AS-Level

Topic : Calculus

May 2013-May 2025

Answer

Question 1

	$y = \frac{8}{\sqrt{x}} - x$			
(i)	$\frac{dy}{dx} = -4x^{-\frac{3}{2}} - 1$	B1		needs both
	$= -\frac{3}{2}$ when $x = 4$.	M1		Subs $x = 4$ into dy/dx
	Eqn of BC $y - 0 = -\frac{3}{2}(x - 4)$	M1		Must be using differential + correct form of line at $B(4,0)$.
	$\rightarrow C(1, 4\frac{1}{2})$	A1	[4]	
(ii)	area under curve $= \int(\frac{8}{\sqrt{x}} - x)$			
	$= \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2$	B1 B1		(both unsimplified)
	Limits 1 to 4 $\rightarrow$ $8\frac{1}{2}$	M1		Using correct limits.
	Area under tangent $= \frac{1}{2} \times 4\frac{1}{2} \times 3 = 6\frac{3}{4}$	M1		Or could use calculus)
	Shaded area $= 1\frac{3}{4}$	A1	[5]	

Question 2

$u = x^2y$ $\quad y + 3x = 9$ $u = x^2(9 - 3x)$ or $\left(\frac{9-y}{3}\right)^2 y$	M1		Expressing u in terms of 1 variable
$\frac{du}{dx} = 18x - 9x^2$ or $\frac{du}{dy} = 27 - 12y + y^2$	DM1A1		Knowing to differentiate.
$= 0$ when $x = 2$ or $y = 3$ $\rightarrow$ $u = 12$	DM1 A1		Setting differential to 0.
$\frac{d^2u}{dx^2} = 18 - 18x$ $\;$ $-$ve	DM1 A1	[7]	Any valid method

Question 3

$\frac{dy}{dx} = \sqrt{2x+5}$			
$\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$	B1 B1		B1 Everything without "÷2". B1 "÷2"
Uses (2, 5) $\rightarrow c = -4$	M1 A1	[4]	Uses point in an integral.

Question 4

$y = \sqrt{1+4x}$		
(i) $\frac{dy}{dx} = \frac{1}{2}(1+4x)^{-\frac{1}{2}} \times 4$	B1 B1	B1 Without "×4". B1 for "×4" even if first B mark lost.
$= 2$ at B (0, 1)		
Gradient of normal $= -\frac{1}{2}$	M1	Use of $m_1m_2 = -1$
Equation $y - 1 = -\frac{1}{2}x$	M1 A1 [5]	Correct method for eqn.
(ii) At A $x = -\frac{1}{4}$	B1	
$\int\sqrt{1+4x}\,dx = \frac{(1+4x)^{\frac{3}{2}}}{\frac{3}{2}} \div 4$	B1 B1	B1 Without the "÷4". For "÷4" even if first B mark lost.
Limits $-\frac{1}{4}$ to 0 $\rightarrow \frac{1}{6}$	B1	
Area $BOC = \frac{1}{2} \times 2 \times 1 = 1$	B1√	For 1 + his "1/6".
$\rightarrow$ Shaded area $= \frac{7}{6}$	[5]	

Question 5

$f(x) = \frac{5}{1-3x}, \quad x \geq 1$		
(i) $f'(x) = \frac{-5}{(1-3x)^2} \times -3$	B1 B1 [2]	B1 without × −3. B1 for ×−3, even if first B mark is incorrect
(ii) $15 > 0$ and $(1-3x)^2 > 0$, $f'(x) > 0$ $\rightarrow$ increasing	B1√ [1]	√ providing ( )² in denominator.
(iii) $y = \frac{5}{1-3x} \rightarrow 3x = 1 - \frac{5}{y}$	M1	Attempt to make x the subject.
$\rightarrow f^{-1}(x) = \frac{x-5}{3x}$ or $\frac{1}{3} - \frac{5}{3x}$	A1	Must be in terms of x.
Range is ≥ 1	B1	must be $\geq$
Domain is $-2.5 \leq x < 0$	B1 B1 [5]	condone $<$

Question 6

(i) $\pi r^2 h = 250\pi \ \rightarrow\ h = \frac{250}{r^2}$ $\rightarrow\ S = 2\pi r h + 2\pi r^2$	M1	Makes h the subject. $\pi r^2 h$ must be right
$\rightarrow\ S = 2\pi r^2 + \frac{500\pi}{r}$	M1 [2]	Ans given – check all formulae..
(ii) $\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2}$	B1 B1	B1 for each term
$= 0$ when $r^3 = 125 \quad \rightarrow\ r = 5$ $\rightarrow\ S = 150\pi$	M1 A1 [4]	Sets differential to 0 + attempt at soln
(iii) $\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3}$ This is positive $\rightarrow$ Minimum	M1 A1 [2]	Any valid method. 2nd differential must be correct – no need for numerical answer or correct r.

Question 7

$\frac{dy}{dx} = \frac{6}{x^2}$ $y = -6x^{-1} + c$	B1	Integration only – unsimplified
Uses $(2, 9) \rightarrow c = 12$	M1	Uses (2, 9) in an integral
$y = -6x^{-1} + 12$	A1 [3]	

Question 8

(i) $\frac{dy}{dx} = 4(x-2)^3$	**B1**	Or $4x^3 - 24x^2 + 48x - 32$
Grad of tangent = –4	**M1**	Sub $x = 1$ into *their* derivative
Eq. of tangent is $y - 1 = -4(x-1)$	**M1**	Line thru (1, 1) and with m from deriv
$\rightarrow B\left(\frac{5}{4}, 0\right)$	**A1**	
Grad of normal = $\frac{1}{4}$	**M1**	Use of $m_1 m_2 = -1$
Eq. of normal is $y - 1 = \frac{1}{4}(x-1) \rightarrow C\left(0, \frac{3}{4}\right)$	**A1** **[6]**	
(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$	**M1**	
$\frac{\sqrt{17}}{4}$	**A1** **[2]**	Allow $\sqrt{\frac{17}{16}}$
(iii) $\int (x-2)^4 dx = \frac{(x-2)^5}{5}$	**B1**	Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$
$\left[0 - \left(-\frac{1}{5}\right)\right] = \frac{1}{5}$	**M1**	Apply limits $1 \rightarrow 2$ for curve
$\Delta = \frac{1}{2} \times 1 \times \left(\textit{their}\ \frac{5}{4} - 1\right) = \frac{1}{8}$	**M1**	Or $\int_1^{\frac{5}{4}} (-4x + 5)\ dx = \frac{1}{8}$
$\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ or 0.075	**A1** **[4]**	

Question 9

(i) $3u+\frac{3}{u}-10=0$	**B1**		Or $3x-10\sqrt{x}+3=0$ Or $(3\sqrt{x}-1)(\sqrt{x}-3)$ or apply formula etc.
$3u^2-10u+3=0\Rightarrow(3u-1)(u-3)=0$	**M1**		
$\sqrt{x}=\frac{1}{3}$ or 3	**A1**		
$\sqrt{x}=\frac{1}{9}$ or 9	**A1**	**[4]**	
(ii) $f''(x)=\frac{3}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$	**B1**		Allow anywhere
At $x=\frac{1}{9}$ $f''(x)=\frac{3}{2}(3)-\frac{3}{2}(27)\,(=-36)<0\rightarrow$ Max	**M1**		Valid method. Allow innac subs, even $3,\ \frac{1}{3}$
At $x=9$ $f''(x)=\frac{3}{2}\times\frac{1}{3}-\frac{3}{2}\times\frac{1}{27}(=\frac{4}{9})>0\rightarrow$ Min	**A1**	**[3]**	Fully correct. No working, no marks.
(iii) $f(x)=2x^{\frac{3}{2}}+6x^{\frac{1}{2}}-10x\ (+c)$	**B2**		B1 for 2/3 terms correct. Allow in **(i)**
$-7=16+12-40+c$	**M1**		Sub (4, –7). c must be present.
$c=5$	**A1**	**[4]**	

Question 10

$f'(x)=(2x-5)^2\times2+1$ or $24\left(x-\frac{5}{2}\right)^2+1$	**B1B1**		**B1** for $3(2x-5)^2$, **B1** for $(\times2+1)$ SC B1 for $24x^2-120x+151$
>0 (allow $\geq$)	**B1**√	[3]	Dep on $k(2x-5)^2+c$ $(k>0)$, $(c\geq0)$ Subst of particular values is B0

Question 11

(i) $\frac{dy}{dx}=\left[\frac{1}{2}(x4+4x+4)^{-\frac{1}{2}}\right]\times\left[4x^3+4\right]$	B1B1		
At $x=0$, $\frac{dy}{dx}=\frac{1}{2}\times\frac{1}{2}\times4=(1)$	M1		Sub $x=0$ and attempt eqn of line following differentiation.
Equation is $y-2=x$	A1	[4]	
(ii) $x+2=\sqrt{x^4+4x+4}\Rightarrow(x+2)^2$	B1		**AG** www
$=x4+4x+4$			
$x^2-x^4=0$ oe	B1		
$x=0,\pm1$	B2,1,0	[4]	
(iii) $(\pi)\left[\frac{x^5}{5}+2x^2+4x\right]$	M1A1		Attempt to integrate y^2
$(\pi)\left[0-\left(\frac{-1}{5}+2-4\right)\right]$	DM1		
$\frac{11\pi}{5}$ (6.91) oe	A1	[4]	Apply limits $-1\rightarrow0$

Question 12

$\frac{dy}{dx}=-k^2(x+2)^{-2}+1=0$	M1A1	Attempt differentiation & set to zero
$x+2=\pm k$	DM1	Attempt to solve
$x=-2\pm k$	A1	cao
$\frac{d^2y}{dx^2}=2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
	M1	Sub their x value with k in it into $\frac{d^2y}{dx^2}$
When $x=-2=k$, $\frac{d^2y}{dx^2}=\left(\frac{2}{k}\right)$ which is (>0) min	A1	Only 1 of bracketed items needed for each
When $x=-2-k$, $\frac{d^2y}{dx^2}=\left(\frac{2}{-k}\right)$ which is (<0) max	A1	but $\frac{d^2y}{dx^2}$ and x need to be correct.
	[8]	

Question 13

$f(x)=2x^{-\frac{1}{2}}+x\ (+c)$	M1A1	Attempt integ $x^{-\frac{1}{2}}$ or $+x$ needed for M
$5=-2\times\frac{1}{2}+4+c$	M1	Sub (4, 5). c must be present
$c=2$	A1 [4]	

Question 14

$y=\frac{8}{x}+2x$

(i) $\frac{dy}{dx}=\frac{-8}{x^2}+2$	M1 A1	Attempt at differentiation. algebraic – unsimplified.
(-6 at A)		
$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dy}{dt}$ $\rightarrow -0.24$	M1 A1 [4]	Ignore notation – needs product of 0.04 and 'his' $\frac{dy}{dx}$.
(ii) $\int y^2=\int\frac{64}{x^2}+4x^2+32$	M1	Use of integral of y^2 (ignore π)
$=(\frac{-64}{x}+\frac{4x^3}{3}+32x\ )$	A3,2,1	3 terms $\rightarrow -1$ each error.
Limits 2 to 5 used correctly	DM1	Uses correct limits correctly.
$\rightarrow 271.2\pi$ or 852 (allow 271π or 851 to 852)	A1 [6]	(omission of π loses last mark)

Question 15

(i) Sim triangles $\frac{y}{16-x}=\frac{12}{16}$ (or trig)	M1	Trig, similarity or eqn of line
$\rightarrow y = 12 - \frac{3}{4}x$	A1	(could also come from eqn of line)
$A = xy = 12x - \frac{3}{4}x^2$.	A1 [3]	ag – check working.
(ii) $\frac{dA}{dx} = 12 - \frac{6x}{4}$	B1	
$= 0$ when $x = 8. \rightarrow A = 48.$	M1 A1	Sets to 0 + solution.
This is a Maximum. From −ve quadratic or 2nd differential.	B1 [4]	Can be deduced without any working. Allow even if '48' incorrect.

Question 16

$y = \frac{2}{\sqrt{5x-6}}$		
(i) $\frac{dy}{dx} = 2 \times -\frac{1}{2} \times (5x-6)^{-\frac{3}{2}} \times 5$	B1 B1	B1 without '×5'. B1 For '×5'
$\rightarrow -\frac{5}{8}$	B1 [3]	Use of '*uv*' or '*u*/*v*' ok.
(ii) integral $= \frac{2\sqrt{5x-6}}{\frac{1}{2}} \div 5$	B1 B1	B1 without '÷5'. B1 for '÷ 5'
Uses 2 to 3 → 2.4 − 1.6 = 0.8	M1 A1 [4]	Use of limits in an integral.

Question 17

(i) $\frac{dy}{dx} = \left[3(3-2x)^2\right] \times [-2]$	**B1B1**	OR $-54 + 72x - 24x^2$ B2,1,0
At $x = \frac{1}{2}, \frac{dy}{dx} = -24$	**M1**	
$y - 8 = -24\left(x - \frac{1}{2}\right)$	**DM1**	
$y = -24x + 20$	**A1** **[5]**	
(ii) Area under curve $= \left[\frac{(3-2x)^4}{4}\right] \times \left[-\frac{1}{2}\right]$	**B1B1**	OR $27x - 27x^2 + 12x^3 - 2x^4$ B2,1,0
$-2 - \left(-\frac{81}{8}\right)$	**M1**	Limits 0→ ½ applied to integral with intention of subtraction shown
Area under tangent $= \int(-24x + 20)$	**M1**	or area trap =½(20 + 8) × ½
$= \left\| -12x^2 + 20x \right\|$ or 7 (from trap)	**A1**	Could be implied
$\frac{9}{8}$ or 1.125	**A1** **[6]**	Dep on both M marks

Question 18

(i) $A = 2xr + \pi r^2$	B1	
$2x + 2\pi r = 400$ $(\Rightarrow\ x = 200 - \pi r)$	B1	
$A = 400r - \pi r^2$	M1A1 [4]	Subst & simplify to **AG** (www)
(ii) $\frac{dA}{dr} = 400 - 2\pi r$	B1	Differentiate
$= 0$	M1	Set to zero and attempt to find r
$r = \frac{200}{\pi}$ oe	A1	
$x = 0 \Rightarrow$ no straight sections **AG**	A1	
$\frac{d^2A}{dr^2} = -2\pi$ (<0) Max	B1 [5]	Dep on -2π, or use of other valid reason

Question 19

Attempt integration	M1	
$f(x) = 2(x+6)^{\frac{1}{2}} - \frac{6}{x}(+c)$	A1A1	Accept unsimplified terms
$2(3) - \frac{6}{3} + c = 1$	M1	Sub. $x = 3$, $y = 1$. c must be present
$c = -3$	A1 [5]	

Question 20

pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow$ $x = 3, 7$	M1A1	Attempt at soln of sim eqns. co
Area of trapezium $= \frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1)\,dx$ from 3 to 7 = 44)	M1A1	Either method ok. co
Area under curve $= -\frac{1}{3}x^3 + 6x^2 - 20x$	B2,1	−1 each term incorrect
Uses 3 to 7 $\rightarrow$ $(54\frac{2}{3})$	DM1	Correct use of limits (Dep 1st M1)
Shaded area $= 10\frac{2}{3}$	A1 [8]	co
OR		
$\int_3^7 \left(-x^2 + 10x - 21\right) = \left(-\frac{x^3}{3} + 5x^2 - 21x\right)$		Functions subtracted before integration
M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1		Subtraction reversed allow A3A0. Limits reversed allow DM1A0

Question 21

(i) $3x^2y = 288$ y is the height	B1	co
$A = 2(3x^2 + xy + 3xy)$	M1	Considers at least 5 faces ($y \neq x$)
Sub for y $\rightarrow$ $A = 6x^2 + \frac{768}{x}$	A1	co answer given
	[3]	
(ii) $\frac{dA}{dx} = 12x - \frac{768}{x^2}$	B1	co
$= 0$ when $x = 4 \rightarrow A = 288$. Allow $(4, 288)$	M1 A1	Sets differential to 0 + solution. co
$\frac{d^2A}{dx^2} = 12 + \frac{1536}{x^3}$	M1	Any valid method
$(= 36) > 0$ Minimum	A1	co www dep on correct f″ and $x = 4$
	[5]	

Question 22

$\frac{dy}{dx} = \frac{12}{\sqrt{4x+a}}$ $P(2, 14)$ Normal $3y + x = 44$		
(i) m of normal $= -\frac{1}{3}$	B1	co
$\frac{dy}{dx} = 3 = \frac{12}{\sqrt{4x+a}}$ $\rightarrow$ $a = 8$	M1 A1	Use of $m_1m_2 = -1$. AG.
	[3]	
(ii) $\int y = 12(4x + a)^{\frac{1}{2}} \div \frac{1}{2}$ $\div 4$ $(+c)$	B1 B1	Correct without "÷4". for "÷4".
Uses (2, 14)	M1	Uses in an integral only. Dep 'c'.
$c = -10$	A1	co All 4 marks can be given in (i)
	[4]	

Question 23

$f(x) = \frac{15}{2x+3}$		
(i) $f'(x) = \frac{-15}{(2x+3)^2} \times 2$	B1 B1	Without the "×2". For "×2" (indep of 1st B1).
( )² always +ve $\rightarrow$ $f'(x) < 0$ (No turning points) – therefore an inverse	B1√	√ providing ( )² in f '(x). 1–1 insuff.
	[3]	
(ii) $y = \frac{15}{2x+3}$ $\rightarrow 2x + 3 = \frac{15}{y}$	M1	Order of ops – allow sign error
$\rightarrow x = \frac{\frac{15}{y} - 3}{2}$ $\rightarrow$ $\frac{15 - 3x}{2x}$	A1	co as function of x. Allow $y = \ldots$
(Range) $0 \leqslant f^{-1}(x) \leqslant 6$. Allow $0 \leqslant y \leqslant 6$, [0,6]	B1	For range/domain ignore letters
(Domain) $1 \leqslant x \leqslant 5$. Allow [1, 5]	B1	unless range/domain not identified
	[4]	

Question 24

$y = 8 - \sqrt{4-x}$		
(i) $\frac{dy}{dx} = -\frac{1}{2}(4-x)^{-\frac{1}{2}} \times -1$	B1 B1	Without (−1). For (×−1).
$\int y\,dx = 8x - \frac{(4-x)^{\frac{3}{2}}}{\frac{3}{2}} \div -1$	3 × B1	B1 for "8x" and +c". B1 for all except ÷(−1). B1 for ÷(−1).
	[5]	(n.b. these 5 marks can be gained in(ii) or (iii))
(ii) Eqn $y - 7 = \frac{1}{2}(x-3)$		
$\rightarrow y = \frac{1}{2}x + 5\frac{1}{2}$	M1A1 [2]	M1 unsimplified. A1 as $y=mx+c$
(iii) Area under curve = ∫ from 0 to 3 (58/3)	M1	Use of limits – needs use of "0"
Area under line = ½(5½ + 7)×3	M1	Correct method
Or $\left[\frac{1}{4}x^2 + \frac{11x}{2}\right]$ from 0 to 3	M1 A1 [4]	M1 Subtraction. A1 co
$\rightarrow \frac{58}{3} - \frac{75}{4} = \frac{7}{12}$		

Question 25

$\frac{d^2y}{dx^2} = 2x - 1$		
$\rightarrow \int \quad \frac{dy}{dx} = x^2 - x \quad + c$	B1	Correct integration (ignore +c)
$= 0$ when $x = 3 \quad \rightarrow c = -6$	M1 A1	Uses a constant of integration. co
$x^2 - x - 6 = 0$ when $x = -2$ (or 3)	A1	Puts dy/dx to 0
$\rightarrow \int \quad y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \quad (+k)$	B1√B1√	√ first 2 terms, √ for cx.
$= -10$ when $x = 3$	M1	Correct method for k
$\rightarrow k = 3\frac{1}{2}$		
$\rightarrow y = 10\frac{5}{6}$	A1 [8]	Co –r 10.8

Question 26

(i) $y = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + (c)$ oe	**B1B1**	Attempt to integrate
$\frac{2}{3} = \frac{16}{3} - 4 + c$	**M1**	Sub $\left(4, \frac{2}{3}\right)$. Dependent on c present
$c = -\frac{2}{3}$	**A1** **[4]**	
(ii) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ oe	**B1B1** **[2]**	
(iii) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 0 \rightarrow \frac{x-1}{\sqrt{x}} = 0$	**M1**	Equate to zero and attempt to solve
$x = 1$	**A1**	
When $x = 1$, $y = \frac{2}{3} - 2 - \frac{2}{3} = -2$	**M1A1**	Sub. *their* '1' into *their* 'y'
When $x = 1$, $\frac{d^2y}{dx^2}(=1) > 0$ Hence minimum	**B1** **[5]**	Everything correct on final line. Also dep on correct (ii). Accept other valid methods

Question 27

$\frac{dy}{dx} = [-2 \times 4(3x+1)^{-3}] \times [3]$	B1B1	$[-2 \times 4u^{-3}] \times [3]$ is B0B1 unless resolved
When $x = -1$, $\frac{dy}{dx} = 3$	B1	
When $x = -1, y = 1$ soi	B1	
$y - 1 = 3(x + 1)$ $(\rightarrow y = 3x + 4)$	B1 √ [5]	Ft on *their* '3' only (not $-\frac{1}{3}$). Dep on diffn

Question 28

(a) **(i)** $(a+b)^{\frac{1}{3}} = 2$, $(9a+b)^{\frac{2}{3}} = 16$	B1B1	Ignore 2nd soln (–9, 17) throughout
$a+b=8$, $9a+b=64$	M1	Cube etc. & attempt to solve
$a=7$, $b=1$	A1	Correct answers without any working 0/4
	[4]	
(ii) $x=(7y+1)^{\frac{1}{3}}$ (*x*/*y* interchange as first or last step)	B1√	ft on from *their a, b* or in terms of *a, b*
$x^3 = 7y+1$ or $y^3 = 7x+1$	B1√	ft on from *their a, b* or in terms of *a, b*
$f^{-1}(x) = \frac{1}{7}(x^3 - 1)$ cao	B1	A function of x required
Domain of f^{-1} is $x \geqslant 1$ cao	B1 [4]	Accept $>$. Must be x
(b) $\frac{dy}{dx} = \left[\frac{1}{3}(7x^2+1)^{-\frac{2}{3}}\right] \times [14x]$	B1B1	
When $x=3$, $\frac{dy}{dx} = \frac{1}{3} \times (64)^{-\frac{2}{3}} \times 42$ $\left(=\frac{7}{8}\right)$	M1	
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{7}{8} \times 8$	DM1	Use chain rule
7	A1 [5]	

Question 29

(i) $x-3\sqrt{x}+2$ or k^2-3k+2 or $\left(3\sqrt{x}\right)^2=(x+2)^2$	**M1**	**OR** attempt to eliminate x eg sub $x=\frac{y^2}{9}$
$\sqrt{x}=1$ or 2 or $k=1$ or 2 or $x^2-5x+4(=0)$	**A1**	$y^2-9y+18=0$
$x=1$ or 4	**A1**	$y=3$ or 6
$y=3$ or 6	**A1** **[4]**	$x=1$ or 4

(ii) $\int 3x^{\frac{1}{2}}\,dx-\left[\int(x+2)\,dx \text{ or attempt at trapezium}\right]$	**M1DM1**	Attempt to integrate. Subtract at some stage
$2x^{\frac{3}{2}}-\left[\left(\frac{1}{2}x^2+2x\right) \text{or} \frac{1}{2}(y_2+y_1)(x_2-x_1)\right]$	**A1A1**	Where $(x_1,y_1),(x_2,y_2)$ is *their* (1, 3), (4, 6)
$(16-2)-\left[\left[(8+8)-\left(\frac{1}{2}+2\right)\right] \text{or } their\ \frac{1}{2}\times9\times3\right]$	**DM1**	Apply *their* 1→4 limits correctly to curve
$\frac{1}{2}$	**A1** **[6]**	For A mark allow reverse subtn→ $-\frac{1}{2}\to\frac{1}{2}$ but not reversed limits
OR		
$\left[\int(y-2)\,dy \text{ or attempt at trap}\right]-\int\frac{y^2}{9}dy$	**M1DM1**	
$\left[\frac{1}{2}y^2-2y \text{ or } \frac{1}{2}(x_1+x_2)(y_2-y_1)\right]-\frac{y^3}{27}$	**A1A1**	
$\left[(18-12)-\left(4\frac{1}{2}-6\right) \text{or } \frac{1}{2}\times5\times3\right]-[8-1]$	**DM1**	Apply *their* 3→6 limits correctly to curve
$\frac{1}{2}$	**A1**	

Question 30

(i) Minimum since f ″(3) (= 4/3) > 0 www	**B1** **[1]**	
(ii) $f'(x)=-18x^{-2}\ (+c)$	**B1**	
$0=-2+c$	**M1**	Sub f′(3) = 0. (dep c present)
$c=2\ \left(\to f'(x)=-18x^{-2}+2\right)$	**A1**	$c=2$ sufficient at this stage
$f(x)=18x^{-1}+2x\ (+k)$	**B1√B1√**	Allow cx at this stage
$7=6+6+k$	**M1**	Sub f(3) = 3 (k present & numeric (or no) c)
$k=-5\to\left(f(x)=18x^{-1}+2x-5\right)$ cao	**A1** **[7]**	

Question 31

(i) $(3x-2)^2+1$	**B1B1B1** **[3]**	For either of 1[st] 2 marks bracket must be in the form $(ax+b)^2$ except for SCB2 for $9\left(x-\frac{2}{3}\right)^2+1$
(ii) $f'(x)=9x^2-12x+5$	**B1**	
$=$ their $(3x-2)^2+1$	**M1**	Ft from (i). Some reference/recognition
>0 (or $\geqslant 1$) hence an increasing function	**A1** **[3]**	Allow >1. Allow *their* 1 provided positive. Allow a complete alt method (2/2 or 0/2)

Question 32

$\frac{d^2y}{dx^2}=\frac{24}{x^3}-4$		
(i) (If $x=2$) it's negative $\rightarrow$ Max	B1 [1]	www
(ii) $\left(\frac{dy}{dx}=\right)-12x^{-2}-4x+(A)$	B2,1,0	oe one per term
$=0$ when $x=2$	M1	Attempt at the constant A after ∫n
$\rightarrow A=11$	A1 [4]	co
(iii) $(y=)\ 12x^{-1}-2x^2+Ax+(c)$	B2,1,0 √	oe Doesn't need $+c$, but does need a term A to give "Ax".
$y=13$ when $x=1 \rightarrow c=-8$	M1	Attempt at c after ∫n
(If $x=2$) $y=12$	A1 [4]	co

Question 33

$y=x^3+ax^2+bx$		
(i) $\frac{dy}{dx}=3x^2+2ax+b$	B1	co
(ii) $b^2-4ac=4a^2-12b\ (<0)$	M1	Use of discriminant on their quadratic $\frac{dy}{dx}$ or other valid method
$\rightarrow a^2<3b$	A1 [3]	co – answer given
(iii) $y=x^3-6x^2+9x$		
$\frac{dy}{dx}=3x^2-12x+9<0$	M1	Attempt at differentiation
$=0$ when $x=1$ and 3	A1	co
$\rightarrow 1<x<3$	A1 [3]	condone $\leqslant$

Question 34

$y = \frac{12}{3-2x}$

(i) Differential $= -12(3-2x)^{-2} \times -2$	B1 B1 [2]	co co (even if 1st B mark lost)
(ii) $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 0.4 \div 0.15$	M1	Chain rule used correctly (AEF)
$\rightarrow \frac{24}{(3-2x)^2} = \frac{8}{3}$	M1	Equates their $\frac{dy}{dx}$ with their $\frac{8}{3}$ or $\frac{3}{8}$
$\rightarrow x = 0$ or 3	A1 A1 [4]	co co

Question 35

Vol = (π) $\int x^2 dy = (\pi) \int (y-1)\,dy$	M1 A1	Use of $\int x^2$ – not $\int y^2$ – ignore π co
Integral is $\frac{1}{2}y^2 - y$ or $\frac{(y-1)^2}{2}$	B1	Sight of an integral sign with 1 and 5
Limits for y are 1 to 5		
$\rightarrow 8\pi$ or 25.1(AWRT)	A1 [4]	co (no π max 3/4)

Question 36

(i)	For $y = (4x+1)^{\frac{1}{2}}$, $\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right] \times [4]$	**B1B1**	
	When $x = 2$, gradient $m_1 = \frac{2}{3}$	**B1**√	Ft from *their* derivative above
	For $y = \frac{1}{2}x^2 + 1$, $\frac{dy}{dx} = x \rightarrow$ gradient $m_2 = 2$	**B1**	
	$\alpha = \tan^{-1} m_2 - \tan^{-1} m_1$	**M1**	
	$\alpha = 63.43 - 33.69 = 29.7$ cao	**A1** **[6]**	
(ii)	$\int (4x+1)^{\frac{1}{2}} dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{2/3}\right] \div [4]$	**B1B1**	
	$\int \left(\frac{1}{2}x^2 + 1\right) dx = \frac{1}{6}x^3 + x$	**B1**	
	$\int_0^2 (4x+1)^{\frac{1}{2}} dx = \frac{1}{6}[27-1]$, $\int_0^2 \left(\frac{1}{2}x^2 + 1\right) dx = \left[\frac{8}{6} + 2\right]$	**M1**	Apply limits $0 \rightarrow 2$ to at least the 1st integral
	$\frac{13}{3} - \frac{10}{3}$	**M1**	Subtract the integrals (at some stage)
	1	**A1** **[6]**	

Question 37

(i)	$f'(2) = 4 - \frac{1}{2} = \frac{7}{2} \to$ gradient of normal $= -\frac{2}{7}$ $y - 6 = -\frac{2}{7}(x-2)$ AEF	**B1M1** **A1**√ **[3]**	 Ft from their $f'(2)$
(ii)	$f(x) = x^2 + \frac{2}{x}(+c)$ $6 = 4 + 1 + c \Rightarrow c = 1$	**B1B1** **M1A1** **[4]**	 Sub (2, 6) – dependent on *c* being present
(iii)	$2x - \frac{2}{x^2} = 0 \Rightarrow 2x^3 - 2 = 0$ $x = 1$ $f''(x) = 2 + \frac{4}{x^3}$ or any valid method $f''(1) = 6$ OR > 0 hence minimum	**M1** **A1** **M1** **A1** **[4]**	Put $f'(x) = 0$ and attempt to solve Not necessary for last A mark as $x > 0$ given Dependent on everything correct

Question 38

(i)	$\frac{dy}{dx} = 6 - 6x$ At $x = 2$, gradient $= -6$ soi $y - 9 = -6(x-2)$ oe Expect $y = -6x + 21$ When $y = 0$, $x = 3\frac{1}{2}$ cao	**B1** **B1**√ **M1** **A1** **[4]**	 Line through (2, 9) and with gradient *their* −6
(ii)	Area under curve: $\int 9 + 6x - 3x^2 dx = 9x + 3x^2 - x^3$ $(27 + 27 - 27) - (18 + 12 - 8)$ Area under tangent: $\frac{1}{2} \times \frac{3}{2} \times 9 (= \frac{27}{4})$ Area required $\frac{27}{4} - 5 = \frac{7}{4}$	**B2,1,0** **M1** **B1**√ **A1** **[5]**	Allow unsimplified terms Apply limits 2,3. Expect 5 OR $\int_2^{7/2}(-6x + 21)\,dx \,(\to \frac{27}{4})$. Ft on *their* $-6x + 21$ and/or *their* 7/2.

Question 39

(i)	$-(x+1)^{-2} - 2(x+1)^{-3}$	**M1A1** **A1** **[3]**	M1 for recognisable attempt at differentn. Allow $\frac{-x^2 - 4x - 3}{(x+1)^4}$ from Q rule. (A2,1,0)
(ii)	$f'(x) < 0$ hence decreasing	**B1** **[1]**	Dep. on *their* (i) < 0 for $x >$ 1
(iii)	$\frac{-1}{(x+1)^2} - \frac{2}{(x+1)^3} = 0$ **or** $\frac{-x^2 - 4x - 3}{(x+1)^4} = 0$ $\frac{-(x+1) - 2}{(x+1)^3} = 0 \to -x - 1 - 2 = 0$ or $-x^2 - 4x - 3 = 0$ $x = -3$, $y = -1/4$	**M1*** **M1** **Dep*** **A1A1** **[4]**	Set $\frac{dy}{dx}$ to 0 OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e.×mult) × multn $\to -(x+1)^3 - 2(x+1)^2 = 0$ (−3, −1/4) www scores 4/4

Question 40

$\left[\dfrac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷2] (+c)	B1B1	
$7 = 9 + c$	M1	Attempt subst $x = 4, y = 7$. c must be there. Dep. on attempt at integration.
$y = \dfrac{(2x+1)^{\frac{3}{2}}}{3} - 2$ or unsimplified	A1 [4]	$c = -2$ sufficient

Question 41

	$y = \dfrac{4}{2x-1}$.	B1	Correct without the ÷2
(i)	$\int \dfrac{16}{(2x-1)^2}\,dx = \dfrac{-16}{2x-1} \div 2$	B1	For the ÷2 even if first B1 is lost
	Vol $= \pi\left[\dfrac{-8}{2x-1}\right]$ with limits 1 and 2	M1	Use of limits in a changed expression.
	$\to \dfrac{16\pi}{3}$	A1 [4]	co
(ii)	$m = \frac{1}{2}m$ of tangent $= -2$	M1	Use of $m_1m_2 = -1$
	$\dfrac{dy}{dx} = \dfrac{-4}{(2x-1)^2} \times 2$	B1 B1	Correct without the ×2 For the ×2 even if first B1 is lost
	Equating their $\dfrac{dy}{dx}$ to -2	DM1	
	$\to x = \frac{3}{2}$ or $-\frac{1}{2}$	A1	co
	$(y = 2$ or $-2)$ $\to c = \frac{5}{2}$ or $-\frac{7}{2}$	A1 [6]	co

Question 42

$u = 2x(y-x)$ and $x + 3y = 12$, $u = 2x\left(\dfrac{12-x}{3} - x\right)$ $= 8x - \dfrac{8x^2}{3}$	M1 A1	Expresses u in terms of x
$\dfrac{du}{dx} = 8 - \dfrac{16x}{3}$	M1	Differentiate candidate's quadratic, sets to 0 + attempt to find x, or other valid method
$= 0$ when $x = 1\frac{1}{2}$	A1	
$\to (y = 3\frac{1}{2})$ $\to u = 6$	A1 [5]	Complete method that leads to u Co

Question 43

$f'(x) = 5 - 2x^2$ and (3, 5)		
$f(x) = 5x - \frac{2x^3}{3}$ (+c)	B1	For integral
Uses (3, 5)	M1	Uses the point in an integral
$\to c = 8$	A1 [3]	co

Question 44

	$y = \frac{8}{\sqrt{3x+4}}$		
(i)	$\frac{dy}{dx} = \frac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef	B1 B1	Without the "×3" For "×3" even if 1st B mark lost.
	$\rightarrow m_{(x=0)} = -\frac{3}{2}$ Perpendicular $m_{(x=0)} = \frac{2}{3}$	M1	Use of $m_1m_2 = -1$ after attempting to find $\frac{dy}{dx}_{(x=0)}$
	Eqn of normal $y - 4 = \frac{2}{3}(x - 0)$	M1	Unsimplified line equation
	Meets $x = 4$ at $B\left(4, \frac{20}{3}\right)$	A1 [5]	cao
(ii)	$\int \frac{8}{\sqrt{(3x+4)}} dx = \frac{8\sqrt{(3x+4))}}{\frac{1}{2}} \div 3$	B1 B1	Without "÷3". For "÷3"
	Limits from 0 to 4 $\rightarrow$ Area $P = \frac{32}{3}$	M1 A1	Correct use of correct limits. cao
	Area Q = Trapezium – P Area of Trapezium = $\frac{1}{2}\left(4 + \frac{20}{3}\right) \times 4 = \frac{64}{3}$	M1	Correct method for area of trapezium
	$\rightarrow$ Areas of P and Q are both $\frac{32}{3}$	A1 [6]	All correct.

Question 45

	$y = x^3 + px^2$		
(i)	$\frac{dy}{dx} = 3x^2 + 2px$	B1	cao
	Sets to 0 $\rightarrow$ $x = 0$ or $-\frac{2p}{3}$	M1	Sets differential to 0
	$\rightarrow$ (0, 0) or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	A1 A1 [4]	cao cao, first A1 for any correct turning point or any correct pair of x values. 2nd A1 for 2 complete TPs
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$	M1	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve
	At (0, 0) $\rightarrow$ $2p$ +ve Minimum	A1	www
	At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$ $\rightarrow -2p$ –ve Maximum	A1 [3]	
(iii)	$y = x^3 + px^2 + px \rightarrow 3x^2 + 2px + p\ (= 0)$	B1	
	Uses $b^2 - 4ac$ $\rightarrow 4p^2 - 12p < 0$	M1	Any correct use of discriminant
	$\rightarrow$ $0 < p < 3$ aef	A1 [3]	cao (condone $\leqslant$)

Question 46

(i)	$24 = r + r + r\theta$		(May not use θ)
	$\rightarrow \theta = \frac{24 - 2r}{r}$	M1	Attempt at $s = r\theta$ linked with 24 and r
	$A = \frac{1}{2}r^2\theta = \frac{24r}{2} - r^2 = 12r - r^2$. aef, ag	M1A1 [3]	Uses A formula with θ as f(r). cao
(ii)	$(A =)36 - (r - 6)^2$	B1 B1 [2]	cao
(iii)	Greatest value of $A = 36$	B1√	Ft on **(ii)**.
	$(r = 6) \rightarrow \theta = 2$	B1 [2]	cao, may use calculus or the discriminant on $12r - r^2$

Question 47

	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$		
(i)	$A = \frac{1}{2} \times (2 + p) \times 2p^2 \; (= 2p^2 + p^3)$	M1 A1 [2]	Attempt at base and height in terms of p and use of $\frac{bh}{2}$
(ii)	$\frac{dA}{dp} = 4p + 3p^2$	B1	cao
	$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt} = 0.02 \times 20 = 0.4$	M1 A1	any correct method, cao
	or $\frac{dA}{dt} = 4p\frac{dp}{dt} + 3p^2\frac{dp}{dt}$	[3]	

Question 48

(i)	$f'(x) = 2 - 2(x+1)^{-3}$	**B1**	
	$f''(x) = 6(x+1)^{-4}$	**B1**	
	$f'0 = 0$ hence stationary at $x = 0$	**B1**	**AG**
	$f''0 = 6 > 0$ hence minimum	**B1** [4]	www. Dependent on correct $f''(x)$ except $-6(x+1)^{-4} \to < 0$ MAX scores SC1
(ii)	$AB^2 = (3/2)^2 + (3/4)^2$	**M1**	
	$AB = 1.68$ or $\sqrt{45/4}$ oe	**A1** [2]	
(iii)	Area under curve $= \int f(x) = x^2 - (x+1)^{-1}$	**B1**	Ignore $+c$ even if evaluated Do not penalise reversed limits
	$= \left(1 - \frac{1}{2}\right) - \left(\frac{1}{4} - 2\right) = 9/4$		
	(Apply limits $-\frac{1}{2} \to 1$)	**M1A1**	Allow reversed subtn if final ans positive
	Area trap. $= \frac{1}{2}(3 + \frac{9}{4}) \times \frac{3}{2}$	**M1**	
	$= 63/16$ or 3.94	**A1**	
	Shaded area $63/16 - 9/4 + 27/16$ or 1.69	**A1** [6]	
	ALT eqn AB is $y = -\frac{1}{2}x + 11/4$	**B1**	
	Area $= \int -\frac{1}{2}x + 11/4 - \int 2x + (x+1)^{-2}$	**M1**	Attempt integration of at least one
	$= \left[-\frac{1}{4}x^2 + \frac{11}{4}x\right] - \left[x^2 - (x+1)^{-1}\right]$	**A1A1**	Ignore $+c$ even if evaluated Dep. on integration having taken place
	Apply limits $-\frac{1}{2} \to 1$ to both integrals	**M1**	Allow reversed subtn if final ans positive
	27/16 or 1.69	**A1**	

Question 49

(i)	At $x=4$, $\frac{dy}{dx}=2$	**B1**	
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=2\times 3=6$	**M1A1** [3]	Use of Chain rule
(ii)	$(y)=x+4x^{\frac{1}{2}}(+c)$	**B1**	
	Sub $x=4,\ y=6\rightarrow 6=4+(4\times 4^{\frac{1}{2}})+c$	**M1**	Must include c
	$c=-6\rightarrow(y=x+4x^{\frac{1}{2}}-6$	**A1** [3]	
(iii)	Eqn of tangent is $y-6=2(x-4)$ or $(6-0)/(4-x)=2$	**M1**	Correct eqn thru (4, 6) & with $m =$ *their* 2
		A1	
	$B=(1,0)$ (Allow $x=1$)	**M1**	[Expect eqn of normal: $y=-½x+8$]
	Gradient of normal $=-1/2$	**A1**	
	$C=(16,0)$ (Allow $x=16$)	**A1**	
	Area of triangle $=\frac{1}{2}\times 15\times 6=45$	[5]	Or $AB=\sqrt{45}$, $AC=\sqrt{180}\rightarrow$ Area = 45.0

Question 50

(i)	$[3]\ [(x-1)^2]\ [-1]$	**B1B1B1** [3]	
(ii)	$f'(x)=3x^2-6x+7$	**B1**	Ft *their* **(i)** + 5
	$=3(x-1)^2+4$	**B1√**	
	>0 hence increasing	**DB1** [3]	Dep B1√ unless other valid reason

Question 51

	$y=\sqrt{(9-2x^2)}$ $P(2, 1)$		
(i)	$\frac{dy}{dx}=\frac{1}{2\sqrt{(9-2x^2)}}\times -4x$	**B1**	Without "$\times -4x$"
		B1	Allow even if B0 above.
	At P, $x=2$, $m=-4$ Normal grad = ¼	**M1**	For $m_1m_2=-1$ calculus needed
	Eqn AP $y-1=¼(x-2)$	**M1**	Normal, not tangent
	$\rightarrow A\ (-2, 0)$ or $B\ (0, ½)$	**A1**	
	Midpoint AP also $(0, ½)$	**A1** [6]	Full justification.
(ii)	$\int x^2\,dy=\int\left(\frac{9}{2}-\frac{y^2}{2}\right)dy$	**M1**	Attempt to integrate x^2
	$=\frac{9y}{2}-\frac{y^3}{6}$	**A1**	Correct integration
	Upper limit = 3	**B1**	Evaluates upper limit
	Uses limits 1 to 3	**DM1**	Uses both limits correctly
	$\rightarrow$ volume = $4\frac{2}{3}\pi$	**A1** [5]	

Question 52

(i)	$f''(x) = \frac{12}{x^3}$		
	$f'(x) = -\frac{6}{x^2} \quad (+c)$	**B1**	Correct integration
	$= 0$ when $x = 2 \rightarrow c = \frac{3}{2}$	**M1 A1**	Uses $x = 2$, $f'(x = 0)$
	$f(x) = \frac{6}{x} + \frac{3x}{2} \quad (+A)$	**B1√ B1√**	For each integral
	$= 10$ when $x = 2 \rightarrow A = 4$	**A1** [6]	
(ii)	$-\frac{6}{x^2} + \frac{3}{2} = 0 \rightarrow x = \pm 2$	**M1**	Sets their 2 term $f'(x)$ to 0.
	Other point is $(-2, -2)$	**A1** [2]	
(iii)	At $x = 2$, $f''(x) = 1.5$ Min	**B1**	
	At $x = -2$, $f''(x) = -1.5$ Max	**B1** [2]	

Question 53

(i)	$\tan 60 = \frac{x}{h} \rightarrow x = h \tan 60$	**B1**	Any correct unsimplified length
	$A = h \times x$	**M1**	Correct method for area
	$V = 40\sqrt{(3h^2)}$	**A1** [3]	ag
(ii)	$\frac{dV}{dh} = 80\sqrt{(3h)}$	**B1**	B1
	If $h = 5$, $\frac{dh}{dt} = \frac{1}{2\sqrt{(3)}}$ or 0.289	**M1A1** [3]	M1 (must be ÷, not ×).

Question 54

(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2}\right] \times [4]$	**B1B1**	
	At $x = 6, \frac{dy}{dx} = \frac{2}{5}$	**B1**	
	Gradient of normal at $P = -\frac{1}{2}$	**B1**√	**OR** eqn of norm $y - 5 = their - \frac{5}{2}(x-6)$ When $y = 0, x = 8$ hence result
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	**B1** [5]	
(ii)	Vol for curve $= (\pi)\int(1+4x)$ and attempt to integrate y^2	**M1**	
	$= (\pi)\left[x + 2x^2\right]$ ignore '+ c'	**A1**	
	$= (\pi)[6 + 72 - 0]$	**DM1**	Apply limits $0 \to 6$ (allow reversed if corrected later)
	$= 78(\pi)$	**A1**	
	Vol for line $= \frac{1}{3} \times (\pi) \times 5^2 \times 2$	**M1**	**OR** $(\pi)\left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}}\right]_6^8$
	$= \frac{50}{3}(\pi)$	**A1**	
	Total Vol $= 78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	**A1** [7]	

Question 55

(i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao	**B1B1**	
	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	**B1** [3]	
(ii)	$-\frac{8}{x^2} + 2 = 0 \to 2x^2 - 8 = 0$	**M1**	Set = 0 and rearrange to quadratic form
	$x = \pm 2$	**A1**	
	$y = \pm 8$	**A1**	If A0A0 scored, SCA1 for just (2, 8)
	$\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM	**B1**√	Ft for "correct" conclusion if $\frac{d^2y}{dx^2}$ incorrect or any valid method inc. a good sketch
	$\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	**B1**√ [5]	

Question 56

$f(x)=x^3-7x(+c)$	B1	
$5=27-21+c$	M1	Sub $x=3$, $y=5$. Dep. on c present
$c=-1 \rightarrow f(x)=x^3-7x-1$	A1	
	[3]	

Question 57

(i)	$x=1/3$	**B1**	
		[1]	
(ii)	$\frac{dy}{dx}=\left[\frac{2}{16}(3x-1)\right][3]$	**B1B1**	
	When $x=3$ $\frac{dy}{dx}=3$ soi	**M1**	
	Equation of QR is $y-4=3(x-3)$	**M1**	
	When $y=0$ $x=5/3$	**A1**	
		[5]	
(iii)	Area under curve $=\left[\frac{1}{16\times 3}(3x-1)^3\right]\left[\times\frac{1}{3}\right]$	**B1B1**	
	$\frac{1}{16\times 9}\left[8^3-0\right]=\frac{32}{9}$	**M1A1**	Apply limits: *their* $\frac{1}{3}$ and 3
	Area of $\Delta=8/3$	**B1**	
	Shaded area $=\frac{32}{9}-\frac{8}{3}=\frac{8}{9}$ (or 0.889)	**A1**	
		[6]	

Question 58

(i)	$A=2\pi r^2+2\pi rh$	**B1**	
	$\pi r^2 h=1000 \rightarrow h=\frac{1000}{\pi r^2}$	**M1**	
	Sub for h into A $\rightarrow$ $A=2\pi r^2+\frac{2000}{r}$ **AG**	**A1**	
		[3]	
(ii)	$\frac{dA}{dr}=0 \Rightarrow 4\pi r-\frac{2000}{r^2}=0$	**M1A1**	Attempt differentiation & set = 0
	$r==$ 5.4	**DM1 A1**	Reasonable attempt to solve to $r^3=$
	$\frac{d^2A}{dr^2}=4\pi+\frac{4000}{r^3}$		
	>0 hence MIN hence MOST EFFICIENT **AG**	**B1**	Or convincing alternative method
		[5]	

Question 59

$y=\frac{3x^3}{3}-\frac{2x^{-2}}{-2}$ $(+c)$	**B1B1**	
$3=-1+1+c$	**M1**	Sub $x=-1, y=3$. c must be present
$y=x^3+x^{-2}+3$	**A1**	Accept $c=3$ www
	[4]	

Question 60

$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$	**B1**	
$\frac{dy}{dx} = 2$	**B1**	
$2x - 5x^{1/2} + 5 = 2$	**M1**	Equate their dy/dx to *their* 2 or ½.
$2x - 5x^{1/2} + 3(=0)$ or equivalent 3-term quadratic	**A1**	
Attempt to solve for $x^{1/2}$ e.g. $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$	**DM1**	Dep. on 3-term quadratic
$x^{1/2} = 3/2$ and 1	**A1**	ALT
$x = 9/4$ and 1	**A1** [7]	$5x^{½} = 2x + 3 \to 25x = (2x+3)^2$ $4x^2 - 13x + 9(=0)$ $x = 9/4$ and 1

Question 61

$(\pi)\int(x^3 + 1)\,dx$	**M1**	Attempt to resolve y^2 and attempt to integrate
$(\pi)\left[\frac{x^4}{4} + x\right]$	**A1**	
6π or 18.8	**DM1A1** [4]	Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is 6π)

Question 62

(i)	$6 + k = 2 \to k = -4$	**B1** [1]	
(ii)	$(y) = \frac{6x^3}{3} - \frac{4}{-2}x^{-2}$ $(+c)$	**B1B1√**	ft on *their k*. Accept $+\frac{k}{-2}x^{-2}$
	$9 = 2 + 2 + c$ *c* must be present	**M1**	Sub (1,9) with numerical *k*. Dep on attempt $\int$
	$(y) = 2x^3 + 2x^{-2} + 5$	**A1** [4]	Equation needs to be seen Sub (2, 3) →$c = -13½$ scores M1A0

Question 63

$\frac{dy}{dx} = [8] + [-2]\,[(2x-1)^{-2}]$	**B2,1,0**	
$= 0 \to 4(2x-1)^2 = 1$ oe eg $16x^2 - 16x + 3 = 0$	**M1**	Set to zero, simplify and attempt to solve soi
$x = \frac{1}{4}$ and $\frac{3}{4}$	**A1**	Needs both *x* values. Ignore *y* values
$\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$	**B1√***	ft to $k(2x-1)^{-3}$ where $k > 0$
When $x = \frac{1}{4}$, $\frac{d^2y}{dx^2}(=-64)$ and/or < 0 MAX	**DB1**	Alt. methods for last 3 marks (values either side of 1/4 & 3/4) must indicate <u>which</u> *x*-values and cannot use $x = 1/2$. (M1A1A1)
When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2}(=64)$ and/or > 0 MIN	**DB1** [7]	

Question 64

	$y=\frac{8}{x}+2x.$		
(i)	$\frac{dy}{dx}=-8x^{-2}+2$	B1	unsimplified ok
	$\frac{d^2y}{dx^2}=16x^{-3}$	B1	unsimplified ok
	$\int y^2 dx=-64x^{-1}$oe$+32x$ oe $+\frac{4x^3}{3}$ **oe** $(+c)$	3 × B1 [5]	B1 for each term – unsimplified ok
(ii)	sets $\frac{dy}{dx}$ to 0 $\rightarrow x=\pm 2$	M1	Sets to 0 and attempts to solve
	$\rightarrow M(2, 8)$	A1	Any pair of correct values A1
	Other turning point is $(-2, -8)$	A1	Second pair of values A1
	If $x=-2$, $\frac{d^2y}{dx^2}<0$	M1	Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x<0$
	∴Maximum	A1 [5]	
(iii)	Vol $=\pi\times$ [part (i)] from 1 to 2	M1	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
	$\frac{220\pi}{3}, 73.3\pi, 230$	A1 [2]	

Question 65

$f'(x)=\frac{8}{(5-2x)^2}$		
$f(x)=\frac{8(5-2x)^{-1}}{-1}\div -2\ (+c)$	B1 B1	Correct without (÷ by −2) An attempt at integration (÷ by−2)
Uses $x=2$, $y=7$,	M1	Substitution of correct values into an integral to find c
$c=3$	A1 [4]	

Question 66

(i)	$A=2y\times 4x(=8xy)$ $10y+12x=480$ $\rightarrow A=384x-9.6x^2$	B1 B1 B1 [3]	answer given
(ii)	$\frac{dA}{dx}=384-19.2x$	B1	
	$=0$ when $x=20$	M1	Sets to 0 and attempt to solve oe Might see completion of square
	$\rightarrow x=20, y=24.$	A1	Needs both x and y
	Uses $x=-\frac{b}{2a}=\frac{-384}{-19.2}=20$, **M1, A1** $y=24$, A1 From graph: **B1** for $x=20$, **M1, A1** for $y=24$	[3]	Trial and improvement **B3**.

Question 67

(i)	$\frac{dy}{dx}=2-8(3x+4)^{-½}$ $(x=0, \rightarrow \frac{dy}{dx}=-2)$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt} \rightarrow -0.6$	**M1A1** [2]	Ignore notation. Must be $\frac{dy}{dx}\times 0.3$
(ii)	$y=\{2x\}\left\{-\frac{8\sqrt{3x+4}}{\frac{1}{2}}\div 3\right\}\ (+c)$	**B1 B1**	No need for $+c$.
	$x=0, y=\frac{4}{3} \rightarrow c=12.$	**M1 A1** [4]	Uses x, y values after ∫ with c

Question 68

$x=\frac{12}{y^2}-2.$ Vol = $(\pi)\times\int x^2\,dy$ $\rightarrow\left[\frac{-144}{3y^3}+4y+\frac{48}{y}\right]$	**M1** **3 × A1**	Ignore omission of π at this stage Attempt at integration Un-simplified
Limits 1 to 2 used $\rightarrow 22\pi$	**A1** [5]	only from correct integration

Question 69

(i)	Attempt diffn. and equate to 0 $\frac{dy}{dx}=-k(kx-3)^{-2}+k=0$	***M1**		Must contain $(kx-3)^{-2}$ + other term(s)
	$(kx-3)^2=1$ or $k^3x^2-6k^2x+8k(=0)$	**DM1**		Simplify to a quadratic
	$x=\frac{2}{k}$ or $\frac{4}{k}$	***A1*A1**		Legitimately obtained
	$\frac{d^2y}{dx^2}=2k^2(kx-3)^{-3}$	**B1√**		Ft must contain $Ak^2(kx-3)^{-3}$ where A>0
	When $x=\frac{2}{k}$, $\frac{d^2y}{dx^2}=(-2k^2)<0$ MAX All previous	**DB1**		Convincing alt. methods (values either side) must show which values used & cannot use $x=3/k$
	When $x=\frac{4}{k}$, $\frac{d^2y}{dx^2}=(2k^2)>0$ MIN working correct	**DB1**	[7]	
(ii)	$V=(\pi)\int\left[(x-3)^{-1}+(x-3)\right]^2 dx$	***M1**		Attempt to expand y^2 and then integrate
	$=(\pi)\int[(x-3)^{-2}+(x-3)^2+2]dx$	**A1**		
	$=(\pi)\left[-(x-3)^{-1}+\frac{(x-3)^3}{3}(+2x)\right]$ Condone missing $2x$	**A1**		Or $\left[-(x-3)^{-1}+\frac{x^3}{3}-3x^2+9x+2x\right]$
	$=(\pi)\left[1-\frac{1}{3}+4-\left(\frac{1}{3}-9+0\right)\right]$	**DM1**		Apply limits 0→2
	$=40\pi/3$ oe or 41.9	**A1**	[5]	2 missing → $28\pi/3$ scores M1A0A1M1A0

Question 70

(i)	at $x=a^2$, $\frac{dy}{dx}=\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}\left(=\frac{3}{a^2}\text{ or }3a^{-2}\right)$	**B1**		$\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}$ seen anywhere in **(i)**
	$y-3=\frac{3}{a^2}\left(x-a^2\right)$ or $y=\frac{3}{a^2}x+c\rightarrow 3=\frac{3}{a^2}a^2+c$	**M1**		Through $(a^2,3)$ & with *their* grad as f(a)
	$y=\frac{3}{a^2}x$ or $3a^{-2}x$ cao	**A1**	[3]	
(ii)	$(y)=\frac{2}{a}\frac{x^{½}}{½}+\frac{ax^{-½}}{-½}$ $(+c)$	**B1B1**		
	sub $x=a^2$, $y=3$ into $\int dy/dx$	**M1**		c must be present. Expect $3=4-2+c$
	$c=1$ $\left(y=\frac{4x^{½}}{a}-2ax^{-½}+1\right)$	**A1**	[4]	
(iii)	sub $x=16$, $y=8\rightarrow 8=\frac{4}{a}\times 4-2a\times\frac{1}{4}+1$	***M1**		Sub into *their y*
	$a^2+14a-32(=0)$	**A1**		
	$a=2$	**A1**		Allow −16 in addition
	$A=(4,3)$, $B=(16,8)$ $AB^2=12^2+5^2\rightarrow AB=13$	**DM1A1**	[5]	

Question 71

$f'(x)=3x^2-6x-9$ soi	**B1**		
Attempt to solve $f'(x)=0$ or $f'(x)>0$ or $f'(x)\geqslant 0$ soi	**M1**		
$(3)(x-3)(x+1)$ or 3,−1 seen or 3 only seen	**A1**		With or without equality/inequality signs
Least possible value of n is 3. Accept $n=3$. Accept $n\geqslant 3$	**A1**	[4]	Must be in terms of n

Question 72

(i)	$\frac{dy}{dx}=\frac{-3}{(2x-1)^2}\times 2$	**B1**		B1for a single correct term (unsimplified) without ×2.
		B1	[2]	
(ii)	e.g. Solve for $\frac{dy}{dx}=0$ is impossible.	**B1**√	[1]	Satisfactory explanation.
(iii)	If $x=2$, $\frac{dy}{dx}=\frac{-6}{9}$ and $y=3$	**M1***		Attempt at both needed.
	Perpendicular has $m=\frac{9}{6}$	**M1***		Use of $m_1m_2=-1$ numerically.
	$\rightarrow y-3=\frac{3}{2}(x-2)$	**DM1**		Line equation using (2, their 3) and their m.
	Shows when x=0 then y=0 AG	**A1**	[4]	
(iv)	$\frac{dx}{dt}=-0.06$			
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\rightarrow -\frac{2}{3}\times -0.06=0.04$	**M1 A1**	[2]	

Question 73

$(y) = 8(4x+1)^{\frac{1}{2}} \div ½ \div 4\ (+c)$	B1 B1		Correct integrand (unsimplified) without ÷4 ÷4. Ignore *c*.
Uses $x = 2$ and $y = 5$	M1		Substitution of correct values into an integrand to find c.
$c = -7$	A1	[4]	$y = 4\sqrt{4x+1} - 7$

Question 74

(i)	$3z - \frac{2}{z} = -1 \Rightarrow 3z^2 + z - 2 = 0$	M1		Express as 3-term quad. Accept $x^{1/2}$ for z
	$x^{1/2}\ (\text{or } z) = 2/3 \text{ or } -1$	A1		(OR
	$x = 4/9$ only	A1	[3]	$3x - 1 = -\sqrt{x},\ 9x^2 - 13x + 4 = 0$ **M1, A1,A1** $x = 4/9$)
(ii)	$f(x) = \frac{3x^{3/2}}{3/2} - \frac{2x^{1/2}}{1/2}\quad (+c)$	B1B1		
	Sub $x = 4, y = 10 \quad 10 = 16 - 8 + c \Rightarrow c = 2$	M1A1		*c* must be present
	When $x = \frac{4}{9},\ y = 2\left(\frac{4}{9}\right)^{3/2} - 4\left(\frac{4}{9}\right)^{1/2} + 2$	M1		Substituting x value from part **(i)**
	$-2/27$	A1	[6]	

Question 75

(i)	$\frac{dy}{dx} = -(x-1)^{-2} + 9(x-5)^{-2}$	M1A1		May be seen in part **(ii)**
	$m_{\text{tangent}} = -\frac{1}{4} + \frac{9}{4} = 2$	B1		
	Equation of normal is $y - 5 = -½(x-3)$	M1		Through (3, 5) and with $m = -1/m_{tangent}$
	$x = 13$	A1	[5]	
(ii)	$(x-5)^2 = 9(x-1)^2$	B1		Set $\frac{dy}{dx} = 0$ and simplify
	$x - 5 = (\pm)3(x-1)$ or $(8)(x^2 - x - 2) = 0$	M1		Simplify further and attempt solution
	$x = -1$ or 2	A1		
	$\frac{d^2y}{dx^2} = 2(x-1)^{-3} - 18(x-5)^{-3}$	B1		If change of sign used, x values close to the roots must be used and all must be correct
	When $x = -1,\ \frac{d^2y}{dx^2} = -\frac{1}{6} < 0$ MAX	B1		
	When $x = 2,\ \frac{d^2y}{dx^2} = \frac{8}{3} > 0$ MIN	B1	[6]	

Question 76

(i)	$A = (\frac{1}{2}, 0)$	**B1**	[1]	Accept $x = 0\, at\; y = 0$
(ii)	$\int (1-2x)^{\frac{1}{2}}\,dx = \left[\frac{(1-2x)^{3/2}}{3/2}\right][\div(-2)]$	**B1B1**		May be seen in a single expression
	$\int (2x-1)^2\,dx = \left[\frac{(2x-1)^3}{3}\right][\div 2]$	**B1B1**		May use $\int_a^1 x\,dy$, may expand $(2x-1)^2$
	$[0-(-1/3)]-[0-(-1/6)]$	**M1**		Correct use of *their* limits
	1/6	**A1**	[6]	

Question 77

(i)	$2x - 2/x^3 = 0$	M1	Set = 0.
	$x^4 = 1 \Rightarrow x = 1$ at A cao	A1	Allow 'spotted' $x = 1$
	Total:	**2**	
(ii)	$f(x) = x^2 + 1/x^2 (+c)$ cao	B1	
	$\frac{189}{16} = 16 + 1/16 + c$	M1	Sub (4, $\frac{189}{16}$). c must be present. Dep. on integration
	$c = -17/4$	A1	
	Total:	**3**	
(iii)	$x^2 + 1/x^2 - 17/4 = 0 \Rightarrow 4x^4 - 17x^2 + 4\ (=0)$	M1	Multiply by $4x^2$ (or similar) to transform into 3-term quartic.
	$(4x^2-1)(x^2-4)\ (=0)$	M1	Treat as quadratic in x^2 and attempt solution or factorisation.
	$x = \frac{1}{2},\ 2$	A1A1	Not necessary to distinguish. Ignore negative values. No working scores 0/4
	Total:	**4**	
(iv)	$\int (x^2 + x^{-2} - 17/4)dx = \frac{x^3}{3} - \frac{1}{x} - \frac{17x}{4}$	B2,1,0√	Mark final integral
	$(8/3 - 1/2 - 17/2) - (1/24 - 2 - 17/8)$	M1	Apply *their* limits from (iii) (Seen). Dep. on integration of at least 1 term of y
	Area = $9/4$	A1	Mark final answer. $\int y^2$ scores 0/4
	Total:	**4**	

Question 78

(i)	$\frac{dy}{dx} = 2x - 2$. At $x = 2$, $m = 2$	B1B1	Numerical m
	Equation of tangent is $y - 2 = 2(x - 2)$	B1	Expect $y = 2x - 2$
	Total:	3	
(ii)	Equation of normal $y - 2 = -½(x - 2)$	M1	Through (2, 2) with gradient $= -1/m$. Expect $y = -½x + 3$
	$x^2 - 2x + 2 = -½x + 3 \rightarrow 2x^2 - 3x - 2 = 0$	M1	Equate and simplify to 3-term quadratic
	$x = -½,\ y = 3¼$	A1A1	Ignore answer of (2, 2)
	Total:	4	
(iii)	At $x = -½$, grad $= 2(-½) - 2 = -3$	B1√	Ft *their* −½.
	Equation of tangent is $y - 3¼ = -3(x + ½)$	*M1	Through *their* *B* with grad *their* −3 (not m_1 or m_2). Expect $y = -3x + 7/4$
	$2x - 2 = -3x + 7/4$	DM1	Equate *their* tangents or attempt to solve simultaneous equations
	$x = 3/4,\ y = -½$	A1	Both required.
	Total:	4	

Question 79

(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right]$ [4]	B1B1	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6 \times 1/2 \times (4x+1)^{-1/2} \times 4$	B1√	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* f ′(x).
	Total:	3	
(ii)	f(2), f ′(2), $kf''(2)$ = 27, 18, 4*k* OR 12	B1B1√B1√	Ft dependent on attempt at differentiation
	27/18 = 18/4*k* oe OR $kf''(2) = 12 \Rightarrow k = 3$	M1A1	
	Total:	5	

Question 80

(i)	$V = \frac{1}{12}h^3$ oe	B1	
	Total:	1	
(ii)	$\frac{dV}{dh} = \frac{1}{4}h^2$ or $\frac{dh}{dV} = 4(12v)^{-2/3}$	M1A1	Attempt differentiation. Allow incorrect notation for M. For A mark accept *their* letter for volume - but otherwise correct notation. Allow *V′*
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \quad = \frac{4}{h^2} \times 20$ soi	DM1	Use chain rule correctly with $\frac{d(V)}{dt} = 20$. Any equivalent formulation Accept non-explicit chain rule (or nothing at all)
	$\left(\frac{dh}{dt}\right) = \frac{4}{10^2} \times 20 = 0.8$ or equivalent fraction	A1	
	Total:	4	

Question 81

(i)	$f'(x) = [(4x+1)^{1/2} \div ½]\ [\div 4]\ (+c)$	**B1 B1**	Expect $½(4x+1)^{1/2}\ (+c)$
	$f'(2) = 0 \Rightarrow \frac{3}{2} + c = 0 \Rightarrow c = -\frac{3}{2}$ (Sufficient)	**B1 FT**	Expect $½(4x+1)^{1/2} - \frac{3}{2}$. FT on *their* $f'(x) = k(4x+1)^{1/2} + c$. (i.e. $c = -3k$)
	Total:	**3**	
(ii)	$f''(0) = 1$ SOI	**B1**	
	$f'(0) = 1/2 - 1½ = -1$ SOI	**B1 FT**	Substitute $x = 0$ into *their* $f'(x)$ but must not involve c otherwise **B0B0**
	$f(0) = -3$	**B1 FT**	FT for 3 terms in AP. FT for 3rd **B1** dep on 1st **B1**. Award marks for the AP method only.
	Total:	**3**	
(iii)	$f(x) = [½(4x+1)^{3/2} \div 3/2 \div 4] - [1½x]\ (+k)$	**B1 FT** **B1 FT**	Expect $(1/12)(4x+1)^{3/2} - 1½x\ (+k)$. FT from *their* $f'(x)$ but c numerical.
	$-3 = 1/12 - 0 + k \Rightarrow k = -37/12$ CAO	**M1A1**	Sub $x = 0, y = their\ f(0)$ into *their* $f(x)$. Dep on cx & k present (c numerical)
	Minimum value = $f(2) = \frac{27}{12} - 3 - \frac{37}{12} = -\frac{23}{6}$ or -3.83	**A1**	
	Total:	**5**	

Question 82

(a)(i)	Attempt to integrate $V = (\pi)\int(y+1)dy$	**M1**	Use of h in integral e.g. $\int(h+1) = ½h^2 + h$ is **M0**. Use of $\int y^2 dx$ is **M0**
	$= (\pi)\left[\frac{y^2}{2} + y\right]$	**A1**	
	$= \pi\left[\frac{h^2}{2} + h\right]$	**A1**	**AG**. Must be from clear use of limits $0 \to h$ somewhere.
	Total:	**3**	
(ii)	$\int(y+1)^{1/2} dy$ **ALT** $6 - \int(x^2 - 1)dx$	**M1**	Correct variable and attempt to integrate
	$⅔(y+1)^{3/2}$ oe **ALT** $6 - (⅓x^3 - x)$ CAO	***A1**	Result of integration must be shown
	$⅔[8-1]$ **ALT** $6 - [\left(\frac{8}{3} - 1\right) - \left(\frac{1}{3} - 1\right)]$	**DM1**	Calculation seen with limits $0 \to 3$ for y. For ALT, limits are $1 \to 2$ and rectangle.
	14/3 **ALT** $6 - 4/3 = 14/3$	**A1**	16/3 from $⅔ \times 8$ gets **DM1A0** provided work is correct up to applying limits.
	Total:	**4**	
(b)	Clear attempt to differentiate wrt h	**M1**	Expect $\frac{dV}{dh} = \pi(h+1)$. Allow $h + 1$. Allow h.
	Derivative = 4π SOI	***A1**	
	$\frac{2}{their\ derivative}$. Can be in terms of h	**DM1**	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	**A1**	
	Total:	**4**	

Question 83

Gradient of normal is – 1/3 → gradient of tangent is 3 SOI	**B1 B1 FT**	FT from *their* gradient of normal.
$dy/dx = 2x - 5 = 3$	**M1**	Differentiate and set = *their* 3 (numerical).
$x = 4$	***A1**	
Sub $x = 4$ into line → $y = 7$ & sub *their* (4, 7) into curve	**DM1**	OR sub $x = 4$ into curve → $y = k - 4$ and sub *their*$(4, k - 4)$ into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
$k = 11$	**A1**	
Total:	**6**	

Question 84

(i)	$\frac{dy}{dx} = 4x^{-½} - 2$	**B1**	Accept unsimplified.
	$= 0$ when $\sqrt{x} = 2$		
	$x = 4, y = 8$	**B1B1**	
	Total:	**3**	
(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	**B1FT**	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right)$ → Maximum	**B1**	Correct $\frac{d^2y}{dx^2}$ and x=4 in (i) are required. Followed by"< 0 or negative" is sufficient" but $\frac{d^2y}{dx^2}$ must be correct evaluated.
	Total:	**2**	
iii)	*EITHER:* Recognises a quadratic in $\sqrt{x}$	**(M1**	Eg $\sqrt{x} = u \rightarrow 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	
	OR: Rearranges then squares	**(M1**	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow x^2 - 10x + 9 = 0$ oe	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	Both correct by trial and improvement gets 3/3
	Total:	**3**	
iv)	$k > 8$	**B1**	
	Total:	**1**	

Question 85

$\text{Vol} = \pi\int(5-x)^2\,dx - \pi\int\frac{16}{x^2}dx$	**M1***	Use of volume formula at least once, condone omission of π and limits dx.
	DM1	Subtracting volumes somewhere must be after squaring.
$\int(5-x)^2dx = \frac{(5-x)^3}{3} \div -1$	**B1 B1**	**B1** Without $\div(-1)$. **B1** for $\div(-1)$
(or $25x - 10x^2/2 + \frac{1}{3}x^3$)	**(B2,1,0)**	−1 for each incorrect term
$\int\frac{16}{x^2}dx = -\frac{16}{x}$	**B1**	
Use of limits 1 and 4 in an integrated expression and subtracted.	**DM1**	Must have used "y^2" at least once. Need to see values substituted.
$\to 9\pi$ or 28.3	**A1**	
Total:	**7**	

Question 86

(i)	Crosses x-axis at (6, 0)	**B1**	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0+) - 12(2-x)^{-2} \times (-1)$	**B2,1,0**	−1 for each incorrect term of the three or addition of + C.
	Tangent $y = \frac{3}{4}(x-6)$ or $4y = 3x - 18$	**M1 A1**	Must use dy/dx, x = their 6 but not $x = 0$ (which gives $m = 3$), and correct form of line equation.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**5**	
(ii)	If $x = 4$, $dy/dx = 3$		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	**M1 A1FT**	**M1** for ("their m" from $\frac{dy}{dx}$ and $x = 4$) × 0.04. Be aware: use of $x = 0$ gives the correct answer but gets **M0**.
	Total:	**2**	

Question 87

(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	**B1 B1**	**B1** without $\times(-3)$ **B1** For $\times(-3)$
	Gradient of tangent = 3, Gradient of normal $-\frac{1}{3}$	***M1**	Use of $m_1m_2 = -1$ after calculus
	$\to$ eqn: $y - 2 = -\frac{1}{3}(x-1)$	**DM1**	Correct form of equation, with (1, their y), not (1,0)
	$\to y = -\frac{1}{3}x + \frac{7}{3}$	**A1**	This mark needs to have come from $y = 2$, y must be subject
	Total:	**5**	
(ii)	$\text{Vol} = \pi\int_0^1 \frac{16}{(5-3x)^2}dx$	**M1**	Use of $V = \pi\int y^2dx$ with an attempt at integration
	$\pi\left[\frac{-16}{(5-3x)} \div -3\right]$	**A1 A1**	**A1** without$(\div -3)$, **A1** for $(\div -3)$
	$= (\pi\left(\frac{16}{6} - \frac{16}{15}\right)) = \frac{8\pi}{5}$ (if limits switched must show – to +)	**M1 A1**	Use of both correct limits **M1**
	Total:	**5**	

Question 88

(i)	$y = 7x - \frac{x^3}{3} - \frac{6x^2}{2}$ (+ c)	**B1**	CAO
	Uses (3, − 10) → $c = 5$	**M1 A1**	Uses the given point to find c
	Total:	**3**	
(ii)	$7 - x^2 - 6x = 16 - (x+3)^2$	**B1 B1**	**B1** $a = 16$, **B1** $b = 3$.
	Total:	**2**	
(iii)	$16 - (x+3)^2 > 0 \rightarrow (x+3)^2 < 16$, and solve	**M1**	or factors $(x + 7)(x - 1)$
	End-points $x = 1$ or -7	**A1**	
	$\rightarrow -7 < x < 1$	**A1**	needs <, not ⩽. (SR $x < 1$ only, or $x > -7$ only **B1** i.e. 1/3)
	Total:	**3**	

Question 89

(i)	Volume $= \left(\frac{1}{2}\right)x^2\frac{\sqrt{3}}{2}h = 2000 \rightarrow h = \frac{8000}{\sqrt{3}x^2}$	**M1**	Use of (area of triangle, with attempt at ht) × h =2000, $h = f(x)$
	$A = 3xh + (2)\times\left(\frac{1}{2}\right)\times x^2 \times \frac{\sqrt{3}}{2}$	**M1**	Uses 3 rectangles and at least one triangle
	Sub for $h \rightarrow A = \frac{\sqrt{3}}{2}x^2 + \frac{24000}{\sqrt{3}}x^{-1}$	**A1**	AG
	Total:	**3**	
(ii)	$\frac{dA}{dx} = \frac{\sqrt{3}}{2}2x - \frac{24000}{\sqrt{3}}x^{-2}$	**B1**	CAO, allow decimal equivalent
	= 0 when $x^3 = 8000 \rightarrow x = 20$	**M1 A1**	Sets their $\frac{dA}{dx}$ to 0 and attempt to solve for x
	Total:	**3**	
(iii)	$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}2 + \frac{48000}{\sqrt{3}}x^{-3} > 0$	**M1**	Any valid method, ignore value of $\frac{d^2A}{dx^2}$ providing it is positive
	→ Minimum	**A1 FT**	FT on their x providing it is positive
	Total:	**2**	

Question 90

1(i)	Gradient of $AB = \frac{1}{2}$	B1	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	B1	
		2	
(ii)	$\frac{dy}{dx} = ½(x-1)^{-\frac{1}{2}}$	B1	
	$½(x-1)^{-\frac{1}{2}} = ½$. Equate *their* $\frac{dy}{dx}$ to *their* ½	*M1	
	$x = 2, \quad y = 1$	A1	
	$y - 1 = ½(x - 2)$ (thro' *their*(2,1) & *their* ½) → $y = ½x$	DM1 A1	
		5	
1(iii)	*EITHER:* $\sin\theta = \frac{d}{1} \quad \to \quad d = \sin\theta$	(M1	Where θ is angle between AB and the x-axis
	gradient of $AB = ½ \Rightarrow \tan\theta = ½ \Rightarrow \theta = 26.5(7)°$	B1	
	$d = \sin 26.5(7)° = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR1: Perpendicular through O has equation $y = -2x$	(M1	
	Intersection with AB: $-2x = ½x - ½ \to \left(\frac{1}{5}, \frac{-2}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR2: Perpendicular through (2, 1) has equation $y = -2x + 5$	(M1	
	Intersection with AB: $-2x + 5 = ½x - ½ \to \left(\frac{11}{5}, \frac{3}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$)	A1)	
1(iii)	*OR3:* ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	(B1	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \to d = \frac{1}{\sqrt{5}}$	M1 A1)	
		3	

Question 91

(i)	$ax^2 + bx = 0 \to x(ax+b) = 0 \to x = \frac{-b}{a}$	**B1**	
	Find $f''(x)$ and attempt sub *their* $\frac{-b}{a}$ into *their* $f''(x)$	**M1**	
	When $x = \frac{-b}{a}$, $f''(x) = 2a\left(\frac{-b}{a}\right) + b = -b$ MAX	**A1**	
		3	
(ii)	Sub $f'(-2) = 0$	**M1**	
	Sub $f'(1) = 9$	**M1**	
	$a = 3 \quad b = 6$	***A1**	Solve simultaneously to give both results.
	$f'(x) = 3x^2 + 6x \to f(x) = x^3 + 3x^2 \ (+c)$	***M1**	Sub *their a, b* into $f'(x)$ and integrate 'correctly'. Allow $\frac{ax^3}{3} + \frac{bx^2}{2}(+c)$
	$-3 = -8 + 12 + c$	**DM1**	Sub $x = -2,\ y = -3$. Dependent on c present. Dependent also on a, b substituted.
	$f(x) = x^3 + 3x^2 - 7$	**A1**	
		6	

Question 92

(i)	*EITHER:* $4 - 3\sqrt{x} = 3 - 2x \to 2x - 3\sqrt{x} + 1\ (=0)$ or e.g. $2k^2 - 3k + 1\ (=0)$	**(M1**	Form 3-term quad & attempt to solve for $\sqrt{x}$.
	$\sqrt{x} = ½, 1$	**A1**	Or $k = ½$ or 1 (where $k = \sqrt{x}$).
	$x = ¼, 1$	**A1)**	
	OR1: $(3\sqrt{x})^2 = (1 + 2x)^2$	**(M1**	
	$4x^2 - 5x + 1\ (=0)$	**A1**	
	$x = ¼, 1$	**A1)**	
	OR2: $\frac{3-y}{2} = \left(\frac{4-y}{3}\right)^2 \ (\to 2y^2 - 7y + 5 (=0))$	**(M1**	Eliminate x
	$y = \frac{5}{2}, 1$	**A1**	
	$x = ¼, 1$	**A1)**	
		3	

(ii)			
	EITHER: Area under line = $\int(3-2x)dx = 3x - x^2$	(B1	
	$= \left[(3-1)-\left(\frac{3}{4}-\frac{1}{16}\right)\right]$	M1	Apply *their* limits (e.g. ¼ → 1) after integn.
	Area under curve $= \int(4-3x^{1/2})dx \;=\; 4x-2x^{3/2}$	B1	
	$[(4-2)-(1-¼)]$	M1	Apply *their* limits (e.g. ¼ → 1) after integration.
	Required area = $\frac{21}{16} - \frac{5}{4} = \frac{1}{16}$ (or 0.0625)	A1)	
	OR: $+/-\int(3-2x)-\left(4-3x^{\frac{1}{2}}\right) = +/-\int(-1-2x+3x^{\frac{1}{2}})$	(*M1	Subtract functions and then attempt integration
	$+/-\left[-x-x^2+\frac{3x^{3/2}}{3/2}\right]$	A2, 1, 0 FT	FT on *their* subtraction. Deduct 1 mark for each term incorrect
	$+/-\left[-1-1+2-\left(-\frac{1}{4}+\frac{1}{16}+\frac{1}{8}\right)\right] = \frac{1}{16}$ (or 0.0625)	DM1 A1)	Apply *their* limits ¼ → 1
		5	

Question 93

$f'(x) = \left[\left(\frac{3}{2}\right)(2x-1)^{1/2}\right]\times[2]-[6]$	B2, 1, 0	Deduct 1 mark for each [...] incorrect.
$f'(x)<0$ or $\leqslant 0$ or $=0$ SOI	M1	
$(2x-1)^{1/2}<2$ or $\leqslant 2$ or $=2$ OE	A1	Allow with k used instead of x
Largest value of k is $\frac{5}{2}$	A1	Allow $k \leqslant \frac{5}{2}$ or $k=\frac{5}{2}$ Answer must be in terms of k (not x)
	5	

Question 94

(i)	$\frac{dy}{dx}=\frac{1}{2}\times(5x-1)^{-\frac{1}{2}}\times 5 \quad (=\frac{5}{6})$	B1 B1	B1 Without × 5 B1 × 5 of an attempt at differentiation
	m of normal $= -\frac{6}{5}$	M1	Uses $m_1m_2=-1$ with their numeric value from their dy/dx
	Equation of normal $y-3=-\frac{6}{5}(x-2)$ OE or $5y+6x=27$ or $y=\frac{-6}{5}x+\frac{27}{5}$	A1	Unsimplified. Can use $y=mx+c$ to get $c=5.4$ ISW
(ii)	*EITHER:*	(B1	Correct expression without ÷5
	For the curve $(\int)\sqrt{5x-1}dx = \frac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} \div 5$	B1	For dividing an attempt at integration of y by 5
	Limits from $\frac{1}{5}$ to 2 used → 3.6 or $\frac{18}{5}$ OE	M1 A1	Using $\frac{1}{5}$ and 2 to evaluate an integrand (may be $\int y^2$)
	Normal crosses x-axis when $y = 0$, → x= (4½)	M1	Uses their equation of normal, NOT tangent
	Area of triangle = 3.75 or $\frac{15}{4}$ OE	A1	This can be obtained by integration
	Total area=3.6 + 3.75 = 7.35, $\frac{147}{20}$ OE	A1)	
	OR: For the curve: $(\int)\frac{1}{5}(y^2+1)dy = \frac{1}{5}\left(\frac{y^3}{3}+y\right)$	(B2, 1, 0	–1 each error or omission.
	Limits from 0 to 3 used → 2.4 or $\frac{12}{5}$ OE	M1 A1	Using 0 and 3 to evaluate an integrand
	Uses their equation of normal, NOT tangent.	M1	Either to find side length for trapezium or attempt at integrating between 0 and 3
	Area of trapezium = $\frac{1}{2}(2+4½)\times 3=\frac{39}{4}$ *or* $9\frac{3}{4}$	A1	This can be obtained by integration
	Shaded area = $\frac{39}{4}-\frac{12}{5}=7.35, \frac{147}{20}$ OE	A1)	

Question 95

(i)	$\frac{dy}{dx}=0$	M1	Sets $\frac{dy}{dx}$ to 0 and attempts to solve leading to two values for x.
	$x = 1, x = 4$	A1	Both values needed
		2	
(ii)	$\frac{d^2y}{dx^2}=-2x+5$	B1	
	Using both of their x values in their $\frac{d^2y}{dx^2}$	M1	Evidence of any valid method for both points.
	x = 1 → (3) → Minimum, x = 4 →(-3) → Maximum	A1	
		3	
(iii)	$y=-\frac{x^3}{3}+\frac{5x^2}{2}-4x \quad (+c)$	B2, 1, 0	$+c$ not needed. –1 each error or omission.
	Uses x =6, y = 2 in an integrand to find c → c = 8	M1 A1	Statement of the final equation not required.
		4	

Question 96

(i)	$\frac{dy}{dx} = 2x - 4 = 0$		Can use completing the square.
	$\rightarrow x = 2,\ y = 3$	**B1 B1**	
	Midpoint of *AB* is (3, 5)	**B1 FT**	**FT** on (*their* 2, *their* 3) with (4,7)
	$\rightarrow m = \frac{7}{3}$ (or 2.33)	**B1**	
		4	
(ii)	Simultaneous equations $\rightarrow\ x^2 - 4x - mx + 9(=0)$	***M1**	Equates and sets to 0 must contain m
	Use of $b^2 - 4ac \rightarrow (m+4)^2 - 36$	**DM1**	Any use of $b^2 - 4ac$ on equation set to 0 must contain m
	Solves = 0 → −10 or 2	**A1**	Correct end-points.
	$-10 < m < 2$	**A1**	Don't condone $\leqslant$ at either or both end(s). Accept $-10 < m,\ m <$
		4	

Question 97

(i)	Area $= \int ½(x^4 - 1)\,dx = ½\left[\frac{x^5}{5} - x\right]$	***B1**	
	$½\left[\frac{1}{5} - 1\right] - 0 \quad = \quad (-)\frac{2}{5}$	**DM1A1**	Apply limits 0→1
		3	
(ii)	Vol $= \pi\int y^2\,dx \quad = \quad ¼(\pi)\int(x^8 - 2x^4 + 1)\,dx$	**M1**	(If middle term missed out can only gain the M marks)
	$¼(\pi)\left[\frac{x^9}{9} - \frac{2x^5}{5} + x\right]$	***A1**	
	$¼(\pi)[\left(\frac{1}{9} - \frac{2}{5} + 1\right] - 0$	**DM1**	
	$\frac{8\pi}{45}$ or 0.559	**A1**	
		4	
)(iii)	Vol $= \pi\int x^2\,dy \quad = \quad (\pi)\int(2y+1)^{1/2}\,dy$	**M1**	Condone use of x if integral is correct
	$(\pi)\left[\frac{(2y+1)^{3/2}}{3/2}\right][\div 2]$	***A1A1**	Expect $(\pi)\left[\frac{(2y+1)^{3/2}}{3}\right]$
	$(\pi)\left[\frac{1}{3} - 0\right]$	**DM1**	
	$\frac{\pi}{3}$ or 1.05	**A1**	Apply $-\frac{1}{2} \rightarrow 0$
		5	

Question 98

(i)	$V=\frac{1}{3}\pi r^2(18-r)=6\pi r^2-\frac{1}{3}\pi r^3$	B1	AG
		1	
(ii)	$\frac{dV}{dr}=12\pi r-\pi r^2=0$	M1	Differentiate and set = 0
	$\pi r(12-r)=0\rightarrow r=12$	A1	
	$\frac{d^2V}{dr^2}=12\pi-2\pi r$	M1	
	Sub r = 12 → $12\pi-24\pi=-12\pi$ → MAX	A1	AG
		4	
(iii)	Sub $r=12,\ h=6\rightarrow$Max $V=288\pi$ or 905	B1	
		1	

Question 99

$\frac{dy}{dx}=3x^{1/2}-3-2x^{-1/2}$	B2,1,0	
at x = 4, $\frac{dy}{dx}$ = 6 – 3 – 1 = 2	M1	
Equation of tangent is $y=2(x-4)$ OE	A1FT	Equation through (4, 0) with *their* gradient
	4	

Question 100

$f'(x)=3x^2-2x-8$	M1	Attempt differentiation
$-\frac{4}{3}$, 2 SOI	A1	
$f'(x)>0\Rightarrow x<-\frac{4}{3}$ SOI	M1	Accept $x>2$ in addition. FT *their* solutions
Largest value of a is $-\frac{4}{3}$	A1	Statement in terms of a. Accept $a\leqslant-\frac{4}{3}$ or $a<-\frac{4}{3}$. Penalise extra solutions
	4	

Question 101

(i)	$dy/dx=[-2]-\left[3(1-2x)^2\right]\times[-2]$ $(=4-24x+24x^2)$	B2,1,0	Award for the accuracy within each set of square brackets
	At x = ½ $dy/dx=-2$	B1	
	Gradient of line $y=1-2x$ is –2 (hence AB is a tangent) AG	B1	
		4	
(ii)	Shaded region = $\int_0^{½}(1-2x)-\int_0^{½}[1-2x-(1-2x)^3]$ oe	M1	Note: If area triangle OAB – area under the curve is used the first part of the integral for the area under the curve must be evaluated
	$=\int_0^{½}(1-2x)^3\,dx$ AG	A1	
		2	
(iii)	Area = $\left[\frac{(1-2x)^4}{4}\right][\div-2]$	*B1B1	
	0 – (–1/8) = 1/8	DB1	OR $\int 1-6x+12x^2-8x^3=x-3x^2+4x^3-2x^4$ (**B2,1,0**) Applying limits 0 → ½
		3	

Question 102

$f'(x)=\frac{-8}{(x-2)^2}$	**B1**	SOI
$y=\frac{8}{x-2}+2 \to y-2=\frac{8}{x-2} \to x-2=\frac{8}{y-2}$	**M1**	Order of operations correct. Accept sign errors
$f^{-1}(x)=\frac{8}{x-2}+2$	**A1**	SOI
$\frac{-48}{(x-2)^2}+\frac{16}{x-2}+4-5\ (<0) \quad \to \quad x^2-20x+84 \quad (<0)$	**M1**	Formation of 3-term quadratic in $x,(x-2)$ or $1/(x-2)$
$(x-6)(x-14)$ or 6, 14	**A1**	SOI
$2<x<6$, $x>14$	**A1**	CAO
	6	

Question 103

(i)	$dy/dx = x-6x^{½}+8$	**B2,1,0**	
	Set to zero and attempt to solve a quadratic for $x^{½}$	**M1**	Could use a substitution for $x^{½}$ or rearrange and square correctly*
	$x^{½}=4$ or $x^{½}=2$ [$x=2$ *and* $x=4$ gets **M1 A0**]	**A1**	Implies **M1**. 'Correct' roots for *their* dy/dx also implies **M1**
	$x=16$ or 4	**A1FT**	Squares of their solutions *Then **A1**,**A1** for each answer
		5	
(ii)	$d^2y/dx^2 = 1-3x^{-½}$	**B1FT**	FT on *their* dy/dx, providing a fractional power of x is present
		1	
(iii)	(When $x=16$) $d^2y/dx^2 = 1/4>0$ hence MIN	**M1**	Checking both of their values in their d^2y/dx^2
	(When $x=4$) $d^2y/dx^2 = -1/2<0$ hence MAX	**A1**	All correct Alternative methods ok but must be explicit about values of x bein considered
		2	

Question 104

$(y)=\frac{x^{½}}{½}-3x\ (+c)$	**B1B1**	
Sub (4, –6) $-6=4-12+c \quad \to \quad c=2$	**M1A1**	Expect $(y)=2x^{½}-3x+2$
	4	

Question 105

(i)	$\frac{dy}{dx}=2(x+1)-(x+1)^{-2}$	B1	
	Set = 0 and obtain $2(x+1)^3=1$ convincingly www **AG**	B1	
	$\frac{d^2y}{dx^2}=2+2(x+1)^{-3}$ www	B1	
	Sub, e.g., $(x+1)^{-3}=2$ OE or $x=\left(\frac{1}{2}\right)^{\frac{1}{3}}-1$	M1	Requires exact method – otherwise scores M0
	$\frac{d^2y}{dx^2}=6$ CAO www	A1	and exact answer – otherwise scores A0
		5	
(ii)	$y^2=(x+1)^4+(x+1)^{-2}+2(x+1)$ SOI	B1	OR $y^2=(x^4+4x^3+6x^2+4x+1)+(2x+2)+(x+1)^{-2}$
	$(\pi)\int y^2dx=(\pi)\left[\frac{(x+1)^5}{5}\right]+\left[\frac{(x+1)^{-1}}{-1}\right]+\left[\frac{2(x+1)^2}{2}\right]$ OR $(\pi)\left[\frac{x^5}{5}+x^4+2x^3+2x^2+x\right]+\left[x^2+2x\right]+\left[-\frac{1}{x+1}\right]$	B1B1B1	Attempt to integrate y^2. Last term might appear as (x^2+2x)
	$(\pi)\left[\frac{32}{5}-\frac{1}{2}+4-\left(\frac{1}{5}-1+1\right)\right]$	M1	Substitute limits $0\to1$ into an attempted integration of y^2. Do not condone omission of value when $x=0$
	9.7π or 30.5	A1	Note: omission of $2(x+1)$ in first line $\to 6.7\pi$ scores 3/6. Ignore initially an extra volume, e.g. $(\pi)\int(4½)^2$. Only take into account for the final answer
		6	

Question 106

(i)	$\frac{dy}{dx}=3x^2-18x+24$	M1A1	Attempt to differentiate. All correct for A mark
	$3x^2-18x+24=-3$	M1	Equate *their* $\frac{dy}{dx}$ to -3
	$x=3$	A1	
	$y=6$	A1	
	$y-6=-3(x-3)$	A1FT	FT on *their A*. Expect $y=-3x+15$
		6	
(ii)	$(3)(x-2)(x-4)$ SOI or $x=2,\ 4$ Allow $(3)(x+2)(x+4)$	M1	Attempt to factorise or solve. Ignore a RHS, e.g. = 0 or > 0, etc.
	Smallest value of k is 4	A1	Allow $k\geqslant 4$. Allow $k=4$. Must be in terms of k
		2	

Question 107

$f(x)=\left[\frac{(3x-1)^{\frac{2}{3}}}{\frac{2}{3}}\right][\div 3]\ (+c)$	B1B1	
$1=\frac{8^{\frac{2}{3}}}{2}+c$	M1	Sub $y=1, x=3$ Dep. on attempt to integrate and c present
$c=-1\to y=\frac{1}{2}(3x-1)^{\frac{2}{3}}-1$ SOI	A1	
When $x=0$, $y=\frac{1}{2}(-1)^{\frac{2}{3}}-1 \quad =-\frac{1}{2}$	DM1A1	Dep. on previous M1
	6	

Question 108

1(i)	$y=\frac{x}{2}+\frac{6}{x}=4 \to x=2$ or 6	B1 B1	Inspection or guesswork OK
	$\frac{dy}{dx}=\frac{1}{2}-\frac{6}{x^2}$	B1	Unsimplified OK
	When $x=2$, $m=-1 \to x+y=6$ When $x=6$, $m=\frac{1}{3} \to y=\frac{1}{3}x+2$	*M1	Correct method for either tangent
	Attempt to solve simultaneous equations	DM1	Could solve BOTH equations separately with $y=x$ and get $x=3$ both times.
	(3,3)	A1	Statement about $y=x$ not required.
		6	
1(ii)	$V=(\pi)\int\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)(dx)$	*M1	Integrate using $\pi\int y^2dx$ (doesn't need π or dx). Allow incorrect squaring. Not awarded for $\pi\int\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2dx$. Integration indicated by increase in any power by 1.
	Integration $\to \frac{x^3}{12}+6x-\frac{36}{x}$	A2,1	3 things wanted —1 each error, allow + C. (Doesn't need π)
	Using limits '*their 2*' to '*their 6*' ($53\frac{1}{3}\pi$, $\frac{160}{3}\pi$, 168 awrt)	DM1	Evidence of their values 6 and 2 from **(i)** substituted into their integrand and then subtracted. $48-\left(-\frac{16}{3}\right)$ is enough.
	Vol for line: integration or cylinder ($\to 64\pi$)	M1	Use of πr^2h or integration of 4^2 (could be from $\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2$)
	Subtracts $\to 10\frac{2}{3}\pi$ oe $\left(\text{e.g.}\frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	
(ii)	**OR**		
	$V=(\pi)\int 4^2-\left(\frac{x}{2}+\frac{6}{x}\right)^2(dx)$	M1 *M1	Integrate using $\pi\int y^2dx$ (doesn't need π or dx) Integration indicated by increase in any power by 1.
	$=(\pi)\int 16-\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)(dx)$		
	$=(\pi)\left[16x-\left(\frac{x^3}{12}+6x-\frac{36}{x}\right)\right](dx)$	A2,1	Or $\left[10x-\frac{x^3}{12}+\frac{36}{x}\right]$
	$=(\pi)(48-37\frac{1}{3})$	DM1	Evidence of their values 6 and 2 from (i) substituted
	$=10\frac{2}{3}\pi$ oe $\left(\text{eg}\frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	
		6	

Question 109

(i)	$y=\frac{2}{3}(4x+1)^{\frac{3}{2}} \div 4\,(+C)\left(=\frac{(4x+1)^{\frac{3}{2}}}{6}\right)$	B1 B1	B1 without ÷ 4. B1 for ÷ 4 oe. Unsimplified OK
	Uses $x = 2$, $y = 5$	M1	Uses (2, 5) in an integral (indicated by an increase in power by 1).
	$\rightarrow c = ½$ oe isw	A1	No isw if candidate now goes on to produce a straight line equatio
		4	
(ii)	$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$		
	$\frac{dx}{dt} = 0.06 \div 3$	M1	Ignore notation. Must be 0.06÷3 for M1.
	= 0.02 oe	A1	Correct answer with no working scores 2/2
		2	
iii)	$\frac{d^2y}{dx^2} = ½\,(4x+1)^{-½} \times 4$	B1	
	$\frac{d^2y}{dx^2}\times\frac{dy}{dx}=\frac{2}{\sqrt{4x+1}}\times\sqrt{4x+1}\quad(=2)$	B1FT	Must either show the algebraic product and state that it results in a constant or evaluate it as '= 2'. Must not evaluate at x =2. ft to apply only if $\frac{d^2y}{dx^2}$ is of the form $k(4x+1)^{-½}$
		2	

Question 110

0	$y = x^3 - 2x^2 + 5x$		
(i)	$\frac{dy}{dx} = 3x^2 - 4x + 5$	B1	CAO
	Using $b^2 - 4ac \rightarrow 16 - 60 \rightarrow$ negative $\rightarrow$ some explanation or completed square and explanation	M1 A1	Uses discriminant on equation (set to 0). CAO
		3	
(ii)	$m = 3x^2 - 4x + 5$ $\frac{dm}{dx} = 6x - 4\ (= 0)$ (must identify as $\frac{dm}{dx}$)	B1FT	FT providing differentiation is equivalent
	$\rightarrow x = \frac{2}{3}$, $m=\frac{11}{3}$ or $\frac{dy}{dx}=\frac{11}{3}$ Alt1: $m=3\left(x-\frac{2}{3}\right)^2+\frac{11}{3}$, $m=\frac{11}{3}$ Alt2: $3x^2-4x+5-m=0$, $b^2-4ac=0$, $m=\frac{11}{3}$	M1 A1	Sets to 0 and solves. A1 for correct m. Alt1: B1 for completing square, M1A1 for ans Alt2: B1 for coefficients, M1A1 for ans
	$\frac{d^2m}{dx^2} = 6$ +ve $\rightarrow$ Minimum value or refer to sketch of curve or check values of m either side of $x = \frac{2}{3}$,	M1 A1	M1 correct method. A1 (no errors anywhere)
		5	
0(iii)	Integrate $\rightarrow \frac{x^4}{4}-\frac{2x^3}{3}+\frac{5x^2}{2}$	B2,1	Loses a mark for each incorrect term
	Uses limits 0 to 6 $\rightarrow$ 270 (may not see use of lower limit)	M1 A1	Use of limits on an integral. CAO Answer only 0/4
		4	

Question 111

$\frac{dy}{dx}=\frac{12}{(2x+1)^2} \rightarrow y=\frac{-12}{2x+1} \div 2\ (+c)$	**B1 B1**	Correct without " $\div 2$". For " $\div 2$". Ignore "c".
Uses (1, 1) $\rightarrow c=3\ (\rightarrow y=\frac{-6}{2x+1}+3)$	**M1 A1**	Finding "c" following integration. CAO
Sets y to 0 and attempts to solve for $x \rightarrow x=\frac{1}{2} \rightarrow ((\frac{1}{2}, 0))$	**DM1 A1**	Sets y to 0. $x=\frac{1}{2}$ is sufficient for A1.
	6	

Question 112

$y=2x+\frac{5}{x} \rightarrow \frac{dy}{dx}=2-\frac{5}{x^2} = -3$ (may be implied) when $x = 1$.	**M1 A1**	Reasonable attempt at differentiation CAO (-3)
$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt} \rightarrow -0.06$	**M1 A1**	Ignore notation, but needs to multiply $\frac{dy}{dx}$ by 0.02.
	4	

Question 113

$f'(x)=3x^2+4x-4$	**B1**	
Factors or crit. values or sub any 2 values $(x \neq -2)$ into $f'(x)$ soi	**M1**	Expect $(x+2)(3x-2)$ or -2, $\frac{2}{3}$ or any 2 subs (excluding $x=-2$).
For $-2<x<\frac{2}{3}$, $f'(x)<0$; for $x>\frac{2}{3}$, $f'(x)>0$ soi Allow $\leqslant, \geqslant$	**M1**	Or at least 1 specific value ($\neq -2$) in each interval giving opp signs Or $f'(\frac{2}{3})=0$ and $f''(\frac{2}{3})\neq 0$ (i.e. gradient changes sign at $x=\frac{2}{3}$)
Neither www	**A1**	Must have 'Neither'
ALT 1 At least 3 values of f(x)	**M1**	e.g. f(0) = 7, f(1) = 6, f(2) = 15
At least 3 <u>correct</u> values of f(x)	**A1**	
At least 3 <u>correct</u> values of f(x) spanning $x=\frac{2}{3}$	**A1**	
Shows a decreasing and then increasing pattern. Neither www	**A1**	Or similar wording. Must have 'Neither'
ALT 2 $f'(x)=3x^2+4x-4 = 3(x+\frac{2}{3})^2-\frac{16}{3}$	**B1B1**	Do not condone sign errors
$f'(x)\geqslant -\frac{16}{3}$	**M1**	
$f'(x)<0$ for some values and >0 for other values. Neither www	**A1**	Or similar wording. Must have 'Neither'
	4	

Question 114

(i)	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x \ (+c)$	B1	
	$11 = 0 + 0 + 0 + c$	M1	Sub $x = 0$, $y = 11$ into an integrated expression. c must be present
	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x + 11$	A1	
		3	
(ii)	$4a + 2b - 4 = 0$	M1	Sub $x = 2$, $dy/dx = 0$
	$\frac{1}{3}(8a) + 2b - 8 + 11 = 3$	M1	Sub $x = 2$, $y = 3$ into an integrated expression. Allow if 11 missing
	Solve simultaneous equations	DM1	Dep. on both M marks
	$a = 3,\ b = -4$	A1A1	Allow if no working seen for simultaneous equations
		5	

Question 115

$V = 4(\pi)\int(3x-1)^{-2/3}\,dx \quad = 4(\pi)\left[\frac{(3x-1)^{1/3}}{1/3}\right][\div 3]$	M1A1A1	Recognisable integration of y^2 (M1) Independent A1, A1 for [ ] [ ]
$4(\pi)[2-1]$	DM1	Expect $4(\pi)(3x-1)^{\frac{1}{3}}$
4π or 12.6	A1	Apply limits ⅔ → 3. Some working must be shown.
	5	

$dy/dx = (-2/3)(3x-1)^{-4/3} \times 3$	B1	Expect $-2(3x-1)^{-4/3}$
When $x = 2/3$, $y = 2$ soi $dy/dx = -2$	B1B1	2nd B1 dep. on correct expression for dy//dx
Equation of normal is $y - 2 = \frac{1}{2}(x - \frac{2}{3})$	M1	Line through (⅔, *their* 2) and with grad $-1/m$. Dep on m from diffn
$y = \frac{1}{2}x + \frac{5}{3}$	A1	
	5	

Question 116

Integrate $\rightarrow \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C)	B1 B1	B1 for each term correct – allow unsimplified. C not required.
$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \rightarrow \frac{40}{3} - \frac{14}{3}$	M1	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
$= \frac{26}{3}$ oe	A1	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
	4	

Question 117

(i)	*P* is $(t, 5t)$ *Q* is $(t, t(9-t^2)) \rightarrow 4t - t^3$	B1 B1	B1 for both *y* coordinates which can be implied by subsequent working. B1 for *PQ* allow $\lvert 4t - t^3 \rvert$ or $\lvert t^3 - 4t \rvert$. **Note:** $4x - x^3$ from equating line and curve 0/2 even if *x* the replaced by *t*.
		[2]	

$\frac{d(PQ)}{dt} = 4 - 3t^2$	**B1FT**	B1FT for differentiation of their *PQ*, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from **(i)** but not the equation of the curve.
$= 0 \rightarrow t = +\frac{2}{\sqrt{3}}$	**M1**	Setting their differential of *PQ* to 0 and attempt to solve for t or *x*.
$\rightarrow$ **Maximum** $PQ = \frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$	**A1**	Allow 3.08 awrt. If answer comes from wrong method in **(i)** award A0. Correct answer from correct expression by T&I scores 3/3.
	3	

Question 118

$\frac{dy}{dx} = \left[\frac{3}{2}\times(4x+1)^{-\frac{1}{2}}\right] [\times 4] [-2] \left(\frac{6}{\sqrt{4x+1}} - 2\right)$	**B2,1,0**	Looking for 3 components
$\int y dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right] [\div 4] [-\frac{2x^2}{2}] (+C)$ $\left(= \frac{(4x+1)^{\frac{3}{2}}}{2} - x^2\right)$	**B1 B1 B1**	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '÷4'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating.
	5	
At *M*, $\frac{dy}{dx} = 0 \rightarrow \frac{6}{\sqrt{4x+1}} = 2$	**M1**	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as $x = k$)
$x = 2, y = 5$	**A1 A1**	
	3	

Question 119

(i)	$0 = 9a + 3a^2$	**M1**	Sub $\frac{dy}{dx} = 0$ and $x = 3$
	$a = -3$ only	**A1**	
		2	
(ii)	$\frac{dy}{dx} = -3x^2 + 9x \rightarrow y = -x^3 + \frac{9x^2}{2} \quad (+c)$	**M1A1FT**	Attempt integration. $\frac{1}{3}ax^3 + \frac{1}{2}a^2x^2$ scores M1. Ft on *their a*.
	$9\frac{1}{2} = -27 + 40\frac{1}{2} + c$	**DM1**	Sub $x = 3, y = 9\frac{1}{2}$. Dependent on c present
	$c = -4$	**A1**	Expect $y = -x^3 + \frac{9x^2}{2} - 4$
		4	
(iii)	$\frac{d^2y}{dx^2} = -6x + 9$	**M1**	$2ax + a^2$ scores M1
	At $x = 3$, $\frac{d^2y}{dx^2} = -9 < 0$ MAX www	**A1**	Requires at least one of –9 or < 0. Other methods possible.
		2	

Question 120

7(i)	$2 = k(8 - 28 + 24) \rightarrow k = 1/2$	**B1**	
		1	
7(ii)	When $x = 5$, $y = [\frac{1}{2}](125 - 175 + 60) = 5$	**M1**	Or solve $[\frac{1}{2}](x^3 - 7x^2 + 12x) = x \Rightarrow x = 5$ $[x = 0, 2]$
	Which lies on $y = x$, oe	**A1**	
		2	
(iii)	$\int [\frac{1}{2}(x^3 - 7x^2 + 12x) - x]dx$.	**M1**	Expect $\int \frac{1}{2}x^3 - \frac{7}{2}x^2 + 5x$
	$\frac{1}{8}x^4 - \frac{7}{6}x^3 + \frac{5}{2}x^2$	**B2,1,0FT**	Ft on their k
	2 – 28/3 +10	**DM1**	Apply limits $0 \rightarrow 2$
	8/3	**A1**	
	OR $\frac{1}{8}x^4 - \frac{7}{6}x^3 + 3x^2$	**B2,1,0FT**	Integrate to find area under curve, Ft on their k
	2 – 28/3 +12	**M1**	Apply limits $0 \rightarrow 2$. Dep on integration attempted
	Area $\Delta = \frac{1}{2} \times 2 \times 2$ or $\int_0^2 x\,dx = [\frac{1}{2}x^2] = 2$	**M1**	
	8/3	**A1**	
		5	

Question 121

)(i)(a)	$\frac{dy}{dx}=\left[-½(4x-3)^{-2}\right]\times[4]$	B1B1	Can gain this in part **(b)(ii)**
	When $x=1,\ m=-2$	B1FT	Ft from *their* $\frac{dy}{dx}$
	Normal is $y-½=½(x-1)$	M1	Line with gradient $-1/m$ and through A
	$y=½x$ soi	A1	Can score in part **(b)**
		5	
)(i)(b)	$\frac{1}{2(4x-3)}=\frac{x}{2}\to 2x(4x-3)=2\to(2)(4x^2-3x-1)\ (=0)$	M1A1	$x/2$ seen on RHS of equation can score *previous* A1
	$x=-1/4$	A1	Ignore $x=1$ seen in addition
		3	
.0(ii)	*Use* of chain rule: $\frac{dy}{dt}=(\textit{their}\ -2)\times(\pm)0.3=0.6$	M1A1	Allow +0.3 or −0.3 for M1
		2	

Question 122

$y=⅓kx^3-x^2\ (+c)$	M1A1	Attempt integration for M mark
Sub (0, 2)	DM1	Dep on c present. Expect c = 2
Sub (3, −1) → $-1=9k-9+\textit{their}\ c$	DM1	
k = 2/3	A1	
	5	

Question 123

(i)	$dy/dx=-2(2x-1)^{-2}+2$	B2,1,0	Unsimplified form ok (−1 for each error in '−2', '$(2x-1)^{-2}$' and '2')
	$d^2y/dx^2=8(2x-1)^{-3}$	B1	Unsimplified form ok
		3	
(ii)	Set dy/dx to zero and attempt to solve – at least one correct step	M1	
	$x=0,\ 1$	A1	Expect $(2x-1)^2=1$
	When x = 0, $d^2y/dx^2=-8$ (or < 0). Hence MAX	B1	
	When x = 1, $d^2y/dx^2=8$ (or > 0). Hence MIN	B1	Both final marks dependent on correct x and correct d^2y/dx^2 and no errors May use change of sign of dy/dx but not at $x=1/2$
		4	

Question 124

(i)	$V=(\pi)\int(x^3+x^2)(\mathrm{d}x)$	M1	Attempt $\int y^2\mathrm{d}x$
	$(\pi)\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_0^3$	A1	
	$(\pi)\left[\frac{81}{4}+9\quad(-0)\right]$	DM1	May be implied by a correct answer
	$\frac{117\pi}{4}$ oe	A1	Accept 91.9 If additional areas rotated about x-axis, maximum of M1A0DM1A0
		4	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}(x^3+x^2)^{-1/2}\times(3x^2+2x)$	B2,1,0	Omission of $3x^2+2x$ is one error
	(At $x=3$,) $y=6$	B1	
	At $x=3$, $m=\frac{1}{2}\times\frac{1}{6}\times33=\frac{11}{4}$ soi	DB1ft	Ft on *their* $\mathrm{d}y/\mathrm{d}x$ providing differentiation attempted
	Equation of normal is $y-6=-\frac{4}{11}(x-3)$	DM1	Equation through (3, *their* 6) and with gradient $-1/$*their* m
	When $x=0$, $y=7\frac{1}{11}$ oe	A1	
		6	

Question 125

$f'(-1)=0\to3-a+b=0$ $f'(3)=0\to27+3a+b=0$	M1	Stationary points at $x=-1$ & $x=3$ gives sim. equations in a & b
$a=-6$	A1	Solve simultaneous equation
$b=-9$	A1	
Hence $f'(x)=3x^2-6x-9\to f(x)=x^3-3x^2-9x(+c)$	B1	**FT** correct integration for *their a,b* (numerical a, b)
$2=-1-3+9+c$	M1	Sub $x=-1$, $y=2$ into *their* integrated f(x). c must be present
$c=-3$	A1	**FT** from *their* f(x)
$f(3)=k\to k=27-27-27-3$	M1	Sub $x=3$, $y=k$ into *their* integrated f(x) (Allow c omitted)
$k=-30$	A1	
	8	

Question 126

(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	B1	oe
	$\frac{dy}{dx} = \left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right] \times 3$	B1	Must have '$\times 3$'
	At $x = 4$, $\frac{dy}{dx} = \frac{3}{8}$ soi	B1	
	Line through (4, *their*4) with gradient *their* $\frac{3}{8}$	M1	If $y \neq 4$ is used then clear evidence of substitution of $x = 4$ is needed
	Equation of tangent is $y - 4 = \frac{3}{8}(x-4)$ or $y = \frac{3}{8}x + \frac{5}{2}$	A1	oe
		5	

(ii)	Area under line $= \frac{1}{2}\left(4 + \frac{5}{2}\right) \times 4 = 13$	B1	OR $\int_0^4 \frac{3}{8}x + \frac{5}{2} = \left[\frac{3}{16}x^2 + \frac{5}{2}x\right] = [3+10] = 13$
	Area under curve: $\int (3x+4)^{\frac{1}{2}} = \left[\frac{(3x+4)^{3/2}}{3/2}\right] [\div 3]$	B1B1	Allow if seen as part of the difference of 2 integrals First B1 for integral without $[\div 3]$ Second B1 must have $[\div 3]$
	$\frac{128}{9} - \frac{16}{9} = \frac{112}{9} = 12\frac{4}{9}$	M1	Apply limits $0 \to 4$ to an integrated expression
	Area $= 13 - 12\frac{4}{9} = \frac{5}{9}$ (or 0.556)	A1	
	Alternative method for question 10(ii)		
	Area for line $= 1/2 \times 4 \times 3/2 = 3$	B1	OR $\int_{5/2}^{4} \frac{1}{3}(8y - 20) = \frac{1}{3}\left[4y^2 - 20\right] = \frac{1}{3}[-16 + 25] = 3$
	Area for curve $= \int \frac{1}{3}(y^2 - 4) = \left[\frac{y^3}{9}\right] - \left[\frac{4y}{3}\right]$	B1B1	
	$\left(\frac{64}{9} - \frac{16}{3}\right) - \left(\frac{8}{9} - \frac{8}{3}\right) = \frac{32}{9}$	M1	Apply limits $2 \to 4$ to an integrated expression for curve
	Area $= \frac{32}{9} - 3 = \frac{5}{9}$ (or 0.556)	A1	
		5	

(iii)	$\frac{dy}{dx} = \frac{1}{2}$	B1	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}} = \frac{1}{2}$	M1	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}} = 2$.
	$(3x+4)^{\frac{1}{2}} = 3 \to 3x + 4 = 9 \to x = \frac{5}{3}$ oe	A1	
		3	

Question 127

(i)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=7\times-0.05$	**M1**	Multiply numerical gradient at $x=2$ by ±0.05.
	−0.35 (units/s) **or** Decreasing at a rate of (+) 0.35	**A1**	Ignore notation and omission of units
		2	
(ii)	$(y)=\frac{x^4}{4}+\frac{4}{x}$ $(+c)$ oe	**B1**	Accept unsimplified
	Uses (2, 9) in an integral to find c.	**M1**	The power of at least one term increase by 1.
	$c=3$ **or** $(y=)\frac{x^4}{4}+\frac{4}{x}+3$ oe	**A1**	A0 if candidate continues to a final equation that is a straight line.
		3	

Question 128

1(i)	$\frac{dy}{dx}=\left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right][\times 4]\left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right][\times 4]$	**B1B1B1**	B1 B1 for each, without × 4. B1 for ×4 twice.
	$\left(\frac{2}{\sqrt{4x+1}}-\frac{18}{\left(\sqrt{4x+1}\right)^3}\ or\ \frac{8x-16}{(4x+1)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of $(4x+1)$ correct.
	$\int y\,dx=\left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right][\div 4]+\left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right][\div 4]\ (+C)$	**B1B1B1**	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6}+\frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of $(4x+1)$ correct.
		6	
(ii)	$\frac{dy}{dx}=0\rightarrow\frac{2}{\sqrt{4x+1}}-\frac{18}{(4x+1)^{\frac{3}{2}}}=0$	**M1**	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x+1=9$ or $(4x+1)^2=81$	**A1**	Must be from correct differential.
	$x=2$, $y=6$ **or** M is (2, 6) only.	**A1**	Both values required. Must be from correct differential.
		3	
(iii)	Realises area is $\int y\,dx$ **and** attempt to use their 2 and sight of 0.	***M1**	Needs to use their integral and to see '*their 2*' substituted.
	Uses limits 0 to 2 correctly → [4.5 + 13.5] − [$\frac{1}{6}$ + 4.5] (= 13⅓)	**DM1**	Uses both 0 and '*their 2*' and subtracts. Condone wrong way round.
	(Area =) 1⅓ **or** 1.33	**A1**	Must be from a correct differential and integral.
		3	13⅓ or 1⅓ with little or no working scores M1DM0A0.

Question 129

)(i)	integrating → $\frac{dy}{dx} = x^2 - 5x\ (+c)$	B1	
	$= 0$ when $x = 3$	M1	Uses the point to find c after ∫ = 0.
	$c = 6$	A1	
	integrating again → $y = \frac{x^3}{3} - \frac{5x^2}{2} + 6x \quad (+d)$	B1	**FT** Integration again FT if a numerical constant term is present.
	use of (3, 6)	M1	Uses the point to find d after ∫ = 0.
	$d = 1½$	A1	
		6	
)(ii)	$\frac{dy}{dx} = x^2 - 5x + 6 = 0 \to x = 2$	B1	
		1	
)(iii)	$x = 3,\ \frac{d^2y}{dx^2} = 1$ and/or +ve Minimum. $x = 2,\ \frac{d^2y}{dx^2} = -1$ and/or −ve Maximum	B1	www
	May use shape of '$+x^3$' curve or change in sign of $\frac{dy}{dx}$	B1	www SC: $x = 3$, minimum, $x = 2$, maximum, B1
		2	

Question 130

(i)	$3 \times -½ \times (1+4x)^{-\frac{3}{2}}$	B1	
	$\frac{dy}{dx} = 3 \times -½ \times (1+4x)^{-\frac{3}{2}} \times 4$	B1	Must have '× 4'
	If $x = 2$, $m = -\frac{2}{9}$, Perpendicular gradient = $\frac{9}{2}$	M1	Use of $m_1.m_2 = -1$
	Equation of normal is $y - 1 = \frac{9}{2}(x-2)$	M1	Correct use of line eqn (could use y=0 here)
	Put $y = 0$ or on the line before → $\frac{16}{9}$	A1	AG
		5	
l(ii)	Area under the curve = $\int_0^2 \frac{3}{\sqrt{1+4x}}\,dx = \frac{3\sqrt{1+4x}}{\frac{1}{2}} \div 4$	B1 B1	Correct without '÷4'. For 2nd B1, ÷4'.
	Use of limits 0 to 2 → 4½ − 1½	M1	Use of correct limits in an integral.
	3	A1	
	Area of the triangle = ½ × 1 × $\frac{2}{9} = \frac{1}{9}$ or attempt to find $\int_{16/9}^{2} \left(\frac{9}{2}x - 8\right)dx$	M1	Any correct method.
	Shaded area = $3 - \frac{1}{9} = 2\frac{8}{9}$	A1	
		6	

Question 131

(i)	$\frac{dy}{dx} = -2(x-1)^{-3}$	B1	
	When $x = 2$, $m = -2 \rightarrow$ gradient of normal $= -\frac{1}{m}$	M1	m must come from differentiation
	Equation of normal is $y - 3 = ½(x-2) \rightarrow y = ½x + 2$	A1	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	
(ii)	$(\pi)\int y_1^2(dx), (\pi)\int y_2^2(dx)$	*M1	Attempt to integrate y^2 for at least one of the functions
	$(\pi)\int\left(\frac{1}{2}x+2\right)^2$ or $\left(\frac{1}{4}x^2+2x+4\right)$ $(\pi)\int\left((x-1)^{-4}+4(x-1)^{-2}+4\right)$	A1A1	A1 for $\left(\frac{1}{2}x+2\right)^2$ depends on an attempt to integrate this form later
	$(\pi)\left[\frac{2}{3}\left(\frac{1}{2}x+2\right)^3 \text{ or } \frac{1}{12}x^3+x^2+4x\right]$ $(\pi)\left[\frac{(x-1)^{-3}}{-3}+\frac{4(x-1)^{-1}}{-1}+4x\right]$	A1A1	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18-\frac{125}{12} \text{ or } \frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)\right\}$ $\left\{\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)\right\}$	DM1	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi\left\{7\frac{7}{12}+6\frac{7}{24}\right\}$	DM1	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	A1	$\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)$ $\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)$
		8	

Question 132

$(y =)\ \frac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\left(=\frac{k\sqrt{x}}{\frac{1}{2}}\right)(+c)$	B1	OE
Substitutes both points into an integrated expression with a '+c' and solve as far as a value for one variable.	M1	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
$k = 2½$ and $c = -6$	A1	WWW
$y = 5\sqrt{x} - 6$	A1	OE From correct values of both k & c and correct integral.
	4	

Question 133

Use of Pythagoras $\rightarrow r^2 = 15^2 - h^2$	M1	
$V = \frac{1}{3}\pi(225 - h^2) \times h \rightarrow \frac{1}{3}\pi(225h - h^3)$	A1	AG WWW e.g. sight of $r = 15 - h$ gets A0.
	2	
$\left(\frac{dv}{dh} =\right) \frac{\pi}{3}(225 - 3h^2)$	B1	
Their $\frac{dv}{dh} = 0$	M1	Differentiates, sets *their* differential to 0 and attempts to solve at least as far as $h^2 \neq 0$.
$(h =)\ \sqrt{75}$, $5\sqrt{3}$ or AWRT 8.66	A1	Ignore $-\sqrt{75}$ OE and ISW for both A marks
$\frac{d^2h}{dh^2} = \frac{\pi}{3}\ (-6h)\ (\rightarrow -\text{ve})$	M1	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
$\rightarrow$ Maximum	A1FT	Correct conclusion from correct 2nd differential, value for h not required, or any other valid complete method. FT for *their* h, if used, as long as it is positive.
		SC Omission of π or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
	5	

Question 134

At A, $x = \frac{1}{2}$.	B1	Ignore extra answer $x = -1.5$
$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(= -\frac{1}{2})$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1m_2 = -1$
Equation of normal: $y - 0 = -\frac{1}{2}(x - \frac{1}{2})$ or $y - 0 = -\frac{1}{2}(0 - \frac{1}{2})$ or $0 = -\frac{1}{2}\times\frac{1}{2} + c$	DM1	Use of *their* x at A and *their* normal gradient.
$B\ (0, \frac{1}{4})$	A1	
	4	

At A, $x = \frac{1}{2}$.	B1	Ignore extra answer $x = -1.5$
$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(= -\frac{1}{2})$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1m_2 = -1$
Equation of normal: $y - 0 = -\frac{1}{2}(x - \frac{1}{2})$ or $y - 0 = -\frac{1}{2}(0 - \frac{1}{2})$ or $0 = -\frac{1}{2}\times\frac{1}{2} + c$	DM1	Use of *their* x at A and *their* normal gradient.
$B\ (0, \frac{1}{4})$	A1	
	4	

)(iii)	$\int_0^{\frac{1}{2}} 1-\frac{4}{(2x+1)^2}(\mathrm{d}x)$	*M1	$\int y\,\mathrm{d}x$ SOI with 0 and *their* positive x coordinate of A.
	$[½ + 1] - [0 + 2] = (-½)$	DM1	Substitutes both 0 and *their* ½ into *their* $\int y\mathrm{d}x$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}}-\frac{1}{2}(\mathrm{d}y)$	*M1	$\int x\,\mathrm{d}y$ SOI. Where x is of the form $k\left(1-y\right)^{-\frac{1}{2}}+c$ with 0 and *their* negative y intercept of curve.
	$[-2]-\left[-4+\frac{3}{2}\right]=(½)$	DM1	Substitutes both 0 and *their* –3 into *their* $\int x\mathrm{d}y$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)

Question 135

Attempt to solve $f'(x)=0$ or $f'(x)>0$ or $f'(x)\geqslant 0$	M1	SOI
$(x-2)(x-4)$	A1	2 and 4 seen
(Least possible value of n is) 4	A1	Accept $n = 4$ or $n \geqslant 4$
	3	

Question 136

)(i)	$y=[(5x-1)^{1/2}\div\frac{3}{2}\ \div 5]\ [-2x]$	B1 B1	
	$3=\frac{27}{(3/2)\times 5}-4+c$	M1	Substitute $x = 2, y = 3$
	$c=7-\frac{18}{5}=\frac{17}{5}\ \rightarrow\left(y=\frac{2(5x-1)^{\frac{3}{2}}}{15}-2x+\frac{17}{5}\right)$	A1	
)(ii)	$\mathrm{d}^2y/\mathrm{d}x^2=\left[½(5x-1)^{-1/2}\right]\ [\times 5]$	B1 B1	
(iii)	$(5x-1)^{1/2}-2=0\ \rightarrow\ 5x-1=4$ $x=1$	M1A1	Set $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and attempt solution (M1)
	$y=\frac{16}{25}-2+\frac{17}{5}=\frac{37}{15}$	A1	Or 2.47 or $\left(1,\ \frac{37}{15}\right)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^x}=\frac{5}{2}\times\frac{1}{2}=\frac{5}{4}$ (> 0) hence minimum	A1	OE

Question 137

(i)	$(y=)\ (x+2)^2-1$	B1 DB1	2nd B1 dependent on 2 in bracket
	$x+2=(\pm)(y+1)^{1/2}$	M1	
	$x=-2+(y+1)^{1/2}$	A1	
(ii)	$x^2=4+(y+1)-/+4(y+1)^{\frac{1}{2}}$	*M1A1	SOI. Attempt to find x^2. The last term can be – or + at this stage
	$(\pi)\int x^2\,(dy)=(\pi)\left[5y+\frac{y^2}{2}-\frac{4(y+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$	A2,1,0	
	$(\pi)\left[15+\frac{9}{2}-\frac{64}{3}-\left(-5+\frac{1}{2}\right)\right]$	DM1	Apply y limits
	$\frac{8\pi}{3}$ or 8.38	A1	

Question 138

$f'(x)=\left[-(3x+2)^{-2}\right]\times[3]+[2x]$	B2, 1, 0	
<0 hence decreasing	B1	Dependent on at least B1 for $f'(x)$ and must include <0 or '(always) neg'
	3	

Question 139

$(\pi)\int(y-1)dy$	*M1	SOI Attempt to integrate x^2 or $(y-1)$
$(\pi)\left[\frac{y^2}{2}-y\right]$	A1	
$(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$	DM1	Apply limits $1\to5$ to an integrated expression
8π or AWRT 25.1	A1	
	4	

Question 140

$\frac{dy}{dx}=2x-2$	B1	
$\frac{dy}{dx}=\frac{4}{6}$	B1	OE, SOI
$their\,(2x-2)=their\,\frac{4}{6}$	M1	LHS and RHS must be *their* $\frac{dy}{dx}$ expression and value
$x=\frac{4}{3}$ oe	A1	
	4	

Question 141

(a)	$2(a+3)^{\frac{1}{2}} - a = 0$	M1	SOI. Set $\frac{dy}{dx} = 0$ when $x = a$. Can be implied by an answer in terms of a
	$4(a+3) = a^2 \rightarrow a^2 - 4a - 12 = 0$	M1	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \rightarrow a = 6$	A1	Must show factors, or formula or completing square. Ignore $a = -2$ SC If a is never used maximum of M1A1 for $x = 6$,with visible solution
		3	
(b)	$\frac{d^2y}{dx^2} = (x+3)^{-\frac{1}{2}} - 1$	B1	
	Sub *their* $a \rightarrow \frac{d^2y}{dx^2} = \frac{1}{3} - 1 = -\frac{2}{3}$ $(or < 0) \rightarrow$ MAX	M1A1	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
(c)	$(y =)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}x^2 \ (+c)$	B1B1	
	Sub $x =$ *their* a and $y = 14$ $\rightarrow 14 = \frac{4}{3}(9)^{\frac{3}{2}} - 18 + c$	M1	Substitute into an integrated expression. c must be present. Expect $c = -4$
	$y = \frac{4}{3}(x+3)^{\frac{3}{2}} - \frac{1}{2}x^2 - 4$	A1	Allow $f(x) = ...$
		4	

Question 142

$(y) = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} \ (+c)$	B1 B1
$7 = 16 - 12 + c$ (**M1** for subsituting $x = 4, y = 7$ into *their* integrated expansion)	M1
$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 3$	A1
	4

Question 143

$\frac{dy}{dx} = \left[\frac{1}{2}(5x-1)^{-1/2}\right] \times [5]$	B1 B1
Use $\frac{dy}{dt} = 2 \times \left(\text{their } \frac{dy}{dx} \text{ when } x = 1\right)$	M1
$\frac{5}{2}$	A1
	4

$2 \times \text{their } \frac{5}{2}(5x-1)^{-1/2} = \frac{5}{8}$ oe	M1
$(5x-1)^{1/2} = 8$	A1
$x = 13$	A1
	3

Question 144

(a)	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	$3x^2 - 4bx + b^2 = 0 \rightarrow (3x-b)(x-b)\ (=0)$	M1
	$x = \frac{b}{3}$ or b	A1
	$a = \frac{b}{3} \rightarrow b = 3a$ **AG**	A1
	Alternative method for question 11(a)	
	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	Sub $b = 3a$ & obtain $\frac{dy}{dx} = 0$ when $x = a$ and when $x = 3a$	M1
	$\frac{d^2y}{dx^2} = 6x - 12a$	A1
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ **AG**	A1
		4
(b)	Area under curve $= \int (x^3 - 6ax^2 + 9a^2x)dx$	M1
	$\frac{x^4}{4} - 2ax^3 + \frac{9a^2x^2}{2}$	B2,1,0
	$\frac{a^4}{4} - 2a^4 + \frac{9a^4}{2}\ \left(= \frac{11a^4}{4}\right)$ (**M1** for applying limits $0 \rightarrow a$)	M1
	When $x = a,\ y = a^3 - 6a^3 + 9a^3 = 4a^3$	B1
	Area under line $= \frac{1}{2}a \times \mathit{their}\ 4a^3$	M1
	Shaded area $= \frac{11a^4}{4} - 2a^4 = \frac{3}{4}a^4$	A1
		7

Question 145

Volume after 30 s = 18000 $\quad \frac{4}{3}\pi r^3 = 18000$	M1
$r = 16.3$ cm	A1
	2
$\frac{dV}{dr} = 4\pi r^2$	B1
$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	M1
$\frac{dr}{dt} = 0.181$ cm per second	A1
	3

Question 146

(a)	Volume $= \pi \int x^2 dy = \pi \int \frac{36}{y^2} dy$	*M1
	$= \pi\left[\frac{-36}{y}\right]$	A1
	Uses limits 2 to 6 correctly $\rightarrow (12\pi)$	DM1
	Vol of cylinder $= \pi.1^2.4$ or $\int 1^2.dy \quad = [y]$ from 2 to 6	M1
	Vol $= 12\pi - 4\pi = 8\pi$	A1
		5
(b)	$\frac{dy}{dx} = \frac{-6}{x^2}$	B1
	$\frac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

Question 147

(a)	$\frac{dy}{dx} = 54 - 6(2x-7)^2$	B2,1
	$\frac{d^2y}{dx^2} = -24(2x-7)$ (FT only for omission of '×2 ' from the bracket)	B2,1 FT
		4
(b)	$\frac{dy}{dx} = 0 \rightarrow (2x-7)^2 = 9$	M1
	$x = 5, y = 243$ or $x = 2, y = 135$	A1 A1
		3
(c)	$x = 5 \ \frac{d^2y}{dx^2} = -72 \rightarrow$ Maximum (FT only for omission of '×2 ' from the bracket)	B1FT
	$x = 2 \ \frac{d^2y}{dx^2} = 72 \rightarrow$ Minimum (FT only for omission of '×2 ' from the bracket)	B1FT
		2

Question 148

(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2)	**B1B1**
	$\frac{d^2y}{dx^2} = -12(3-2x)\times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without − 2)	**B1FT** **B1**
		4
(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \rightarrow 3-2x = \pm 2$	**M1**
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	**A1A1**
		3
(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	**B1FT**
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	**B1FT**
		2

Question 149

(a)	Simultaneous equations $\frac{8}{x+2} = 4 - ½x$	**M1**
	$x = 0$ or $x = 6 \rightarrow A$ (0, 4) and B (6, 1)	**B1A1**
	At C $\frac{-8}{(x+2)^2} = -\frac{1}{2} \rightarrow C$ (2, 2) (**B1** for the differentiation. **M1** for equating and solving)	**B1** **M1A1**
		6
(b)	Volume under line $= \pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\frac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$ (**M1** for volume formula. **A2,1** for integration)	**M1** **A2,1**
	Volume under curve $= \pi\int\left(\frac{8}{x+2}\right)^2 dx = \pi\left[\frac{-64}{x+2}\right] = (24\pi)$	**A1**
	Subtracts and uses 0 to 6 $\rightarrow 18\pi$	**M1A1**
		6

Question 150

(a)	$\frac{dy}{dx} = \left[\frac{x^{-1/2}}{2k}\right] - \left[\frac{x^{-3/2}}{2}\right] + ([0])$	B2, 1, 0	$([0])$ implies that more than 2 terms counts as an error
	Sub $\frac{dy}{dx} = 3$ when $x = \frac{1}{4}$ Expect $3 = \frac{1}{k} - 4$	M1	
	$k = \frac{1}{7}$ (or 0.143)	A1	
		4	

(b)	$\int \frac{1}{k}x^{1/2} + x^{-1/2} + \frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right] + \left[2x^{1/2}\right] + \left[\frac{x}{k^2}\right]$	B2, 1, 0	OE
	$\left(\frac{2k^2}{3} + 2k + 1\right) - \left(\frac{k^2}{12} + k + \frac{1}{4}\right)$	M1	Apply limits $\frac{k^2}{4} \to k^2$ to an integrated expression. Expect $\frac{7}{12}k^2 + k + \frac{3}{4}$
	$\frac{7}{12}k^2 + k + \frac{3}{4} = \frac{13}{12}$	M1	Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2 + 12k - 4\ (= 0)$
	$k = \frac{2}{7}$ only (or 0.286)	A1	Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square.
		5	

Question 151

(a)	$\frac{dy}{dx} = [2] \quad [-2(2x+1)^{-2}]$	B1 B1	
	$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$	B1	
		3	
(b)	Set *their* $\frac{dy}{dx} = 0$ and attempt solution	M1	
	$(2x+1)^2 = 1 \to 2x + 1 = (\pm)\,1$ or $4x^2 + 4x = 0 \to (4)x(x+1) = 0$	M1	Solving as far as $x =$ ....
	$x = 0$	A1	WWW. Ignore other solution.
	(0, 2)	A1	One solution only. Accept $x = 0$, $y = 2$ only.
	$\frac{d^2y}{dx^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	B1	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

Question 152

(a)	$\frac{-2}{x+2}$	B1	Integrate f(x). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\frac{2}{3}\right) = \frac{2}{3}$	M1 A1	Apply limit(s) to an integrated expansion. CAO for A1
		3	
(b)	$-1=-2+c$	M1	Substitute $x=-1, y=-1$ into *their* integrated expression (c present)
	$y=\frac{-2}{x+2}+1$	A1	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

Question 153

(a)	$\left(\frac{dy}{dx}\right)=[8]\times[(3-2x)^{-3}]+[-1]$ $\left(=\frac{8}{(3-2x)^3}-1\right)$	B2, 1, 0	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2}=-3\times8\times(3-2x)^{-4}\times(-2)$ $\left(=\frac{48}{(3-2x)^4}\right)$	B1 FT	FT providing *their* bracket is to a negative power
	$\int y dx = [(3-2x)^{-1}]\ [2\div(-1\times-2)]\ [-\frac{1}{2}x^2]\ (+c)$ $\left(=\frac{1}{3-2x}-\frac{1}{2}x^2+c\right)$	B1 B1 B1	Simplification not needed, B1 for each correct element
		6	
(b)	$\frac{dy}{dx}=0\rightarrow(3-2x)^3=8\rightarrow3-2x=k\rightarrow x=$	M1	Setting *their* 2-term differential to 0 and attempts to solve as far as $x=$
	$\frac{1}{2}$	A1	
	Alternative method for question 10(b)		
	$y=0\rightarrow\frac{2}{(3-2x)^2}-x=0\rightarrow(x-2)(2x-1)^2=0\rightarrow x=$	M1	Setting y to 0 and attempts to solve a cubic as far as $x=$ (3 factors needed)
	$\frac{1}{2}$	A1	
		2	
(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)}-\frac{\left(\frac{1}{2}\right)^2}{2}\right]-\left[\frac{1}{3-2\times0}-0\right]$	M1	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	A1	
		2	

Question 154

(a)	$f'(4)\left(=\frac{5}{2}\right)$	*M1	Substituting 4 into $f'(x)$
	$\left(\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\right)\rightarrow\left(\frac{dy}{dt}\right)=\frac{5}{2}\times 0.12$	DM1	Multiplies *their* $f'(4)$ by 0.12
	$\left(\frac{dy}{dt}=\right)0.3$	A1	OE
		3	
(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	B1 B1	B1 for each unsimplified integral.
	Uses (4, 7) leading to $c=(-21)$	M1	Uses (4, 7) to find a c value
	y or $f(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	A1	Need to see y or f(x) = somewhere in *their* solution and 12 and 8
		4	

Question 155

(a)	$4x^{\frac{1}{2}}-2x=3-x\ \rightarrow x-4x^{\frac{1}{2}}+3(=0)$	*M1	3-term quadratic. Can be expressed as e.g. u^2-4u+3 (=0)
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$	DM1	Or quadratic formula or completing square
	$x^{\frac{1}{2}}=1, 3$	A1	SOI
	$x=1, 9$	A1	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$	*M1	Isolate $x^{\frac{1}{2}}$
	$16x=9+6x+x^2\ \rightarrow\ x^2-10x+9\ (=0)$	A1	3-term quadratic
	$(x-1)(x-9)\ (=0)$	DM1	Or formula or completing square on a quadratic obtained by a correct method
	$x=1, 9$	A1	
		4	
(b)	$\frac{dy}{dx}=2x^{1/2}-2$	*B1	
	$\frac{dy}{dx}$ or $2x^{1/2}-2=0$ when $x=1$ hence B is a stationary point	DB1	
		2	

(c)	Area of correct triangle = $\frac{1}{2}$ (9 – 3) × 6	**M1**	or $\int_3^9 (3-x)(\mathrm{d}x) = \left[3x - \frac{1}{2}x^2\right] \rightarrow -18$
	$\int(4x^{\frac{1}{2}} - 2x)(\mathrm{d}x) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	**B1 B1**	
	$(72-81) - \left(\frac{64}{3} - 16\right)$	**M1**	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	**A1**	OE
	Shaded region = $18 - 14\frac{1}{3} = 3\frac{2}{3}$	**A1**	OE
		6	

Question 156

$\frac{dy}{dx} = \left[\frac{1}{2}\left(25 - x^2\right)^{-1/2}\right] \times [-2x]$	B1 B1	
$\frac{-x}{\left(25 - x^2\right)^{1/2}} = \frac{4}{3} \rightarrow \frac{x^2}{25 - x^2} = \frac{16}{9}$	M1	Set = $\frac{4}{3}$ and square both sides
$16\left(25 - x^2\right) = 9x^2 \rightarrow 25x^2 = 400 \rightarrow x = (\pm)4$	A1	
When $x = -4$, $y = 5$ $\rightarrow$ $(-4, 5)$	A1	
	5	

Question 157

(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	B1	SOI Award this mark for $\frac{dr}{dV}$
$\frac{50}{\textit{their}\text{ derivative}}$	M1	Can be in terms of r
$\frac{1}{8\pi}$ or 0.0398	A1	AWRT
	3	

Question 158

$(y =) \left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]$ $(+c)$	B1 B1	
$7 = 1 + 2 + c$	M1	Substitute $x = 2$, $y = 7$ into an integrated expansion (c present). Expect $c = 4$
$y = -(x-3)^{-1} + \frac{1}{2}x^2$ $+4$	A1	OE
	4	

Question 159

(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	M1	OE. Set y to zero and attempt to solve.
	$x=4$ **only**	A1	From use of a correct method.
		2	
(b)	$\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x=4$ gradient $=9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	M1	Using *their* $x=4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	A1	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	M1	Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve.
	$x=12$	A1	Condone $(\pm)12$ from use of a correct method.
		2	
(d)	$\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)dx=9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	M1	Apply limits *their* 4 → 9 to an integrated expression with no consideration of other areas.
	6	A1	Use of π scores A0
		4	

Question 160

(a)	At $x=1$, $\frac{dy}{dx}=6$	B1	
	$\frac{dx}{dt}=\left(\frac{dx}{dy}\times\frac{dy}{dt}\right)=\frac{1}{6}\times 3=\frac{1}{2}$	M1 A1	Chain rule used correctly. Allow alternative and minimal notation.
		3	
(b)	$[y=]\left(\frac{6(3x-2)^{-2}}{-2}\right)\div(3)\ [+c]$	B1 B1	
	$-3=-1+c$	M1	Substitute $x=1$, $y=-3$. c must be present.
	$y=-(3x-2)^{-2}-2$	A1	OE. Allow f(x)=
		4	

Question 161

(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2} = 0$ leading to $\frac{1}{2}x^{-1/2} = \frac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
(b)	When $x = 4k^2$, $\frac{dy}{dx} = \left[\frac{1}{4k} - \frac{1}{16k} =\right]\frac{3}{16k}$	B1	OE
	$y = \left[2k + k^2 \times \frac{1}{2k}\right] = \frac{5k}{2}$	B1	OE. Accept $2k + \frac{k}{2}$
	Equation of tangent is $y - \frac{5k}{2} = \frac{3}{16k}(x - 4k^2)$ or $y = mx + c \rightarrow \frac{5k}{2} = \frac{3}{16k}(4k^2) + c$	M1	Use of line equation with *their* gradient and $(4k^2, \textit{their } y)$,
	When $x = 0$, $y = \left[\frac{5k}{2} - \frac{3k}{4} =\right]\frac{7k}{4}$ or from $y = mx + c$, $c = \frac{7k}{4}$	A1	OE
		4	
(c)	$\int\left(x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}\right)dx = \frac{2x^{\frac{3}{2}}}{3} + 2k^2x^{\frac{1}{2}}$	B1	Any unsimplified form
	$\left(\frac{16k^3}{3} + 4k^3\right) - \left(\frac{9k^3}{4} + 3k^3\right)$	M1	Apply limits $\frac{9}{4}k^2 \rightarrow 4k^2$ to an integration of y. M0 if volume attempted.
	$\frac{49k^3}{12}$	A1	OE. Accept $4.08\,k^3$
		3	

Question 162

$[f^{-1}(x) =]\ \left((2x-1)^{1/2}\right) \times \left(\frac{1}{3} \times 2 \times \frac{3}{2}\right)(-2)$	B2, 1, 0	Expect $(2x-1)^{1/2} - 2$
$(2x-1)^{1/2} - 2 \leqslant 0 \rightarrow 2x - 1 \leqslant 4$ or $2x - 1 < 4$	M1	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0' at this stage but do not allow '≥ 0' or '> 0' If '–2' missed then must see $\leqslant$ or $<$ for the M1
Value [of a] is 2½ or $a = 2½$	A1	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
	4	

Question 163

$[f(x) =]\ 2x^3 + \frac{8}{x}\ [+c]$	B1	Allow any correct form
$7 = 16 + 4 + c$	M1	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
$f(x) = 2x^3 + \frac{8}{x} - 13$	A1	Allow $y =$, $f(x)$ or y can appear earlier in answer
	3	

Question 164

(a)	At stationary point $\frac{dy}{dx}=0$ so $6(3\times2-5)^3-k\times2^2=0$	M1	Setting given $\frac{dy}{dx}=0$ and substituting $x=2$ into it.
	$[k=]\frac{3}{2}$	A1	OE
		2	
(b)	$[y=]\frac{6}{4\times3}(3x-5)^4-\frac{1}{3}kx^3\ [+c]$.	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$. FT *their* non zero k.
	$-\frac{7}{2}=\frac{1}{2}(3\times2-5)^4-\frac{1}{3}\times\frac{3}{2}\times2^3+c$ [leading to $-3.5+c=-3.5$]	DM1	Using (2,-3.5) in an integrated expression. + c needed. Substitution needs to be seen, simply stating $c=0$ is DM0.
	$y=\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$	A1	$y=$ or f(x)= must be seen somewhere in solution.
(b)	**Alternative method for Question 11(b)**		
	$[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or $-270x^3-k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their k*
	$-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$	DM1	Using (2, -3.5) in an integrated expression. + c needed
	$y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$	A1	$y=$ or f(x)= must be seen somewhere in solution.
		4	
(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their k.* Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2-1623x+1350$ or $-1620x-2kx$	B2,1,0 FT	FT *their k.* B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
(d)	[At $x=2$] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48	M1	OE. Substituting $x=2$ into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

Question 165

Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
Volume $= [\pi]\int(x-2)[\mathrm{d}x]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
$[\pi]\left[\frac{1}{2}x^2 - 2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
$=[\pi]\left[\left(\frac{5^2}{2} - 2\times 5\right) - \left(\frac{their\,3^2}{2} - 2\times their\,3\right)\right]$ $=[\pi]\left[\frac{5}{2} + \frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
Volume of cylinder $= \pi\times 1^2\times(5 - their\,3)[= 2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and *their* 3 used).
[Volume of solid $= 4\pi - 2\pi =]2\pi$ or 6.28	A1	AWRT

Question 166

(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3(3x+4)^{-0.5} - 1$	**B1 B1**	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent $= -\frac{1}{4}$ and Gradient of normal = 4	***M1**	Substituting $x = 4$ into a differentiated expression and using $m_1m_2 = -1$
	Equation of line is $(y-4) = 4(x-4)$ **or** evaluate c	**DM1**	With (4, 4) and *their* gradient of normal
	So $y = 4x - 12$	**A1**	
		5	
(b)	$3(3x+4)^{-0.5} - 1 = 0$	**M1**	Setting *their* $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
	Solving as far as $x =$	**M1**	Where $\frac{\mathrm{d}y}{\mathrm{d}x}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x = \frac{5}{3}$, $y = 2\left(3\times\frac{5}{3} + 4\right)^{0.5} - \frac{5}{3} = \frac{13}{3}$	**A1**	
		3	
(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{9}{2}(3x+4)^{-1.5}$	**M1**	Differentiating *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ OR checking $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find +ve and -ve either side of their $x = \frac{5}{3}$
	At $x = \frac{5}{3}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is negative so the point is a maximum	**A1**	
		2	
(d)	Area $= \left[\int 2(3x+4)^{0.5} - x\,\mathrm{d}x =\right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	**B1 B1**	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2\right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	**M1**	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	**A1**	Or $\frac{152}{9}$
		4	

Question 167

$[y=]-\frac{1}{x^3}+8x^4\ [+c]$	**B1 B1**	OE. Accept unsimplified.
$4=-8+\frac{1}{2}+c$	**M1**	Substituting $\left(\frac{1}{2},4\right)$ into an integrated expression
$y=-\frac{1}{x^3}+8x^4+\frac{23}{2}$	**A1**	OE. Accept $-x^{-3}$; must be 8; y = must be seen in working.
	4	

Question 168

(a)	$\left\{5(y-3)^2\right\}\quad\{+5\}$	**B1 B1**	Accept $a=-3$, $b=5$
		2	
(b)	$[f'(x)=]5x^4-30x^2+50$	**B1**	
	$5(x^2-3)^2+5$ or $b^2<4ac$ and at least one value of $f'(x)>0$	**M1**	
	>0 and increasing	**A1**	WWW
		3	

Question 169

(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$	**M1**	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)dx-\int\left(x^{-1/2}\right)dx$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	**A1 A1 A1**	If two separate integrals with no subtraction **SC B1** for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	**DM1**	Substitute limits $\frac{1}{4}\to 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	**A1**	WWW. Cannot be awarded if π appears in any integral.
		6	
(b)	$\left[\frac{dy}{dx}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	**B1**	
	When $x=1$, $m=-\frac{1}{2}$	**M1**	Substitute $x=1$ into a differential.
	[Equation of normal is] $y-1=2(x-1)$	**M1**	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
	[When $x=0$,] $p=-1$	**A1**	WWW
		4	

Question 170

)(a)	$f''(x)=-\left(\frac{1}{2}x+k\right)^{-3}$	**B1**	
	$f''(2)>0\Rightarrow-(1+k)^{-3}>0$	**M1**	Allow for solving *their* $f''(2)>0$
	$k<-1$	**A1**	WWW
		3	

(b)	$\left[f(x)=\int\left(\left(\tfrac{1}{2}x-3\right)^{-2}-(-2)^{-2}\right)dx=\right]\left\{\dfrac{\left(\tfrac{1}{2}x-3\right)^{-1}}{-1\times\tfrac{1}{2}}\right\}\left\{-\dfrac{x}{4}\right\}$	B1 B1	Allow $-2\left(\dfrac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\tfrac{1}{2}=1-\tfrac{1}{2}+c$	M1	Substitute $x=2$, $y=3\tfrac{1}{2}$ into *their* integral with c present.
	$f(x)=\dfrac{-2}{\left(\tfrac{1}{2}x-3\right)}-\dfrac{x}{4}+3$	A1	OE
		4	
(c)	$\left(\tfrac{1}{2}x-3\right)^{-2}-(-2)^{-2}=0$	M1	Substitute $k=-3$ and set to zero.
	leading to $\left(\tfrac{1}{2}x-3\right)^{2}=4$ $\left[\tfrac{1}{2}x-3=(\pm)2\right]$ leading to $x=10$	A1	
	$(10, -\tfrac{1}{2})$	A1	Or when $x=10$, $y=-1-2\tfrac{1}{2}+3=-\tfrac{1}{2}$
	$f''(10)\left[=-(5-3)^{-3}\rightarrow\right]<0\rightarrow$ MAXIMUM	A1	WWW
		4	

Question 171

(a)	$\dfrac{dy}{dx}=\dfrac{1}{2}+\dfrac{1}{3(x-2)^{\frac{4}{3}}}$	B1	OE. Allow unsimplified.
	Attempt at evaluating *their* $\dfrac{dy}{dx}$ at $x=3$ $\left[\dfrac{1}{2}+\dfrac{1}{3(3-2)^{\frac{4}{3}}}=\dfrac{5}{6}\right]$	*M1	Substituting $x=3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal $=\dfrac{-1}{their\dfrac{dy}{dx}}$ $\left[=-\dfrac{6}{5}\right]$	*DM1	Negative reciprocal of *their* evaluated $\dfrac{dy}{dx}$.
	Equation of normal $y-\dfrac{6}{5}=(their\text{ normal gradient})(x-3)$ $\left[y=-\dfrac{6}{5}x+4.8\Rightarrow 5y=-6x+24\right]$	DM1	Using *their* normal gradient and A in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y=0$,] $x=4$	A1	or (4, 0)
		5	
(b)	Area under curve $=\int\left(\dfrac{1}{2}x+\dfrac{7}{10}-\dfrac{1}{(x-2)^{\frac{1}{3}}}\right)[dx]$	M1	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\dfrac{1}{4}x^2+\dfrac{7}{10}x-\dfrac{3(x-2)^{\frac{2}{3}}}{2}$	A1	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\dfrac{9}{4}+2.1-\dfrac{3}{2}\right)-\left(\dfrac{6.25}{4}+1.75-\dfrac{3\times0.5^{\frac{2}{3}}}{2}\right)$	M1	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	A1	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	B1	OE
	[Total area =] 1.08	A1	Dependent on the first M1 and WWW.
		6	

Question 172

(a)	[f'(x) =] $2x - \frac{k}{x^2}$	B1	
	$f'(2) = 0$ $\left[2\times 2 - \frac{k}{2^2} = 0\right] \Rightarrow k = \ldots$	M1	Setting *their* 2-term f'(2) = 0, at least one term correct and attempting to solve as far as $k =$.
	$k = 16$	A1	
		3	
(b)	$f''(2) =$ e.g. $2 + \frac{2k}{2^3}$	M1	Evaluate a two term f″(2) with at least one term correct. Or other valid method.
	$\left[2 + \frac{2k}{2^3}\right] > 0 \Rightarrow$ minimum or $= 6 \Rightarrow$ minimum	A1 FT	WWW. FT on positive k value.
		2	
(c)	When $x = 2$, $f(x) = 14$	B1	SOI
	[Range is or y or $f(x)$] $\geqslant$ *their* f(2)	B1 FT	Not $x \geqslant$ *their* f(2)
		2	

Question 173

(a)	$\left[\frac{dV}{dr} =\right]$ $\frac{9}{2}\left(r - \frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1.5}{their \frac{dV}{dr}}$ $\left[= \frac{1.5}{\frac{9}{2}\left(5.5 - \frac{1}{2}\right)^2} = \frac{1.5}{112.5}\right]$	M1	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{dV}{dr}$ and using $r = 5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
(b)	$\frac{dV}{dr}$ or *their* $\frac{dV}{dr} = \frac{1.5}{0.1}$ or 15 OR $0.1 = \frac{1.5}{their \frac{dV}{dr}}$ $\left[= \frac{2\times 1.5}{9\left(r - \frac{1}{2}\right)^2}\right.$ OE$\left.\right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or *their* $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r - \frac{1}{2}\right)^2 = 15 \Rightarrow\right]$ $r = \frac{1}{2} + \sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3 + 5\sqrt{30}}{3}$. CAO.
		3	

Question 174

$y = -\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
	DB1	For $-\frac{8}{3}$
$5\frac{2}{3} = -\frac{\frac{8}{3}}{(3\times 2+2)} + c$	M1	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = -1, or dividing by their power. + c needed
$y = -\frac{8}{3(3x+2)} + 6$	A1	OE e.g. $y = -\frac{8}{3}(3x+2)^{-1} + 6$
	4	

Question 175

(a)	$\left\{\dfrac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	B2, 1, 0	Attempt to integrate
	$-\frac{2}{3}[0-1]$	M1	M1 for applying limits $1 \to \infty$ to an integrated expression (either correct power or dividing by their power).
	$\dfrac{2}{3}$	A1	
		4	
(b)	$[\pi]\int y^2 \mathrm{d}x = [\pi]\int (3x-2)^{-3}\,\mathrm{d}x = [\pi]\dfrac{(3x-2)^{-2}}{-2\times 3}$	*M1 A1	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\dfrac{1}{6}\right]\left[\dfrac{1}{16}-1\right]$	DM1	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\dfrac{5\pi}{32}$	A1	OE
		4	
(c)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{3}{2}\times 3(3x-2)^{-\frac{5}{2}}$	M1	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{9}{2}$	*M1	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y-1 = \dfrac{2}{9}(x-1)$ OR evaluates c	DM1	Forms equation of line or evaluates c using (1, 1) and gradient $\dfrac{-1}{their\dfrac{\mathrm{d}y}{\mathrm{d}x}}$.
	At A, $y = \dfrac{7}{9}$	A1	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

Question 176

(a)	$\mathrm{f}(x)=\frac{2}{3}x^3-7x+4x^{-1}$ $[+c]$	**B2, 1, 0**	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$	**M1**	Substitute f(1) = $-\frac{1}{3}$ into an integrated expression and evaluate *c*.
	$\mathrm{f}(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	**A1**	OE.
		4	
(b)	$2x^4-7x^2-4\,[=0]$	**M1**	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2-7y-4[=0]$.
	$(2x^2+1)(x^2-4)\,[=0]$	**M1**	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. *y*. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x=[\pm]2$	**A1**	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3-7(2)+\frac{4}{2}+2 \text{ leading to}\right]\left(2,-\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3-7(-2)+\frac{4}{-2}+2 \text{ leading to}\right]\left(-2,\frac{26}{3}\right)$	**B1 B1**	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
(c)	$\mathrm{f}''(x)=4x+8x^{-3}$	**B1**	OE
		1	
(d)	$\mathrm{f}''(2)=9>0$ MINIMUM at $x=$ *their* 2	**B1 FT**	FT on *their* $x=[\pm]2$ provided $\mathrm{f}''(x)$ is correct. Must have correct value of $\mathrm{f}''(x)$ if $x=2$.
	$\mathrm{f}''(-2)=-9<0$ MAXIMUM at $x=$ *their* -2	**B1 FT**	FT on *their* $x=[\pm]2$ provided $\mathrm{f}''(x)$ is correct. Must have correct value of $\mathrm{f}''(x)$ if $x=-2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $\mathrm{f}''(2)>0$ MIN and $\mathrm{f}''(-2)<0$ MAX
	Alternative method for question 9(d)		
	Evaluate $\mathrm{f}'(x)$ for *x*-values either side of 2 and –2	**M1**	FT on *their* $x=[\pm]2$
	MINIMUM at $x=$ *their* 2, MAXIMUM at $x=$ *their* 2	**A1 FT**	FT on *their* $x=[\pm]2$. Must have correct values of $\mathrm{f}'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $\mathrm{f}'(2)$ −/0/+ MIN and $\mathrm{f}'(-2)$ +/0/− MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	**B1 B1**	Need correct coordinates in **(b)** for this method.
		2	

Question 177

(a)	$\frac{dy}{dx} = \{-k(3x-k)^{-2}\}\ \{\times 3\}\ \{+3\}$	B2, 1, 0	
	$\frac{-3k}{(3x-k)^2}+3=0$ leading to $(3)(3x-k)^2=(3)k$ leading to $3x-k=[\pm]\sqrt{k}$	M1	Set $\frac{dy}{dx}=0$ and remove the denominator
	$x=\frac{k\pm\sqrt{k}}{3}$	A1	OE
		4	
(b)	$a=\frac{4\pm\sqrt{4}}{3}$ leading to $a=2$	B1	Substitute $x = a$ when $k = 4$. Allow $x = 2$.
	$f''(x)=f'\left[-12(3x-4)^{-2}+3\right] = 72(3x-4)^{-3}$	B1	Allow $18k(3x-k)^{-3}$
	> 0 (or 9) when $x = 2 \to$ minimum	B1 FT	FT on *their* $x = 2$, providing their $x \geqslant \frac{3}{2}$ and $f''(x)$ is correct
		3	
(c)	Substitute $k=-1$ leading to $g'(x)=\frac{3}{(3x+1)^2}+3$	M1	Condone one error.
	$g'(x)>0$ or $g'(x)$ always positive, hence g is an increasing function	A1	WWW. A0 if the conclusion depends on substitution of values into $g'(x)$.
	Alternative method for question 11(c)		
	$x=\frac{k\pm\sqrt{k}}{3}$ when $k=-1$ has no solutions, so g is increasing or decreasing	M1	Allow the statement 'no turning points' for increasing or decreasing
	Show g'(x) is positive for any value of x, hence g is an increasing function	A1	Or show $g(b) > g(a)$ for $b > a \to$ g, hence g is an increasing function
		2	

Question 178

(a)	$(-2)^2+y^2=8$ leading to $y=2$ leading to $A=(0,2)$	B1	
	Substitute $y = their\, 2$ into circle leading to $(x-2)^2+4=8$	M1	Expect $x = 4$.
	$B = (4, 2)$	A1	
		3	
(b)	Attempt to find $[\pi]\int\left(8-(x-2)^2\right)dx$	*M1	
	$[\pi]\left[8x-\frac{(x-2)^3}{3}\right]$ or $[\pi]\left[8x-\left(\frac{x^3}{3}-2x^2+4x\right)\right]$	A1	
	$[\pi]\left(32-\frac{16}{3}\right)$ or $[\pi]\left[32-\left(\frac{64}{3}-32+16\right)\right]$	DM1	Apply limits $0 \to$ *their* 4.
	Volume of cylinder = $\pi\times 2^2\times 4=16\pi$	B1 FT	OR from $\pi\int 2^2 dx$ with *their* limits from **(a)**. FT on *their* A and B
	$\left[\text{Volume of revolution} = 26\tfrac{2}{3}\pi-16\pi=\right]10\tfrac{2}{3}\pi$	A1	Accept 33.5
		5	

Question 179

$\left[f(x)=\right]\dfrac{2x^{\frac{2}{3}}}{\frac{2}{3}}-\dfrac{x^{\frac{4}{3}}}{\frac{4}{3}}$ $[+c]$	B1 B1	$\frac{2}{3}$ and $\frac{4}{3}$ may be seen as sums of 1 and a fraction.
$5 = 12 - 12 + c$	M1	Substituting (8,5) into an integral.
$\left[f(x)=\right]3x^{\frac{2}{3}}-\frac{3}{4}x^{\frac{4}{3}}+5$	A1	Fractions in the denominators scores A0.
	4	

Question 180

(a)	$\left\{\dfrac{(4x+2)^{-1}}{-1}\right\}\{\div 4\}$ or eg $\left\{\dfrac{1}{16}\right\}\left\{-(x+0.5)^{-1}\right\}$ or $\dfrac{-1}{(16x+8)}$	B1 B1	OE If more than one function of x present then B0 B0.
	$0-(-1/24)$	M1	Apply limits to an integral, ∞ must be used correctly.
	1/24	A1	Allow 0.0417 AWRT.
		4	
(b)	$\dfrac{dy}{dx}=\left\{-2(4x+2)^{-3}\right\}$ $\{\times 4\}$	B1 B1	Allow unsimplified forms.
	Recognise $\dfrac{dy}{dx}$ = -1	B1	SOI
	their $\dfrac{-8}{(4x+2)^3}$ = *their* -1	M1	Must be numerical. Must be some attempt to solve *their* equation and $\dfrac{dy}{dx}\neq 0$.
	(0, ¼)	A1 A1	Accept $x = 0$, $y = ¼$. $y = ¼$ must be from $x = 0$ not $x = -1$.
		6	

Question 181

(a)	$\left[\dfrac{dy}{dx}=\right]\frac{1}{2}x^{-1/2}-2x^{-3/2}$	B1 B1	Allow unsimplified versions.
	At $x=1$, $\dfrac{dy}{dx}=\dfrac{1}{2}-2=-\dfrac{3}{2}$	M1	Substitute $x = 1$ into a differentiated y.
	Equation of tangent is $y-5=-\dfrac{3}{2}(x-1)$	A1	WWW Or $y=-\dfrac{3}{2}x+\dfrac{13}{2}$.
		4	
(b)	$\dfrac{x^{3/2}}{3/2}+8x^{1/2}$	B1	OE Integrate to find area under curve, allow unsimplified versions.
	$\left[\left(\dfrac{128}{3}+32\right)-\left(\dfrac{2}{3}+8\right)\right]$	M1	Apply limits 1 → 16 to an integrated expression.
	Area under line = 15 × 5 = 75	B1	Or by $\int_1^{16} 5\,dx$.
	Required area = 75 – 66 = 9	A1	
		4	

Question 182

(a)	$\frac{dy}{dx} = \{3\} + \left\{-4 \times \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3\right\}\left[= 3 - 6(3x+1)^{-\frac{1}{2}}\right]$	B1 B1	Correct differentiation of $3x+1$ and no other terms and correct differentiation of $-4(3x+1)^{\frac{1}{2}}$. Accept unsimplified.
	$\left[\frac{d^2y}{dx^2} = \right] -\frac{1}{2} \times -6(3x+1)^{-\frac{3}{2}} \times 3\ [= 9(3x+1)^{-\frac{3}{2}}]$	B1	WWW. Accept unsimplified. Do not award if $\frac{dy}{dx}$ is incorrect.
		3	
(b)	$\frac{dy}{dx} = 0$ leading to $3 - 6(3x+1)^{-\frac{1}{2}} = 0$	M1	Setting *their* $\frac{dy}{dx} = 0$.
	$(3x+1)^{\frac{1}{2}} = 2 \Rightarrow 3x+1 = 4$ leading to $x = 1$	A1	CAO – do not ISW for a second answer.
	$y = -4$ [coordinates $(1, -4)$]	A1	Condone inclusion of second value from a second answer.
	$\frac{d^2y}{dx^2} = 9(3 \times 1 + 1)^{-\frac{3}{2}} = \frac{9}{8}$ or > 0 so minimum	A1	Some evidence of substitution needed but $\frac{d^2y}{dx^2}$. Do not award if $\frac{d^2y}{dx^2}$ is incorrect or wrongly evaluated. Accept correct consideration of gradients either side of $x = 1$.
		4	

Question 183

Line meets curve when: $2x + 2 = 5x^{\frac{1}{2}}$ leading to $2x - 5x^{\frac{1}{2}} + 2[= 0]$ **or** $4x^2 + 8x + 4 = 25x$ leading to $4x^2 - 17x + 4[= 0]$ **or** $x = \frac{y^2}{25}$ leading to $2y^2 - 25y + 50[= 0]$	M1	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are: $(2x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
$x = \frac{1}{4}, x = 4$	A1	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
Area $= \int 5x^{\frac{1}{2}} - (2x+2)\,dx = \int 5x^{\frac{1}{2}} - 2x - 2\ dx$	*M1	Intention to integrate and subtract areas. Condone missing brackets and/or subtraction wrong way around.
$= \left[\frac{10}{3}x^{\frac{3}{2}} - x^2 - 2x\right]_{\frac{1}{4}}^{4} = \left(\left(\frac{10}{3} \times 8 - 16 - 8\right) - \left(\frac{10}{3} \times \frac{1}{8} - \frac{1}{16} - \frac{1}{2}\right)\right)$	DM1	Integrating($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something else).
$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	A1	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.

Question 184

$[y =]\left\{\frac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2} \times 4}\right\} + \left\{-\frac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right\}\left[\Rightarrow \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}}\right][+c]$	B1 B1	Marks can be awarded for correct unsimplified expressions ISW.
$\frac{5}{2} = \frac{1}{2}(9)^{\frac{3}{2}} - 8 \times 4^{\frac{1}{2}} + c \quad [\Rightarrow c = 5]$	M1	Using $(4, \frac{5}{2})$ in an integrated expression (defined by at least one correct power) including $+ c$.
$y = \frac{3}{6}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + 5$.	A1	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution. Coefficients must not contain unresolved double fractions.
	4	

Question 185

(a)	$\frac{d^2y}{dx^2}=6(-1)^2-\frac{4}{(-1)^3}>0\therefore$ minimum **or** $\frac{d^2y}{dx^2}=10\ \therefore$ minimum	**B1**	Sub $x=-1$ into $\frac{d^2y}{dx^2}$, correct conclusion. WWW
		1	
(b)	$\frac{dy}{dx}=2x^3+\frac{2}{x^2}[+c]$	***M1**	Integrating $\frac{d^2y}{dx^2}$ (at least one term correct).
	$0=-2+2+c$ leading to $c=[0]$	**DM1**	Substituting $x=-1, \frac{dy}{dx}=0$ (need to see) to evaluate c. DM0 if simply state $c=0$ or omit $+c$.
	$y=\frac{1}{2}x^4-\frac{2}{x}+(\textit{their } c)x+k$	**A1 FT**	Integrated. FT *their* non-zero value of c if DM1 awarded.
	$\frac{9}{2}=\frac{1}{2}+2+k$ leading to $k=[2]$	**DM1**	Substituting $x=-1, y=\frac{9}{2}$ to evaluate k (dep on *M1).
	$y=\frac{1}{2}x^4-\frac{2}{x}+2$	**A1**	OE e.g. $2x^{-1}$ or $\frac{4}{2}$. A0 (wrong process) if c not evaluated but correct answer obtained.
		5	
(c)	$\frac{dy}{dx}=2x^3+\frac{2}{x^2}=0$	**M1**	*Their* $\frac{dy}{dx}=0$.
	Leading to $x^5=-1$	**M1**	Reaching equation of the form $x^5=a$.
	So only stationary point is when $x=-1$	**A1**	$x=-1$ and stating e.g. 'only' or 'no other solutions.
		3	
(d)	At $x=1$, $\frac{dy}{dx}=[4]$	***M1**	Substituting $x=1$ into *their* $\frac{dy}{dx}$.
	$\frac{dx}{dt}=\frac{dx}{dy}\times\frac{dy}{dt}=\frac{1}{4}\times 5$	**DM1**	OE Using chain rule correctly SOI.
	$\frac{5}{4}$	**A1**	OE e.g. 1.25.
		3	

Question 186

(a)	$(3x-2)^{\frac{1}{2}}=\frac{1}{2}x+1 \Rightarrow 3x-2=\left(\frac{1}{2}x+1\right)^2=\frac{1}{4}x^2+x+1$	M1	Equating curve and line, attempt to square; $\frac{1}{4}x^2+1$ M0
	$\Rightarrow \frac{1}{4}x^2-2x+3[=0]\left[\Rightarrow x^2-8x+12=0\right]\Rightarrow(x-6)(x-2)[=0]$	M1	Forming and solving a 3TQ by factorisation, formula or completing the square – see guidance.
	(2, 2) and (6, 4)	A1 A1	A1 for each point, or A1 A0 for two correct *x*-values. If M0 for solving, **SC B2** possible: B1 for each point or B1 B0 for two correct *x*-values.
		4	
(b)	Area $=\pm\int_{[2]}^{[6]}\left((3x-2)^{\frac{1}{2}}-\left(\frac{1}{2}x+1\right)\right)[dx]$	*M1	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}-\left(\frac{1}{4}x^2+x\right)\right]_2^6$	B1 B1	B1 for each bracket integrated correctly (in any form).
	$\pm\left(\left[\frac{2}{9}(16)^{\frac{3}{2}}-\left(\frac{1}{4}\times36+6\right)\right]-\left[\frac{2}{9}(4)^{\frac{3}{2}}-\left(\frac{1}{4}\times4+2\right)\right]\right)$	DM1	±(F(*their* 6) – F(*their* 2)) with *their* integral. Allow 1 sign error.
	$\frac{4}{9}$	A1	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.

Question 187

$\left[\frac{dv}{dx}\right]=(9-x)^2$	B1	Allow unsimplified forms. Allow any or no notation
Substitute $x = 4$ into *their* differentiated V,	*M1	Expect 25.
$\frac{dx}{dt}=\frac{1}{\textit{their}\text{ derivative}}\times3.6$ (accept $\frac{dt}{dx}=\frac{\textit{their}\text{ derivative}}{3.6}$)	M1	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
$=\frac{1}{\textit{their numerical}\text{ derivative}}\times3.6\times\frac{100}{60}$	DM1	Correct use of the conversion factors.
$=\frac{1}{25}\times3.6\times\frac{100}{60}=0.24$	A1	
	5	

Question 188

(a)	$\frac{-3}{(a+2)^4} = -\frac{16}{27}$ → e.g. $16(a+2)^4 = 81$	M1	Equate first derivative and $-\frac{16}{27}$ and move term in a (or x) into the numerator.
	$\to (a+2)^2 = \frac{9}{4} \to a+2 = [\pm]\frac{3}{2}$	M1	Solve for $(a+2)$ or $(x+2)$
	$a = -\frac{1}{2}$ or $-\frac{7}{2}$	A1 A1	Allow 'x ='
		4	
(b)	$[f(x)] = \frac{1}{(x+2)^3}$ $[+c]$	B1	Allow unsimplified form and '$y =$'
	$5 = 1 + c$	M1	Sub $x = -1$, $y = 5$ into an integral.
	$[f(x)] = \frac{1}{(x+2)^3} + 4$	A1	Allow '$y =$'
		3	

Question 189

(a)	$x^2 + (2x-1)^2 - 2[=0] \to 5x^2 - 4x - 1\ [=0]$	*M1 A1	Or $5y^2 + 2y - 7\ [=0]$.
	$(5x+1)(x-1)[=0]$ or $(5y+7)(y-1)[=0]$	DM1	May see factors or formula or completing square.
	$x = 1$, $y = 1$ or (1, 1) only	A1	May be implied on the diagram.
		4	
(b)	$(\pi)\int(2-x^2)\mathrm{d}x = (\pi)\left(2x - \frac{x^3}{3}\right)$	*M1 A1	Attempt integration of y^2, allow $\int(2-y^2)\mathrm{d}y$.
	$(\pi)\left(2\sqrt{2} - \frac{(\sqrt{2})^3}{3}\right) - \left(2 - \frac{1}{3}\right))$	DM1	Apply limits $1 \to \sqrt{2}$.
	$\frac{\pi}{3}(4\sqrt{2} - 5)$	A1	CAO, allow $\frac{\pi}{3}(2\sqrt{8} - 5)$, must be in given form.
		4	
(c)	Arc length = $\frac{1}{8}(2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter = $\sqrt{2}$ + *their* arc length	B1 FT	Must be exact, do not allow inverse trig functions.
		2	

Question 190

(a)	$[y=]\left\{\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+\left\{-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]\left[=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right]$	B1 B1	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5=2\times3^{\frac{3}{2}}-6\times3^{\frac{1}{2}}+c$	M1	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+5$	A1	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	
(b)	$3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}=0$	M1	Setting given differential to 0.
	$[x=]$ 1	A1	CAO WWW Condone extra solution of —1 only if it is rejected.
		2	
(c)	$x>1$ or $x>$ "*their* 8(b)"	B1FT	Allow $\geqslant$
		1	

Question 191

(a)	$\left[\frac{dy}{dx}=\right]\frac{9}{2}x-12\,[=0]$ or $[y=]\ \frac{9}{4}\left\{\left(x-\frac{8}{3}\right)^2+\frac{8}{9}\right\}$ or $\frac{9}{4}\left(x-\frac{8}{3}\right)^2+2$	B1	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
	$x=\frac{24}{9}$	B1	OE Condone 2.67 AWRT.
	$y=2$	B1	CAO Note: $x=\frac{-b}{2a}=\frac{8}{3}$ Bl; substitute $\frac{8}{3}$ for x in $y=$ B1; $y=2$ B1.
		3	
(b)	[Area =] $\int\left(18-\frac{3}{8}x^{\frac{5}{2}}-\left(\frac{9}{4}x^2-12x+18\right)\right)dx$	M1	Intention to integrate **and** subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}\right)$, $\left(\frac{9x^3}{4\times3}-\frac{12x^2}{2}[+18x]\right)$ Or combined $[18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}-\frac{9x^3}{4\times3}+\frac{12x^2}{2}[-18x]$	B1,B1	One mark for correct integration of each curve, allow unsimplified. $\left([18x]-\frac{3}{28}x^{\frac{7}{2}}\right)$ $\left(\frac{3}{4}x^3-6x^2[+18x]\right)$ or $[18x]-\frac{3}{28}x^{\frac{7}{2}}-\frac{3}{4}x^3+6x^2[-18x]$ **BUT** condone sign errors that are only due to missing brackets.
	$=\left(-\frac{3}{28}\times4^{\frac{7}{2}}-\frac{3}{4}\times4^3+6\times4^2\right)\ [-(0)]$	M1	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$=\frac{240}{7}$ or **34.3** AWRT	A1	**SC:** If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

(c)	$\left[\frac{dy}{dx}=\right]\frac{-5\times 3}{2\times 8}x^{\frac{3}{2}}\left[=-\frac{15}{16}x^{\frac{3}{2}}\right]$	B1	Allow unsimplified.
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=-\frac{15}{16}\times 8\times 2$	M1	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
	-15	A1	Accept decreasing [at/by] 15
		3	**Note:** If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2}-12)\times 2[=12]$

Question 192

(a)	$12\left(\frac{1}{2}\times 6-1\right)^{-4}\left[=12(2)^{-4}=\frac{3}{4}\right]$	M1	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
	$y-4=\frac{3}{4}(x-6)$ OR evaluates $c=-\frac{1}{2}$	A1	OE e.g. $y=\frac{3}{4}x-\frac{1}{2}$ or evaluates c in $y=\frac{3}{4}x+c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
		2	
(b)	$[y=]\left(\frac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3}\right)\div\frac{1}{2}\left[=-8\left(\frac{1}{2}x-1\right)^{-3}\right]$	B2, 1, 0	
	$4=\frac{12\times\left(\frac{1}{2}\times 6-1\right)^{-3}}{\frac{1}{2}\times -3}+c\left[\Rightarrow 4=-8\times 2^{-3}+c\right]\Rightarrow c=[5]$	M1	Must have $+c$. Substitute $y=4,\ x=6$ and solve for c in an integrated expression. May be unsimplified.
	$[y=]-8\left(\frac{1}{2}x-1\right)^{-3}+5$	A1	OE Must see '$y=$' or '$f(x)=$' in the working.
		4	

Question 193

$\frac{dy}{dx}=\frac{1}{2}ax^{-\frac{1}{2}}-2$	B2, 1, 0	
$0=\frac{1}{2}a(9)^{-\frac{1}{2}}-2\Rightarrow\frac{a}{6}-2=0\Rightarrow a=[12]$	M1	Substitute $x=9$ and $\frac{dy}{dx}=0$ into *their* derivative and solve a linear equation for a.
$[a=]12$	A1	
$\left[y=their\ a\times(9)^{\frac{1}{2}}-18=\right]18$	A1 FT	FT on *their* a.
	5	

Question 194

(a)	$f'(x) = -3(-1)(4)(4x-p)^{-2}\left[=\dfrac{12}{(4x-p)^2}\right]$	**B2, 1, 0**	
	>0 Hence increasing function	**B1FT**	Correct conclusion from *their* $f'(x)$.
		3	
(b)	$y = 2-\dfrac{3}{4x-p}$ $\Rightarrow$ $(y-2)(4x-p)=-3$ **or** $4xy-py=8x-2p-3$	**M1**	OE Form horizontal equation. Sign errors only, no missing terms. May go directly to $4y = p-\dfrac{3}{x-2}$ OE M1 M1
	$4xy-8x = py-2p-3 \Rightarrow 4x(y-2) = p(y-2)-3$ **or** $4x = -\dfrac{3}{x-2}+p$	**M1**	OE Factorise out $[4]x$ or $[4]y$.
	$x = \dfrac{p(y-2)-3}{4(y-2)}$ $\left[\Rightarrow x = \dfrac{p}{4}-\dfrac{3}{4y-8}\right]$ **or** $\dfrac{-\dfrac{3}{x-2}+p}{4}$	**M1**	OE Make x (or y) the subject.
	$\left[f^{-1}(x)=\right]\dfrac{p}{4}-\dfrac{3}{4x-8}$	**A1**	OE in correct form (must be in terms of x).
		4	
(c)	$[p=]8$	**B1**	
		1	

Question 195

(a)	$\pm\int\left(2x^{1/2}+1\right)-\left(\dfrac{1}{2}x^2-x+1\right)\,dx$ $\left[=\pm\int 2x^{1/2}-\dfrac{1}{2}x^2+x\,dx\right]$	***M1**	
	$\pm\left(\dfrac{4x^{3/2}}{3}+x-\left(\dfrac{x^3}{6}-\dfrac{x^2}{2}+x\right)\right)$ or $\pm\left(\dfrac{4x^{3/2}}{3}-\dfrac{x^3}{6}+\dfrac{x^2}{2}\right)$	**B2, 1, 0**	OE Coefficients may be unsimplified.
	$\pm\left(\dfrac{32}{3}-\dfrac{32}{3}+8\right)$ or $\pm\left(\dfrac{44}{3}-0-\dfrac{20}{3}+0\right)$	**DM1**	$\pm$(F(4) – F(0)) using *their* integral(s).
	$=8$	**A1**	Depends on all previous marks. If *M1 B2 DM0 **and** limits stated, **SC B1** for +8
		5	
(b)	Upper curve: $\dfrac{dy}{dx}=x^{-\frac{1}{2}}$. Lower curve: $\dfrac{dy}{dx}=x-1$	**M1 A1**	Attempt at differentiating one function. A1 if both correct.
	At $x = 4$: gradient of upper curve $=\dfrac{1}{2}$, gradient of lower curve $=3$	**M1**	Evaluate two gradients using $x=4$.
	$\alpha = \tan^{-1}3-\tan^{-1}\dfrac{1}{2}$ $[=71.57-26.57]$	**M1**	Use inverse tan to find angles then subtract. **OR** find equations of both tangents then Pythagoras using a point on each e.g. on axes. **OR** cosine rule using intercepts or proportion.
	$[\alpha=]45°$	**A1**	AWRT
		5	

Question 196

$\frac{dy}{dx} = \left\{\frac{1}{60}(3x+1)\times 2\right\} \times \{3\}$	B1 B1	May see $\frac{1}{60}(18x+6)$.
$\frac{1}{10}(3x+1) = 1$	M1	Equate *their* $\frac{dy}{dx}$ to 1.
$x = 3$	A1	
	4	

Question 197

(a)	$-\frac{3}{2} = \frac{1}{2} + k$ leading to $k = -2$	B1	**AG** Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
(b)	$[y] = 2x^{\frac{1}{2}} - 2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2} - 2x$.
	$-1 = 4 - 8 + c$	M1	Substitute $x = 4$, $y = -1$ (c present) Expect $c = 3$.
	$y = 2x^{\frac{1}{2}} - 2x + 3$ or $y = 2\sqrt{x} - 2x + 3$	A1	Allow if f(x) = or y = anywhere in the solution.
		4	
(c)	$x^{-1/2} - 2 = 0$	M1	Set *their* $\frac{dy}{dx}$ to zero.
	$x = \frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2 = \pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4}, 3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	
(d)	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	< 0 (or −4) hence Maximum	DB1	WWW Ignore extra solutions from $x = -\frac{1}{4}$.
		2	

Question 198

(a)	Gradient of $AB = \frac{2-(-1)}{5-2}$	M1	Expect 1, must be from $\Delta y / \Delta x$.
	Equation of AB is $y-2=1(x-5)$ or $y+1=1(x-2)$	A1	OE. Expect $y=x-3$.
		2	
(b)	$[\pi]\int x^2 dy = [\pi]\int (y^2+1)^2 dy = [\pi]\int (y^4+2y^2+1)dy$	M1	For curve: Attempt to square y^2+1 and attempt integration. Subtracting curve equation from line equation before squaring is M0. Integration before squaring M0.
	$[\pi]\left(\frac{y^5}{5}+\frac{2y^3}{3}+y\right)$	A2, 1, 0	
	$[\pi]\int (y+3)^2 dy = [\pi]\int (y^2+6y+9)dy$	M1	For line: Attempt to square *their* $y+3$ and attempt integration.
	$[\pi]\left(\frac{y^3}{3}+3y^2+9y\right)$ or $[\pi]\left(\frac{(y+3)^3}{3}\right)$	A2, 1, 0	Not available for incorrect line equations.
	$[\pi]\left\{\frac{8}{3}+12+18-\left(-\frac{1}{3}+3-9\right)\right\}$ or $[\pi]\left\{\frac{32}{5}+\frac{16}{3}+2-\left(-\frac{1}{5}-\frac{2}{3}-1\right)\right\}$	DM1	Apply limits $-1 \to 2$ to either integral providing they have been awarded M1. Expect $15\frac{3}{5}$ $[\pi]$ and/or $39[\pi]$. Some evidence of substitution of both -1 and 2 must be seen. Dependent on at least one of the first 2 M1 marks.
	Volume = $[\pi](39-15\frac{3}{5})$	DM1	Appropriate subtraction. Dependent on at least one of the first 2 M1 marks.
	$= 23\frac{2}{5}\pi$ or $\frac{117}{5}\pi$ or awrt 73.5[1327]	A1	
		9	

Question 199

(a)	$[y=]\ \{x\}\ \{+(x-1)^{-2}\ \}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0$, $y = 3$ leading to 3 = 0 + 1 + c	M1	Substitution into an integral, expect $c = 2$.
	$y = x+(x-1)^{-2}+2$ or $f(x) = x+(x-1)^{-2}+2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
(b)	[Gradient of tangent =] f′(0) = 3	B1	
	Equation of tangent is $y-3=$ *their* gradient at $x=0(x-0)$	M1*	Expect $y=3x+3$, normal gets M0.
	Intersection given by $3x+3=x+(x-1)^{-2}+2$	DM1	FT *their* equation from part **(a)**.
	$2x+1=\frac{1}{(x-1)^2}$ → $(2x+1)(x-1)^2-1=0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	
(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\frac{3}{2}$ $y = 7½$	B1	
		2	

Question 200

(a)	$\left[\frac{dy}{dx}=\right]\{9\}+\left\{-\frac{3}{2}(2x+1)^{1/2}\times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9-3(2x+1)^{1/2}=0$ leading to $2x+1=9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2-3^2(2x+1)=0$ M0A0. $2x+1=\sqrt{3}$ *or* $2x+1=\pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
(b)	When x = 1½, shows substitution or $\frac{dy}{dx}=3$	M1	Substituting x = 1½ into their $\frac{dy}{dx}$.
	Gradient of *AB* is $\frac{5½-3½}{1½-7½}\left[=\frac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\frac{1}{3}\text{x}3=-1$. [Hence *AB* is the normal]	A1	
	Alternative method for Question 10(b)		
	When x = 1½ $\frac{dy}{dx}=3$, [perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y=\frac{-x}{3}+6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or $48.325-3.725$	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ [= 27]	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

Question 201

$[y]=\frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	B1	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
$5=\frac{4}{-2}(4-3)^{-2}+c$ or $5=\frac{4}{-2(4-3)^2}+c$ leading to $c=$	M1	Correct use of $(4, 5)$ to find c in an integrated expression (defined by the correct power and no extra x's or terms).
$y=\frac{-2}{(x-3)^2}+7$ or $y=-2(x-3)^{-2}+7$	A1	OE $-\frac{4}{2}$ must be simplified to **–**2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
	3	

Question 202

$\left[\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=\right]\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}\left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	B1 B1	B1 for contents of each { } then ISW.
$=\left(their\frac{20}{3}\times 8-32\right)[-0]$	M1	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their ($\frac{160}{3}-32$).
[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	A1	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
	4	

Question 203

(a)	$\frac{dy}{dx}=\left\{k\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right\}\{\times 4\}\{-1\}$	B 2,1,0	OE e.g. $2k(4x+1)^{-\frac{1}{2}}-1$ B2 Three correct unsimplified { } and no others. B1 Two correct { } or three correct { } and an additional term e.g. +5. B0 More than one error.
		2	
(b)	$2k(4x+1)^{-\frac{1}{2}}-1=0$ leading to $(4x+1)^{\frac{1}{2}}=2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}}=1$	M1	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}}-1$ where a = 2 or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x=\frac{4k^2-1}{4}$	A1	CAO OE simplified expression ISW.
		2	
(c)	$2\times 10.5(4x+1)^{-\frac{1}{2}}-1=2$	M1	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7=(4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	A1	If M1 earned **SCB1** available for $x=\frac{33}{64}$ from $a=\frac{1}{2}$.
	$y=\left[10.5\sqrt{4x+1}-x+5=\right]66.5$ [leading to (12, 66.5)]	A1	
	$y-66.5=-\frac{1}{2}(x-12)$	A1	OE
		4	

Question 204

$\frac{dy}{dx}=\frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	*M1	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
$[y=]\frac{1}{2x^2}x-\frac{1}{2x^2}=-\frac{1}{2x}$ or $\frac{1}{x}=\frac{1}{2x^2}$ $[\Rightarrow 2x^2-x=0]$	DM1	Sub *their* $\frac{dy}{dx}$ into equation of line or set gradient = k to form equation in x.
$x=\frac{1}{2}$ only	A1	If DM0 then $x=\frac{1}{2}$, award A0XP then B0 B0.
$y=\left[2\times\frac{1}{2}-2\right]=-1$	B1	
$k=2$	B1	
	5	

Question 205

	Answer	Marks	Guidance
(a)	$\frac{dV}{dh}=\frac{4}{3}\times3(25+h)^2$ [= 4900 when h = 10]	B1	Correct expression for $\frac{dV}{dh}$.
	$\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}\Rightarrow$ *their* "$4(25+10)^2$"$\times\frac{dh}{dt}=500\Rightarrow\frac{dh}{dt}=\left[\frac{500}{4900}\right]$	M1	Use chain rule correctly to find a numerical expression for $\frac{dh}{dt}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{dh}{dt}=0.102\left[\text{cms}^{-1}\right]$	A1	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
(b)	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}\Rightarrow500=$ *their* "$4(25+h)^2$"$\times0.075$	*M1	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2=\frac{5000}{3}\right]\Rightarrow h=[15.8\,248\ldots]$	DM1	Solve quadratic to find h. Exact value of h is $\sqrt{\frac{5000}{3}}-25$ or $\frac{50\sqrt{6}}{3}-25$ $h+25=40.82\ldots$
	$V=69900\text{ cm}^3$	A1	AWRT ISW Look for 698(88.5).
		3	

Question 206

	Answer	Marks	Guidance
(a)	$[\pi]\int\frac{16}{(2x-1)^4}[dx]=[\pi]\int16(2x-1)^{-4}[dx]=[\pi]\left(-\frac{16}{3\times2\times(2x-1)^3}\right)$	*M1	Integrate y^2 (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}\boldsymbol{x}(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times2\times(2x-1)^3}\right)$	A1	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times8}+\frac{16}{6\times1}\right)\left[=[\pi]\frac{112}{48}=[\pi]\frac{7}{3}\right]$	DM1	Sub correct limits into *their* integral: $F\left(\frac{3}{2}\right)-F(1)$. Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33π or 7.33.
	Volume of cylinder $\left[=\pi\times1^2\times\frac{1}{2}\right]=\frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5}1[dx]=\frac{1}{2}\pi$	B1	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[=\frac{7}{3}\pi-\frac{1}{2}\pi\right]=\frac{11}{6}\pi$	A1	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, **SC B1** for $\frac{11}{6}\pi$.
		5	
(b)	$\left[\frac{dy}{dx}=\right]\left\{-8(2x-1)^{-3}\right\}\{\times2\}$	B2, 1, 0	OE B1 for each correct element in {}.
	At B gradient = -2	B1	
	Eqn of tangent $y-1=$ *their* "-2"$\left(x-\frac{3}{2}\right)$ OR Eqn of normal $y-1=$ *their* "$\frac{1}{2}$"$\left(x-\frac{3}{2}\right)$	M1	SOI Following differentiation OE e.g. $y=-2x+4$ or $y=\frac{1}{2}x+\frac{1}{4}$. (Must have $m_N=-\frac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 **or** normal crosses x-axis at $-\frac{1}{2}$	A1	SOI For at least one intercept correct or correct integration.
	Area = $\frac{5}{4}$	A1	From intercepts: $\frac{1}{2}\times\frac{5}{2}\times1=\frac{5}{4}$ or $1+\frac{1}{4}=\frac{5}{4}$, from lengths: $\frac{1}{2}\times\sqrt{5}\times\frac{\sqrt{5}}{2}=\frac{5}{4}$ or by integration.
		6	

Question 207

(a)	$6a^2 - 30a + 6a = 0$ [$\Rightarrow 6a(a-4) = 0$]	B1	Sub $x = a$ into $\frac{dy}{dx} = 0$. May see $a^2 - 5a + a = 0$.
	$a = 4$ only	B1	
		2	
(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
(c)	$[y =]\ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\textit{their } a)x[+c]$	B1 FT	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(\textit{their}\ "4")^3 - 15(\textit{their}\ "4")^2 + 6(\textit{their}\ "4")^2 + c$	M1	Sub $x = \textit{their}"4"$, $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ [$\Rightarrow c = 1$].
	$y = 2x^3 - 15x^2 + 24x + 1$	A1	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\textit{their}\ "4")[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	A1	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	

Question 208

(a)	$\frac{dy}{dx}=\left\{-2\times2\times(2x-1)^{-3}\times2\right\}+\{1\}$	B1B1	Expect $\frac{-8}{(2x-1)^3}+1$.
	Substitute $x=\frac{3}{2}$ leading to $\frac{dy}{dx}=\frac{-8}{8}+1=0$. Hence x-coordinate of R is $\frac{3}{2}$	DB1	AG. Or correct solution of $\frac{dy}{dx}=0$.
	When $x=\frac{3}{2}, y=\frac{2}{4}+\frac{3}{2}=2$	B1	Answer only is acceptable.
		4	
(b)	y-coordinate of $P=3$, y-coordinate of $Q=\frac{20}{9}$	B1	Both required.
	$\left\{\frac{2(2x-1)^{-1}}{-1\times2}\right\}+\left\{\frac{1}{2}x^2\right\}$	B1 B1	Area below curve.
	$\left[\left(-\frac{1}{3}+2\right)-\left(-1+\frac{1}{2}\right)\right]=\frac{5}{3}-\left(-\frac{1}{2}\right)$	M1	Apply limits 1→2 to an integral. Expect $\frac{13}{6}$.
	$\frac{1}{2}\left(3+\frac{20}{9}\right)=\frac{47}{18}$	M1	Area of trapezium, only allow errors in y-coordinate of Q.
	$\frac{47}{18}-\frac{13}{6}=\frac{4}{9}$	A1	Shaded region.
		6	
	Alternative method 1: Changes the award of the first M1		
	Their equation of line PQ:[$y=\frac{-7}{9}x+\frac{34}{9}$]. Integrating between 1 and 2.	M1	Must be some evidence of use of limits.
	Alternative method 2: Changes the award of the first M1, a B1 and the second M1		
	Combining line and curve: $\int\left(\frac{-16}{9}x+\frac{34}{9}-\frac{2}{(2x-1)^2}\right)dx$	M1	For area under the line if *their* $\frac{34}{9}$ is seen integrated correctly and limits used. Correct first and 3rd terms.
	$=\frac{-8}{9}x^2+\frac{34}{9}x+\frac{1}{(2x-1)}$	B1 B1	
	Use of limits on the whole integral	M1	

Question 209

(a)	$\frac{dy}{dx} = x^{-\frac{1}{2}} \rightarrow m = \frac{1}{2}$ at $x = 4$	**B1**	
	Equation of normal is $y - 3 = -2(x - 4)$	**M1**	Through (4, 3) with gradient $-\frac{1}{their\ m}$. (Dependent on differentiation used).
	$y = -2x + 11$	**A1**	Only acceptable answer.
		3	
(b)	$\frac{dy}{dt} = their\ \frac{1}{2} \times 3$	**M1**	Correct use of the chain rule with a numerical gradient.
	$\frac{3}{2}$	**A1**	
		2	
(c)	Required gradient $\left[= \frac{dy}{dx}\right] = -2$	**B1FT**	SOI. FT from *their* part **(a)** if a normal gradient has been found from $m_1 m_2 = -1$ and differentiation.
	$\frac{dx}{dt} = \frac{1}{their\ normal\ gradient} \times -5$	**M1**	Correct use of the chain rule . Allow method mark also for +5, must be numerical. *Their* normal gradient must come from $m_1 m_2 = -1$ and differentiation in part(a) unless 'restarted' here.
	$\frac{5}{2}$	**A1**	
		3	

Question 210

$cx^2 + 2x - 3 = 6x - c$ leading to $cx^2 - 4x + (c - 3)$ $[= 0]$	**B1**	3-term quadratic.
$16 - 4c(c - 3) = 0$	***M1**	Apply $b^2 - 4ac = 0$ ('= 0' may be implied in subsequent work). *Their* coefficients must be substituted correctly
$4c^2 - 12c - 16[= 0]$ leading to $[4](c - 4)(c + 1)$ $[= 0]$ leading to $c = 4$ and -1	**A1**	Dependent on factorisation oe.
When $c = 4$, $4x^2 - 4x + 1[= 0]$ $\left[(2x - 1)^2 = 0\right]$	**DM1**	OE. Substituting *their* $c = 4$ into *their* quadratic equation.
$x = \frac{1}{2}$, $y = -1$	**A1**	Both required.
When $c = -1$, $x^2 + 4x + 4[= 0]$ $\left[(x + 2)^2 = 0\right]$	**DM1**	OE. Substituting *their* $c = -1$ into *their* quadratic equation.
$x = -2$, $y = -11$	**A1**	Both required.
Alternative method for Question 6		
$\frac{dy}{dx} = 2cx + 2$	**B1**	
$2cx + 2 = 6$	**M1**	Equating *their* curve gradient and 6.
$c = \frac{2}{x}$	**A1**	SOI
$2x^2 + 3x - 2$ $[= 0]$	**DM1**	Substitute $c = \frac{2}{x}$ into $cx^2 + 2x - 3 = 6x - c$. Simplify to 3-term quadratic.

Question 211

$[y]=\left\{\dfrac{x^2}{2}\right\}\ \left\{\dfrac{-3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]$	**B1 B1**	Any unsimplified correct form, ISW for extra terms, allow lists.
$1 = 8 - 12 + c$	**M1**	Substitute (into an integrated expression) $x = 4$, $y = 1$. c must be present. Expect $c = 5$.
$y=\dfrac{1}{2}x^2-6x^{\frac{1}{2}}+5$, allow f($x$) =	**A1**	Condone $c = 5$ as the final line so long as '$y =$' present.
	4	

Question 212

(a)	$\dfrac{dy}{dx}=\left\{\dfrac{5}{3}(4x-3)^{\frac{2}{3}}\right\}\ \{\times 4\}\left\{-\dfrac{20}{3}\right\}$	**B2,1,0**	B2 Three correct unsimplified { } and no others. B1 Two correct { }or three correct { } and an additional term e.g. + c. B0 More than one error.
	$\left[\dfrac{20}{3}(4x-3)^{\frac{2}{3}}-\dfrac{20}{3}=0\right]$ leading to $(4x-3)^2=k,\ k>0$ leading to $4x-3=\pm m$	**M1**	Equating *their* $\dfrac{dy}{dx}$ to 0 and using a valid method to arrive at 2 answers.
	$[4x-3=\pm 1]\ \ [x=]\dfrac{1}{2},1$	**A1**	
	$\dfrac{d^2y}{dx^2}=\dfrac{40}{9}(4x-3)^{-\frac{1}{3}}\times 4$	**B1**	OE
	$\left[x=\dfrac{1}{2}\right]\ \dfrac{d^2y}{dx^2}=\left(\dfrac{160}{9}\right)(4x-3)^{-\frac{1}{3}}<0$ or $-\dfrac{160}{9}$ or -17.8 so max $[x=1]\ \dfrac{d^2y}{dx^2}=\left(\dfrac{160}{9}\right)(4x-3)^{-\frac{1}{3}}>0$ or $\dfrac{160}{9}$ or 17.8 so min	**B1**	If $\dfrac{d^2y}{dx^2}$ evaluated the answers for both must be correct OR Clear use of change in sign of $\dfrac{dy}{dx}$ correctly for both B1's. If B1M1A0B0B0 scored then **SCB1** can be awarded for: $\dfrac{dy}{dx}=\left\{\dfrac{5}{3}(4x-3)^{\frac{2}{3}}\right\}-\left\{\dfrac{20}{3}\right\}$ leading to $(4x-3)^2=64$ leading to $x=-\dfrac{5}{4},\dfrac{11}{4}$. $\dfrac{d^2y}{dx^2}=\dfrac{10}{9}(4x-3)^{-\frac{1}{3}}$, $x=-\dfrac{5}{4}$, $\dfrac{d^2y}{dx^2}<0$ so max, $x=\dfrac{11}{4}$, $\dfrac{d^2y}{dx^2}>0$ so min.
		6	
(b)	$x<\frac{1}{2},\ x>1$	**B1**	Allow $\leqslant$ and/or $\geqslant$. FT only from special case $x<-\frac{5}{4},\ x>\frac{11}{4}$ Condone: $1<x<\frac{1}{2}$.
		1	

Question 213

(a)	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}} \Rightarrow x-6x^{\frac{1}{2}}+5=0$	*M1	OE Equating the two expressions in x and then multiplying each term by $x^{\frac{1}{2}}$ or by their substitution for $x^{\frac{1}{2}}$. Coefficients need to be retained but condone +/– sign errors. Allow $x^{\frac{1}{2}}$ replaced by x.
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-5\right)$ [= 0] or [x=] $\frac{6\pm\sqrt{36-4\times1\times5}}{2}$	DM1	OE Solving their three-term quadratic.
	Alternative method for first 2 marks of Question 9(a)		
	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}}$ leading to $2x+10=12x^{\frac{1}{2}}$	*M1	Equating the two expressions in x and isolating their term in $x^{\frac{1}{2}}$.
	$(2x+10)^2=144x$ leading to $[4]\left(x^2-26x+25\right)[=0]$ leading to $[4](x-25)(x-1)$ [= 0] or [x=] $\frac{26\pm\sqrt{676-4\times1\times25}}{2}$	DM1	OE Squaring both sides, rearranging and solving a three-term quadratic.
	$x=1$ and 25, $y=15$ and $12\frac{3}{5}$	A1, A1	A1 for both x-values and A1 for both y values. If M1DM0 scored then **SCB1B1** is available for final answers.
		4	Answers without working score 0/4
(b)	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)-\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[dx]$ $\left[=-2x^{\frac{1}{2}}+12-10x^{-\frac{1}{2}}\right]$	M1	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+12x\left\{-\frac{10x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	B1 B1	B1 for either { }. B1 for completely correct integration of their expression following through +/– sign errors from the subtraction.
	$\left(-\frac{4}{3}(\textit{their}\ 25)^{\frac{3}{2}}+12(\textit{their}\ 25)-20(\textit{their}\ 25)^{\frac{1}{2}}\right)-$ $\left(-\frac{4}{3}(\textit{their}\ 1)^{\frac{3}{2}}+12(\textit{their}\ 1)-20(\textit{their}\ 1)^{\frac{1}{2}}\right)$	M1	OE Substitution of *their* positive limits from part **(a)** in *their* integrated expression, defined by at least one correct fractional power, and subtraction.

Question 214

(a)	[Gradient of normal =] $\frac{-1}{\text{Their}\ \frac{11}{2}}$ $\left[\frac{-1}{\frac{11}{2}}=-\frac{2}{11}\right]$	M1	Tangent gradient must come from x = 2 substituted into the given expression.
	$\frac{y-8}{x-2}=-\frac{2}{11}$ or $11y+2x=92$ or $y=-\frac{2x}{11}+\frac{92}{11}$	A1	OE
		2	
(b)	$[y=]\left\{\frac{1}{2}x^2\div2\right\}\left\{+\frac{72}{x^3}\div-3\right\}$ $[+c]$ $\left[\frac{x^2}{4}-\frac{24}{x^3}+c\right]$	B1, B1	One mark for each correct unsimplified { }.
	$8=\frac{1}{4}\times4-\frac{24}{8}+c$	M1	Substitution of x = 2, y = 8 into *their* integrated expression, defined by at least one correct power. Two terms and + c needed.
	$y=\left(\frac{1}{4}\text{ or }0.25\right)x^2-\frac{24}{x^3}+10$	A1	Both coefficients must be simplified but allow x^{-3}. Condone c = 10 as line as long as either y or f(x) = is seen elsewhere.
		4	

Question 215

(a)	$\frac{dy}{dx} = 3x^2 \ [+c]$	B1	
	$3 \times 2^2 + c = 0$	M1	Substitute $x = 2$ and $\frac{dy}{dx} = 0$ into an integral (c must be present).
	$\frac{dy}{dx} = 3x^2 - 12$	A1	
		3	
(b)	$y = x^3 - 12x \ [+k]$	B1 FT	FT on *their* non-zero c (dependent on c being found at some stage).
	$-10 = 2^3 - 12 \times 2 + k$	M1	Substitute $x = 2$, $y = -10$ (k present).
	$y = x^3 - 12x + 6$	A1	Must be $y =$ (unless $y = x^3 - 12x + k$ stated earlier).
		3	
(c)	$3x^2 - 12 = 0$ [leading to $x = -2$]	M1	Set *their* two term $\frac{dy}{dx} = 0$. Expect $x = -2$. Ignore $x = 2$ given in addition.
	$y = (-2)^3 - 12 \times (-2) + 6 = 22$ leading to $(-2, 22)$	A1	
	When $x = -2$, $\frac{d^2y}{dx^2} < 0$ (or -12) hence Maximum	A1	Can be from correct conclusion from $\frac{dy}{dx}$ **sign** diagram if $\frac{dy}{dx}$ calculated correctly. Do not allow concave downward for final A1. Can be awarded if the only error is incorrect or missing y-coordinate.
		3	
(d)	At $x = 0$, $\frac{dy}{dx} = -12$, $y = 6$	M1	Both required. FT on *their* $\frac{dy}{dx}$ and y.
	$y - 6 = -12x$	A1	OE
		2	

Question 216

(a)	$u = 2x - 3$ leading to $2u^4 = u^2 + 1$ leading to $2u^4 - u^2 - 1[= 0]$	B1	
	$(2u^2 + 1)(u^2 - 1)[= 0]$	M1	Factors or formula or completing square must be shown.
	$u = \pm 1$ leading to $2x - 3 = \pm 1$ leading to $x = 1$ or 2	A1	
	(1, 2), (2, 2)	A1	**Special case:** If B1 M0 scored then **SC B2** can be awarded for correct coordinates or **SC B1** for correct x values only.
			Special case $2(2x - 3)^4 = (2x - 3)^2 + 1$ $32x^4 - 192x^3 + 428x^2 - 420x + 152 = 0$ $x = 1, 2$ finding both from a correct quartic **SC B1** (1, 2), (2, 2) **SC DB1** **Special case:** Trial and improvement without quartic. Both x values correct B1, both coordinates correct B2.
		4	

(b)	$\left\{\frac{(2x-3)^3}{3\times 2}+x\right\}[-]\left\{\frac{2(2x-3)^5}{5\times 2}\right\}$	B1 B1	Integrate the 2 functions.
	$\left(\frac{1}{6}+2\right)-\left(-\frac{1}{6}+1\right)-\left\{\frac{1}{5}-\left(-\frac{1}{5}\right)\right\}$	M1	Apply *their* limits 1 → 2 (must be shown) to an integral. Some evidence of substitution. Minimum $(\frac{13}{6}-\frac{5}{6})-(\frac{1}{5}+\frac{1}{5})$ or equivalent. Allow 1 sign error for 1st M1.
	$\frac{4}{3}-\frac{2}{5}$	M1	Subtract (at some point) the 2 areas. Must subtract **areas** and not just integrals.
	$\frac{14}{15}$	A1	**Special case:** If M0 for substitution of limits can award **SC B1** for correct answer. Condone $-\frac{14}{15}$ if corrected.
			If subtraction is the wrong way round award B1 B1 M1 M1 A0. $\int y^2\,dx$ or $\int x\,dy$ scores 0 /5. $\pi\int y\,dx$ used. Award B1 B1 M1 M1 A0.

(b)	**Alternative method for Question 8(b)**		
	u = 2x – 3 $\int(u^2+1-2u^4)\,du$ $\left\{\frac{1}{2}\right\}\left(\left\{\frac{1}{3}u^3+u\right\}-\left\{\frac{2}{5}u^5\right\}\right)$	B2,1,0	
	$\frac{1}{2}\left(\left(\frac{1}{3}+1-\frac{2}{5}\right)-\left(\frac{-1}{3}-1+\frac{2}{5}\right)\right)$	M1	Applies limits –1 → 1.
		M1	Subtract (at some point) the 2 areas.
	$\frac{1}{2}\left(\frac{14}{15}+\frac{14}{15}\right)$ $\frac{14}{15}$	A1	
		5	

Question 217

$\frac{dV}{dx}=3x^2$	B1	SOI
$\frac{dV}{dt}\left[=\frac{dV}{dx}\times\frac{dx}{dt}\right]=3\times 20^2\times 0.01$	M1	Correct use of chain rule with x = 20 substituted into $\frac{dV}{dx}$.
12	A1	
	3	

Question 218

(a)	Differentiate to obtain $-\frac{4}{3}x^{-\frac{5}{3}}+x^{-\frac{4}{3}}$ **or** rewrite as a quadratic equation in $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$	B1	Expect quadratic $2\left(x^{-\frac{1}{3}}\right)^2-3x^{-\frac{1}{3}}+1$ OE Allow $2x^2-3x+1$.
	Equate first derivative to zero and reach a solution for $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$ with no error in use of indices **or** complete square to find minimum point $2\left(a-\frac{3}{4}\right)^2-\frac{1}{8}$ where $a=x^{-\frac{1}{3}}$	M1	Substitution SOI if dealt with correctly later
	Obtain $x=\frac{64}{27}$	A1	Or exact equivalent. **SC B1** if no working shown. Ignore extra solution $x=0$.
	$y=-\frac{1}{8}$ seen	B1	Or exact equivalent. Allow -0.125.
		4	
(b)	Recognise equation as quadratic in $x^{-\frac{1}{3}}$ or equivalent and attempt solution	M1	$2a^2-3a+1[=0]$ where $a=x^{-\frac{1}{3}}$.
	Obtain $x^{-\frac{1}{3}}=1$ and $x^{-\frac{1}{3}}=\frac{1}{2}$	A1	OE **SC B1** if no M mark awarded.
	Obtain 1 and 8	A1	**SC B1** if no M mark awarded.
	Integrate to obtain form $k_1x^{\frac{1}{3}}+k_2x^{\frac{2}{3}}+x$ or 2 out of 3 correct terms	*M1	Expect $6x^{\frac{1}{3}}-\frac{9}{2}x^{\frac{2}{3}}+x$.
	Obtain correct $6x^{\frac{1}{3}}-\frac{9}{2}x^{\frac{2}{3}}+x$	A1	No other terms from a second integral.
	Apply *their* limits correctly	DM1	*Their* limits must be from *their* working.
	[Obtain –0.5 and conclude area is] 0.5	A1	
		7	

Question 219

Differentiate to obtain form $kx(2x^2-5)^{-2}$	M1	
Obtain correct $-12x(2x^2-5)^{-2}$	A1	OE
Substitute (2, 1) to obtain gradient $-\frac{24}{9}$	A1	OE e.g. $-\frac{8}{3}$. Allow -2.67.
Apply negative reciprocal to *their* numerical gradient to obtain gradient of normal	*M1	Must have been some attempt at differentiation. Expect $\frac{3}{8}$
Attempt equation of normal using *their* gradient of the normal and (2, 1)	DM1	Expect $y-1=\frac{3}{8}(x-2)$.
Obtain $3x-8y+2=0$ (allow multiples)	A1	Or equivalent of requested form e.g. $8y-3x-2=0$.
	6	

Question 220

Integrate to obtain form $k(4x+5)^{\frac{3}{2}}$	*M1	
Obtain correct $\frac{1}{2}(4x+5)^{\frac{3}{2}}$	A1	Or (unsimplified) equivalent. Condone missing … $+c$ so far.
Substitute $x=1,\ y=9$ to form an equation in c	DM1	
Obtain or imply $[y=]\ \frac{1}{2}(4x+5)^{\frac{3}{2}}-\frac{9}{2}$	A1	May be implied by $[a=]\frac{1}{2}(4(1)+5)^{\frac{3}{2}}-\frac{9}{2}$.
Substitute $x=5$ to obtain $a=58$	A1	
	5	

Question 221

Integrate to obtain $-2x^{-1}$	B1	OE
Substitute limits correctly with clear indication seen that upper limit gives 0	M1	For integral of form $-k\,x^{-n}$, where $k>0$, $n>0$.
Obtain $\frac{2}{3}$	A1	WWW Accept 0.667.
	3	

Question 222

(a)	Differentiate to obtain form $kx^2(2x^3+10)^{-\frac{1}{2}}$	M1	OE
	$3x^2(2x^3+10)^{-\frac{1}{2}}$	A1	Or unsimplified equivalent.
	Substitute $x=3$ in first derivative and evaluate to find gradient	*M1	Expect $\frac{27}{8}$. Allow if first derivative of forms $k(2x^3+10)^{\frac{-1}{2}}$, $kx(2x^3+10)^{\frac{-1}{2}}$ or $kx^2(2x^3+10)^{\frac{-1}{2}}$.
	Attempt equation of tangent at $(3,8)$ with numerical gradient	DM1	Use of gradient of the normal is DM0.
	$[\pm](27x-8y-17)=0$ or integer multiples	A1	
		5	
(b)	State or imply volume is $\pi\int(2x^3+10)\,dx$	B1	Implied if π appears only at the end. Do not allow an unsimplified: $\pi\int\left(\left(2x^3+10\right)^{1/2}\right)^2$.
	Integrate to obtain $k_1x^4+k_2x$ and evaluate using limits 1 and 3	M1	Where $k_1k_2\neq 0$.
	60π	A1	OE Allow from a correct integral and sight of limits. Allow numerical answers in the range 188-189.
		3	

Question 223

(a)	Integrate to obtain form $k(5x-3)^{-1}$	***M1**	OE
	$4(5x-3)^{-1}$	**A1**	Or unsimplified equivalent. Condone absence of $\ldots + c$ so far.
	Substitute $x = \frac{4}{5}$ and $y = -3$ to attempt value of c	**DM1**	DM0 for substituting $\left(-3, \dfrac{4}{5}\right)$.
	$y = 4(5x-3)^{-1} - 7$ allow f(x) *or* $f = 4(5x-3)^{-1} - 7$	**A1**	OE Condone $c = -7$ as the final answer providing $y = \text{or } f(x) = \dfrac{4}{(5x-3)} + c$ OE is seen earlier. Attempts to write equation in $y = mx + c$ form scores A0. Do not ISW. Gains max 3/4.
		4	
(b)	Carry out stretch by replacing x by $2x$ in *their* equation	**M1**	Award if given as the second transformation. Do not ignore sign errors.
	Carry out translation by replacing x by $x-2$ and y by $y-10$	**M1**	OE Award if given as the first transformation. Do not ignore sign errors.
	$y = \dfrac{4}{10x-23} + 3$	**A1**	Or similarly simplified equivalent, WWW.
		3	

Question 224

(a)	Differentiate to obtain $4x + \frac{1}{2}x^{-2}$	**B1**	OE Condone '+c'.
	Equate first derivative to zero and solve $4x + \dfrac{K}{x^2} = 0$ as far as $x^3 = k$, K and k non-zero	**M1**	Not given if '+c' used.
	$x = -\frac{1}{2}$ and $y = \frac{9}{2}$	**A1**	OE **B1 SC** if no visible solution of the cubic.
		3	
(b)	Differentiate *their* first derivative, substitute *their* x value. Substitution may be implied by a correct inequality or correct value,	**M1**	Must differentiate one term correctly. Expect $4 - x^{-3} = 12$ at $x = \dfrac{-1}{2}$ Alternative: substitute values of x into $\dfrac{dy}{dx}$. One value $x < -\dfrac{1}{2}$ and one value $-\dfrac{1}{2} < x < 0$.
	conclude minimum	**A1**	Following correct work only
		2	
(c)	State increasing …	**B1**	
	… with clear reference to first derivative always being positive [for $x > 0$]	**B1**	Dependent on first derivative being correct. It is not sufficient to substitute values of x.
		2	

Question 225

(a)	$[x=]-2$	B1	
	$\frac{dy}{dx}=k(5-2x)^{\frac{1}{2}}$ $\left[=-2\times\frac{3}{2}(5-2x)^{\frac{1}{2}}\right]$	M1*	OE Differentiating to get $k(5-2x)^{\frac{1}{2}}$ only.
	$\left[\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}\text{ leading to}\right]$ $-9=\pm 5\times\frac{dt}{dx}$	DM1	Correct statement linking *their* numerical expression for $\frac{dy}{dx}$ with $\frac{dt}{dx}$ and ± 5.
	$\frac{5}{9}$ or 0.556 =	A1	AWRT
		4	
(b)	$k(5-2x)^{\frac{1}{2}}=-3$	M1	Equating *their* $\frac{dy}{dx}$ of the form $k(5-2x)^{\frac{1}{2}}$ to -3.
	[B is] $(2,6)$	A1	
	Gradient $AB=m_1=\frac{32-6}{-2-2}$,gradient of perpendicular $=-\frac{1}{m_1}=\frac{4}{26}$	M1*	For A, y must be 32. Clear use of $\frac{\text{difference in y co-ordinates}}{\text{difference in x co-ordinates}}$ for points A and B, condone inconsistent order, and using $m_1m_2=-1$. If incorrect values or another complete method used, then working must be clear.
	Mid point is $\left(\frac{2-2}{2},\frac{6+32}{2}\right)=(0,19)$	M1*	Finding the midpoint of AB using A and B. If incorrect values used then all working must be clear. For A, y must be 32.
	$y-19=\frac{2}{13}(x-0)$	DM1	Finding the equation of the perpendicular bisector using *their* midpoint and *their* perpendicular gradient.
	$2x-13y+247=0$ or $\pm$ integer multiples of this.	A1	
		6	

Question 226

	Answer	Mark	Partial Marks
(a)	$6(2x-3)^2 - 6x < 0$ or $= 0$	**B1***	Condone $\leqslant 0$. If $6(2x-3)^2$ only or $6(2x-3)^2 - 6$ is used, do not treat as a MR.
	$24x^2 - 78x + 54$ or $4x^2 - 13x + 9$ or $(x-1)(4x-9)$ OR $6(2x-3)^2 < 6x$ leading to $(2x-3) < \sqrt{x}$ leading to $2x - \sqrt{x} - 3$	**M1**	Expanding brackets and collecting terms to arrive at a three term quadratic, only condone sign errors.
	$[x=]1, \frac{9}{4}$	**B1**	
	$1 < x < \frac{9}{4}$ or $x > 1$ and $x < \frac{9}{4}$ or $\left(1, \frac{9}{4}\right)$	**DB1FT**	OE Condone consistent use of $\leqslant$ and $\geqslant$ or [ ]. Do not allow $x > 1$ or $x < \frac{9}{4}$ nor $x > 1, x < \frac{9}{4}$. FT on *their* values coming from a correct initial statement.
		4	
(b)	$[f(x) =]\left\{\frac{6}{3\times 2}(2x-3)^3\right\}\left\{-\frac{6}{2}x^2\right\}$ $[+C]$	**B1 B1**	B1 for each {Correct integral}.
	$-1 = (-1)^3 - 3\times 1^2 + C$	**M1**	$f(x) = -1$ equated to *their* integrated expression, defined by two terms with at least one correct power + C, with $x = 1$.
	$[f(x) =](2x-3)^3 - 3x^2 + 3$	**A1**	CAO Only condone $C = 3$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y = mx + c$.
	Alternative method for Question 9(b)		
	$[f'(x) = 24x^2 - 78x + 54$ leading to$]$ $[f(x) =]8x^3 - 39x^2 + 54x[+C]$	**(B2,1,0)**	B2 completely correct, B1 any two correct terms.
	$-1 = 8 - 39 + 54 + C$	**(M1)**	$f(x) = -1$ equated to *their* integrated expression, defined by three terms with at least one correct power + C, with $x = 1$.
	$[f(x) =]$ $8x^3 - 39x^2 + 54x - 24$	**(A1)**	Only condone $C = -24$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y = mx + c$.
		4	

Question 227

	Answer	Mark	Guidance
(a)	$\frac{dy}{dx} = 2 - \frac{1}{2} \times 8x^{-\frac{1}{2}}$	B1	
	$2 - 4x^{-\frac{1}{2}} = 0$	M1	Equating *their* two term $\frac{dy}{dx}$, with at least one term correct, to 0.
	[A is] $(4, -8)$ or $x = 4, y = -8$	A1	
	[B is] $(16, 0)$ or $x = 16, y = 0$	B1	
		4	Note: Correct answers without use of $\frac{dy}{dx}$ can be awarded 4/4.
(b)	$[\pm]\ \frac{2x^2}{2} - \frac{8}{\frac{3}{2}}x^{\frac{3}{2}}\ [+C]$	B1	Seen correct in unsimplified form or better.
	$[\pm]\frac{x^2 - 32x}{3}$ or $\frac{(2x-32)^2}{12}\ [+C]$	B1	Seen correct in unsimplified form or better.
	Attempt to integrate, defined by at least one correct power in each expression, and then subtract.	M1	Multiplying by 3 before integration scores M0.
	$\left\{\left(16^2 - \frac{8}{\frac{3}{2}}.16^{\frac{3}{2}}\right) - \left(4^2 - \frac{8}{\frac{3}{2}}.4^{\frac{3}{2}}\right)\right\}\ [-]\ \left\{\left(\frac{16^2 - 32\times16}{3}\right) - \left(\frac{4^2 - 32\times4}{3}\right)\right\}$	M1	Use of *their* x values, > 0, from **(a)** as limits in *their* integrated expressions. Allow, for correct limits, sight of $\pm\left(\left\{\left(-\frac{256}{3}\right) - \left(-\frac{80}{3}\right)\right\}[-]\left\{\left(-\frac{256}{3}\right) - \left(-\frac{112}{3}\right)\right\}\right)$. If incorrect limits are used, then clear substitution must be seen.
(b)	**Alternative Method 1 for first 4 marks of Question 6(b)**		
	$\left[[\pm]\int 2x - 8x^{\frac{1}{2}}\,dx =\right]\ x^2 - \frac{8}{\frac{3}{2}}x^{\frac{3}{2}}\ [+C]$	(B1)	Seen correct in unsimplified form or better.
	[Area of triangle =] 48	(B1)	
	Attempt to integrate, defined by at least one correct power, and then subtract *their* triangle area.	(M1)	
	$\left\{\left(16^2 - \frac{8}{\frac{3}{2}}.16^{\frac{3}{2}}\right) - \left(4^2 - \frac{8}{\frac{3}{2}}.4^{\frac{3}{2}}\right)\right\}$	(M1)	Use of *their* x values, > 0, from **(a)** as limits in *their* integrated expression. Allow sight of $\pm\left(\left\{\left(-\frac{256}{3}\right) - \left(-\frac{80}{3}\right)\right\}\right)$. If incorrect limits are used, then clear substitution must be seen.

(b)	**Alternative Method 2 for first 4 marks of Question 6(b)**		
	Subtract and then integrate, defined by at least two correct powers. Condone functions being the wrong way round.	(M1)	If terms in x have not been combined use the first scheme.
	$[\pm]\left(\frac{4}{3\times 2}x^2-\frac{8}{\frac{3}{2}}x^{\frac{3}{2}}+\frac{32x}{3}\right)$	(B2,1,0)	B2 for 3 correct terms, B1 for any 2 correct terms.
	$[\pm]\left(\left(\frac{4}{3\times 2}\times 16^2-\frac{8}{\frac{3}{2}}\times 16^{\frac{3}{2}}+\frac{32\times 16}{3}\right)-\left(\frac{4}{3\times 2}\times 4^2-\frac{8}{\frac{3}{2}}\times 4^{\frac{3}{2}}+\frac{32\times 4}{3}\right)\right)$	(M1)	Use of *their* x values, >0, from **(a)** as limits in *their* integrated expression. Allow sight of $\pm\left(0-\frac{32}{3}\right)$. If incorrect limits are used, then clear substitution must be seen.
	$\frac{32}{3}$, $10\frac{2}{3}$ or 10.7	(B1)	AWRT Allow $-\frac{32}{3}$ or $-\frac{32}{3}$ changed to $+\frac{32}{3}$ for this mark.
		(5)	Condone the inclusion of π for the first 4 marks but use of $\int y^2$ scores a maximum of B1 for the triangle.

Question 228

(a)	$\frac{dy}{dx}=-\frac{12}{x^4}+\frac{3}{x^2}$	B1	
	$\frac{dy}{dx}=-\frac{12}{x^4}+\frac{3}{x^2}=0$ leading to $3x^4-12x^2=0$ or $-12+3x^2=0$	M1	Set = 0 or uses $<$, $\leqslant$ and simplifies. Must be from $\frac{dy}{dx}=\frac{A}{x^4}+\frac{B}{x^2}$.
	$3x^2\left(x^2-4\right)=0$ leading to $x=\pm 2$ only	A1	**SC B1** for $x=\pm 2$ if M0 scored.
	$-2<x<0$ and $0<x<2$ or (-2, 0) and (0, 2) or $-2<x<2$ and $x\neq 0$	B1FT	Allow and/or.
		B1FT	Allow $-2\leqslant x<0$ and / or $0<x\leqslant 2$ but only B1B0 if 0 included in either or both. Allow [–2, 0) and (0, 2]. Allow B1B0 for $-2<x<2$ or (–2, 2). Must be from $\frac{dy}{dx}=\frac{A}{x^4}+\frac{B}{x^2}$.
		5	B marks only available if $\frac{dy}{dx}=\frac{A}{x^4}+\frac{B}{x^2}$.
(b)	[At $x=1$] $y=3$ and m tan $=-9$	*M1	Using *their* $\frac{dy}{dx}$.
	$m\,\text{norm}=-\frac{1}{-9}=\frac{1}{9}$	DM1	
	Equation of normal is $y-3=\frac{1}{9}(x-1)$ $\left[\text{leading to } y=\frac{1}{9}x+\frac{26}{9}\right]$	A1	
	At $x=-1, y=1, m=-9$	M1	
	Equation of tangent is $y-1=-9(x+1)$ [leading to $y=-9x-8$]	A1	
	Meet when $\frac{1}{9}x+\frac{26}{9}=-9x-8$ $\left[\text{leading to } x=-1.19512\ldots,\frac{-49}{41}\right]$	M1	Equates *their* tangent and *their* normal.
	Area $=\frac{1}{2}\times\textit{their}\,1.19512\ldots\times\textit{their}\left(\frac{26}{9}+8\right)$	M1	If $\int y_2-y_1$ is used integration must be correct and substitution shown.
	6.51	A1	AWRT Accept fraction wrt 6.51
		8	

Question 229

Volume of cylinder = $\pi \times 1^2 \times \frac{7}{5} = \frac{7}{5}\pi$	**B1**	May be done using $\int_1^{2.4} 1$. This would be the only mark available if candidate integrates *y*.
Volume under curve $= [\pi]\int \frac{1}{(5x-4)^{\frac{2}{3}}}\,dx$	**M1**	No further marks available if $\int y$.
$= [\pi]\left\{\frac{3}{5}\right\}\left\{(5x-4)^{\frac{1}{3}}\right\}$	**B1 B1**	Calculator used for integration scores no further marks.
$= [\pi]\frac{3}{5}\left(8^{\frac{1}{3}} - 1\right)\ \left[= \frac{3}{5}\pi\right]$	**M1**	Uses limits 1, 2.4 in an integral of y^2.
Volume $= \frac{7}{5}\pi - \frac{3}{5}\pi = \frac{4}{5}\pi$	**A1**	**SC B1** if the only error is not showing substitution.
	6	

Question 230

(a)	$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - 8x$	**B1**	
	$\frac{d^2y}{dx^2} = -\frac{1}{4}kx^{-\frac{3}{2}} - 8$	**B1**	
		2	
(b)	$x^{-\frac{1}{2}} - 8x = 0 \Rightarrow 1 - 8x^{\frac{3}{2}} = 0$ or $x^{-1} = 64x^2$ $\left[\Rightarrow x^3 = \frac{1}{64}\text{ or } 8x^{\frac{3}{2}} = 1\right]$ Setting their $\frac{dy}{dx}$ to zero and solving, providing their only error(s) are incorrect coefficients	**M1**	OE Award if working leads to $x = \frac{1}{4}$ WWW. Squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$ gets M0.
	$x = \frac{1}{4}$ only	**A1**	If $x = 0$ included, A0 and max of 3/4. **SC B1** only for $x = \frac{1}{4}$ only from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ directly to $x^{-1} - 64x^2 = 0$ (**SC B1** replacing the M1A1).
	$y = \frac{11}{4}$	**A1**	**SC B1** for $y = \frac{11}{4}$ from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$.
	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}} - 8$ which is negative, so maximum	**B1 FT**	WWW FT *their* *x*-value and *their* $\frac{d^2y}{dx^2}$. No FT if $x = 0$ is the only solution.
		4	

(c)	When $x=1$, attempting to find $y=k-2$ and gradient $=\frac{1}{2}k-8$	M1*	OE **SC B1** if both correct gradients only, or both correct y-coordinates only.
	Equation of tangent is $y-k+2=\left(\frac{1}{2}k-8\right)(x-1)$	A1	OE, e.g. $y=\left(\frac{k}{2}-8\right)x+\frac{k}{2}+6$ or $y=\frac{k}{2}x-8x+\frac{k}{2}+6$.
	When $x=\frac{1}{4}$, attempting to find $y=\frac{1}{2}k+1.75$ and gradient $=k-2$	M1*	OE
	Equation of tangent is $y-\frac{1}{2}k-1.75=(k-2)(x-0.25)$	A1	OE, e.g. $y=(k-2)x+\frac{k}{4}+\frac{9}{4}$ or $y=kx-2x+\frac{k}{4}+\frac{9}{4}$.
	Meet at $\left(\frac{1}{2}k-8\right)(0.6-1)+k-2=(k-2)(0.6-0.25)+\frac{1}{2}k+1.75$ Equate two tangent equations and substitute $x=0.6$	DM1	OE, e.g. $\left(\frac{k}{2}-8\right)0.6+\frac{k}{2}+6=(k-2)0.6+\frac{k}{4}+\frac{9}{4}$. M0 if constants in both equations are the same.
	$\Rightarrow [-0.2k+k+3.2-2=0.35k-0.7+0.5k+1.75]$ $\Rightarrow 0.05k=0.15$ $k=3$	A1	
		6	

Question 231

(a)	$y=x^3-3x+3$ and $y=2x^3-4x^2+3 \Rightarrow x^3-4x^2+3x[=0]$	M1	Reducing to 3-term cubic or quadratic if x cancelled.
	$[x](x-1)(x-3)[=0]$	DM1	Factorising the cubic or quadratic.
	$x=0$, 1 and 3 {$x=0$ may be seen in the working}	A1	**SC B1** for $x=1, 3$ only, with no M marks awarded.
		3	
(b)	Attempt at integration of both functions. Can be before or after subtraction of the functions or integrals	M1	Expect integration of $\int\left(\left(x^3-3x+3\right)-\left(2x^3-4x^2+3\right)\right)dx$ or $\int\left(-x^3+4x^2-3x\right)dx$. At this stage, subtraction can be done either way.
	$=\pm\left(-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right)$ or $\pm\left\{\left(\frac{x^4}{4}-\frac{3}{2}x^2+3x\right)-\left(\frac{2}{4}x^4-\frac{4}{3}x^3+3x\right)\right\}$	A1	OE $\pm$ covers A1 being awarded to those who subtract the 'other' way.
	$=\left[\left(-\frac{81}{4}+\frac{108}{3}-\frac{27}{2}\right)-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right]$, or $\left(\frac{81}{4}-\frac{27}{2}+9\right)-\left(\frac{1}{4}-\frac{3}{2}+3\right)-\left\{\left(\frac{81}{2}-\frac{108}{3}+9\right)-\left(\frac{1}{2}-\frac{4}{3}+3\right)\right\}$	DM1	OE Minimum required is $\left(\frac{63}{4}-\frac{7}{4}\right)-\left(\frac{27}{2}-\frac{13}{6}\right)$, i.e. four fractions. Correctly apply limits *their* 1 and 3. Do not allow if $x=0$ used. Need at least one correct substitution in every bracket. If two integrals, need to see substitution into both. Allow one sign error only in each expression, if brackets are not shown.
	$=\frac{8}{3}$	A1	Accept if this comes from use of limits $f(1)-f(3)$ or $\int\left(x^3-4x^2+3x\right)dx$, if $\left\lvert\frac{-8}{3}\right\rvert$ used. Only dependent on the first method mark. Accept AWRT 2.67.
		4	

Question 232

(a)	-18	**B1**	SOI
	$\frac{1}{18}$	**M1**	Use of $m_1m_2=-1$ from $f'(x)$ with $x=1$.
	$\frac{y\,[-0]}{x-1}=\frac{1}{18}$	**A1**	OE ISW
		3	
(b)	$[f(x)=]\left\{8(2x-3)^{\frac{4}{3}}.\frac{1}{2}.\frac{1}{\frac{4}{3}}\right\}\left\{-10x^{\frac{5}{3}}.\frac{1}{\frac{5}{3}}\right\}[+c]$ $\left[3(2x-3)^{\frac{4}{3}}-6x^{\frac{5}{3}}+c\right]$	**B1B1**	B1 for each unsimplified {}. Can be implied by equivalent simplified or partly simplified versions.
	$0=3(2(1)-3)^{\frac{4}{3}}-6(1)^{\frac{5}{3}}+c \quad [0=3-6+c]$	**M1**	Use of $x=1$ and $y=0$ in *their* integrated $f'(x)$, defined as an expression with at least one correct power, which must contain $+c$.
	$[f(x)\text{ or }y=]\ 3(2x-3)^{\frac{4}{3}}-6x^{\frac{5}{3}}+3$	**A1**	Only condone $c=3$ as their final answer if all coefficients have previously been simplified in a correct statement.
		4	
(c)	$b^2-4ac=128^2-4\times125\times192$ and stating "< 0" OR use of the quadratic formula and stating "No solutions" OR completing the square for the given quadratic and stating positive or > 0. OR sketch of the given quadratic and stating positive.	**M1***	$b^2-4ac=-79616$ can be accepted in place of working.
	No turning points [in the original function.]	**DM1**	
	Decreasing because $f'(\text{any positive } x \text{ value})<0$	**A1**	WWW e.g. $f'(1)=-18$.
		3	

Question 233

(a)	$-2\left((x \pm p)^2 \pm q\right)$ or $-2(x \pm p)^2 \pm q$	**M1***	$p \neq 0$.
	$-2\left((x-2)^2 \pm q\right)$ or $-2(x-2)^2 \pm q$	**DM1**	
	$-2(x-2)^2 + 19$ and (2, 19)	**A1**	Accept $x = 2, y = 19$ or 2, 19.
		3	
(b)	**Method 1**		
	$[x =] \pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	**M1***	
	$\left[\int\left(-2x^2+2\right)\mathrm{d}x\right] - \frac{2}{3}x^3 + 2x$	**B1***	Both terms correct.
	$\left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right)$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve to their integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3} \rightarrow \frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 2		
	$[x =] \pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly their area of a trapezium.
	$\left\{\frac{-2x^3}{3}+\frac{8}{2}x^2+11x\right\}[-]\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	OE All terms correct. The second integral can be replaced by $\frac{1}{2}(1+17)\times 2$ OE.
(b)	$\left\{\left(\frac{-2}{3}+4+11\right)-\left(\frac{2}{3}+4-11\right)\right\}$ $[-]$ $\{(4+9)-(4-9)\}$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expressions. If the trapezium has been used, the second integral can be replaced by *their* 18.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3} \rightarrow \frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 3		
	$[x =] \pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	**M1***	
	$-\frac{2}{3}(x-2)^3 - \frac{8}{2}x^2 + 10x$	**B1***	All terms correct.
	$\left(\frac{2}{3}-4+10\right)-(18-4-10)$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW.

(b)	**Method 4**		
	$[x=]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly *their* area of a trapezium.
	$\left\{-\frac{2}{3}(x-2)^3+19x\right\}$ [−] $\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	All terms correct. The second integral can be replaced with $\frac{1}{2}(1+17)\times 2$ OE.
	$\left\{\left(\frac{2}{3}+19\right)-(18-19)\right\}$ [−] $\{(4+9)-(4-9)\}$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression. If the trapezium has been used the second integral can be replaced with *their* 18 OE.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
		5	

Question 234

(a)	$[\mathrm{f}(2+h)=]\ 2(2+h)^2-3$	**B1**	SOI
	$\frac{\left(2(2+h)^2-3\right)-5}{(2+h)-2}$ $\left[=\frac{2h^2+8h}{h}\right]$	**M1**	$\frac{\left\{their\left(2(2+h)^2-3\right)\right\}-their\,5}{(2+h)-2}$ can be implied by the simplified expression or the correct answer. *Their* 5 must come from $2(2)^2-3$.
	$2h+8$ or $2(h+4)$	**A1**	
		3	
(b)	$h\to 0$, or chord [AB] $\to$ tangent [at A]	**B1**	Either of these statements or any sight of $h=0$.
	8	**B1FT**	Could come from anywhere except wrong working. Either correct or FT their linear expression from **(a)**.
		2	

Question 235

(a)	Differentiate to obtain $5+12x-9x^2$	**B1**	
	Attempt to find two critical values by solving quadratic equation or inequality	**M1**	
	Obtain values $-\frac{1}{3}$ and $\frac{5}{3}$	**A1**	**SC B1** if no method for solving the quadratic.
	Conclude $x<-\frac{1}{3},\ x>\frac{5}{3}$	**A1FT**	**SC B1** if no method for solving the quadratic.
		4	
(b)	Equate first derivative to 9 and simplify to 3 term quadratic	***M1**	
	Obtain $x=\frac{2}{3}$	**A1**	**SC B1** for solving $5+12x-9x^2=9$ without simplifying to a 3-term quadratic.
	Use x-value and corresponding y-value to determine value of k	**DM1**	
	Obtain $k=\frac{28}{9}$	**A1**	**SC B1** for $k=\frac{28}{9}$ from solving $5+12x-9x^2=9$ without simplifying to a 3-term quadratic.
		4	

Question 236

(a)	Differentiate to obtain form $k_1(2x+1)^{-\frac{4}{3}}$	M1	
	Obtain correct $-8(2x+1)^{-\frac{4}{3}}$ or unsimplified equivalent	A1	
	Attempt equation of tangent at $\left(\frac{7}{2}, 6\right)$ with numerical gradient	M1	Gradient must come from a differentiated expression.
	Obtain $y=-\frac{1}{2}x+\frac{31}{4}$ or equivalent of requested form	A1	
		4	
(b)	Integrate to obtain form $k_2(2x+1)^{\frac{2}{3}}$	M1	
	Obtain correct $9(2x+1)^{\frac{2}{3}}$ or unsimplified equivalent	A1	
	Use correct limits correctly to find area	M1	Substitute correct limits into an integrated expression. 36 – 9 minimum working required.
	Obtain 27	A1	**SC B1** if M1 A1 M0 scored.
		4	

Question 237

(a)	Attempt correct process for solving 3-term quadratic equation in $\sqrt{x}$	M1	Accept $8y^2-6y-9 \to (2y-3)(4y+3)$, if $y=\sqrt{x}$ specified.
	Obtain at least $2\sqrt{x}-3=0$ or equivalent	A1	Ignore $4\sqrt{x}+3=0$. **SC B1** for $\sqrt{x}=\frac{3}{2}$ with no method shown for solving the 3-term quadratic.
	Conclude $x=\frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
	Alternative Method for Q5(a)		
	$3\sqrt{x}=4x-\frac{9}{2} \to 16x^2-45x+\frac{81}{4}$ o.e and attempt correct process to solve	M1	
	Obtain $x=\frac{9}{4}$ or $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
	$x=\frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
		3	
(b)	Integrate to obtain form $k_1x^2+k_2x^{\frac{3}{2}}+k_3x$ where $k_1k_2k_3 \neq 0$	M1	
	Obtain correct $2x^2-2x^{\frac{3}{2}}+x$ or equivalent	A1	Allow unsimplified.
	Substitute $x=4$ and $y=11$ to attempt value of c	M1	Dependent on at least 2 correct terms involving x.
	Obtain $y=2x^2-2x^{\frac{3}{2}}+x-9$	A1	Must be simplified. Allow 'f(x) ='. Allow y missing if y appears previously.
		4	

Question 238

Differentiate to obtain $2x + ax^{-2}$ or equivalent	**B1**	
Equate first derivative to zero, substitute $x = -3$ and attempt value of a	**M1**	Must be an attempt at differentiation.
Obtain $a = 54$	**A1**	
Obtain $b = 27$	**A1**	
	4	

Question 239

(a)	$\left\{\frac{2}{3} \times \frac{4}{3}(3x+4)^{\frac{3}{2}}\right\}\left\{-\frac{2x^2}{2} - 6x\right\}$	**B1 B1**	B1 for each correct { }.
	$[A =] \left(\frac{8}{9}(21+4)^{\frac{3}{2}} - 7^2 - 6 \times 7\right) - \left(\frac{8}{9}(4)^{\frac{3}{2}}\right)$	**M1**	Correct use of 7 and 0 in an expression with at least two terms with two correct powers.
	13	**A1**	
		4	
(b)	$\left(2(3x+4)^{-\frac{1}{2}} \times 3\right) - 2$	**B1**	
	$y - 0 = \left(\textit{their} \frac{dy}{dx} \textit{ with } x = 7\right)(x - 7)$	***M1**	Using $x = 7$, *their* value for $\frac{dy}{dx}$ and then any form of the equation of a straight line using $(7, 0)$.
	Either:		
	$[Q \text{ is}] \left(0, \frac{28}{5}\right)$	**DM1**	Use $x = 0$ in *their* equation of PQ.
	[Area of OPQ =] $\frac{1}{2} \times 7 \times \left(\textit{their } \frac{28}{5}\right)$	**DM1**	
	$\left[\frac{1}{2} \times 7 \times \left(\textit{their } \frac{28}{5}\right) - (\textit{their } 13 \text{ from } (\mathbf{a}))\right] = \frac{33}{5}$	**A1 FT**	Only FT, following B0M1DM1DM1, from *their* **(a)** if $(\textit{their } 13 \text{ from } (\mathbf{a})) < \frac{98}{5}$ and the area is > 0.
	Or:		
	$\int \left(\textit{their}\left(-\frac{4}{5}(x-7)\right)\right) dx$	**DM1**	
	Evaluating *their* $\left(-\frac{4}{5}\left(\frac{x^2}{2} - 7x\right)\right)$ with limits 7 and 0	**DM1**	
(b)	$\left[(\textit{their} \text{ area of } OPQ) - (\textit{their } 13 \textit{ from } (\boldsymbol{a}))\right] = \frac{33}{5}$	**A1 FT**	Only FT, following B0M1DM1DM1, from their **(a)** if $(\textit{their } 13 \textit{ from } (a)) < \frac{98}{5}$ and the area is > 0.
		5	

Question 240

(a)	$\dfrac{6}{\left(\frac{1}{2}\right)^4}-\dfrac{5}{\left(\frac{1}{2}\right)^3}$	**M1**	Substitute $x=\frac{1}{2}$ and evaluate second derivative.
	$96-40[=56]>0 \Rightarrow$ Minimum	**A1**	CWO Evidence and conclusion. **SC B1** for $\dfrac{d^2y}{dx^2}>0$ without sight of $96-40$ or 56.
		2	
(b)	$\left\{\dfrac{6}{-3}x^{-3}\right\}\left\{-\dfrac{5}{-2}x^{-2}\right\}[+c_1]$	**B1 B1**	OE B1 for each correct { }.
	$0=\dfrac{6}{-3}\left(\dfrac{1}{2}\right)^{-3}-\dfrac{5}{-2}\left(\dfrac{1}{2}\right)^{-2}+c_1$	**M1**	Substitute $x=\frac{1}{2}$ into two terms of an integrated expression (at least one correct power), now with c_1, and equate to 0 to find c_1.
	$c_1=6$	**A1**	
	$k_1x^{-2}+k_2x^{-1}+k_3x\ \ [+c_2]$	**M1**	Integration of *their* $\dfrac{dy}{dx}$ to produce at least two terms with correct powers, $k_1, k_2 \neq 0$.
	$9=\dfrac{1}{\left(\frac{1}{2}\right)^2}-\dfrac{5}{2\left(\frac{1}{2}\right)}+6\left(\dfrac{1}{2}\right)+c_2$	**M1**	OE Substitute $\left(\frac{1}{2},9\right)$ into integrated expression (at least two correct powers) to find c_2.
	$y=x^{-2}-\dfrac{5}{2}x^{-1}+6x+7$	**A1**	OE Condone their final answer being $c_2=7$ if a completely correct simplified expression for the equation containing c_2 has been stated previously.
		7	

Question 241

(a)	$4x+\dfrac{5}{x^2}$	**B1**	OE
	$\left[\dfrac{dy}{dx}=\right]9$	**B1 FT**	Correct use of $x=1$ in *their* two-term differentiated expression, defined as an expression with one correct power.
		2	
(b)	*Their* $\left(4x+\dfrac{5}{x^2}\right)=0$ and valid method as far as '$x=\ldots$'	**M1**	Equate their derivative of the form $Ax\pm\dfrac{B}{x^2}$ to zero, where $A,\ B\neq 0$, and solve. If no working is seen, this can be implied by a correct answer for x.
	$x=-1.08$	**A1**	AWRT
	$y=9.96$	**A1**	AWRT
		3	

Question 242

(a)	$-9\times 2(2x-5)^{-2}+2$	**B1**	Correct differential.
	$\left(their\ -18(2x-5)^{-2}+2\right)=0$ and rearrange to form a quadratic. $\left[(2x-5)^2=9 \text{ or } 8x^2-40x+32=0\right]$	**M1**	Equating a two term $\frac{dy}{dx}$ to 0 and dealing correctly with the negative power. Their two term $\frac{dy}{dx}$ must contain $(2x-5)^{-2}$.
	$(1,\ -6)$ **and** $(4,6)$	**A1, A1**	A1 for two correct x-values or one correct point, second A1 for all correct.
		4	
(b)	$-18\times -2\times 2(2x-5)^{-3}\left[=72(2x-5)^{-3} \text{ or } \frac{144x-360}{(2x-5)^4}\right]$	**B1 FT**	Following through on *their* first derivative which must contain $(2x-5)^{-2}$.
	Use $\left(their\ x=1 \text{ and } x=4\right)$ in $\left(their\ 72(2x-5)^{-3}\right)$ To determine the nature of both turning points.	**M1**	Substitute x-coordinate of each stationary point and determine their nature. Nature of the turning points must correctly follow from *their* values of $\frac{d^2y}{dx^2}$.
	For $x=1$, $\left[\frac{d^2y}{dx^2}=\right]-\frac{72}{27}$ or $<0 \Rightarrow$ maximum For $x=4$, $\left[\frac{d^2y}{dx^2}=\right]\frac{72}{27}$ or $>0 \Rightarrow$ minimum	**A1**	CWO
		3	
(c)(i)	$(1,\ -13)$	**B1**	
		1	
(c)(ii)	$[y=]\frac{9}{2(x\pm 3)-5}+2(x\pm 3)-5\pm 7$	**M1**	Application of $\begin{pmatrix}-3\\7\end{pmatrix}$ to the original expression for C but condone +/− sign errors.
	$[y=]-\left(\frac{9}{2(x\pm 3)-5}+2(x\pm 3)-5\pm 7\right)$	**M1**	**SC B1** for $[y=]-\left(\frac{9}{2x-5}+2x-5\right)$.
	$y=-\frac{9}{2x+1}-2x-8$	**A1**	Answer must be in this format; the '$y=$' can be implied by earlier inclusion.
		3	

Question 243

(a)	$3x^2+10x-8<0$ or $3\left(x+\frac{5}{3}\right)^2-\frac{49}{3}<0$	***B1**	OE Condone = or ⩽ 0, but use of > or ⩾ can only score B0B1B0. If there is no < , = or ⩽ 0, then a correct final answer implies < 0 here.
	-4 and $\frac{2}{3}$	**B1**	
	$-4<x<\frac{2}{3}$	**DB1 FT**	OE Use of ⩽ sign(s) gets DB0. Condone $x>-4,\ x<\frac{2}{3}$, $x>-4$ and $x<\frac{2}{3}$ but not $x>-4$ or $x<\frac{2}{3}$. FT on *their* critical values.
		3	
(b)	Identify $x=-4$ as the x-value of the maximum point.	**B1 FT**	SOI FT on *their* lower critical value from part **(a).**
	$[y=]\frac{3}{3}x^3+\frac{10}{2}x^2-8x[+c]$	**B1**	Correct integration of the three terms given. Condone $f(x)=$.
	Use of $y=27,\ x=$ *their* $\left(-4 \text{ or } \frac{2}{3}\right)$ in *their* integral.	**M1**	*Their* integral must be a cubic, *their* $\left(-4 \text{ or } \frac{2}{3}\right)$ must come from **(a)** or a restart.
	$c=-21 \Rightarrow y=x^3+5x^2-8x-21$	**A1**	Do not ISW if they continue to find a straight line. Condone omission of final statement if y or $f(x)=x^3+5x^2-8x+c$ has been seen earlier.
		4	

Question 244

$\left(\frac{1}{-1\times4}\right)a(4x-3)^{-1}+2x$	**B1**	OE Do not accept $(-2+1)$ as equivalent to -1.
Apply correct limits, $x=3$ & 1, to *their* integral	***M1**	*Their* integral must contain $(4x-3)^{-1}$. Condone using $x=1$ and 3.
$\frac{-a}{36}+6-\left(\frac{-a}{4}+2\right)=12 \Rightarrow \left[\frac{8a}{36}+4=12\right]$	**DM1**	OE Equate *their* linear unsimplified expression in a to 12.
$a=36$	**A1**	
	4	

Question 245

(a)	$\left[\frac{dy}{dx}=\right]\frac{-24x^{-2}}{-2}\left[=\frac{12}{x^2}\right][+c]$	B1	May be unsimplified.
	$\frac{12}{(-2)^2}+c=0\ [\Rightarrow c=-3]$	M1	Substituting $x=-2$ into *their* $\frac{dy}{dx}$ of the form $kx^{-2}+c$, and setting = 0. Condone missing bracket.
	$\left[\frac{dy}{dx}=\right]\frac{12}{x^2}-3$ or $12x^{-2}-3$	A1	$\frac{-24}{-2}$ must be simplified to 12, but accept $c=-3$ as the final answer.
		3	
(b)	$\left[\frac{12}{x^2}-3=0\Rightarrow x^2=4\Rightarrow x=\pm2\Rightarrow\right]x=2$	B1	May be done by inspection or calculator, but must be WWW. Condone ±2.
	$\left[\frac{d^2y}{dx^2}=\right]-\frac{24}{(2)^3}$ or -3 or $<0\Rightarrow$ maximum	B1	WWW. Or other valid method. Ignore any working for $x=-2$.
		2	
(c)	$[y=]-12x^{-1}+\textit{their }cx\left[=-\frac{12}{x}-3x\right][+d]$	B1FT	May be unsimplified. FT *their* nonzero *c*.
	$19=-\frac{12}{-2}-3\times(-2)+d\ [\Rightarrow d=7]$	M1	Substituting $(-2, 19)$ into their integrated expression (defined by having at least one correct power) and including $+d$.
	$y=-\frac{12}{x}-3x+7$	A1	Accept $d=7$ as final answer if $y=\ldots$ seen previously. Allow '$f(x)=$' in place of '$y=$'.
		3	
(d)	$\frac{12}{x^2}-3=-\frac{9}{4}\Rightarrow\frac{12}{x^2}=\frac{3}{4}\Rightarrow x=\ldots\ [\Rightarrow x^2=16\Rightarrow x=4, y=-8]$	*M1	Setting *their* $\frac{dy}{dx}$ of the form $kx^{-2}+c=-\frac{9}{4}$, and correctly solving for *x* from their equation where $x^2>0$.
	Gradient of normal = $\frac{4}{9}$	B1	
	$\frac{y+8}{(x-4)}=\frac{4}{9}$	DM1	For using *their* stated positive *x* and *their* stated *y* and a changed gradient to correctly form a line equation; condone $-\frac{4}{9}$ or $\frac{9}{4}$.
	$4x-9y-88=0$	A1	OE in correct form.
		4	

Question 246

$6-3x=3\Rightarrow x$-coordinate of point of intersection = 1	B1	
$\left[\frac{18}{5}(5x+4)^{\frac{1}{2}}\right]_0^1$	B1 B1	B1 for $k(5x+4)^{\frac{1}{2}}$
Area $=\frac{18}{5}\times3-\frac{18}{5}\times2\left[=\frac{18}{5}\right]$	M1	
Line crosses *x*-axis at $x=2$, so triangle area $=\frac{1}{2}\times3\times1$	M1	
Total area $=\frac{51}{10}$	A1	
	6	

Question 247

(a)	$\left[\frac{dy}{dx}\right] = \frac{3}{2}ax^{\frac{1}{2}} - 12$	*M1	For attempt at differentiation; at least one correct term needed. Condone poor notation throughout.
	$\left[\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \right]\left(\frac{3}{2}a \times 9^{\frac{1}{2}} - 12\right) \times 5$	DM1	For correct use of chain rule with 5, $x = 9$ and *their* $\frac{dy}{dx}$. Condone missing brackets and allow errors in their working.
	$\left[\frac{dy}{dt} = \right] 5\left(\frac{9}{2}a - 12\right)$ or $\frac{45}{2}a - 60$ or $22.5a - 60$ or $\frac{45a - 120}{2}$ or $\frac{15}{2}(3a - 8)$	A1	OE simplified form.
		3	
(b)	$\frac{3}{2}a \times \left(\frac{1}{4}\right)^{\frac{1}{2}} - 12 = 0$	M1	For setting *their* 2 term $\frac{dy}{dx}$ with at least one term correct = 0 and substituting $x = 0.25$. Condone missing brackets. Allow a restart for $\frac{dy}{dx}$ if 2 terms seen and at least one term correct.
	$\Rightarrow a = 16$	A1	
		2	

Question 248

(a)	$\left[\frac{dy}{dx} = \right] 8x - \frac{18}{x^3}$	B1	OE Accept unsimplified.
	At $x = 2, \frac{dy}{dx} = \frac{55}{4}$	M1	OE For evaluating *their* $\frac{dy}{dx}$ or $\frac{dx}{dy}$.
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{55}{4} = -5 \times \frac{dt}{dx}$	M1	For correct use of chain rule with ±5 and *their* $\frac{dy}{dx}$ (may be algebraic). Condone missing brackets.
	$\left[\frac{dx}{dt} = \right] -\frac{4}{11}$	A1	Or decreasing at a rate of $\frac{4}{11}$. AWRT −0.364.
		4	
(b)	$8x - \frac{18}{x^3} = 0 \left[\Rightarrow x^4 = \frac{9}{4}\right]$	M1	Equating *their* 2-term $\frac{dy}{dx}$ to zero.
	$x = \pm\sqrt{\frac{3}{2}}$ or $\pm\frac{\sqrt{6}}{2}$	A1	AWRT ± 1.22.
	$y = 4$ (for both)	A1	A0 A1 if one point correct. AWRT 4.00.
	$\left[\frac{d^2y}{dx^2}\right] = 8 + \frac{54}{x^4}$	M1	For differentiation. At least one correct term needed.
	So both are minima	A1	No need for reason. WWW on x-values.
		5	

Question 249

Attempt to integrate $(x-16)-\left(5x^{\frac{3}{2}}-20x\right)\left[=-5x^{\frac{3}{2}}+21x-16\right]$	*M1	Attempt to integrate both terms and subtract areas. Accept subtraction either way round.
$\left\{\frac{1}{2}x^2-16x\right\}[-]\left\{\left(2x^{\frac{5}{2}}-10x^2\right)\right\}\left[=-2x^{\frac{5}{2}}+\frac{21}{2}x^2-16x\right]$	B1 B1	B1 for each integral. B2,1, 0 for $-2x^{\frac{5}{2}}+\frac{21}{2}x^2-16x$.
Use of limits 1 and 16	DM1	Limits either way round. Minimum –7.5 – 384. E.g. (($\frac{1}{2}$ – 16) – (2 – 10)) – ((128 – 256) – (2048 – 2560)) = (15.5 + 8) – (–128 + 512) = –7.5 – 384 = –391.5 Or (–2 + 10.5 – 16) – (–2048 + 2688 – 256) = –7.5 – 384 = –391.5 Or $-504+\frac{225}{2}=-391.5$
391.5 or $\frac{783}{2}$	A1	CAO **SC B1** for correct answer if M1 B1 B1 DM0 scored.

Alternative Method for Question 4

Height of triangle = 15	B1	
Area of triangle = $\frac{1}{2}\times 15\times$ *their* height $=112.5$	M1	
Integrates $\pm\left(5x^{\frac{3}{2}}-20x\right)$ to $\pm\left(2x^{\frac{5}{2}}-10x^2\right)$	B1	
Use of limits 1 and 16 on their integral and subtracts area of triangle	DM1	Limits either way round.
391.5	A1	CAO **SC B1** for correct answer if M1 B1 B1 DM0 scored.
	5	

Question 250

(a)	[Gradient of tangent] = $4(2\times4-5)^3-9\times4^{\frac{1}{2}}[=90]$	M1	Substitute $x=4$ into $\frac{dy}{dx}$. $\frac{-11}{2}=4(2\times4-5)^3-9\times4^{\frac{1}{2}}$ is M0 unless they reach $-\frac{1}{90}$.
	[Gradient of normal] = $-\frac{1}{90}$	A1	AWRT –0.0111.
		2	
(b)	$[y]=\left\{\frac{1}{2}(2x-5)^4\right\}\left\{-6x^{\frac{3}{2}}\right\}[+c]$	B1 B1	Accept unsimplified.
	$-\frac{11}{2}=\frac{1}{2}\times(2\times4-5)^4-6\times4^{\frac{3}{2}}+c$	M1	Sub $x=4, y=-\frac{11}{2}$ into an integrated expression and attempt to find c.
	$y=\frac{1}{2}(2x-5)^4-6x^{\frac{3}{2}}+2$	A1	Condone $c=2$ as final answer if '$y=\dots$' seen previously. Fractions must be simplified. Accept f(x) in place of y.
		4	