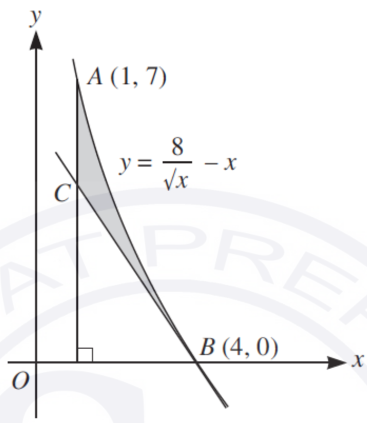
AS-Level

Pure Mathematics P1

Topic : Calculus

May 2013- May 2023

Question 1



The diagram shows part of the curve $y = \frac{8}{\sqrt{x}} - x$ and points $A\,(1,\ 7)$ and $B\,(4,\ 0)$ which lie on the curve. The tangent to the curve at B intersects the line $x = 1$ at the point C.

(i) Find the coordinates of C. [4]

(ii) Find the area of the shaded region. [5]

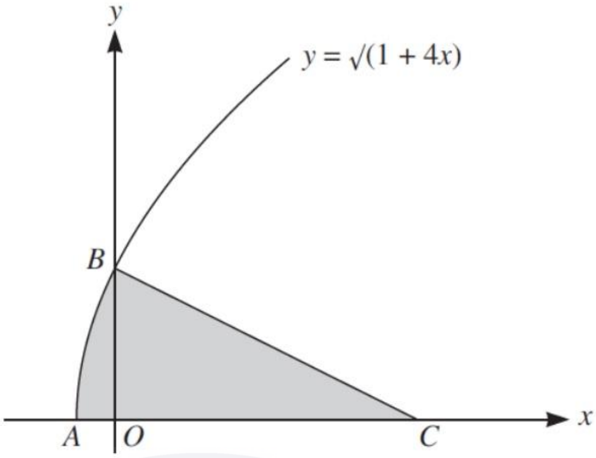
Question 2

The non-zero variables x, y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

Question 3

A curve is such that $\frac{dy}{dx} = \sqrt{(2x + 5)}$ and $(2,\ 5)$ is a point on the curve. Find the equation of the curve. [4]

Question 4



The diagram shows the curve $y = \sqrt{(1 + 4x)}$, which intersects the x-axis at A and the y-axis at B. The normal to the curve at B meets the x-axis at C. Find

(i) the equation of BC, [5]

(ii) the area of the shaded region. [5]

Question 5

A function f is defined by $\mathrm{f}(x) = \dfrac{5}{1 - 3x}$, for $x \geqslant 1$.

(i) Find an expression for $\mathrm{f}'(x)$. [2]

(ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

(iii) Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of f^{-1}. [5]

Question 6

The volume of a solid circular cylinder of radius r cm is 250π cm^3.

(i) Show that the total surface area, S cm^2, of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}.$$ [2]

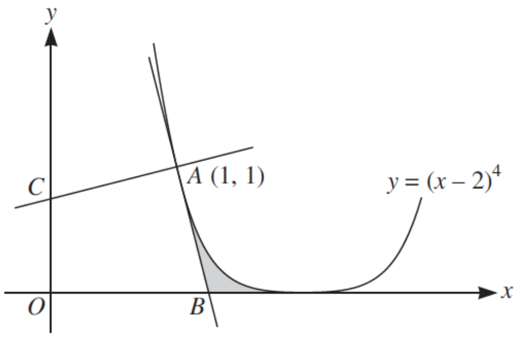
(ii) Given that r can vary, find the stationary value of S. [4]

(iii) Determine the nature of this stationary value. [2]

Question 7

A curve is such that $\frac{dy}{dx} = \frac{6}{x^2}$ and (2, 9) is a point on the curve. Find the equation of the curve. [3]

Question 8



The diagram shows part of the curve $y = (x-2)^4$ and the point A (1, 1) on the curve. The tangent at A cuts the x-axis at B and the normal at A cuts the y-axis at C.

(i) Find the coordinates of B and C. [6]

(ii) Find the distance AC, giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. [2]

(iii) Find the area of the shaded region. [4]

Question 9

A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]

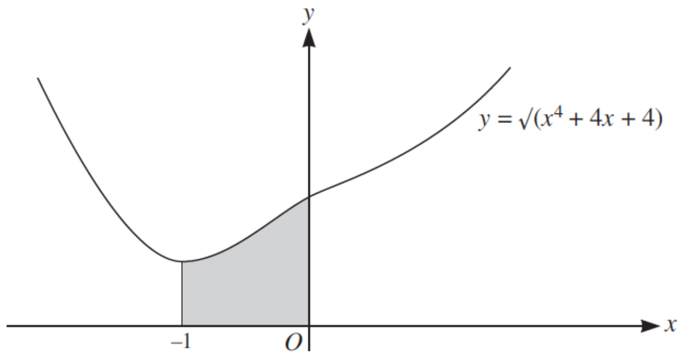
(ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

Question 10

It is given that $f(x) = (2x-5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

Question 11



The diagram shows the curve $y = \sqrt{(x^4 + 4x + 4)}$.

(i) Find the equation of the tangent to the curve at the point (0, 2). [4]

(ii) Show that the x-coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x-coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x-axis. Find the volume of revolution. [4]

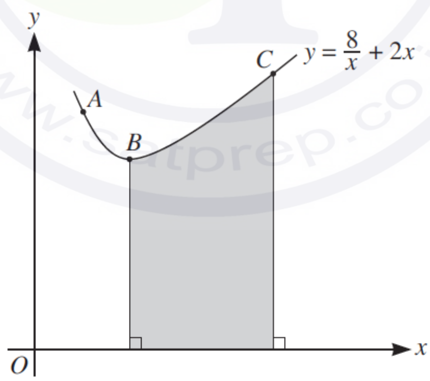
Question 12

A curve has equation $y = \frac{k^2}{x + 2} + x$, where k is a positive constant. Find, in terms of k, the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

Question 13

A curve has equation $y = f(x)$. It is given that $f'(x) = x^{-\frac{3}{2}} + 1$ and that $f(4) = 5$. Find $f(x)$. [4]

Question 14

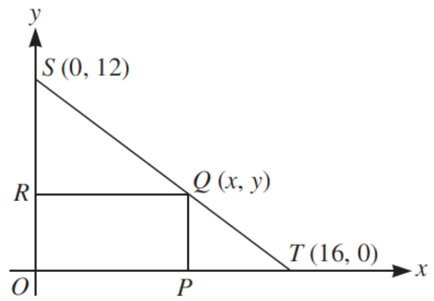


The diagram shows part of the curve $y = \frac{8}{x} + 2x$ and three points A, B and C on the curve with x-coordinates 1, 2 and 5 respectively.

(i) A point P moves along the curve in such a way that its x-coordinate increases at a constant rate of 0.04 units per second. Find the rate at which the y-coordinate of P is changing as P passes through A. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Question 15



In the diagram, S is the point (0, 12) and T is the point (16, 0). The point Q lies on ST, between S and T, and has coordinates (x, y). The points P and R lie on the x-axis and y-axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A, of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

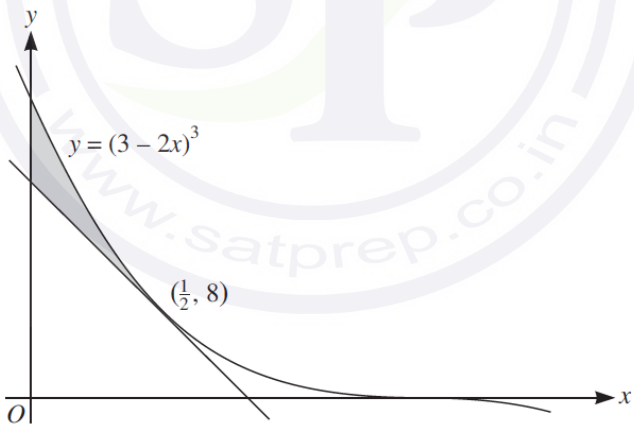
Question 16

The equation of a curve is $y = \dfrac{2}{\sqrt{(5x-6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find $\displaystyle\int \frac{2}{\sqrt{(5x-6)}}\,dx$ and hence evaluate $\displaystyle\int_2^3 \frac{2}{\sqrt{(5x-6)}}\,dx$. [4]

Question 17

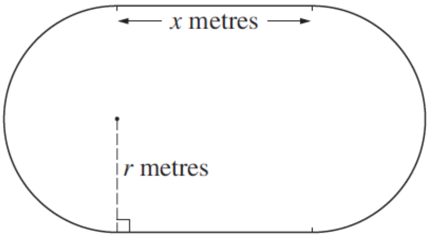


The diagram shows the curve $y = (3 - 2x)^3$ and the tangent to the curve at the point $(\frac{1}{2}, 8)$.

(i) Find the equation of this tangent, giving your answer in the form $y = mx + c$. [5]

(ii) Find the area of the shaded region. [6]

Question 18



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

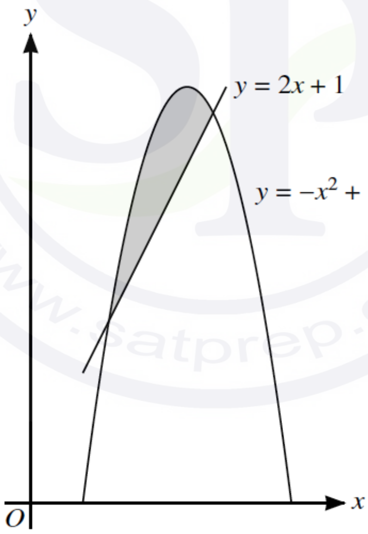
(i) Show that the area, A m^2, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]

(ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

Question 19

A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = \dfrac{1}{\sqrt{(x+6)}} + \dfrac{6}{x^2}$ and that $\mathrm{f}(3) = 1$. Find $\mathrm{f}(x)$. [5]

Question 20



The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

Question 21

The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

(i) Show that the total surface area of the cuboid, $A \text{ cm}^2$, is given by

$$A = 6x^2 + \frac{768}{x}.$$ [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [5]

Question 22

A curve is such that $\frac{dy}{dx} = \frac{12}{\sqrt{(4x+a)}}$, where a is a constant. The point $P(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 5$.

(i) Show that $a = 8$. [3]

(ii) Find the equation of the curve. [4]

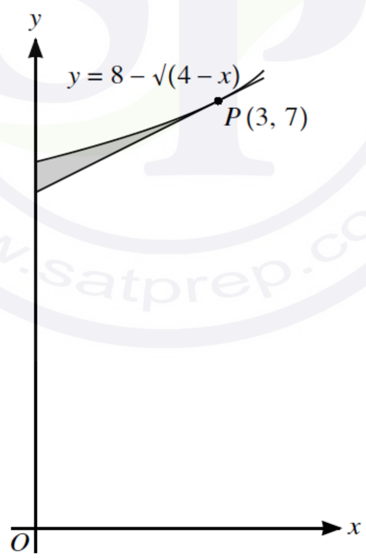
Question 23

A function f is such that $f(x) = \frac{15}{2x+3}$ for $0 \leqslant x \leqslant 6$.

(i) Find an expression for $f'(x)$ and use your result to explain why f has an inverse. [3]

(ii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1}. [4]

Question 24



The diagram shows part of the curve $y = 8 - \sqrt{(4-x)}$ and the tangent to the curve at $P(3, 7)$.

(i) Find expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [5]

(ii) Find the equation of the tangent to the curve at P in the form $y = mx + c$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

Question 25

The equation of a curve is such that $\frac{d^2y}{dx^2} = 2x - 1$. Given that the curve has a minimum point at $(3, -10)$, find the coordinates of the maximum point. [8]

Question 26

A curve is such that $\frac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$. The curve passes through the point $(4, \frac{2}{3})$.

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point and determine its nature. [5]

Question 27

A curve has equation $y = \frac{4}{(3x+1)^2}$. Find the equation of the tangent to the curve at the point where the line $x = -1$ intersects the curve. [5]

Question 28

(a) The functions f and g are defined for $x \geqslant 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$
$$g : x \mapsto x^2.$$

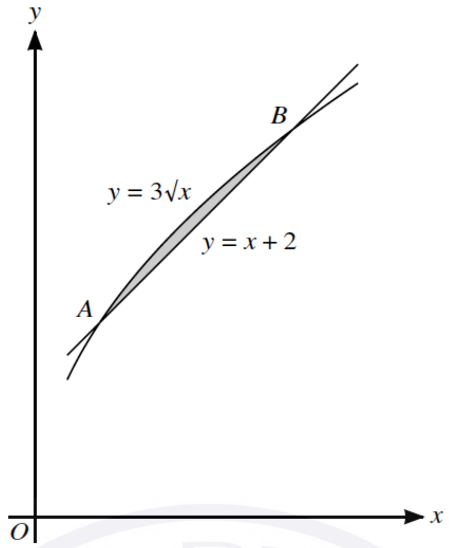
Given that $fg(1) = 2$ and $gf(9) = 16$,

(i) calculate the values of a and b, [4]

(ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(b) A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x-coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y-coordinate of P at the instant when P is at the point $(3, 4)$. [5]

Question 29



The diagram shows parts of the graphs of $y = x + 2$ and $y = 3\sqrt{x}$ intersecting at points A and B.

(i) Write down an equation satisfied by the x-coordinates of A and B. Solve this equation and hence find the coordinates of A and B. [4]

(ii) Find by integration the area of the shaded region. [6]

Question 30

A curve $y = \mathrm{f}(x)$ has a stationary point at $(3, 7)$ and is such that $\mathrm{f}''(x) = 36x^{-3}$.

(i) State, with a reason, whether this stationary point is a maximum or a minimum. [1]

(ii) Find $\mathrm{f}'(x)$ and $\mathrm{f}(x)$. [7]

Question 31

(i) Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]

(ii) Determine whether $3x^3 - 6x^2 + 5x - 12$ is an increasing function, a decreasing function or neither. [3]

Question 32

A curve is such that $\dfrac{d^2y}{dx^2} = \dfrac{24}{x^3} - 4$. The curve has a stationary point at P where $x = 2$.

(i) State, with a reason, the nature of this stationary point. [1]

(ii) Find an expression for $\dfrac{dy}{dx}$. [4]

(iii) Given that the curve passes through the point $(1, 13)$, find the coordinates of the stationary point P. [4]

Question 33

The equation of a curve is $y = x^3 + ax^2 + bx$, where a and b are constants.

(i) In the case where the curve has no stationary point, show that $a^2 < 3b$. [3]

(ii) In the case where $a = -6$ and $b = 9$, find the set of values of x for which y is a decreasing function of x. [3]

Question 34

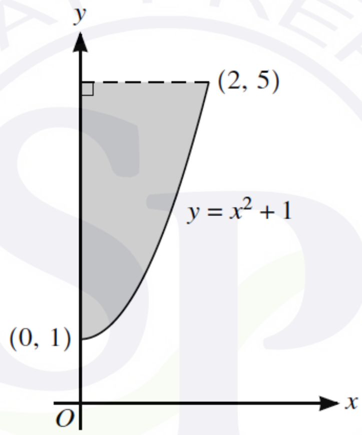
A curve has equation $y = \dfrac{12}{3 - 2x}$.

(i) Find $\dfrac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

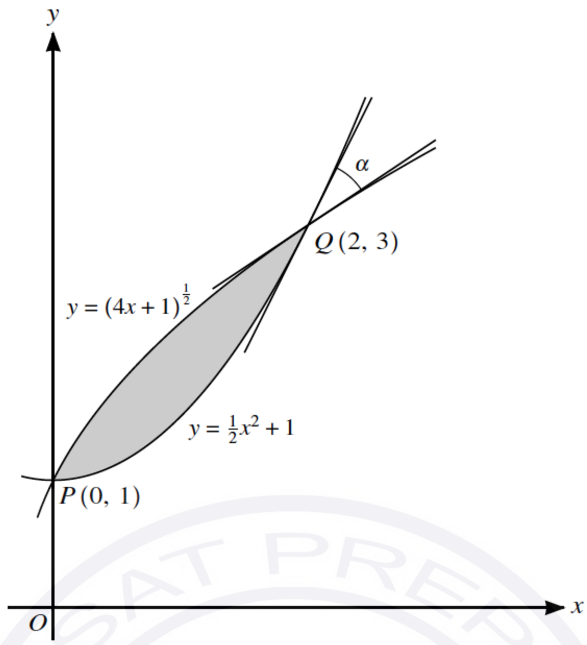
(ii) Find the possible x-coordinates of A. [4]

Question 35



The diagram shows part of the curve $y = x^2 + 1$. Find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [4]

Question 36



The diagram shows parts of the curves $y = (4x+1)^{\frac{1}{2}}$ and $y = \frac{1}{2}x^2 + 1$ intersecting at points $P\,(0,\ 1)$ and $Q\,(2,\ 3)$. The angle between the tangents to the two curves at Q is α.

(i) Find α, giving your answer in degrees correct to 3 significant figures. [6]

(ii) Find by integration the area of the shaded region. [6]

Question 37

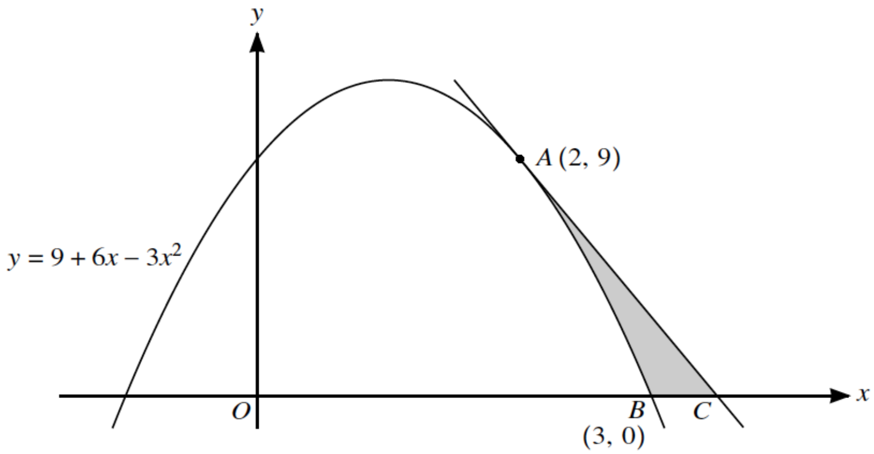
The function f is defined for $x > 0$ and is such that $f'(x) = 2x - \dfrac{2}{x^2}$. The curve $y = f(x)$ passes through the point $P\,(2,\ 6)$.

(i) Find the equation of the normal to the curve at P. [3]

(ii) Find the equation of the curve. [4]

(iii) Find the x-coordinate of the stationary point and state with a reason whether this point is a maximum or a minimum. [4]

Question 38



Points A (2, 9) and B (3, 0) lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at A intersects the x-axis at C. Showing all necessary working,

(i) find the equation of the tangent AC and hence find the x-coordinate of C, [4]

(ii) find the area of the shaded region ABC. [5]

Question 39

The function f is defined by $\mathrm{f}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x > -1$.

(i) Find $\mathrm{f}'(x)$. [3]

(ii) State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

The function g is defined by $\mathrm{g}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x < -1$.

(iii) Find the coordinates of the stationary point on the curve $y = \mathrm{g}(x)$. [4]

Question 40

A curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^{\frac{1}{2}}$ and the point (4, 7) lies on the curve. Find the equation of the curve. [4]

Question 41

The equation of a curve is $y = \dfrac{4}{2x-1}$.

(i) Find, showing all necessary working, the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [4]

(ii) Given that the line $2y = x + c$ is a normal to the curve, find the possible values of the constant c. [6]

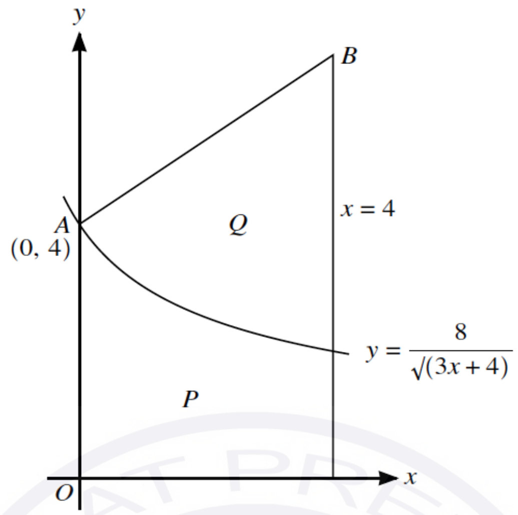
Question 42

Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

Question 43

The function f is such that $f'(x) = 5 - 2x^2$ and (3, 5) is a point on the curve $y = f(x)$. Find $f(x)$. [3]

Question 44



The diagram shows part of the curve $y = \dfrac{8}{\sqrt{(3x+4)}}$. The curve intersects the y-axis at A (0, 4). The normal to the curve at A intersects the line $x = 4$ at the point B.

(i) Find the coordinates of B. [5]

(ii) Show, with all necessary working, that the areas of the regions marked P and Q are equal. [6]

Question 45

The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

Question 46

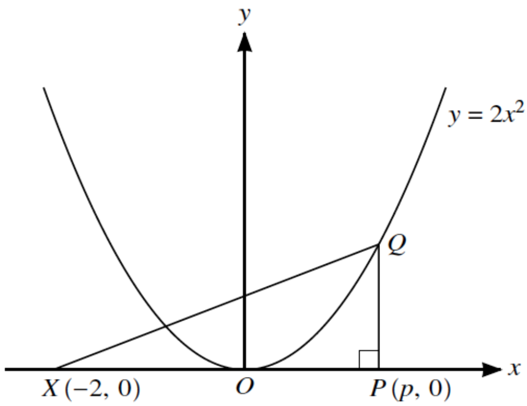
A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r - b)^2$, where a and b are constants. [2]

(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

Question 47



The diagram shows the curve $y = 2x^2$ and the points $X(-2, 0)$ and $P(p, 0)$. The point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the area, A, of triangle XPQ in terms of p. [2]

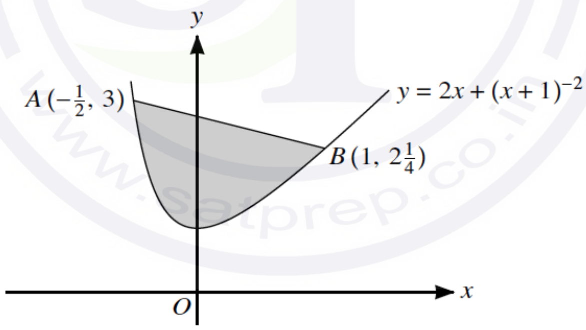
The point P moves along the x-axis at a constant rate of 0.02 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is increasing when $p = 2$. [3]

Question 48

The function f is defined by $\mathrm{f}(x) = 2x + (x+1)^{-2}$ for $x > -1$.

(i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$ and hence verify that the function f has a minimum value at $x = 0$. [4]



The points $A(-\frac{1}{2}, 3)$ and $B(1, 2\frac{1}{4})$ lie on the curve $y = 2x + (x+1)^{-2}$, as shown in the diagram.

(ii) Find the distance AB. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]

Question 49

A curve passes through the point $A\,(4,\ 6)$ and is such that $\frac{dy}{dx} = 1 + 2x^{-\frac{1}{2}}$. A point P is moving along the curve in such a way that the x-coordinate of P is increasing at a constant rate of 3 units per minute.

(i) Find the rate at which the y-coordinate of P is increasing when P is at A. [3]

(ii) Find the equation of the curve. [3]

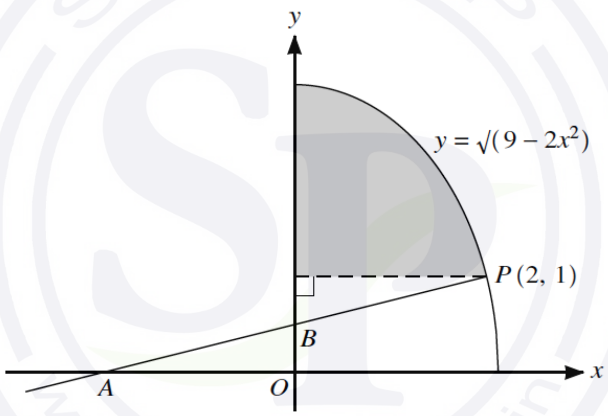
(iii) The tangent to the curve at A crosses the x-axis at B and the normal to the curve at A crosses the x-axis at C. Find the area of triangle ABC. [5]

Question 50

(i) Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f, where $f(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $f'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]

Question 51



The diagram shows part of the curve $y = \sqrt{(9 - 2x^2)}$. The point $P\,(2,\ 1)$ lies on the curve and the normal to the curve at P intersects the x-axis at A and the y-axis at B.

(i) Show that B is the mid-point of AP. [6]

The shaded region is bounded by the curve, the y-axis and the line $y = 1$.

(ii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [5]

Question 52

The curve $y = f(x)$ has a stationary point at $(2,\ 10)$ and it is given that $f''(x) = \frac{12}{x^3}$.

(i) Find $f(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

Question 53

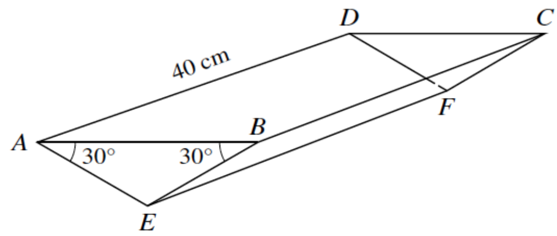


Fig. 1

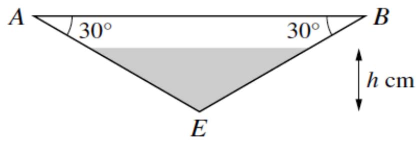


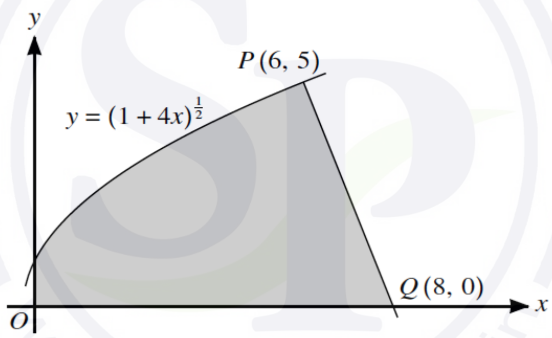
Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles. Angle ABE = angle $BAE = 30°$. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of 200 $\text{cm}^3\,\text{s}^{-1}$. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

(i) Show that, when the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by $V = (40\sqrt{3})h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [3]

Question 54



The diagram shows part of the curve $y = (1 + 4x)^{\frac{1}{2}}$ and a point $P(6, 5)$ lying on the curve. The line PQ intersects the x-axis at $Q(8, 0)$.

(i) Show that PQ is a normal to the curve. [5]

(ii) Find, showing all necessary working, the exact volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [7]

[In part **(ii)** you may find it useful to apply the fact that the volume, V, of a cone of base radius r and vertical height h, is given by $V = \frac{1}{3}\pi r^2 h$.]

Question 55

A curve has equation $y = \frac{8}{x} + 2x$.

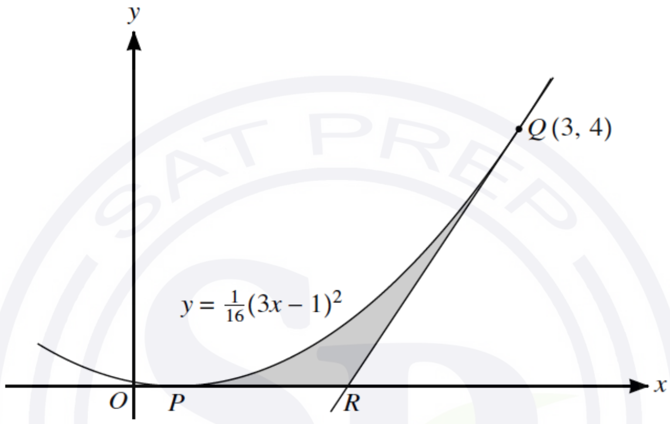
(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

Question 56

The function f is such that $f'(x) = 3x^2 - 7$ and $f(3) = 5$. Find $f(x)$. [3]

Question 57



The diagram shows part of the curve $y = \frac{1}{16}(3x-1)^2$, which touches the x-axis at the point P. The point $Q(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

Question 58

A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \frac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

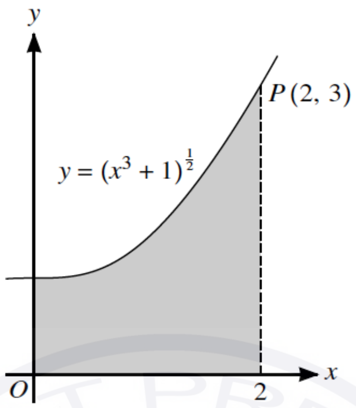
Question 59

A curve for which $\frac{dy}{dx} = 3x^2 - \frac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

Question 60

The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

Question 61



The diagram shows part of the curve $y = (x^3 + 1)^{\frac{1}{2}}$ and the point $P(2, 3)$ lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Question 62

A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 + \frac{k}{x^3}$ and passes through the point $P(1, 9)$. The gradient of the curve at P is 2.

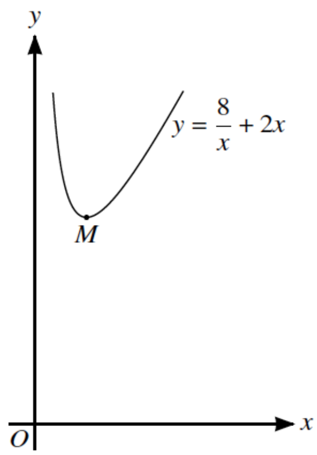
(i) Find the value of the constant k. [1]

(ii) Find the equation of the curve. [4]

Question 63

A curve has equation $y = 8x + (2x - 1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

Question 64



The diagram shows the part of the curve $y = \frac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y^2 \, dx$. [5]

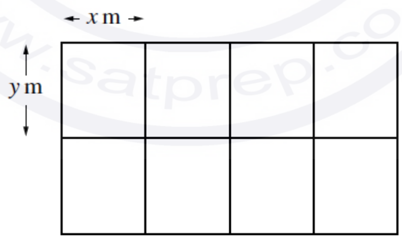
(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

Question 65

A curve is such that $\frac{dy}{dx} = \frac{8}{(5 - 2x)^2}$. Given that the curve passes through (2, 7), find the equation of the curve. [4]

Question 66



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, A m^2, is given by

$$A = 384x - 9.6x^2.$$ [3]

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

Question 67

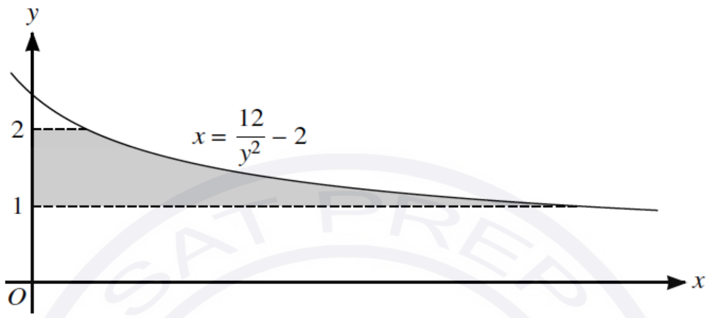
A curve is such that $\frac{dy}{dx} = 2 - 8(3x+4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

Question 68



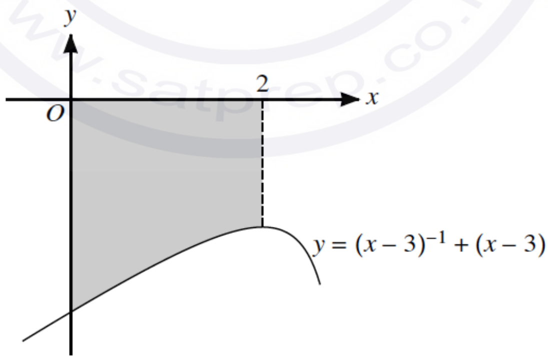
The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

Question 69

A curve has equation $y = (kx-3)^{-1} + (kx-3)$, where k is a non-zero constant.

(i) Find the x-coordinates of the stationary points in terms of k, and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x-axis, the y-axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x-axis. [5]

Question 70

A curve is such that $\frac{dy}{dx} = \frac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

It is now given that $B\,(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB. [5]

Question 71

The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

Question 72

The equation of a curve is $y = 2 + \frac{3}{2x - 1}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

Question 73

A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x + 1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

Question 74

A curve has equation $y = f(x)$ and it is given that $f'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1.

(i) Find the x-coordinate of A. [3]

(ii) Given that the curve also passes through the point (4, 10), find the y-coordinate of A, giving your answer as a fraction. [6]

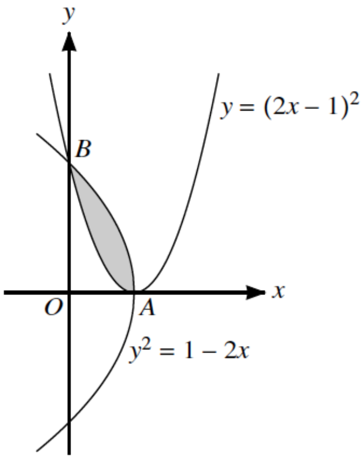
Question 75

The point $P\,(3, 5)$ lies on the curve $y = \frac{1}{x - 1} - \frac{9}{x - 5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis. [5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

Question 76

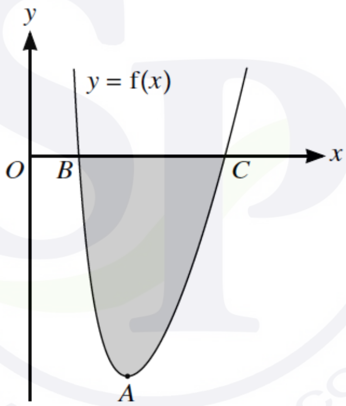


The diagram shows parts of the curves $y = (2x - 1)^2$ and $y^2 = 1 - 2x$, intersecting at points A and B.

(i) State the coordinates of A. [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

Question 77



The diagram shows the curve $y = f(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\frac{dy}{dx} = 2x - \frac{2}{x^3}$ and that the curve passes through the point $(4, \frac{189}{16})$.

(i) Find the x-coordinate of A. [2]

(ii) Find $f(x)$. [3]

(iii) Find the x-coordinates of B and C. [4]

(iv) Find, showing all necessary working, the area of the shaded region. [4]

Question 78

The point $A\,(2,\ 2)$ lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

Question 79

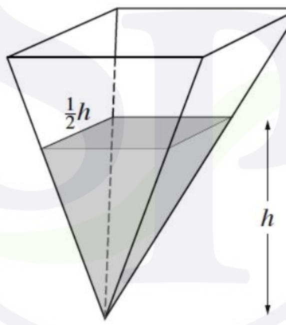
The function f is defined for $x \geqslant 0$ by $f(x) = (4x + 1)^{\frac{3}{2}}$.

(i) Find $f'(x)$ and $f''(x)$. [3]

The first, second and third terms of a geometric progression are respectively $f(2)$, $f'(2)$ and $kf''(2)$.

(ii) Find the value of the constant k. [5]

Question 80



The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

Water is steadily dripping into the container at a constant rate of $20\,\text{cm}^3$ per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is 10 cm. [4]

Question 81

The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $f''(x) = (4x + 1)^{-\frac{1}{2}}$.

(i) Find $f'(x)$. [3]

It is now given that $f''(0)$, $f'(0)$ and $f(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $f(0)$. [3]

(iii) Find $f(x)$, and hence find the minimum value of f. [5]

Question 82

(a)

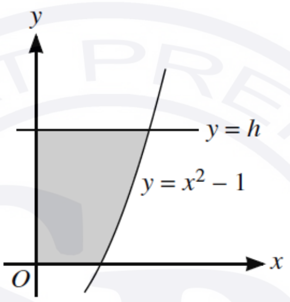


Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

(b)

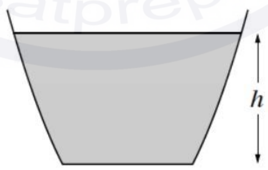


Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, V cm^3, of water is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. Water is poured into the bowl at a constant rate of 2 cm^3 s^{-1}. Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

Question 83

The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

Question 84

The equation of a curve is $y = 8\sqrt{x} - 2x$.

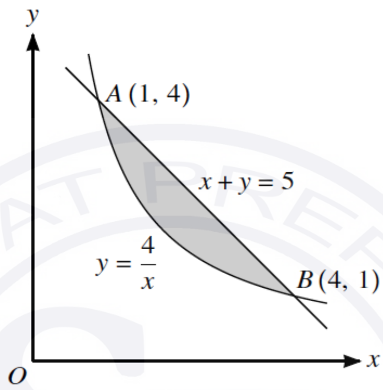
(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

Question 85



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \frac{4}{x}$ at the points $A\,(1, 4)$ and $B\,(4, 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

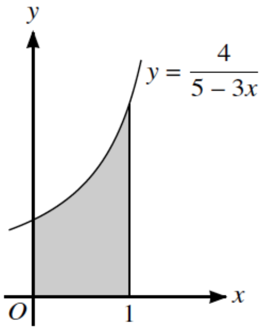
Question 86

A curve has equation $y = 3 + \frac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

Question 87



The diagram shows part of the curve $y = \dfrac{4}{5-3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through 360° about the x-axis. [5]

Question 88

A curve for which $\dfrac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

(iii) Find the area of the shaded region. [3]

Question 89

The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is 2000 cm^3.

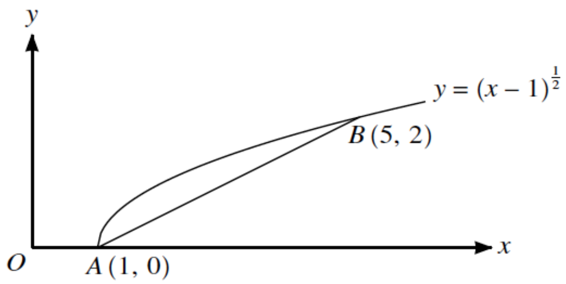
(i) Express h in terms of x and show that the total surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

Question 90



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A\,(1, 0)$ and $B\,(5, 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

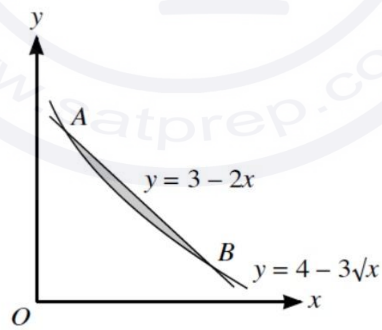
Question 91

A curve has equation $y = f(x)$ and it is given that $f'(x) = ax^2 + bx$, where a and b are positive constants.

(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $f(x)$. [6]

Question 92



The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

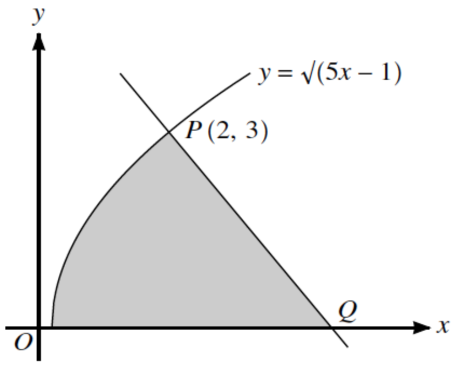
(i) Find by calculation the x-coordinates of A and B. [3]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

Question 93

The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

Question 94



The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P(2, 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

(ii) Find, showing all necessary working, the area of the shaded region. [7]

Question 95

A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

(iii) Given that the curve passes through the point $(6, 2)$, find the equation of the curve. [4]

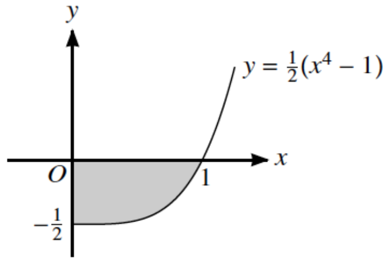
Question 96

Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

(ii) Find the set of values of m for which L does not meet the curve. [4]

Question 97



The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geqslant 0$.

(i) Find, showing all necessary working, the area of the shaded region. [3]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

Question 98

Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm^3, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

(iii) Find the maximum volume of a cone that can be made by these machines. [1]

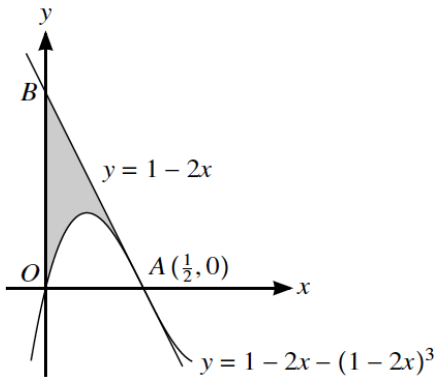
Question 99

A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point (4, 0). [4]

Question 100

A function f is defined by $\mathrm{f} : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a. [4]

Question 101



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1 - 2x)^3 \, dx$. [2]

(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

Question 102

$$f(x) = \frac{8}{x - 2} + 2 \quad \text{for } x > 2,$$

Find the set of values of x satisfying the inequality $6f'(x) + 2f^{-1}(x) - 5 < 0$. [6]

Question 103

A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

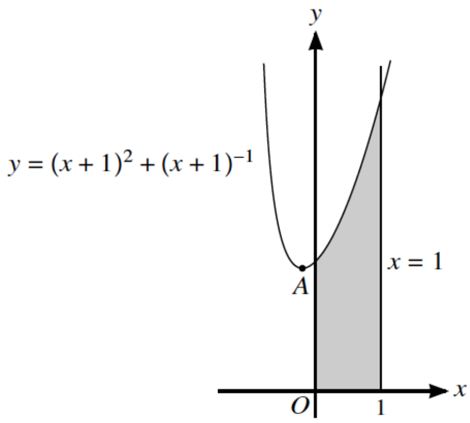
(ii) Find $\frac{d^2y}{dx^2}$. [1]

(iii) Find, showing all necessary working, the nature of each stationary point. [2]

Question 104

A curve passes through the point $(4, -6)$ and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

Question 105



The diagram shows part of the curve $y = (x+1)^2 + (x+1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x+1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Question 106

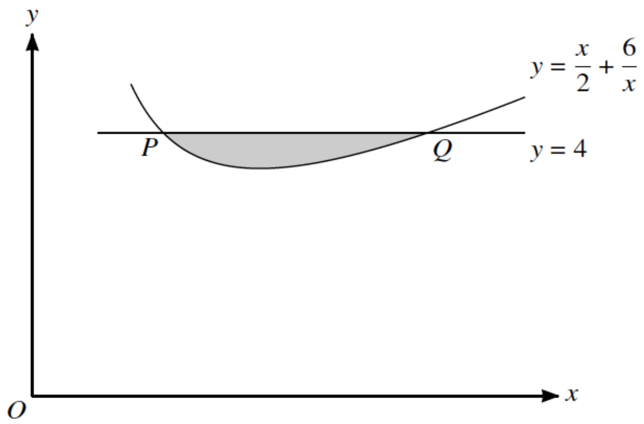
(i) The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

Question 107

A curve with equation $y = f(x)$ passes through the point $A\ (3,\ 1)$ and crosses the y-axis at B. It is given that $f'(x) = (3x-1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

Question 108



The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

Question 109

A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and (2, 5) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

Question 110

The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

Question 111

A curve is such that $\frac{dy}{dx} = \frac{12}{(2x + 1)^2}$. The point (1, 1) lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

Question 112

A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

Question 113

The function f is defined by $f(x) = x^3 + 2x^2 - 4x + 7$ for $x \geqslant -2$. Determine, showing all necessary working, whether f is an increasing function, a decreasing function or neither. [4]

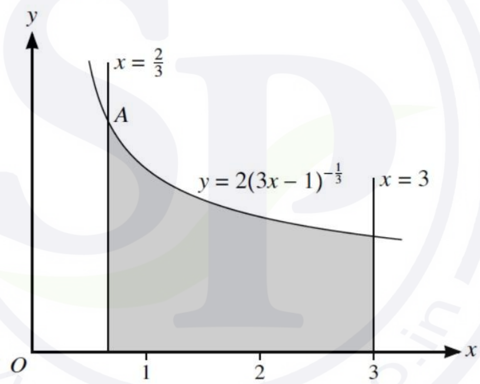
Question 114

A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

(ii) It is now given that the curve has a stationary point at (2, 3). Find the values of a and b. [5]

Question 115



The diagram shows part of the curve $y = 2(3x - 1)^{-\frac{1}{3}}$ and the lines $x = \frac{2}{3}$ and $x = 3$. The curve and the line $x = \frac{2}{3}$ intersect at the point A.

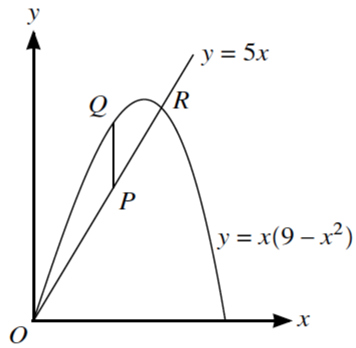
(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [5]

(ii) Find the equation of the normal to the curve at A, giving your answer in the form $y = mx + c$. [5]

Question 116

Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) dx$. [4]

Question 117

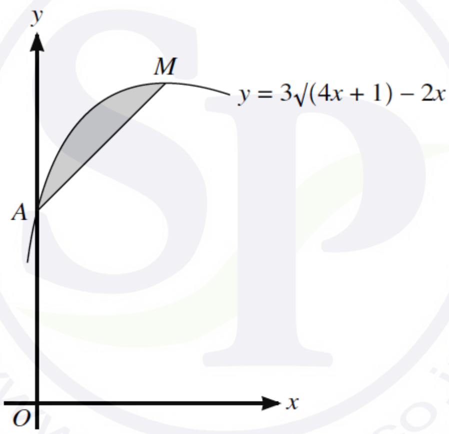


The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

Question 118



The diagram shows part of the curve $y = 3\sqrt{(4x + 1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [5]

(ii) Find the coordinates of M. [3]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

Question 119

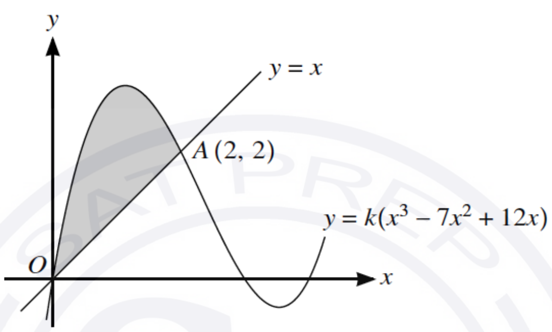
A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{\mathrm{d}y}{\mathrm{d}x} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

(ii) Find the equation of the curve. [4]

(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

Question 120



The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\ (2, 2)$.

(i) Find the value of k. [1]

(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [5]

Question 121

A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) **(a)** Find and simplify the equation of the normal through A. [5]

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

Question 122

A curve with equation $y = \mathrm{f}(x)$ passes through the points $(0, 2)$ and $(3, -1)$. It is given that $\mathrm{f}'(x) = kx^2 - 2x$, where k is a constant. Find the value of k. [5]

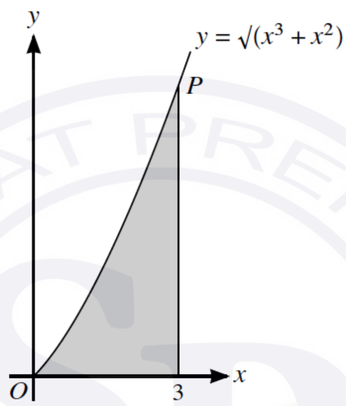
Question 123

A curve has equation $y = (2x - 1)^{-1} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinates of the stationary points and, showing all necessary working, determine the nature of each stationary point. [4]

Question 124



The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

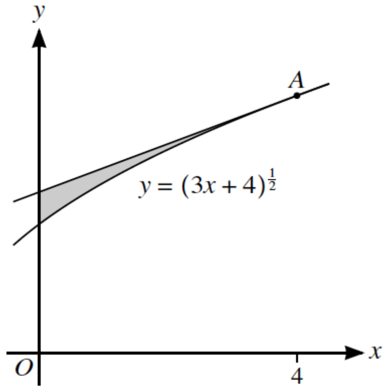
(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

Question 125

A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

Question 126



The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

(iii) A point is moving along the curve. At the point P the y-coordinate is increasing at half the rate at which the x-coordinate is increasing. Find the x-coordinate of P. [3]

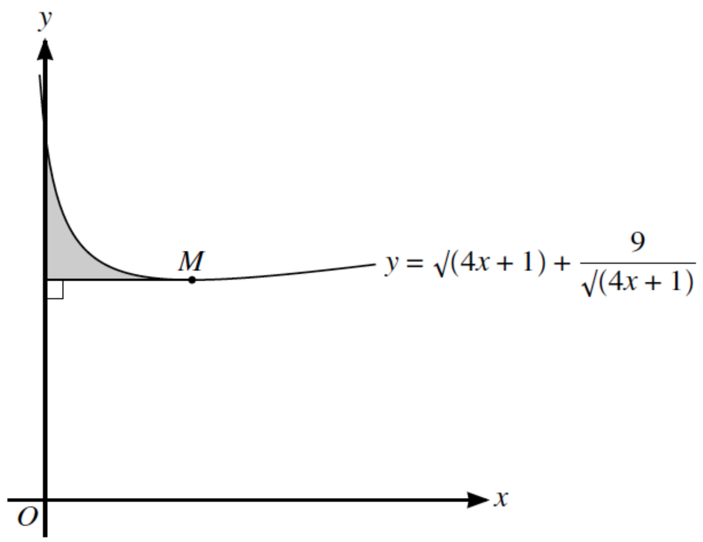
Question 127

A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

(ii) Find the equation of the curve. [3]

Question 128



The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\int y\,dx$. [6]

(ii) Find the coordinates of M. [3]

The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

Question 129

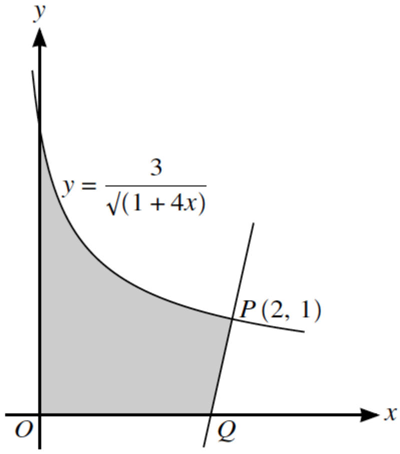
A curve for which $\dfrac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

(ii) Find the x-coordinate of the other stationary point on the curve. [1]

(iii) Determine the nature of each of the stationary points. [2]

Question 130

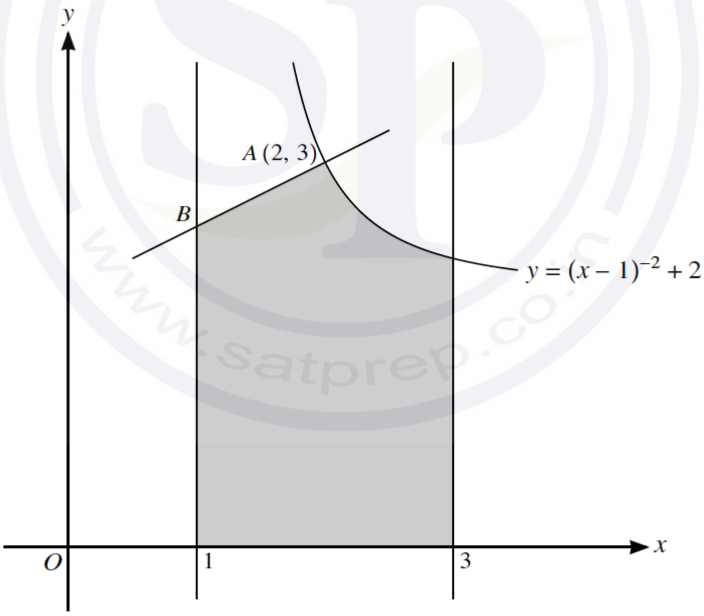


The diagram shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$ and a point $P(2, 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

Question 131



The diagram shows part of the curve $y = (x-1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates $(2, 3)$. The normal to the curve at A crosses the line $x = 1$ at B.

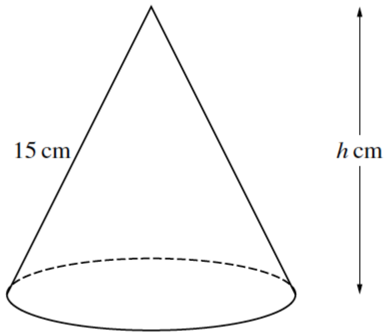
(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

Question 132

A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

Question 133



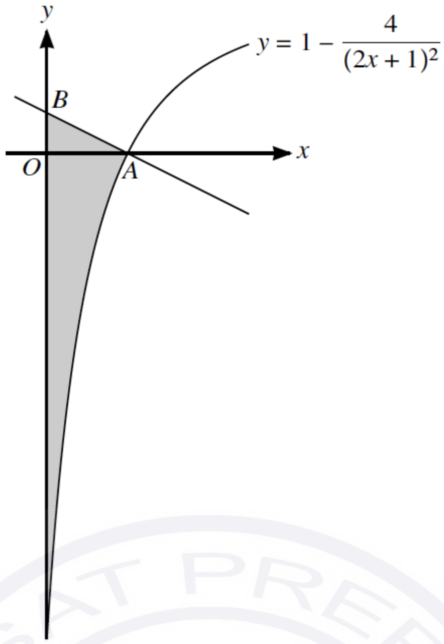
The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

Question 134



The diagram shows part of the curve $y = 1 - \frac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [4]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

(ii) Find the coordinates of B. [4]

Question 135

An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

Question 136

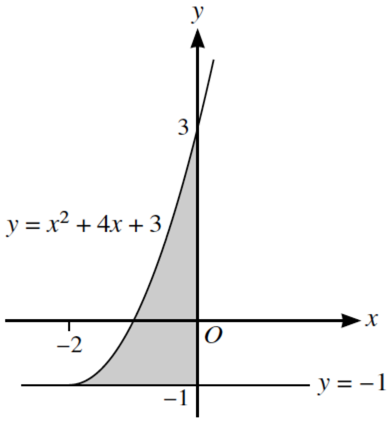
A curve for which $\frac{dy}{dx} = (5x-1)^{\frac{1}{2}} - 2$ passes through the point (2, 3).

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

Question 137



The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

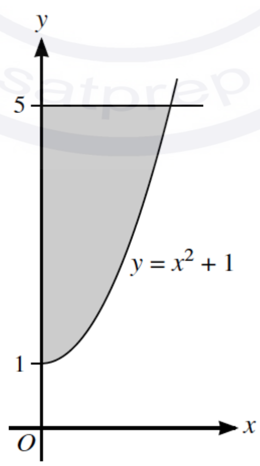
(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

Question 138

The function f is defined by $\mathrm{f}(x) = \dfrac{1}{3x + 2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

Question 139



The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

Question 140

A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

Question 141

The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

(c) Find the equation of the curve. [4]

Question 142

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point $(4, 7)$ lies on the curve.

Find the equation of the curve. [4]

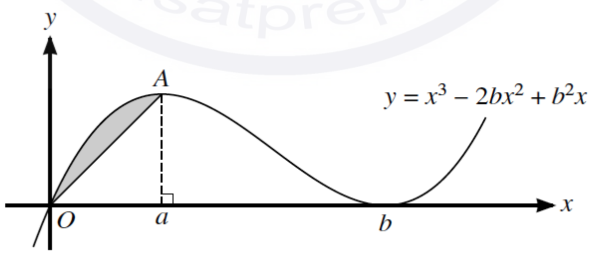
Question 143

A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

Question 144



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

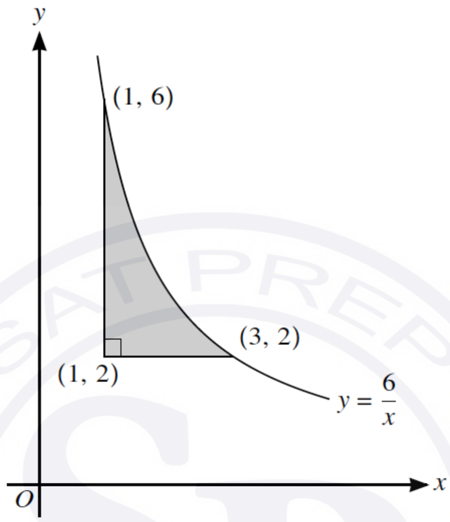
Question 145

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\ \text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

(b) Find the rate of increase of the radius after 30 seconds. [3]

Question 146



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

Question 147

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

Question 148

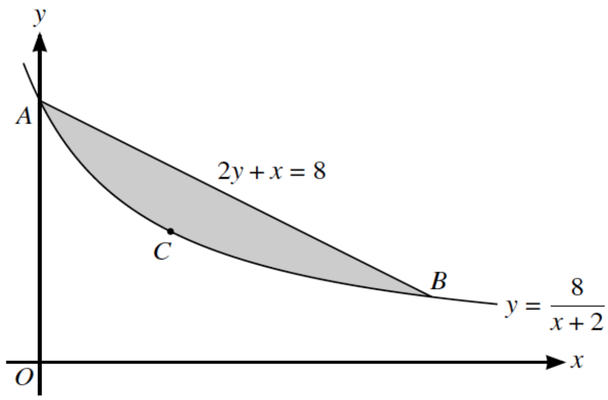
The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each stationary point. [2]

Question 149



The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

Question 150

A curve has equation $y = \dfrac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \dfrac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\displaystyle\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}\right) dx = \frac{13}{12}$.

Find the value of k. [5]

Question 151

The equation of a curve is $y = 2x + 1 + \dfrac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

Question 152

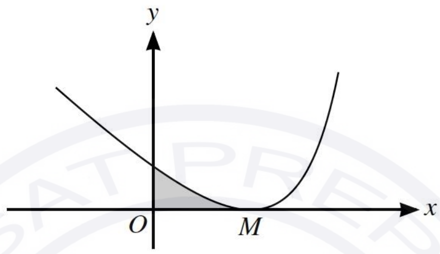
The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^{\infty} f(x)\,dx$. [3]

(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

Question 153



The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\displaystyle\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

Question 154

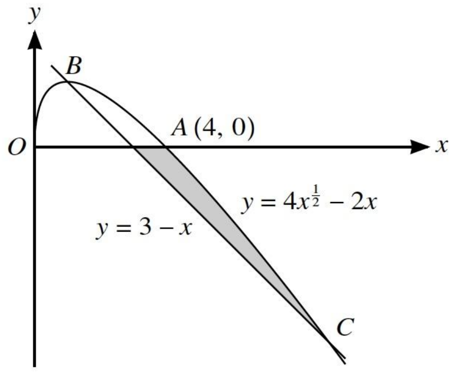
The point $(4, 7)$ lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

Question 155



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4,\ 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

(b) Show that B is a stationary point on the curve. [2]

(c) Find the area of the shaded region. [6]

Question 156

The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

Question 157

Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

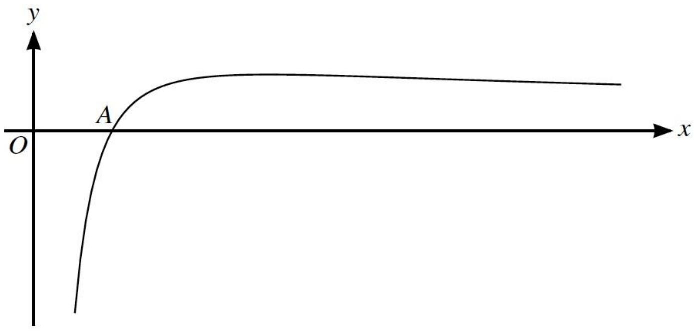
Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

Question 158

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point $(2,\ 7)$.

Find the equation of the curve. [4]

Question 159



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

(b) Find the equation of the tangent to the curve at A. [4]

(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

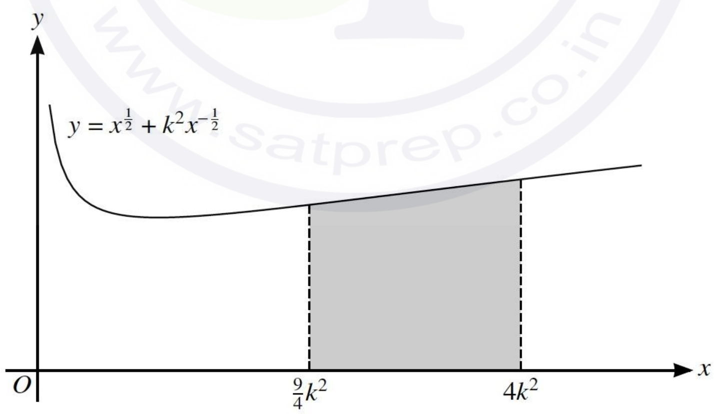
Question 160

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

Question 161



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

Question 162

The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

Question 163

A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point (2, 7).

Find $f(x)$. [3]

Question 164

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x-5)^3 - kx^2$, where k is a constant. The curve has a stationary point at (2, −3.5).

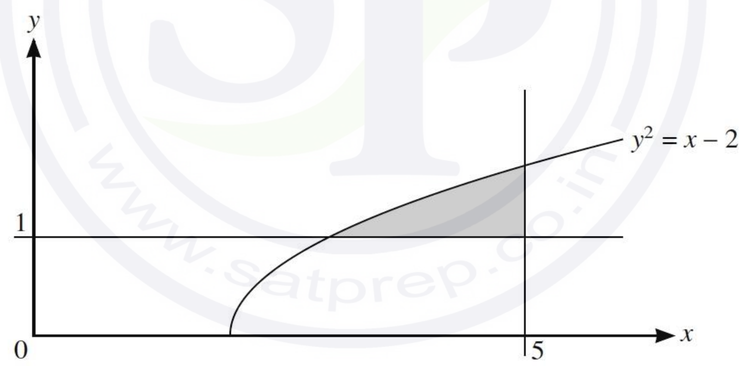
(a) Find the value of k. [2]

(b) Find the equation of the curve. [4]

(c) Find $\frac{d^2y}{dx^2}$. [2]

(d) Determine the nature of the stationary point at (2, −3.5). [2]

Question 165



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

Question 166

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

Question 167

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve. [4]

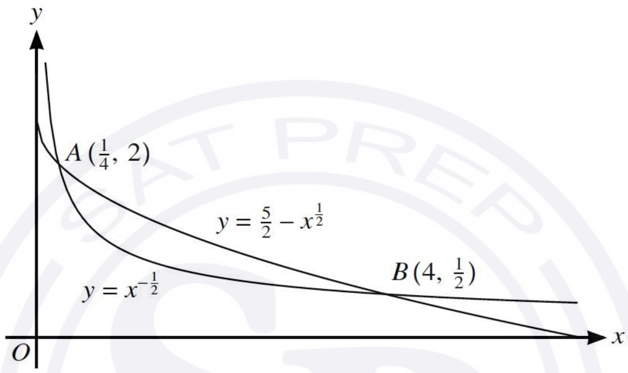
Question 168

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

Question 169



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A(\frac{1}{4}, 2)$ and $B(4, \frac{1}{2})$.

(a) Find the area of the region between the two curves. [6]

(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1, 1)$ intersects the y-axis at the point $(0, p)$.

Find the value of p. [4]

Question 170

A curve has equation $y = f(x)$ and it is given that

$$f'(x) = (\tfrac{1}{2}x + k)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

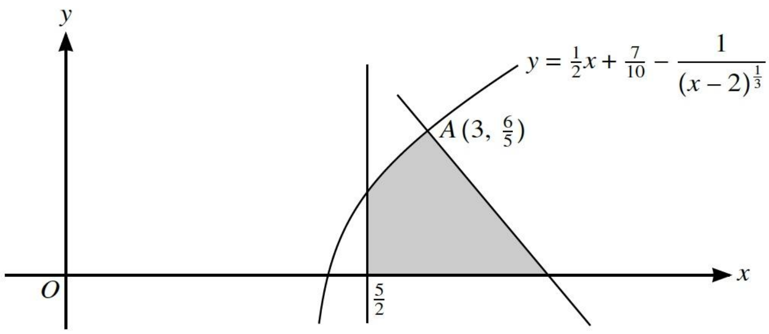
(a) Find $f''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $f(x)$. [4]

(c) Find the coordinates of the other stationary point and determine its nature. [4]

Question 171



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\left(3, \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

Question 172

The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

(c) Given that this is the only stationary point of the curve, find the range of f. [2]

Question 173

The volume $V\,\text{m}^3$ of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\text{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

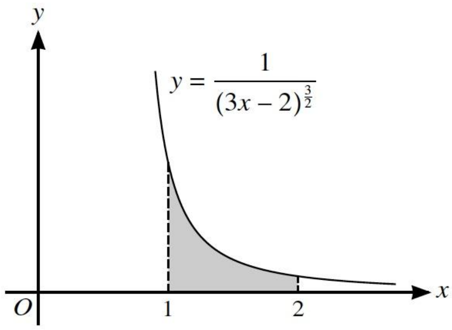
Question 174

A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $\left(2, 5\frac{2}{3}\right)$.

Find the equation of the curve. [4]

Question 175

(a) Find $\displaystyle\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]



The diagram shows the curve with equation $y = \dfrac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

The normal to the curve at the point $(1,\ 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

Question 176

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \dfrac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

(b) Find the coordinates of the stationary points on the curve. [5]

(c) Find $f''(x)$. [1]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

Question 177

It is given that a curve has equation $y = k(3x-k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

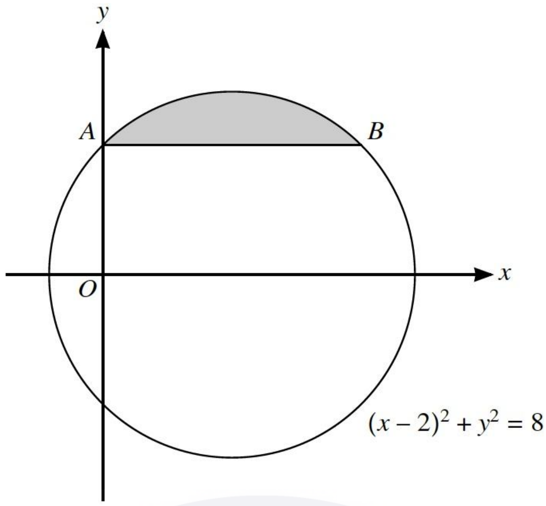
$$f(x) = 4(3x-4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

(c) The function g is defined by $g(x) = -(3x+1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

Question 178



The diagram shows the circle with equation $(x-2)^2+y^2=8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

Question 179

A curve with equation $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $\mathrm{f}(8) = 5$.

Find $\mathrm{f}(x)$. [4]

Question 180

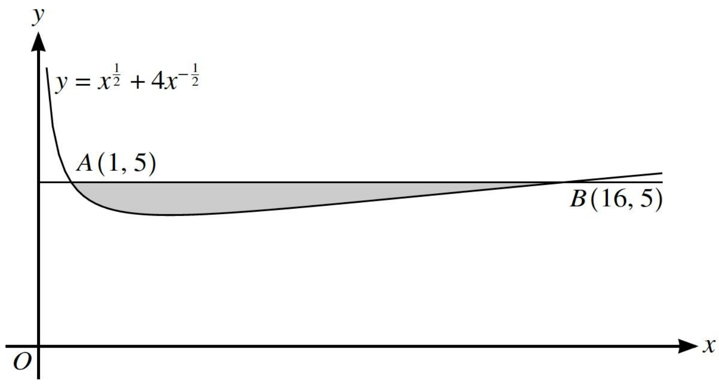
The function f is defined by $\mathrm{f}(x) = (4x+2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} \mathrm{f}(x)\,\mathrm{d}x$. [4]

A point is moving along the curve $y = \mathrm{f}(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

Question 181



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1,\ 5)$ and $B\,(16,\ 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

(b) Calculate the area of the shaded region. [4]

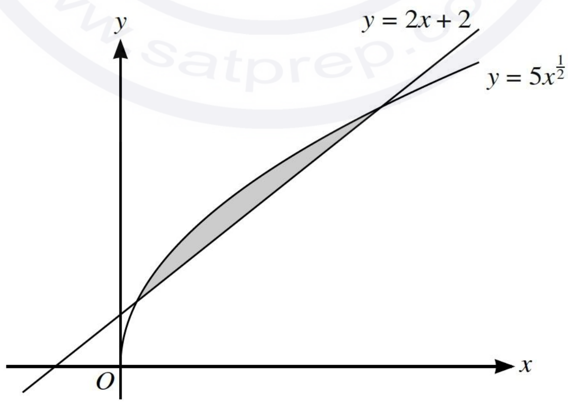
Question 182

The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

Question 183



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

Question 184

The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$.

Find the equation of the curve. [4]

Question 185

The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

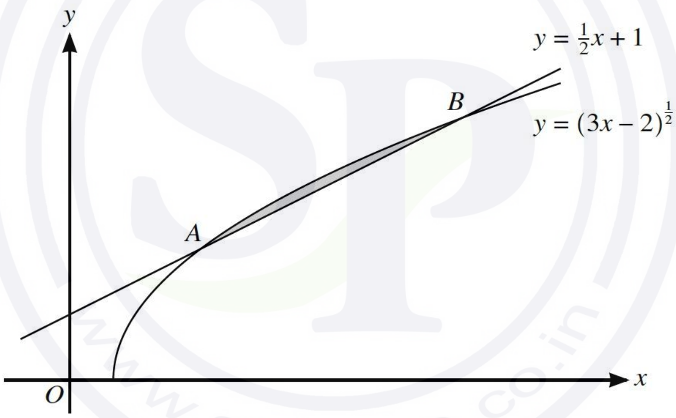
(b) Find the equation of the curve. [5]

(c) Show that the curve has no other stationary points. [3]

(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

Question 186



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

(b) Hence find the area of the region enclosed between the curve and the line. [5]

Question 187

A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

Question 188

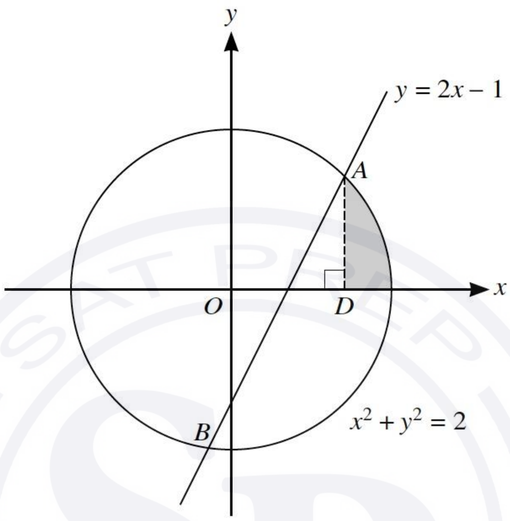
The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

(b) Find $\mathrm{f}(x)$ given that the curve passes through the point $(-1, 5)$. [3]

Question 189



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\dfrac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

Question190

The equation of a curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point $(3, 5)$.

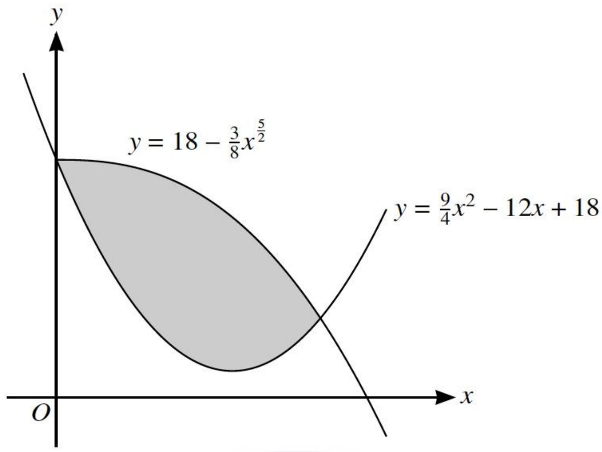
(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

(c) State the set of values of x for which y increases as x increases. [1]

Question 191

(a) Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points $(0, 18)$ and $(4, 6)$.

(b) Find the area of the shaded region. [5]

(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

Question 192

The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

(b) Find the equation of the curve. [4]

Question 193

A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

Question 194

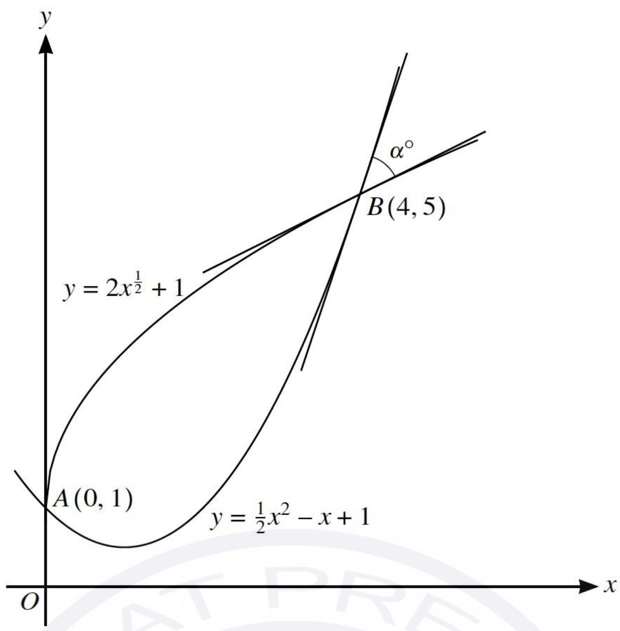
The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

Question 195



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

Question 196

A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

Question 197

At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

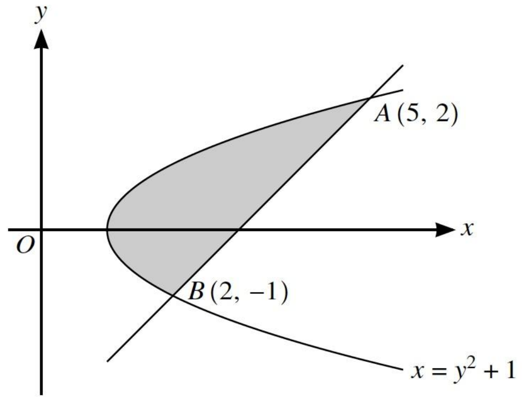
(a) Show that $k = -2$. [1]

(b) Find the equation of the curve. [4]

(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

Question 198



The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5, 2)$ and $B\,(2, -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

Question 199

A curve which passes through $(0, 3)$ has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

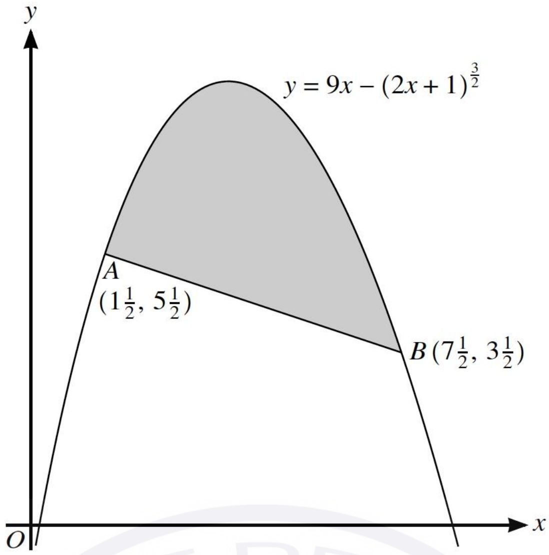
(a) Find the equation of the curve. [4]

The tangent to the curve at $(0, 3)$ intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

Question 200



The diagram shows the points $A\,(1\frac{1}{2}, 5\frac{1}{2})$ and $B\,(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

(b) Verify that the line AB is the normal to the curve at A. [3]

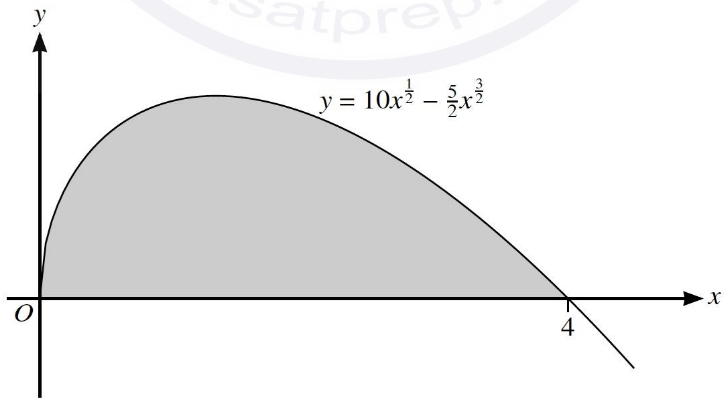
(c) Find the area of the shaded region. [5]

Question 201

The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

Question 202



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points (0, 0) and (4, 0).

Find the area of the shaded region. [4]

Question 203

The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the x-coordinate of the stationary point in terms of k. [2]

(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

Question 204

The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

Question 205

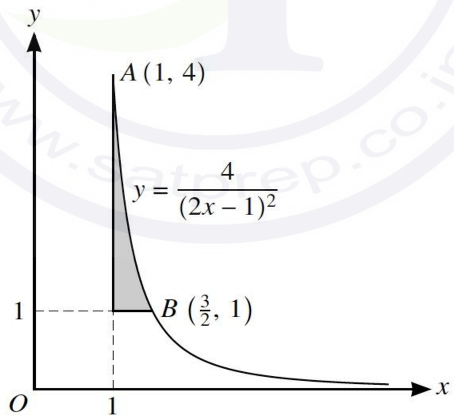
Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25+h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

Question 206



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, (\frac{3}{2}, 1)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

Question 207

The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

(b) Determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]