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	A1	
		3	
(d)	Set $x = 20$ and obtain answer $t = 2.15$	B1	
		1	

Question 53

Separate variables correctly	B1	$\int \frac{1}{y^2}\,dy = \int 4xe^{-2x}\,dx$
$\int \frac{1}{y^2}\,dy = -\frac{1}{y}$	B1	OE
Commence the other integration and reach $axe^{-2x} + b\int e^{-2x}\,dx$	M1	
Obtain $-2xe^{-2x} + 2\int e^{-2x}\,dx$ or $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$	A1	SOI (might have taken out factor of 4)
Complete integration and obtain $-2xe^{-2x} - e^{-2x}$	A1	
Evaluate a constant or use $x = 0$ and $y = 1$ as limits in a solution containing terms of the form $\frac{p}{y}$, qxe^{-2x}, re^{-2x}, or equivalent.	M1	
Obtain $y = \frac{e^{2x}}{2x+1}$, or equivalent expression for y	A1	ISW
	7	

Question 54

(a)	Show sufficient working to justify the given answer	B1	
		1	
(b)	Correct separation of variables	B1	e.g. $-\int \frac{1}{t}\mathrm{d}t = \int \frac{1}{x\ln x}\mathrm{d}x$
	Obtain term $\ln(\ln x)$	B1	
	Obtain term $-\ln t$	B1	
	Evaluate a constant or use $x = \mathrm{e}$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$	M1	
	Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$	A1	
	Use log laws to enable removal of logarithms	M1	
	Obtain answer $x = \mathrm{e}^{\frac{2}{t}}$, or simplified equivalent	A1	
		7	
(c)	State that x tends to 1 coming from $x = \mathrm{e}^{\frac{k}{t}}$	B1	
		1	

Question 55

Correctly separate variables and integrate at least one side	M1	To obtain $a\ln y$ or $b\ln(x+1) + c\ln(3x+1)$
Obtain term $\ln y$ from integral of $1/y$	B1	
State or imply the form $\frac{A}{x+1} + \frac{B}{3x+1}$ and use a correct method to find a constant	M1	
Obtain $A = -\frac{1}{2}$ and $B = \frac{3}{2}$	A1	
Obtain terms $-\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(3x+1)$ or $-\frac{1}{2}\ln(2x+2) + \frac{1}{2}\ln(6x+2)$ or combination of these terms	A1 FT + A1 FT	The FT is on the values of A and B.
Use $x = 1$ and $y = 1$ to evaluate a constant, or expression for a constant, (decimal equivalent of ln terms allowed) or as limits, in a solution containing terms $a\ln y$, $b\ln(x+1)$ and $c\ln(3x+1)$, where $abc \neq 0$	*M1	e.g. $\ln y = -\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(3x+1) - \frac{1}{2}\ln 2$
Obtain an expression for y or y^2 and substitute $x = 3$	DM1	Do not accept decimal equivalent of ln terms
Obtain answer $y = \frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or $\sqrt{\frac{10}{8}}$	A1	ISW. Must be simplified and exact, do not allow 1.118 or $\mathrm{e}^{\frac{1}{2}\ln\frac{5}{4}}$.
	9	

Question 56

Part	Answer	Mark	Guidance
(a)	Separate variables correctly	B1	$\frac{dN}{N^{\frac{3}{2}}} = (k\cos 0.02t)\,dt$ Allow without integral signs.
	Obtain term $-\frac{2}{\sqrt{N}}$	B1	OE Ignore position of k.
	Obtain term $50\sin 0.02t$	B1	OE Ignore position of k.
	Use $t = 0$, $N = 100$ to evaluate a constant, or as limits, in a solution containing terms $\frac{a}{\sqrt{N}}$ and $b\sin 0.02t$, where $ab \neq 0$	M1	$\left[\text{e.g. } c = -0.2 \quad \text{or} \quad c = \frac{-0.2}{k}\right]$
	Obtain correct solution in any form, e.g. $-\frac{2}{\sqrt{N}} = 50k\sin 0.02t - 0.2$	A1	OE ISW e.g. $N = \frac{1}{(25k\sin 0.02t - 0.1)^2}$ $\quad -2N^{-\frac{1}{2}} = \frac{k}{0.02}\sin 0.02t - \frac{1}{5}$ $50k\sin 0.02t = -\frac{2}{\sqrt{N}} + \frac{1}{5}$ $\quad \frac{1}{\sqrt{N}} = -\frac{1}{2}k(50\sin 0.02t) + \frac{1}{10}$ $50\sin\left(\frac{1}{50}t\right) = -\frac{2\sqrt{N}}{kN} + \frac{20}{100k}$
		5	
(b)	Use the substitution $N = 625$ and $t = 50$ in expression of appropriate form to evaluate k	M1	Expression must contain $a + bk\sin 0.02t$, $(\sqrt{N})^{\pm n}$, where $n = -1, 1, 3$ or 5 and a and b are constants $ab \neq 0$ or $(a + bk\sin 0.02t)^{\pm 2}$ and $(N)^{\pm n}$. Allow with k replaced by $\frac{1}{k}$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. If invert term by term when 3 terms shown then M0.
	Obtain $k = 0.00285[2148]$	A1	Must evaluate sin1. Degrees $k = 0.138$ M1 A0.
		2	
(c)	Rearrange and obtain $N = 4(0.2 - 0.142(607)\sin 0.02t)^{-2}$ Substitution for k required	M1	Anything of the form $N = c(d - ek\sin 0.02t)^{-2}$, where c, d and e are constants $cde \neq 0$ and value of k substituted. Allow with k replaced by $1/k$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. OE ISW e.g. $N = \left(-\frac{10}{0.7125\sin 0.02t - 1}\right)^2$ $\quad N = \frac{1}{(-0.0713\sin 0.02t + 0.1)^2}$ $N = \frac{100}{\left(\left(\frac{0.6}{\sin 1}\right)\sin 0.02t - 1\right)^2}$ $\quad N = \frac{1}{\left(\frac{3}{-50\sin 1} \times \sin 0.02t + \frac{1}{10}\right)^2}$ $N = \left(-\frac{0.06}{\sin 1}\sin 0.02t + 0.1\right)^{-2}$ $\quad N = \left(\frac{800}{80 - 57\sin 0.02t}\right)^2$ Do not need to substitute for $\sin(0.02t) = 1$, but must substitute for k.
	Accept answers between 1209 and 1215	A1	ISW Substitute $\sin 0.02t = 1$ or $t = 50\sin^{-1} 1$ or 78.5 or 25π. Answer with no working (rubric) 0/2. **SC** $N = \ldots$ not seen but correct numerical answer **B1** 1/2.
		2	

Question 57

(a)	Correct separation of variables	B1	$\int e^{-y}dy = \int xe^{-x}dx$ Condone missing integral signs.
	Obtain term $-e^{-y}$	B1	
	Commence integration by parts and reach $\pm xe^{-x} \pm \int e^{-x}\,dx$	*M1	M0 if clearly using differentiation of a product.
	Complete integration and obtain $-xe^{-x} - e^{-x}$	A1	
	Use $x = 0$ and $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms ae^{-y}, bxe^{-x} and ce^{-x}, where $abc \neq 0$	DM1	Must see working for this. In a correct solution they should have $-e^{-y} + C = -xe^{-x} - e^{-x}$ or equivalent. If they take logarithms before finding the constant, the constant must be of the right form.
	Correct solution in any form Must follow from correct working	A1	e.g. $-e^{-y} = -xe^{-x} - e^{-x}$ A0 if constant of integration ignored or assumed to be zero.
	Obtain final answer $y = -\ln\left((x+1)e^{-x}\right)$ from correct working	A1	OE e.g. $y = x - \ln(x+1)$, $y = \ln\left(\frac{e^x}{x+1}\right)$. A0 if constant of integration ignored or assumed to be zero.
		7	
(b)	Obtain answer $(y =)1 - \ln 2$	B1	Must follow from at least 6 or7 obtained in part **6(a).**
		1	

Question 58

Separate variables correctly	B1	$\int \frac{x}{1+x^2}dx = \int \frac{1}{y}dy$ Accept without integral signs.
Obtain term ln y	B1	
State term of the form $k\ln\left(1+x^2\right)$	M1	
State correct term $\frac{1}{2}\ln\left(1+x^2\right)$	A1	OE
Evaluate a constant, or use limits $x = 0$, $y = 2$ in a solution containing terms $a\ \ln y$ and $b\ln\left(1+x^2\right)$ where $ab \neq 0$	M1	If they remove logs first the constant must be of the correct form.
Obtain correct solution in any form	A1	e.g $\ln y + \ln\frac{1}{2} = \frac{1}{2}\ln\left(1+x^2\right)$
Simplify and obtain $y = 2\sqrt{1+x^2}$	A1	OE The question asks for simplification, so need to deal with $\exp(\ln(...))$.
	7	

Question 59

(a)	$a = 30$ and $b = 0.01$	**B1**	
		1	

(b)	Separate variables and integrate one side	**M1**	
	Obtain terms $-100\ln(30-0.01V)$ and t, or equivalent	**A1 FT** **+ A1 FT**	FT *their a* and *b*.
	Evaluate a constant, or use $t = 0$, $V = 0$ as limits, in a solution containing terms $c\ln(30-0.01V)$ and dt where $cd \neq 0$	**M1**	
	Obtain solution $100\ln 30 - 100\ln(30-0.01V) = t$, or equivalent	**A1**	
	Substitute $V = 1000$ and obtain answer $t = 40.5$	**A1**	
		6	
(c)	Obtain $V = 3000(1-e^{-0.01t})$	**B1**	OE
	State that V approaches 3000	**B1**	
		2	

Question 60

(a)	Show sufficient working to justify the given statement	**B1**	e.g. see $2\cot\theta \times -\operatorname{cosec}^2\theta$ in the working or express in terms of $\sin\theta$ and $\cos\theta$ and use quotient rule to obtain the given result. Solution must have θ present throughout and must reach the given answer.
		1	

(b)	Separate variables correctly Check for relevant working in **(a)**	**B1**	$\int x\,dx = \int \frac{\tan^2\theta}{\sin^2\theta} - \frac{2\cot\theta}{\sin^2\theta}\,d\theta$ Condone incorrect notation e.g. missing dx. Need either the integral sign or the dx, dθ.
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain terms $\tan\theta + \cot^2\theta$	**B1 + B1**	Alternative: $\int \frac{2\cot\theta}{\sin^2\theta}\,d\theta = \int \frac{2\cos\theta}{\sin^3\theta}\,d\theta = -\frac{1}{\sin^2\theta}(+C)$
	Form an equation for the constant of integration, or use limits $x = 2$, $\theta = \frac{1}{4}\pi$, in a solution with **at least two correctly obtained terms** of the form ax^2, $b\tan\theta$ and $c\cot^2\theta$, where $abc \neq 0$	**M1**	Need to have 3 terms. Constant of correct form.
	State correct solution in any form, e.g. $\frac{1}{2}x^2 = \tan\theta + \cot^2\theta$	**A1**	or $\frac{1}{2}x^2 = \tan\theta + \operatorname{cosec}^2\theta - 1$ If everything else is correct, allow a correct final answer to imply this A1.
	Substitute $\theta = \frac{1}{6}\pi$ and obtain answer $x = 2.67$	**A1**	2.6748… $\sqrt{\frac{18+2\sqrt{3}}{3}}$ If see a correctly rounded value ISW.
		7	

Question 61

(a)	Separate variables correctly	B1	$\int \frac{1}{x}\,dx = \int k e^{-0.1t}\,dt$
	Obtain term $\ln x$	B1	
	Obtain term $-10ke^{-0.1t}$	B1	Not from $\int xe^{-0.1t}\,dt$
	Use $x = 20$, $t = 0$ to evaluate a constant or as limits in a solution containing terms $a\ln x, be^{-0.1t}$ where $ab \neq 0$	M1	
	Obtain $\ln x = 10k\left(1 - e^{-0.1t}\right) + \ln 20$	A1	or equivalent ISW
		5	
(b)	Use $x = 40$, $t = 10$ to find k or $10k$	M1	Available for their function of the correct structure even if they found no constant in **(a)**.
	Obtain $10k = 1.09654$	A1	or equivalent e.g. $10k = \dfrac{\ln 2}{1 - e^{-1}}$
	State that x tends to 59.9	A1	Need a number, not an expression for that value 3 sf or better 59.87595…..
		3	

Question 62

Separate variables correctly and obtain e^{-3y} and $\sin^2 2x$ on the opposite sides	B1	
Obtain term $-\dfrac{1}{3}e^{-3y}$	B1	
Use correct double angle formula for $\sin^2 2x = (1/2)[1 - \cos 4x]$	M1	
Obtain terms $\dfrac{1}{2}\left[x - \dfrac{1}{4}\sin 4x\right]$ oe	A1	
Use $x = 0$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form ax and $b\sin 4x$ and $ce^{\pm 3y}$	M1	
Obtain correct answer in any form e.g. $-\dfrac{1}{3}e^{-3y} = \dfrac{1}{2}\left[x - \dfrac{1}{4}\sin 4x\right] - \dfrac{1}{3}$	A1	
Substitute $x = \frac{1}{2}$ and obtain $y = 0.175$ or $-\frac{1}{3}\ln\left(\frac{1}{4} + \frac{3}{8}\sin 2\right)$	A1	OE ISW
	7	

Question 63

Separate the variables correctly	B1	$\frac{y+4}{y^2+4}\mathrm{d}y = \frac{1}{x}\mathrm{d}x$.
Obtain $\ln x$	B1	
Split the fraction and integrate to obtain $p\ln\left(y^2+4\right)$ or $q\tan^{-1}\frac{y}{2}$ correctly	*M1	Only following subdivision into $\frac{y}{y^2+4}+\frac{4}{y^2+4}$. If no subdivision seen then both terms $p\ln\left(y^2+4\right)$ and $q\tan^{-1}\frac{y}{2}$ must be present.
Obtain $\frac{1}{2}\ln\left(y^2+4\right)$	A1	
Obtain $2\tan^{-1}\frac{y}{2}$	A1	
Use $\left(4, 2\sqrt{3}\right)$ in an expression containing at least 2 of $a\ln x, b\ln\left(y^2+4\right)$ and $c\tan^{-1}\frac{y}{2}$ to obtain constant of integration	DM1	Allow one sign or arithmetic error e.g. $\frac{2\pi}{3}$. May use $\left(4, 2\sqrt{3}\right)$ and $(x, 2)$ as limits to find x for the final 3 marks.
Correct solution (any form) e.g. $\frac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\frac{y}{2}=\ln x+\frac{2\pi}{3}$ or $\frac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\frac{y}{2}=\ln x+2\tan^{-1}\sqrt{3}+\frac{1}{2}\ln 16-\ln 4$	A1	However solution not asked for so allow $\frac{1}{2}\ln 8+2\tan^{-1}1=\ln x+2\tan^{-1}\sqrt{3}+\frac{1}{2}\ln 16-\ln 4$.
Obtain $\sqrt{8}e^{-\frac{1}{6}\pi}$ or 1.68 or more accurate or $2\sqrt{2}e^{-\frac{1}{6}\pi}$ or $\frac{\sqrt{8}}{e^{\frac{1}{6}\pi}}$ or $e^{0.516}$	A1	ISW Must remove ln so $x = e^{(\ln 2\sqrt{2} - \pi/6)}$ A0.

Alternative method for first *M1 A1 A1

$p\left(\left(y+4\right)\tan^{-1}\frac{y}{2}-\int\tan^{-1}\frac{y}{2}\mathrm{d}y\right)$	*M1	Allow sign error.
$\left(y+4\right)\frac{1}{2}\tan^{-1}\frac{y}{2}-\frac{y}{2}\tan^{-1}\frac{y}{2}+\int\frac{y}{y^2+4}\mathrm{d}y$	A1	
Obtain $2\tan^{-1}\frac{y}{2}+\frac{1}{2}\ln\left(y^2+4\right)$	A1	
	8	

Question 64

(a)	Separate variables correctly	B1	$\int \frac{1}{4+9y^2}\,dy = \int e^{-(2x+1)}\,dx$. Condone missing integral signs **or** dx and dy missing.
	Obtain term $-\frac{1}{2}e^{-2x-1}$	B1	OE e.g. $-\frac{1}{2e}e^{-2x}$.
	Obtain term of the form $a\tan^{-1}\left(\frac{3y}{2}\right)$	M1	
	Obtain term $\frac{1}{6}\tan^{-1}\left(\frac{3y}{2}\right)$	A1	OE e.g. $\frac{1}{9}\times\frac{3}{2}\tan^{-1}\frac{3y}{2}$.
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms of the form $a\tan^{-1}(by)$ and $ce^{\pm(2x+1)}$	M1	If they rearrange before evaluating the constant, the constant must be of the correct form.
	Obtain correct answer in any form	A1	e.g. $\frac{1}{6}\tan^{-1}\frac{3y}{2} = \frac{1}{2}e^{-3} - \frac{1}{2}e^{-(2x+1)}$.
	Obtain final answer $y = \frac{2}{3}\tan\left(3e^{-3} - 3e^{-2x-1}\right)$	A1	OE Allow with $3e^{-3} = 0.149....$
		7	
(b)	State that y approaches $\frac{2}{3}\tan\left(3e^{-3}\right)$	B1 FT	Or exact equivalent. The FT is on correct work on a solution containing e^{-2x-1}. Condone $y = \ldots$ Accept correct answer stated with minimal wording. 0.10032… is not exact so B0.
		1	

Question 65

Correct separation of variables	B1	$\int \sin^2 3y\,dy = \int 4\sec 2x\tan 2x\,dx$ or equivalent. Condone missing integral signs or dx and dy.
Integrate to obtain $k\sec 2x$	M1	
Obtain $2\sec 2x$	A1	
Use double angle formula and integrate to obtain $py + q\sin 6y$	M1	Or two cycles of integration by parts.
Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y$	A1	
Use $y = 0$, $x = \frac{\pi}{6}$ in a solution containing terms $\lambda\sec 2x$ and $\mu\sin 6y$ to find the constant of integration	M1	
Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y = 2\sec 2x - 4$	A1	Or equivalent seen or implied by $\frac{\pi}{2}\left(-\frac{1}{12}\sin\pi\right) = 2\sec 2x - 4$.
Obtain $x = 0.541$	A1	From correct working (not by using the calculator to integrate).
	8	

Question 66

Separate variables correctly and reach $a\sec^2 3y$ or be^{-4x}	B1	Condone missing integral signs or dy and dx, but allow if recognisable integrals follow. Not for $1/\cos^2 3y$ and $1/e^{4x}$.
Obtain term $-\frac{1}{4}e^{-4x}$	B1	Can recover the previous B1 if de^{-4x} seen here.
Obtain only a term of the form $a\tan 3y$	M1	Can recover the first B1 if $a\tan 3y$ seen here.
Obtain term $\frac{1}{3}\tan 3y$	A1	
Use $x = 2$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan by$ and $ce^{\pm 4x}$	M1	May see $\tan by$ and $e^{\pm 4x}$ here.
Obtain correct answer in any form	A1	e.g. $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + \frac{1}{4}e^{-8}$ or $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + 8.39 \times 10^{-5}$
Obtain final answer $y = \frac{1}{3}\tan^{-1}\left(\frac{3}{4}e^{-8} - \frac{3}{4}e^{-4x}\right)$	A1	ISW OE e.g. $y = \frac{1}{3}\tan^{-1}\left(2.52\times10^{-4} - \frac{3}{4}e^{-4x}\right)$
	7	

Question 67

(a)	Correct separation of variables.	B1	$\int \frac{1}{y^2+y}\,dy = \int -\frac{1}{x^2}\,dx$. Condone missing integral signs or missing dx, dy, but not both.
	Obtain $\frac{1}{x}$	B1	
	Express $\frac{1}{y^2+y}$ in partial fractions or express the denominator of the fraction as a difference of two squares	*M1	Allow for the correct split of $\frac{\pm 1}{(y^2 \pm y)}$.
	Obtain $\frac{1}{y} - \frac{1}{y+1}$ or $\frac{1}{\left(y+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$	A1	Allow if coefficients for the partial fractions are correct but followed by an error.
	Obtain $\ln y - \ln(y+1)$	A1	Or equivalent, dependent on where they left the minus sign.
	Use $x=1, y=1$ to find constant of integration or as limits in a definite integral in an expression containing terms of the form $\frac{p}{x}, q\ln y$ and $r\ln(1+y)$	DM1	$\ln\frac{1}{2} = 1 + C$ If they rearrange the equation before finding the constant of integration then the constant must be of the correct form.
	Correct equation in x and y	A1	$\ln\frac{y}{1+y} = \frac{1}{x} - 1 + \ln\frac{1}{2}$.
	Obtain $y = \frac{e^{\frac{1}{x}-1}}{2 - e^{\frac{1}{x}-1}}$	A1	Or equivalent e.g. $y = \frac{1}{2e^{1-\frac{1}{x}} - 1}$, $y = \frac{1}{e^{1-\frac{1}{x}+\ln 2} - 1}$. Accept with decimal value for e^{-1}.
		8	

(b)	State that y approaches $\frac{1}{2e-1}$	B1 FT	Or exact equivalent. Condone $y=\frac{1}{2e-1}$. FT on an expression in $e^{\frac{1}{x}}$.
		1	

Question 68

Separate variables correctly	B1	$\int \tan\theta\,d\theta = \int \frac{x}{x^2+3}\,dx$. Condone missing integral signs **or** missing dx, dθ. Can be implied by later work.
Obtain term $-\ln(\cos\theta)$	B1	Or equivalent e.g. $\ln(\sec\theta)$.
Obtain term of the form $a\ln\left(x^2+3\right)$	M1	
Obtain term $\frac{1}{2}\ln\left(x^2+3\right)$	A1	
Use $x = 1$, $\theta = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\ln\left(x^2+3\right)$ and $b\ln(\cos\theta)$	M1	If they have rearranged then the constant must be of the correct form.
Obtain correct answer in any form	A1	$\frac{1}{2}\ln\left(x^2+3\right) = -\ln\cos\theta + \ln 2$.
Obtain final answer $x^2 = \frac{4}{\cos^2\theta} - 3$	A1	Or equivalent e.g. $x^2 = 4\sec^2 x - 3$. lns removed.
	7	

Question 69

Separate variables correctly	**B1**	$\int (1+y)e^{-3y}\,dy = \int \frac{1}{1+\cos 2\theta}\,d\theta$. Allow $1/e^{3y}$ and missing integral signs.
Integrate to obtain $p(1+y)e^{-3y} + \int qe^{-3y}\,dy$	**M1**	Allow unless clear evidence that formula used has a + sign.
Obtain $\frac{-1}{3}(1+y)e^{-3y} + \int \frac{1}{3}e^{-3y}\,dy$	**A1**	Allow unsimplified.
Obtain $\frac{-1}{3}(1+y)e^{-3y} - \frac{1}{9}e^{-3y}\,(+A)$	**A1**	Condone no constant of integration.
Use correct double angle formula to obtain $\int \frac{1}{2\cos^2\theta}\,d\theta$	**B1**	
Obtain $k\tan[+B]$	**B1**	Condone no constant of integration.
Use $y=0,\ \theta=\frac{\pi}{4}$ to evaluate a constant of integration in an expression of the form αye^{-3y}, βe^{-3y} and $\gamma\tan\theta$ only.	**M1***	$\frac{1}{2} = -\frac{1}{3} - \frac{1}{9} + C \quad \left(C = \frac{17}{18}\right)$ Allow αye^{3y} and βe^{3y}. Must have integrated LHS twice.
Use $y=1$	**DM1**	$\frac{-(1+1)}{3e^3} - 1\left(9e^3\right) = \frac{1}{2}\tan\theta - \frac{17}{18}$. Must have integrated LHS.
Obtain $\tan\theta = \frac{17}{9} - \frac{14}{9}e^{-3}$	**A1**	Or exact equivalent . Exact ISW. Allow $\theta = \tan^{-1}\left(\frac{17}{9} - \frac{14}{9}e^{-3}\right)$. If x instead of θ then withhold final **A1**.
	9	

Question 70

(a)

	Obtain $\frac{dV}{dt} = [\pm]\frac{k}{t}$ or $\frac{dV}{dt} = [\pm]\frac{1}{kt}$	**B1**	
	Obtain $\frac{dV}{dx} = 20x - 3x^2$	**B1**	
	Correct use of chain rule involving k	**M1**	Use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$. Expressions for $\frac{dV}{dt}$ and $\frac{dV}{dx}$ must be seen to get M1.
	Obtain $\frac{dx}{dt} = [\pm]\frac{k}{t(20x-3x^2)}$ or equivalent,	**A1**	If this expression is first seen with numerical values, allow A1 when their value of k is substituted back into the general expression.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = -\frac{20}{37}$ to obtain given answer which must be stated $\frac{dx}{dt} = -\frac{20}{37}$ needed to score final A1	**A1**	$\frac{dx}{dt} = \frac{-1}{2t(20x-3x^2)}$ AG Need to at least see $-\frac{20}{37} = \frac{k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $\frac{k}{t}$ or $-\frac{20}{37} = \frac{-k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct k. $\frac{dx}{dt} = \frac{20}{37}$ seen anywhere, then A0.
		5	

(b)

	Separate variables correctly & integrate at least one side correctly	**B1**	
	Obtain terms $10x^2 - x^3$	**B1**	May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2} - \frac{3x^3}{3}$.
	Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a \ln t$ for $\frac{a}{t}$.	**B1FT**	FT sign and position of 2 from their separation but B0 if error from later manipulation.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form x^2, x^3 and $\ln t$ (or $\ln 2t$)	**M1**	Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for c or e^c.
	Obtain correct answer in any form, for example $10x^2 - x^3 = -\frac{\ln t}{2} + \frac{19}{8} + \frac{\ln 0.1}{2}$	**A1**	$10x^2 - x^3 = -\frac{\ln 2t}{2} + \frac{19}{8} + \frac{\ln 0.2}{2}$ or $10x^2 - x^3 = -\frac{\ln t}{2} + 2.5 - 0.125 - 1.15\ldots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45
	Obtain answer $t = \frac{1}{10}e^{2x^3 - 20x^2 + \frac{19}{4}}$ or equivalent	**A1**	ISW Need $t = \ldots\ldots\ldots$ E.g. $\frac{0.1}{e^{20x^2 - 2x^3 - \frac{19}{4}}}$, $\frac{e^{2x^3 + \frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10}e^{\frac{19}{4}}e^{2x^3 - 20x^2}$. Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3 - 20x^2 + 2.45}$. A0 if $e^{\ln\frac{1}{10}}$ present in final answer.
		6	

Question 71

(a)	Use of correct chain rule (and correct quotient rule) and $\cos^{-3}\theta$	**M1**	Obtain $k\times(\cos\theta)^{-4}\times\sin\theta$ or equivalent.
	$\frac{dy}{d\theta}=-3\times-\sin\theta(\cos\theta)^{-4}=3\sin\theta\sec^4\theta$ Must be expressed in the given form	**A1**	Obtain given answer from full and correct working (signs must be shown), but condone $\frac{d}{d\theta}(\sec^3\theta)=\ldots$ and $y'(\theta)$.
		2	
(b)	Separate variables: $\int\frac{\sin\theta}{\cos^4\theta}d\theta=\int\frac{(x+3)}{(x^2+9)}dx$	**B1**	Or $\int\frac{3\sin\theta}{\cos^4\theta}d\theta=\int\frac{3x+9}{x^2+9}dx$. Condone missing integral signs or missing dx or dθ, but not both.
	Obtain $p\sec^3\theta(+A)$	**B1**	Correct form, p any constant but not 0.
	Use $\int\frac{3x+9}{x^2+9}dx=\int\left(\frac{3x}{x^2+9}+\frac{9}{x^2+9}\right)dx$ and obtain $q\ln(x^2+9)$ or $r\tan^{-1}\frac{x}{3}(+C)$.	***M1**	Might have one third of both sides. Alt: substitute $x=3\tan\phi$ to obtain $q\int 1+\tan\phi\, d\phi$; condone if have θ in place of ϕ in this method.
	Obtain $q\ln(x^2+9)$ and $r\tan^{-1}\frac{x}{3}(+C)$	**DM1**	Obtain $q(\phi\mp\ln(\cos\phi))$ OE.
	Obtain $\sec^3\theta=\frac{3}{2}\ln(x^2+9)+3\tan^{-1}\frac{x}{3}(+C)$ or equivalent	**A1**	Or might see a third of both sides. Must have 2 different variables.
	Use $\theta=\frac{1}{3}\pi, x=3$ in an equation including $p\sec^3\theta$, $q\ln(x^2+9)$ and $r\tan^{-1}\frac{x}{3}$ to evaluate the constant of integration	**M1**	Or as limits in a definite integral. Limits for ϕ are 0 and $\frac{1}{4}\pi$.
	Obtain constant = $8-\frac{3}{2}\ln 18-\frac{3}{4}\pi$	**A1**	OE, e.g. 1.308… to at least 3sf.
	Obtain $\cos\theta=0.601$	**A1**	Accept AWRT 0.601.
		8	

Question 72

(a)	State or imply equation of the form $\frac{dx}{dt} = kx(300-x)$ and use $\frac{dx}{dt} = 0.2$ and $x = 1$	**M1**	M0 for verification.
	Obtain $k = \frac{1}{1495}$ and rearrange to the given answer	**A1**	$1495\frac{dx}{dt} = x(300-x)$.
		2	
(b)	Separate variables correctly	**B1**	$\int \frac{1}{x(300-x)}dx = \int \frac{1}{1495}dt$
	Correct integration of t term	**B1**	E.g. obtain t or $\frac{t}{1495}$.
	State or imply partial fractions of the form $\frac{A}{x} + \frac{B}{300-x}$	**B1**	
	Correct method to find A or B	**M1**	$A = \frac{1}{300}$ and $B = \frac{1}{300}$. May see $A = B = \frac{1495}{300} = \frac{299}{60}$.
	Obtain terms $\frac{1495}{300}\ln x - \frac{1495}{300}\ln(300-x)$	**A1**	OE. May see $\frac{1}{300}\ln x - \frac{1}{300}\ln(300-x)$.
	Use $t = 0$, $x = 1$ to evaluate a constant or as limits in a solution containing terms of the form $\ln x$, $\ln(300-x)$ and t.	**M1**	
	Obtain correct answer in any form	**A1**	E.g. $\frac{1495}{300}[\ln x - \ln(300-x)] = t - \frac{1495}{300}\ln 299$.
	Use law of logarithms twice to obtain an expression for t	**M1**	
	Obtain final answer $t = \frac{299}{60}\ln\frac{299x}{300-x}$ or equivalent single logarithm	**A1**	
		9	

Question 73

(a)	State that $\frac{dV}{dt} = 50000 - 600h$	**B1**	May be seen as $\frac{dV}{dt} = 50000$ and $\frac{dV}{dt} = [-]600h$. When put together (may be in the chain rule) B1 can be awarded.
	[Use $V = 40000h$ to] obtain $\frac{dV}{dh} = 40000$ and use this and *their* $\frac{dV}{dt}$ in the correct chain rule to obtain $\frac{dh}{dt}$ or [Use $V = 40000h$ to] obtain $\frac{dV}{dt} = 40000\frac{dh}{dt}$ and equate to *their* $\frac{dV}{dt}$	**M1**	$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ E.g. $\frac{50000-600h}{40000} = \frac{dh}{dt}$ E.g. $40000\frac{dh}{dt} = 50000 - 600h$

	Answer	Mark	Guidance
(a)	Obtain $200\frac{dh}{dt} = 250 - 3h$ from full and correct working	A1	AG $\frac{dh}{dt} = \frac{50000 - 600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen **SC B1** (1/3). $\frac{dV}{dt} = 50000 - 600h$ B1 followed by $\frac{dh}{dt} = \frac{50000 - 600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen, **SC B1** 2/3. $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ followed by $50000 - 600h = 40000\frac{dh}{dt}$ OE, leading to given answer with no other working, or no incorrect working seen, B1 for implied $\frac{dV}{dt}$ and **SC B1** 2/3.
		3	
(b)	Separate variables correctly and integrate one side correctly	**M1**	E.g. $\int \frac{1}{250 - 3h} dh = \int \frac{1}{200} dt$. Integral signs may be omitted, 200 may be on opposite side.
	Obtain $-\frac{1}{3}\ln\lvert 250 - 3h\rvert = \frac{t}{200} (+C)$	**A1**	OE Condone missing "$+C$" and lack of modulus signs.
	Use $t = 0$, $h = 50$ in an expression containing $\ln(250 - 3h)$ or $\ln\lvert 250 - 3h\rvert$ to find the constant of integration.	**M1**	Or equivalent use of limits 50 and 80.
	Obtain $C = -\frac{1}{3}\ln 100$	**A1**	OE, e.g. $\frac{1}{3}\ln\frac{100}{250 - 3h} = \frac{t}{200}$, or $-\frac{200}{3}\ln\left(\frac{10}{100}\right)$. With or without modulus signs on the log terms.
	$t = 150$	**A1**	
		5	

Question 74

(a)	Obtain $\frac{dV}{dt} = 40\pi - 0.8\pi r$ or equivalent	B1	Need a complete correct statement seen or implied.
	Obtain $\frac{dV}{dr} = 4\pi r^2$ or equivalent e.g. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	B1	Need a complete correct statement seen or implied.
	Use the chain rule to obtain **given answer (including the derivative)**	B1	Allow if $\frac{dr}{dt} = \frac{50 - r}{5r^2}$ follows $\frac{dr}{dt} = \frac{40 - 0.8r}{4r^2}$ without further explanation (π already cancelled) and no incorrect statements seen.
		3	
(b)	Commence division and reach quotient of the form $-5r \pm 250$ or $5r^2 = (50 - r)(Ar + B) + C$ and reach $A = -5$ and $B = \pm 250$	M1	Allow M1 if divide by $r - 50$ to obtain $5r \pm 250$.
	Obtain quotient $-5r - 250$	A1	Do not need to state which is quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder 12 500	A1	Note: 12 500 following division by $r - 50$ is correct and scores this A1 ISW.
			SC B1 only for correct use of remainder theorem to obtain correct remainder.
		3	
(c)	Prepare to integrate e.g. separate variables correctly Or express in the form $\frac{dt}{dr} = \frac{5r^2}{50 - r}\left(= -(5r + 250) + \frac{12500}{50 - r}\right)$	B1FT	$\int \frac{5r^2}{50 - r} dr = \int 1 dt$ Condone missing dr, dt or missing integral signs, but not both. Follow their division in **(b)** if substitute before separating.
	Obtain term t	DB1	
	Obtain terms $\frac{A}{2}r^2 + Br - C\ln(50 - r)$	M1	From *their* $Ar + B + \frac{C}{50 - r}$ in **(b)** where $ABC \neq 0$. Allow a single slip in the coefficients.
	Obtain terms $-\frac{5}{2}r^2 - 250r - 12500\ln(50 - r)$	A1FT	FT *their* **(b)**, provided of the correct form.
	Use $t = 0$, $r = 0$ to evaluate a constant or as limits in a solution containing terms of the form r^2, r, $\ln(50 - r)$ and t	M1	
	Obtain final answer $t = -\frac{5}{2}r^2 - 250r - 12500\ln(50 - r) + 12500\ln 50$	A1	OE Must be $t = \ldots$.. Allow with $12500\ln 50 = 48900$ or better.
		6	
(d)	Obtain $t = 70.5$	B1	May be more accurate (70.4605…).
		1	

Question 75

(a)	$\frac{dV}{dt} = \pm k\sqrt{V}$ or $\frac{dV}{dt} = 16\pi\frac{dh}{dt}$	B1	SOI
	Correct use of chain rule and $V = 16\pi h$	M1	OE, e.g. $\frac{dV}{dt} = 16\pi\frac{dh}{dt}$.
	Obtain $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{-k\sqrt{16\pi h}}{16\pi}$	A1	Any equivalent form in terms of h.
	$= -\left(\frac{k}{4\sqrt{\pi}}\right)\sqrt{h} = -\lambda\sqrt{h}$ since $\frac{k}{4\sqrt{\pi}}$ is constant	A1	Obtain given answer from full and correct working.
		4	
(b)	Separate variables correctly and commence integration	*M1	$\int \frac{1}{\sqrt{h}} dh = \int -\lambda dt$ OE
	Obtain $-\lambda t = 2\sqrt{h}(+C)$	A1	
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form an equation in λ and/or C	DM1	OE, e.g. $0 = 4 + C$ or $-20\lambda = 2 \times 1.5 + C$.
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form a second equation in λ and/or C and solve	DM1	$C = -4, \lambda = \frac{1}{20}$ OE, e.g. $-\frac{t}{20} = 2\sqrt{h} - 4$.
	Hence $t = 80 - 40\sqrt{h}$	A1	Must be seen.
	Time to empty the tank is 80 minutes	A1	
		6	

Question 76

Use correct double angle formula to express $\sin^2 2\theta$ in terms of $\cos 4\theta$	B1	$\sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$
Separate variables correctly and reasonable attempt at integration of at least one side	M1	Position of $(x + 5)$ or $\left(\frac{1}{5}x+1\right)$ and $\sin^2 2\theta$ sufficient for correct separation.
Obtain term $5\ln\left(\frac{1}{5}x+1\right)$	B1	OE May see $5\ln(x+5)$.
Obtain term $\frac{1}{2}(\theta - \frac{1}{4}\sin 4\theta)$	B1 FT	Allow $\frac{1}{2}\left(\pm\theta \pm \frac{1}{4}\sin 4\theta\right)$ from $= \frac{1}{2}(\pm 1 \pm \cos 4\theta)$.
Use $x = 5$ when $\theta = 0$ to evaluate a constant or as limits in a solution containing terms of the form $\ln\left(\frac{1}{5}x+1\right)$, θ and $\sin 4\theta$	M1	OE
Obtain correct answer in any form	A1	E.g. $5\ln(x+5) = \frac{1}{2}(\theta - \frac{1}{4}\sin 4\theta) + 5\ln 10$
Obtain final answer $x = 10\exp\left[\frac{1}{10}\left(\theta - \frac{1}{4}\sin 4\theta\right)\right] - 5$ or equivalent with ln removed	A1 FT	$x = 10\exp\left[\frac{1}{2}\left(\pm\theta \pm \frac{1}{4}\sin 4\theta\right)\right] - 5$ Must remove ln from $\ln(x+5) = \frac{1}{2}(\pm\theta \pm \frac{1}{4}\sin 4\theta) + \alpha\ln 10$.
	7	

Question 77

(a)	Separate variables correctly	B1	
	Use correct double angle formula to simplify integral in y	*M1	$\int \frac{\sin 4y}{\sin 2y} \mathrm{d}y = \int 2\cos 2y \, \mathrm{d}y$
	Obtain $\sin 2y$	A1	
	Commence integration by parts and obtain $px\cos 3x + q\int \cos 3x \, \mathrm{d}x$	*M1	
	Obtain $-\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x \, \mathrm{d}x$	A1	
	Complete integration and obtain $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$	A1	
	Use $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$ in an expression with $\sin 2y$, $x\cos 3x$ and $\sin 3x$ to obtain the constant of integration	DM1	$\frac{1}{2} = 0 - \frac{1}{9} + c$
	Obtain $\sin 2y = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + \frac{11}{18}$	A1	OE ISW
		8	
(b)	Solve $\sin 2y = \frac{11}{18}$ to obtain one solution, e.g. 0.329	M1	AWRT Allow for *their* constant from a solution involving $\sin 2y$, $x\cos 3x$ and $\sin 3x$. M0 for answers in degrees.
	Obtain a second solution, e.g. 1.24, and no others in range	A1	AWRT
		2	

Question 78

Separate variables correctly	**B1**	$\int \frac{1}{4x+3}\,\mathrm{d}x = \int \frac{\cos 2\theta}{\sin 2\theta}\,\mathrm{d}\theta$ Can be implied by obtaining both correct integrals.
Obtain term $\frac{1}{4}\ln(4x+3)$	**B1**	OE or $\frac{1}{4}\ln\left(x+\frac{3}{4}\right)$.
Obtain term of the form $A\ln(\sin 2\theta)$	**M1**	Or $A\ln(k\sin 2\theta)$ OE, e.g. $P\ln a\sin\theta + Q\ln b\cos\theta$ from using the $\tan 2\theta$ formula. Or expanding $\cos 2\theta$ as $\cos^2\theta - \sin^2\theta$.
Obtain term $\frac{1}{2}\ln(\sin 2\theta)$	**A1**	OE Correct in any form, e.g. $\frac{1}{2}\ln a\sin\theta + \frac{1}{2}\ln b\cos\theta$.
Use $x = 0$ when $\theta = \frac{1}{12}\pi$ to evaluate a constant or as limits in a solution containing terms of the form $\ln(\sin 2\theta)$ and $\ln(4x+3)$ c = ... seen or implied	**M1**	E.g. $c = \frac{1}{4}\ln 3 - \frac{1}{2}\ln\sin\left(\frac{1}{6}\pi\right)$ $\frac{1}{4}\ln(4x+3) - \frac{1}{4}\ln 3 = \frac{1}{2}\ln(\sin 2\theta) - \frac{1}{2}\ln\frac{1}{2}$. Note that the constant may be expressed as a logarithm
Obtain correct answer in any form with the trigonometry evaluated	**A1**	E.g. $\frac{1}{4}\ln(4x+3) = \frac{1}{2}\ln(\sin 2\theta) + \frac{1}{4}\ln 12$ $\frac{1}{4}\ln(4x+3) = \frac{1}{2}\ln(\sin 2\theta) + 0.621$ $\frac{1}{4}\ln\frac{4x+3}{3} = \frac{1}{2}\ln(2\sin 2\theta)$
Obtain final answer $x = \frac{12\sin^2 2\theta - 3}{4}$	**A1**	OE Allow $\frac{e^{2.48...}\sin^2 2\theta - 3}{4}$. Allow 2.48. Allow $x = \frac{1}{4}\left(e^{2\ln(\sin 2\theta)+2.48...} - 3\right)$.
	7	

Question 79

(a)	Divide by (x^2-3) to obtain $x+k\ \ (k \neq 0)$	**M1**	$\begin{array}{r\|rrrr} & & & x & +5 \\ \hline x^2-3 & x^3 & +5x^2 & -2x & -15 \\ & x^3 & & -3x & \\ \hline & & +5x^2 & +x & \\ \hline & & +5x^2 & & -15 \\ \hline & & & x & \end{array}$
	Obtain quotient $x+5$	**A1**	
	Obtain remainder x	**A1**	ISW Allow $\frac{x}{x^2-3}$.
		3	
(b)	Separate variables correctly and obtain $\int 6y\,dy = 3y^2$	**B1**	OE from $\int 6y\,dy = \int \frac{x^3+5x^2-2x-15}{x^2-3}\,dx$.
	Obtain $\frac{1}{2}x^2 + 5x$	**B1ft**	Follow *their* linear quotient.
	Obtain $\frac{1}{2}\ln\left(x^2-3\right)$	**B1ft**	From the x term in *their* remainder $ax+b$.
	Use $y=2,\ x=2$ to evaluate the constant of integration in an integral containing $k\ln\left(x^2-3\right)$	**M1**	$12 = 2 + 10 + \frac{1}{2}\ln 1 + C \ \Rightarrow\ C = 0$
	Obtain $y^2 = \frac{1}{6}x^2 + \frac{5}{3}x + \frac{1}{6}\ln\left(x^2-3\right)$	**A1**	OE
		5	