

A-level
Topic : Partial Fraction
May 2013-May 2023
Questions

Question 1

Express $\frac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

Question 2

(i) Express $\frac{1}{x^2(2x + 1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x + 1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x + 1) \frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$.
Give your value of y in a form not involving logarithms. [7]

Question 3

Let $f(x) = \frac{2x^2 - 7x - 1}{(x - 2)(x^2 + 3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 4

(i) Express $\frac{7x^2 + 8}{(1 + x)^2(2 - 3x)}$ in partial fractions. [5]

(ii) Hence expand $\frac{7x^2 + 8}{(1 + x)^2(2 - 3x)}$ in ascending powers of x up to and including the term in x^2 ,
simplifying the coefficients. [5]

Question 5

(i) Express $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 6

Let $f(x) = \frac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $f(x)$ in the form $\frac{A}{2 - x} + \frac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\int_{-1}^1 f(x) dx = 3 \ln 3$. [5]

Question 7

Let $f(x) = \frac{x^2 - 8x + 9}{(1 - x)(2 - x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 8

Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 9

$$\text{Let } f(x) = \frac{11x + 7}{(2x - 1)(x + 2)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_1^2 f(x) dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

Question 10

(i) Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6} dx$. [8]

Question 11

$$\text{Let } f(x) = \frac{3x^3 + 6x - 8}{x(x^2 + 2)}.$$

(i) Express $f(x)$ in the form $A + \frac{B}{x} + \frac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x) dx = 3 - \ln 4$. [5]

Question 12

$$\text{Let } f(x) = \frac{4x^2 + 12}{(x + 1)(x - 3)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 13

$$\text{Let } f(x) = \frac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x) dx = 8 - \ln 3$. [5]

Question 14

$$\text{Let } f(x) = \frac{10x - 2x^2}{(x + 3)(x - 1)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 15

$$\text{Let } f(x) = \frac{3x^2 + x + 6}{(x + 2)(x^2 + 4)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 16

$$\text{Let } f(x) = \frac{x(6 - x)}{(2 + x)(4 + x^2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 17

(i) Express $\frac{1}{x(2x+3)}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

Question 18

$$\text{Let } f(x) = \frac{5x^2 - 7x + 4}{(3x+2)(x^2+5)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 19

$$\text{Let } f(x) = \frac{3x^2 - 4}{x^2(3x+2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_1^2 f(x) dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

Question 20

$$\text{Let } f(x) = \frac{4x^2 + 9x - 8}{(x+2)(2x-1)}.$$

(i) Express $f(x)$ in the form $A + \frac{B}{x+2} + \frac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 f(x) dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

Question 21

$$\text{Let } f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}.$$

(i) Express $f(x)$ in the form $A + \frac{B}{x + 2} + \frac{C}{2x - 1}$. [4]

(ii) Hence show that $\int_1^4 f(x) dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

Question 22

$$\text{Let } f(x) = \frac{5x^2 + x + 27}{(2x + 1)(x^2 + 9)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence find $\int_0^4 f(x) dx$, giving your answer in the form $\ln c$, where c is an integer. [5]

Question 23

$$\text{Let } f(x) = \frac{12x^2 + 4x - 1}{(x - 1)(3x + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 24

$$\text{Let } f(x) = \frac{x - 4x^2}{(3 - x)(2 + x^2)}.$$

(i) Express $f(x)$ in the form $\frac{A}{3 - x} + \frac{Bx + C}{2 + x^2}$. [4]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . [5]

Question 25

(i) Express $\frac{1}{4-y^2}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x \frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x . [6]

Question 26

$$\text{Let } f(x) = \frac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^0 f(x) dx = 1 + \frac{1}{2} \ln\left(\frac{3}{4}\right)$. [5]

Question 27

$$\text{Let } f(x) = \frac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 28

$$\text{Let } f(x) = \frac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^0 f(x) dx = 1 + \frac{1}{2} \ln\left(\frac{3}{4}\right)$. [5]

Question 29

$$\text{Let } f(x) = \frac{12 + 12x - 4x^2}{(2+x)(3-2x)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 30

$$\text{Let } f(x) = \frac{16 - 17x}{(2 + x)(3 - x)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 31

$$\text{Let } f(x) = \frac{10x + 9}{(2x + 1)(2x + 3)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x) dx = \frac{1}{2} \ln \frac{9}{5} + \frac{1}{5}$. [5]

Question 32

$$\text{Let } f(x) = \frac{2x(5 - x)}{(3 + x)(1 - x)^2}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 . [5]

Question 33

$$\text{Let } f(x) = \frac{x^2 + x + 6}{x^2(x + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\int_1^4 f(x) dx$ is $\frac{9}{4}$. [5]

Question 34

$$\text{Let } f(x) = \frac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x) dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

Question 35

$$\text{Let } f(x) = \frac{2 + 11x - 10x^2}{(1 + 2x)(1 - 2x)(2 + x)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 36

$$\text{Let } f(x) = \frac{2}{(2x - 1)(2x + 1)}.$$

(a) Express $f(x)$ in partial fractions. [2]

(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x - 1)^2} - \frac{1}{2x - 1} + \frac{1}{2x + 1} + \frac{1}{(2x + 1)^2}. \quad [2]$$

(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

Question 37

$$\text{Let } f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 38

$$\text{Let } f(x) = \frac{7x + 18}{(3x + 2)(x^2 + 4)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x) dx$. [6]

Question 39

$$\text{Let } f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 40

$$\text{Let } f(x) = \frac{5a}{(2x - a)(3a - x)}, \text{ where } a \text{ is a positive constant.}$$

(a) Express $f(x)$ in partial fractions. [3]

Question 41

$$\text{Let } f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}.$$

(a) Express $f(x)$ in partial fractions. [3]

Question 42

$$\text{Let } f(x) = \frac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 43

The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x . [11]

Question 44

Express $\frac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions. [5]

Question 45

$$\text{Let } f(x) = \frac{5x^2 + 8x - 3}{(x - 2)(2x^2 + 3)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

Question 46

$$\text{Let } f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}.$$

Express $f(x)$ in partial fractions. [5]

Question 47

$$\text{Let } f(x) = \frac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}.$$

Express $f(x)$ in partial fractions. [5]

Question 48

$$\text{Let } f(x) = \frac{4 - x + x^2}{(1 + x)(2 + x^2)}.$$

Express $f(x)$ in partial fractions. [5]

Question 49

$$\text{Let } f(x) = \frac{2x^2 + 7x + 8}{(1 + x)(2 + x)^2}.$$

Express $f(x)$ in partial fractions. [5]

Question 50

$$\text{Let } f(x) = \frac{5x^2 + x + 11}{(4 + x^2)(1 + x)}.$$

Express $f(x)$ in partial fractions. [5]

Question 51

$$\text{Let } f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}.$$

Express $f(x)$ in partial fractions. [5]

Question 52

$$\text{Let } f(x) = \frac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}.$$

Express $f(x)$ in partial fractions.

[5]

Question 53

$$\text{Let } f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}.$$

Express $f(x)$ in partial fractions.

[5]

