A-level

Topic :Vector

May 2013-May 2023

Answer

Question 1

(i)		State or imply A is $(1, 4, -2)$	B1	
		State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent	B1	
		Use QP as normal and A as mid-point to find equation of plane	M1	
		Obtain $12x + 6y - 6z = 48$ or equivalent	A1	[4]
(ii)	<u>Either</u>	State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$	B1	
		Set up and solve a relevant equation for λ.	M1	
		Obtain $\lambda = -9$ and hence B is $(-2, 7, -5)$	A1	
		Use correct method to find distance between A and B.	M1	
		Obtain 5.20	A1	
	<u>Or</u>	Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent	B1	
		Use correct method involving moduli, scalar product and cosine to find angle APB	M1	
		Obtain $35.26°$ or equivalent	A1	
		Use relevant trigonometry to find AB	M1	
		Obtain 5.20	A1	[5]

Question 2

(i)	Carry out a correct method for finding a vector equation for AB	M1	
	Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
	Substitute components in equation of p and solve for λ or for μ	M1	
	Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
(ii)	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
	Obtain $3 + b - c = 0$, or equivalent	A1	
	Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
	Obtain correct equation in any form, e.g. $\frac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
	Solve simultaneous equations for b or for c	M1 (dep*)	
	Obtain $b = -4$ and $c = -1$	A1	
	Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent (The f.t. is on b and c.)	A1$\sqrt{}$	[7]

Question 3

(i)	Equate scalar product of direction vector of l and p to zero	M1	
	Solve for a and obtain $a = -6$	A1	[2]
(ii)	Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$	B1	
	Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ	M1	
	Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4) = 0$ or $(1+\mu)(a-4) = 0$	A1	
	Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied	A1	[4]
(iii)	Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation	M1*	
	Use $\frac{2}{\sqrt{5}}$ as sine of the angle	A1	
	State equation in any form, e.g. $\frac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \frac{2}{\sqrt{5}}$	A1	
	Solve for a	M1 (dep*)	
	Obtain answers for $a = 0$ and $a = \frac{60}{31}$, or equivalent	A1	[5]

Question 4

(i) *EITHER:* Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ — B1

Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$ — M1

Obtain two correct equations in *a, b, c* — A1

Solve to obtain ratio $a : b : c$ — M1

Obtain $a:b:c = 3:1:-2$, or equivalent — A1

Obtain equation $3x + y - 2z = 1$, or equivalent — A1

OR1: Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$ — B1

Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations — M1

Obtain two correct equations in three unknowns, e.g. in *a, b, c* — A1

Solve to obtain their ratio, e.g. $a : b : c$ — M1

Obtain $a:b:c = 3:1:-2$, $a:c:d = 3:-2:1$, $a:b:d = 3:1:1$ or $b:c:d = -1:-2:1$ — A1

Obtain equation $3x + y - 2z = 1$, or equivalent — A1

(ii) Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent — B1 **[1]**

(iii) *EITHER:* Use $\dfrac{\overrightarrow{OA}.\overrightarrow{OD}}{\left|\overrightarrow{OD}\right|}$ to find projection *ON* of *OA* onto *OD* — M1

Obtain $ON = \dfrac{4}{3}$ — A1

Use Pythagoras in triangle *OAN* to find *AN* — M1

Obtain the given answer — A1

OR1: Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$ — M1

Obtain answer $6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ — A1

Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$ — M1

Obtain the given answer — A1

OR2: Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA*, or setting the derivative of $\left|\overrightarrow{AP}\right|$ to zero — M1

Solve and obtain $\lambda = \dfrac{4}{9}$ — A1

Carry out method to calculate *AP* when $\lambda = \dfrac{4}{9}$ — M1

Obtain the given answer — A1

Question 5

(i)	Find scalar product of the normals to the planes	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result.	M1	
	Obtain 67.8° (or 1.18 radians)	A1	[3]

(ii) <u>EITHER</u> Carry out complete method for finding point on line — M1

Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or … — A1…

<u>Either</u>

State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent	B1
Attempt to solve for one ratio, e.g. $a:b$	M1
Obtain $a:b:c=1:7:2$ or equivalent	A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√

<u>Or 1</u>

Obtain a second point on the line	A1
Subtract position vectors to obtain direction vector	M1
Obtain [1, 7, 2] or equivalent	A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√

Question 6

(i)	Obtain $2x-3y+6z$ for LHS of equation	B1	
	Obtain $2x-3y+6z=23$	B1	[2]

(ii) <u>Either</u>

Use correct formula to find perpendicular distance	M1	
Obtain unsimplified value $\dfrac{\pm 23}{\sqrt{2^2+(-3)^2+6^2}}$, following answer to (i)	A1√	
Obtain $\dfrac{23}{7}$ or equivalent	A1	[3]

	<u>OR 1</u>	Use scalar product of (4, –1, 2) and a vector normal to the plane	M1	
		Use unit normal to plane to obtain $\pm\dfrac{(8+3+12)}{\sqrt{49}}$	A1	
		Obtain $\dfrac{23}{7}$ or equivalent	A1	[3]
	<u>OR 2</u>	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
		Obtain $\mu = \dfrac{23}{49}$ [and $\left(\dfrac{46}{49}, -\dfrac{69}{49}, \dfrac{138}{49}\right)$ as foot of perpendicular]	A1	
		Obtain distance $\dfrac{23}{7}$ or equivalent	A1	[3]
(iii)	<u>Either</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
		Obtain $\dfrac{\lvert 23-k\rvert}{7} = 14$ or equivalent	A1	
		Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
	<u>OR</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
		Obtain $2x - 3y + 6z = 121$	A1	
		Obtain $2x - 3y + 6z = -75$	A1	[3]

Question 7

(i)	*EITHER:*	State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	
		Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent	M1	
		Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
		Obtain answer $\frac{20}{21}$	A1	
	OR:	Use correct method to find lengths of all sides of triangle *ABC*	M1	
		Apply cosine rule correctly to find the cosine of angle *BAC*	M1	
		Obtain answer $\frac{20}{21}$	A1	**4**
(ii)		State an exact value for the sine of angle *BAC*, e.g. $\sqrt{41}/21$	B1$\sqrt{}$	
		Use correct area formula to find the area of triangle *ABC*	M1	
		Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	**3**
		[SR: Allow use of a vector product, e.g. $\overrightarrow{AB}\times\overrightarrow{AC} = -6\mathbf{i}+2\mathbf{j}-\mathbf{k}$ B1$\sqrt{}$. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
(iii)	*EITHER*:	State or obtain $b = 0$	B1	
		Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
		Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
		Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
		Obtain answer $4x + z = 9$, or equivalent	A1	
	*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j})\times(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i}-\mathbf{k}$	A1	
		Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
		Obtain $4x + z = 9$, or equivalent	A1	
	*OR*2:	Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate μ	M1	
		Obtain answer $4x + z = 9$, or equivalent	A1	

Question 8

(i)		Express general point of l in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda)$	B1	
		Substitute in given equation of p and solve for λ	M1	
		Obtain final answer $-\frac{1}{2}\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or equivalent, from $\lambda=-\frac{1}{2}$	A1	**3**
(ii)		State or imply a vector normal to the plane, e.g. $2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
		Obtain answer 23.2° (or 0.404 radians)	A1	**4**
(iii)	*EITHER*:	State $2a + 3b - 5c = 0$ or $3a - 2b + 2c = 0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 4 : 19 : 13$, or equivalent	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$, and evaluate d	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
	*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})\times(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i} -19\mathbf{j} -13\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	

Question 9

(i)	*EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i} - 17\mathbf{j} + 4\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	B1	
		Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
		Solve and obtain $\lambda = 3$	A1	
		Carry out a complete method for finding the length of AP	M1	
		Obtain the given answer 15 correctly	A1	
	*OR*1:	Calling $(4, -9, 9)$ B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$	B1	
		Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	M1	
		Obtain correct answer, e.g. $-30\mathbf{i} + 6\mathbf{j} + 33\mathbf{k}$	A1	
		Divide the modulus of the product by that of the direction vector	M1	
		Obtain the given answer correctly	A1	

(ii) *EITHER*:	Substitute coordinates of a general point of l in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in a and b	M1*	
	Obtain a correct equation, e.g. $4a - 9b - 27 + 1 = 0$	A1	
	Obtain a second correct equation, e.g. $-2a + b + 6 = 0$	A1	
	Solve for a or for b	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	
OR:	Substitute coordinates of a point of l and obtain a correct equation, e.g. $4a - 9b = 26$	B1	
	EITHER: Find a second point on l and obtain an equation in a and b	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for l and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a + b + 6 = 0$	A1	
	Solve for a or for b	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	**[5]**

Question 11

(i)	Obtain $\pm\begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$ as direction vector of l_1	B1	
	State that two direction vectors are not parallel	B1	
	Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
	Equate at least two pairs of components and solve for λ or for μ	M1	
	Obtain correct answers for λ and μ	A1	
	Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1	
	Use correct process for finding modulus and evaluating inverse cosine	M1	
	Obtain $79.5°$ or 1.39 radians	A1	[3]

Question 12

(i) Carry out a correct method for finding a vector equation for *AB* M1

Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent A1

Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ M1

Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$;

or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$ A1

Verify that not all three pairs of equations are satisfied and that the lines fail to intersect A1 **[5]**

(ii) *EITHER*: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $3a + b - c = 0$ B1

Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$ M1

Obtain final answer $a : b : c = 1 : 4 : 7$ A1

Use coordinates of a relevant point and values of *a*, *b* and *c* in general equation and find *d* M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

*OR*1: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ M1

Obtain two correct components A1

Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$ A1

Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find *d* M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

Question 13

(i) State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ B1

Carry out correct process for evaluating the scalar product of two normal vectors M1

Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result M1

Obtain answer 85.9° or 1.50 radians A1 **4**

(ii) *EITHER*: Carry out a complete strategy for finding a point on *l* M1

Obtain such a point, e.g. (0, 2, 1) A1

EITHER: State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for *l*, e.g. $a + 3b - 2c = 0$ and $2a + b + 3c = 0$ B1

Solve for one ratio, e.g. $a : b$ M1

Obtain $a : b : c = 11 : -7 : -5$ A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ A1√

*OR*1: Obtain a second point on *l*, e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$ B1

Subtract position vectors and obtain a direction vector for l M1

Obtain $22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k}$, or equivalent A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k})$ A1√

OR2: Attempt to find the vector product of the two normal vectors M1

Obtain two correct components A1

Obtain $11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, or equivalent A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ A1√

OR3: Express one variable in terms of a second M1

Obtain a correct simplified expression, e.g. $x = (22 - 11y)/7$ A1

Express the same variable in terms of the third M1

Obtain a correct simplified expression, e.g. $x = (11 - 11z)/5$ A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$ A1√

Question 14

(i) Use correct method to form a vector equation for *AB* **M1**

Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ **A1** [2]

(ii) Using a direction vector for *AB* and a relevant point, obtain an equation for ***m*** in any form **M1**

Obtain answer $2x - 2y + z = 4$, or equivalent **A1** [2]

(iii) Express general point of *AB* in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$ **B1√**

Substitute in equation of ***m*** and solve for λ or for μ **M1**

Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of *N*, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$ **A1**

Carry out a correct method for finding *CN* **M1**

Obtain the given answer $\sqrt{13}$ **A1** [5]

Question 15

(i) Express a general point on the line in single component form, e.g. $(\lambda, 2 - 3\lambda, -8 + 4\lambda)$, substitute in equation of plane and solve for λ **M1**

Obtain $\lambda = 3$ **A1**

Obtain $(3, -7, 4)$ **A1** [3]

(ii) State or imply normal vector to plane is $4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ **B1**

Carry out process for evaluating scalar product of two relevant vectors **M1**

Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result. **M1**

Obtain $54.8°$ or 0.956 radians **A1** [4]

(iii)	Either	Find at least one position of C by translating by appropriate multiple of direction vector $\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ from A or B	**M1**	
		Obtain $(-3, 11, -20)$	**A1**	
		Obtain $(9, -25, 28)$	**A1**	
	Or	Form quadratic equation in λ by considering $BC^2 = 4AB^2$	**M1**	
		Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	**A1**	
		Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	**A1**	[3]

Question 16

(i)	*EITHER*:	Substitute for $\mathbf{r}$ in the given equation of p and expand scalar product	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
	*OR*1:	Substitute coordinates of a general point of l in the Cartesian equation of plane p	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
	*OR*2:	Expand scalar product of the normal to p and the direction vector of l	**M1**	
		Verify scalar product is zero	**A1**	
		Verify that one point of l does not lie in the plane	**A1**	
	*OR*3:	Use correct method to find the perpendicular distance of a general point of l from p	**M1**	
		Obtain a correct unsimplified expression in terms of λ	**A1**	
		Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all λ	**A1**	
(ii)	*EITHER*:	Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$	**B1**	
		State equation $2a-b-c=0$	**B1**	
		Solve for one ratio, e.g. $a:b$	**M1**	
		Obtain ratio $a:b:c = 1:4:-2$, or equivalent	**A1**	
	OR:	Attempt to calculate the vector product of the direction vector of l and the normal vector of the plane p, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	**M2**	
		Obtain two correct components of the product	**A1**	
		Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
		Form line equation with relevant vectors	**M1**	
		Obtain answer $\mathbf{r} = 5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent	**A1**√	[6]

Question 17

(i)	*EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$	**B1**
	Use scalar product to obtain an equation in a, b, c e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$	**M1**
	State two correct equations	**A1**
	Solve to obtain ratio $a : b : c$	**M1**
	Obtain $a : b : c = 5 : -2 : 3$	**A1**
	Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1**
	*OR*1: Substitute for two points, e.g. A and B, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$	**(B1**
	Substitute for another point, e.g. C, to obtain a third equation and eliminate one unknown entirely from all three equations	**M1**
	Obtain two correct equations in three unknowns, e.g. in a, b, c	**A1**
	Solve to obtain their ratio	**M1**
	Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$	**A1**
	Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1)**
		[6]
(ii)	Correctly form an equation for the line through D parallel to OA	**M1**
	Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	**A1**
	Substitute components in the equation of the plane and solve for λ	**M1**
	Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for P	**A1**
	Obtain the given answer correctly	**A1**
		[5]

Question 18

(i)	*Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$	**B1**	
	Use a correct method for finding the position vector of D	**M1**	
	Obtain answer $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
	EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and BC	**M1**	
	Show that $ABCD$ has a pair of adjacent sides that are equal	**A1**	
	OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent	**M1**	
	Show that $ABCD$ has perpendicular diagonals	**A1**	[5]

(ii)	*EITHER*: State $a + 2b + 3c = 0$ or $2a + b - 2c = 0$	**B1**	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = -7 : 8 : -3$, or equivalent	**A1**	
	Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$, and evaluate	**M1**	
	Obtain answer $-7x + 8y - 3z = 29$, or equivalent	**A1**	
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $-7\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$	**A1**	
	Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$ and evaluate d	**M1**	
	Obtain answer $-7x + 8y - 3z = 29$, or equivalent	**A1**	[5]

Question 19

(i)	State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	**B1**	
	Equate at least two pairs of components of AB and l and solve for λ or for μ	**M1**	
	Obtain correct answer for λ or for μ, e.g. $\lambda = -1$ or $\mu = 2$	**A1**	
	Show that not all three equations are not satisfied and that the lines do not intersect	**A1**	[4]
(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l, e.g. $(1-\mu)\mathbf{i} + (-3+2\mu)\mathbf{j} + (-2+\mu)\mathbf{k}$	**B1**	
	Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	**M1**	
	Solve and obtain $\mu = \frac{3}{2}$	**A1**	
	Carry out a method to calculate AP when $\mu = \frac{3}{2}$	**M1**	
	Obtain the given answer $\frac{1}{\sqrt{2}}$ correctly	**A1**	
	OR 1: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l	**(B1**	
	Use correct method to express AP^2 (or AP) in terms of μ	**M1**	
	Obtain a correct expression in any form, e.g. $(1-\mu)^2 + (-3+2\mu)^2 + (-2+\mu)^2$	**A1**	
	Carry out a complete method for finding its minimum	**M1**	
	Obtain the given answer correctly	**A1)**	
	OR 2: Calling $(2, -2, -1)$ C, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	**(B1**	
	Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l	**M1**	
	Obtain correct answer in any form, e.g. $\frac{9}{\sqrt{6}}$	**A1**	
	Use Pythagoras to find the perpendicular	**M1**	
	Obtain the given answer correctly	**A1)**	

Question 20

(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$	**B1**	
	Use correct method to calculate their scalar product	**M1**	
	Show value is zero and planes are perpendicular	**A1**	[3]
(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection	**M1**	
	Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0)	**A1**	
	EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$	**B1**	
	Solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = 1 : -7 : -4$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*1: Obtain a second point on l, e.g. (1, 0, 1)	**B1**	
	Subtract vectors and obtain a direction vector for l	**M1**	
	Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	**A1**√	
	*OR*2: Attempt to find the vector product of the two normal vectors	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*1: Express one variable in terms of a second variable	**M1**	
	Obtain a correct simplified expression, e.g. $y = 7 - 7x$	**A1**	
	Express the third variable in terms of the second	**M1**	
	Obtain a correct simplified expression, e.g. $z = 5 - 4x$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*2: Express one variable in terms of a second variable	**M1**	
	Obtain a correct simplified expression, e.g. $z=5-4x$	**A1**	
	Express the same variable in terms of the third	**M1**	
	Obtain a correct simplified expression e.g. $z=(7+4y)/7$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**A1**√	[6]

Question 21

(i)	Express general point of l in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$	**B1**	
	Using the correct process for the modulus form an equation in λ	**M1***	
	Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$	**A1**	
	Solve for λ (usual requirements for solution of a quadratic)	**DM1**	
	Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$	**A1**	[5]
(ii)	Using the correct process, find the scalar product of a direction vector for l and a normal for p	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$	**M1**	
	State a correct equation in any form, e.g. $\dfrac{2a-1+1}{\sqrt{(a^2+1+1)}.\sqrt{(2^2+(-1)^2+1)}}=\pm\dfrac{2}{3}$	**A1**	
	Solve for a^2	**M1**	
	Obtain answer $a=\pm 2$	**A1**	[5]

Question 22

(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x+2y-z=k$	**B1**
	Obtain answer $2x+2y-z=2$	**B1**
	Total:	**3**

Question 23

(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x+2y-z=k$	**B1**
	Obtain answer $2x+2y-z=2$	**B1**
	Total:	**3**

Question 24

(i)	*EITHER*: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g. $(8+3\lambda, -3-\lambda, 4+2\lambda)$	**(B1**
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	**M1**
	Obtain $\lambda=-\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**A1)**
	OR: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g. $(8+3\lambda, -3-\lambda, 4+2\lambda)$	**(B1**
	Differentiate $\lvert AP\rvert^2$ and solve for λ at minimum	**M1**
	Obtain $\lambda=-\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**A1)**
	Total:	**5**

(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $3a - b + 2c = 0$	(B1
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	A1)
	*OR*1: Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	A1)
	*OR*2: Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	State 3 correct equations in x, y, z, s and t	A1
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	A1)
	*OR*3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	(M1
	Expand a correct determinant and obtain two correct cofactors	A1
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	A1)
	Total:	**3**
(iii)	*EITHER:* Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	(M1
	Obtain a correct expression in any form, e.g. $\dfrac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	A1 FT
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	OR1: Obtain equation of the parallel plane through A, e.g. $x + y - z = -1$ [The f.t. is on the plane found in part (**ii**).]	(B1 FT
	Use correct method to find its distance from the origin	M1
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)

Question 25

(i)	Carry out a correct method for finding a vector equation for AB	M1
	Obtain $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$, or equivalent	A1
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda = \frac{5}{7}$ or $\mu = \frac{3}{7}$	A1
	Obtain $m = 3$	A1
	Total:	**5**

(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $a - 2b - 4c = 0$	(B1
	Form a second relevant equation, e.g. $2a + 3b - c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 14 : -7 : 7$	A1
	Use coordinates of a relevant point and values of a, b and c and find d	M1
	Obtain answer $14x - 7y + 7z = 42$, or equivalent	A1)
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}) \times (2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $14\mathbf{i} - 7\mathbf{j} + 7\mathbf{k}$	A1
	Substitute coordinates of a relevant point in $14x - 7y + 7z = d$, or equivalent, and find d	M1
	Obtain answer $14x - 7y + 7z = 42$, or equivalent	A1)

Question 26

(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	A1
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1
		3
(ii)	Carry out correct process for evaluating scalar product of direction vectors for l and m	*M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	DM1
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	A1
		3
(iii)	*EITHER:* Use scalar product to obtain a relevant equation in a, b and c, e.g. $-a + b + 4c = 0$	B1
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	A1
	Substitute $(3, -2, -1)$ and values of a, b and c in general equation and find d	M1
	Obtain answer $2x - 2y + z = 9$, or equivalent	A1
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
	Substitute $(3, -2, -1)$ in $-6x + 6y - 3z = d$, or equivalent, and find d	M1
	Obtain answer $-2x + 2y - z = -9$, or equivalent	A1)

Question 27

)(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$	**B1**
	Carry out correct process for evaluating the scalar product of two normal vectors	**M1**
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	**M1**
	Obtain final answer 72.5° or 1.26 radians	**A1**
		4
(ii)	*EITHER*: Substitute $y = 2$ in both plane equations and solve for x or for z	**(M1**
	Obtain $x = 3$ and $z = 1$	**A1)**
	OR: Find the equation of the line of intersection of the planes	
	Substitute $y = 2$ in line equation and solve for x or for z	**(M1**
	Obtain $x = 3$ and $z = 1$	**A1)**

EITHER:	Use scalar product to obtain an equation in a, b and c, e.g. $a+b+3c=0$	**(B1**
	Form a second relevant equation, e.g. $2a-2b+c=0$, and solve for one ratio, e.g. $a : b$	***M1**
	Obtain final answer $a : b : c = 7 : 5 : -4$	**A1**
	Use coordinates of A and values of a, b and c in general equation and find d	**DM1**
	Obtain answer $7x+5y-4z=27$, or equivalent	**A1 FT)**
*OR*1:	Calculate the vector product of relevant vectors, e.g. $(\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-2\mathbf{j}+\mathbf{k})$	**(*M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $7\mathbf{i}+5\mathbf{j}-4\mathbf{k}$	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x+5y-4z=27$, or equivalent	**A1 FT)**

Question 28

)(i)	Express general point of l in component form, e.g. $\mathbf{r} = (4 + \mu)\mathbf{i} + (3 + 2\mu)\mathbf{j} + (-1 - 2\mu)\mathbf{k}$, or equivalent NB: Calling the vector $\mathbf{a} + \mu\mathbf{b}$, the **B1** is earned by a correct reduction of the sum to a single vector or by expressing the substitution as a distributed sum $\mathbf{a.n} + \mu\mathbf{b.n}$	**B1**
	Substitute in given equation of p and solve for μ	**M1**
	Obtain final answer $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ from $\mu = -2$	**A1**
		3
)(ii)	Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	**M1**
	Obtain answer $10.3°$ (or 0.179 radians)	**A1**
		3
(iii)	*EITHER*: State $a + 2b - 2c = 0$ or $2a - 3b - c = 0$	**(B1**
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 8 : 3 : 7$, or equivalent	**A1**
	Substitute a, b, c and given point and evaluate d	**M1**
	Obtain answer $8x + 3y + 7z = 5$	**A1)**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components of the product	**A1**
	Obtain correct product, e.g. $8\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	**A1**
	Use the product and the given point to find d	**M1**
	Obtain answer $8x + 3y + 7z = 5$, or equivalent	**A1)**

Question 29

)(a)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\,\mathbf{i}+(4+2\mu)\mathbf{j}+(4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu=-\dfrac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
)(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $a + 2b + 3c = 0$	**B1**
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	**M1**
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	**A1**
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**A1**
	Substitute a relevant point and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**

Question 30

(i)	Equate at least two pairs of components and solve for s or for t	**M1**	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for s or t, e.g. $s = -6$, $t = -11$	**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect	**A1**	
	State that the lines are not parallel	**B1**	
		4	
(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a + 3b - c = 0$	**B1**	
	Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
	OR: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	**A1**	
		3	
(iii)	*EITHER*: State position vector or coordinates of the mid-point of a line segment joining points on l and m, e.g. $\frac{3}{2}\mathbf{i} + \mathbf{j} + \frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing l and m B1
	Use the result of (ii) and the mid-point to find d	**M1**	Use average of distances to find equation of p. M1
	Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR: Using the result of part **(ii)**, form an equation in d by equating perpendicular distances to the plane of a point on l and a point on m	**M1**	
	State a correct equation, e.g. $\left\lvert \frac{14 - d}{\sqrt{35}} \right\rvert = \left\lvert \frac{-d}{\sqrt{35}} \right\rvert$	**A1**	
	Solve for d and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
		3	

Question 31

(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components AB and l and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
(ii)	State or imply a direction vector for AP has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of t, and obtain an equation in terms of t	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for P	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**

Question 32

(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**
	Verify scalar product is zero	**A1**
	Verify that one point of l does not lie in the plane	**A1**
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**
	Obtain correct equation in λ in any form	**A1**
	Verify that the equation is not satisfied for any value of λ	**A1**
		3

(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**
	Obtain answer 74.5° or 1.30 radians	**A1**
		3
(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**
	Obtain a correct modular or non-modular equation in any form	**A1**
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**
		4

Question 33

(i)	Substitute for $\mathbf{r}$ and expand the scalar product to obtain an equation in λ	**M1***
	Solve a linear equation for λ	**M1(dep*)**
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	**A1**
		3
(ii)	State or imply a normal vector of p is $3\mathbf{i} + \mathbf{j} + \mathbf{k}$, or equivalent	**B1**
	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(3\mathbf{i} + \mathbf{j} + \mathbf{k})$	**M1**
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**
	Obtain answer 14.3° or 0.249 radians	**A1**

(iii)	Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state a relevant equation in a, b, c, e.g. $3a + b + c = 0$	B1
	State a second relevant equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 3 : -2 : -7$, or equivalent	A1
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	A1ft

Question 34

(i)	State or imply a correct normal vector to either plane, e.g. $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, or $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1
	Carry out correct process for evaluating the scalar product of two normal vectors	M1
	Using the correct process for the moduli, divide the scalar product of the two normal vectors by the product of their moduli and evaluate the inverse cosine of the result	M1
	Obtain answer 56.9° or 0.994 radians	A1
		4
(ii)	*EITHER*: Carry out a complete strategy for finding a point on the line (call the line l)	M1
	Obtain such a point, e.g. (1, 1, 4)	A1
	EITHER: State a correct equation for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $2a + 3b - c = 0$	B1
	State a second equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 1 : -3 : -7$, or equivalent	A1
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$	A1
	*OR*1: Attempt to calculate the vector product of the two normal vectors	M1
	Obtain two correct components	A1
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	A1
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	A1

Question 35

(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	**B1**
	Use scalar product to obtain an equation in a, b, c,	**M1**
	Obtain two correct equations in a, b, c	**A1**
	Solve to obtain $a : b : c$,	**M1**
	Obtain $a : b : c = 5 : -10 : -1$,	**A1**
	Obtain equation $5x - 10y - z = -25$,	**A1**
(ii)	State or imply a normal vector for the plane $OABC$ is $\mathbf{k}$	**B1**
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(5\mathbf{i} - 10\mathbf{j} - \mathbf{k}).(\mathbf{k})$	**M1**
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**
	Obtain answer 84.9° or 1.48 radians	**A1**
		4

Question 36

(i)	Carry out correct method for finding a vector equation for AB	M1
	Obtain $(\mathbf{r} =)\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+2\mathbf{k})$, or equivalent	A1
	Equate two pairs of components of general points on *their* AB and l and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$	A1
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$) Conclusion needs to follow correct values	A1
		5
(ii)	State or imply midpoint has position vector $2\mathbf{i}+\frac{3}{2}\mathbf{j}$	B1
	Substitute in $2x - y + 2z = d$ and find d	M1
	Obtain plane equation $4x - 2y + 4z = 5$	A1
	Substitute components of l in plane equation and solve for μ	M1
	Obtain $\mu = -\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ for the point P	A1
		5

Question 37

(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	**B1**
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	**M1**
	Solve for μ and obtain $\mu = 2$	**A1**
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	**M1**
	Obtain answer 3	**A1**
(ii)	Substitute coordinates of a general point of l in the plane equation and equate constant terms	**M1**
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**
	Equate the coefficient of μ to zero	**M1**
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	**A1**
	Substitute (1, 2, 3) in the plane equation	**M1**
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**

Question 38

(i)	Express general point of l or m in component form e.g. $(a+\lambda,\ 2-2\lambda,\ 3+3\lambda)$ or $(2+2\mu,\ 1-\mu,\ 2+\mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ **or** $\mu = \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ **or** $\mu = \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	

(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	**B1**	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	**M1**	
	Obtain $a : b : c = 1 : 5 : 3$	**A1**	OE
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (i), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	**M1**	e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	**A1**	
	Substitute a relevant point and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (i), if used

Question 39

(i)	Express general point of l in component form e.g. $(1+\lambda, 3-2\lambda, -2+3\lambda)$	**B1**
	Substitute in equation of p and solve for λ	**M1**
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda=\frac{2}{3}$	**A1**
		3

(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+3\mathbf{k}).(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	**M1**
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**
	Obtain answer $40.0°$ or 0.698 radians	**A1**
		3
	Alternative method for question 10(ii)	
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+3\mathbf{k})\text{x}(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	**M1**
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	**M1**
	Obtain answer $40.0°$ or 0.698 radians	**A1**
		3

Question 40

(i)	Substitute coordinates (5, 2, – 2) in $x + 4y - 8z = d$	**M1**
	Obtain plane equation $x + 4y - 8z = 29$, or equivalent	**A1**
		2
(ii)	Attempt to use perpendicular formula to find perpendicular from (5, 2, – 2) to m	**M1**
	Obtain a correct unsimplified expression, e.g. $\dfrac{5+8+16-2}{\sqrt{(1+16+64)}}$	**A1**
	Obtain answer 3	**A1**
(iii)	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use a scalar product to form a relevant equation in a, b and c, e.g. $a + 4b - 8c = 0$ or $5a + 2b - 2z = 0$	**B1**
	Solve two relevant equations for the ratio $a : b : c$	**M1**
	Obtain $a : b : c = 4 : -19 : -9$	**A1**
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**

Question 41

(a)	Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$	**B1**
	Use a correct method to find $\overrightarrow{MN}$	**M1**
	Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	**A1**
		3
(b)	Use a correct method to form an equation for MN	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	**A1**
		2

(c)	Find $\overrightarrow{DP}$ for a point P on MN with parameter λ, e.g. $(2-\lambda, 1+2\lambda, -2+2\lambda)$	**B1**
	Equate scalar product of $\overrightarrow{DP}$ and a direction vector for MN to zero and solve for λ	**M1**
	Obtain $\lambda = \frac{4}{9}$	**A1**
	State that the position vector of P is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$	**A1**
		4

Question 42

(a)	State $\overrightarrow{AB}$ (or $\overrightarrow{BA}$) and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle ABC is a right angle	**A1**
		3
(b)	Use correct method to calculate the lengths of AB and BC	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
(c)	State a correct equation for the line through B and C, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(-2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	**B1**
	Taking a general point of BC to be P, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle OBP (or OCP), *or* setting the derivative of $\lvert\overrightarrow{OP}\rvert$ to zero	**M1**
	Solve and obtain $\lambda = -\frac{5}{9}$	**A1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**

Question 43

(a)	State that the position vector of M is $3\mathbf{i}+\mathbf{j}$	B1
	Use a correct method to find the position vector of N	M1
	Obtain answer $\frac{10}{3}\mathbf{i}+2\mathbf{j}+2\mathbf{k}$	A1
	Use a correct method to form an equation for MN	M1
	Obtain correct answer in any form, e.g. $\mathbf{r}=3\mathbf{i}+\mathbf{j}+\lambda\left(\frac{1}{3}\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$	A1
		5
(b)	State or imply $\mathbf{r}=\mu(2\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ as equation for OB	B1
	Equate sufficient components of MN and OB and solve for λ or for μ	M1
	Obtain $\lambda=3$ or $\mu=2$ and position vector $4\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ for P	A1
		3
(c)	Carry out correct process for evaluating the scalar product of direction vectors for OP and MP, or equivalent	M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1
	Obtain answer 21.6°	A1
		3

Question 44

(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	B1
	Use a correct method for finding the position vector of C	M1
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	A1
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and AD	M1
	Show that $ABCD$ has a pair of unequal adjacent sides	A1
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	B1
	Use a correct method for finding the position vector of C	M1
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	A1
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	M1
	Show that the diagonals of $ABCD$ are not perpendicular	A1
		5
(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	M1
	Obtain answer 100.3°	A1
		3

(c)	Use a correct method to calculate the area, e.g. calculate $AB.AC \sin BAD$	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

Question 45

(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu \quad 2 + 2\lambda = 1 - \mu \quad 1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\frac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	
(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) . (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5) \quad 138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0 \quad (23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question 46

<table>
<tr><td>(a)</td><td>Obtain $\overrightarrow{AB}=\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\overrightarrow{CD}=\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1</td><td>Or equivalent seen or implied</td></tr>
<tr><td></td><td>Use the correct process for calculating the modulus of both vectors to obtain AB and CD</td><td>M1</td><td>$AB=\sqrt{24},\ CD=\sqrt{6}$</td></tr>
<tr><td></td><td>Using exact values, verify that $AB=2CD$</td><td>A1</td><td>Obtain given statement from correct work
Allow from $BA=2DC$, OE</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>(b)</td><td>Use the correct process to calculate the scalar product of the relevant vectors (their $\overrightarrow{AB}$ and $\overrightarrow{CD}$)</td><td>M1</td><td>$\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ or $\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}4\\2\\2\end{pmatrix}$</td></tr>
<tr><td></td><td>Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain answer $99.6°$ (or 1.74 radians) or better</td><td>A1</td><td>Do not ISW if go on to subtract from 180°
(99.594…, 1.738...) Accept 260.4°</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>(c)</td><td>State correct vector equations for AB and CD in any form,
e.g. $(\mathbf{r}=)\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r}=)\begin{pmatrix}4\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}3\\2\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><td>A</td><td>λ</td><td>μ</td><td></td><td>B</td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0\neq2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0\neq2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5\neq-5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5\neq-5$</td></tr>
</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3}\neq\frac{7}{3}$ or $\frac{17}{3}=\frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

Question 47

(a)	Express general point of a line in component form, e.g. $(1 + 2s, 3 - s, 2 + 3s)$ or $(2 + t, 1 - t, 4 + 4t)$	**B1**
	Equate at least two pairs of components and solve for s or for t	**M1**
	Obtain correct answer for s or for t (possible answers are $-1, 6, \frac{2}{5}$ for s and $-3, 4, -\frac{1}{5}$ for t)	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Show that the lines are not parallel and are thus skew	**A1**
		5
(b)	Carry out correct process for evaluating the scalar product of the direction vectors	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**
	Obtain answer $19.1°$ or 0.333 radians	**A1**
		3

Question 48

(a)	State or imply $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**B1**
	Carry out a correct method to find $\overrightarrow{OD}$	**M1**
	Obtain answer $-4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$	**A1**
		3
(b)	State $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**B1FT**
		1

(c)	For a general point P on AB, state $\overrightarrow{CP}$ or $\overrightarrow{DP}$ in component form, e.g. $\overrightarrow{CP} = (3-2\lambda, -\lambda, -6+2\lambda)$	***M1**
	Equate a relevant scalar product to zero *or* equate derivative of $\left\|\overrightarrow{CP}\right\|$ to zero *or* use Pythagoras in a relevant triangle and solve for λ	**DM1**
	Obtain $\lambda = 2$	**A1**
	Show the perpendicular is of length 3	**A1**
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**

Question 49

(a)	Show that $OA = OB = \sqrt{5}$	**B1**
	Evaluate the scalar product of the correct position vectors	**M1**
	Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer 101.5°	**A1**
		4

(b)	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**
	Taking a general point of OM to have position vector $\lambda\mathbf{i} - \lambda\mathbf{k}$, express $AP = \sqrt{7}\ OA$ as an equation in λ	***M1**
	State a correct equation in any form	**A1**
	Reduce to $\lambda^2 - 2\lambda - 15 = 0$	**A1**
	Solve a quadratic and state a position vector	**DM1**
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**

Question 50

(a)	State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$	**B1**
	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for l	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer $83.7°$ or 1.46 radians	**A1**
		4

8(b)	State or imply ± $\overrightarrow{AP}$ and ± $\overrightarrow{BP}$ in component form, i.e. $(1 + \lambda, 1 - 2\lambda, \lambda)$ and $(-1 + \lambda, 2 - 2\lambda, 3 + \lambda)$, or equivalent	B1
	Form an equation in λ by equating moduli or by using $\cos BAP = \cos ABP$	*M1
	Obtain a correct equation in any form $(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$	A1
	Solve for λ and obtain position vector	DM1
	Obtain correct position vector for P in any form, e.g. $(8, -9, 7)$ or $8\mathbf{i} - 9\mathbf{j} + 7\mathbf{k}$	A1
		5

Question 51

(a)	State $\overrightarrow{OM} = 4\mathbf{i} + 2\mathbf{j}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	
	Obtain answer $3\mathbf{j} + \mathbf{k}$	A1	
	Use a correct method to find a line equation for MN	M1	
	Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, or equivalent	A1	
		5	
(b)	Taking a general point P on MN, form an equation in λ by *either* equating a relevant scalar product to zero *or* equating the derivative of $\overrightarrow{OP}$ to zero *or* using Pythagoras in triangle OPM or OPN	M1	
	Obtain $\lambda = \frac{2}{9}$	A1	OE
	Use correct method to find OP	M1	
	Obtain the given answer correctly	A1	
	Alternative method to Question 8(b)		
	Use a scalar product to find the projection of OM (or ON) on MN	M1	
	Obtain answer $\frac{14}{\sqrt{18}}$ $\left(\text{or } \frac{4}{\sqrt{18}}\right)$	A1	
	Use Pythagoras to obtain the perpendicular	M1	
	Obtain the given answer correctly	A1	
		4	

Question 52

(a)	Obtain direction vector $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$, or equivalent	B1	Accept answers as column vectors throughout.
	Use a correct method to form a vector equation	M1	
	State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, or equivalent correct form	A1	e.g. $\mathbf{r}=\begin{pmatrix}0\\3\\1\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ Allow $\begin{pmatrix}x\\y\\z\end{pmatrix}$ for $\mathbf{r}$.
		3	
(b)	Use a correct method to find the position vector of C	M1	e.g. $\mathbf{OC}=\mathbf{OA}+\mathbf{AC}=\begin{pmatrix}1-3\\2+3\\-1+6\end{pmatrix}$
	Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		2	
(c)	State $\overrightarrow{OP}$ in component form	B1 FT	
	Form an equation in λ by equating the modulus of OP to $\sqrt{14}$, or equivalent	M1	
	Simplify and obtain $3\lambda^2-\lambda-4=0$, or equivalent	A1	$3\lambda^2+\lambda-4=0$ if using $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ in **(a)**. $3\mu^2+5\mu-2=0$ if using $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ in **(a)** and OB.
	Solve a 3-term quadratic and find a position vector	M1	$\left(\lambda=-1,\frac{4}{3}\text{ or }\lambda=1,-\frac{4}{3}\text{ or }\mu=\frac{1}{3},-2\text{ or }\mu=-\frac{1}{3},2\right)$
	Obtain answers $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i}+\frac{10}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		5	

Question 53

(a)	Use correct method to evaluate the scalar product of relevant vectors	**M1**	$(-4-2+6)$
	Obtain answer zero and deduce the given statement	**A1**	Need a conclusion or a statement in advance that the scalar product will be zero.
		2	
(b)	Express general point of l or m in component form, e.g. $(3 + 4s,\ 2 - s,\ 5 + 3s)$ or $(1 - t,\ -1 + 2t,\ -2 + 2t)$	**B1**	
	Equate at least two pairs of components and solve for s or for t	**M1**	
	Obtain correct answer $s = -1$ and $t = 2$	**A1**	
	Verify that all three equations are satisfied	**A1**	
	State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, or equivalent	**A1**	Can come from 1 correct value and no contradictory statement.
		5	
(c)	Taking a general point P on m, form an equation in t by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\lvert\overline{OP}\rvert$ to zero, ***or*** taking a specific point Q on m, e.g. $(1, -1, -2)$, using Pythagoras in triangle OPQ	***M1**	e.g. $\begin{pmatrix} 1-t \\ -1+2t \\ -2+2t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$
	Obtain $t = \dfrac{7}{9}$	**A1**	
	Carry out correct method to find OP	**DM1**	
	Obtain $\dfrac{\sqrt{5}}{3}$	**A1**	Obtain the **given answer** from full and correct working.

Question 54

(a)	Obtain direction vector $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$	**B1**	OE
	Use a correct method to form a vector equation	**M1**	
	Obtain answer $\mathbf{r}=2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$ or $\mathbf{r}=\ \mathbf{i}\ 2\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$	**A1**	Need **r** or r on LHS
		3	
(b)	Carry out the correct process for evaluating the scalar product of the direction vectors.	**M1**	(–1, – 3, 1) **.** (1, –3, –2) = – 1 + 9 – 2
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result for any 2 vectors	**M1**	$\cos^{-1}\left(\frac{1+9\ 2}{(1+9+1)(1+9+4))}\right)$
	Obtain answer 61.1 °	**A1**	61.086°
		3	
(c)	Express general point of *AB* or *l* in component form, e.g. (2 – λ, 1 – 3λ, 1 + λ) or (1 + μ, 2 – 3μ, – 3 – 2μ)	**B1**	
	Equate at least two pairs of components and solve for λ or for μ	**M1**	
	Obtain a correct answer for λ or μ, e.g. $\lambda = 6$, $\frac{1}{3}$, or $-\frac{14}{9}$; $\mu = -5$, $\frac{2}{3}$ or $-\frac{11}{9}$	**A1**	
	Verify that all three equations are not satisfied, and the lines do not intersect	**A1**	
	Express general point of *AB* or *l* in component form, e.g. (1 – λ*, – 2 – 3λ*, 2 + λ*) or (1 + μ*, 2 – 3μ*, – 3 – 2μ*)	**4**	

Question 55

(a)	Using the correct process find the scalar product of direction vectors of *l* and **OA**	**M1**	(1, 5, 6).(– 1, 2, 3) = –1.1 + 5.2 + 6.3 = – 1 + 10 + 18
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result	**M1**	*Their* scalar product ÷ [$\sqrt{(1^2+5^2+6^2)}\sqrt{((-1)^2+2^2+3^2)}$]. Angle = $\cos^{-1}\frac{27}{\sqrt{62}\sqrt{14}}$.
	Obtain answer 23.6°.	**A1**	AWRT 23.6°. 23.5889°. Radians 0.412 scores A0 (0.4117…).
		3	
(b)	Taking a general point *P* on *l*, state **AP** (or **PA**) in component form, e.g. (3 – λ, – 5 + 2λ, – 5 + 3λ)	**B1**	Note: (4, 1, 0) or (4, 1, 1), for **4i + k** is not MR, but M1 possible.
	Either equate scalar product of **AP** and direction vector of *l* to zero and solve for λ *or* use Pythagoras in a relevant triangle and solve for λ	**M1**	(3 – λ, – 5 + 2λ, – 5 + 3λ).(–1, 2, 3) = 0 –3 –10 –15 + λ + 4 λ + 9 λ = 0 or let **OQ** = (4, 0, 1) so **AQ** = (3, – 5, – 5), **QP** = (– λ, 2λ, 3λ), **AP** = (3 – λ, –5 + 2λ, –5 + 3λ) hence $3^2+(-5)^2+(-5)^2=$ $(3-\lambda)^2+(-5+2\lambda)^2+(-5+3\lambda)^2+(-\lambda)^2+(2\lambda)^2+(3\lambda)^2$ Other alternative approaches are possible, e.g. minimise *AP* or AP^2, either by completing the square or by differentiating.
	Obtain $\lambda = 2$	**A1**	$\lambda = 2$
	State that the position vector **OP*** of the foot is 2**i + 4j + 7k**	**A1**	OE Condone coordinates.
		4	

(c)	Set up a correct method for finding the position vector of the reflection of A in l	M1	For all methods, allow a sign error in one component only: **OA′** = **OP*** + (**OP*** – **OA**) $their(2,4,7)+(their\,2,4,7-1,5,6)$ or **OA′** = **OP*** – (**OA** – **OP***) $their(2,4,7)-(1,5,6-their\,2,4,7)$ or **OA′** = **OA** + 2(**OP*** – **OA**) $\begin{pmatrix}1+2(their\,2-1)\\5+2(their\,4-5)\\6+2(their\,7-6)\end{pmatrix}$ or midpoint **OP*** = (**OA** + **OA′**)/2 with *their* λ value substituted. $\frac{1+x}{2}=their\,2\quad\frac{5+y}{2}=their\,4\quad\frac{6+z}{2}=their\,7$
	Obtain answer $3\mathbf{i}+3\mathbf{j}+8\mathbf{k}$ or $3\left(\mathbf{i}+\mathbf{j}+\frac{8}{3}\right)$	A1	OE Condone coordinates $x=3, y=3, z=8$ A1. No method shown and correct answer 2/2.
		2	

Question 56

(a)	Express general point of l or m in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	B1	
	Equate components and eliminate either λ or μ	M1	e.g. $\mu=\frac{2}{1-b}$, $\lambda=\frac{-1-b}{1-b}$, $\mu=\frac{-4}{2+a}$, $\lambda=\frac{a+6}{a+2}$
	Eliminate the other parameter or obtain a second expression in the first	M1	λ and μ are not required to be the subject of the equations.
	Show intermediate steps to obtain $2b-a=4$	A1	AG
(b)	Using the correct process equate the scalar product of the direction vectors to zero	*M1	$(2\mathbf{i}-\mathbf{j}-\mathbf{k}).(a\mathbf{i}+b\mathbf{j}+\mathbf{k})=0$ SOI.
	Obtain $2a-b-1=0$	A1	OE e.g. $2(2b-4)-b-1=0$
	Solve simultaneous equations for a or for b	DM1	
	Obtain $a = 2, b = 3$	A1	
		4	
(c)	Substitute found values in component equations and solve for λ or for μ	M1	
	Obtain answer $3\mathbf{i}+\mathbf{j}+2\mathbf{k}$ from either $\lambda = 2$ or $\mu = -1$	A1	Accept as coordinates or equivalent.
		2	

Question 57

(a)	Obtain $\overrightarrow{OM} = \mathbf{i}+2\mathbf{j}+2\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	e.g. $\overrightarrow{MO}+\overrightarrow{OA}+\overrightarrow{AN}$ or $\overrightarrow{MO}+\overrightarrow{ON}$
	Obtain $\overrightarrow{MN} = \mathbf{i}+\mathbf{j}-2\mathbf{k}$	A1	Accept any notation.
		3	
(b)	Use a correct method to form an equation for MN	M1	Allow without $\mathbf{r} = \ldots$
	Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i}+\mathbf{j}-2\mathbf{k})$	A1 FT	OE e.g. $\mathbf{r} = \mathbf{i}+2\mathbf{j}+2\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-2\mathbf{k})$ Must have $\mathbf{r} = \ldots$ Follow *their* answers to part **9(a)**.
		2	
(c)	State $\overrightarrow{OP}$ for a general point P on MN in component form, e.g. $(2+\lambda, 3+\lambda, -2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{OP}$ and a direction vector for MN to zero and solve for λ	M1	
	Obtain $\lambda = -\frac{5}{6}$	A1	OE e.g. $\mu = \frac{1}{6}$
	Obtain $\sqrt{\frac{53}{6}}$ correctly	A1	AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2+\left(\frac{13}{6}\right)^2+\left(\frac{5}{3}\right)^2}$
		4	

Question 58

(a)	State $\overrightarrow{OM} = \begin{pmatrix}2\\1\\0\end{pmatrix}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	
	Obtain answer $\begin{pmatrix}3\\-2\\-1\end{pmatrix}$	A1	
		3	
(b)	Carry out a correct method to form a vector equation for MN	M1	
	Obtain a correct equation in any form, e.g. $\mathbf{r} = \begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\-1\end{pmatrix}$	A1	OE
		2	

(c)	State a correct vector equation for AB in any form, e.g. $\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}-1\\5\\1\end{pmatrix}$	**B1**	
	Equate components of AB and MN and solve for λ or for μ	**M1**	
	Obtain $\lambda = -3$ or $\mu = 2$	**A1**	
	Obtain position vector $\begin{pmatrix}-1\\10\\3\end{pmatrix}$, or equivalent, for Q	**A1**	
		4	

Question 59

(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form	**B1**	$\left(\overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k},\quad \overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{k}\right)$.
	Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$,	**M1**	or $\overrightarrow{BA}.\overrightarrow{CA}$ $(8 - 3 = 5)$. M0 for $\overrightarrow{AB}.\overrightarrow{CA}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of *their* moduli to obtain $\cos\theta$ or θ	**M1**	$\left(\dfrac{5}{\sqrt{9}\sqrt{25}}\right)$ Independent of the first M1.
	Obtain answer $\dfrac{1}{3}$	**A1**	ISW. Need to see a value for $\cos\theta$. Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1}\frac{1}{3}$ alone is not sufficient)
		4	
(b)	Use correct method to find an **exact** value for the sine of angle BAC from *their* **(a)**	**M1**	$\left(\sqrt{1 - \frac{1}{9}}\right)$
	Obtain answer $\dfrac{2}{3}\sqrt{2}$, or equivalent	**A1**	
	Use correct area formula to find the area of triangle ABC with *their* versions of relevant vectors	**M1**	$\left(\frac{1}{2}\sqrt{9}\sqrt{25} \times \textit{their} \sin\theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25} \times \sin\left(\cos^{-1}\frac{1}{3}\right)$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	Only ISW
	Alternative method 1 for question 6(b)		
	Use correct method to find the perpendicular distance from A to BC (or B to AC or C to AB)	**M1**	$\begin{pmatrix}2+2\lambda\\-2+2\lambda\\1-4\lambda\end{pmatrix}.\begin{pmatrix}2\\2\\-4\end{pmatrix} = 0 \Rightarrow \lambda = \dfrac{1}{6}$
	Obtain $\frac{1}{3}\sqrt{75}$	**A1**	$\left(\left\|\frac{7}{3}\mathbf{i} - \frac{5}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right\|\right)$
	Use correct area formula to find the area of triangle ABC	**M1**	$\left(\frac{1}{2} \times \textit{their}\sqrt{24} \times \textit{their}\,\frac{1}{3}\sqrt{75}\right)$ The length they use for *their* base must be found correctly.
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	

Question 60

(a)	State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent	**B1**	Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$.
	Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	**B1**	
	Use a correct method to find $\overrightarrow{ON}$	**M1**	e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$
	Obtain answer $3\mathbf{j} + \mathbf{k}$	**A1**	
		4	
(b)	Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result	**M1**	$\cos^{-1}\left(\dfrac{-6+3}{\sqrt{10}\sqrt{17}}\right)$
	Obtain final answer $103.3°$	**A1**	
		3	
(c)	Taking a general point P of ON to have position vector $\lambda(3\mathbf{j}+\mathbf{k})$, form an equation in λ by *either* equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, *or* applying Pythagoras to triangle OMP, *or* equating the derivative of $\lvert\overrightarrow{MP}\rvert$ to zero	**M1**	e.g. $\begin{pmatrix} -2 \\ -2+3\lambda \\ \lambda \end{pmatrix} . \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0$
	Solve and obtain $\lambda = \dfrac{3}{5}$	**A1**	
	Substitute for λ and calculate MP	**M1**	$\overrightarrow{MP} = -2\mathbf{i} - \frac{1}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$
	Obtain $\sqrt{\dfrac{22}{5}}$	**A1**	AG
	Alternative method for question 11(c)		
	Use a scalar product to find the projection OQ of OM on OM	**M1**	
	Obtain $OQ = \dfrac{6}{\sqrt{10}}$	**A1**	
	Use Pythagoras in triangle OMQ to find MQ	**M1**	
	Obtain $\sqrt{\dfrac{22}{5}}$	**A1**	AG
		4	

Question 61

(a)	Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	**M1**	$\pm(3, -1, 2).(1, 2, -3) = \pm(3-2-6) = [-5]$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain $\cos^{-1}\{\pm(3-2-6)/[\sqrt{(3^2+(-1)^2+2^2)}\ \sqrt{(1^2+2^2+(-3)^2)}]\}$	**A1**	
	Obtain answer 110.9° or 1.94^c	**A1**	
		3	
(b)	Use a correct method to form an equation for line through AB	**M1**	
	Obtain $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu_1(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	**A1**	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu_2(-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Need $\mathbf{r}$ or (x, y, z).
		2	
(c)	Obtain a correct equation for line through CD e.g. $[\mathbf{r} =]\ \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + \lambda_1(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$	**B1**	OE e.g. $[\mathbf{r} =]\ 5\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} + \lambda_2(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$. $\mathbf{r}$ can be omitted or another symbol used.
	Equate two pairs of components of general points on *their* l and *their* CD and solve for λ or for μ	**M1**	
	Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$	**A1**	
	Obtain position vector $9\mathbf{i} - 10\mathbf{j} + 17\mathbf{k}$	**A1**	Condone $(9, -10, 17)$ but not $(9\mathbf{i}, -10\mathbf{j}, 17\mathbf{k})$.
		4	

Question 62

(a)	Perform scalar product of direction vectors and set result equal to zero	**M1**	$2c + 6 + 4 = 0$.
	Use P to find the value of λ	**M1**	$3 - 2\lambda = 7 \Rightarrow \lambda = -2$ $[a + \lambda c = 4, b + 4\lambda = -2]$. Equation for line l may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda = 2$ all marks available.
	Obtain $c = -5$ or $b = 6$	**A1**	
	$a = -6$, $b = 6$ and $c = -5$ all correct	**A1**	
		4	**SC1**: Use P to find the value of λ M1 Substitute $\lambda = -2$ into point P, so $a - 2c = 4$, and put $\mu = -1$ and $\lambda = -1$ into l so $a - c = -1$, then solve to obtain $a = -6$, $b = 6$ and $c = -5$. All 3 values correct A1. Max 2/4.

(b)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on m $= \pm((1+2\mu, 2-3\mu, 3+\mu)-(a+\lambda c, 3-2\lambda, b+4\lambda))$	**B1**	$\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$ Could be *their* a, b, c and λ values provided M1 M1 gained in **(a)**. Allow expression in answer column.
	Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for m to zero and obtain an equation in μ	**M1***	$(2(-3+2\mu)-3(-5-3\mu)+(5+\mu))=0$. Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem.
	Solve and obtain $\mu=-1$	**A1**	$PQ^2 = (-3+2\mu)^2 + (-5-3\mu)^2 + (5+\mu)^2$. $[= 14(\mu+1)^2 + 45]$. Min when $\mu = -1$ or by differentiation.
	Obtain $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Must be labelled correctly	**A1**	The working may be in **(a)** provided at least this result is used in **(b)**.
	Carry out a method to find the position vector of R **Alternative method for DM1** $\overrightarrow{OR} = (4, 7, -2) + t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$ Solve $\lvert \boldsymbol{QR}\rvert^2 = \frac{9}{4}\lvert \boldsymbol{PQ}\rvert^2$ or $\lvert \boldsymbol{QR}\rvert = \frac{3}{2}\lvert \boldsymbol{PQ}\rvert$ $t = 2.5$	**DM1**	e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \frac{5}{2}\overrightarrow{OQ} - \frac{3}{2}\overrightarrow{OP}$ or $2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3\overrightarrow{PQ}$ where $\overrightarrow{OR} = (x, y, z)$. $\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$.
(b)	Obtain $-\frac{17}{2}\boldsymbol{i}+2\boldsymbol{j}+8\boldsymbol{k}$ from correct working	**A1**	Accept coordinates. Don't accept $-\frac{17}{2}\mathbf{i}+\frac{4}{2}\mathbf{j}+\frac{16}{2}\mathbf{k}$.
		6	**SC2** Equate lines, attempt to find $\mu=-1$ or $\lambda = -1$ M1* $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ A1. Attempt to find $\overrightarrow{OQ}$ using other parameter value DM1. $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ therefore intersect A1. Then use main scheme for the final DM1 A1.
			First **DM1 A1** are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overrightarrow{OQ}$ using the other parameter value and then showing intersection

Question 63

(a)	Carry out correct method for finding a vector equation for *AB*	**M1**	
	Obtain [**r** =] **i** + 2**j** – 2**k** + λ (**i** – 3**j** + 3**k**)	**A1**	OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.
	Equate two pairs of components of general points on *their AB* and *l* and evaluate λ or μ	**M1**	$\begin{pmatrix} 1+\lambda \\ 2-3\lambda \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ -1-3\mu \\ 3+4\mu \end{pmatrix}$.
	Obtain correct answer for λ or μ, e.g. $\lambda = -1$, $\mu = -2$	**A1**	Correct value from two correct component equations.
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $0 \neq -3$).	**A1**	Conclusion needs to follow correct values. Hybrid versions are possible e.g. using **j** and **k** to get one parameter and then **i** to obtain the other. or e.g. solving two pairs of simultaneous equations and showing that the results are not the same. Alternatives: (see table below)
		5	

Alternatives:

A	λ	μ		***B***	λ	μ	
ij	2	1	$4 \neq 7$	**ij**	1	1	$4 \neq 7$
ik	5	5/2	$-13 \neq -17/2$	**ik**	4	5/2	$-13 \neq -17/2$
jk	–1	–2	$0 \neq -3$	**jk**	–2	–2	$0 \neq -3$

(b)	Find $\overrightarrow{AP}$ for a general point *P* on *l*, e.g. –3**j** + 5**k** + μ(2**i** – 3**j** + 4**k**)	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Calculate scalar product of *their* $\overrightarrow{AP}$ and a direction vector for *l* and equate the result to zero	**M1**	e.g. $4\mu + (9+9\mu) + (20+16\mu) = 0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through *A*.
	Obtain $\mu = -1$	**A1**	
	Obtain answer –**i** + 2**j** – **k**	**A1**	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point *P* on *l*, e.g. –3**j** + 5**k** + μ(2**i** – 3**j** + 4**k**)	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	**M1**	$\frac{\mathrm{d}}{\mathrm{d}\mu}\left(4\mu^2 + 9(\mu+1)^2 + (4\mu+5)^2\right) = 0$.
	Obtain $\mu = -1$	**A1**	
	Obtain answer –**i** + 2**j** – **k**	**A1**	Accept coordinates in place of position vector.
		4	

Question 64

(a)	Obtain a vector for one side of the parallelogram	**B1**	e.g. $\overrightarrow{AB}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\-6\end{pmatrix}$.
	Correct method to obtain $\pm\overrightarrow{OD}$	**M1**	e.g. $\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{BC}$. MO if use $\overrightarrow{AB}=\overrightarrow{CD}$ or $\overrightarrow{BC}=\overrightarrow{DA}$.
	Obtain $\overrightarrow{OD}=\mathbf{i}-4\mathbf{j}-3\mathbf{k}$	**A1**	Any equivalent form. Accept coordinates.
		3	
(b)	Using the correct process, evaluate the scalar product $\overrightarrow{BA}.\overrightarrow{BC}$	**M1**	$(2+10-6)$ Scalar product of two relevant vectors. OE
	Using the correct process for the moduli, divide the scalar product by the product of the moduli.	**M1**	$\dfrac{2+10-6}{\sqrt{9}\times\sqrt{62}}$.
	Obtain answer $\dfrac{2}{\sqrt{62}}$	**A1**	ISW Or simplified equivalent i.e. $\dfrac{\sqrt{62}}{31}$.
		3	
(c)	State or imply $\sin\theta=\sqrt{\dfrac{58}{62}}$	**B1 FT**	Follow *their* $\cos\theta$.
	Use correct method to find the area of *ABCD*	**M1**	**e.g.** $2\times\frac{1}{2}BA\times BC\sin\theta$. Condone decimals.
	Correct unsimplified expression for the area	**A1 FT**	**e.g.** $2\times\frac{1}{2}\times3\times\sqrt{62}\times\sin\theta$. Condone decimals. Follow *their* sides and angle.
	Obtain answer $3\sqrt{58}$	**A1**	Correct only.
		4	