AS-Level

Topic : Calculus

May 2013-May 2023

Answer

Question 1

	$y=\frac{8}{\sqrt{x}}-x$			
(i)	$\frac{dy}{dx}=-4x^{-\frac{3}{2}}-1$	B1		needs both
	$=-\frac{3}{2}$ when $x=4$.	M1		Subs $x=4$ into dy/dx
	Eqn of BC $y-0=-\frac{3}{2}(x-4)$	M1 A1		Must be using differential + correct form of line at $B(4,0)$.
	$\rightarrow C(1, 4\frac{1}{2})$		[4]	
(ii)	area under curve = $\int(\frac{8}{\sqrt{x}}-x)$			
	$=\frac{8x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{1}{2}x^2$	B1 B1		(both unsimplified)
	Limits 1 to 4 $\rightarrow$ $8\frac{1}{2}$	M1		Using correct limits.
	Area under tangent = $\frac{1}{2}\times4\frac{1}{2}\times3=6\frac{3}{4}$	M1		Or could use calculus)
	Shaded area = $1\frac{3}{4}$	A1	[5]	

Question 2

	$u=x^2y \quad y+3x=9$ $u=x^2(9-3x)$ or $\left(\frac{9-y}{3}\right)^2y$	M1		Expressing u in terms of 1 variable
	$\frac{du}{dx}=18x-9x^2$ or $\frac{du}{dy}=27-12y+y^2$	DM1A1		Knowing to differentiate.
	$=0$ when $x=2$ or $y=3$ $\rightarrow$ $u=12$	DM1 A1		Setting differential to 0.
	$\frac{d^2u}{dx^2}=18-18x$ $-$ve	DM1 A1	[7]	Any valid method

Question 3

$\frac{dy}{dx} = \sqrt{2x+5}$			
$\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$	B1 B1		B1 Everything without "÷2". B1 "÷2"
Uses (2, 5) $\rightarrow c = -4$	M1 A1	[4]	Uses point in an integral.

Question 4

$y = \sqrt{1+4x}$		
(i) $\frac{dy}{dx} = \frac{1}{2}(1+4x)^{-\frac{1}{2}} \times 4$ $= 2$ at B (0, 1) Gradient of normal $= -½$ Equation $y - 1 = -½\,x$	B1 B1 M1 M1 A1 [5]	B1 Without "×4". B1 for "×4" even if first B mark lost. Use of $m_1m_2 = -1$ Correct method for eqn.
(ii) At A $x = -¼$ $\int\sqrt{1+4x}\,dx = \frac{(1+4x)^{\frac{3}{2}}}{\frac{3}{2}} \div 4$ Limits $-¼$ to 0 $\rightarrow \frac{1}{6}$ Area $BOC = ½ \times 2 \times 1 = 1$ $\rightarrow$ Shaded area $= \frac{7}{6}$	B1 B1 B1 B1 B1√ [5]	 B1 Without the "÷4". For "÷4" even if first B mark lost. For 1 + his "1/6".

Question 5

$f(x) = \frac{5}{1-3x},\ x \geq 1$		
(i) $f'(x) = \frac{-5}{(1-3x)^2} \times -3$	B1 B1 [2]	B1 without × −3. B1 for ×−3, even if first B mark is incorrect
(ii) $15 > 0$ and $(1-3x)^2 > 0$, $f'(x) > 0$ $\rightarrow$ increasing	B1√ [1]	√ providing $(\ )^2$ in denominator.
(iii) $y = \frac{5}{1-3x} \rightarrow 3x = 1 - \frac{5}{y}$ $\rightarrow f^{-1}(x) = \frac{x-5}{3x}$ or $⅓ - \frac{5}{3x}$ Range is ≥ 1 Domain is $-2.5 \leq x < 0$	M1 A1 B1 B1 B1 [5]	Attempt to make x the subject. Must be in terms of x. must be $\geq$ condone $<$

Question 6

(i) $\pi r^2 h = 250\pi \rightarrow h = \frac{250}{r^2}$		
$\rightarrow S = 2\pi rh + 2\pi r^2$	M1	Makes h the subject. $\pi r^2 h$ must be right
$\rightarrow S = 2\pi r^2 + \frac{500\pi}{r}$	M1 [2]	Ans given – check all formulae..
(ii) $\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2}$	B1 B1	B1 for each term
$= 0$ when $r^3 = 125 \rightarrow r = 5$	M1	Sets differential to 0 + attempt at soln
$\rightarrow S = 150\pi$	A1 [4]	
(iii) $\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3}$	M1	Any valid method.
This is positive $\rightarrow$ Minimum	A1 [2]	2nd differential must be correct – no need for numerical answer or correct r.

Question 7

$\frac{dy}{dx} = \frac{6}{x^2}$		
$y = -6x^{-1} + c$	B1	Integration only – unsimplified
Uses (2, 9) $\rightarrow c = 12$	M1	Uses (2, 9) in an integral
$y = -6x^{-1} + 12$	A1 [3]	

Question 8

(i) $\frac{dy}{dx} = 4(x-2)^3$	**B1**	Or $4x^3 - 24x^2 + 48x - 32$
Grad of tangent = –4	**M1**	Sub $x = 1$ into *their* derivative
Eq. of tangent is $y - 1 = -4(x-1)$	**M1**	Line thru (1, 1) and with ***m*** from deriv
$\rightarrow$ B $(\frac{5}{4}, 0)$	**A1**	
Grad of normal = $\frac{1}{4}$	**M1**	Use of $m_1 m_2 = -1$
Eq. of normal is $y - 1 = \frac{1}{4}(x-1) \rightarrow$ C$(0, \frac{3}{4})$	**A1** **[6]**	
(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$	**M1**	
$\frac{\sqrt{17}}{4}$	**A1** **[2]**	Allow $\sqrt{\frac{17}{16}}$
(iii) $\int (x-2)^4 dx = \frac{(x-2)^5}{5}$	**B1**	Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$
$\left[0 - (-\frac{1}{5})\right] = \frac{1}{5}$	**M1**	Apply limits $1 \rightarrow 2$ for curve
$\Delta = \frac{1}{2} \times 1 \times (their\,\frac{5}{4} - 1) = \frac{1}{8}$	**M1**	Or $\int_1^{\frac{5}{4}} (-4x+5)\,dx = \frac{1}{8}$
$\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ or 0.075	**A1** **[4]**	

Question 9

(i) $3u+\frac{3}{u}-10=0$	**B1**	Or $3x-10\sqrt{x}+3=0$ Or $(3\sqrt{x}-1)(\sqrt{x}-3)$ or apply formula etc.
$3u^2-10u+3=0 \Rightarrow (3u-1)(u-3)=0$	**M1**	
$\sqrt{x}=\frac{1}{3}$ or 3	**A1**	
$\sqrt{x}=\frac{1}{9}$ or 9	**A1** **[4]**	
(ii) $f''(x)=\frac{3}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$	**B1**	Allow anywhere
At $x=\frac{1}{9}$ $f''(x)=\frac{3}{2}(3)-\frac{3}{2}(27)\ (=-36)<0\rightarrow$ Max	**M1**	Valid method. Allow innac subs, even $3,\ \frac{1}{3}$
At $x=9$ $f''(x)=\frac{3}{2}\times\frac{1}{3}-\frac{3}{2}\times\frac{1}{27}(=\frac{4}{9})>0\rightarrow$ Min	**A1** **[3]**	Fully correct. No working, no marks.
(iii) $f(x)=2x^{\frac{3}{2}}+6x^{\frac{1}{2}}-10x\ (+c)$	**B2**	B1 for 2/3 terms correct. Allow in **(i)**
$-7=16+12-40+c$	**M1**	Sub $(4,-7)$. c must be present.
$c=5$	**A1** **[4]**	

Question 10

$f'(x)=(2x-5)^2\times 2+1$ or $24\left(x-\frac{5}{2}\right)^2+1$	**B1B1**	**B1** for $3(2x-5)^2$, **B1** for $(\times 2+1)$ SC B1 for $24x^2-120x+151$
>0 (allow $\geq$)	**B1**√ **[3]**	Dep on $k(2x-5)^2+c\ (k>0),\ (c\geq 0)$ Subst of particular values is B0

Question 11

(i) $\frac{dy}{dx}=\left[\frac{1}{2}(x4+4x+4)^{-\frac{1}{2}}\right]\times\left[4x^3+4\right]$	B1B1	
At $x=0$, $\frac{dy}{dx}=\frac{1}{2}\times\frac{1}{2}\times 4=(1)$	M1	Sub $x=0$ and attempt eqn of line following differentiation.
Equation is $y-2=x$	A1 [4]	
(ii) $x+2=\sqrt{x^4+4x+4}\Rightarrow (x+2)^2$ $=x4+4x+4$	B1	**AG** www
$x^2-x^4=0$ oe	B1	
$x=0,\pm 1$	B2,1,0 [4]	
(iii) $(\pi)\left[\frac{x^5}{5}+2x^2+4x\right]$	M1A1	Attempt to integrate y^2
$(\pi)\left[0-\left(\frac{-1}{5}+2-4\right)\right]$	DM1	
$\frac{11\pi}{5}$ (6.91) oe	A1 [4]	Apply limits $-1\rightarrow 0$

Question 12

$\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$	M1A1	Attempt differentiation & set to zero
$x + 2 = \pm k$	DM1	Attempt to solve
$x = -2 \pm k$	A1	cao
$\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
	M1	Sub their x value with k in it into $\frac{d^2y}{dx^2}$
When $x = -2 + k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min	A1	Only 1 of bracketed items needed for each
When $x = -2 - k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) max	A1	but $\frac{d^2y}{dx^2}$ and x need to be correct.
	[8]	

Question 13

$f(x) = 2x^{-\frac{1}{2}} + x\ (+c)$	M1A1	Attempt integ $x^{-\frac{1}{2}}$ or $+x$ needed for M
$5 = -2 \times \frac{1}{2} + 4 + c$	M1	Sub (4, 5). c must be present
$c = 2$	A1 [4]	

Question 14

$y = \frac{8}{x} + 2x$

(i) $\frac{dy}{dx} = \frac{-8}{x^2} + 2$	M1	Attempt at differentiation.
$(-6$ at $A)$	A1	algebraic – unsimplified.
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dy}{dt}$	M1	Ignore notation – needs product of 0.04
$\rightarrow -0.24$	A1 [4]	and 'his' $\frac{dy}{dx}$.
(ii) $\int y^2 = \int \frac{64}{x^2} + 4x^2 + 32$	M1	Use of integral of y^2 (ignore π)
$= \left(\frac{-64}{x} + \frac{4x^3}{3} + 32x\right)$	A3,2,1	3 terms $\rightarrow -1$ each error.
Limits 2 to 5 used correctly	DM1	Uses correct limits correctly.
$\rightarrow 271.2\pi$ or 852	A1	
(allow 271π or 851 to 852)	[6]	(omission of π loses last mark)

Question 15

(i) Sim triangles $\frac{y}{16-x}=\frac{12}{16}$ (or trig)	M1	Trig, similarity or eqn of line
$\rightarrow y = 12 - \frac{3}{4}x$	A1	(could also come from eqn of line)
$A = xy = 12x - \frac{3}{4}x^2$.	A1 [3]	ag – check working.
(ii) $\frac{dA}{dx} = 12 - \frac{6x}{4}$	B1	
$= 0$ when $x = 8$. $\rightarrow A = 48$.	M1 A1	Sets to 0 + solution.
This is a Maximum. From −ve quadratic or 2nd differential.	B1 [4]	Can be deduced without any working. Allow even if '48' incorrect.

Question 16

$y = \frac{2}{\sqrt{5x-6}}$

(i) $\frac{dy}{dx} = 2 \times -\frac{1}{2} \times (5x-6)^{-\frac{3}{2}} \times 5$	B1 B1	B1 without '×5'. B1 For '×5'
$\rightarrow -\frac{5}{8}$	B1 [3]	Use of 'uv' or 'u/v' ok.
(ii) integral $= \frac{2\sqrt{5x-6}}{\frac{1}{2}} \div 5$	B1 B1	B1 without '÷5'. B1 for '÷ 5'
Uses 2 to 3 $\rightarrow 2.4 - 1.6 = 0.8$	M1 A1 [4]	Use of limits in an integral.

Question 17

(i) $\frac{dy}{dx} = \left[3(3-2x)^2\right] \times [-2]$	**B1B1**	OR $-54+72x-24x^2$ B2,1,0
At $x = \frac{1}{2}, \frac{dy}{dx} = -24$	**M1**	
$y - 8 = -24\left(x - \frac{1}{2}\right)$	**DM1**	
$y = -24x + 20$	**A1** **[5]**	
(ii) Area under curve $= \left[\frac{(3-2x)^4}{4}\right] \times \left[-\frac{1}{2}\right]$	**B1B1**	OR $27x - 27x^2 + 12x^3 - 2x^4$ B2,1,0
$-2 - \left(-\frac{81}{8}\right)$	**M1**	Limits 0→ ½ applied to integral with intention of subtraction shown
Area under tangent $= \int(-24x+20)$	**M1**	or area trap =½(20 + 8) × ½
$= \left\|-12x^2 + 20x\right\|$ or 7 (from trap)	**A1**	Could be implied
$\frac{9}{8}$ or 1.125	**A1** **[6]**	Dep on both M marks

Question 18

(i) $A = 2xr + \pi r^2$	B1	
$2x + 2\pi r = 400$ $(\Rightarrow\ x = 200 - \pi r)$	B1	
$A = 400r - \pi r^2$	M1A1 [4]	Subst & simplify to **AG** (www)
(ii) $\frac{dA}{dr} = 400 - 2\pi r$	B1	Differentiate
$= 0$	M1	Set to zero and attempt to find r
$r = \frac{200}{\pi}$ oe	A1	
$x = 0 \Rightarrow$ no straight sections **AG**	A1	
$\frac{d^2A}{dr^2} = -2\pi$ (<0) Max	B1 [5]	Dep on -2π, or use of other valid reason

Question 19

Attempt integration	M1	
$f(x) = 2(x+6)^{\frac{1}{2}} - \frac{6}{x}(+c)$	A1A1	Accept unsimplified terms
$2(3) - \frac{6}{3} + c = 1$	M1	Sub. $x = 3, y = 1$. c must be present
$c = -3$	A1 [5]	

Question 20

pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow$ $x = 3, 7$	M1A1	Attempt at soln of sim eqns. co
Area of trapezium = $\frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1)\,dx$ from 3 to 7 = 44)	M1A1	Either method ok. co
Area under curve = $-\frac{1}{3}x^3 + 6x^2 - 20x$	B2,1	−1 each term incorrect
Uses 3 to 7 $\rightarrow$ $(54\frac{2}{3})$	DM1	Correct use of limits (Dep 1st M1)
Shaded area = $10\frac{2}{3}$	A1 [8]	co
OR		
$\int_3^7 \left(-x^2 + 10x - 21\right) = -\frac{x^3}{3} + 5x^2 - 21x$		Functions subtracted before integration
M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1		Subtraction reversed allow A3A0. Limits reversed allow DM1A0

Question 21

(i) $3x^2y = 288$ y is the height	B1	co
$A = 2(3x^2 + xy + 3xy)$	M1	Considers at least 5 faces ($y \neq x$)
Sub for y $\rightarrow$ $A = 6x^2 + \dfrac{768}{x}$	A1	co answer given
	[3]	
(ii) $\dfrac{dA}{dx} = 12x - \dfrac{768}{x^2}$	B1	co
$= 0$ when $x = 4 \rightarrow A = 288$. Allow $(4, 288)$	M1 A1	Sets differential to 0 + solution. co
$\dfrac{d^2A}{dx^2} = 12 + \dfrac{1536}{x^3}$	M1	Any valid method
$(= 36) > 0$ Minimum	A1	co www dep on correct f″ and $x = 4$
	[5]	

Question 22

$\dfrac{dy}{dx} = \dfrac{12}{\sqrt{4x + a}}$ $P(2, 14)$ Normal $3y + x = 44$		
(i) m of normal $= -\dfrac{1}{3}$	B1	co
$\dfrac{dy}{dx} = 3 = \dfrac{12}{\sqrt{4x + a}}$ $\rightarrow$ $a = 8$	M1 A1	Use of $m_1 m_2 = -1$. AG.
	[3]	
(ii) $\int y = 12(4x + a)^{\frac{1}{2}} \div \dfrac{1}{2}$ $\div 4$ $(+c)$	B1 B1	Correct without "÷4". for "÷4".
Uses (2, 14) $c = -10$	M1 A1	Uses in an integral only. Dep 'c'. co All 4 marks can be given in (i)
	[4]	

Question 23

$f(x) = \dfrac{15}{2x + 3}$		
(i) $f'(x) = \dfrac{-15}{(2x + 3)^2} \times 2$	B1 B1	Without the "×2". For "×2" (indep of 1st B1).
$(\ )^2$ always +ve $\rightarrow$ $f'(x) < 0$ (No turning points) – therefore an inverse	B1√	√ providing $(\ )^2$ in $f'(x)$. 1–1 insuff.
	[3]	
(ii) $y = \dfrac{15}{2x + 3}$ $\rightarrow 2x + 3 = \dfrac{15}{y}$	M1	Order of ops – allow sign error
$\rightarrow x = \dfrac{\frac{15}{y} - 3}{2}$ $\rightarrow$ $\dfrac{15 - 3x}{2x}$	A1	co as function of x. Allow $y = \ldots$
(Range) $0 \leqslant f^{-1}(x) \leqslant 6$. Allow $0 \leqslant y \leqslant 6$, [0,6] (Domain) $1 \leqslant x \leqslant 5$. Allow [1, 5]	 B1 B1	 For range/domain ignore letters unless range/domain not identified
	[4]	

Question 24

$y = 8 - \sqrt{4-x}$		
(i) $\frac{dy}{dx} = -\frac{1}{2}(4-x)^{-\frac{1}{2}} \times -1$	B1 B1	Without (−1). For (×−1).
$\int y\,dx = 8x - \frac{(4-x)^{\frac{3}{2}}}{\frac{3}{2}} \div -1$	3 × B1	B1 for "8x" and +c". B1 for all except ÷(−1). B1 for ÷(−1).
	[5]	(n.b. these 5 marks can be gained in(ii) or (iii))
(ii) Eqn $y - 7 = \frac{1}{2}(x-3)$ → $y = \frac{1}{2}x + 5\frac{1}{2}$	M1A1 [2]	M1 unsimplified. A1 as $y=mx+c$
(iii) Area under curve = ∫ from 0 to 3 (58/3)	M1	Use of limits – needs use of "0"
Area under line = $\frac{1}{2}(5\frac{1}{2} + 7)\times 3$	M1	Correct method
Or $\left[\frac{1}{4}x^2 + \frac{11x}{2}\right]$ from 0 to 3	M1 A1 [4]	M1 Subtraction. A1 co
→ $\frac{58}{3} - \frac{75}{4} = \frac{7}{12}$		

Question 25

$\frac{d^2y}{dx^2} = 2x - 1$		
→ ∫ $\frac{dy}{dx} = x^2 - x \quad + c$	B1	Correct integration (ignore +c)
= 0 when $x = 3$ → $c = -6$	M1 A1	Uses a constant of integration. co
$x^2 - x - 6 = 0$ when $x = -2$ (or 3)	A1	Puts dy/dx to 0
→ ∫ $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \quad (+k)$	B1√B1√	√ first 2 terms, √ for cx.
= −10 when x = 3	M1	Correct method for k
→ $k = 3\frac{1}{2}$		
→ $y = 10\frac{5}{6}$	A1 [8]	Co −r 10.8

Question 26

(i) $y = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + (c)$ oe	**B1B1**	Attempt to integrate
$\frac{2}{3} = \frac{16}{3} - 4 + c$	**M1**	Sub $\left(4, \frac{2}{3}\right)$. Dependent on c present
$c = -\frac{2}{3}$	**A1** **[4]**	
(ii) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ oe	**B1B1** **[2]**	
(iii) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 0 \rightarrow \frac{x-1}{\sqrt{x}} = 0$	**M1**	Equate to zero and attempt to solve
$x = 1$	**A1**	
When $x = 1$, $y = \frac{2}{3} - 2 - \frac{2}{3} = -2$	**M1A1**	Sub. *their* '1' into *their* 'y'
When $x = 1$, $\frac{d^2y}{dx^2}(=1) > 0$ Hence minimum	**B1** **[5]**	Everything correct on final line. Also dep on correct (ii). Accept other valid methods

Question 27

$\frac{dy}{dx} = [-2 \times 4(3x+1)^{-3}] \times [3]$	B1B1	$[-2 \times 4u^{-3}] \times [3]$ is B0B1 unless resolved
When $x = -1$, $\frac{dy}{dx} = 3$	B1	
When $x = -1, y = 1$ soi	B1	
$y - 1 = 3(x + 1)$ $(\rightarrow y = 3x + 4)$	B1 √ [5]	Ft on *their* '3' only (not $-\frac{1}{3}$). Dep on diffn

Question 28

(a) **(i)** $(a+b)^{\frac{1}{3}} = 2$, $(9a+b)^{\frac{2}{3}} = 16$	B1B1	Ignore 2nd soln (–9, 17) throughout
$a+b=8$, $9a+b=64$	M1	Cube etc. & attempt to solve
$a=7$, $b=1$	A1	Correct answers without any working 0/4
	[4]	
(ii) $x=(7y+1)^{\frac{1}{3}}$ (*x/y* interchange as first or last step)	B1√	ft on from *their a, b* or in terms of *a, b*
$x^3 = 7y+1$ or $y^3 = 7x+1$	B1√	ft on from *their a, b* or in terms of *a, b*
$f^{-1}(x) = \frac{1}{7}(x^3 - 1)$ cao	B1	A function of x required
Domain of f^{-1} is $x \geqslant 1$ cao	B1 [4]	Accept >. Must be x
(b) $\frac{dy}{dx} = \left[\frac{1}{3}(7x^2+1)^{-\frac{2}{3}}\right] \times [14x]$	B1B1	
When $x=3$, $\frac{dy}{dx} = \frac{1}{3} \times (64)^{-\frac{2}{3}} \times 42$ $\left(=\frac{7}{8}\right)$	M1	
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{7}{8} \times 8$	DM1	Use chain rule
7	A1 [5]	

Question 29

(i) $x-3\sqrt{x}+2$ or k^2-3k+2 or $\left(3\sqrt{x}\right)^2=(x+2)^2$	**M1**	**OR** attempt to eliminate x eg sub $x=\frac{y^2}{9}$
$\sqrt{x}=1$ or 2 or $k=1$ or 2 or $x^2-5x+4(=0)$	**A1**	$y^2-9y+18=0$
$x=1$ or 4	**A1**	$y=3$ or 6
$y=3$ or 6	**A1**	$x=1$ or 4
	[4]	

(ii) $\int 3x^{\frac{1}{2}}\,dx-\left[\int(x+2)\,dx \text{ or attempt at trapezium}\right]$	**M1DM1**	Attempt to integrate. Subtract at some stage
$2x^{\frac{3}{2}}-\left[\left(\frac{1}{2}x^2+2x\right) \text{or} \frac{1}{2}(y_2+y_1)(x_2-x_1)\right]$	**A1A1**	Where $(x_1,y_1),(x_2,y_2)$ is *their* (1, 3), (4, 6)
$(16-2)-\left[\left[(8+8)-\left(\frac{1}{2}+2\right)\right] \text{or } \textit{their } \frac{1}{2}\times 9\times 3\right]$	**DM1**	Apply *their* 1→4 limits correctly to curve
$\frac{1}{2}$	**A1**	For A mark allow reverse subtn→ $-\frac{1}{2}\rightarrow\frac{1}{2}$ but not reversed limits
	[6]	
OR		
$\left[\int(y-2)\,dy \text{ or attempt at trap}\right]-\int\frac{y^2}{9}\,dy$	**M1DM1**	
$\left[\frac{1}{2}y^2-2y \text{ or } \frac{1}{2}(x_1+x_2)(y_2-y_1)\right]-\frac{y^3}{27}$	**A1A1**	
$\left[(18-12)-\left(4\frac{1}{2}-6\right) \text{or } \frac{1}{2}\times5\times3\right]-[8-1]$	**DM1**	Apply *their* 3→6 limits correctly to curve
$\frac{1}{2}$	**A1**	

Question 30

(i) Minimum since f ″(3) (= 4/3) > 0 www	**B1**	
	[1]	
(ii) $f'(x)=-18x^{-2}\ (+c)$	**B1**	
$0=-2+c$	**M1**	Sub $f'(3)=0$. (dep c present)
$c=2\ \left(\rightarrow f'(x)=-18x^{-2}+2\right)$	**A1**	$c=2$ sufficient at this stage
$f(x)=18x^{-1}+2x\ (+k)$	**B1√B1√**	Allow cx at this stage
$7=6+6+k$	**M1**	Sub f(3) = 3 (k present & numeric (or no) c)
$k=-5\rightarrow\left(f(x)=18x^{-1}+2x-5\right)$ cao	**A1**	
	[7]	

Question 31

(i) $(3x-2)^2+1$	**B1B1B1**	For either of 1st 2 marks bracket must be in the form $(ax+b)^2$ except for
		SCB2 for $9\left(x-\frac{2}{3}\right)^2+1$
	[3]	
(ii) $f'(x)=9x^2-12x+5$	**B1**	
$=$ their $(3x-2)^2+1$	**M1**	Ft from (i). Some reference/recognition
>0 (or $\geqslant 1$) hence an increasing function	**A1**	Allow >1. Allow *their* 1 provided positive.
	[3]	Allow a complete alt method (2/2 or 0/2)

Question 32

$\frac{d^2y}{dx^2}=\frac{24}{x^3}-4$		
(i) (If $x=2$) it's negative $\rightarrow$ Max	B1	www
	[1]	
(ii) $\left(\frac{dy}{dx}=\right)-12x^{-2}-4x+(A)$	B2,1,0	oe one per term
$=0$ when $x=2$	M1	Attempt at the constant A after ∫n
$\rightarrow A=11$	A1	co
	[4]	
(iii) $(y=)\ 12x^{-1}-2x^2+Ax+(c)$	B2,1,0 √	oe Doesn't need $+c$, but does need a term A to give "Ax".
$y=13$ when $x=1 \rightarrow c=-8$	M1	Attempt at c after ∫n
(If $x=2$) $y=12$	A1	co
	[4]	

Question 33

$y=x^3+ax^2+bx$		
(i) $\frac{dy}{dx}=3x^2+2ax+b$	B1	co
(ii) $b^2-4ac=4a^2-12b\ (<0)$	M1	Use of discriminant on their quadratic $\frac{dy}{dx}$ or other valid method
$\rightarrow a^2<3b$	A1	co – answer given
	[3]	
(iii) $y=x^3-6x^2+9x$		
$\frac{dy}{dx}=3x^2-12x+9<0$	M1	Attempt at differentiation
$=0$ when $x=1$ and 3	A1	co
$\rightarrow 1<x<3$	A1	condone $\leqslant$
	[3]	

Question 34

$y=\dfrac{12}{3-2x}$		
(i) Differential $= -12(3-2x)^{-2} \times -2$	B1 B1 [2]	co co (even if 1st B mark lost)
(ii) $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}=0.4\div 0.15$	M1	Chain rule used correctly (AEF)
$\rightarrow \dfrac{24}{(3-2x)^2}=\dfrac{8}{3}$	M1	Equates their $\dfrac{dy}{dx}$ with their $\dfrac{8}{3}$ or $\dfrac{3}{8}$
$\rightarrow x=0$ or 3	A1 A1 [4]	co co

Question 35

Vol = $(\pi)\int x^2 dy = (\pi)\int (y-1)\,dy$	M1 A1	Use of $\int x^2$ – not $\int y^2$ – ignore π co
Integral is $\frac{1}{2}y^2 - y$ or $\dfrac{(y-1)^2}{2}$ Limits for y are 1 to 5	B1	Sight of an integral sign with 1 and 5
$\rightarrow 8\pi$ or 25.1(AWRT)	A1 [4]	co (no π max 3/4)

Question 36

(i)	For $y=(4x+1)^{\frac{1}{2}}$, $\dfrac{dy}{dx}=\left[\tfrac{1}{2}(4x+1)^{-\frac{1}{2}}\right]\times[4]$	**B1B1**	
	When $x = 2$, gradient $m_1=\frac{2}{3}$	**B1**√	Ft from *their* derivative above
	For $y=\frac{1}{2}x^2+1$, $\dfrac{dy}{dx}=x\rightarrow$ gradient $m_2=2$	**B1**	
	$\alpha=\tan^{-1}m_2-\tan^{-1}m_1$	**M1**	
	$\alpha=63.43-33.69=29.7$ cao	**A1** **[6]**	
(ii)	$\int(4x+1)^{\frac{1}{2}}\,dx=\left[\dfrac{(4x+1)^{\frac{3}{2}}}{2/3}\right]\div[4]$	**B1B1**	
	$\int\left(\frac{1}{2}x^2+1\right)dx=\frac{1}{6}x^3+x$	**B1**	
	$\int_0^2(4x+1)^{\frac{1}{2}}\,dx=\frac{1}{6}[27-1]$, $\int_0^2\left(\frac{1}{2}x^2+1\right)dx=\left[\frac{8}{6}+2\right]$	**M1**	Apply limits $0\rightarrow 2$ to at least the 1st integral
	$\dfrac{13}{3}-\dfrac{10}{3}$	**M1**	Subtract the integrals (at some stage)
	1	**A1** **[6]**	

Question 37

(i)	$f'(2) = 4 - \frac{1}{2} = \frac{7}{2} \to$ gradient of normal $= -\frac{2}{7}$ $y - 6 = -\frac{2}{7}(x-2)$ AEF	B1M1 A1√ [3]	Ft from their $f'(2)$
(ii)	$f(x) = x^2 + \frac{2}{x}(+c)$ $6 = 4 + 1 + c \Rightarrow c = 1$	B1B1 M1A1 [4]	Sub (2, 6) – dependent on *c* being present
(iii)	$2x - \frac{2}{x^2} = 0 \Rightarrow 2x^3 - 2 = 0$ $x = 1$ $f''(x) = 2 + \frac{4}{x^3}$ or any valid method $f''(1) = 6$ OR > 0 hence minimum	M1 A1 M1 A1 [4]	Put $f'(x) = 0$ and attempt to solve Not necessary for last A mark as $x > 0$ given Dependent on everything correct

Question 38

(i)	$\frac{dy}{dx} = 6 - 6x$ At $x = 2$, gradient $= -6$ soi $y - 9 = -6(x-2)$ oe Expect $y = -6x + 21$ When $y = 0$, $x = 3\frac{1}{2}$ cao	B1 B1√ M1 A1 [4]	Line through (2, 9) and with gradient *their* −6
(ii)	Area under curve: $\int 9 + 6x - 3x^2 dx = 9x + 3x^2 - x^3$ $(27 + 27 - 27) - (18 + 12 - 8)$ Area under tangent: $\frac{1}{2} \times \frac{3}{2} \times 9 (= \frac{27}{4})$ Area required $\frac{27}{4} - 5 = \frac{7}{4}$	B2,1,0 M1 B1√ A1 [5]	Allow unsimplified terms Apply limits 2,3. Expect 5 OR $\int_2^{7/2}(-6x+21)\,dx \ (\to \frac{27}{4})$. Ft on *their* $-6x + 21$ and/or *their* 7/2.

Question 39

(i)	$-(x+1)^{-2} - 2(x+1)^{-3}$	M1A1 A1 [3]	M1 for recognisable attempt at differentn. Allow $\frac{-x^2 - 4x - 3}{(x+1)^4}$ from Q rule. (A2,1,0)
(ii)	$f'(x) < 0$ hence decreasing	B1 [1]	Dep. on *their* (i) < 0 for $x >$ 1
(iii)	$\frac{-1}{(x+1)^2} - \frac{2}{(x+1)^3} = 0$ **or** $\frac{-x^2 - 4x - 3}{(x+1)^4} = 0$ $\frac{-(x+1) - 2}{(x+1)^3} = 0 \to -x - 1 - 2 = 0$ or $-x^2 - 4x - 3 = 0$ $x = -3$, $y = -1/4$	M1* M1 Dep* A1A1 [4]	Set $\frac{dy}{dx}$ to 0 OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e.×mult) × multn $\to -(x+1)^3 - 2(x+1)^2 = 0$ (−3, −1/4) www scores 4/4

Question 40

$\left[\dfrac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷2] (+c)	**B1B1**	
$7 = 9 + c$	**M1**	Attempt subst $x = 4, y = 7$. c must be there. Dep. on attempt at integration.
$y = \dfrac{(2x+1)^{\frac{3}{2}}}{3} - 2$ or unsimplified	**A1** **[4]**	$c = -2$ sufficient

Question 41

	$y = \dfrac{4}{2x-1}$.	B1	Correct without the ÷2
(i)	$\displaystyle\int \frac{16}{(2x-1)^2}\,dx = \frac{-16}{2x-1} \div 2$	B1	For the ÷2 even if first B1 is lost
	Vol $= \pi\left[\dfrac{-8}{2x-1}\right]$ with limits 1 and 2	M1	Use of limits in a changed expression.
	$\rightarrow \dfrac{16\pi}{3}$	A1 [4]	co
(ii)	$m = \frac{1}{2}m$ of tangent $= -2$	M1	Use of $m_1 m_2 = -1$
	$\dfrac{dy}{dx} = \dfrac{-4}{(2x-1)^2} \times 2$	B1	Correct without the ×2
		B1	For the ×2 even if first B1 is lost
	Equating their $\dfrac{dy}{dx}$ to -2	DM1	
	$\rightarrow x = \frac{3}{2}$ or $-\frac{1}{2}$	A1	co
	$(y = 2$ or $-2)$		
	$\rightarrow c = \frac{5}{2}$ or $-\frac{7}{2}$	A1	co
		[6]	

Question 42

$u = 2x(y - x)$ and $x + 3y = 12$,		
$u = 2x\left(\dfrac{12 - x}{3} - x\right)$	M1 A1	Expresses u in terms of x
$= 8x - \dfrac{8x^2}{3}$		
$\dfrac{du}{dx} = 8 - \dfrac{16x}{3}$	M1	Differentiate candidate's quadratic, sets to 0 + attempt to find x, or other valid method
$= 0$ when $x = 1\frac{1}{2}$	A1	
$\rightarrow (y = 3\frac{1}{2})$	A1	Complete method that leads to u
$\rightarrow u = 6$	[5]	Co

Question 43

$f'(x) = 5 - 2x^2$ and (3, 5)		
$f(x) = 5x - \frac{2x^3}{3}$ (+c)	B1	For integral
Uses (3, 5)	M1	Uses the point in an integral
$\rightarrow c = 8$	A1	co
	[3]	

Question 44

	$y = \dfrac{8}{\sqrt{3x+4}}$		
(i)	$\dfrac{dy}{dx} = \dfrac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef	B1 B1	Without the "×3" For "×3" even if 1st B mark lost.
	$\rightarrow m_{(x=0)} = -\dfrac{3}{2}$ Perpendicular $m_{(x=0)} = \dfrac{2}{3}$	M1	Use of $m_1m_2 = -1$ after attempting to find $\dfrac{dy}{dx}_{(x=0)}$
	Eqn of normal $y - 4 = \dfrac{2}{3}(x - 0)$	M1	Unsimplified line equation
	Meets $x = 4$ at $B\left(4, \dfrac{20}{3}\right)$	A1 [5]	cao
(ii)	$\int \dfrac{8}{\sqrt{(3x+4)}}dx = \dfrac{8\sqrt{(3x+4))}}{\frac{1}{2}} \div 3$	B1 B1	Without "÷3". For "÷3"
	Limits from 0 to 4 $\rightarrow$ Area $P = \dfrac{32}{3}$	M1 A1	Correct use of correct limits. cao
	Area Q = Trapezium – P Area of Trapezium = $\dfrac{1}{2}\left(4 + \dfrac{20}{3}\right) \times 4 = \dfrac{64}{3}$	M1	Correct method for area of trapezium
	$\rightarrow$ Areas of P and Q are both $\dfrac{32}{3}$	A1 [6]	All correct.

Question 45

	$y = x^3 + px^2$		
(i)	$\frac{dy}{dx} = 3x^2 + 2px$	B1	cao
	Sets to 0 $\to$ $x = 0$ or $-\frac{2p}{3}$	M1	Sets differential to 0
	$\to$ (0, 0) or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	A1 A1 [4]	cao cao, first A1 for any correct turning point or any correct pair of x values. 2nd A1 for 2 complete TPs
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$	M1	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve
	At (0, 0) $\to$ $2p$ +ve Minimum	A1	www
	At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right) \to -2p$ –ve Maximum	A1 [3]	
(iii)	$y = x^3 + px^2 + px \to 3x^2 + 2px + p\ (= 0)$	B1	
	Uses $b^2 - 4ac$	M1	Any correct use of discriminant
	$\to 4p^2 - 12p < 0$		
	$\to$ $0 < p < 3$ aef	A1 [3]	cao (condone $\leqslant$)

Question 46

(i)	$24 = r + r + r\theta$		(May not use θ)
	$\to$ $\theta = \frac{24 - 2r}{r}$	M1	Attempt at $s = r\theta$ linked with 24 and r
	$A = \frac{1}{2}r^2\theta = \frac{24r}{2} - r^2 = 12r - r^2$. aef, ag	M1A1 [3]	Uses A formula with θ as f(r). cao
(ii)	$(A =) 36 - (r - 6)^2$	B1 B1 [2]	cao
(iii)	Greatest value of $A = 36$	B1√	Ft on **(ii).**
	$(r = 6) \to \theta = 2$	B1 [2]	cao, may use calculus or the discriminant on $12r - r^2$

Question 47

	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$		
(i)	$A = \frac{1}{2} \times (2 + p) \times 2p^2\ (= 2p^2 + p^3)$	M1 A1 [2]	Attempt at base and height in terms of p and use of $\frac{bh}{2}$
(ii)	$\frac{dA}{dp} = 4p + 3p^2$	B1	cao
	$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt} = 0.02 \times 20 = 0.4$	M1 A1	any correct method, cao
	or $\frac{dA}{dt} = 4p\frac{dp}{dt} + 3p^2\frac{dp}{dt}$	[3]	

Question 48

(i)	$f'(x) = 2 - 2(x+1)^{-3}$	**B1**	
	$f''(x) = 6(x+1)^{-4}$	**B1**	
	$f'0 = 0$ hence stationary at $x = 0$	**B1**	**AG**
	$f''0 = 6 > 0$ hence minimum	**B1** [4]	www. Dependent on correct $f''(x)$ except $-6(x+1)^{-4} \to < 0$ MAX scores SC1
(ii)	$AB^2 = (3/2)^2 + (3/4)^2$	**M1**	
	$AB = 1.68$ or $\sqrt{45/4}$ oe	**A1** [2]	
(iii)	Area under curve $= \int f(x) = x^2 - (x+1)^{-1}$	**B1**	Ignore $+c$ even if evaluated Do not penalise reversed limits
	$= \left(1 - \frac{1}{2}\right) - \left(\frac{1}{4} - 2\right) = 9/4$ (Apply limits $-\frac{1}{2} \to 1$)	**M1A1**	Allow reversed subtn if final ans positive
	Area trap. $= \frac{1}{2}(3 + \frac{9}{4}) \times \frac{3}{2}$	**M1**	
	$= 63/16$ or 3.94	**A1**	
	Shaded area $63/16 - 9/4 + 27/16$ or 1.69	**A1** [6]	
	ALT eqn AB is $y = -\frac{1}{2}x + 11/4$	**B1**	
	Area $= \int -\frac{1}{2}x + 11/4 - \int 2x + (x+1)^{-2}$	**M1**	Attempt integration of at least one
	$= \left[-\frac{1}{4}x^2 + \frac{11}{4}x\right] - \left[x^2 - (x+1)^{-1}\right]$	**A1A1**	Ignore $+c$ even if evaluated Dep. on integration having taken place
	Apply limits $-\frac{1}{2} \to 1$ to both integrals	**M1**	Allow reversed subtn if final ans positive
	27/16 or 1.69	**A1**	

Question 49

(i)	At $x=4$, $\frac{dy}{dx}=2$	**B1**	
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=2\times3=6$	**M1A1** [3]	Use of Chain rule
(ii)	$(y)=x+4x^{\frac{1}{2}}(+c)$	**B1**	
	Sub $x=4$, $y=6\rightarrow6=4+(4\times4^{\frac{1}{2}})+c$	**M1**	Must include c
	$c=-6\rightarrow(y=x+4x^{\frac{1}{2}}-6$	**A1** [3]	
(iii)	Eqn of tangent is $y-6=2(x-4)$ or $(6-0)/(4-x)=2$	**M1**	Correct eqn thru (4, 6) & with $m=$ *their* 2
		A1	
	$B=(1,0)$ (Allow $x=1$)	**M1**	[Expect eqn of normal: $y=-\frac{1}{2}x+8$]
	Gradient of normal $=-1/2$	**A1**	
	$C=(16,0)$ (Allow $x=16$)	**A1**	
	Area of triangle $=\frac{1}{2}\times15\times6=45$	[5]	Or $AB=\sqrt{45}$, $AC=\sqrt{180}\rightarrow$ Area = 45.0

Question 50

(i)	$[3]\left[(x-1)^2\right][-1]$	**B1B1B1** [3]	
(ii)	$f'(x)=3x^2-6x+7$	**B1**	Ft *their* **(i)** + 5
	$=3(x-1)^2+4$	**B1**√	
	>0 hence increasing	**DB1** [3]	Dep B1√ unless other valid reason

Question 51

	$y=\sqrt{(9-2x^2)}$ $P(2, 1)$		
(i)	$\frac{dy}{dx}=\frac{1}{2\sqrt{(9-2x^2)}}\times-4x$	**B1**	Without "$\times -4x$"
		B1	Allow even if B0 above.
	At P, $x=2$, $m=-4$ Normal grad = ¼	**M1**	For $m_1m_2=-1$ calculus needed
	Eqn AP $y-1=\frac{1}{4}(x-2)$	**M1**	Normal, not tangent
	$\rightarrow A(-2, 0)$ or $B(0, \frac{1}{2})$	**A1**	
	Midpoint AP also $(0, \frac{1}{2})$	**A1** [6]	Full justification.
(ii)	$\int x^2\,dy=\int\left(\frac{9}{2}-\frac{y^2}{2}\right)dy$	**M1**	Attempt to integrate x^2
	$=\frac{9y}{2}-\frac{y^3}{6}$	**A1**	Correct integration
	Upper limit = 3	**B1**	Evaluates upper limit
	Uses limits 1 to 3	**DM1**	Uses both limits correctly
	$\rightarrow$ volume $=4\frac{2}{3}\pi$	**A1** [5]	

Question 52

(i)	$f''(x) = \frac{12}{x^3}$		
	$f'(x) = -\frac{6}{x^2} \quad (+c)$	**B1**	Correct integration
	$= 0$ when $x = 2 \rightarrow c = \frac{3}{2}$	**M1 A1**	Uses $x = 2$, $f'(x = 0)$
	$f(x) = \frac{6}{x} + \frac{3x}{2} \quad (+A)$	**B1√B1√**	For each integral
	$= 10$ when $x = 2 \rightarrow A = 4$	**A1** [6]	
(ii)	$-\frac{6}{x^2} + \frac{3}{2} = 0 \rightarrow x = \pm 2$	**M1**	Sets their 2 term $f'(x)$ to 0.
	Other point is $(-2, -2)$	**A1** [2]	
(iii)	At $x = 2$, $f''(x) = 1.5$ Min	**B1**	
	At $x = -2$, $f''(x) = -1.5$ Max	**B1** [2]	

Question 53

(i)	$\tan 60 = \frac{x}{h} \rightarrow x = h \tan 60$	**B1**	Any correct unsimplified length
	$A = h \times x$	**M1**	Correct method for area
	$V = 40\sqrt{(3h^2)}$	**A1** [3]	ag
(ii)	$\frac{dV}{dh} = 80\sqrt{(3h)}$	**B1**	B1
	If $h = 5$, $\frac{dh}{dt} = \frac{1}{2\sqrt{(3)}}$ or 0.289	**M1A1** [3]	M1 (must be ÷, not ×).

Question 54

(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2}\right] \times [4]$	**B1B1**	
	At $x = 6, \frac{dy}{dx} = \frac{2}{5}$	**B1**	
	Gradient of normal at $P = -\frac{1}{2}$	**B1√**	**OR** eqn of norm $y - 5 = their -\frac{5}{2}(x-6)$
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal,		When $y = 0, x = 8$ hence result
	or $m_1 m_2 = -1$	**B1** [5]	
(ii)	Vol for curve $= (\pi)\int(1+4x)$ and attempt to integrate y^2	**M1**	
	$= (\pi)[x + 2x^2]$ ignore '+ c'	**A1**	
	$= (\pi)[6 + 72 - 0]$	**DM1**	Apply limits $0 \to 6$ (allow reversed if corrected later)
	$= 78(\pi)$	**A1**	
	Vol for line $= \frac{1}{3} \times (\pi) \times 5^2 \times 2$	**M1**	**OR** $(\pi)\left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}}\right]_6^8$
	$= \frac{50}{3}(\pi)$	**A1**	
	Total Vol $= 78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	**A1** [7]	

Question 55

(i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao	**B1B1**	
	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	**B1** [3]	
(ii)	$-\frac{8}{x^2} + 2 = 0 \to 2x^2 - 8 = 0$	**M1**	Set = 0 and rearrange to quadratic form
	$x = \pm 2$	**A1**	
	$y = \pm 8$	**A1**	If A0A0 scored, SCA1 for just (2, 8)
	$\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM	**B1√**	Ft for "correct" conclusion if $\frac{d^2y}{dx^2}$ incorrect or any valid method inc. a good sketch
	$\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	**B1√** [5]	

Question 56

$f(x) = x^3 - 7x (+c)$	**B1**	
$5 = 27 - 21 + c$	**M1**	Sub $x = 3$, $y = 5$. Dep. on c present
$c = -1 \rightarrow f(x) = x^3 - 7x - 1$	**A1** [3]	

Question 57

(i)	$x = 1/3$	**B1** [1]	
(ii)	$\frac{dy}{dx} = \left[\frac{2}{16}(3x-1)\right][3]$	**B1B1**	
	When $x = 3$ $\frac{dy}{dx} = 3$ soi	**M1**	
	Equation of QR is $y - 4 = 3(x-3)$	**M1**	
	When $y = 0$ $x = 5/3$	**A1** [5]	
(iii)	Area under curve $= \left[\frac{1}{16 \times 3}(3x-1)^3\right]\left[\times\frac{1}{3}\right]$	**B1B1**	
	$\frac{1}{16 \times 9}\left[8^3 - 0\right] = \frac{32}{9}$	**M1A1**	Apply limits: *their* $\frac{1}{3}$ and 3
	Area of $\Delta = 8/3$	**B1**	
	Shaded area $= \frac{32}{9} - \frac{8}{3} = \frac{8}{9}$ (or 0.889)	**A1** [6]	

Question 58

(i)	$A = 2\pi r^2 + 2\pi rh$	**B1**	
	$\pi r^2 h = 1000 \rightarrow h = \frac{1000}{\pi r^2}$	**M1**	
	Sub for h into A $\rightarrow$ $A = 2\pi r^2 + \frac{2000}{r}$ **AG**	**A1** [3]	
(ii)	$\frac{dA}{dr} = 0 \Rightarrow 4\pi r - \frac{2000}{r^2} = 0$	**M1A1**	Attempt differentiation & set = 0
	$r == 5.4$	**DM1 A1**	Reasonable attempt to solve to $r^3 =$
	$\frac{d^2A}{dr^2} = 4\pi + \frac{4000}{r^3}$		
	> 0 hence MIN hence MOST EFFICIENT **AG**	**B1** [5]	Or convincing alternative method

Question 59

$y = \frac{3x^3}{3} - \frac{2x^{-2}}{-2}$ $(+c)$	**B1B1**	
$3 = -1 + 1 + c$	**M1**	Sub $x = -1, y = 3$. c must be present
$y = x^3 + x^{-2} + 3$	**A1** [4]	Accept $c = 3$ www

Question 60

$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$	B1	
$\frac{dy}{dx} = 2$	B1	
$2x - 5x^{1/2} + 5 = 2$	M1	Equate their dy/dx to *their* 2 or ½.
$2x - 5x^{1/2} + 3(=0)$ or equivalent 3-term quadratic	A1	
Attempt to solve for $x^{1/2}$ e.g. $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$	DM1	Dep. on 3-term quadratic
$x^{1/2} = 3/2$ and 1	A1	ALT
$x = 9/4$ and 1	A1 [7]	$5x^{½} = 2x + 3 \to 25x = (2x+3)^2$ $4x^2 - 13x + 9(=0)$ $x = 9/4$ and 1

Question 61

$(\pi)\int(x^3 + 1)\,dx$	M1	Attempt to resolve y^2 and attempt to integrate
$(\pi)\left[\frac{x^4}{4} + x\right]$	A1	
6π or 18.8	DM1A1 [4]	Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is 6π)

Question 62

(i)	$6 + k = 2 \to k = -4$	B1 [1]	
(ii)	$(y) = \frac{6x^3}{3} - \frac{4}{-2}x^{-2}$ $(+c)$	B1B1√	ft on *their k*. Accept $+\frac{k}{-2}x^{-2}$
	$9 = 2 + 2 + c$ *c* must be present	M1	Sub (1,9) with numerical *k*. Dep on attempt $\int$
	$(y) = 2x^3 + 2x^{-2} + 5$	A1 [4]	Equation needs to be seen Sub (2, 3) $\to c = -13½$ scores M1A0

Question 63

$\frac{dy}{dx} = [8] + [-2]\,[(2x-1)^{-2}]$	B2,1,0	
$= 0 \to 4(2x-1)^2 = 1$ oe eg $16x^2 - 16x + 3 = 0$	M1	Set to zero, simplify and attempt to solve soi
$x = \frac{1}{4}$ and $\frac{3}{4}$	A1	Needs both *x* values. Ignore *y* values
$\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$	B1√*	ft to $k(2x-1)^{-3}$ where $k > 0$
When $x = \frac{1}{4}$, $\frac{d^2y}{dx^2}(=-64)$ and/or < 0 MAX	DB1	Alt. methods for last 3 marks (values either side of 1/4 & 3/4) must indicate <u>which</u> *x*-values and cannot use $x = 1/2$. (M1A1A1)
When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2}(=64)$ and/or > 0 MIN	DB1 [7]	

Question 64

	$y=\frac{8}{x}+2x.$		
(i)	$\frac{dy}{dx}=-8x^{-2}+2$	**B1**	unsimplified ok
	$\frac{d^2y}{dx^2}=16x^{-3}$	**B1**	unsimplified ok
	$\int y^2dx=-64x^{-1}$ oe $+32x$ oe $+\frac{4x^3}{3}$ **oe** $(+c)$	**3 × B1** [5]	B1 for each term – unsimplified ok

(ii)	sets $\frac{dy}{dx}$ to $0\rightarrow x=\pm2$	**M1**	Sets to 0 and attempts to solve
	$\rightarrow M(2, 8)$	**A1**	Any pair of correct values A1
	Other turning point is $(-2, -8)$	**A1**	Second pair of values A1
	If $x=-2$, $\frac{d^2y}{dx^2}<0$	**M1**	Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x<0$
	∴Maximum	**A1** [5]	
(iii)	Vol $=\pi\times$ [part (i)] from 1 to 2	**M1**	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
	$\frac{220\pi}{3}, 73.3\pi, 230$	**A1** [2]	

Question 65

$f'(x)=\frac{8}{(5-2x)^2}$		
$f(x)=\frac{8(5-2x)^{-1}}{-1}\div-2\ (+c)$	**B1**	Correct without (÷ by −2)
	B1	An attempt at integration (÷ by−2)
Uses $x=2$, $y=7$,	**M1**	Substitution of correct values into an integral to find c
$c=3$	**A1** [4]	

Question 66

(i)	$A=2y\times4x(=8xy)$	**B1**	
	$10y+12x=480$	**B1**	
	$\rightarrow A=384x-9.6x^2$	**B1** [3]	answer given

(ii)	$\frac{dA}{dx}=384-19.2x$	**B1**	
	$=0$ when $x=20$	**M1**	Sets to 0 and attempt to solve oe Might see completion of square
	$\rightarrow x=20, y=24.$	**A1**	Needs both x and y
	Uses $x=-\frac{b}{2a}=\frac{-384}{-19.2}=20$, **M1, A1** $y=24$, A1 From graph: **B1** for $x=20$, **M1**, **A1** for $y=24$	[3]	Trial and improvement **B3**.

Question 67

(i)	$\frac{dy}{dx}=2-8(3x+4)^{-½}$ $(x=0, \to \frac{dy}{dx}=-2)$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt} \to -0.6$	M1A1 [2]	Ignore notation. Must be $\frac{dy}{dx}\times 0.3$
(ii)	$y=\{2x\}\left\{-\frac{8\sqrt{3x+4}}{\frac{1}{2}}\div 3\right\} \quad (+c)$	**B1 B1**	No need for $+c$.
	$x=0, y=\frac{4}{3} \to c=12.$	**M1 A1** [4]	Uses x, y values after ∫ with c

Question 68

$x=\frac{12}{y^2}-2.$ Vol = $(\pi)\times\int x^2\,dy$ $\to\left[\frac{-144}{3y^3}+4y+\frac{48}{y}\right]$	M1 3 × A1	Ignore omission of π at this stage Attempt at integration Un-simplified
Limits 1 to 2 used $\to 22\pi$	**A1** [5]	only from correct integration

Question 69

(i)	Attempt diffn. and equate to 0 $\frac{dy}{dx}=-k(kx-3)^{-2}+k=0$	***M1**		Must contain $(kx-3)^{-2}$ + other term(s)
	$(kx-3)^2=1$ or $k^3x^2-6k^2x+8k(=0)$	**DM1**		Simplify to a quadratic
	$x=\frac{2}{k}$ or $\frac{4}{k}$	***A1*A1**		Legitimately obtained
	$\frac{d^2y}{dx^2}=2k^2(kx-3)^{-3}$	**B1**√		Ft must contain $Ak^2(kx-3)^{-3}$ where $A>0$
	When $x=\frac{2}{k}$, $\frac{d^2y}{dx^2}=(-2k^2)<0$ MAX All previous	**DB1**		Convincing alt. methods (values either side) must show which values used & cannot use $x=3/k$
	When $x=\frac{4}{k}$, $\frac{d^2y}{dx^2}=(2k^2)>0$ MIN working correct	**DB1**	[7]	
(ii)	$V=(\pi)\int\left[(x-3)^{-1}+(x-3)\right]^2 dx$	***M1**		Attempt to expand y^2 and then integrate
	$=(\pi)\int[(x-3)^{-2}+(x-3)^2+2]dx$	**A1**		
	$=(\pi)\left[-(x-3)^{-1}+\frac{(x-3)^3}{3}(+2x)\right]$ Condone missing $2x$	**A1**		Or $\left[-(x-3)^{-1}+\frac{x^3}{3}-3x^2+9x+2x\right]$
	$=(\pi)\left[1-\frac{1}{3}+4-\left(\frac{1}{3}-9+0\right)\right]$	**DM1**		Apply limits 0→2
	$=40\pi/3$ oe or 41.9	**A1**	[5]	2 missing → $28\pi/3$ scores M1A0A1M1A0

Question 70

(i)	at $x=a^2$, $\frac{dy}{dx}=\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}\left(=\frac{3}{a^2}\text{ or }3a^{-2}\right)$	**B1**		$\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}$ seen anywhere in (**i**)
	$y-3=\frac{3}{a^2}\left(x-a^2\right)$ or $y=\frac{3}{a^2}x+c\to 3=\frac{3}{a^2}a^2+c$	**M1**		Through $(a^2,3)$ & with *their* grad as f(a)
	$y=\frac{3}{a^2}x$ or $3a^{-2}x$ cao	**A1**	[3]	
(ii)	$(y)=\frac{2}{a}\frac{x^{½}}{½}+\frac{ax^{-½}}{-½}$ $(+c)$	**B1B1**		
	sub $x=a^2$, $y=3$ into $\int dy/dx$	**M1**		c must be present. Expect $3=4-2+c$
	$c=1$ $\left(y=\frac{4x^{½}}{a}-2ax^{-½}+1\right)$	**A1**	[4]	
(iii)	sub $x=16$, $y=8\to 8=\frac{4}{a}\times 4-2a\times\frac{1}{4}+1$	***M1**		Sub into *their y*
	$a^2+14a-32(=0)$	**A1**		
	$a=2$	**A1**		Allow -16 in addition
	$A=(4,3)$, $B=(16,8)$ $AB^2=12^2+5^2\to AB=13$	**DM1A1**	[5]	

Question 71

$f'(x)=3x^2-6x-9$ soi	**B1**		
Attempt to solve $f'(x)=0$ or $f'(x)>0$ or $f'(x)\geqslant 0$ soi	**M1**		
$(3)(x-3)(x+1)$ or $3,-1$ seen or 3 only seen	**A1**		With or without equality/inequality signs
Least possible value of n is 3. Accept $n=3$. Accept $n\geqslant 3$	**A1**	[4]	Must be in terms of n

Question 72

(i)	$\frac{dy}{dx}=\frac{-3}{(2x-1)^2}\times 2$	**B1**		B1for a single correct term (unsimplified) without ×2.
		B1	[2]	
(ii)	e.g. Solve for $\frac{dy}{dx}=0$ is impossible.	**B1√**	[1]	Satisfactory explanation.
(iii)	If $x=2$, $\frac{dy}{dx}=\frac{-6}{9}$ and $y=3$	**M1***		Attempt at both needed.
	Perpendicular has $m=\frac{9}{6}$	**M1***		Use of $m_1m_2=-1$ numerically.
	$\to$ $y-3=\frac{3}{2}(x-2)$	**DM1**		Line equation using (2, their 3) and their m.
	Shows when x=0 then y=0 AG	**A1**	[4]	
(iv)	$\frac{dx}{dt}=-0.06$			
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\to$ $-\frac{2}{3}\times -0.06=0.04$	**M1 A1**	[2]	

Question 73

$(y) = 8(4x+1)^{\frac{1}{2}} \div ½ \div 4\ (+c)$	B1 B1		Correct integrand (unsimplified) without ÷4 ÷4. Ignore *c*.
Uses $x = 2$ and $y = 5$	M1		Substitution of correct values into an integrand to find c.
$c = -7$	A1	[4]	$y = 4\sqrt{4x+1} - 7$

Question 74

(i)	$3z - \frac{2}{z} = -1 \Rightarrow 3z^2 + z - 2 = 0$	M1		Express as 3-term quad. Accept $x^{1/2}$ for z
	$x^{1/2}$ (or z) $= 2/3$ or -1	A1		(OR
	$x = 4/9$ only	A1	[3]	$3x - 1 = -\sqrt{x},\ 9x^2 - 13x + 4 = 0$ **M1, A1,A1** $x = 4/9$)
(ii)	$f(x) = \frac{3x^{3/2}}{3/2} - \frac{2x^{1/2}}{1/2}\quad (+c)$	B1B1		
	Sub $x = 4, y = 10\quad 10 = 16 - 8 + c \Rightarrow c = 2$	M1A1		c must be present
	When $x = \frac{4}{9},\ y = 2\left(\frac{4}{9}\right)^{3/2} - 4\left(\frac{4}{9}\right)^{1/2} + 2$	M1		Substituting x value from part **(i)**
	$-2/27$	A1	[6]	

Question 75

(i)	$\frac{dy}{dx} = -(x-1)^{-2} + 9(x-5)^{-2}$	M1A1		May be seen in part **(ii)**
	$m_{tangent} = -\frac{1}{4} + \frac{9}{4} = 2$	B1		
	Equation of normal is $y - 5 = -½(x-3)$	M1		Through (3, 5) and with $m = -1/m_{tangent}$
	$x = 13$	A1	[5]	
(ii)	$(x-5)^2 = 9(x-1)^2$	B1		Set $\frac{dy}{dx} = 0$ and simplify
	$x - 5 = (\pm)3(x-1)$ or $(8)(x^2 - x - 2) = 0$	M1		Simplify further and attempt solution
	$x = -1$ or 2	A1		
	$\frac{d^2y}{dx^2} = 2(x-1)^{-3} - 18(x-5)^{-3}$	B1		If change of sign used, x values close to the roots must be used and all must be correct
	When $x = -1,\ \frac{d^2y}{dx^2} = -\frac{1}{6} < 0$ MAX	B1		
	When $x = 2,\ \frac{d^2y}{dx^2} = \frac{8}{3} > 0$ MIN	B1	[6]	

Question 76

(i)	$A = (½, 0)$	**B1**	[1]	Accept $x = 0 \, at \, y = 0$
(ii)	$\int (1-2x)^{\frac{1}{2}} dx = \left[\frac{(1-2x)^{3/2}}{3/2}\right][\div(-2)]$	**B1B1**		May be seen in a single expression
	$\int (2x-1)^2 dx = \left[\frac{(2x-1)^3}{3}\right][\div 2]$	**B1B1**		May use $\int_a^1 x \, dy$, may expand
	$[0-(-1/3)]-[0-(-1/6)]$	**M1**		$(2x-1)^2$
	1/6	**A1**	[6]	Correct use of *their* limits

Question 77

(i)	$2x - 2/x^3 = 0$	M1	Set = 0.
	$x^4 = 1 \Rightarrow x = 1$ at A cao	A1	Allow 'spotted' $x = 1$
	Total:	2	
(ii)	$f(x) = x^2 + 1/x^2 (+c)$ cao	B1	
	$\frac{189}{16} = 16 + 1/16 + c$	M1	Sub (4, $\frac{189}{16}$). c must be present. Dep. on integration
	$c = -17/4$	A1	
	Total:	3	
(iii)	$x^2 + 1/x^2 - 17/4 = 0 \Rightarrow 4x^4 - 17x^2 + 4 \ (=0)$	M1	Multiply by $4x^2$ (or similar) to transform into 3-term quartic.
	$(4x^2-1)(x^2-4) \ (=0)$	M1	Treat as quadratic in x^2 and attempt solution or factorisation.
	$x = ½, \ 2$	A1A1	Not necessary to distinguish. Ignore negative values. No working scores 0/4
	Total:	4	
(iv)	$\int (x^2 + x^{-2} - 17/4) dx = \frac{x^3}{3} - \frac{1}{x} - \frac{17x}{4}$	B2,1,0√	Mark final integral
	$(8/3 - 1/2 - 17/2) - (1/24 - 2 - 17/8)$	M1	Apply *their* limits from (iii) (Seen). Dep. on integration of at least 1 term of y
	Area = $9/4$	A1	Mark final answer. $\int y^2$ scores 0/4
	Total:	4	

Question 78

(i)	$\frac{dy}{dx} = 2x - 2$. At $x = 2$, $m = 2$	B1B1	Numerical m
	Equation of tangent is $y - 2 = 2(x - 2)$	B1	Expect $y = 2x - 2$
	Total:	3	
(ii)	Equation of normal $y - 2 = -½(x - 2)$	M1	Through (2, 2) with gradient $= -1/m$. Expect $y = -½x + 3$
	$x^2 - 2x + 2 = -½x + 3 \rightarrow 2x^2 - 3x - 2 = 0$	M1	Equate and simplify to 3-term quadratic
	$x = -½,\ y = 3¼$	A1A1	Ignore answer of (2, 2)
	Total:	4	
(iii)	At $x = -½$, grad$= 2(-½) - 2 = -3$	B1√	Ft *their* $-½$.
	Equation of tangent is $y - 3¼ = -3(x + ½)$	*M1	Through *their B* with grad *their* -3 (not m_1 or m_2). Expect $y = -3x + 7/4$
	$2x - 2 = -3x + 7/4$	DM1	Equate *their* tangents or attempt to solve simultaneous equations
	$x = 3/4,\ y = -½$	A1	Both required.
	Total:	4	

Question 79

(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right]\ [4]$	B1B1	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6 \times 1/2 \times (4x+1)^{-1/2} \times 4$	B1√	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* $f'(x)$.
	Total:	3	
(ii)	$f(2)$, $f'(2)$, $kf''(2) = 27, 18, 4k$ OR 12	B1B1√B1√	Ft dependent on attempt at differentiation
	$27/18 = 18/4k$ oe OR $kf''(2) = 12 \Rightarrow k = 3$	M1A1	
	Total:	5	

Question 80

(i)	$V = \frac{1}{12}h^3$ oe	B1	
	Total:	1	
(ii)	$\frac{dV}{dh} = \frac{1}{4}h^2$ or $\frac{dh}{dV} = 4(12v)^{-2/3}$	M1A1	Attempt differentiation. Allow incorrect notation for M. For A mark accept *their* letter for volume - but otherwise correct notation. Allow V'
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \quad = \frac{4}{h^2} \times 20$ soi	DM1	Use chain rule correctly with $\frac{d(V)}{dt} = 20$. Any equivalent formulation Accept non-explicit chain rule (or nothing at all)
	$\left(\frac{dh}{dt}\right) = \frac{4}{10^2} \times 20 = 0.8$ or equivalent fraction	A1	
	Total:	4	

Question 81

(i)	$f'(x)=[(4x+1)^{1/2} \div ½] [\div 4]$ $(+c)$	**B1 B1**	Expect $½(4x+1)^{1/2}$ $(+c)$
	$f'(2)=0 \Rightarrow \frac{3}{2}+c=0 \Rightarrow c=-\frac{3}{2}$ (Sufficient)	**B1 FT**	Expect $½(4x+1)^{1/2}-\frac{3}{2}$. FT on *their* $f'(x)=k(4x+1)^{1/2}+c$. (i.e. $c = -3k$)
	Total:	**3**	
(ii)	$f''(0)=1$ SOI	**B1**	
	$f'(0)=1/2-1½=-1$ SOI	**B1 FT**	Substitute x = 0 into *their* f ′(x) but must not involve c otherwise **B0B0**
	f(0) = – 3	**B1 FT**	FT for 3 terms in AP. FT for 3rd **B1** dep on 1st **B1**. Award marks for the AP method only.
	Total:	**3**	
(iii)	$f(x)=\left[½(4x+1)^{3/2} \div 3/2 \div 4\right]-\left[1½x\right]$ $(+k)$	**B1 FT** **B1 FT**	Expect $(1/12)(4x+1)^{3/2}-1½x$ $(+k)$. FT from *their* f ′(x) but c numerical.
	$-3=1/12-0+k \Rightarrow k=-37/12$ CAO	**M1A1**	Sub $x=0, y=$ *their* f(0) into *their* f(x). Dep on cx & k present (c numerical)
	Minimum value = f(2) = $\frac{27}{12}-3-\frac{37}{12}=-\frac{23}{6}$ or – 3.83	**A1**	
	Total:	**5**	

Question 82

(a)(i)	Attempt to integrate $V=(\pi)\int(y+1)dy$	**M1**	Use of h in integral e.g. $\int(h+1)=½h^2+h$ is **M0**. Use of $\int y^2 dx$ is **M0**
	$=(\pi)\left[\frac{y^2}{2}+y\right]$	**A1**	
	$=\pi\left[\frac{h^2}{2}+h\right]$	**A1**	**AG**. Must be from clear use of limits 0→ h somewhere.
	Total:	**3**	
0(ii)	$\int(y+1)^{1/2}dy$ **ALT** $6-\int(x^2-1)dx$	**M1**	Correct variable and attempt to integrate
	$⅔(y+1)^{3/2}$ oe **ALT** $6-(⅓x^3-x)$ CAO	***A1**	Result of integration must be shown
	$⅔[8-1]$ **ALT** $6-[\left(\frac{8}{3}-1\right)-\left(\frac{1}{3}-1\right)]$	**DM1**	Calculation seen with limits 0→3 for y. For ALT, limits are 1→2 and rectangle.
	14/3 **ALT** 6 – 4/3 = 14/3	**A1**	16/3 from $⅔ \times 8$ gets **DM1A0** provided work is correct up to applying limits.
	Total:	**4**	
(b)	Clear attempt to differentiate wrt h	**M1**	Expect $\frac{dV}{dh}=\pi(h+1)$. Allow h + 1. Allow h.
	Derivative = 4π SOI	***A1**	
	$\frac{2}{\textit{their}\text{ derivative}}$. Can be in terms of h	**DM1**	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	**A1**	
	Total:	**4**	

Question 83

Gradient of normal is – 1/3 → gradient of tangent is 3 SOI	**B1 B1 FT**	FT from *their* gradient of normal.
$dy/dx = 2x - 5 = 3$	**M1**	Differentiate and set = *their* 3 (numerical).
$x = 4$	***A1**	
Sub $x = 4$ into line → $y = 7$ & sub *their* (4, 7) into curve	**DM1**	OR sub $x = 4$ into curve → $y = k - 4$ and sub *their*$(4, k - 4)$ into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
$k = 11$	**A1**	
Total:	**6**	

Question 84

(i)	$\frac{dy}{dx} = 4x^{-½} - 2$	**B1**	Accept unsimplified.
	$= 0$ when $\sqrt{x} = 2$		
	$x = 4, y = 8$	**B1B1**	
	Total:	**3**	
(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	**B1FT**	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right)$ → Maximum	**B1**	Correct $\frac{d^2y}{dx^2}$ and x=4 in (i) are required. Followed by"< 0 or negative" is sufficient" but $\frac{d^2y}{dx^2}$ must be correct evaluated.
	Total:	**2**	
iii)	*EITHER:* Recognises a quadratic in $\sqrt{x}$	**(M1**	Eg $\sqrt{x} = u \rightarrow 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	
	OR: Rearranges then squares	**(M1**	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow x^2 - 10x + 9 = 0$ oe	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	Both correct by trial and improvement gets 3/3
	Total:	**3**	
iv)	$k > 8$	**B1**	
	Total:	**1**	

Question 85

$\text{Vol} = \pi \int (5-x)^2\,dx - \pi \int \frac{16}{x^2}\,dx$	**M1***	Use of volume formula at least once, condone omission of π and limits dx .
	DM1	Subtracting volumes somewhere must be after squaring.
$\int (5-x)^2 dx = \frac{(5-x)^3}{3} \div -1$	**B1 B1**	**B1** Without $\div(-1)$. **B1** for $\div(-1)$
(or $25x - 10x^2/2 + \frac{1}{3}x^3$)	**(B2,1,0)**	-1 for each incorrect term
$\int \frac{16}{x^2} dx = -\frac{16}{x}$	**B1**	
Use of limits 1 and 4 in an integrated expression and subtracted.	**DM1**	Must have used "y^2" at least once. Need to see values substituted.
$\rightarrow 9\pi$ or 28.3	**A1**	
Total:	**7**	

Question 86

(i)	Crosses x-axis at (6, 0)	**B1**	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0 +) -12\,(2-x)^{-2} \times (-1)$	**B2,1,0**	-1 for each incorrect term of the three or addition of + C.
	Tangent $y = \frac{3}{4}(x-6)$ or $4y = 3x - 18$	**M1 A1**	Must use dy/dx, $x =$ their 6 but not $x = 0$ (which gives $m = 3$), and correct form of line equation.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**5**	
(ii)	If $x = 4$, $dy/dx = 3$		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	**M1 A1FT**	**M1** for ("their m" from $\frac{dy}{dx}$ and $x = 4$) × 0.04. Be aware: use of $x = 0$ gives the correct answer but gets **M0**.
	Total:	**2**	

Question 87

(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	**B1 B1**	**B1** without $\times(-3)$ **B1** For $\times(-3)$
	Gradient of tangent = 3, Gradient of normal $-\frac{1}{3}$	***M1**	Use of $m_1 m_2 = -1$ after calculus
	$\rightarrow$ eqn: $y - 2 = -\frac{1}{3}(x-1)$	**DM1**	Correct form of equation, with (1, their y), not (1,0)
	$\rightarrow y = -\frac{1}{3}x + \frac{7}{3}$	**A1**	This mark needs to have come from $y = 2$, y must be subject
	Total:	**5**	
(ii)	$\text{Vol} = \pi \int_0^1 \frac{16}{(5-3x)^2} dx$	**M1**	Use of $V = \pi \int y^2 dx$ with an attempt at integration
	$\pi \left[\frac{-16}{(5-3x)} \div -3 \right]$	**A1 A1**	**A1** without$(\div -3)$, **A1** for $(\div -3)$
	$= (\pi \left(\frac{16}{6} - \frac{16}{15} \right)) = \frac{8\pi}{5}$ (if limits switched must show – to +)	**M1 A1**	Use of both correct limits **M1**
	Total:	**5**	

Question 88

(i)	$y = 7x - \frac{x^3}{3} - \frac{6x^2}{2}$ $(+c)$	**B1**	CAO
	Uses (3, − 10) → $c = 5$	**M1 A1**	Uses the given point to find c
	Total:	**3**	
(ii)	$7 - x^2 - 6x = 16 - (x+3)^2$	**B1 B1**	**B1** $a = 16$, **B1** $b = 3$.
	Total:	**2**	
(iii)	$16 - (x+3)^2 > 0 \rightarrow (x+3)^2 < 16$, and solve	**M1**	or factors $(x + 7)(x − 1)$
	End-points $x = 1$ or -7	**A1**	
	$\rightarrow -7 < x < 1$	**A1**	needs <, not ⩽. (SR $x < 1$ only, or $x > -7$ only **B1** i.e. 1/3)
	Total:	**3**	

Question 89

(i)	Volume $= \left(\frac{1}{2}\right)x^2\frac{\sqrt{3}}{2}h = 2000 \rightarrow h = \frac{8000}{\sqrt{3}x^2}$	**M1**	Use of (area of triangle, with attempt at ht) × h =2000, $h = \mathrm{f}(x)$
	$A = 3xh + (2)\times\left(\frac{1}{2}\right)\times x^2 \times \frac{\sqrt{3}}{2}$	**M1**	Uses 3 rectangles and at least one triangle
	Sub for $h \rightarrow A = \frac{\sqrt{3}}{2}x^2 + \frac{24000}{\sqrt{3}}x^{-1}$	**A1**	AG
	Total:	**3**	
(ii)	$\frac{dA}{dx} = \frac{\sqrt{3}}{2}2x - \frac{24000}{\sqrt{3}}x^{-2}$	**B1**	CAO, allow decimal equivalent
	$= 0$ when $x^3 = 8000 \rightarrow x = 20$	**M1 A1**	Sets their $\frac{dA}{dx}$ to 0 and attempt to solve for x
	Total:	**3**	
(iii)	$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}2 + \frac{48000}{\sqrt{3}}x^{-3} > 0$	**M1**	Any valid method, ignore value of $\frac{d^2A}{dx^2}$ providing it is positive
	→ Minimum	**A1 FT**	FT on their x providing it is positive
	Total:	**2**	

Question 90

1(i)	Gradient of $AB = \frac{1}{2}$	B1	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	B1	
		2	
(ii)	$\frac{dy}{dx} = ½(x-1)^{-\frac{1}{2}}$	B1	
	$½(x-1)^{-\frac{1}{2}} = ½$. Equate *their* $\frac{dy}{dx}$ to *their* ½	*M1	
	$x = 2, \quad y = 1$	A1	
	$y - 1 = ½(x - 2)$ (thro' *their*(2,1) & *their* ½) $\rightarrow y = ½x$	DM1 A1	
		5	
1(iii)	*EITHER:* $\sin\theta = \frac{d}{1} \rightarrow d = \sin\theta$	(M1	Where θ is angle between AB and the x-axis
	gradient of $AB = ½ \Rightarrow \tan\theta = ½ \Rightarrow \theta = 26.5(7)°$	B1	
	$d = \sin 26.5(7)° = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR1: Perpendicular through O has equation $y = -2x$	(M1	
	Intersection with AB: $-2x = ½x - ½ \rightarrow \left(\frac{1}{5}, \frac{-2}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR2: Perpendicular through (2, 1) has equation $y = -2x + 5$	(M1	
	Intersection with AB: $-2x + 5 = ½x - ½ \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$)	A1)	
1(iii)	*OR3:* ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	(B1	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \rightarrow d = \frac{1}{\sqrt{5}}$	M1 A1)	
		3	

Question 91

(i)	$ax^2+bx=0 \rightarrow x(ax+b)=0 \rightarrow x=\frac{-b}{a}$	B1	
	Find $f''(x)$ and attempt sub *their* $\frac{-b}{a}$ into *their* $f''(x)$	M1	
	When $x=\frac{-b}{a}$, $f''(x)=2a\left(\frac{-b}{a}\right)+b=-b$ MAX	A1	
		3	
(ii)	Sub $f'(-2)=0$	M1	
	Sub $f'(1)=9$	M1	
	$a=3 \quad b=6$	*A1	Solve simultaneously to give both results.
	$f'(x)=3x^2+6x \rightarrow f(x)=x^3+3x^2\ (+c)$	*M1	Sub *their a, b* into $f'(x)$ and integrate 'correctly'. Allow $\frac{ax^3}{3}+\frac{bx^2}{2}(+c)$
	$-3=-8+12+c$	DM1	Sub $x=-2,\ y=-3$. Dependent on c present. Dependent also on a, b substituted.
	$f(x)=x^3+3x^2-7$	A1	
		6	

Question 92

(i)	*EITHER:* $4-3\sqrt{x}=3-2x \rightarrow 2x-3\sqrt{x}+1\ (=0)$ or e.g. $2k^2-3k+1\ (=0)$	(M1	Form 3-term quad & attempt to solve for $\sqrt{x}$.
	$\sqrt{x}=½,\ 1$	A1	Or $k=½$ or 1 (where $k=\sqrt{x}$).
	$x=¼,\ 1$	A1)	
	OR1: $(3\sqrt{x})^2=(1+2x)^2$	(M1	
	$4x^2-5x+1\ (=0)$	A1	
	$x=¼,\ 1$	A1)	
	OR2: $\frac{3-y}{2}=\left(\frac{4-y}{3}\right)^2\ \left(\rightarrow 2y^2-7y+5(=0)\right)$	(M1	Eliminate x
	$y=\frac{5}{2},\ 1$	A1	
	$x=¼,\ 1$	A1)	
		3	

(ii)	*EITHER:* Area under line $= \int (3-2x)\,dx = 3x - x^2$	(B1	
	$= \left[(3-1) - \left(\frac{3}{4} - \frac{1}{16}\right)\right]$	M1	Apply *their* limits (e.g. ¼ → 1) after integn.
	Area under curve $= \int \left(4 - 3x^{1/2}\right) dx \;=\; 4x - 2x^{3/2}$	B1	
	$\left[(4-2) - (1 - ¼)\right]$	M1	Apply *their* limits (e.g. ¼ → 1) after integration.
	Required area $= \frac{21}{16} - \frac{5}{4} = \frac{1}{16}$ (or 0.0625)	A1)	
	OR: $+/-\int (3-2x) - \left(4 - 3x^{\frac{1}{2}}\right) = +/-\int (-1 - 2x + 3x^{\frac{1}{2}})$	(*M1	Subtract functions and then attempt integration
	$+/-\left[-x - x^2 + \frac{3x^{3/2}}{3/2}\right]$	A2, 1, 0 FT	FT on *their* subtraction. Deduct 1 mark for each term incorrect
	$+/-\left[-1 - 1 + 2 - \left(-\frac{1}{4} + \frac{1}{16} + \frac{1}{8}\right)\right] = \frac{1}{16}$ (or 0.0625)	DM1 A1)	Apply *their* limits ¼ → 1
		5	

Question 93

$f'(x) = \left[\left(\frac{3}{2}\right)(2x-1)^{1/2}\right] \times [2] - [6]$	B2, 1, 0	Deduct 1 mark for each [...] incorrect.
$f'(x) < 0$ or $\leqslant 0$ or $= 0$ SOI	M1	
$(2x-1)^{1/2} < 2$ or $\leqslant 2$ or $= 2$ OE	A1	Allow with k used instead of x
Largest value of k is $\frac{5}{2}$	A1	Allow $k \leqslant \frac{5}{2}$ or $k = \frac{5}{2}$ Answer must be in terms of k (not x)
	5	

Question 94

(i)	$\frac{dy}{dx} = \frac{1}{2} \times (5x-1)^{-\frac{1}{2}} \times 5 \quad (= \frac{5}{6})$	B1 B1	**B1** Without × 5 **B1** × 5 of an attempt at differentiation
	m of normal = $-\frac{6}{5}$	M1	Uses $m_1 m_2 = -1$ with their numeric value from their dy/dx
	Equation of normal $y - 3 = -\frac{6}{5}(x-2)$ OE or $5y + 6x = 27$ or $y = \frac{-6}{5}x + \frac{27}{5}$	A1	Unsimplified. Can use $y = mx + c$ to get $c = 5.4$ ISW
(ii)	*EITHER:* For the curve $(\int)\sqrt{5x-1}dx = \frac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} \div 5$	(B1 B1	Correct expression without ÷5 For dividing an attempt at integration of y by 5
	Limits from $\frac{1}{5}$ to 2 used → 3.6 or $\frac{18}{5}$ OE	M1 A1	Using $\frac{1}{5}$ and 2 to evaluate an integrand (may be $\int y^2$)
	Normal crosses x-axis when $y = 0$, → x= (4½)	M1	Uses their equation of normal, NOT tangent
	Area of triangle = 3.75 or $\frac{15}{4}$ OE	A1	This can be obtained by integration
	Total area=3.6 + 3.75 = 7.35, $\frac{147}{20}$ OE	A1)	
	OR: For the curve: $(\int)\frac{1}{5}(y^2+1)dy = \frac{1}{5}\left(\frac{y^3}{3} + y\right)$	(B2, 1, 0	–1 each error or omission.
	Limits from 0 to 3 used → 2.4 or $\frac{12}{5}$ OE	M1 A1	Using 0 and 3 to evaluate an integrand
	Uses their equation of normal, NOT tangent.	M1	Either to find side length for trapezium or attempt at integrating between 0 and 3
	Area of trapezium = $\frac{1}{2}(2 + 4½) \times 3 = \frac{39}{4}$ or $9\frac{3}{4}$	A1	This can be obtained by integration
	Shaded area = $\frac{39}{4} - \frac{12}{5} = 7.35, \frac{147}{20}$ OE	A1)	

Question 95

(i)	$\frac{dy}{dx} = 0$	M1	Sets $\frac{dy}{dx}$ to 0 and attempts to solve leading to two values for x.
	$x = 1, x = 4$	A1	Both values needed
		2	
(ii)	$\frac{d^2y}{dx^2} = -2x + 5$	B1	
	Using both of their x values in their $\frac{d^2y}{dx^2}$	M1	Evidence of any valid method for both points.
	$x = 1$ → (3) → Minimum, $x = 4$ →(-3) → Maximum	A1	
		3	
(iii)	$y = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \quad (+c)$	B2, 1, 0	$+c$ not needed. –1 each error or omission.
	Uses x =6, y = 2 in an integrand to find c → c = 8	M1 A1	Statement of the final equation not required.
		4	

Question 96

(i)	$\frac{dy}{dx} = 2x - 4 = 0$		Can use completing the square.
	$\to x = 2,\ y = 3$	**B1 B1**	
	Midpoint of AB is (3, 5)	**B1 FT**	**FT** on (*their* 2, *their* 3) with (4,7)
	$\to m = \frac{7}{3}$ (or 2.33)	**B1**	
		4	
(ii)	Simultaneous equations $\to$ $x^2 - 4x - mx + 9(=0)$	***M1**	Equates and sets to 0 must contain m
	Use of $b^2 - 4ac \to (m+4)^2 - 36$	**DM1**	Any use of $b^2 - 4ac$ on equation set to 0 must contain m
	Solves = 0 $\to$ −10 or 2	**A1**	Correct end-points.
	$-10 < m < 2$	**A1**	Don't condone $\leqslant$ at either or both end(s). Accept $-10 < m,\ m <$ 2
		4	

Question 97

(i)	Area $= \int ½(x^4 - 1)dx = ½\left[\frac{x^5}{5} - x\right]$	***B1**	
	$½\left[\frac{1}{5} - 1\right] - 0 \quad = \quad (-)\frac{2}{5}$	**DM1A1**	Apply limits 0→1
		3	
(ii)	Vol $= \pi \int y^2 dx \ = \ ¼(\pi)\int(x^8 - 2x^4 + 1)dx$	**M1**	(If middle term missed out can only gain the M marks)
	$¼(\pi)\left[\frac{x^9}{9} - \frac{2x^5}{5} + x\right]$	***A1**	
	$¼(\pi)[\left(\frac{1}{9} - \frac{2}{5} + 1\right] - 0$	**DM1**	
	$\frac{8\pi}{45}$ or 0.559	**A1**	
		4	
(iii)	Vol $= \pi \int x^2 dy \ = \ (\pi)\int(2y+1)^{1/2} dy$	**M1**	Condone use of x if integral is correct
	$(\pi)\left[\frac{(2y+1)^{3/2}}{3/2}\right][\div 2]$	***A1A1**	Expect $(\pi)\left[\frac{(2y+1)^{3/2}}{3}\right]$
	$(\pi)\left[\frac{1}{3} - 0\right]$	**DM1**	
	$\frac{\pi}{3}$ or 1.05	**A1**	Apply $-\frac{1}{2} \to 0$
		5	

Question 98

(i)	$V = \frac{1}{3}\pi r^2(18-r) = 6\pi r^2 - \frac{1}{3}\pi r^3$	B1	AG
		1	
(ii)	$\frac{dV}{dr} = 12\pi r - \pi r^2 = 0$	M1	Differentiate and set = 0
	$\pi r(12-r) = 0 \to r = 12$	A1	
	$\frac{d^2V}{dr^2} = 12\pi - 2\pi r$	M1	
	Sub r = 12 $\to$ $12\pi - 24\pi = -12\pi \to$ MAX	A1	AG
		4	
(iii)	Sub $r = 12,\ h = 6 \to \text{Max}\ V = 288\pi$ or 905	B1	
		1	

Question 99

$\frac{dy}{dx} = 3x^{1/2} - 3 - 2x^{-1/2}$	B2,1,0	
at x = 4, $\frac{dy}{dx} = 6 - 3 - 1 = 2$	M1	
Equation of tangent is $y = 2(x-4)$ OE	A1FT	Equation through (4, 0) with *their* gradient
	4	

Question 100

$f'(x) = 3x^2 - 2x - 8$	M1	Attempt differentiation
$-\frac{4}{3}$, 2 SOI	A1	
$f'(x) > 0 \Rightarrow x < -\frac{4}{3}$ SOI	M1	Accept $x > 2$ in addition. FT *their* solutions
Largest value of a is $-\frac{4}{3}$	A1	Statement in terms of a. Accept $a \leqslant -\frac{4}{3}$ or $a < -\frac{4}{3}$. Penalise extra solutions
	4	

Question 101

(i)	$dy/dx = [-2] - \left[3(1-2x)^2\right] \times [-2]\ \ (= 4 - 24x + 24x^2)$	B2,1,0	Award for the accuracy within each set of square brackets
	At $x = ½$ $dy/dx = -2$	B1	
	Gradient of line $y = 1 - 2x$ is -2 (hence AB is a tangent) AG	B1	
		4	
(ii)	Shaded region = $\int_0^{½}(1-2x) - \int_0^{½}[1 - 2x - (1-2x)^3]$ oe	M1	Note: If area triangle OAB – area under the curve is used the first part of the integral for the area under the curve must be evaluated
	$= \int_0^{½}(1-2x)^3\,dx$ AG	A1	
		2	
(iii)	Area = $\left[\frac{(1-2x)^4}{4}\right][\div -2]$	*B1B1	
	0 – (–1/8) = 1/8	DB1	OR $\int 1 - 6x + 12x^2 - 8x^3 = x - 3x^2 + 4x^3 - 2x^4$ **(B2,1,0)** Applying limits 0 $\to$ ½
		3	

Question 102

$f'(x)=\frac{-8}{(x-2)^2}$	**B1**	SOI
$y=\frac{8}{x-2}+2 \rightarrow y-2=\frac{8}{x-2} \rightarrow x-2=\frac{8}{y-2}$	**M1**	Order of operations correct. Accept sign errors
$f^{-1}(x)=\frac{8}{x-2}+2$	**A1**	SOI
$\frac{-48}{(x-2)^2}+\frac{16}{x-2}+4-5\ (<0) \rightarrow x^2-20x+84\ (<0)$	**M1**	Formation of 3-term quadratic in $x, (x-2)$ or $1/(x-2)$
$(x-6)(x-14)$ or 6, 14	**A1**	SOI
$2<x<6$, $x>14$	**A1**	CAO
	6	

Question 103

(i)	$dy/dx = x - 6x^{½} + 8$	**B2,1,0**	
	Set to zero and attempt to solve a quadratic for $x^{½}$	**M1**	Could use a substitution for $x^{½}$ or rearrange and square correctly*
	$x^{½}=4$ or $x^{½}=2$ [$x=2$ *and* $x=4$ gets **M1 A0**]	**A1**	Implies **M1**. 'Correct' roots for *their* dy/dx also implies **M1**
	$x = 16$ or 4	**A1FT**	Squares of their solutions *Then **A1**,**A1** for each answer
		5	
(ii)	$d^2y/dx^2 = 1 - 3x^{-½}$	**B1FT**	FT on *their* dy/dx, providing a fractional power of x is present
		1	
(iii)	(When $x = 16$) $d^2y/dx^2 = 1/4 > 0$ hence MIN	**M1**	Checking both of their values in their d^2y/dx^2
	(When $x = 4$) $d^2y/dx^2 = -1/2 < 0$ hence MAX	**A1**	All correct Alternative methods ok but must be explicit about values of x bein considered
		2	

Question 104

$(y)=\frac{x^{½}}{½}-3x\ (+c)$	**B1B1**	
Sub (4, –6) $-6=4-12+c \rightarrow c=2$	**M1A1**	Expect $(y)=2x^{½}-3x+2$
	4	

Question 105

(i)	$\frac{dy}{dx}=2(x+1)-(x+1)^{-2}$	B1	
	Set = 0 and obtain $2(x+1)^3=1$ convincingly www **AG**	B1	
	$\frac{d^2y}{dx^2}=2+2(x+1)^{-3}$ www	B1	
	Sub, e.g., $(x+1)^{-3}=2$ OE or $x=\left(\frac{1}{2}\right)^{\frac{1}{3}}-1$	M1	Requires exact method – otherwise scores M0
	$\frac{d^2y}{dx^2}=6$ CAO www	A1	and exact answer – otherwise scores A0
		5	
(ii)	$y^2=(x+1)^4+(x+1)^{-2}+2(x+1)$ SOI	B1	OR $y^2=(x^4+4x^3+6x^2+4x+1)+(2x+2)+(x+1)^{-2}$
	$(\pi)\int y^2dx=(\pi)\left[\frac{(x+1)^5}{5}\right]+\left[\frac{(x+1)^{-1}}{-1}\right]+\left[\frac{2(x+1)^2}{2}\right]$ OR $(\pi)\left[\frac{x^5}{5}+x^4+2x^3+2x^2+x\right]+\left[x^2+2x\right]+\left[-\frac{1}{x+1}\right]$	B1B1B1	Attempt to integrate y^2. Last term might appear as (x^2+2x)
	$(\pi)\left[\frac{32}{5}-\frac{1}{2}+4-\left(\frac{1}{5}-1+1\right)\right]$	M1	Substitute limits $0\to1$ into an attempted integration of y^2. Do not condone omission of value when $x=0$
	9.7π or 30.5	A1	Note: omission of $2(x+1)$ in first line $\to 6.7\pi$ scores 3/6. Ignore initially an extra volume, e.g. $(\pi)\int(4½)^2$. Only take into account for the final answer
		6	

Question 106

(i)	$\frac{dy}{dx}=3x^2-18x+24$	M1A1	Attempt to differentiate. All correct for A mark
	$3x^2-18x+24=-3$	M1	Equate *their* $\frac{dy}{dx}$ to –3
	$x=3$	A1	
	$y=6$	A1	
	$y-6=-3(x-3)$	A1FT	FT on *their A*. Expect $y=-3x+15$
		6	
(ii)	$(3)(x-2)(x-4)$ SOI or $x=2,\ 4$ Allow $(3)(x+2)(x+4)$	M1	Attempt to factorise or solve. Ignore a RHS, e.g. = 0 or > 0, etc.
	Smallest value of k is 4	A1	Allow $k\geqslant4$. Allow $k=4$. Must be in terms of k
		2	

Question 107

$f(x)=\left[\frac{(3x-1)^{\frac{2}{3}}}{\frac{2}{3}}\right][\div3]\ (+c)$	B1B1	
$1=\frac{8^{\frac{2}{3}}}{2}+c$	M1	Sub $y=1, x=3$ Dep. on attempt to integrate and c present
$c=-1\to y=\frac{1}{2}(3x-1)^{\frac{2}{3}}-1$ SOI	A1	
When $x=0$, $y=\frac{1}{2}(-1)^{\frac{2}{3}}-1\quad=-\frac{1}{2}$	DM1A1	Dep. on previous M1
	6	

Question 108

1(i)	$y=\frac{x}{2}+\frac{6}{x}=4\to x=2$ or 6	B1 B1	Inspection or guesswork OK
	$\frac{dy}{dx}=\frac{1}{2}-\frac{6}{x^2}$	B1	Unsimplified OK
	When $x=2$, $m=-1\to x+y=6$ When $x=6$, $m=\frac{1}{3}\to y=\frac{1}{3}x+2$	*M1	Correct method for either tangent
	Attempt to solve simultaneous equations	DM1	Could solve BOTH equations separately with $y=x$ and get $x=3$ both times.
	(3,3)	A1	Statement about $y=x$ not required.
		6	
1(ii)	$V=(\pi)\int\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)(dx)$	*M1	Integrate using $\pi\int y^2dx$ (doesn't need π or dx). Allow incorrect squaring. Not awarded for $\pi\int\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2dx$. Integration indicated by increase in any power by 1.
	Integration $\to\frac{x^3}{12}+6x-\frac{36}{x}$	A2,1	3 things wanted —1 each error, allow + C. (Doesn't need π)
	Using limits '*their 2*' to '*their 6*' ($53\frac{1}{3}\pi$, $\frac{160}{3}\pi$, 168 awrt)	DM1	Evidence of their values 6 and 2 from **(i)** substituted into their integrand and then subtracted. $48-\left(-\frac{16}{3}\right)$ is enough.
	Vol for line: integration or cylinder ($\to 64\pi$)	M1	Use of πr^2h or integration of 4^2 (could be from $\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2$)
	Subtracts $\to 10\frac{2}{3}\pi$ oe $\left(\text{e.g.}\frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	
(ii)	**OR**		
	$V=(\pi)\int 4^2-\left(\frac{x}{2}+\frac{6}{x}\right)^2(dx)$	M1 *M1	Integrate using $\pi\int y^2dx$ (doesn't need π or dx) Integration indicated by increase in any power by 1.
	$=(\pi)\int 16-\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)(dx)$		
	$=(\pi)\left[16x-\left(\frac{x^3}{12}+6x-\frac{36}{x}\right)\right](dx)$	A2,1	Or $\left[10x-\frac{x^3}{12}+\frac{36}{x}\right]$
	$=(\pi)(48-37\frac{1}{3})$	DM1	Evidence of their values 6 and 2 from (i) substituted
	$=10\frac{2}{3}\pi$ oe $\left(\text{eg}\frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	
		6	

Question 109

(i)	$y = \frac{2}{3}(4x+1)^{\frac{3}{2}} \div 4\ (+\text{C}) \left(= \frac{(4x+1)^{\frac{3}{2}}}{6}\right)$	**B1 B1**	B1 without ÷ 4. B1 for ÷ 4 oe. Unsimplified OK
	Uses $x = 2$, $y = 5$	**M1**	Uses (2, 5) in an integral (indicated by an increase in power by 1).
	→ $c = ½$ oe isw	**A1**	No isw if candidate now goes on to produce a straight line equatio
		4	
(ii)	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		
	$\frac{dx}{dt} = 0.06 \div 3$	**M1**	Ignore notation. Must be 0.06÷3 for M1.
	= 0.02 oe	**A1**	Correct answer with no working scores 2/2
		2	
iii)	$\frac{d^2y}{dx^2} = ½\ (4x+1)^{-½} \times 4$	**B1**	
	$\frac{d^2y}{dx^2} \times \frac{dy}{dx} = \frac{2}{\sqrt{4x+1}} \times \sqrt{4x+1} \quad (= 2)$	**B1FT**	Must either show the algebraic product and state that it results in a constant or evaluate it as '= 2'. Must not evaluate at $x = 2$. ft to apply only if $\frac{d^2y}{dx^2}$ is of the form $k(4x+1)^{-½}$
		2	

Question 110

0	$y = x^3 - 2x^2 + 5x$		
(i)	$\frac{dy}{dx} = 3x^2 - 4x + 5$	**B1**	CAO
	Using $b^2 - 4ac$ → 16 – 60 → negative → some explanation or completed square and explanation	**M1 A1**	Uses discriminant on equation (set to 0). CAO
		3	
(ii)	$m = 3x^2 - 4x + 5$ $\frac{dm}{dx} = 6x - 4\ (= 0)$ (must identify as $\frac{dm}{dx}$)	**B1FT**	FT providing differentiation is equivalent
	→ $x = \frac{2}{3}$, $m = \frac{11}{3}$ or $\frac{dy}{dx} = \frac{11}{3}$ Alt1: $m = 3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3}$, $m = \frac{11}{3}$ Alt2: $3x^2 - 4x + 5 - m = 0$, $b^2 - 4ac = 0$, $m = \frac{11}{3}$	**M1 A1**	Sets to 0 and solves. A1 for correct m. Alt1: B1 for completing square, M1A1 for ans Alt2: B1 for coefficients, M1A1 for ans
	$\frac{d^2m}{dx^2} = 6$ +ve → Minimum value or refer to sketch of curve or check values of m either side of $x = \frac{2}{3}$,	**M1 A1**	M1 correct method. A1 (no errors anywhere)
		5	
0(iii)	Integrate → $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2}$	**B2,1**	Loses a mark for each incorrect term
	Uses limits 0 to 6 → 270 (may not see use of lower limit)	**M1 A1**	Use of limits on an integral. CAO Answer only 0/4
		4	

Question 111

$\frac{dy}{dx}=\frac{12}{(2x+1)^2} \rightarrow y=\frac{-12}{2x+1} \div 2\ (+c)$	B1 B1	Correct without " ÷ 2". For " ÷ 2". Ignore "c".
Uses (1, 1) $\rightarrow c=3\ (\rightarrow y=\frac{-6}{2x+1}+3)$	M1 A1	Finding "c" following integration. CAO
Sets y to 0 and attempts to solve for $x \rightarrow x=\frac{1}{2} \rightarrow ((\frac{1}{2}, 0))$	DM1 A1	Sets y to 0. $x=\frac{1}{2}$ is sufficient for A1.
	6	

Question 112

$y=2x+\frac{5}{x} \rightarrow \frac{dy}{dx}=2-\frac{5}{x^2}$ = −3 (may be implied) when $x=1$.	M1 A1	Reasonable attempt at differentiation CAO (−3)
$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt} \rightarrow -0.06$	M1 A1	Ignore notation, but needs to multiply $\frac{dy}{dx}$ by 0.02.
	4	

Question 113

$f'(x)=3x^2+4x-4$	B1	
Factors or crit. values or sub any 2 values $(x\neq-2)$ into $f'(x)$ soi	M1	Expect $(x+2)(3x-2)$ or -2, $\frac{2}{3}$ or any 2 subs (excluding $x=-2$).
For $-2<x<\frac{2}{3}$, $f'(x)<0$; for $x>\frac{2}{3}$, $f'(x)>0$ soi Allow $\leqslant,\geqslant$	M1	Or at least 1 specific value $(\neq -2)$ in each interval giving opp signs Or $f'(\frac{2}{3})=0$ and $f''(\frac{2}{3})\neq 0$ (i.e. gradient changes sign at $x=\frac{2}{3}$)
Neither www	A1	Must have 'Neither'
ALT 1 At least 3 values of f(x)	M1	e.g. f(0) = 7, f(1) = 6, f(2) = 15
At least 3 <u>correct</u> values of f(x)	A1	
At least 3 <u>correct</u> values of f(x) spanning $x=\frac{2}{3}$	A1	
Shows a decreasing and then increasing pattern. Neither www	A1	Or similar wording. Must have 'Neither'
ALT 2 $f'(x)=3x^2+4x-4=3(x+\frac{2}{3})^2-\frac{16}{3}$	B1B1	Do not condone sign errors
$f'(x)\geqslant-\frac{16}{3}$	M1	
$f'(x)<0$ for some values and >0 for other values. Neither www	A1	Or similar wording. Must have 'Neither'
	4	

Question 114

(i)	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x \quad (+c)$	B1	
	$11 = 0+0+0+c$	M1	Sub $x = 0$, $y = 11$ into an integrated expression. c must be present
	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x + 11$	A1	
		3	
(ii)	$4a + 2b - 4 = 0$	M1	Sub $x = 2$, $dy/dx = 0$
	$\frac{1}{3}(8a) + 2b - 8 + 11 = 3$	M1	Sub $x = 2$, $y = 3$ into an integrated expression. Allow if 11 missing
	Solve simultaneous equations	DM1	Dep. on both M marks
	$a = 3,\ b = -4$	A1A1	Allow if no working seen for simultaneous equations
		5	

Question 115

$V = 4(\pi)\int(3x-1)^{-2/3}\,dx \quad = 4(\pi)\left[\frac{(3x-1)^{1/3}}{1/3}\right][\div 3]$	M1A1A1	Recognisable integration of y^2 (M1) Independent A1, A1 for [ ] [ ]
$4(\pi)[2-1]$	DM1	Expect $4(\pi)(3x-1)^{\frac{1}{3}}$
4π or 12.6	A1	Apply limits ⅔ → 3. Some working must be shown.
	5	
$dy/dx = (-2/3)(3x-1)^{-4/3} \times 3$	B1	Expect $-2(3x-1)^{-4/3}$
When $x = 2/3$, $y = 2$ soi $dy/dx = -2$	B1B1	2nd B1 dep. on correct expression for $dy//dx$
Equation of normal is $y - 2 = \frac{1}{2}(x - \frac{2}{3})$	M1	Line through (⅔, *their* 2) and with grad $-1/m$. Dep on m from diffn
$y = \frac{1}{2}x + \frac{5}{3}$	A1	
	5	

Question 116

Integrate $\rightarrow \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C)	B1 B1	B1 for each term correct – allow unsimplified. C not required.
$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \rightarrow \frac{40}{3} - \frac{14}{3}$	M1	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
$= \frac{26}{3}$ oe	A1	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
	4	

Question 117

(i)	P is $(t, 5t)$ Q is $(t, t(9-t^2)) \rightarrow 4t - t^3$	B1 B1	B1 for both y coordinates which can be implied by subsequent working. B1 for PQ allow $\lvert 4t - t^3 \rvert$ or $\lvert t^3 - 4t \rvert$. **Note:** $4x - x^3$ from equating line and curve 0/2 even if x the replaced by t.
		[2]	

$\frac{d(PQ)}{dt} = 4 - 3t^2$	B1FT	B1FT for differentiation of their PQ, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from (i) but not the equation of the curve.
$= 0 \rightarrow t = +\frac{2}{\sqrt{3}}$	M1	Setting their differential of PQ to 0 and attempt to solve for t or x.
$\rightarrow$ **Maximum** $PQ = \frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$	A1	Allow 3.08 awrt. If answer comes from wrong method in (i) award A0. Correct answer from correct expression by T&I scores 3/3.
	3	

Question 118

$\frac{dy}{dx} = \left[\frac{3}{2}\times(4x+1)^{-\frac{1}{2}}\right]$ $[\times 4]$ $[-2]$ $\left(\frac{6}{\sqrt{4x+1}} - 2\right)$	B2,1,0	Looking for 3 components
$\int y\,dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right]$ $[\div 4]$ $[-\frac{2x^2}{2}]$ $(+C)$ $\left(= \frac{(4x+1)^{\frac{3}{2}}}{2} - x^2\right)$	B1 B1 B1	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '$\div 4$'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating.
	5	
At M, $\frac{dy}{dx} = 0 \rightarrow \frac{6}{\sqrt{4x+1}} = 2$	M1	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as $x = k$)
$x = 2, y = 5$	A1 A1	
	3	

Question 119

(i)	$0 = 9a + 3a^2$	**M1**	Sub $\frac{dy}{dx} = 0$ and $x = 3$
	$a = -3$ only	**A1**	
		2	
(ii)	$\frac{dy}{dx} = -3x^2 + 9x \rightarrow y = -x^3 + \frac{9x^2}{2}$ $(+c)$	**M1A1FT**	Attempt integration. $\frac{1}{3}ax^3 + \frac{1}{2}a^2x^2$ scores M1. Ft on *their a*.
	$9\frac{1}{2} = -27 + 40\frac{1}{2} + c$	**DM1**	Sub $x = 3, y = 9\frac{1}{2}$. Dependent on c present
	$c = -4$	**A1**	Expect $y = -x^3 + \frac{9x^2}{2} - 4$
		4	
(iii)	$\frac{d^2y}{dx^2} = -6x + 9$	**M1**	$2ax + a^2$ scores M1
	At $x = 3, \frac{d^2y}{dx^2} = -9 < 0$ MAX www	**A1**	Requires at least one of -9 or < 0. Other methods possible.
		2	

Question 120

7(i)	$2 = k(8 - 28 + 24) \rightarrow k = 1/2$	**B1**	
		1	
7(ii)	When $x = 5, y = [\frac{1}{2}](125 - 175 + 60) = 5$	**M1**	Or solve $[\frac{1}{2}](x^3 - 7x^2 + 12x) = x \Rightarrow x = 5$ $[x = 0, 2]$
	Which lies on $y = x$, oe	**A1**	
		2	
(iii)	$\int[\frac{1}{2}(x^3 - 7x^2 + 12x) - x]dx$.	**M1**	Expect $\int \frac{1}{2}x^3 - \frac{7}{2}x^2 + 5x$
	$\frac{1}{8}x^4 - \frac{7}{6}x^3 + \frac{5}{2}x^2$	**B2,1,0FT**	Ft on their k
	$2 - 28/3 + 10$	**DM1**	Apply limits $0 \rightarrow 2$
	$8/3$	**A1**	
	OR $\frac{1}{8}x^4 - \frac{7}{6}x^3 + 3x^2$	**B2,1,0FT**	Integrate to find area under curve, Ft on their k
	$2 - 28/3 + 12$	**M1**	Apply limits $0 \rightarrow 2$. Dep on integration attempted
	Area $\Delta = \frac{1}{2} \times 2 \times 2$ or $\int_0^2 x\,dx = \left[\frac{1}{2}x^2\right] = 2$	**M1**	
	$8/3$	**A1**	
		5	

Question 121

)(i)(a)	$\frac{dy}{dx}=\left[-½(4x-3)^{-2}\right]\times[4]$	**B1B1**	Can gain this in part **(b)(ii)**
	When $x=1$, $m=-2$	**B1FT**	Ft from *their* $\frac{dy}{dx}$
	Normal is $y-½=½(x-1)$	**M1**	Line with gradient $-1/m$ and through A
	$y=½x$ soi	**A1**	Can score in part **(b)**
		5	
)(i)(b)	$\frac{1}{2(4x-3)}=\frac{x}{2}\rightarrow 2x(4x-3)=2\rightarrow(2)(4x^2-3x-1)\ (=0)$	**M1A1**	$x/2$ seen on RHS of equation can score *previous* A1
	$x=-1/4$	**A1**	Ignore $x=1$ seen in addition
		3	
.0(ii)	*Use* of chain rule: $\frac{dy}{dt}=(\textit{their}-2)\times(\pm)0.3=0.6$	**M1A1**	Allow +0.3 or –0.3 for M1
		2	

Question 122

$y=⅓kx^3-x^2\ (+c)$	**M1A1**	Attempt integration for M mark
Sub (0, 2)	**DM1**	Dep on c present. Expect $c=2$
Sub (3, –1) → $-1=9k-9+\textit{their } c$	**DM1**	
$k=2/3$	**A1**	
	5	

Question 123

(i)	$dy/dx=-2(2x-1)^{-2}+2$	**B2,1,0**	Unsimplified form ok (–1 for each error in '–2', '$(2x-1)^{-2}$' and '2')
	$d^2y/dx^2=8(2x-1)^{-3}$	**B1**	Unsimplified form ok
		3	
(ii)	Set dy/dx to zero and attempt to solve – at least one correct step	**M1**	
	$x=0,\ 1$	**A1**	Expect $(2x-1)^2=1$
	When $x=0$, $d^2y/dx^2=-8$ (or <0). Hence MAX	**B1**	
	When $x=1$, $d^2y/dx^2=8$ (or >0). Hence MIN	**B1**	Both final marks dependent on correct x and correct d^2y/dx^2 and no errors May use change of sign of dy/dx but not at $x=1/2$
		4	

Question 124

(i)	$V=(\pi)\int(x^3+x^2)(\mathrm{d}x)$	M1	Attempt $\int y^2\mathrm{d}x$
	$(\pi)\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_0^3$	A1	
	$(\pi)\left[\frac{81}{4}+9\quad(-0)\right]$	DM1	May be implied by a correct answer
	$\frac{117\pi}{4}$ oe	A1	Accept 91.9 If additional areas rotated about x-axis, maximum of M1A0DM1A0
		4	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}(x^3+x^2)^{-1/2}\times(3x^2+2x)$	B2,1,0	Omission of $3x^2+2x$ is one error
	(At $x=3$,) $y=6$	B1	
	At $x=3$, $m=\frac{1}{2}\times\frac{1}{6}\times 33=\frac{11}{4}$ soi	DB1ft	Ft on *their* $\mathrm{d}y/\mathrm{d}x$ providing differentiation attempted
	Equation of normal is $y-6=-\frac{4}{11}(x-3)$	DM1	Equation through (3, *their* 6) and with gradient $-1/$*their* m
	When $x=0$, $y=7\frac{1}{11}$ oe	A1	
		6	

Question 125

$f'(-1)=0\rightarrow 3-a+b=0$ $f'(3)=0\rightarrow 27+3a+b=0$	M1	Stationary points at $x=-1$ & $x=3$ gives sim. equations in a & b
$a=-6$	A1	Solve simultaneous equation
$b=-9$	A1	
Hence $f'(x)=3x^2-6x-9\rightarrow f(x)=x^3-3x^2-9x(+c)$	B1	**FT** correct integration for *their* a,b (numerical a, b)
$2=-1-3+9+c$	M1	Sub $x=-1$, $y=2$ into *their* integrated f(x). c must be present
$c=-3$	A1	**FT** from *their* f(x)
$f(3)=k\rightarrow k=27-27-27-3$	M1	Sub $x=3$, $y=k$ into *their* integrated f(x) (Allow c omitted)
$k=-30$	A1	
	8	

Question 126

(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	B1	oe
	$\frac{dy}{dx}=\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]\times 3$	B1	Must have '$\times 3$'
	At $x=4$, $\frac{dy}{dx}=\frac{3}{8}$ soi	B1	
	Line through (4, *their*4) with gradient *their* $\frac{3}{8}$	M1	If $y\neq 4$ is used then clear evidence of substitution of $x=4$ is needed
	Equation of tangent is $y-4=\frac{3}{8}(x-4)$ or $y=\frac{3}{8}x+\frac{5}{2}$	A1	oe
		5	
(ii)	Area under line $=\frac{1}{2}\left(4+\frac{5}{2}\right)\times 4=13$	B1	OR $\int_0^4 \frac{3}{8}x+\frac{5}{2}=\left[\frac{3}{16}x^2+\frac{5}{2}x\right]=[3+10]=13$
	Area under curve: $\int(3x+4)^{\frac{1}{2}}=\left[\frac{(3x+4)^{3/2}}{3/2}\right][\div 3]$	B1B1	Allow if seen as part of the difference of 2 integrals First B1 for integral without $[\div 3]$ Second B1 must have $[\div 3]$
	$\frac{128}{9}-\frac{16}{9}=\frac{112}{9}=12\frac{4}{9}$	M1	Apply limits $0\to 4$ to an integrated expression
	Area $=13-12\frac{4}{9}=\frac{5}{9}$ (or 0.556)	A1	
	Alternative method for question 10(ii)		
	Area for line $=1/2\times 4\times 3/2=3$	B1	OR $\int_{5/2}^{4}\frac{1}{3}(8y-20)=\frac{1}{3}\left[4y^2-20\right]=\frac{1}{3}[-16+25]=3$
	Area for curve $=\int \frac{1}{3}(y^2-4)=\left[\frac{y^3}{9}\right]-\left[\frac{4y}{3}\right]$	B1B1	
	$\left(\frac{64}{9}-\frac{16}{3}\right)-\left(\frac{8}{9}-\frac{8}{3}\right)=\frac{32}{9}$	M1	Apply limits $2\to 4$ to an integrated expression for curve
	Area $=\frac{32}{9}-3=\frac{5}{9}$ (or 0.556)	A1	
		5	

(iii)	$\frac{dy}{dx}=\frac{1}{2}$	B1	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}}=\frac{1}{2}$	M1	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}}=2$.
	$(3x+4)^{\frac{1}{2}}=3\ \to\ 3x+4=9\ \to\ x=\frac{5}{3}$ oe	A1	
		3	

Question 127

(i)	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 7 \times -0.05$	M1	Multiply numerical gradient at $x = 2$ by ± 0.05.
	-0.35 (units/s) **or** Decreasing at a rate of (+) 0.35	A1	Ignore notation and omission of units
		2	
(ii)	$(y) = \frac{x^4}{4} + \frac{4}{x}$ $(+c)$ oe	B1	Accept unsimplified
	Uses (2, 9) in an integral to find c.	M1	The power of at least one term increase by 1.
	$c = 3$ **or** $(y =)\frac{x^4}{4} + \frac{4}{x} + 3$ oe	A1	A0 if candidate continues to a final equation that is a straight line.
		3	

Question 128

I(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right]$ [× 4] $\left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right]$ [× 4]	B1B1B1	B1 B1 for each, without × 4. B1 for ×4 twice.
	$\left(\frac{2}{\sqrt{4x+1}} - \frac{18}{\left(\sqrt{4x+1}\right)^3} \text{ or } \frac{8x-16}{(4x+1)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of $(4x+1)$ correct.
	$\int y dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷ 4] + $\left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]$ [÷ 4] (+C)	B1B1B1	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6} + \frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of $(4x+1)$ correct.
		6	
(ii)	$\frac{dy}{dx} = 0 \rightarrow \frac{2}{\sqrt{4x+1}} - \frac{18}{(4x+1)^{\frac{3}{2}}} = 0$	M1	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x + 1 = 9$ or $(4x + 1)^2 = 81$	A1	Must be from correct differential.
	$x = 2$, $y = 6$ **or** M is (2, 6) only.	A1	Both values required. Must be from correct differential.
		3	
(iii)	Realises area is $\int y\, dx$ **and** attempt to use their 2 and sight of 0.	*M1	Needs to use their integral and to see '*their 2*' substituted.
	Uses limits 0 to 2 correctly → $[4.5 + 13.5] - [\frac{1}{6} + 4.5]$ (= 13⅓)	DM1	Uses both 0 and '*their 2*' and subtracts. Condone wrong way round.
	(Area =) 1⅓ **or** 1.33	A1	Must be from a correct differential and integral.
		3	13⅓ or 1⅓ with little or no working scores M1DM0A0.

Question 129

(i)	integrating $\rightarrow \frac{dy}{dx} = x^2 - 5x\ (+c)$	B1	
	$= 0$ when $x = 3$	M1	Uses the point to find c after $\int = 0$.
	$c = 6$	A1	
	integrating again $\rightarrow y = \frac{x^3}{3} - \frac{5x^2}{2} + 6x \quad (+d)$	B1	**FT** Integration again FT if a numerical constant term is present.
	use of (3, 6)	M1	Uses the point to find d after $\int = 0$.
	$d = 1½$	A1	
		6	
(ii)	$\frac{dy}{dx} = x^2 - 5x + 6 = 0 \rightarrow x = 2$	B1	
		1	
(iii)	$x = 3,\ \frac{d^2y}{dx^2} = 1$ and/or +ve Minimum. $x = 2,\ \frac{d^2y}{dx^2} = -1$ and/or −ve Maximum	B1	www
	May use shape of '$+x^3$' curve or change in sign of $\frac{dy}{dx}$	B1	www SC: $x = 3$, minimum, $x = 2$, maximum, B1
		2	

Question 130

(i)	$3 \times -½ \times (1+4x)^{-\frac{3}{2}}$	B1	
	$\frac{dy}{dx} = 3 \times -½ \times (1+4x)^{-\frac{3}{2}} \times 4$	B1	Must have '× 4'
	If $x = 2$, $m = -\frac{2}{9}$, Perpendicular gradient $= \frac{9}{2}$	M1	Use of $m_1.m_2 = -1$
	Equation of normal is $y - 1 = \frac{9}{2}(x-2)$	M1	Correct use of line eqn (could use y=0 here)
	Put $y = 0$ or on the line before $\rightarrow \frac{16}{9}$	A1	AG
		5	
(ii)	Area under the curve $= \int_0^2 \frac{3}{\sqrt{1+4x}}\,dx = \frac{3\sqrt{1+4x}}{\frac{1}{2}} \div 4$	B1 B1	Correct without '÷4'. For 2nd B1, ÷4'.
	Use of limits 0 to 2 $\rightarrow 4½ - 1½$	M1	Use of correct limits in an integral.
	3	A1	
	Area of the triangle $= ½ \times 1 \times \frac{2}{9} = \frac{1}{9}$ or attempt to find $\int_{16/9}^{2} \left(\frac{9}{2}x - 8\right)dx$	M1	Any correct method.
	Shaded area $= 3 - \frac{1}{9} = 2\frac{8}{9}$	A1	
		6	

Question 131

(i)	$\frac{dy}{dx} = -2(x-1)^{-3}$	B1	
	When $x = 2$, $m = -2 \rightarrow$ gradient of normal $= -\frac{1}{m}$	M1	m must come from differentiation
	Equation of normal is $y - 3 = ½(x-2) \rightarrow y = ½x + 2$	A1	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	
(ii)	$(\pi)\int y_1^2 (dx), (\pi)\int y_2^2 (dx)$	*M1	Attempt to integrate y^2 for at least one of the functions
	$(\pi)\int\left(\frac{1}{2}x+2\right)^2$ or $\left(\frac{1}{4}x^2+2x+4\right)$ $(\pi)\int\left((x-1)^{-4}+4(x-1)^{-2}+4\right)$	A1A1	A1 for $\left(\frac{1}{2}x+2\right)^2$ depends on an attempt to integrate this form later
	$(\pi)\left[\frac{2}{3}\left(\frac{1}{2}x+2\right)^3 \text{ or } \frac{1}{12}x^3+x^2+4x\right]$ $(\pi)\left[\frac{(x-1)^{-3}}{-3}+\frac{4(x-1)^{-1}}{-1}+4x\right]$	A1A1	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18-\frac{125}{12} \text{ or } \frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)\right\}$ $\left\{\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)\right\}$	DM1	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi\left\{7\frac{7}{12}+6\frac{7}{24}\right\}$	DM1	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	A1	$\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)$ $\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)$
		8	

Question 132

$(y =)\ \frac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\left(=\frac{k\sqrt{x}}{\frac{1}{2}}\right)(+c)$	B1	OE
Substitutes both points into an integrated expression with a '+c' and solve as far as a value for one variable.	M1	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
$k = 2½$ **and** $c = -6$	A1	WWW
$y = 5\sqrt{x} - 6$	A1	OE From correct values of both k & c and correct integral.
	4	

Question 133

Use of Pythagoras $\rightarrow r^2 = 15^2 - h^2$	M1	
$V = ⅓\pi(225 - h^2) \times h \rightarrow ⅓\pi(225h - h^3)$	A1	AG WWW e.g. sight of $r = 15 - h$ gets A0.
	2	
$\left(\frac{dv}{dh} =\right)\frac{\pi}{3}(225 - 3h^2)$	B1	
Their $\frac{dv}{dh} = 0$	M1	Differentiates, sets *their* differential to 0 and attempts to solve at least as far as $h^2 \neq 0$.
$(h =)\ \sqrt{75}$, $5\sqrt{3}$ or AWRT 8.66	A1	Ignore $-\sqrt{75}$ OE and ISW for both A marks
$\frac{d^2h}{dh^2} = \frac{\pi}{3}\ (-6h)\ (\rightarrow -\text{ve})$	M1	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
$\rightarrow$ Maximum	A1FT	Correct conclusion from correct 2nd differential, value for h not required, or any other valid complete method. FT for *their* h, if used, as long as it is positive.
		SC Omission of π or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
	5	

Question 134

At A, $x = ½$.	B1	Ignore extra answer $x = -1.5$
$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(= -½)$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1m_2 = -1$
Equation of normal: $y - 0 = -½(x - ½)$ or $y - 0 = -½\ (0 - ½)$ or $0 = -½ \times ½ + c$	DM1	Use of *their* x at A and *their* normal gradient.
$B\ (0, ¼)$	A1	
	4	

At A, $x = ½$.	B1	Ignore extra answer $x = -1.5$
$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(= -½)$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1m_2 = -1$
Equation of normal: $y - 0 = -½(x - ½)$ or $y - 0 = -½\ (0 - ½)$ or $0 = -½ \times ½ + c$	DM1	Use of *their* x at A and *their* normal gradient.
$B\ (0, ¼)$	A1	
	4	

)(iii)	$\int_0^{\frac{1}{2}} 1-\frac{4}{(2x+1)^2}(\mathrm{d}x)$	*M1	$\int y\,\mathrm{d}x$ SOI with 0 and *their* positive x coordinate of A.
	$[½ + 1] – [0 + 2] = (–½)$	DM1	Substitutes both 0 and *their* ½ into *their* $\int y\mathrm{d}x$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}}-\frac{1}{2}(\mathrm{d}y)$	*M1	$\int x\,\mathrm{d}y$ SOI. Where x is of the form $k\left(1-y\right)^{-\frac{1}{2}}+c$ with 0 and *their* negative y intercept of curve.
	$[-2]-\left[-4+\frac{3}{2}\right]=(½)$	DM1	Substitutes both 0 and *their* –3 into *their* $\int x\mathrm{d}y$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)

Question 135

Attempt to solve $f'(x)=0$ or $f'(x)>0$ or $f'(x)\geqslant 0$	M1	SOI
$(x-2)(x-4)$	A1	2 and 4 seen
(Least possible value of n is) 4	A1	Accept $n = 4$ or $n \geqslant 4$
	3	

Question 136

)(i)	$y = [(5x-1)^{1/2} \div \frac{3}{2} \div 5]\ [-2x]$	B1 B1	
	$3=\frac{27}{(3/2)\times 5}-4+c$	M1	Substitute $x = 2, y = 3$
	$c=7-\frac{18}{5}=\frac{17}{5} \rightarrow \left(y=\frac{2(5x-1)^{\frac{3}{2}}}{15}-2x+\frac{17}{5}\right)$	A1	
)(ii)	$\mathrm{d}^2y/\mathrm{d}x^2=\left[½(5x-1)^{-1/2}\right]\ [\times 5]$	B1 B1	
(iii)	$(5x-1)^{1/2}-2=0 \rightarrow 5x-1=4$ $x=1$	M1A1	Set $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and attempt solution (M1)
	$y=\frac{16}{25}-2+\frac{17}{5}=\frac{37}{15}$	A1	Or 2.47 or $\left(1, \frac{37}{15}\right)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^x}=\frac{5}{2}\times\frac{1}{2}=\frac{5}{4}$ (> 0) hence minimum	A1	OE

Question 137

(i)	$(y=)\ (x+2)^2-1$	B1 DB1	2nd B1 dependent on 2 in bracket
	$x+2=(\pm)(y+1)^{1/2}$	M1	
	$x=-2+(y+1)^{1/2}$	A1	
(ii)	$x^2=4+(y+1)-/+4(y+1)^{\frac{1}{2}}$	*M1A1	SOI. Attempt to find x^2. The last term can be – or + at this stage
	$(\pi)\int x^2\,(\mathrm{d}y)=(\pi)\left[5y+\frac{y^2}{2}-\frac{4(y+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$	A2,1,0	
	$(\pi)\left[15+\frac{9}{2}-\frac{64}{3}-\left(-5+\frac{1}{2}\right)\right]$	DM1	Apply *y* limits
	$\frac{8\pi}{3}$ or 8.38	A1	

Question 138

$\mathrm{f}'(x)=\left[-(3x+2)^{-2}\right]\times[3]+[2x]$	B2, 1, 0	
<0 hence decreasing	B1	Dependent on at least B1 for $\mathrm{f}'(x)$ and must include <0 or '(always) neg'
	3	

Question 139

$(\pi)\int(y-1)\mathrm{d}y$	*M1	SOI Attempt to integrate x^2 or $(y-1)$
$(\pi)\left[\frac{y^2}{2}-y\right]$	A1	
$(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$	DM1	Apply limits $1\to5$ to an integrated expression
8π or AWRT 25.1	A1	
	4	

Question 140

$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2$	B1	
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4}{6}$	B1	OE, SOI
their $(2x-2)=$ *their* $\frac{4}{6}$	M1	LHS and RHS must be *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression and value
$x=\frac{4}{3}$ oe	A1	
	4	

Question 141

(a)	$2(a+3)^{\frac{1}{2}} - a = 0$	M1	SOI. Set $\frac{dy}{dx} = 0$ when $x = a$. Can be implied by an answer in terms of a
	$4(a+3) = a^2 \rightarrow a^2 - 4a - 12 = 0$	M1	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \rightarrow a = 6$	A1	Must show factors, or formula or completing square. Ignore $a = -2$ SC If a is never used maximum of M1A1 for $x = 6$, with visible solution
		3	
(b)	$\frac{d^2y}{dx^2} = (x+3)^{-\frac{1}{2}} - 1$	B1	
	Sub *their* $a \rightarrow \frac{d^2y}{dx^2} = \frac{1}{3} - 1 = -\frac{2}{3}$ $(or < 0) \rightarrow$ MAX	M1A1	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
(c)	$(y =)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}x^2 \ (+c)$	B1B1	
	Sub $x =$ *their* a and $y = 14$ $\rightarrow 14 = \frac{4}{3}(9)^{\frac{3}{2}} - 18 + c$	M1	Substitute into an integrated expression. c must be present. Expect $c = -4$
	$y = \frac{4}{3}(x+3)^{\frac{3}{2}} - \frac{1}{2}x^2 - 4$	A1	Allow $f(x) = \ldots$
		4	

Question 142

$(y) = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} \ (+c)$	B1 B1
$7 = 16 - 12 + c$ (**M1** for subsituting $x = 4$, $y = 7$ into *their* integrated expansion)	M1
$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 3$	A1
	4

Question 143

$\frac{dy}{dx} = \left[\frac{1}{2}(5x-1)^{-1/2}\right] \times [5]$	B1 B1
Use $\frac{dy}{dt} = 2 \times \left(\textit{their} \frac{dy}{dx} \text{ when } x = 1\right)$	M1
$\frac{5}{2}$	A1
	4

$2 \times \textit{their} \frac{5}{2}(5x-1)^{-1/2} = \frac{5}{8}$ oe	M1
$(5x-1)^{1/2} = 8$	A1
$x = 13$	A1
	3

Question 144

(a)	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	$3x^2 - 4bx + b^2 = 0 \rightarrow (3x-b)(x-b)\ (=0)$	M1
	$x = \frac{b}{3}$ or b	A1
	$a = \frac{b}{3} \rightarrow b = 3a$ **AG**	A1
	Alternative method for question 11(a)	
	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	Sub $b = 3a$ & obtain $\frac{dy}{dx} = 0$ when $x = a$ and when $x = 3a$	M1
	$\frac{d^2y}{dx^2} = 6x - 12a$	A1
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ **AG**	A1
		4
(b)	Area under curve $= \int (x^3 - 6ax^2 + 9a^2x)dx$	M1
	$\frac{x^4}{4} - 2ax^3 + \frac{9a^2x^2}{2}$	B2,1,0
	$\frac{a^4}{4} - 2a^4 + \frac{9a^4}{2}\ \left(= \frac{11a^4}{4}\right)$ (**M1** for applying limits $0 \rightarrow a$)	M1
	When $x = a$, $y = a^3 - 6a^3 + 9a^3 = 4a^3$	B1
	Area under line $= \frac{1}{2} a \times$ *their* $4a^3$	M1
	Shaded area $= \frac{11a^4}{4} - 2a^4 = \frac{3}{4}a^4$	A1
		7

Question 145

Volume after 30 s = 18000 $\quad \frac{4}{3}\pi r^3 = 18000$	M1
$r = 16.3$ cm	A1
	2
$\frac{dV}{dr} = 4\pi r^2$	B1
$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	M1
$\frac{dr}{dt} = 0.181$ cm per second	A1
	3

Question 146

(a)	Volume $= \pi \int x^2 dy = \pi \int \frac{36}{y^2} dy$	*M1
	$= \pi \left[\frac{-36}{y} \right]$	A1
	Uses limits 2 to 6 correctly $\rightarrow (12\pi)$	DM1
	Vol of cylinder $= \pi .1^2.4$ or $\int 1^2.dy \quad = [y]$ from 2 to 6	M1
	Vol $= 12\pi - 4\pi = 8\pi$	A1
		5
(b)	$\frac{dy}{dx} = \frac{-6}{x^2}$	B1
	$\frac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

Question 147

(a)	$\frac{dy}{dx} = 54 - 6(2x - 7)^2$	B2,1
	$\frac{d^2y}{dx^2} = -24(2x - 7)$ (FT only for omission of '×2 ' from the bracket)	B2,1 FT
		4
(b)	$\frac{dy}{dx} = 0 \rightarrow (2x - 7)^2 = 9$	M1
	$x = 5, y = 243$ or $x = 2, y = 135$	A1 A1
		3
(c)	$x = 5 \; \frac{d^2y}{dx^2} = -72 \rightarrow$ Maximum (FT only for omission of '×2 ' from the bracket)	B1FT
	$x = 2 \; \frac{d^2y}{dx^2} = 72 \rightarrow$ Minimum (FT only for omission of '×2 ' from the bracket)	B1FT
		2

Question 148

(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2)	**B1B1**
	$\frac{d^2y}{dx^2} = -12(3-2x)\times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without − 2)	**B1FT** **B1**
		4
(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \rightarrow 3-2x = \pm 2$	**M1**
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	**A1A1**
		3
(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	**B1FT**
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	**B1FT**
		2

Question 149

(a)	Simultaneous equations $\frac{8}{x+2} = 4 - ½x$	**M1**
	$x = 0$ or $x = 6 \rightarrow A$ (0, 4) and B (6, 1)	**B1A1**
	At C $\frac{-8}{(x+2)^2} = -\frac{1}{2} \rightarrow C$ (2, 2) (**B1** for the differentiation. **M1** for equating and solving)	**B1** **M1A1**
		6
(b)	Volume under line $= \pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\frac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$ (**M1** for volume formula. **A2,1** for integration)	**M1** **A2,1**
	Volume under curve $= \pi\int\left(\frac{8}{x+2}\right)^2 dx = \pi\left[\frac{-64}{x+2}\right] = (24\pi)$	**A1**
	Subtracts and uses 0 to 6 → 18π	**M1A1**
		6

Question 150

(a)	$\frac{dy}{dx} = \left[\frac{x^{-1/2}}{2k}\right] - \left[\frac{x^{-3/2}}{2}\right] + ([0])$	**B2, 1, 0**	$([0])$ implies that more than 2 terms counts as an error
	Sub $\frac{dy}{dx} = 3$ when $x = \frac{1}{4}$ Expect $3 = \frac{1}{k} - 4$	**M1**	
	$k = \frac{1}{7}$ (or 0.143)	**A1**	
		4	

(b)	$\int \frac{1}{k}x^{1/2} + x^{-1/2} + \frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right] + \left[2x^{1/2}\right] + \left[\frac{x}{k^2}\right]$	**B2, 1, 0**	OE
	$\left(\frac{2k^2}{3} + 2k + 1\right) - \left(\frac{k^2}{12} + k + \frac{1}{4}\right)$	**M1**	Apply limits $\frac{k^2}{4} \to k^2$ to an integrated expression. Expect $\frac{7}{12}k^2 + k + \frac{3}{4}$
	$\frac{7}{12}k^2 + k + \frac{3}{4} = \frac{13}{12}$	**M1**	Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2 + 12k - 4\ (= 0)$
	$k = \frac{2}{7}$ only (or 0.286)	**A1**	Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square.
		5	

Question 151

(a)	$\frac{dy}{dx} = [2]\quad [-2(2x+1)^{-2}]$	**B1 B1**	
	$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$	**B1**	
		3	
(b)	Set *their* $\frac{dy}{dx} = 0$ and attempt solution	**M1**	
	$(2x+1)^2 = 1 \to 2x + 1 = (\pm)\,1$ or $4x^2 + 4x = 0 \to (4)x(x+1) = 0$	**M1**	Solving as far as $x = ...$
	$x = 0$	**A1**	WWW. Ignore other solution.
	(0, 2)	**A1**	One solution only. Accept $x = 0$, $y = 2$ only.
	$\frac{d^2y}{dx^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	**B1**	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

Question 152

(a)	$\frac{-2}{x+2}$	**B1**	Integrate f(*x*). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\frac{2}{3}\right) = \frac{2}{3}$	**M1 A1**	Apply limit(s) to an integrated expansion. CAO for A1
		3	
(b)	$-1=-2+c$	**M1**	Substitute $x=-1, y=-1$ into *their* integrated expression (*c* present)
	$y=\frac{-2}{x+2}+1$	**A1**	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

Question 153

(a)	$\left(\frac{dy}{dx}\right) = [8] \times [(3-2x)^{-3}] + [-1]$ $\left(= \frac{8}{(3-2x)^3} - 1\right)$	**B2, 1, 0**	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2} = -3\times 8\times(3-2x)^{-4}\times(-2)$ $\left(= \frac{48}{(3-2x)^4}\right)$	**B1 FT**	FT providing *their* bracket is to a negative power
	$\int y dx = [(3-2x)^{-1}]\ [2 \div (-1 \times -2)]\ [-\tfrac{1}{2}x^2]\ (+c)$ $\left(= \frac{1}{3-2x} - \frac{1}{2}x^2 + c\right)$	**B1 B1 B1**	Simplification not needed, B1 for each correct element
		6	

(b)	$\frac{dy}{dx} = 0 \to (3-2x)^3 = 8 \to 3-2x = k \to x =$	**M1**	Setting *their* 2-term differential to 0 and attempts to solve as far as $x =$
	$\frac{1}{2}$	**A1**	
	Alternative method for question 10(b)		
	$y=0 \to \frac{2}{(3-2x)^2} - x = 0 \to (x-2)(2x-1)^2 = 0 \to x =$	**M1**	Setting *y* to 0 and attempts to solve a cubic as far as $x =$ (3 factors needed)
	$\frac{1}{2}$	**A1**	
		2	
(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times 0} - 0\right]$	**M1**	Using *their* integral, *their* positive *x* limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	**A1**	
		2	

Question 154

(a)	$f'(4)\left(=\frac{5}{2}\right)$	***M1**	Substituting 4 into $f'(x)$
	$\left(\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\right)\rightarrow\left(\frac{dy}{dt}\right)=\frac{5}{2}\times 0.12$	**DM1**	Multiplies *their* $f'(4)$ by 0.12
	$\left(\frac{dy}{dt}=\right)0.3$	**A1**	OE
		3	
(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	**B1 B1**	B1 for each unsimplified integral.
	Uses (4, 7) leading to $c = (-21)$	**M1**	Uses (4, 7) to find a c value
	y or $f(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	**A1**	Need to see y or $f(x)$ = somewhere in *their* solution and 12 and 8
		4	

Question 155

(a)	$4x^{\frac{1}{2}}-2x=3-x\ \rightarrow x-4x^{\frac{1}{2}}+3(=0)$	***M1**	3-term quadratic. Can be expressed as e.g. $u^2-4u+3\ (=0)$
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$	**DM1**	Or quadratic formula or completing square
	$x^{\frac{1}{2}}=1, 3$	**A1**	SOI
	$x=1, 9$	**A1**	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$	***M1**	Isolate $x^{\frac{1}{2}}$
	$16x=9+6x+x^2\ \rightarrow\ x^2-10x+9\ (=0)$	**A1**	3-term quadratic
	$(x-1)(x-9)\ (=0)$	**DM1**	Or formula or completing square on a quadratic obtained by a correct method
	$x=1, 9$	**A1**	
		4	
(b)	$\frac{dy}{dx}=2x^{1/2}-2$	***B1**	
	$\frac{dy}{dx}$ or $2x^{1/2}-2=0$ when $x=1$ hence B is a stationary point	**DB1**	
		2	

(c)	Area of correct triangle = $\frac{1}{2}$ (9 – 3) × 6	**M1**	or $\int_3^9 (3-x)(dx) = \left[3x - \frac{1}{2}x^2\right] \rightarrow -18$
	$\int (4x^{\frac{1}{2}} - 2x)(dx) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	**B1 B1**	
	$(72-81) - \left(\frac{64}{3} - 16\right)$	**M1**	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	**A1**	OE
	Shaded region = $18 - 14\frac{1}{3} = 3\frac{2}{3}$	**A1**	OE
		6	

Question 156

$\frac{dy}{dx}=\left[\frac{1}{2}\left(25-x^2\right)^{-1/2}\right]\times[-2x]$	**B1 B1**	
$\frac{-x}{\left(25-x^2\right)^{1/2}}=\frac{4}{3}\rightarrow\frac{x^2}{25-x^2}=\frac{16}{9}$	**M1**	Set = $\frac{4}{3}$ and square both sides
$16\left(25-x^2\right)=9x^2\rightarrow 25x^2=400\rightarrow x=(\pm)4$	**A1**	
When $x=-4$, $y=5$ $\rightarrow$ $(-4, 5)$	**A1**	
	5	

Question 157

(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	**B1**	SOI Award this mark for $\frac{dr}{dV}$
$\frac{50}{\textit{their}\text{ derivative}}$	**M1**	Can be in terms of r
$\frac{1}{8\pi}$ or 0.0398	**A1**	AWRT
	3	

Question 158

$(y=)\left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]$ $(+c)$	**B1 B1**	
$7=1+2+c$	**M1**	Substitute $x=2$, $y=7$ into an integrated expansion (c present). Expect $c=4$
$y=-(x-3)^{-1}+\frac{1}{2}x^2$ $+4$	**A1**	OE
	4	

Question 159

(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	**M1**	OE. Set y to zero and attempt to solve.
	$x=4$ **only**	**A1**	From use of a correct method.
		2	
(b)	$\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	**B2, 1, 0**	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x=4$ gradient $=9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	**M1**	Using *their* $x=4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	**A1**	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	**M1**	Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve.
	$x=12$	**A1**	Condone $(\pm)12$ from use of a correct method.
		2	
(d)	$\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)dx=9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	**B2, 1, 0**	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	**M1**	Apply limits *their* 4 $\rightarrow$ 9 to an integrated expression with no consideration of other areas.
	6	**A1**	Use of π scores A0
		4	

Question 160

(a)	At $x=1$, $\frac{dy}{dx}=6$	**B1**	
	$\frac{dx}{dt}=\left(\frac{dx}{dy}\times\frac{dy}{dt}\right)=\frac{1}{6}\times3=\frac{1}{2}$	**M1 A1**	Chain rule used correctly. Allow alternative and minimal notation.
		3	
(b)	$[y=]\left(\frac{6(3x-2)^{-2}}{-2}\right)\div(3)\ [+c]$	**B1 B1**	
	$-3=-1+c$	**M1**	Substitute $x=1$, $y=-3$. c must be present.
	$y=-(3x-2)^{-2}-2$	**A1**	OE. Allow f(x)=
		4	

Question 161

(a)	$\frac{dy}{dx}=\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}=0$ leading to $\frac{1}{2}x^{-1/2}=\frac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$\left(k^2,2k\right)$	A1	
		4	
(b)	When $x = 4k^2$, $\frac{dy}{dx}=\left[\frac{1}{4k}-\frac{1}{16k}=\right]\frac{3}{16k}$	B1	OE
	$y=\left[2k+k^2\times\frac{1}{2k}\right]=\frac{5k}{2}$	B1	OE. Accept $2k+\frac{k}{2}$
	Equation of tangent is $y-\frac{5k}{2}=\frac{3}{16k}\left(x-4k^2\right)$ or $y=mx+c\rightarrow\frac{5k}{2}=\frac{3}{16k}\left(4k^2\right)+c$	M1	Use of line equation with *their* gradient and $(4k^2, their\ y)$,
	When $x=0$, $y=\left[\frac{5k}{2}-\frac{3k}{4}=\right]\frac{7k}{4}$ or from $y=mx+c$, $c=\frac{7k}{4}$	A1	OE
		4	
(c)	$\int\left(x^{\frac{1}{2}}+k^2x^{-\frac{1}{2}}\right)dx=\frac{2x^{\frac{3}{2}}}{3}+2k^2x^{\frac{1}{2}}$	B1	Any unsimplified form
	$\left(\frac{16k^3}{3}+4k^3\right)-\left(\frac{9k^3}{4}+3k^3\right)$	M1	Apply limits $\frac{9}{4}k^2\rightarrow4k^2$ to an integration of y. M0 if volume attempted.
	$\frac{49k^3}{12}$	A1	OE. Accept $4.08\,k^3$
		3	

Question 162

$\left[f^{-1}(x)=\right]\left((2x-1)^{1/2}\right)\times\left(\frac{1}{3}\times2\times\frac{3}{2}\right)(-2)$	B2, 1, 0	Expect $(2x-1)^{1/2}-2$
$(2x-1)^{1/2}-2\leqslant0\ \rightarrow\ 2x-1\leqslant4$ or $2x-1<4$	M1	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0' at this stage but do not allow '≥ 0' or '>0' If '–2' missed then must see $\leqslant$ or $<$ for the M1
Value [of a] is 2½ or $a=2½$	A1	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
	4	

Question 163

$\left[f(x)=\right]\ 2x^3+\frac{8}{x}\ [+c]$	B1	Allow any correct form
$7 = 16 + 4 + c$	M1	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
$f(x)=\ 2x^3+\frac{8}{x}-13$	A1	Allow $y =$, $f(x)$ or y can appear earlier in answer
	3	

Question 164

(a)	At stationary point $\frac{dy}{dx}=0$ so $6(3\times2-5)^3-k\times2^2=0$	M1	Setting given $\frac{dy}{dx}=0$ and substituting $x = 2$ into it.
	$[k=]\frac{3}{2}$	A1	OE
		2	
(b)	$[y=]\frac{6}{4\times3}(3x-5)^4-\frac{1}{3}kx^3$ $[+c]$.	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$. FT *their* non zero k.
	$-\frac{7}{2}=\frac{1}{2}(3\times2-5)^4-\frac{1}{3}\times\frac{3}{2}\times2^3+c$ [leading to $-3.5+c=-3.5$]	DM1	Using (2,-3.5) in an integrated expression. + c needed. Substitution needs to be seen, simply stating $c = 0$ is DM0.
	$y=\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$	A1	y = or f(x)= must be seen somewhere in solution.
(b)	**Alternative method for Question 11(b)**		
	$[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or $-270x^3-k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their k*
	$-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$	DM1	Using (2, -3.5) in an integrated expression. + c needed
	$y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$	A1	y = or f(x)= must be seen somewhere in solution.
		4	
(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their k*. Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2-1623x+1350$ or $-1620x-2kx$	B2,1,0 FT	FT *their k*. B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
(d)	[At $x = 2$] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48	M1	OE. Substituting $x = 2$ into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

Question 165

Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
Volume $= [\pi]\int(x-2)[\mathrm{d}x]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
$[\pi]\left[\frac{1}{2}x^2-2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
$=[\pi]\left[\left(\frac{5^2}{2}-2\times5\right)-\left(\frac{their\,3^2}{2}-2\times their\,3\right)\right]$ $=[\pi]\left[\frac{5}{2}+\frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
Volume of cylinder = $\pi\times1^2\times(5-their\,3)[=2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and *their* 3 used).
[Volume of solid $= 4\pi-2\pi=$]2π or 6.28	A1	AWRT

Question 166

(a)	$\frac{dy}{dx}=3(3x+4)^{-0.5}-1$	**B1 B1**	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent $= -\frac{1}{4}$ and Gradient of normal = 4	***M1**	Substituting $x = 4$ into a differentiated expression and using $m_1m_2 = -1$
	Equation of line is $(y-4) = 4(x-4)$ **or** evaluate c	**DM1**	With (4, 4) and *their* gradient of normal
	So $y = 4x - 12$	**A1**	
		5	
(b)	$3(3x+4)^{-0.5}-1=0$	**M1**	Setting *their* $\frac{dy}{dx}=0$
	Solving as far as $x=$	**M1**	Where $\frac{dy}{dx}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x=\frac{5}{3}$, $y=2\left(3\times\frac{5}{3}+4\right)^{0.5}-\frac{5}{3}=\frac{13}{3}$	**A1**	
		3	
(c)	$\frac{d^2y}{dx^2}=-\frac{9}{2}(3x+4)^{-1.5}$	**M1**	Differentiating *their* $\frac{dy}{dx}$ OR checking $\frac{dy}{dx}$ to find +ve and -ve either side of their $x=\frac{5}{3}$
	At $x=\frac{5}{3}$ $\frac{d^2y}{dx^2}$ is negative so the point is a maximum	**A1**	
		2	
(d)	Area $=\left[\int 2(3x+4)^{0.5}-x\,dx=\right]\frac{4}{9}(3x+4)^{1.5}-\frac{1}{2}x^2$	**B1 B1**	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5}-\frac{1}{2}(4)^2\right)-\frac{4}{9}(4)^{1.5}=\frac{256}{9}-8-\frac{32}{9}$	**M1**	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	**A1**	Or $\frac{152}{9}$
		4	

Question 167

$[y=]-\frac{1}{x^3}+8x^4\ [+c]$	**B1 B1**	OE. Accept unsimplified.
$4=-8+\frac{1}{2}+c$	**M1**	Substituting $\left(\frac{1}{2},4\right)$ into an integrated expression
$y=-\frac{1}{x^3}+8x^4+\frac{23}{2}$	**A1**	OE. Accept $-x^{-3}$; must be 8; $y=$ must be seen in working.
	4	

Question 168

(a)	$\{5(y-3)^2\}\quad\{+5\}$	**B1 B1**	Accept $a=-3$, $b=5$
		2	
(b)	$[f'(x)=]5x^4-30x^2+50$	**B1**	
	$5(x^2-3)^2+5$ or $b^2<4ac$ and at least one value of $f'(x)>0$	**M1**	
	>0 and increasing	**A1**	WWW
		3	

Question 169

(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$	**M1**	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)dx-\int\left(x^{-1/2}\right)dx$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	**A1 A1 A1**	If two separate integrals with no subtraction **SC B1** for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	**DM1**	Substitute limits $\frac{1}{4}\rightarrow 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	**A1**	WWW. Cannot be awarded if π appears in any integral.
		6	
(b)	$\left[\frac{dy}{dx}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	**B1**	
	When $x=1$, $m=-\frac{1}{2}$	**M1**	Substitute $x=1$ into a differential.
	[Equation of normal is] $y-1=2(x-1)$	**M1**	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
	[When $x=0$,] $p=-1$	**A1**	WWW
		4	

Question 170

)(a)	$f''(x)=-\left(\frac{1}{2}x+k\right)^{-3}$	**B1**	
	$f''(2)>0\Rightarrow-(1+k)^{-3}>0$	**M1**	Allow for solving *their* $f''(2)>0$
	$k<-1$	**A1**	WWW
		3	

(b)	$\left[f(x)=\int\left(\left(\tfrac{1}{2}x-3\right)^{-2}-(-2)^{-2}\right)dx=\right]\left\{\dfrac{\left(\tfrac{1}{2}x-3\right)^{-1}}{-1\times\tfrac{1}{2}}\right\}\left\{-\dfrac{x}{4}\right\}$	B1 B1	Allow $-2\left(\dfrac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\tfrac{1}{2}=1-\tfrac{1}{2}+c$	M1	Substitute $x = 2$, $y = 3\tfrac{1}{2}$ into *their* integral with c present.
	$f(x)=\dfrac{-2}{\left(\tfrac{1}{2}x-3\right)}-\dfrac{x}{4}+3$	A1	OE
		4	
(c)	$\left(\tfrac{1}{2}x-3\right)^{-2}-(-2)^{-2}=0$	M1	Substitute $k=-3$ and set to zero.
	leading to $\left(\tfrac{1}{2}x-3\right)^{2}=4$ $\left[\tfrac{1}{2}x-3=(\pm)2\right]$ leading to $x=10$	A1	
	$(10, -\tfrac{1}{2})$	A1	Or when $x = 10$, $y = -1 - 2\tfrac{1}{2} + 3 = -\tfrac{1}{2}$
	$f''(10)\left[=-(5-3)^{-3}\rightarrow\right]<0\rightarrow$ MAXIMUM	A1	WWW
		4	

Question 171

(a)	$\dfrac{dy}{dx}=\dfrac{1}{2}+\dfrac{1}{3(x-2)^{\frac{4}{3}}}$	B1	OE. Allow unsimplified.
	Attempt at evaluating *their* $\dfrac{dy}{dx}$ at $x = 3$ $\left[\dfrac{1}{2}+\dfrac{1}{3(3-2)^{\frac{4}{3}}}=\dfrac{5}{6}\right]$	*M1	Substituting $x = 3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal = $\dfrac{-1}{their\dfrac{dy}{dx}}$ $\left[=-\dfrac{6}{5}\right]$	*DM1	Negative reciprocal of *their* evaluated $\dfrac{dy}{dx}$.
	Equation of normal $y-\dfrac{6}{5}=(their\text{ normal gradient})(x-3)$ $\left[y=-\dfrac{6}{5}x+4.8\Rightarrow 5y=-6x+24\right]$	DM1	Using *their* normal gradient and A in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	A1	or (4, 0)
		5	
(b)	Area under curve = $\int\left(\dfrac{1}{2}x+\dfrac{7}{10}-\dfrac{1}{(x-2)^{\frac{1}{3}}}\right)[dx]$	M1	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\dfrac{1}{4}x^2+\dfrac{7}{10}x-\dfrac{3(x-2)^{\frac{2}{3}}}{2}$	A1	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\dfrac{9}{4}+2.1-\dfrac{3}{2}\right)-\left(\dfrac{6.25}{4}+1.75-\dfrac{3\times0.5^{\frac{2}{3}}}{2}\right)$	M1	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	A1	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	B1	OE
	[Total area =] 1.08	A1	Dependent on the first M1 and WWW.
		6	

Question 172

(a)	$[f'(x) =]\ 2x - \frac{k}{x^2}$	B1	
	$f'(2) = 0\ \left[2\times 2 - \frac{k}{2^2} = 0\right] \Rightarrow k = \ldots$	M1	Setting *their* 2-term f'(2) = 0, at least one term correct and attempting to solve as far as $k =$.
	$k = 16$	A1	
		3	
(b)	$f''(2) =$ e.g. $2 + \frac{2k}{2^3}$	M1	Evaluate a two term f″(2) with at least one term correct. Or other valid method.
	$\left[2 + \frac{2k}{2^3}\right] > 0 \Rightarrow$ minimum or $= 6 \Rightarrow$ minimum	A1 FT	WWW. FT on positive k value.
		2	
(c)	When $x = 2$, $f(x) = 14$	B1	SOI
	[Range is or y or $f(x)$] $\geqslant$ *their* f(2)	B1 FT	Not $x \geqslant$ *their* f(2)
		2	

Question 173

(a)	$\left[\frac{dV}{dr} =\right]\ \frac{9}{2}\left(r - \frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1.5}{their\frac{dV}{dr}}\ \left[= \frac{1.5}{\frac{9}{2}\left(5.5 - \frac{1}{2}\right)^2} = \frac{1.5}{112.5}\right]$	M1	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{dV}{dr}$ and using $r = 5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
(b)	$\frac{dV}{dr}$ or $their\frac{dV}{dr} = \frac{1.5}{0.1}$ or 15 OR $0.1 = \frac{1.5}{their\frac{dV}{dr}}\ \left[= \frac{2\times 1.5}{9\left(r - \frac{1}{2}\right)^2}\text{OE}\right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or *their* $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r - \frac{1}{2}\right)^2 = 15 \Rightarrow\right]\ r = \frac{1}{2} + \sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3 + 5\sqrt{30}}{3}$. CAO.
		3	

Question 174

$y = -\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
	DB1	For $-\frac{8}{3}$
$5\frac{2}{3} = -\frac{\frac{8}{3}}{(3\times 2+2)} + c$	M1	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = -1, or dividing by their power. + c needed
$y = -\frac{8}{3(3x+2)} + 6$	A1	OE e.g. $y = -\frac{8}{3}(3x+2)^{-1} + 6$
	4	

Question 175

(a)	$\left\{\dfrac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	B2, 1, 0	Attempt to integrate
	$-\frac{2}{3}[0-1]$	M1	M1 for applying limits $1 \rightarrow \infty$ to an integrated expression (either correct power or dividing by their power).
	$\dfrac{2}{3}$	A1	
		4	
(b)	$[\pi]\int y^2 dx = [\pi]\int (3x-2)^{-3} dx = [\pi]\dfrac{(3x-2)^{-2}}{-2\times 3}$	*M1 A1	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\dfrac{1}{6}\right]\left[\dfrac{1}{16}-1\right]$	DM1	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\dfrac{5\pi}{32}$	A1	OE
		4	
(c)	$\dfrac{dy}{dx} = -\dfrac{3}{2}\times 3(3x-2)^{-\frac{5}{2}}$	M1	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\dfrac{dy}{dx} = -\dfrac{9}{2}$	*M1	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y-1=\dfrac{2}{9}(x-1)$ OR evaluates c	DM1	Forms equation of line or evaluates c using (1, 1) and gradient $\dfrac{-1}{their\dfrac{dy}{dx}}$.
	At A, $y=\dfrac{7}{9}$	A1	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

Question 176

(a)	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}\ [+c]$	**B2, 1, 0**	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$	**M1**	Substitute $f(1)=-\frac{1}{3}$ into an integrated expression and evaluate c.
	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	**A1**	OE.
		4	
(b)	$2x^4-7x^2-4\ [=0]$	**M1**	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2-7y-4[=0]$.
	$(2x^2+1)(x^2-4)\ [=0]$	**M1**	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. y. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x=[\pm]2$	**A1**	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3-7(2)+\frac{4}{2}+2\right.$ leading to $\left.\right]\left(2,-\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3-7(-2)+\frac{4}{-2}+2\right.$ leading to $\left.\right]\left(-2,\frac{26}{3}\right)$	**B1 B1**	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
(c)	$f''(x)=4x+8x^{-3}$	**B1**	OE
		1	
(d)	$f''(2)=9>0$ MINIMUM at $x=$ *their* 2	**B1 FT**	FT on *their* $x=[\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=2$.
	$f''(-2)=-9<0$ MAXIMUM at $x=$ *their* -2	**B1 FT**	FT on *their* $x=[\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=-2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2)>0$ MIN and $f''(-2)<0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	**M1**	FT on *their* $x=[\pm]2$
	MINIMUM at $x=$ *their* 2, MAXIMUM at $x=$ *their* 2	**A1 FT**	FT on *their* $x=[\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ −/0/+ MIN and $f'(-2)$ +/0/− MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	**B1 B1**	Need correct coordinates in **(b)** for this method.
		2	

Question 177

(a)	$\frac{dy}{dx}=\left\{-k(3x-k)^{-2}\right\}\{\times 3\}\{+3\}$	B2, 1, 0	
	$\frac{-3k}{(3x-k)^2}+3=0$ leading to $(3)(3x-k)^2=(3)k$ leading to $3x-k=[\pm]\sqrt{k}$	M1	Set $\frac{dy}{dx}=0$ and remove the denominator
	$x=\frac{k\pm\sqrt{k}}{3}$	A1	OE
		4	
(b)	$a=\frac{4\pm\sqrt{4}}{3}$ leading to $a=2$	B1	Substitute $x=a$ when $k=4$. Allow $x=2$.
	$f''(x)=f'\left[-12(3x-4)^{-2}+3\right]=72(3x-4)^{-3}$	B1	Allow $18k(3x-k)^{-3}$
	>0 (or 9) when $x=2\rightarrow$ minimum	B1 FT	FT on *their* $x=2$, providing their $x\geqslant\frac{3}{2}$ and $f''(x)$ is correct
		3	
(c)	Substitute $k=-1$ leading to $g'(x)=\frac{3}{(3x+1)^2}+3$	M1	Condone one error.
	$g'(x)>0$ or $g'(x)$ always positive, hence g is an increasing function	A1	WWW. A0 if the conclusion depends on substitution of values into $g'(x)$.
	Alternative method for question 11(c)		
	$x=\frac{k\pm\sqrt{k}}{3}$ when $k=-1$ has no solutions, so g is increasing or decreasing	M1	Allow the statement 'no turning points' for increasing or decreasing
	Show g'(x) is positive for any value of x, hence g is an increasing function	A1	Or show $g(b)>g(a)$ for $b>a\rightarrow$ g, hence g is an increasing function
		2	

Question 178

(a)	$(-2)^2+y^2=8$ leading to $y=2$ leading to $A=(0,2)$	B1	
	Substitute $y=their\,2$ into circle leading to $(x-2)^2+4=8$	M1	Expect $x=4$.
	$B=(4, 2)$	A1	
		3	
(b)	Attempt to find $[\pi]\int\left(8-(x-2)^2\right)dx$	*M1	
	$[\pi]\left[8x-\frac{(x-2)^3}{3}\right]$ or $[\pi]\left[8x-\left(\frac{x^3}{3}-2x^2+4x\right)\right]$	A1	
	$[\pi]\left(32-\frac{16}{3}\right)$ or $[\pi]\left[32-\left(\frac{64}{3}-32+16\right)\right]$	DM1	Apply limits $0\rightarrow$ *their* 4.
	Volume of cylinder = $\pi\times 2^2\times 4=16\pi$	B1 FT	OR from $\pi\int 2^2\,dx$ with *their* limits from **(a)**. FT on *their* A and B
	$\left[\text{Volume of revolution}=26\tfrac{2}{3}\pi-16\pi=\right]10\tfrac{2}{3}\pi$	A1	Accept 33.5
		5	

Question 179

$[f(x)=]\frac{2x^{\frac{2}{3}}}{\frac{2}{3}}-\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\ [+c]$	**B1 B1**	$\frac{2}{3}$ and $\frac{4}{3}$ may be seen as sums of 1 and a fraction.
$5=12-12+c$	**M1**	Substituting (8,5) into an integral.
$[f(x)=]3x^{\frac{2}{3}}-\frac{3}{4}x^{\frac{4}{3}}+5$	**A1**	Fractions in the denominators scores A0.
	4	

Question 180

(a)	$\left\{\frac{(4x+2)^{-1}}{-1}\right\}\{\div 4\}$ or eg $\left\{\frac{1}{16}\right\}\left\{-(x+0.5)^{-1}\right\}$ or $\frac{-1}{(16x+8)}$	**B1 B1**	OE If more than one function of x present then B0 B0.
	$0-(-1/24)$	**M1**	Apply limits to an integral, ∞ must be used correctly.
	1/24	**A1**	Allow 0.0417 AWRT.
		4	
(b)	$\frac{dy}{dx}=\left\{-2(4x+2)^{-3}\right\}\quad\{\times 4\}$	**B1 B1**	Allow unsimplified forms.
	Recognise $\frac{dy}{dx}=-1$	**B1**	SOI
	their $\frac{-8}{(4x+2)^3}=$ *their* -1	**M1**	Must be numerical. Must be some attempt to solve *their* equation and $\frac{dy}{dx}\neq 0$.
	(0, ¼)	**A1 A1**	Accept $x=0$, $y=$ ¼. $y=$ ¼ must be from $x=0$ not $x=-1$.
		6	

Question 181

(a)	$\left[\frac{dy}{dx}=\right]\frac{1}{2}x^{-1/2}-2x^{-3/2}$	**B1 B1**	Allow unsimplified versions.
	At $x=1$, $\frac{dy}{dx}=\frac{1}{2}-2=-\frac{3}{2}$	**M1**	Substitute $x=1$ into a differentiated y.
	Equation of tangent is $y-5=-\frac{3}{2}(x-1)$	**A1**	WWW Or $y=-\frac{3}{2}x+\frac{13}{2}$.
		4	
(b)	$\frac{x^{3/2}}{3/2}+8x^{1/2}$	**B1**	OE Integrate to find area under curve, allow unsimplified versions.
	$\left[\left(\frac{128}{3}+32\right)-\left(\frac{2}{3}+8\right)\right]$	**M1**	Apply limits 1 → 16 to an integrated expression.
	Area under line = 15 × 5 = 75	**B1**	Or by $\int_1^{16}5\,dx$.
	Required area = 75 – 66 = 9	**A1**	
		4	

Question 182

(a)	$\frac{dy}{dx} = \{3\} + \left\{-4 \times \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3\right\}\left[= 3 - 6(3x+1)^{-\frac{1}{2}}\right]$	B1 B1	Correct differentiation of $3x+1$ and no other terms and correct differentiation of $-4(3x+1)^{\frac{1}{2}}$. Accept unsimplified.
	$\left[\frac{d^2y}{dx^2} =\right] -\frac{1}{2} \times -6(3x+1)^{-\frac{3}{2}} \times 3\ [= 9(3x+1)^{-\frac{3}{2}}]$	B1	WWW. Accept unsimplified. Do not award if $\frac{dy}{dx}$ is incorrect.
		3	
(b)	$\frac{dy}{dx} = 0$ leading to $3 - 6(3x+1)^{-\frac{1}{2}} = 0$	M1	Setting *their* $\frac{dy}{dx} = 0$.
	$(3x+1)^{\frac{1}{2}} = 2 \Rightarrow 3x+1 = 4$ leading to $x = 1$	A1	CAO – do not ISW for a second answer.
	$y = -4$ [coordinates (1, −4)]	A1	Condone inclusion of second value from a second answer.
	$\frac{d^2y}{dx^2} = 9(3 \times 1 + 1)^{-\frac{3}{2}} = \frac{9}{8}$ or > 0 so minimum	A1	Some evidence of substitution needed but $\frac{d^2y}{dx^2}$. Do not award if $\frac{d^2y}{dx^2}$ is incorrect or wrongly evaluated. Accept correct consideration of gradients either side of $x = 1$.
		4	

Question 183

Line meets curve when: $2x + 2 = 5x^{\frac{1}{2}}$ leading to $2x - 5x^{\frac{1}{2}} + 2[= 0]$ **or** $4x^2 + 8x + 4 = 25x$ leading to $4x^2 - 17x + 4[= 0]$ **or** $x = \frac{y^2}{25}$ leading to $2y^2 - 25y + 50[= 0]$	M1	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are: $(2x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
$x = \frac{1}{4}, x = 4$	A1	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
Area = $\int 5x^{\frac{1}{2}} - (2x+2)\,dx = \int 5x^{\frac{1}{2}} - 2x - 2\ dx$	*M1	Intention to integrate and subtract areas. Condone missing brackets and/or subtraction wrong way around.
$= \left[\frac{10}{3}x^{\frac{3}{2}} - x^2 - 2x\right]_{\frac{1}{4}}^{4} = \left(\left(\frac{10}{3} \times 8 - 16 - 8\right) - \left(\frac{10}{3} \times \frac{1}{8} - \frac{1}{16} - \frac{1}{2}\right)\right)$	DM1	Integrating($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something else).
$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	A1	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.

Question 184

$[y =]\left\{\frac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2} \times 4}\right\} + \left\{-\frac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right\}\left[\Rightarrow \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}}\right][+c]$	B1 B1	Marks can be awarded for correct unsimplified expressions ISW.
$\frac{5}{2} = \frac{1}{2}(9)^{\frac{3}{2}} - 8 \times 4^{\frac{1}{2}} + c\quad [\Rightarrow c = 5]$	M1	Using $(4, \frac{5}{2})$ in an integrated expression (defined by at least one correct power) including + *c*.
$y = \frac{3}{6}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + 5$.	A1	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution. Coefficients must not contain unresolved double fractions.
	4	

Question 185

(a)	$\dfrac{d^2y}{dx^2}=6(-1)^2-\dfrac{4}{(-1)^3}>0 \therefore$ minimum **or** $\dfrac{d^2y}{dx^2}=10 \;\therefore$ minimum	**B1**	Sub $x=-1$ into $\dfrac{d^2y}{dx^2}$, correct conclusion. WWW
		1	
(b)	$\dfrac{dy}{dx}=2x^3+\dfrac{2}{x^2}[+c]$	***M1**	Integrating $\dfrac{d^2y}{dx^2}$ (at least one term correct).
	$0=-2+2+c$ leading to $c=[0]$	**DM1**	Substituting $x=-1, \dfrac{dy}{dx}=0$ (need to see) to evaluate c. DM0 if simply state $c=0$ or omit $+c$.
	$y=\dfrac{1}{2}x^4-\dfrac{2}{x}+(\textit{their } c)x+k$	**A1 FT**	Integrated. FT *their* non-zero value of c if DM1 awarded.
	$\dfrac{9}{2}=\dfrac{1}{2}+2+k$ leading to $k=[2]$	**DM1**	Substituting $x=-1, y=\dfrac{9}{2}$ to evaluate k (dep on *M1).
	$y=\dfrac{1}{2}x^4-\dfrac{2}{x}+2$	**A1**	OE e.g. $2x^{-1}$ or $\dfrac{4}{2}$. A0 (wrong process) if c not evaluated but correct answer obtained.
		5	
(c)	$\dfrac{dy}{dx}=2x^3+\dfrac{2}{x^2}=0$	**M1**	*Their* $\dfrac{dy}{dx}=0$.
	Leading to $x^5=-1$	**M1**	Reaching equation of the form $x^5=a$.
	So only stationary point is when $x=-1$	**A1**	$x=-1$ and stating e.g. 'only' or 'no other solutions.
		3	
(d)	At $x=1$, $\dfrac{dy}{dx}=[4]$	***M1**	Substituting $x=1$ into *their* $\dfrac{dy}{dx}$.
	$\dfrac{dx}{dt}=\dfrac{dx}{dy}\times\dfrac{dy}{dt}=\dfrac{1}{4}\times 5$	**DM1**	OE Using chain rule correctly SOI.
	$\dfrac{5}{4}$	**A1**	OE e.g. 1.25.
		3	

Question 186

(a)	$(3x-2)^{\frac{1}{2}}=\frac{1}{2}x+1 \Rightarrow 3x-2=\left(\frac{1}{2}x+1\right)^2=\frac{1}{4}x^2+x+1$	M1	Equating curve and line, attempt to square; $\frac{1}{4}x^2+1$ M0
	$\Rightarrow \frac{1}{4}x^2-2x+3[=0]\left[\Rightarrow x^2-8x+12=0\right]\Rightarrow(x-6)(x-2)[=0]$	M1	Forming and solving a 3TQ by factorisation, formula or completing the square – see guidance.
	(2, 2) and (6, 4)	A1 A1	A1 for each point, or A1 A0 for two correct *x*-values. If M0 for solving, **SC B2** possible: B1 for each point or B1 B0 for two correct *x*-values.
		4	
(b)	Area $=\pm\int_{[2]}^{[6]}\left((3x-2)^{\frac{1}{2}}-\left(\frac{1}{2}x+1\right)\right)[dx]$	*M1	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}-\left(\frac{1}{4}x^2+x\right)\right]_2^6$	B1 B1	B1 for each bracket integrated correctly (in any form).
	$\pm\left(\left[\frac{2}{9}(16)^{\frac{3}{2}}-\left(\frac{1}{4}\times 36+6\right)\right]-\left[\frac{2}{9}(4)^{\frac{3}{2}}-\left(\frac{1}{4}\times 4+2\right)\right]\right)$	DM1	±(F(*their* 6) – F(*their* 2)) with *their* integral. Allow 1 sign error.
	$\frac{4}{9}$	A1	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.

Question 187

$\left[\frac{dv}{dx}\right]=(9-x)^2$	B1	Allow unsimplified forms. Allow any or no notation
Substitute $x = 4$ into *their* differentiated V,	*M1	Expect 25.
$\frac{dx}{dt}=\frac{1}{\textit{their}\text{ derivative}}\times 3.6$ (accept $\frac{dt}{dx}=\frac{\textit{their}\text{ derivative}}{3.6}$)	M1	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
$=\frac{1}{\textit{their numerical}\text{ derivative}}\times 3.6\times\frac{100}{60}$	DM1	Correct use of the conversion factors.
$=\frac{1}{25}\times 3.6\times\frac{100}{60}=0.24$	A1	
	5	

Question 188

(a)	$\frac{-3}{(a+2)^4} = -\frac{16}{27} \rightarrow$ e.g. $16(a+2)^4 = 81$	M1	Equate first derivative and $-\frac{16}{27}$ and move term in a (or x) into the numerator.
	$\rightarrow (a+2)^2 = \frac{9}{4} \rightarrow a+2 = [\pm]\frac{3}{2}$	M1	Solve for $(a+2)$ or $(x+2)$
	$a = -\frac{1}{2}$ or $-\frac{7}{2}$	A1 A1	Allow 'x ='
		4	
(b)	$[\mathrm{f}(x)] = \frac{1}{(x+2)^3}$ $[+c]$	B1	Allow unsimplified form and 'y ='
	$5 = 1 + c$	M1	Sub $x=-1,\ y=5$ into an integral.
	$[\mathrm{f}(x)] = \frac{1}{(x+2)^3} + 4$	A1	Allow 'y ='
		3	

Question 189

(a)	$x^2 + (2x-1)^2 - 2[=0] \rightarrow 5x^2 - 4x - 1\ [=0]$	*M1 A1	Or $5y^2 + 2y - 7\ [=0]$.
	$(5x+1)(x-1)[=0]$ or $(5y+7)(y-1)[=0]$	DM1	May see factors or formula or completing square.
	$x=1,\ y=1$ or (1, 1) only	A1	May be implied on the diagram.
		4	
(b)	$(\pi)\int(2-x^2)\mathrm{d}x = (\pi)\left(2x - \frac{x^3}{3}\right)$	*M1 A1	Attempt integration of y^2, allow $\int(2-y^2)\mathrm{d}y$.
	$(\pi)\left(2\sqrt{2} - \frac{(\sqrt{2})^3}{3}\right) - \left(2 - \frac{1}{3}\right))$	DM1	Apply limits $1 \rightarrow \sqrt{2}$.
	$\frac{\pi}{3}(4\sqrt{2} - 5)$	A1	CAO, allow $\frac{\pi}{3}(2\sqrt{8} - 5)$, must be in given form.
		4	
(c)	Arc length = $\frac{1}{8}(2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter = $\sqrt{2} +$ *their* arc length	B1 FT	Must be exact, do not allow inverse trig functions.
		2	

Question 190

(a)	$[y=]\left\{\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+\left\{-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]\left[=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right]$	**B1 B1**	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5=2\times3^{\frac{3}{2}}-6\times3^{\frac{1}{2}}+c$	**M1**	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+5$	**A1**	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	
(b)	$3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}=0$	**M1**	Setting given differential to 0.
	$[x=]\ 1$	**A1**	CAO WWW Condone extra solution of —1 only if it is rejected.
		2	
(c)	$x>1$ or $x>$ "*their* 8(b)"	**B1FT**	Allow $\geqslant$
		1	

Question 191

(a)	$\left[\frac{dy}{dx}=\right]\frac{9}{2}x-12\ [=0]$ or $[y=]\ \frac{9}{4}\left\{\left(x-\frac{8}{3}\right)^2+\frac{8}{9}\right\}$ or $\frac{9}{4}\left(x-\frac{8}{3}\right)^2+2$	**B1**	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
	$x=\frac{24}{9}$	**B1**	OE Condone 2.67 AWRT.
	$y=2$	**B1**	CAO Note: $x=\frac{-b}{2a}=\frac{8}{3}$ B1; substitute $\frac{8}{3}$ for x in $y=$ B1; $y=2$ B1.
		3	
(b)	[Area =] $\int\left(18-\frac{3}{8}x^{\frac{5}{2}}-\left(\frac{9}{4}x^2-12x+18\right)\right)dx$	**M1**	Intention to integrate **and** subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}\right)$, $\left(\frac{9x^3}{4\times3}-\frac{12x^2}{2}[+18x]\right)$ Or combined $[18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}-\frac{9x^3}{4\times3}+\frac{12x^2}{2}[-18x]$	**B1,B1**	One mark for correct integration of each curve, allow unsimplified. $\left([18x]-\frac{3}{28}x^{\frac{7}{2}}\right)$ $\left(\frac{3}{4}x^3-6x^2[+18x]\right)$ or $[18x]-\frac{3}{28}x^{\frac{7}{2}}-\frac{3}{4}x^3+6x^2[-18x]$ **BUT** condone sign errors that are only due to missing brackets.
	$=\left(-\frac{3}{28}\times4^{\frac{7}{2}}-\frac{3}{4}\times4^3+6\times4^2\right)\ [-(0)]$	**M1**	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$=\frac{240}{7}$ or **34.3** AWRT	**A1**	**SC:** If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

(c)	$\left[\frac{dy}{dx}=\right]\frac{-5\times 3}{2\times 8}x^{\frac{3}{2}}\left[=-\frac{15}{16}x^{\frac{3}{2}}\right]$	B1	Allow unsimplified.
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=-\frac{15}{16}\times 8\times 2$	M1	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
	-15	A1	Accept decreasing [at/by] 15
		3	**Note:** If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2}-12)\times 2[=12]$

Question 192

(a)	$12\left(\frac{1}{2}\times 6-1\right)^{-4}\left[=12(2)^{-4}=\frac{3}{4}\right]$	M1	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
	$y-4=\frac{3}{4}(x-6)$ OR evaluates $c=-\frac{1}{2}$	A1	OE e.g. $y=\frac{3}{4}x-\frac{1}{2}$ or evaluates c in $y=\frac{3}{4}x+c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
		2	
(b)	$[y=]\left(\frac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3}\right)\div\frac{1}{2}\left[=-8\left(\frac{1}{2}x-1\right)^{-3}\right]$	B2, 1, 0	
	$4=\frac{12\times\left(\frac{1}{2}\times 6-1\right)^{-3}}{\frac{1}{2}\times -3}+c\ \left[\Rightarrow 4=-8\times 2^{-3}+c\right]\Rightarrow c=[5]$	M1	Must have $+c$. Substitute $y=4,\ x=6$ and solve for c in an integrated expression. May be unsimplified.
	$[y=]-8\left(\frac{1}{2}x-1\right)^{-3}+5$	A1	OE Must see '$y=$' or '$f(x)=$' in the working.
		4	

Question 193

$\frac{dy}{dx}=\frac{1}{2}ax^{-\frac{1}{2}}-2$	B2, 1, 0	
$0=\frac{1}{2}a(9)^{-\frac{1}{2}}-2\Rightarrow\frac{a}{6}-2=0\Rightarrow a=[12]$	M1	Substitute $x = 9$ and $\frac{dy}{dx}=0$ into *their* derivative and solve a linear equation for a.
$[a=]12$	A1	
$\left[y=\textit{their}\ a\times(9)^{\frac{1}{2}}-18=\right]18$	A1 FT	FT on *their* a.
	5	

Question 194

(a)	$f'(x) = -3(-1)(4)(4x-p)^{-2}\left[= \dfrac{12}{(4x-p)^2}\right]$	**B2, 1, 0**	
	> 0 Hence increasing function	**B1FT**	Correct conclusion from *their* $f'(x)$.
		3	
(b)	$y = 2 - \dfrac{3}{4x-p} \Rightarrow (y-2)(4x-p) = -3$ **or** $4xy - py = 8x - 2p - 3$	**M1**	OE Form horizontal equation. Sign errors only, no missing terms. May go directly to $4y = p - \dfrac{3}{x-2}$ OE M1 M1
	$4xy - 8x = py - 2p - 3 \Rightarrow 4x(y-2) = p(y-2) - 3$ **or** $4x = -\dfrac{3}{x-2} + p$	**M1**	OE Factorise out $[4]x$ or $[4]y$.
	$x = \dfrac{p(y-2)-3}{4(y-2)}\left[\Rightarrow x = \dfrac{p}{4} - \dfrac{3}{4y-8}\right]$ **or** $\dfrac{-\dfrac{3}{x-2}+p}{4}$	**M1**	OE Make x (or y) the subject.
	$\left[f^{-1}(x) =\right]\dfrac{p}{4} - \dfrac{3}{4x-8}$	**A1**	OE in correct form (must be in terms of x).
		4	
(c)	$[p=]8$	**B1**	
		1	

Question 195

(a)	$\pm\int\left(2x^{1/2}+1\right) - \left(\dfrac{1}{2}x^2 - x + 1\right)\,dx\ [= \pm\int 2x^{1/2} - \dfrac{1}{2}x^2 + x\,dx]$	***M1**	
	$\pm\left(\dfrac{4x^{3/2}}{3} + x - \left(\dfrac{x^3}{6} - \dfrac{x^2}{2} + x\right)\right)$ or $\pm\left(\dfrac{4x^{3/2}}{3} - \dfrac{x^3}{6} + \dfrac{x^2}{2}\right)$	**B2, 1, 0**	OE Coefficients may be unsimplified.
	$\pm\left(\dfrac{32}{3} - \dfrac{32}{3} + 8\right)$ or $\pm\left(\dfrac{44}{3} - 0 - \dfrac{20}{3} + 0\right)$	**DM1**	± (F(4) – F(0)) using *their* integral(s).
	$= 8$	**A1**	Depends on all previous marks. If *M1 B2 DM0 **and** limits stated, **SC B1** for +8
		5	
(b)	Upper curve: $\dfrac{dy}{dx} = x^{-\frac{1}{2}}$. Lower curve: $\dfrac{dy}{dx} = x - 1$	**M1 A1**	Attempt at differentiating one function. A1 if both correct.
	At $x = 4$: gradient of upper curve $= \dfrac{1}{2}$, gradient of lower curve $= 3$	**M1**	Evaluate two gradients using $x = 4$.
	$\alpha = \tan^{-1}3 - \tan^{-1}\dfrac{1}{2}\ [= 71.57 - 26.57]$	**M1**	Use inverse tan to find angles then subtract. **OR** find equations of both tangents then Pythagoras using a point on each e.g. on axes. **OR** cosine rule using intercepts or proportion.
	$[\alpha =]45°$	**A1**	AWRT
		5	

Question 196

$\frac{dy}{dx} = \left\{\frac{1}{60}(3x+1)\times 2\right\} \times \{3\}$	B1 B1	May see $\frac{1}{60}(18x+6)$.
$\frac{1}{10}(3x+1) = 1$	M1	Equate *their* $\frac{dy}{dx}$ to 1.
$x = 3$	A1	
	4	

Question 197

(a)	$-\frac{3}{2} = \frac{1}{2} + k$ leading to $k = -2$	B1	**AG** Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
(b)	$[y] = 2x^{\frac{1}{2}} - 2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2} - 2x$.
	$-1 = 4 - 8 + c$	M1	Substitute $x = 4$, $y = -1$ (c present) Expect $c = 3$.
	$y = 2x^{\frac{1}{2}} - 2x + 3$ or $y = 2\sqrt{x} - 2x + 3$	A1	Allow if f(x) = or y = anywhere in the solution.
		4	
(c)	$x^{-1/2} - 2 = 0$	M1	Set *their* $\frac{dy}{dx}$ to zero.
	$x = \frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2 = \pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4}, 3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	
(d)	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	< 0 (or −4) hence Maximum	DB1	WWW Ignore extra solutions from $x = -\frac{1}{4}$.
		2	

Question 198

(a)	Gradient of $AB = \dfrac{2-(-1)}{5-2}$	M1	Expect 1, must be from $\Delta y / \Delta x$.
	Equation of AB is $y-2=1(x-5)$ or $y+1=1(x-2)$	A1	OE. Expect $y=x-3$.
		2	
(b)	$[\pi]\int x^2 dy = [\pi]\int (y^2+1)^2 dy = [\pi]\int (y^4+2y^2+1)dy$	M1	For curve: Attempt to square y^2+1 and attempt integration. Subtracting curve equation from line equation before squaring is M0. Integration before squaring M0.
	$[\pi]\left(\dfrac{y^5}{5}+\dfrac{2y^3}{3}+y\right)$	A2, 1, 0	
	$[\pi]\int (y+3)^2 dy = [\pi]\int (y^2+6y+9)dy$	M1	For line: Attempt to square *their* $y+3$ and attempt integration.
	$[\pi]\left(\dfrac{y^3}{3}+3y^2+9y\right)$ or $[\pi]\left(\dfrac{(y+3)^3}{3}\right)$	A2, 1, 0	Not available for incorrect line equations.
	$[\pi]\left\{\dfrac{8}{3}+12+18-\left(-\dfrac{1}{3}+3-9\right)\right\}$ or $[\pi]\left\{\dfrac{32}{5}+\dfrac{16}{3}+2-\left(-\dfrac{1}{5}-\dfrac{2}{3}-1\right)\right\}$	DM1	Apply limits $-1 \to 2$ to either integral providing they have been awarded M1. Expect $15\frac{3}{5}$ $[\pi]$ and/or $39[\pi]$. Some evidence of substitution of both -1 and 2 must be seen. Dependent on at least one of the first 2 M1 marks.
	Volume = $[\pi](39 - 15\frac{3}{5})$	DM1	Appropriate subtraction. Dependent on at least one of the first 2 M1 marks.
	$= 23\dfrac{2}{5}\pi$ or $\dfrac{117}{5}\pi$ or awrt 73.5[1327]	A1	
		9	

Question 199

(a)	$[y=]\ \{x\}\ \{+(x-1)^{-2}\ \}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0$, $y = 3$ leading to 3 = 0 + 1 + c	M1	Substitution into an integral, expect $c = 2$.
	$y=x+(x-1)^{-2}+2$ or $f(x)=x+(x-1)^{-2}+2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
(b)	[Gradient of tangent =] f′(0) = 3	B1	
	Equation of tangent is $y-3=$ *their* gradient at $x=0(x-0)$	M1*	Expect $y=3x+3$, normal gets M0.
	Intersection given by $3x+3=x+(x-1)^{-2}+2$	DM1	FT *their* equation from part **(a)**.
	$2x+1=\frac{1}{(x-1)^2}$ → $(2x+1)(x-1)^2-1=0$ or solve equation before given form reached and show solution ($x=3/2$) satisfies given result	A1	WWW AG
		4	
(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\frac{3}{2}$ $y = 7½$	B1	
		2	

Question 200

(a)	$\left[\frac{dy}{dx}=\right]\{9\}+\left\{-\frac{3}{2}(2x+1)^{1/2}\times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9-3(2x+1)^{1/2}=0$ leading to $2x+1=9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2-3^2(2x+1)=0$ M0A0. $2x+1=\sqrt{3}$ *or* $2x+1=\pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
(b)	When x = 1½, shows substitution or $\frac{dy}{dx}=3$	M1	Substituting x = 1½ into their $\frac{dy}{dx}$.
	Gradient of *AB* is $\frac{5½-3½}{1½-7½}\left[=\frac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\frac{1}{3}$x3 $=-1$. [Hence *AB* is the normal]	A1	
	Alternative method for Question 10(b)		
	When x = 1½ $\frac{dy}{dx}=3$,[perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y=\frac{-x}{3}+6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	**B1 B1**	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or 48.325 – 3.725	**M1**	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ [= 27]	**B1**	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	**A1**	**SC B1** if no substitution of the limits seen.
		5	

Question 201

$[y]=\frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	**B1**	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
$5=\frac{4}{-2}(4-3)^{-2}+c$ or $5=\frac{4}{-2(4-3)^2}+c$ leading to $c=$	**M1**	Correct use of $(4, 5)$ to find c in an integrated expression (defined by the correct power and no extra x's or terms).
$y=\frac{-2}{(x-3)^2}+7$ or $y=-2(x-3)^{-2}+7$	**A1**	OE $-\frac{4}{2}$ must be simplified to **–**2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
	3	

Question 202

$[\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=]\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}\left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	**B1 B1**	B1 for contents of each { } then ISW.
$=\left(their\frac{20}{3}\times 8-32\right)[-0]$	**M1**	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their $(\frac{160}{3}-32)$.
[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	**A1**	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
	4	

Question 203

(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left\{k\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right\}\{\times 4\}\{-1\}$	B 2,1,0	OE e.g. $2k(4x+1)^{-\frac{1}{2}}-1$ B2 Three correct unsimplified { } and no others. B1 Two correct { } or three correct { } and an additional term e.g. +5. B0 More than one error.
		2	
(b)	$2k(4x+1)^{-\frac{1}{2}}-1=0$ leading to $(4x+1)^{\frac{1}{2}}=2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}}=1$	M1	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}}-1$ where a = 2 or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x=\frac{4k^2-1}{4}$	A1	CAO OE simplified expression ISW.
		2	
(c)	$2\times 10.5(4x+1)^{-\frac{1}{2}}-1=2$	M1	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7=(4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	A1	If M1 earned **SCB1** available for $x=\frac{33}{64}$ from $a=\frac{1}{2}$.
	$y=\left[10.5\sqrt{4x+1}-x+5=\right]66.5$ [leading to (12, 66.5)]	A1	
	$y-66.5=-\frac{1}{2}(x-12)$	A1	OE
		4	

Question 204

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	*M1	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
$[y=]\frac{1}{2x^2}x-\frac{1}{2x^2}=-\frac{1}{2x}$ or $\frac{1}{x}=\frac{1}{2x^2}$ $[\Rightarrow 2x^2-x=0]$	DM1	Sub *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ into equation of line or set gradient = k to form equation in x.
$x=\frac{1}{2}$ only	A1	If DM0 then $x=\frac{1}{2}$, award A0XP then B0 B0.
$y=\left[2\times\frac{1}{2}-2\right]=-1$	B1	
$k=2$	B1	
	5	

Question 205

(a)	$\frac{dV}{dh} = \frac{4}{3}\times 3(25+h)^2$ [= 4900 when h = 10]	B1	Correct expression for $\frac{dV}{dh}$.
	$\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow$ *their* "$4(25+10)^2$"$\times\frac{dh}{dt} = 500 \Rightarrow \frac{dh}{dt} = \left[\frac{500}{4900}\right]$	M1	Use chain rule correctly to find a numerical expression for $\frac{dh}{dt}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{dh}{dt} = 0.102\left[\text{cms}^{-1}\right]$	A1	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
(b)	$\frac{dV}{dt} = \frac{dV}{dh}\times\frac{dh}{dt} \Rightarrow 500 =$ *their* "$4(25+h)^2$"$\times 0.075$	*M1	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2 = \frac{5000}{3}\right] \Rightarrow h = [15.8\,248\ldots]$	DM1	Solve quadratic to find h. Exact value of h is $\sqrt{\frac{5000}{3}} - 25$ or $\frac{50\sqrt{6}}{3} - 25$ $h + 25 = 40.82\ldots$
	$V = 69900\ \text{cm}^3$	A1	AWRT ISW Look for 698(88.5) .
		3	

Question 206

(a)	$[\pi]\int\frac{16}{(2x-1)^4}[dx] = [\pi]\int 16(2x-1)^{-4}[dx] = [\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	*M1	Integrate y^2 (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}\boldsymbol{x}(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	A1	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times 8} + \frac{16}{6\times 1}\right)$ $\left[= [\pi]\frac{112}{48} = [\pi]\frac{7}{3}\right]$	DM1	Sub correct limits into *their* integral: $\text{F}\left(\frac{3}{2}\right) - \text{F}(1)$. Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33 π or 7.33 .
	Volume of cylinder $\left[= \pi\times 1^2\times\frac{1}{2}\right] = \frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5} 1[dx] = \frac{1}{2}\pi$	B1	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[= \frac{7}{3}\pi - \frac{1}{2}\pi\right] = \frac{11}{6}\pi$	A1	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, **SC B1** for $\frac{11}{6}\pi$.
		5	
(b)	$\left[\frac{dy}{dx} = \right]\left\{-8(2x-1)^{-3}\right\}\{\times 2\}$	B2, 1, 0	OE B1 for each correct element in {}.
	At B gradient = -2	B1	
	Eqn of tangent $y - 1 =$ *their* "-2"$\left(x - \frac{3}{2}\right)$ OR Eqn of normal $y - 1 =$ *their* "$\frac{1}{2}$"$\left(x - \frac{3}{2}\right)$	M1	SOI Following differentiation OE e.g. $y = -2x + 4$ or $y = \frac{1}{2}x + \frac{1}{4}$. (Must have $m_N = -\frac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 **or** normal crosses x-axis at $-\frac{1}{2}$	A1	SOI For at least one intercept correct or correct integration.
	Area = $\frac{5}{4}$	A1	From intercepts: $\frac{1}{2}\times\frac{5}{2}\times 1 = \frac{5}{4}$ or $1 + \frac{1}{4} = \frac{5}{4}$, from lengths: $\frac{1}{2}\times\sqrt{5}\times\frac{\sqrt{5}}{2} = \frac{5}{4}$ or by integration.
		6	

Question 207

(a)	$6a^2 - 30a + 6a = 0$ [$\Rightarrow 6a(a-4)=0$]	B1	Sub $x = a$ into $\frac{dy}{dx} = 0$. May see $a^2 - 5a + a = 0$.
	$a = 4$ only	B1	
		2	
(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
(c)	$[y =]\ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\textit{their } a)x[+c]$	B1 FT	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(\textit{their}\ \text{"4"})^3 - 15(\textit{their}\ \text{"4"})^2 + 6(\textit{their}\ \text{"4"})^2 + c$	M1	Sub $x = \textit{their}\text{"4"}, y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ [$\Rightarrow c = 1$].
	$y = 2x^3 - 15x^2 + 24x + 1$	A1	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\textit{their}\ \text{"4"})[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	A1	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	