AS-Level

Pure Mathematics P1

Topic : Binomial Theorem

May 2013- May 2025

Question 1

(i) Find the first three terms in the expansion of $(2 + ax)^5$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x)(2 + ax)^5$ is 240, find the possible values of a. [3]

Question 2

Find the coefficient of x^2 in the expansion of

(i) $\left(2x - \frac{1}{2x}\right)^6$, [2]

(ii) $(1 + x^2)\left(2x - \frac{1}{2x}\right)^6$. [3]

Question 3

(i) In the expression $(1 - px)^6$, p is a non-zero constant. Find the first three terms when $(1 - px)^6$ is expanded in ascending powers of x. [2]

(ii) It is given that the coefficient of x^2 in the expansion of $(1 - x)(1 - px)^6$ is zero. Find the value of p. [3]

Question 4

(i) Find the coefficient of x^8 in the expansion of $(x + 3x^2)^4$. [1]

(ii) Find the coefficient of x^8 in the expansion of $(x + 3x^2)^5$. [3]

(iii) Hence find the coefficient of x^8 in the expansion of $[1 + (x + 3x^2)]^5$. [4]

Question 5

(i) Find the first three terms when $(2 + 3x)^6$ is expanded in ascending powers of x. [3]

(ii) In the expansion of $(1 + ax)(2 + 3x)^6$, the coefficient of x^2 is zero. Find the value of a. [2]

Question 6

Find the coefficient of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^5$. [3]

Question 7

Find the coefficient of x^2 in the expansion of $(1 + x^2)\left(\frac{x}{2} - \frac{4}{x}\right)^6$. [5]

Question 8

Find the term independent of x in the expansion of $\left(4x^3 + \frac{1}{2x}\right)^8$. [4]

Question 9

In the expansion of $(2 + ax)^6$, the coefficient of x^2 is equal to the coefficient of x^3. Find the value of the non-zero constant a. [4]

Question 10

(i) Find the first 3 terms, in ascending powers of x, in the expansion of $(1 + x)^5$. [2]

The coefficient of x^2 in the expansion of $(1 + (px + x^2))^5$ is 95.

(ii) Use the answer to part **(i)** to find the value of the positive constant p. [3]

Question 11

In the expansion of $(2 + ax)^7$, the coefficient of x is equal to the coefficient of x^2. Find the value of the non-zero constant a. [3]

Question 12

(i) Write down the first 4 terms, in ascending powers of x, of the expansion of $(a - x)^5$. [2]

(ii) The coefficient of x^3 in the expansion of $(1 - ax)(a - x)^5$ is -200. Find the possible values of the constant a. [4]

Question 13

(i) Find the coefficients of x^2 and x^3 in the expansion of $(2 - x)^6$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(3x + 1)(2 - x)^6$. [2]

Question 14

(i) Find the first three terms, in ascending powers of x, in the expansion of

(a) $(1 - x)^6$, [2]

(b) $(1 + 2x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $[(1 - x)(1 + 2x)]^6$. [3]

Question 15

Find the coefficient of x in the expansion of $\left(\frac{x}{3} + \frac{9}{x^2}\right)^7$. [4]

Question 16

In the expansion of $(x + 2k)^7$, where k is a non-zero constant, the coefficients of x^4 and x^5 are equal. Find the value of k. [4]

Question 17

In the expansion of $\left(1 - \frac{2x}{a}\right)(a + x)^5$, where a is a non-zero constant, show that the coefficient of x^2 is zero. [3]

Question 18

(i) Find the coefficients of x^4 and x^5 in the expansion of $(1-2x)^5$. [2]

(ii) It is given that, when $(1+px)(1-2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

Question 19

Find the coefficient of x in the expansion of $\left(\frac{1}{x}+3x^2\right)^5$. [3]

Question 20

Find the term that is independent of x in the expansion of

(i) $\left(x-\frac{2}{x}\right)^6$, [2]

(ii) $\left(2+\frac{3}{x^2}\right)\left(x-\frac{2}{x}\right)^6$. [4]

Question 21

Find the term independent of x in the expansion of $\left(x-\frac{3}{2x}\right)^6$. [3]

Question 22

The coefficient of x^3 in the expansion of $(1-3x)^6+(1+ax)^5$ is 100. Find the value of the constant a. [4]

Question 23

In the expansion of $(3-2x)\left(1+\frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

Question 24

Find the term independent of x in the expansion of $\left(2x+\frac{1}{2x^3}\right)^8$. [4]

Question 25

In the expansion of $\left(\frac{1}{ax}+2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

Question 26

The coefficients of x and x^2 in the expansion of $(2+ax)^7$ are equal. Find the value of the non-zero constant a. [3]

Question 27

(i) Find the coefficient of x in the expansion of $\left(2x-\frac{1}{x}\right)^5$. [2]

(ii) Hence find the coefficient of x in the expansion of $(1+3x^2)\left(2x-\frac{1}{x}\right)^5$. [4]

Question 28

The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

Question 29

(i) Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

Question 30

Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

Question 31

(i) Find the coefficients of x^2 and x^3 in the expansion of $(1 - 2x)^7$. [3]

(ii) Hence find the coefficient of x^3 in the expansion of $(2 + 5x)(1 - 2x)^7$. [2]

Question 32

Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

Question 33

The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

Question 34

(i) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant a. [3]

Question 35

Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$. [3]

Question 36

Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

Question 37

The coefficient of x^3 in the expansion of $(1 - px)^5$ is -2160. Find the value of the constant p. [3]

Question 38

(i) In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

Question 39

Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

Question 40

The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

Question 41

(i) Expand $(1 + y)^6$ in ascending powers of y as far as the term in y^2. [1]

(ii) In the expansion of $(1 + (px - 2x^2))^6$ the coefficient of x^2 is 48. Find the value of the positive constant p. [3]

Question 42

The coefficient of x^2 in the expansion of $(4 + ax)\left(1 + \frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

Question 43

Find the term independent of x in the expansion of $\left(2x + \frac{1}{4x^2}\right)^6$. [3]

Question 44

The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

Question 45

(a) Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

Question 46

(a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

Question 47

The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

Question 48

In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

Question 49

The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

Question 50

In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

Question 51

(a) Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

Question 52

(a) Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

Question 53

The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

Question 54

(a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x. [3]

(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

Question 55

(a) Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

Question 56

(a) It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

(b) It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

Question 57

(a) Expand $\left(1 - \frac{1}{2x}\right)^2$. [1]

(b) Find the first four terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

(c) Hence find the coefficient of x in the expansion of $\left(1 - \frac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

Question 58

Find the term independent of x in each of the following expansions.

(a) $\left(3x + \frac{2}{x^2}\right)^6$ [3]

(b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$ [3]

Question 59

The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p. [4]

Question 60

The coefficient of x^4 in the expansion of $(3 + x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

Question 61

The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

(b) Given that $a + b = 216$, find the possible values of k. [3]

Question 62

(a) Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

Question 63

The coefficient of x^2 in the expansion of $\left(1 + \frac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

Find the possible values of the constant p. [6]

Question 64

In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

Question 65

(a) Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

(b) In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

Question 66

The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

Question 67

(a) Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

Question 68

(a) Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1 + 2x)^5$. [1]

(ii) $(1 - ax)^6$, where a is a constant. [2]

In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

Question 69

The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

Question 70

(a) Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

Question 71

It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

Question 72

Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$ [4]

Question 73

The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

Question 74

The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a. [2]

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$. [3]

Question 75

(a) Find the coefficients of x^3 and x^4 in the expansion of $(3-ax)^5$, where a is a constant. Give your answers in terms of a. [3]

(b) Given that the coefficient of x^4 in the expansion of $(ax+7)(3-ax)^5$ is 240, find the positive value of a. [3]

Question 76

Find the term independent of x in the expansion of each of the following:

(a) $\left(x+\frac{3}{x^2}\right)^6$ [2]

(b) $(4x^3-5)\left(x+\frac{3}{x^2}\right)^6$. [4]

Question 77

In the expansion of $\left(kx+\frac{2}{x}\right)^4$, where k is a positive constant, the term independent of x is equal to 150.

Find the value of k and hence determine the coefficient of x^2 in the expansion. [4]

Question 78

(a) Find the complete expansion of $\left(2x-\frac{3}{x}\right)^4$. [4]

(b) Hence determine the coefficient of x^2 in the expansion of $\left(x^2+5\right)\left(2x-\frac{3}{x}\right)^4$. [2]

Question 79

(a) Find the first three terms in the expansion of $\left(2-\frac{3}{2}x\right)^5$ in ascending powers of x. [3]

(b) Use your answer to part **(a)**, with a suitable value of x, to find an approximation to 1.985^5. [3]

Question 80

The coefficient of x^7 in the expansion of $\left(px^2+\frac{4}{p}x\right)^5$ is 1280.

Find the value of the constant p. [4]

Question 81

(a) Find the first three terms, in ascending powers of x, in the expansion of each of the following expressions.

(i) $(2-px)^5$ [2]

(ii) $\left(1-\frac{1}{2}x\right)^4$ [2]

(b) Given that the coefficient of x^2 in the expansion of $(2-px)^5\left(1-\frac{1}{2}x\right)^4$ is 93, find the possible values of the constant p. [3]