

AS-Level

Topic : Binomial Theorem

May 2013-May 2025

Answer

Question 1

(i) $(2 + ax)^5 = 32 + 80ax + 80a^2x^2$	$3 \times \text{B1}$	$[3]$	B1 for each term.
(ii) $\times (1 + 2x)$ $240 = 80a^2 + 160a$ $\rightarrow a = 1 \text{ or } a = -3.$	M1 DM1A1	$[3]$	Realises need to consider 2 terms. Solution of 3-term quadratic.

Question 2

$\left(2x - \frac{1}{2x}\right)^6$ (i) Coeff of $x^2 = 15 \times 16 \times (-\frac{1}{2})^2 = 60$	B1 B1	$[2]$	B1 for 2/3 parts. B1
(ii) Constant term is $20 \times 8x^3 \times (-1 \div 8x^3)$ $\times (1 + x^2)$ needs to consider 2 terms $\rightarrow 60 - 20 = 40$	B1 M1 A1	$[3]$	B1 unsimplified Needs to consider the constant term

Question 3

(i) $1 - 6px + 15p^2x^2$	B1B1	$[2]$	Simplificn of nCr can be scored in (ii)
(ii) $15p^2 \times 1 - 6p \times -1$ $3p(5p + 2) = 0$ $p = -\frac{2}{5}$ oe	M1 DM1 A1	$[3]$	Obtain & attempt to solve quadratic Allow $p = 0$ in addition

Question 4

(i) $81(x^8)$	B1	$[1]$	
(ii) $10 \times 3^3(x^8)$ soi leading to their answer $270(x^8)$	B1B1 B1	$[3]$	B1 for 10, 5C2 or 5C3. B1 for 3^3 . But must be multiplied.
(iii) $k \times$ (i) 405 soi $+$ (ii) $675(x^8)$	M1 A1 DM1 A1	$[4]$	$k \neq 1, 0$

Question 5

<p>(i) $64 + 576x + 2160x^2$</p> <p>(ii) $576a(x^2) + 2160(x^2) = 0$ $a = -\frac{2160}{576}$ oe (eg $-\frac{15}{4}$) or -3.75</p>	<p>B1B1B1 [3]</p> <p>M1</p> <p>A1 [2]</p>	<p>Can score in (ii)</p>
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Question 6

<p>$(x^2 - \frac{2}{x})^5$</p> <p>Term in x is $10 \times (x^2)^2 \times (\frac{-2}{x})^3$</p> <p>Coefficient = $-80(x)$</p>	<p>B1 B1</p> <p>B1</p> <p>[3]</p>	<p>B1 10 or 5C_2 or 5C_3, B1 $(\frac{-2}{x})^3$</p> <p>co Must be identified</p>
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Question 7

<p>$(1+x^2)(\frac{x}{2} - \frac{4}{x})^6$</p> <p>Term in $x^2 = 15 \times \frac{1}{16} \times (-4)^2 = 15$</p> <p>Constant term = $20 \times \frac{1}{8} \times (-4)^3 = -160$</p> <p>Coefficient of $x^2 = -145$</p>	<p>B1 B1</p> <p>B1 B1</p> <p>B1 \checkmark</p> <p>[5]</p>	<p>B1 unsimplified. B1 15.</p> <p>B1 unsimplified. B1 -160</p> <p>Uses 2 terms. \checkmark on previous answers</p>
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Question 8

<p>$[{}^8C_6 \text{ or } 28] \times [16 \text{ or } 4^2] (x^6) \times \left[\frac{1}{(64 \text{ or } 2^6)(x^6)} \right]$</p> <p>7</p>	<p>B1B1B1</p> <p>B1</p> <p>[4]</p>	<p>Seen in expansion ok. Allow 8C_2</p> <p>Identified as answer</p>
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Question 9

<p>$(15 \text{ or } {}^{16}C_2) \times 2^4 \times (ax)^2, (20 \text{ or } {}^6C_3) \times 2^3 \times (ax)^3$</p> <p>$a = \frac{15 \times 2^4}{20 \times 2^3} = \frac{3}{2}$</p>	<p>B1B1</p> <p>M1A1</p> <p>[4]</p>	<p>$240a = 160a$ is M0</p>
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Question 10

<p>(i) $(1+x)^5 = 1 + 5x + 10x^2$</p> <p>(ii) $(1+px+x^2)^5$ $(1+) 5(px+x^2) + 10(px+x^2)^2$</p> <p>Coeff of $x^2 = 5 + 10p^2$ $= 95 \rightarrow p = 3$</p>	<p>B2,1</p> <p>[2]</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[3]</p>	<p>Loses 1 for each error</p> <p>Replace x by $(px+x^2)$ in their expansion</p> <p>Considers 2 terms co – no penalty for ± 3</p>
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Question 11

${}^7C_1 \times 2^6 \times a \quad (=) \quad {}^7C_2 \times 2^5 \times a^2 \quad \text{soi}$ $a = \left(\frac{7 \times 2^6}{21 \times 2^5} \right) = \frac{2}{3} \quad \text{oe}$	<p>B2, 1, 0</p> <p>B1</p> <p>[3]</p>	<p>Treat the same error in each expression as a single error</p>
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Question 12

<p>(i) $a^5 - 5a^4x + 10a^3x^2 - 10a^2x^3 + \dots$</p>	<p>B2,1,0</p> <p>[2]</p>	<p>Ok full expansion (ignore extra terms) Descending: Ok if full expansion but max B1 for 4 terms</p>
<p>(ii) $(1-ax)(\dots 10a^3x^2 - 10a^2x^3 \dots) = (x^3)(-10a^4 - 10a^2)$ $-10a^4 - 10a^2 = -200$ $a^2 = 4$ ignore $a^2 = -5$ $a = \pm 2$ cao</p>	<p>M1</p> <p>A1✓</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Attempt to find coeff. of x^3 from 2 terms Ft from <i>their</i> $10a^3, -10a^2$ from part (i) Attempt soln. for a^2 from 3-term quad. in a^2 Ignore any imaginary solutions</p>

Question 13

<p>(i) $(2-x)^6$ Coeff of x^2 is 240 Coeff of x^3 is $-20 \times 8 = -160$</p>	<p>B1</p> <p>B2,1</p> <p>[3]</p>	<p>co</p> <p>B1 for +160</p>
<p>(ii) $(3x+1)(2-x)^6$ Product needs exactly 2 terms $\rightarrow 720 - 160 = 560$</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>3 \times their 240 + their -160 ✓ for candidate's answers.</p>

Question 14

<p>(i) (a) $(1-x)^2(1+2x)^6$ $(1-x)^6 = 1 - 6x + 15x^2$</p>	<p>B2,1</p> <p>[2]</p>	<p>-1 each error</p>
<p>(b) $(1+2x)^6 = 1 + 12x + 60x^2$</p>	<p>B2,1</p> <p>[2]</p>	<p>-1 each error SC B1 only, in each part, for all 3 correct descending powers SC only one penalty for omission of the '1' in each expansion</p>
<p>(ii) Product of (a) and (b) with >1 term $\rightarrow 60 - 72 + 15 = 3$</p>	<p>M1</p> <p>DM1A1</p> <p>[3]</p>	<p>Must be 2 or more products M1 exactly 3 products. cao, condone $3x^2$</p>

Question 15

$\left[7C_2 \times \left[\left(\frac{x}{3} \right)^5 \right] \times \left[\left(\frac{9}{x^2} \right)^2 \right] \right]$	soi	B2,1,0	Seen
$21 \times \frac{1}{3^5} (x^5) \times 81 \left(\frac{1}{x^4} \right)$	soi	B1	Identified as required term
7		B1	Accept 7x
			[4]

Question 16

$(x + 2k)^7$		B1	
Term in $x^5 = 21 \times 4k^2 = 84k^2$		B1	
Term in $x^4 = 35 \times 8k^3 = 280k^3$		M1 A1	Correct method to obtain k .
Equate and solve $\rightarrow k = 0.3$ or $\frac{3}{10}$			[4]

Question 17

$(a + x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + \dots$	soi	M1	Ignore subsequent terms
$\left(-\frac{2}{a} \times (their\ 5a^4) + (their\ 10a^3) \right) (x^2)$		M1	
0		A1	AG
			[3]

Question 18

(i)	$80(x^4), -32(x^5)$	B1B1	Fully simplified
			[2]
(ii)	$(-32 + 80p)(x^5) = 0$ $p = 2/5 \text{ or } 32/80 \text{ oe}$	M1	Attempt to mult. relevant terms & put = 0
		A1	
			[2]

Question 19

$5C2 \left(\frac{1}{x}\right)^3 (3x^2)^2$	B1	Can be seen in expansion
$10(\times 1) \times 3^2$	B1	Identified as leading to answer
$90(x)$	B1	
		[3]

Question 20

<p>(i)</p> $\left(x - \frac{2}{x}\right)^6$ <p>Term is ${}^6C_3 \times (-2)^3 = (-)160$ -160</p>	<p>B1 B1</p> <p style="text-align: right;">[2]</p>	±160 seen anywhere
<p>(ii)</p> $\left(2 + \frac{3}{x^2}\right) \left(x - \frac{2}{x}\right)^6$ <p>Term in $x^2 = {}^6C_2 (-2)^2 x^2$ = 60 (x^2)</p> <p>Term independent of x: = 2 × (their -160) + 3 × (their 60) -140</p>	<p>B1 B1</p> <p>M1 A1</p> <p style="text-align: right;">[4]</p>	±60 seen anywhere Using 2 products correctly

Question 21

$\left(x - \frac{3}{2x}\right)^6$ <p>Term is ${}^6C_3 \times x^3 \times \left(\frac{-3}{2x}\right)^3$ → -67.5 oe</p>	<p>B1 B1</p> <p>B1</p> <p style="text-align: right;">[3]</p>	B1 for Bin coeff. B1 for rest.
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Question 22

$(+/-)20 \times 3^3 (x^3), \quad 10a^3 (x^3) \text{ soi}$ $-540 + 10a^3 = 100 \text{ oe}$ $a = 4$	<p>B1B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">[4]</p>	Each term can include x^3 Must have 3 terms and include a^3 and 100
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Question 23

Term in $x = \frac{nx}{2}$	B1	Could be implied by use of a numerical n .
$(3 - 2x)(1 + \frac{nx}{2} + \dots) \rightarrow 7 = \frac{3n}{2} - 2$ $\rightarrow n = 6$	M1	(Their 2 terms in x) = 7
Term in $x^2 = \frac{n(n-1)}{2} \left(\frac{x}{2}\right)^2$	A1	
Coefficient of $x^2 = \frac{3n(n-1)}{8} - \frac{2n}{2}$ $= \frac{21}{4}$	B1	May be implied by (their n) \times (their $n-1$) \div 8.
	M1	Considers 2 terms in x^2 .
	A1	aef
		[6]

Question 24

$8C6(2x)^6 \left(\frac{1}{2x^3}\right)^2$ soi	B1	May be seen within a number of terms
$28 \times 64 \times \frac{1}{4}$ oe (powers and factorials evaluated)	B2,1,0	May be seen within a number of terms
448	B1	Identified as answer
		[4]

Question 25

$5C2 \left(\frac{1}{ax}\right)^3 (2ax^2)^2$ soi	B1	Seen or implied. Can be part of an expansion.
$10 \times \frac{1}{a^3} \times 4a^2 = 5$ soi	M1A1	M1 for identifying relevant term and equating to 5, all correct. Ignore extra x
$a = 8$ cao	A1	
Total:	4	

Question 26

$7C1 \times 2^6 \times a(x), 7C2 \times 2^5 \times [a(x)]^2$	B1 B1	SOI Can be part of expansion. Condone ax^2 only if followed by a^2 . ALT $2^7 [1 + ax/2]^7 \rightarrow 7C1 [a(x)/2] = 7C2 [a(x)/2]^2$
$a = \frac{7 \times 2^6}{21 \times 2^5} = \frac{2}{3}$	B1	Ignore extra soln $a = 0$. Allow $a = 0.667$. Do not allow an extra x in the answer
Total:	3	

Question 27

(i)	Coefficient of $x = 80(x)$	B2	Correct value must be selected for both marks. SR +80 seen in an expansion gets B1 or -80 gets B1 if selected .
	Total:	2	
(ii)	Coefficient of $\frac{1}{x} = -40 \left(\frac{1}{x}\right)$	B2	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets B1 .
	Coefficient of $x = (1 \times \text{their } 80) + (3 \times \text{their } -40) = -40(x)$	M1 A1	Links the appropriate 2 terms only for M1 .
	Total:	4	

Question 28

$(3-2x)^6$		
Coeff of $x^2 = 3^4 \times (-2)^2 \times {}_6C_2 = a$	B3,2,1	Mark unsimplified forms. -1 each independent error but powers must be correct. Ignore any 'x' present.
Coeff of $x^3 = 3^3 \times (-2)^3 \times {}_6C_3 = b$		
$\frac{a}{b} = -\frac{9}{8}$	B1	OE. Negative sign must appear before or in the numerator
Total:	4	

Question 29

(i)	$6C_3 \left(\frac{2}{x}\right)^3 (-3x)^3$ SOI also allowed if seen in an expansion	M1	Both x's can be missing.
	-4320 Identified as answer	A1	Cannot be earned retrospectively in (ii).
		2	
(ii)	$6C_2 \left(\frac{2}{x}\right)^4 [(-)3x]^2$ SOI clearly identified as critical term	M1	Both x's and minus sign can be missing.
	$15a \times 16 \times 9 - \text{their } 4320 (=0)$	A1 FT	FT on their 4320.
	$a = 2$	A1	
		3	

Question 30

<i>EITHER:</i> Term is ${}^9C_3 \times 2^6 \times (-1/4)^3$	(B1, B1, B1)	OE
<i>OR1:</i> $\left(\frac{8x^3-1}{4x^2}\right)^9 = \left(\frac{1}{4x^2}\right)^9 (8x^3-1)^9$ or $-\left(\frac{1}{4x^2}\right)^9 (1-8x^3)^9$		
Term is $-\frac{1}{4^9} \times {}^9C_3 \times 8^6$	(B1, B1, B1)	OE
<i>OR2:</i> $(2x)^9 \left(1 - \frac{1}{8x^3}\right)^9$		
Term is $2^9 \times {}^9C_3 \times \left(-\frac{1}{8}\right)^3$	(B1, B1, B1)	OE
Selected term, which must be independent of x = -84	B1	
	4	

Question 31

(i)	${}^7C_2 (+/-2x)^2 + {}^7C_3 (-2x)^3$	M1	SOI, Allow for either term correct. Allow + or - inside first bracket.
	$84(x^2), -280(x^3)$	A1A1	
		3	
(ii)	$2 \times (\text{their} - 280) + 5 \times (\text{their } 84)$ only	M1	
	-140	A1	
		2	

Question 32

${}^5C_3 x^2 \left(\frac{-2}{x}\right)^3$ SOI	B2,1,0	-80 www scores B3. Accept 5C_2 .
-80 Accept $\frac{-80}{x}$	B1	+80 without clear working scores SCB1
	3	

Question 33

Coefficient of x^2 in $\left(2 + \frac{x}{2}\right)^6$ is ${}^6C_2 \times 2^4 \times \left(\frac{1}{2}\right)^2 (x^2)$ (= 60)	B2,1,0	3 things wanted -1 each incorrect component, must be multiplied together. Allow 6C_4 , $\binom{6}{4}$ and factorial equivalents. Marks can be awarded for correct term in an expansion.
Coefficient of x^2 in $(a+x)^5$ is ${}^5C_2 \times a^3 (x^2)$ (= $10a^3$)	B1	Marks can be awarded for correct term in an expansion.
$\rightarrow 60 + 10a^3 = 330$	M1	Forms an equation 'their 60' + 'their $10a^3$ ' = 330. OK with x^2 in all three terms initially. This can be recovered by a correct answer.
$a = 3$	A1	Condone ± 3 as long as +3 is selected.
	5	

Question 34

(i)	$(1-2x)^5 = 1 - 10x + 40x^2$ (no penalty for extra terms)	B2,1	Loses a mark for each incorrect term. Treat $-32x^3 + 80x^4 - 80x^3$ as MR -1
		2	
(ii)	$\rightarrow (1+ax+2x^2)(1-10x+40x^2)$		
	3 terms in $x^2 \rightarrow 40 - 10a + 2$	M1 A1FT	Selects 3 terms in x^2 . FT from (i)
	Equate with 12 $\rightarrow a = 3$	A1	CAO
		3	

Question 35

${}^7C_5 x^2 (-2/x)^5$ soi	B1	Can appear in an expansion. Allow 7C2
21×-32 soi	B1	Identified. Allow $(21x^2) \times (-32x^{-5})$. Implied by correct answer
-672	B1	Allow $\frac{-672}{x^3}$. If 0/3 scored, 672 scores SCB1
	3	

Question 36

For a correctly selected term in $\frac{1}{x^2} : (3x)^4$ or 3^4	B1	Components of coefficient added together 0/4 B1 expect 81
$\times \left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$	B1	B1 expect 8/27
$\times {}^7C_3$ or 7C_4	B1	B1 expect 35
$\rightarrow 840$ or $\frac{840}{x^2}$	B1	All of the first three marks can be scored if the correct term is seen in an expansion and it is selected but then wrongly simplified.
		SC: A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2.
	4	

Question 37

$5C_3 [(-)(px)^3]$ soi	B1	Can be part of expansion. Condone omission of – sign
$(-1)10p^3 = -2160$ then \div and cube root	M1	Condone omission of – sign.
$p = 6$	A1	
	3	

Question 38

(i)	$\frac{-5}{x} + \frac{5}{8x^3} - \frac{1}{32x^5}$ (or $-5x^{-1} + \frac{5}{8}x^{-3} - \frac{1}{32}x^{-5}$)	B1B1B1	B1 for each correct term SCB1 for both $\frac{+5}{x}$ & $-\frac{+1}{32x^5}$
		3	
(ii)	$1 \times 20 + 4 \times \text{their } (-5) = 0$	M1A1	Must be from exactly 2 terms SCB1 for $20 + 20 = 40$
		2	

Question 39

	For $\left(\frac{2}{x} - 3x\right)^5$ term in x is 10 or $5C_3$ or $5C_2 \times \left(\frac{2}{x}\right)^2 \times (-3x)^3$ or $\left(\frac{2}{x}\right)^5 \frac{5.4.3}{3!} \left(-\frac{3}{2}x^2\right)^3$ or $(-3x)^5 \frac{5.4}{2!} \left(\frac{2}{3x^2}\right)^2$	B2,1	3 elements required. –1 for each error with or without x 's. Can be seen in an expansion.
	–1080 identified	B1	Allow –1080x Allow if expansion stops at this term. Allow from expanding brackets.
		3	

Question 40

(i)	Ind term = $(2x)^3 \times \left(\frac{k}{x}\right)^3 \times {}_6C_3$	B2,1,0	Term must be isolated
	= 540 $\rightarrow k = 1\frac{1}{2}$	B1	
		3	
(ii)	Term, in x^2 is $(2x)^4 \times \left(\frac{k}{x}\right)^2 \times {}_6C_2$	B1	All correct – even if k incorrect.
	$15 \times 16 \times k^2 = 540$ (or $540x^2$)	B1	FT For $240k^2$ or $240k^2x^2$
		2	

Question 41

(i)	$1 + 6y + 15y^2$	B1	CAO
		1	
(ii)	$1 + 6(px - 2x^2) + 15(px - 2x^2)^2$	M1	SOI. Allow $6C1 \times 1^5 (px - 2x^2)$, $6C2 \times 1^4 (px - 2x^2)^2$
	$(15p^2 - 12)(x^2) = 48(x^2)$	A1	1 term from each bracket and equate to 48
	$p = 2$	A1	SC: A1 $p = 4$ from $15p - 12 = 48$
		3	

Question 42

$\frac{6x}{2}, 15 \times \frac{x^2}{4}$	B1 B1	OE In or from a correct expansion. Can be implied by correct equation.
$\times (4 + ax) \rightarrow 3a + 15 = 3$	M1	2 terms in x^2 equated to 3 or $3x^2$. Condone x^2 on one side only.
$a = -4$	A1	CAO
	4	

Question 43

$6C2 \times (2x)^4 \times \frac{1}{(4x^2)^2}$	B1	SOI SC: Condone errors in $(4^{-1})^2$ evaluation or interpretation for B1 only
$15 \times 2^4 \times \frac{1}{4^2}$	B1	Identified as required term.
15	B1	
	3	

Question 44

(a)	$5C2 [2(x)]^3 \left[\frac{a}{(x^2)} \right]^2$	B1	SOI Can include correct x 's
	$10 \times 8 \times a^2 \left(\frac{x^3}{x^4} \right) = 720 \left(\frac{1}{x} \right)$	B1	SOI Can include correct x 's
	$a = \pm 3$	B1	
		3	
(b)	$5C4 [2(x)]^4 \left[\frac{\text{their } a}{(x^2)} \right]^4$	B1	SOI Their a can be just <u>one</u> of their values (e.g. just 3). Can gain mark from within an expansion but must use <i>their</i> value of a
	810 identified	B1	Allow with x^{-7}
		2	

Question 45

(a)	$1 + 5a + 10a^2 + 10a^3 + \dots$	B1
		1
(b)	$1 + 5(x + x^2) + 10(x + x^2)^2 + 10(x + x^2)^3 + \dots$ SOI	M1
	$1 + 5(x + x^2) + 10(x^2 + 2x^3 + \dots) + 10(x^3 + \dots) + \dots$ SOI	A1
	$1 + 5x + 15x^2 + 30x^3 + \dots$	A1
		3

Question 46

(a)	$1 + 5a + 10a^2 + 10a^3 + \dots$	B1
		1
(b)	$1 + 5(x + x^2) + 10(x + x^2)^2 + 10(x + x^2)^3 + \dots$ SOI	M1
	$1 + 5(x + x^2) + 10(x^2 + 2x^3 + \dots) + 10(x^3 + \dots) + \dots$ SOI	A1
	$1 + 5x + 15x^2 + 30x^3 + \dots$	A1
		3

Question 47

$\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$	B1B1
Coefficient in $\left(kx + \frac{1}{x}\right)^5 = 10 \times k^2$ (B1 for 10. B1 for k^2)	
Coefficient in $\left(1 - \frac{2}{x}\right)^8 = 8 \times -2$	B2,1,0
$10k^2 - 16 = 74 \rightarrow k = 3$	B1
	5

Question 48

$[7C1a^6b(x)], [7C2a^5b^2(x^2)], [7C4a^3b^4(x^4)]$	B2, 1, 0	SOI, can be seen in an expansion.
$\frac{7C2a^5b^2(x^2)}{7C1a^6b(x)} = \frac{7C4a^3b^4(x^4)}{7C2a^5b^2(x^2)} \rightarrow \frac{21a^5b^2}{7a^6b} = \frac{35a^3b^4}{21a^5b^2}$	M1 A1	M1 for a correct relationship OE (Ft from <i>their</i> 3 terms). For A1 binomial coefficients must be correct & evaluated.
$\frac{a}{b} = \frac{5}{9}$	A1	OE
	5	

Question 49

Coefficient of x^3 in $(1 - 2x)^5$ is -80	B1	Can be seen in an expansion but must be simplified correctly.
Coefficient of x^2 in $(1 - 2x)^5$ is 40	B1	
Coefficient of x^3 in $(1 + kx)(1 - 2x)^5$ is $40k - 80 = 20$	M1	Uses the relevant two terms to form an equation = 20 and solves to find k . Condone x^3 appearing in some terms if recovered.
$(k =) \frac{5}{2}$	A1	
	4	

Question 50

(a)	$6C2 \times [2(x^2)]^4 \times \left[\frac{a}{(x)}\right]^2$, $6C3 \times [2(x^2)]^3 \times \left[\frac{a}{(x)}\right]^3$	B1 B1	SOI Can be seen in an expansion
	$15 \times 2^4 \times a^2 = 20 \times 2^3 \times a^3$	M1	SOI Terms must be from a correct series
	$a = \frac{15 \times 2^4}{20 \times 2^3} = \frac{3}{2}$	A1	OE
		4	
(b)	0	B1	
		1	

Question 51

(a)	$1 + 5x + 10x^2$	B1	
		1	
(b)	$1 - 12x + 60x^2$	B2, 1, 0	B2 all correct, B1 for two correct components.
		2	
(c)	$(1 + 5x + 10x^2)(1 - 12x + 60x^2)$ leading to $60 - 60 + 10$	M1	3 products required
	10	A1	Allow $10x^2$
		2	

Question 52

(a)	$(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \dots$	B2, 1, 0	Allow extra terms. Terms may be listed. Allow a^6x^0 .
		2	
(b)	$\left(1 + \frac{2}{ax}\right)(\dots 15a^4x^2 - 20a^3x^3 + \dots)$ leading to $[x^2](15a^4 - 40a^2)$	M1	Attempting to find 2 terms in x^2
	$15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20 = 0$	A1	Terms on one side of the equation
	$(5a^2 - 10)(3a^2 - 2) = 0$	M1	OE. M1 for attempted factorisation or solving for a^2 or u ($=a^2$) using e.g. formula or completing the square
	$a = \pm\sqrt{2}, \pm\sqrt{\frac{2}{3}}$	B1 B1	OE exact form only If B0B0 scored then SC B1 for $\sqrt{2}, \sqrt{\frac{2}{3}}$ WWW or $\pm 1.41, \pm 0.816$ WWW
		5	

Question 53

[Coefficient of x or $p =$] 480	B1	SOI. Allow $480x$ even in an expansion.
$\left[\text{Term in } \frac{1}{x} \text{ or } q = \right] [10 \times] (2x)^3 \left(\frac{k}{x^2} \right)^2$	M1	Appropriate term identified and selected.
$[10 \times 2^3 k^2 =] 80k^2$	A1	Allow $\frac{80k^2}{x}$
$p = 6q$ used ($480 = 6 \times 80k^2$ or $80 = 80k^2$)	M1	Correct link used for <i>their</i> coefficient of x and $\frac{1}{x}$ (p and q) with no x 's.
$[k^2 = 1 \Rightarrow] k = \pm 1$	A1	A0 if a range of values given. Do not allow $\pm\sqrt{1}$.
	5	

Question 54

(a)	243	B1	
	$-810x$	B1	
	$+1080x^2$	B1	
		3	
(b)	$(4 + x)^2 = 16 + 8x + x^2$	B1	
	Coefficient of x^2 is $16 \times 1080 + 8 \times (-810) + 243$	M1	Allow if at least 2 pairs used correctly
	11043	A1	Allow $11043x^2$
		3	

Question 55

(a)	$1 + 6ax + 15a^2x^2$	B1	Terms must be evaluated.
		1	
(b)	<i>their</i> $15a^2 \pm (3 \times \text{their } 6a)$	*M1	Expect $15a^2 - 18a$.
	$15a^2 - 18a = -3$	A1	
	$(3)(a-1)(5a-1) [= 0]$	DM1	Dependent on 3-term quadratic. Or solve using formula or completing the square.
	$a = 1, \frac{1}{5}$	A1	WWW. If DM0 awarded SC B1 if both answers correct.
		4	

Question 56

(a)	Terms required for x^2 : $-5 \times 2^4 \times ax + 10 \times 2^3 \times a^2 x^2 [= -80ax + 80a^2 x^2]$	B1	Can be seen as part of an expansion or in correct products.
	$2 \times (\pm \text{their coefficient of } x) + 4 \times (\pm \text{their coefficient of } x^2)$	*M1	
	x^2 coefficient is $320a^2 - 160a = -15$ $\Rightarrow 64a^2 - 32a + 3 \Rightarrow (8a-3)(8a-1)$	DM1	Forming a 3-term quadratic in a , with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give <i>their</i> coefficient of a^2 .
	$a = \frac{1}{8}$ or $a = \frac{3}{8}$	A1	OE. Special case: If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	
(b)	$320a^2 - 160a = k \Rightarrow 320a^2 - 160a - k [= 0]$	M1	Forming a 3-term quadratic in a with all terms on the same side. Allow \pm sign errors.
	<i>Their</i> $b^2 - 4ac [= 0]$, $[160^2 - 4 \times 320 \times (-k) = 0]$	M1	Any use of discriminant on a 3-term quadratic.
	$k = -20$	A1	
	$a = \frac{1}{4}$	B1	Condone $a = \frac{1}{4}$ from $k = 20$.
	Alternative method for question 8(b)		
	$320a^2 - 160a = k$ and divide by 320 $\left[a^2 - \frac{a}{2} = \frac{k}{320} \right]$	M1	Allow \pm sign errors.
	Attempt to complete the square $\left[\left(a - \frac{1}{4} \right)^2 - \frac{1}{16} = \frac{k}{320} \right]$	M1	Must have $\left(a - \frac{1}{4} \right)^2$
	$a = \frac{1}{4}$	A1	
	$k = -20$	B1	

Question 57

(a)	$1 - \frac{1}{x} + \frac{1}{4x^2}$	B1	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
(b)	$1 + 12x + 60x^2 + 160x^3$	B2, 1, 0	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
(c)	<i>their</i> $(1 \times 12) + \text{their} (-1 \times 60) + \text{their} \left(\frac{1}{4} \times 160 \right)$	M1	Attempts at least 2 products where each product contains one term from each expansion.
	$[12 - 60 + 40 =] -8$	A1	Allow $-8x$.
		2	

Question 58

(a)	${}^6C_2 \times (3x)^4 \left(\frac{2}{x^2}\right)^2$	B1	Can be seen within an expansion.
	$15 \times 3^4 \times 2^2$	B1	Identified. Powers must be correct.
	4860	B1	Without any power of x
		3	
(b)	<i>Their</i> 4860 and one other relevant term	M1	Using <i>their</i> 4860 and an attempt to find a term in x^{-3}
	Other term = ${}^6C_3 (3x)^3 \left(\frac{2}{x^2}\right)^3$ or ${}^6C_3 \times 3^3 \times 2^3$ or 4320	A1	Must be identified. If M0 scored then SC B1 for 4320 as the only answer.
	$[4860 - 4320 =] 540$	A1	
		3	

Question 59

$4C1 \times p \times \frac{1}{p^3} x^3$	B1	OE soi Can be seen in an expansion.
$\frac{4}{p^2} = 144$	B1	OE Correct with correct power of p and only one p term.
$p = \pm \frac{1}{6}$	B1 B1	OE $\pm \frac{2}{12}$ etc. Allow ± 0.167 for B1 B1. SC B1 for $\pm \sqrt{\frac{1}{36}}$ B1 only,
	4	

Question 60

Coefficient of $x^4 = 15$	B1	Condone inclusion of x^4 . Can be seen as part of an expansion.
Coefficient of $x^2 = 240a^2$	B1	Condone inclusion of x^2 . Can be seen as part of an expansion.
' <i>Their</i> 240' a^2 - ' <i>their</i> 15'	M1	Forming an equation of the form $pa^2 = q$, where p and q are constants. Condone inclusion of powers of x as long as they then disappear.
$a = \frac{1}{4}$ or 0.25	A1	OE Do not condone extra 'answer' of $-\frac{1}{4}$, or allow $\sqrt{\frac{1}{16}}$ or similar.
	4	

Question 61

(a)	x^4 term is $[10 \times](2x^2)^3 \left(\frac{k^2}{x}\right)^2$	M1	For selecting the term in x^4 .
	$80k^4x^4 \Rightarrow a = 80k^4$	A1	For correct value of a . Allow $80k^4x^4$.
	$[x^2$ term is $[6 \times](2kx)^2 \times 1 = 24k^2x^2 \Rightarrow] b = 24k^2$	B1	For correct value of b . Allow $24k^2x^2$.
		3	
(b)	$80k^4 + 24k^2 - 216 = 0 \quad [\Rightarrow 10k^4 + 3k^2 - 27 = 0]$	M1	Forming a 3-term equation in k (all terms on one side) with <i>their</i> a and b and no x 's.
	$(2k^2 - 3)(5k^2 + 9) = 0 \quad [\Rightarrow k^2 = \frac{3}{2} \text{ or } -\frac{9}{5}]$	M1	Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square – see guidance.
	$[k] = \pm\sqrt{\frac{3}{2}}$	A1	OE e.g. $\pm\frac{\sqrt{6}}{2}$, $\pm\sqrt{1.5}$, AWR ± 1.22 Omission of \pm A0. Additional answers A0. If M1 M0, SC B1 can be awarded for correct final answer, max 2/3.
		3	

Question 62

(a)	$1 + 10x + 40x^2$ May be part of a complete expansion	B2, 1, 0	1^5 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
(b)	$1 - 12x + 54x^2$ May be part of a complete expansion	B2, 1, 0	1^4 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
(c)	$54 - 120 + 40$	M1	Forming exactly 3 products correctly using their terms.
	-26	A1	Allow $-26x^2$ If in a list with other terms it must be clear this is the required term otherwise A0.
		2	

Question 63

Coefficient of x^2 in $\left(1 + \frac{2}{p}x\right)^5$ is $10\left(\frac{2}{p}\right)^2 = \frac{10 \times 2^2}{p^2} \left[= \frac{40}{p^2} \right]$	B1	Accept with x^2 present. Must evaluate 5C_2
Coefficient of x^2 in $(1 + px)^6$ is $15(p)^2 \left[= 15p^2 \right]$	B1	Accept with x^2 present. Must evaluate 6C_2
$\frac{40}{p^2} + 15p^2 = 70$	*M1	Forming an equation in p with <i>their</i> coefficients, the given 70, no x terms and no extra terms.
$15p^4 - 70p^2 + 40 \left[= 0 \right]$ or $3p^4 - 14p^2 + 8 \left[= 0 \right]$	DM1	Forming a 3-term equation in p (or another variable) with all terms on one side and <i>their</i> coefficients.
$[5](p^2 - 4)(3p^2 - 2) \left[= 0 \right]$ or $\frac{70 \pm \sqrt{70^2 - 4(15)(40)}}{30}$ or $\frac{14 \pm \sqrt{14^2 - 4(3)(8)}}{6}$	DM1	Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square.
$p = \pm 2, \pm \sqrt{\frac{2}{3}}$	A1	OE e.g. $\pm \frac{\sqrt{6}}{3}$ or AWRT ± 0.816 If *M1 DM1 DM0, allow SC B1 for 4 correct values.
	6	

Question 64

${}_{7C1} \left(\frac{x}{a}\right)^6 \left(\frac{a}{x^2}\right)$ or ${}_{7C6} \left(\frac{x}{a}\right)^6 \left(\frac{a}{x^2}\right)$ ${}_{7C2} \left(\frac{x}{a}\right)^5 \left(\frac{a}{x^2}\right)^2$ or ${}_{7C5} \left(\frac{x}{a}\right)^5 \left(\frac{a}{x^2}\right)^2$	B1 B1	Coefficients x^4 & x . Can be seen in an expansion.
$\left(\frac{7}{a^5}\right) = 3$ $\left(\frac{21}{a^3}\right)$	M1	OE. Allow extraneous x^4 and x at this stage; numerator and denominator must be functions of a . Allow errors in evaluation of the combinations.
	A1	Completely correct.
$a^2 = \frac{1}{9}$	A1	SOI (implied by $a = \frac{1}{3}$).
$a = \pm \frac{1}{3}$	A1	Allow ± 0.333 .
	6	

Question 65

(a)	$x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$ or $x^5 + 10x^3 + 40x + 80x^{-1} + 80x^{-3} + 32x^{-5}$	B2, 1, 0	B2, all terms correct, B1 5 terms correct. Terms must be simplified. Lists of terms allowed.
		2	
(b)	<i>their</i> $40 \times a + (\textit{their coefficient of } x^{-1}) \times b = 0$	M1	Coefficients of a and b must be non-zero, allow x 's so long as they are dealt with correctly.
	$(\textit{their coefficient of } x^{-1}) \times a + (\textit{their coefficient of } x^{-3}) \times b = 80$	M1	Coefficients of a and b must be non-zero, allow x 's as long as they are dealt with correctly.
	$a = 2 \quad b = -1$	A1 A1	Dependent on both M marks, may be seen without working.
		4	

Question 66

[Coefficient of $x^4 = p =$] $15a^2$	B1	May be seen in an expansion or with x^4 .
[Coefficient of $x^2 = q =$] $54a^2$	B1	May be seen in an expansion or with x^2 .
Equating <i>their</i> $p +$ <i>their</i> q to 276 leading to an equation in a^2 only	M1	No x terms and no extra terms. If p and q are not identified then it needs to be clear from the expansion that the appropriate coefficients are being used. $69a^2 = 276$ implies the first 3 marks.
$a = \pm 2$	A1	CAO
	4	

Question 67

(a)	$16 + 96x + 216x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
(b)	$1 - 10x + 40x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
(c)	$(16 \times 40) - (10 \times 96) + (1 \times 216)$	M1	<i>Their</i> 3 products which would give the term in x^2 (FT <i>their</i> values). Look for $640 - 960 + 216$.
	-104	A1	Condone $-104x^2$.
		2	

Question 68

(a)(i)	$1 + 10x + 40x^2$	B1	Ignore any additional terms (ISW). Allow x^0 or $1x^0$ for the first term, allow lists.
		1	
(a)(ii)	$\{1\}\{-6ax\}\{+15a^2x^2\}$	B2, 1, 0	Ignore any additional terms (ISW). Allow x^0 or $1x^0$ for the first term, allow lists.
		2	
(b)	$15a^2 - 60a + 40 = -5$	M1 A1	Correct 3 products from <i>their</i> expansions for M1. Condone inclusion of x^2 for M1.
	$[15](a-1)(a-3)[=0]$ OE	DM1	OE. Rearranging and solving a quadratic.
	$a=1$ and 3	A1	Special case: If M1 A1 DM0 scored then SC B1 can be awarded for correct answers.
		4	

Question 69

[Coefficient of x^3 from $(3+2ax)^5 =$] $10 \times 9 \times 8a^3$ $[= 720a^3]$	B1	May be seen in an expansion or with x^3 .
[Coefficient of x^2 from $(2+ax)^6 =$] $15 \times 16 \times a^2$ $[= 240a^2]$	B1	May be seen in an expansion or with x^2 .
<i>their</i> $(10 \times 9 \times 8a^3) = 6 \times$ <i>their</i> $(15 \times 16 \times a^2)$ $[\Rightarrow 720a^3 = 1440a^2]$	M1	OE Equating <i>their</i> coefficient of x^3 and $6 \times$ <i>their</i> coefficient of x^2 .
$a = 2$	A1	Condone extra solution $a = 0$.
	4	

Question 70

(a)	$1+18x+135x^2$	B2, 1, 0	Accept 1, 18x, 135x ² listed horizontally or vertically or $1x^0+18x+135x^2$.
		2	
(b)	Coefficient of x^2 is $135-7\times 18+1=10$	M1 A1	3 products, allow $10x^2$. If full expansion given, like terms must be collected for M1.
		2	

Question 71

	$\binom{4}{2}2^2(ax)^2, \binom{4}{3}2^1(ax)^3$	B1 B1	OE Expect $24a^2x^2, 8a^3x^3$ (may be seen in an expansion).
	Multiply terms involving x^2 and x^3 by $5-ax$ to obtain x^3 term	*M1	Must find two products only (may be seen in an expansion).
	Equate coefficient of x^3 to 432 and solve for a	DM1	Ignore inclusion of x^3 at this stage.
	Obtain $a=3$ only	A1	
		5	

Question 72

	Correct second term $30x$ in expansion of $(1+3x)^{10}$	B1	WWW, may be implied later.
	Correct third term $+405x^2$	B1	Ignore subsequent terms, may be implied later.
	Multiply $(2-5x)$ by <i>their</i> $30x+405x^2$ to obtain two x^2 terms only	M1	Expect $-150x^2, 810x^2$.
	Coefficient is 660	A1	Must be clearly identified. Allow final answer $660x^2$.
		4	

Question 73

	$240[x^2]$ or $80a^2[x^2]$	B1	May be seen in an expansion.
	$240=12\times 80a^2$	M1	<i>Their</i> 240 equated to $12\times$ <i>their</i> $80a^2$ which must contain a^2 .
	0.5	A1	OE Condone ± 0.5
		3	

Question 74

(a)	$20\times 27\times a^3 [=160]$	M1	Allow $6C3\times 3^3\times a^3 [=160]$. Accept $540a^3$ with no other working for M1.
	$[a]=\frac{2}{3}$	A1	Allow 0.667 AWRP. SC B1 is $a=\frac{2}{3}$ with no other working.
		2	
(b)	Coefficient of x^2 is $15\times 81\times \left(\textit{their}\frac{2}{3}\right)^2 [=540]$	B1 FT	May be in a list. 6C2 and 3^4 must be evaluated but may be implied by later work. Condone 540 with no working.
	$160\times 1-2\times \textit{their} 540$	M1	
	$=-920$	A1	Condone $-920x^3$.
		3	

Question 75

(a)	$x^3 : \binom{5}{3} 3^2 (-ax)^3 [-10 \times 3^2 \times a^3]$ or $x^4 : \binom{5}{4} 3(-ax)^4 [5 \times 3 \times a^4]$	M1	Allow for either term, allow sign error and combination notation.
	$x^3 : -90a^3$	A1	Allow in the full expansion.
	$x^4 : 15a^4$	A1	Allow in the full expansion.
		3	
(b)	Coefficient of x^4 is $a \times \text{their } -90a^3 + 7 \times \text{their } 15a^4 [=15a^4]$	M1	Must select two appropriate terms only.
	$15a^4 = 240$	DM1	Reducing to a simple quartic equation in a .
	$\Rightarrow a^4 = 16 \Rightarrow a = 2$	A1	A0 if $a = -2$ is given as a solution.
		3	

Question 76

(a)	15 or $\binom{6}{2} \times x^4 \left(\frac{3}{x^2}\right)^2$ or $x^6 \times \frac{6 \times 5}{2} \left(\frac{3}{x^3}\right)^2$	B1	OE May be in a list. Allow $\binom{6}{4}$.
	135	B1	Correct term must be identified if in a list. Allow $135x^0$.
		2	
(b)	20 or $\binom{6}{3} \times x^3 \left(\frac{3}{x^2}\right)^3$ or $x^6 \times \frac{6.5.4}{3!} \left(\frac{3}{x^3}\right)^3$	B1	OE May be in a list.
	$= \frac{540}{x^3}$	B1	Identifying $\frac{1}{x^3}$ term. This can be implied by sight of 2160 as part of the constant term.
	$4 \times 540 - 5 \times 135$	M1	$4 \times \text{their } 540 - 5 \times \text{their } 135$
	1485	A1	Allow $1485x^0$.
		4	

Question 77

Identify correct term and obtain $6(kx)^2 \cdot \left(\frac{2}{x}\right)^2$	M1	Needs numerical coefficient or $\frac{4!}{2!2!}$, not 4C_2 .
Equate to 150 and obtain $k = \frac{5}{2}$	A1	Ignore $-\frac{5}{2}$
Identify correct term $4(kx)^3 \cdot \left(\frac{2}{x}\right)$ with their value of k	M1	Needs numerical coefficient or $\frac{4!}{3!1!}$.
Obtain coefficient 125	A1	Accept $125x^2$ as final answer.
	4	

Question 78

(a)	$(2x)^4$ and $\left(\frac{\pm 3}{x}\right)^4$	B1	May be seen in a full expansion. This can be implied by $16x^4$ and $+\frac{81}{x^4}$ unless they are clearly using $+\frac{3}{x}$ throughout.
	$4(2x)^3\left(\frac{-3}{x}\right) + 6(2x)^2\left(\frac{-3}{x}\right)^2 + 4(2x)\left(\frac{-3}{x}\right)^3$	B1	Correct combination of numerical coefficients for the middle three terms. Can be implied by a correct full expansion.
	$[16x^4] + k_1x^2 + k_2[x^0] + k_3x^{-2} [+81x^{-4}]$	M1	OE Powers now simplified correctly with <i>their</i> $k_1, k_2, k_3 \neq 0$.
	$16x^4 - 96x^2 + 216 - 216x^{-2} + 81x^{-4}$	A1	OE
		4	
(b)	Use of (<i>their</i> 216) + 5 × (<i>their</i> -96) only, to arrive at the coefficient of x^2	M1	Other terms may be seen.
	-264	A1	Accept $-264x^2$ as the final answer.
		2	

Question 79

(a)	$\{32[x^0]\} \{-120x\} \{+180x^2\}$	B1B1B1	Coefficients must be simplified. Ignore further terms in the expansion. SC B3: $2^5\left(1 - \frac{15}{4}x + \frac{45}{8}x^2 \dots\right)$ can be awarded full marks, ISW.
		3	
(b)	$x = 0.01$ or $\frac{1}{100}$	B1	Identify 0.01 as the only value to substitute, condone any method.
	(<i>Their</i> $32 - 120x + 180x^2$) with $x =$ (<i>their</i> 0.01)	M1	Clear use of <i>their</i> x -value from <i>their</i> solution of $2 - \frac{3}{2}x = 1.985$ and <i>their</i> answer to (a). Expect $32 - 120(0.01) + 180(0.01)^2 = 32 - 1.2 + 0.018$. Condone substitution into further terms in the expansion.
	30.8[18]	A1	Correct answer to at least 3sf coming from substitution into three or more terms. 'Correct' answers without appropriate working get A0.
		3	

Question 80

	$\binom{5}{3}$ or $\binom{5}{2}$ or $10 \times (px^2)^{5-3} \times \left(\frac{4}{p}x\right)^3$	B1	May be seen in a list. Allow with or without x 's. Condone missing brackets only if recovered later.
	$10p^2 \times \frac{64}{p^3}$	B1	This term must now be identified if given as part of a list. Allow with or without x 's.
	$\frac{\text{Their integer } k}{p} [x^7] = 1280[x^7]$	M1	Condone e.g. $\frac{640x^7}{p} = 1280$ if x^7 then disappears in subsequent work. Must be from $a \times (px^2)^{5-3} \times \left(\frac{4}{p}x\right)^3$.
	$p = \frac{1}{2}$	A1	OE SC B1 for $\binom{5}{4}$ or $\binom{5}{1}$ or $5 \times (px^2)^{5-1} \times \left(\frac{4}{px}\right)^1 \Rightarrow p = 4$.
		4	

Question 81

(a)(i)	$32 - 80px + 80p^2x^2$	B2,1,0	B2 for all correct. B1 for any two correct. May be in a list. Ignore terms with higher powers.
		2	
(a)(ii)	$1 - 2x + \frac{3}{2}x^2$	B2,1,0	OE B2 for all correct. B1 for any two correct. May be in a list. Ignore terms with higher powers.
		2	
(b)	Coefficient of $x^2 = 32 \times \frac{3}{2} + (-2 \times -80p) + 80p^2$	*M1	3 terms FT <i>their</i> values.
	$48 + 160p + 80p^2 = 93 \Rightarrow 5(4p + 9)(4p - 1) = 0$	DM1	Set <i>their</i> 3-term coefficient to 93 and attempt to solve by factorising or other accepted method for solving their 3-term quadratic.
	$p = -\frac{9}{4}$ and $p = \frac{1}{4}$	A1	SC B1 following M0 if method for solving quadratic is not shown.
		3	

