A-level

Topic : Partial Fraction

May 2013-May 2023

Answers

Question 1

State or imply correct form $\frac{A}{x}+\frac{Bx+C}{x^2+1}$	B1	
Use any relevant method to find at least one constant	M1	
Obtain $A=2$	A1	
Obtain $B=5$	A1	
Obtain $C=-3$	A1	[5]

Question 2

(i)	Use any relevant method to determine a constant	M1	
	Obtain one of the values $A = 1$, $B = -2$, $C = 4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[4]
	[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]		
(ii)	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
	Obtain $\ln y = -\frac{1}{2} - 2\ln(2x+1) + c$, or equivalent	A3√	
	Evaluate a constant, or use limits $x = 1$, $y = 1$, in a solution containing at least three terms of the form $k \ln y$, l/x, $m \ln x$ and $n \ln(2x+1)$, or equivalent	M1	
	Obtain solution $\ln y = -\frac{1}{2} - 2\ln x + 2\ln(2x+1) + c$, or equivalent	A1	
	Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]

Question 3

(i)	State or imply partial fractions are of the form $\frac{A}{x-2}+\frac{Bx+C}{x^2+3}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = -1$, $B = 3$, $C = -1$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	**[5]**

(ii) Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$ M1

Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction A1√+A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ M1

Obtain final answer $\frac{1}{6}+\frac{5}{4}x+\frac{17}{72}x^2$, or equivalent A1 **[5]**

Question 4

(i) Either

State or imply form $\frac{A}{1+x}+\frac{B}{(1+x)^2}+\frac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A=-1$ A1

Obtain $B=3$ A1

Obtain $C=4$ A1

Or

State or imply form $\frac{A}{1+x}+\frac{Bx}{(1+x)^2}+\frac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A=2$ A1

Obtain $B=-3$ A1

Obtain $C=4$ A1

(ii) Either

Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of third partial fraction up to x^2 term A1√

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 1

Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Question 5

(i)	Either	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 1$	A1	
		Obtain $B=\frac{3}{2}$	A1	
		Obtain $C=-\frac{1}{2}$	A1	[5]
	Or	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{Dx+E}{(1+2x)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 1$	A1	
		Obtain $D = 3$	A1	
		Obtain $E = 1$	A1	[5]

(ii) Obtain the first two terms of one of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$ $(1+2x)^{-1}$ and $(1+2x)^{-2}$ — M1

Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C — A1$\sqrt{}$ A1$\sqrt{}$ A1$\sqrt{}$

Obtain answer $\frac{4}{3}-\frac{8}{9}x+\frac{1}{27}x^2$ — A1 [5]

[If A, D, E approach used in part (i), give M1A1$\sqrt{}$A1$\sqrt{}$ for the expansions, M1 for multiplying out fully and A1 for final answer]

Question 6

(i)	Use a correct method for finding a constant	M1	
	Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	**4**
(ii)	Integrate and obtain term $-3\ln(2-x)$	B1$\sqrt{}$	
	Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
	Obtain term $\frac{3}{2}\ln(2+x^2)$	A1$\sqrt{}$	
	Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$	M1	
	Obtain given answer after full and correct working	A1	**5**

Question 7

(i)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	B1	
	Use a correct method to determine a constant	M1	
	Obtain one of $A=2$, $B=-1$, $C=3$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	**[5]**
(ii)	Use correct method to find the first two terms of the expansion of $(1-x)^{-1}, (2-x)^{-1}, (2-x)^{-2}, (1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1✓ + A1✓ + A1✓	
	Obtain final answer $\frac{9}{4}+\frac{5}{2}x+\frac{39}{16}x^2$, or equivalent	A1	**[5]**

Question 8

(i)	State or imply the form $\dfrac{A}{3-2x}+\dfrac{Bx+C}{x^2+4}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A=3$, $B=-1$, $C=-2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	**[5]**
(ii)	Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1✓+A1✓	
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	M1	
	Obtain final answer $\frac{1}{2}+\frac{5}{12}x+\frac{41}{72}x^2$, or equivalent	A1	**[5]**

Question 9

(i)	State or imply $f(x) \equiv \dfrac{A}{2x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A=2$, $B=-1$, $C=3$	A1	
	Obtain the remaining values A1 +	A1	**5**

(ii)	Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)-\frac{3}{x+2}$	B1√ + B1√ + B1√	
	Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
	Obtain the given answer following full and exact working	A1	**5**

Question 10

(i)	Either	Substitute $x=-1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
	Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]

(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2+kx$	**M1**	
	Obtain complete quotient $4x^2-5x-6$	**A1**	
	State form $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{4x+3}$	**A1**	
	Use relevant method for finding at least one constant	**M1**	
	Obtain one of $A=-2, B=1, C=8$	**A1**	
	Obtain all three values	**A1**	
	Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
	Obtain $-2\ln(x+1)+\ln(x-2)+2\ln(4x+3)$, condoning no use of modulus signs and absence of … $+c$	**A1**	[8]

Question 11

(i)	State or obtain $A = 3$	**B1**	
	Use a relevant method to find a constant	**M1**	
	Obtain one of $B = -4$, $C = 4$ and $D = 0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[5]

(ii)	Integrate and obtain $3x-4\ln x$	**B1√**	
	Integrate and obtain term of the form $k\ln(x^2+2)$	**M1**	
	Obtain term $2\ln(x^2+2)$	**A1√**	
	Substitute limits in an integral of the form $ax+b\ln x+c\ln(x^2+2)$, where $abc\neq 0$	**M1**	
	Obtain given answer $3-\ln 4$ after full and correct working	**A1**	[5]

Question 12

(i)	State or imply the form $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$	**B1**
	Use a correct method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = 3$, $C = 12$	**A1**
	Obtain a second value	**A1**
	Obtain a third value	**A1**
		[5]
(ii)	Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$, $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1$\surd$ + A1$\surd$ + A1$\surd$**
	Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent	**A1**
		[5]

Question 13

(i)	State or imply the form $A+\frac{B}{2x+1}+\frac{C}{x+2}$	**B1**	
	State or obtain $A = 2$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $B = 1$, $C = -2$	**A1**	
	Obtain the other value	**A1**	[5]
(ii)	Integrate and obtain terms $2x+\frac{1}{2}\ln(2x+1)-2\ln(x+2)$	**B3$\surd$**	
	Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	**M1**	
	Obtain the given answer after full and correct working	**A1**	[5]

Question 14

(i)	State or imply the form $\frac{A}{x+3}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$	**B1**
	Use a correct method to determine a constant	**M1**
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	[Mark the form $\frac{A}{x+3}+\frac{Dx+E}{(x-1)^2}$, where $A = -3$, $D = 1$, $E = 1$, B1M1A1A1A1 as above.]	[5]

(ii)	Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}, (1+\frac{1}{3}x)^{-1}$, $(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$	**M1**
	Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction	**A1**√ + **A1**√ + **A1**√
	Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2$, or equivalent	**A1**
		[5]

Question 15

(i)	State or imply the form $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$	**B1**	
	Use a correct method to determine a constant	**M1**	
	Obtain one of $A = 2$, $B = 1$, $C = -1$	**A1**	
	Obtain a second value	**A1**	
	Obtain a third value	**A1**	[5]
(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1**√ + **A1**√	
	Multiply out fully by $Bx + C$, where $BC \neq 0$	**M1**	
	Obtain final answer $\frac{3}{4} - \frac{1}{4}x + \frac{5}{16}x^2$, or equivalent	**A1**	[5]

Question 16

(i)	State or imply the form $\dfrac{A}{2+x} + \dfrac{Bx+C}{4+x^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = -2$, $B = 1$, $C = 4$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**

(ii)	Use correct method to obtain the first two terms of the expansion of $(1+\frac{1}{2}x)^{-1}$, $(2+x)^{-1}$, $(1+\frac{1}{4}x^2)^{-1}$ or $(4+x^2)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1√ + A1√**
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	**M1**
	Obtain final answer $\frac{3}{4}x - \frac{1}{2}x^2$	**A1**
	[Symbolic binomial coefficients, e.g. ${}_{-1}C_2$, are not sufficient for the first M1. The f.t. is on *A*, *B*, *C*.]	
	[In the case of an attempt to expand $x(6-x)(2+x)^{-1}(4+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	
	Total:	**5**

Question 17

(i)	Carry out a relevant method to obtain *A* and *B* such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	**M1**
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	**A1**
	Total:	**2**
(ii)	Separate variables and integrate one side	**B1**
	Obtain term $\ln y$	**B1**
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	**B2 FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	**A1**
	Obtain answer $y = 1.29$ (3s.f. only)	**A1**
	Total:	**7**

Question 18

(i)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+5}$		B1
	Use a relevant method to determine a constant		M1
	Obtain one of the values $A=2$, $B=1$, $C=-3$		A1
	Obtain a second value		A1
	Obtain the third value		A1
		Total:	**5**
(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient]		M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C. from part (i)		A1FT + A1FT
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$		M1
	Obtain **final answer** $\frac{2}{5}-\frac{13}{10}x+\frac{237}{100}x^2$, or equivalent		A1
		Total:	**5**

Question 19

(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x+2}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A=3$, $B=-2$, $C=-6$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2}+\frac{C}{3x+2}$ using same pattern of marks.]	A1
	Total:	**5**
(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	B3 FT
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	M1
	Obtain the given answer following full and exact working	A1
	Total:	**5**

Question 20

(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question 21

(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question 22

(i)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{Bx+C}{x^2+9}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = 3$, $B = 1$ and $C = 0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(ii)	Integrate and obtain term $\dfrac{3}{2}\ln(2x+1)$ (FT on A value)	**B1 FT**
	Integrate and obtain term of the form $k\ln(x^2+9)$	**M1**
	Obtain term $\dfrac{1}{2}\ln(x^2+9)$ (FT on B value)	**A1 FT**
	Substitute limits correctly in an integral of the form $a\ln(2x+1)+b\ln(x^2+9)$, where $ab\neq 0$	**M1**
	Obtain answer ln 45 after full and correct working	**A1**
		5

Question 23

(i)	State or imply the form $A+\dfrac{B}{x-1}+\dfrac{C}{3x+2}$	**B1**
	State or obtain $A = 4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B = 3$, $C = -1$	**A1**
	Obtain the other value	**A1**
		5

(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\frac{1}{2}-\frac{9}{4}x-\frac{33}{8}x^2$	**A1**
		5

Question 24

(i)	Use a correct method to find a constant	**M1**
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4

(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $(2+x^2)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**
	Multiply out their expansion, up to the terms in x^3, by $Bx + C$, where $BC \neq 0$	**M1**
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**
		5

Question 25

6(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	**M1**
	Obtain $A = B = \frac{1}{4}$	**A1**
	Total:	**2**
6(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2+y)$ or $c \ln (2-y)$	**M1**
	Obtain term $\ln x$	**B1**
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	**A1FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2+y)$ and $c \ln (2-y)$	**M1**
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	**A1**
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	**A1**
	Total:	**6**

Question 26

(i)	State or imply the form $\frac{A}{2-x} + \frac{B}{3+2x} + \frac{C}{(3+2x)^2}$	**B1**
	Use a correct method to find a constant	**M1**
	Obtain one of $A = 1$, $B = -1$, $C = 3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\frac{A}{2-x} + \frac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	**A1**
		5

(ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	B3ft
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1
		5

Question 27

(i)	State or imply the form $\frac{A}{1-2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is "correct")	M1
	Obtain one of $A = 1$, $B = 3$, $C = -2$	A1
	Obtain a second value	A1
	Obtain the third value	A1
	[Mark the form $\frac{A}{1-2x}+\frac{Dx+E}{(2-x)^2}$, where $A = 1$, $D = -3$ and $E = 4$, B1M1A1A1A1 as above.]	
		5

(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3ft
	Obtain **final** answer $2+\frac{9}{4}x+4x^2$	A1
	[For the A, D, E form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]	
		5

Question 28

(i)	State or imply the form $\frac{A}{2-x}+\frac{B}{3+2x}+\frac{C}{(3+2x)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A=1$, $B=-1$, $C=3$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{A}{2-x}+\frac{Dx+E}{(3+2x)^2}$, where $A=1$, $D=-2$ and $E=0$, B1M1A1A1A1 as above.]	A1
		5

ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	**B3ft**
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	**M1**
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**
		5

Question 29

(i)	State or imply the form $A+\frac{B}{2+x}+\frac{C}{3-2x}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=2$, $B=-4$ and $C=6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft +A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $2+\frac{7}{3}x+\frac{7}{18}x^2$	**A1**
		5

Question 30

(i)	State or imply the form $\frac{A}{2+x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $A = 2$, $B = 2$, $C = -7$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5

Question 31

(i)	State or imply the form $\frac{A}{2x+1} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	**B1**
	Use a correct method to find a constant	**M1**
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**
	[Mark the form $\frac{A}{2x+1} + \frac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	
		5

(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\frac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	B1 B1 B1
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	M1
	Obtain the given answer following full and correct working	A1
		5

Question 32

(i)	State or imply the form $\frac{A}{3+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A=-3$, $B=-1$, $C=2$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5

Question 33

(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	M1
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	A1
		A1
		A1
	Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2+\frac{190}{27}x^3$	A1
		5

Question 33

(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A=-1$, $B=3$, $C=2$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5

3(ii)	Integrate and obtain terms $\ln x-\frac{3}{x}+2\ln(x+2)$	B1FT + B1FT + B1FT
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc \neq 0$	M1
	Obtain $\frac{9}{4}$ following full and exact working	A1
		5

Question 34

(i)	State or imply the form $\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A=4$, $B=-1$, $C=0$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(ii)	Integrate and obtain term $2\ln(2x-1)$	B1FT
	Integrate and obtain term of the form $k\ln(x^2+2)$	*M1
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	A1FT
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab\neq 0$	DM1
	Obtain answer ln 27 after full and correct exact working	A1
		5

Question 35

(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{1-2x}+\frac{C}{2+x}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=-2$, $B=1$ and $C=4$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(b)	Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1FT** + **A1FT** + **A1FT**
	Obtain final answer $1+5x-\frac{7}{2}x^2$	**A1**
		5

Question 36

(a)	State or imply the form $\frac{A}{2x-1}+\frac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A=1$, $B=-1$	**A1**
		2
(b)	Square the result of part (a) and substitute the fractions of part (a)	**M1**
	Obtain the given answer correctly	**A1**
		2
(c)	Integrate and obtain $-\frac{1}{2(2x-1)}-\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(2x+1)-\frac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

Question 37

(a)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2+3x}+\dfrac{C}{(2+3x)^2}$	**B1**
	Use a correct method for finding a coefficient	**M1**
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5

(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** + **A1 FT** + **A1 FT**
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**
		5

Question 38

(a)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+4}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=3$, $B=-1$, $C=3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(b)	Integrate and obtain $\ln(3x+2)$...	**B1 FT**
	State a term of the form $k\ln\left(x^2+4\right)$.	**M1**
	$...-\frac{1}{2}\ln\left(x^2+4\right)...$	**A1 FT**
	$...+\frac{3}{2}\tan^{-1}\frac{x}{2}$	**B1 FT**
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\tan^{-1}\left(\frac{x}{2}\right)$, and subtract in correct order	**M1**
	Obtain answer $\frac{3}{2}\ln 2+\frac{3}{8}\pi$, or exact 2-term equivalent	**A1**
		6

Question 39

(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**
	Use a correct method for finding a coefficient	**M1**
	Obtain one of $A=1$, $B=-1$, $C=6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5

(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** + **A1 FT** + **A1 FT**
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**
		5

Question 40

(a)	Carry out a relevant method to determine constants A and B such that $\dfrac{5a}{(2x-a)(3a-x)} = \dfrac{A}{2x-a} + \dfrac{B}{3a-x}$	**M1**
	Obtain $A = 2$	**A1**
	Obtain $B = 1$	**A1**
		3

Question 41

State or imply the form $\dfrac{A}{1+2x} + \dfrac{B}{4-x}$ and use a correct method to find a constant	**M1**
Obtain one of $A = 4$ and $B = -1$	**A1**
Obtain the second value	**A1**
	3

Question 42

(a)	State or imply the form $\frac{A}{2+x}+\frac{B+Cx}{3+x^2}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=4$, $B=1$ and $C=-2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(3+x^2)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1 FT** **A1 FT**
	Multiply out, up to the terms in x^2, by $B+Cx$, where $BC\neq 0$	**M1**
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	**A1**
		5

Question 43

State a suitable form of partial fractions for $\dfrac{1}{x^2(1+2x)}$	**B1**
Use a relevant method to determine a constant	**M1**
Obtain one of $A = -2$, $B = 1$ and $C = 4$	**A1**
Obtain a second value	**A1**
Obtain the third value	**A1**
Separate variables correctly and integrate at least one term	**M1**
Obtain terms $-2\ln x - \dfrac{1}{x} + 2\ln(1+2x)$ and t	**B3 FT**
Evaluate a constant, or use limits $x = 1$, $t = 0$ in a solution containing terms t, $a\ln x$ and $b\ln(1+2x)$, where $ab \neq 0$	**M1**
Obtain a correct expression for t in any form, e.g. $t = -\dfrac{1}{x} + 2\ln\left(\dfrac{1+2x}{3x}\right) + 1$	**A1**
	11

Question 44

State or imply the form $A + \dfrac{B}{2x-1} + \dfrac{C}{x-3}$	**B1**	$\dfrac{Dx+E}{2x-1} + \dfrac{F}{x-3}$ and $\dfrac{P}{2x-1} + \dfrac{Qx+R}{x-3}$ are also valid.
Use a correct method for finding a constant	**M1**	
Obtain one of $A = 2$, $B = -3$ and $C = 2$	**A1**	Allow maximum **M1A1** for one or more 'correct' values after **B0**.
Obtain a second value	**A1**	
Obtain the third value	**A1**	

Question 45

(a)	State or imply the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{2x^2+3}$	B1	If $1-\dfrac{A}{x-2}+\dfrac{Bx+C}{2x^2+3}$ or $\dfrac{A}{x-2}+\dfrac{C}{2x^2+3}$ B0 then M1 A1 (for $A=3$) still possible.
	Use a correct method for finding a constant	M1	
	Obtain one of $A=3$, $B=-1$ and $C=6$	A1	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2x^2+3)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	M1	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2, of each partial fraction	A1 FT A1 FT	The FT is on A, B and C. $-\dfrac{A}{2}\left[1-\left(-\dfrac{x}{2}\right)+\dfrac{(-1)(-2)}{2}\left(-\dfrac{x}{2}\right)^2+...\right]$ $\dfrac{Bx+C}{3}\left[1-\dfrac{2x^2}{3}+...\right]$
	Extract the coefficient 3 correctly from $(2x^2+3)^{-1}$ with expansion to $1\pm\dfrac{2}{3}x^2$ then multiply by $Bx+C$ up to the terms in x^2, where $BC\neq 0$	M1	$\dfrac{C}{3}+\dfrac{Bx}{3}\pm\dfrac{C}{3}\left(\dfrac{2}{3}\right)x^2$ or $\dfrac{1}{3}\left(C+Bx\pm C\left(\dfrac{2}{3}\right)x^2\right)$ Allow a slip in multiplication for M1. Allow miscopies in B and C from **7(a)**.
	Obtain final answer $\dfrac{1}{2}-\dfrac{13}{12}x-\dfrac{41}{24}x^2$	A1	Do not ISW.
		5	

Question 46

State or imply the form $\dfrac{A}{3x-1}+\dfrac{Bx+C}{x^2+3}$	B1	
Use a correct method for finding a constant	M1	
Obtain one of $A=1$, $B=0$ and $C=3$ from correct working	A1	A maximum of M1 A1 is available after B0.
Obtain a second value from correct working	A1	
Obtain the third value from correct working	A1	
	5	

Question 47

State or imply the form $\dfrac{A}{3-x}+\dfrac{Bx+C}{1+3x^2}$	B1	
Use a correct method to find a constant	M1	
Obtain one of $A=2$, $B=0$ and $C=1$	A1	
Obtain a second value	A1	
Obtain the third value	A1	
	5	

Question 48

State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	**B1**	
Use a correct method for finding a constant	**M1**	
Obtain one of $A = 2$, $B = -1$ and $C = 0$	**A1**	**SC**: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
Obtain a second value	**A1**	
Obtain the third value	**A1**	
	5	

Question 49

State or imply the form $\frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$	**B1**	
Use a correct method to find a constant	**M1**	
Obtain one of $A = 3$, $B = -1$ and $C = -2$	**A1**	SR after B0 can score M1A1 for one correct value
Obtain a second value	**A1**	
Obtain the third value	**A1**	$\frac{A}{1+x} + \frac{Dx+E}{(2+x)^2}$, where $A = 3$, $D = -1$ and $E = -4$, is awarded B1 M1 A1 A1 A1 as above.
	5	

Question 50

State or imply the form $\frac{Ax+B}{4+x^2} + \frac{C}{1+x}$	**B1**	
Use a correct method for finding a coefficient	**M1**	$(Ax + B)(1 + x) + C(4 + x^2) = 5x^2 + x + 11$.
Obtain one of $A = 2$, $B = -1$ and $C = 3$	**A1**	If error present in above still allow A1 for C.
Obtain a second value	**A1**	
Obtain the third value	**A1**	If $A = 0$ then max M1 A1 (for C).
	5	

Question 51

State or imply the form $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$.
Use a correct method to find a constant	M1	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1 + 2x)(3 - x)^3$.
Obtain one of $A = 2$, $B = 2$ and $C = -3$	A1	Alternative form: obtain one of $A = 2$, $D = -2$ and $E = 3$.
Obtain a second value	A1	
Obtain the third value	A1	Do not need to substitute values back into original form.
	5	If $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for A, A1 for B and A1 for C and D. If $C = 0$ then recovers B1 from above.

Question 52

State or imply the form $\frac{A}{1+2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(2-x)^2}$
Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2 + B(1 + 2x)(2 - x) + C(1 + 2x)$ $= 2x^2 + 17x - 17$ and compare coefficients or substitute for x. $A(2-x)^3 + B(1 + 2x)(2 - x)^2 + C(1 + 2x)(2 - x)$ $= 2x^2 + 17x - 17$ scores M0.
Obtain one of $A = -4$, $B = -3$ and $C = 5$	A1	
Obtain a second value	A1	
Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A = -4$, $D = 3$ and $E = -1$.
	5	

Question 53

State or imply the form $\frac{A}{2x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$	B1	Accept $\frac{A}{2x+1}+\frac{Dx+E}{(x+2)^2}$.
Use a correct method for finding a constant	M1	
Obtain one of $A = 1$, $B = -2$, $C = 3$	A1	For alternative form: $A = 1$, $D = -2$, $E = -1$.
Obtain a second value	A1	
Obtain the third value	A1	
	5	