A-level

Topic : Partial Fraction

May 2013-May 2025

Questions

Question 1

Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

Question 2

(i) Express $\dfrac{1}{x^2(2x + 1)}$ in the form $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{2x + 1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x + 1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

Question 3

Let $f(x) = \dfrac{2x^2 - 7x - 1}{(x - 2)(x^2 + 3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 4

(i) Express $\dfrac{7x^2 + 8}{(1 + x)^2(2 - 3x)}$ in partial fractions. [5]

(ii) Hence expand $\dfrac{7x^2 + 8}{(1 + x)^2(2 - 3x)}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [5]

Question 5

(i) Express $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

Question 6

Let $\mathrm{f}(x) = \dfrac{6+6x}{(2-x)(2+x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx+C}{2+x^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = 3\ln 3$. [5]

Question 7

Let $\mathrm{f}(x) = \dfrac{x^2-8x+9}{(1-x)(2-x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 8

Let $\mathrm{f}(x) = \dfrac{5x^2+x+6}{(3-2x)(x^2+4)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 9

Express $\dfrac{6x^2-2x+2}{(x-1)(2x+1)}$ in partial fractions. [5]

Question 10

Let $f(x) = \dfrac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_1^2 f(x)\,dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

Question 11

(i) Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \dfrac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6}\,dx$. [8]

Question 12

Let $f(x) = \dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x} + \dfrac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x)\,dx = 3 - \ln 4$. [5]

Question 13

Let $f(x) = \dfrac{4x^2 + 12}{(x + 1)(x - 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 14

Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x)\,dx = 8 - \ln 3$. [5]

Question 15

Let $f(x) = \dfrac{10x - 2x^2}{(x+3)(x-1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 16

Let $f(x) = \dfrac{3x^2 + x + 6}{(x+2)(x^2+4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 17

Let $f(x) = \dfrac{x(6-x)}{(2+x)(4+x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 18

(i) Express $\dfrac{1}{x(2x+3)}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

Question 19

Let $f(x) = \dfrac{5x^2 - 7x + 4}{(3x+2)(x^2+5)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 20

Let $f(x) = \dfrac{3x^2 - 4}{x^2(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_1^2 f(x)\,dx = \ln\left(\tfrac{25}{8}\right) - 1$. [5]

Question 21

Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\displaystyle\int_1^4 f(x)\,dx = 6 + \tfrac{1}{2}\ln\left(\tfrac{16}{7}\right)$. [5]

Question 22

Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 f(x)\,dx = 6 + \frac{1}{2}\ln\left(\frac{16}{7}\right)$. [5]

Question 23

Let $f(x) = \dfrac{5x^2 + x + 27}{(2x+1)(x^2+9)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence find $\int_0^4 f(x)\,dx$, giving your answer in the form $\ln c$, where c is an integer. [5]

Question 24

Let $f(x) = \dfrac{12x^2 + 4x - 1}{(x-1)(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 25

Let $f(x) = \dfrac{x - 4x^2}{(3-x)(2+x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{3-x} + \dfrac{Bx + C}{2+x^2}$. [4]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

Question 26

(i) Express $\dfrac{1}{4-y^2}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

Question 27

Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\displaystyle\int_{-1}^{0} f(x)\,dx = 1 + \tfrac{1}{2}\ln\left(\tfrac{3}{4}\right)$. [5]

Question 28

Let $f(x) = \dfrac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 29

Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\displaystyle\int_{-1}^{0} f(x)\,dx = 1 + \tfrac{1}{2}\ln\left(\tfrac{3}{4}\right)$. [5]

Question 30

Let $f(x) = \dfrac{12 + 12x - 4x^2}{(2+x)(3-2x)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 31

Let $f(x) = \dfrac{16 - 17x}{(2+x)(3-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 32

Let $f(x) = \dfrac{10x + 9}{(2x+1)(2x+3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

Question 33

Let $f(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3. [5]

Question 34

Let $f(x) = \dfrac{x^2 + x + 6}{x^2(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\int_1^4 f(x)\,dx$ is $\frac{9}{4}$. [5]

Question 35

Let $f(x) = \dfrac{2x^2 + x + 8}{(2x-1)(x^2+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x)\,dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

Question 36

Let $f(x) = \dfrac{2 + 11x - 10x^2}{(1 + 2x)(1 - 2x)(2 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 37

Let $f(x) = \dfrac{2}{(2x - 1)(2x + 1)}$.

(a) Express $f(x)$ in partial fractions. [2]

(b) Using your answer to part **(a)**, show that

$$(f(x))^2 = \frac{1}{(2x - 1)^2} - \frac{1}{2x - 1} + \frac{1}{2x + 1} + \frac{1}{(2x + 1)^2}.$$ [2]

(c) Hence show that $\int_1^2 (f(x))^2 \, dx = \frac{2}{5} + \frac{1}{2}\ln(\frac{5}{9})$. [5]

Question 38

Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 39

Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x) \, dx$. [6]

Question 40

Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 41

Let $f(x) = \dfrac{5a}{(2x - a)(3a - x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

Question 42

Let $f(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions. [3]

Question 43

Let $f(x) = \dfrac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 44

The variables x and t satisfy the differential equation $\dfrac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x. [11]

Question 45

Express $\dfrac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions. [5]

Question 46

Let $f(x) = \dfrac{5x^2 + 8x - 3}{(x-2)(2x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 47

Let $f(x) = \dfrac{x^2 + 9x}{(3x-1)(x^2+3)}$.

Express $f(x)$ in partial fractions. [5]

Question 48

Let $f(x) = \dfrac{5 - x + 6x^2}{(3-x)(1+3x^2)}$.

Express $f(x)$ in partial fractions. [5]

Question 49

Let $f(x) = \dfrac{4 - x + x^2}{(1+x)(2+x^2)}$.

Express $f(x)$ in partial fractions. [5]

Question 50

Let $f(x) = \dfrac{2x^2 + 7x + 8}{(1+x)(2+x)^2}$.

Express $f(x)$ in partial fractions. [5]

Question 51

Let $f(x) = \dfrac{5x^2 + x + 11}{(4+x^2)(1+x)}$.

Express $f(x)$ in partial fractions. [5]

Question 52

Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1+2x)(3-x)^2}$.

Express $f(x)$ in partial fractions. [5]

Question 53

Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

Express $f(x)$ in partial fractions. [5]

Question 54

Let $f(x) = \dfrac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

Express $f(x)$ in partial fractions. [5]

Question 55

Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

Question 56

Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Question 57

Let $f(x) = \dfrac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

Express $f(x)$ in partial fractions. [5]

Question 58

Express $\dfrac{6x^2 - 9x - 16}{2x^2 - 5x - 12}$ in partial fractions. [5]

Question 59

Let $f(x) = \frac{7a^2}{(a-2x)(3a+x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

Question 60

Let $f(x) = \frac{5x^2+8x+5}{(1+2x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the coefficient of x^3 in the expansion of $f(x)$. [4]

Question 61

Let $f(x) = \frac{-7x^2+2x-6}{(1+x)(4+x^2)}$.

Express $f(x)$ in partial fractions. [5]

Question 62

Let $f(x) = \frac{3a-5x}{(3a+2x)(2a-x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]