AS-Level

Topic : Circular measure

May 2013-May 2023

Answer

Question 1

(i)	$\frac{1}{2}.3^2\pi = \frac{1}{2}9^2\theta - \frac{1}{2}3^2\theta$	M1 A1		M1 needs $\frac{1}{2}r^2\theta$ once. A1 all correct.
	$\rightarrow \theta = \frac{1}{4}\pi$	A1	[3]	Answer given
(ii)	$P = 6+6+3\times\frac{1}{4}\pi + 9\times\frac{1}{4}\pi = 21.4$ cm.	M1		M1 is for use of $s=r\theta$ once.
	or $12 + 3\pi$	A1	[2]	

Question 2

(i)	$BOC = 2\tan^{-1}\frac{1}{2} = 0.9273$	M1 A1 [2]	Correct trigonometry. (ans given)
(ii)	$OB = \sqrt{(10^2 + 5^2)}$ or $11.2 = r$	B1	Use of trig (or Pyth) for the $OB = \sqrt{125}$.
	Arc $BXC = \sqrt{125} \times 0.9273$	M1	Use of $s = r\theta$ with θ in rads , $r \neq 10$
	$\rightarrow$ Perimeter = 20.4 cm	A1 [3]	
(iii)	Area = $\frac{1}{2}r^2\theta$	M1	Correct formula used with rads, $r \neq 10$.
	$- \frac{1}{2}.10.10 \rightarrow 7.96$ cm².	A1 [2]	Allow 7.95 or 7.96

Question 3

(i)	$(OAB) = \frac{1}{2}\times 8^2\alpha$, $(OAC) = \frac{1}{2}\times\pi\times 4^2$	**B1B1**		Accept 25.1 (for OAC)
	$\alpha = \frac{\pi}{8}$	**B1**	**[3]**	
(ii)	$8 + 8 \times$ their $\alpha + \frac{1}{2}\times 8\times\pi$	**B1** √		23.7 gets B1B0
	$8 + 5\pi$	**B1**	**[2]**	SC B1 for e.g. 5π (omitted OB)

Question 4

(i)	sector areas are $\frac{1}{2}11^2\alpha, \frac{1}{2}5^2\alpha$	B1	Sight of 11^2, 5^2
	$k = \dfrac{\frac{1}{2}\times 11^2\alpha - \frac{1}{2}\times 5^2\alpha}{\frac{1}{2}\times 5^2\alpha}$	M1	Or $\dfrac{11^2 - 5^2}{5^2}$
	$k = \dfrac{96}{25}$ or 3.84	A1 [3]	
(ii)	perimeter shaded region= $11\alpha + 5\alpha + 6 + 6 = 16\alpha + 12$	B1	
	perimeter unshaded region = $5\alpha + 5 + 5 = 5\alpha + 10$	B1	
	$16\alpha + 12 = 2(5\alpha + 10)$	M1	
	$\alpha = 4/3$ or 1.33	A1 [4]	

Question 5

(i)	slant length = 10 cm.	B1	
	circumference of base = 12π	B1	
	arc length = 10θ (= 12π)	B1√	Use of $r\theta$, θ calculated, not 6 or 8.
	$\rightarrow \theta = 1.2\pi$ or 3.77 radians.	B1 [4]	
(ii)	$\frac{1}{2}r^2\theta = 188.5$ cm ^{2} or 60π.	M1 A1√ [2]	Use of $\frac{1}{2}r^2\theta$ with radians and r = calculated '10', not 6 or 8.

Question 6

(i)	$r(2\pi - \alpha)$ $\quad + 2r\alpha + 2r$	**B1B1**	
	$2\pi r + r\alpha + 2r$	**B1**√ **[3]**	ft for $r\alpha$ instead of $2r\alpha$ or omission $2r$ SC1 for $2r\alpha + 4r$. (Plate = shaded part)
(ii)	$\frac{1}{2}(2r)^2\alpha + \pi r^2 - \frac{1}{2}r^2\alpha$	**B1B1**	Either B1 can be scored in (iii)
	$\dfrac{3r^2\alpha}{2} + \pi r^2$	**B1** **[3]**	
(iii)	$\pi r^2 - \frac{1}{2}r^2\alpha = 2r^2\alpha$	**M1**	For equating *their* 2 parts from (ii)
	$\alpha = \frac{2}{5}\pi$	**A1** **[2]**	

Question 7

(i)	$s = r\theta$ Angle of major arc = $2\pi - 2.2 = (4.083)$ Perimeter = 12 + 24.5 = 36.5 or $12\pi - 1.2$ (or full circle − minor arc B1)	M1 B1 A1 [3]	Used with major or minor arc Could be gained in **(ii)**. co
(ii)	Area of major sector = $\frac{1}{2}r^2\theta$ = (73.49) Area of triangle = $\frac{1}{2}.6^2\sin 2.2 = (14.55)$ Ratio = 5.05 : 1 (Allow 5.03 → 5.06)	M1 M1 A1 [3]	Used with major/minor sector. Correct formula or method. $(2\pi - 2.2)/\sin 2.2$ gets M1M1 co

Question 8

(i)	$\frac{1}{2}r^2\theta = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ → $2\sin\theta = \theta$ → p = 2.	B1 B1 [2]	Correct equation. All ok – answer given.
(ii)	Chord length = 8sin1.2 × 2 (14.9) (or from cosine rule) Arc length = 2.4 × 8 (19.2) Perimeter = sum of these = 34.1	M1 B1 A1 [3]	Needs ×2. Any method ok. co

Question 9

(i)	area Δ = $\frac{1}{2} \times 4 \times 4\tan\alpha$ oe soi Area sector = $\frac{1}{2} \times 2^2\alpha$ oe soi Shaded area = $8\tan\alpha - 2\alpha$ cao	**B1** **B1** **B1** **[3]**	$4\tan\alpha = \sqrt{16/\cos^2\alpha - 16}$. (Can also score in answer) Accept θ throughout Little/no working – accept terms in answer
(ii)	$DC = \frac{4}{\cos\alpha} - 2$ oe soi Arc $DE = 2\alpha$ soi anywhere provided clear Perimeter = $\frac{4}{\cos\alpha} + 4\tan\alpha + 2\alpha$ cao	**B1** **B1** **B1** **[3]**	$\frac{4}{\cos\alpha} = \sqrt{16 + 16\tan^2\alpha}$. Can score in answer Little/no working – accept terms in answer

Question 10

(i)	CB or $AB = \dfrac{3}{\tan\frac{\pi}{6}}$ or $3\tan\dfrac{\pi}{3}$	**B1**	Allow throughout for e.g. $3\sqrt{3}$,
		B1	$\sqrt{27}, \sqrt{3^3}, \left(\sqrt{3}\right)^3, \dfrac{9}{\sqrt{3}}$
	Arc or $AC = 3\times\left[\dfrac{2\pi}{3} \text{ or } \dfrac{\pi}{3}\right]$ $\quad(= 2\pi \text{ or } \pi)$	**B1** **[3]**	After B0B0 SCB1 for 16.7
	Perimeter $= 6\sqrt{3} + 2\pi$ oe	**B1**√	*Their AB* in form $k\sqrt{3}$
(ii)	Area *OABC* $(2)\times\dfrac{1}{2}\times 3\times$ *their AB*		
	$(=9\sqrt{3}$ or $\dfrac{9\sqrt{3}}{2})$	**B1**	
	Area *OADC* $\dfrac{1}{2}\times 3^2\times\left(\dfrac{2\pi}{2} \text{ or } \dfrac{\pi}{3}\right)$ $\left(= 3\pi \text{ or } \dfrac{3\pi}{2}\right)$	**B1**	After B0B0 SCB1 for 6.16 or 6.17.
	Shaded area $9\sqrt{3} - 3\pi$ oe	**[3]**	Allow $\left(\sqrt{3}\right)^5 - 3\pi$

Question 11

(i)	$\tan\theta = \dfrac{5}{12}$	M1	Any valid trig method ag
	$\rightarrow (\ \theta = 0.3948\ )$	[1]	
(ii)	Other angle in triangle $= \frac{1}{2}\pi - 0.3948$	B1	Unsimplified OK
	Area of triangle *AOB* $= \frac{1}{2}\times 12\times 5\ (= 30)$	B1	co
	Use of $\frac{1}{2}r^2\theta$ once	M1	With θ in radians and $r = 5$ or 12
	Shaded area = sector + sector – triangle		
	$= \frac{1}{2}\times 12^2\times 0.3948 + \frac{1}{2}5^2\theta - 30$	DM1	Sum of 2 sectors – triangle or any other valid method using the given angle and a different one.
	$= 28.43 + 14.70 - 30 = 13.1$	A1 [5]	co

Question 12

(i)	Arc $AB = 4\alpha$	**B1**	
	Arc $DC = (4\cos\alpha)\alpha$	**B1**	
	AC (or DB) $= 4 - 4\cos\alpha$	**B1**	
	Perimeter $= 4\alpha\cos\alpha + 4\alpha + 8 - 8\cos\alpha$	**B1** **[4]**	
(ii)	$OD = 4\cos\frac{\pi}{6}\left(= 2\sqrt{3}\right)$	**B1**	
	Shaded area $= \left[\ \frac{1}{2}\times 4^2\times\frac{\pi}{6}\ \right]\left[\ -\frac{1}{2}\left(2\sqrt{3}\right)^2\times\frac{\pi}{6}\ \right]$	**B1B1**	
	$\dfrac{\pi}{3}$	**B1** **[4]**	Or $k = \frac{1}{3}$

Question 13

(i)	$OC = r\cos\alpha$ or $AC = r\sin\alpha$ or oe soi (Area $\Delta OAC =$) $\frac{1}{2}r^2 \sin\alpha \cos\alpha$ $\frac{1}{2}r^2 \sin\alpha\cos\alpha = \frac{1}{2}\times\frac{1}{2}r^2\alpha$ oe $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$	**M1** **A1** **M1** **A1** **[4]**	 Or e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{4}r^2\alpha$ $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{2}r^2\cos\alpha\sin\alpha$ AG
(ii)	Perimeter $\Delta OAC = r + r\sin\alpha + r\cos\alpha = 2.4(0)r$ Perim. $ACB = r\alpha + r\sin\alpha + r - r\cos\alpha = 2.18r$ or $2.17r$ Ratio $= \dfrac{2.4(0)}{2.18 \text{ or } 2.17} : 1 = 1.1 : 1$	**M1A1** **M1A1** **A1** **[5]**	Allow with r a number. 2.0164 gets M1A0 Allow with r a number. 0.9644 gets M1A0 Allow 2.2 www. Use of $\cos = 0.6$, $\sin = 0.8$, $\alpha = 0.9$ is PA 1
(iii)	54.3° cao	**B1** **[1]**	

Question 14

Radius of semicircle $= \frac{1}{2}AB = r\sin\theta$ Area of semicircle $= \frac{1}{2}\pi r^2\sin^2\theta = A_1$ Shaded area = semicircle – segment $= A_1 - \frac{1}{2}r^2 2\theta + \frac{1}{2}r^2\sin 2\theta$	B1 B1√ B1B1 [4]	aef Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$ B1 (–sector), B1 for + (triangle)

Question 15

(i)	Sector $OCD = \frac{1}{2}(2r)^2\theta \; (= 2r^2\theta)$ Sector(s) $OAB/OEF = (2)\frac{1}{2}r^2(\pi - \theta)$ Total $= r^2(\pi + \theta)$	**B1** **B1** **B1** [3]	$2r^2\theta$ seen somewhere Accept with/without factor (2) **AG www**
(ii)	Arc $CD = 2r\theta$ Arc(s) AB/EF $(2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	**B1** **B1** **B1** **B1** [4]	Accept with/without factor (2) Must be simplified

Question16

(i)	Length of $OB = \dfrac{6}{\cos 0.6} = 7.270$	**M1** [1]	ag Any valid method
(ii)	$AB = 6\tan 0.6$ or 4.1 Arc length = $7.27 \times (\frac{1}{2}\pi - 0.6) = (7.06)$ Perimeter = $6 + 7.27 + 7.06 + 6\tan 0.6 = 24.4$	**B1** **M1** **A1** [3]	Sight of in **(ii)** Use of $s = r\theta$ with sector angle
(iii)	Area of $AOB = \frac{1}{2} \times 6 \times 7.27 \times \sin 0.6$ Area of $OBC = \frac{1}{2} \times 7.27^2 \times (\frac{1}{2}\pi - 0.6)$ $\rightarrow$ area = $12.31 + 25.65 = 38.0$	**M1** **M1** **A1** [3]	Use of any correct area method Use of $\frac{1}{2}r^2\theta$.

Question 17

(i)	$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$	**B1** [1]	**AG**
(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi	**M1**	Expect $\frac{1}{2}\pi r^2$
	Area $\Delta\, BCAD = \frac{1}{2}(2r)r$	**M1**	Expect r^2 (could be embedded)
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$.oe	**A1**	
	Area semi-circle $CADE = \frac{1}{2}\pi r^2$	**B1**	
	Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$		
	or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$	**DM1**	Depends on the area $\Delta\, BCD$
	$= r^2$	**A1** [6]	

Question 18

(a) (i)	$BAO = OBA = \frac{\pi}{2} - \alpha$ $AOB = \pi - \left(\frac{\pi}{2} - \alpha\right) - \left(\frac{\pi}{2} - \alpha\right) = 2\alpha$ **AG**	**M1A1** **[2]**	Allow use of 90º or 180º Or other valid reasoning
(ii)	$\frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2 \sin 2\alpha$ oe	**B2,1,0** **[2]**	SCB1 for reversed subtraction
(b)	Use of $\alpha = \frac{\pi}{6}$, $r = 4$ 1 segment $S = \left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ Area ABC $T = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $\left(= 4\sqrt{3}\right)$ $T - 3S = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3} - 3$ $\left[\left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}\right]$ $16\sqrt{3} - 8\pi$ cao	**B1B1** **M1** **B1** **M1** **A1** **[6]**	 Ft *their* **(ii)**, α, r OR $AXB = \frac{T}{3} = 4\tan\frac{\pi}{6}$ or $\frac{1}{2}(\frac{4}{\sqrt{3}})^2 \sin\frac{2\pi}{3}\left(= \frac{4\sqrt{3}}{3}\right)$ OR $3\left[\frac{T}{3} - S\right] = 3\left[\frac{4\sqrt{3}}{3} - \left(\frac{8\pi}{3} - 4\sqrt{3}\right)\right]$

Question 19

$BAC = \sin^{-1}(3/5)$ or $\cos^{-1}(4/5)$ or $\tan^{-1}(3/4)$ $ABC = \sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$ or $\tan^{-1}(4/3)$	**B1** **B1**	Accept 36.8(7)º Accept 53.1(3)º
$ACB = \pi/2$ (Allow 90º) Shaded area = ΔABC – sectors ($AEF + BEG + CFG$) $\Delta ABC = \frac{1}{2} \times 4 \times 3$ oe Sum sectors $= \frac{1}{2}\left[3^2 0.6435\right)$ + $2^2 0.9273 + 1^2 1.5708]$ **OR** $\frac{\pi}{360}\left[3^2 36.8(7) + 2^2 53.1(3) + 1^2 90\right]$ $6 - 5.536 = 0.464$	**B1** **M1** **B1** **M1** **A1** [7]	

Question 20

(i)	$PT = r\tan\alpha$	B1	
	$QT = OT - OQ = \dfrac{r}{\cos\alpha} - r$ or $\sqrt{r^2 + r^2\tan^2\alpha} - r$	B1	
	Perimeter = sum of the 3 parts including $r\alpha$	B1 [3]	
(ii)	Area of triangle = $\frac{1}{2}\times 10\times 10\tan\dfrac{\pi}{3}$	M1	Correct formula used, $50\sqrt{3}$,86.6
	Area of sector = $\frac{1}{2}\times 10^2\times\frac{1}{3}\pi$	M1	Correct formula used, $\dfrac{50\pi}{3}$, 52.36
	Shaded region has area 34 (2sf)	A1 [3]	

Question 21

(i)	$CD = r\cos\theta$, $BD = r - r\sin\theta$ oe	B1 B1	allow degrees but not for last B1
	Arc $CB = r(\frac{1}{2}\pi - \theta)$ oe	B1	
	$\rightarrow P = r\cos\theta + r - r\sin\theta + r(\frac{1}{2}\pi - \theta)$ oe	B1√ [4]	√ sum – assuming trig used
(ii)	Sector $= \frac{1}{2}.5^2.(\frac{1}{2}\pi - 0.6)$ (12.135)	M1	Uses $\frac{1}{2}r^2\theta$
	Triangle = $\frac{1}{2}$.5cos0.6.5sin0.6 (5.825)	M1	Uses $\frac{1}{2}bh$ with some use of trig.
	→ Area = 6.31	A1	
	(or $\frac{1}{4}$ circle – triangle – sector)	[3]	

Question 22

(i)	$\tan\left(\frac{\pi}{3}\right) = \dfrac{AC}{2x}$ or $\cos\left(\frac{\pi}{3}\right)\left(=\sin\frac{\pi}{6}\right) = \dfrac{2x}{AB}$ → $AC = 2\sqrt{3}x$ or $AB = 4x$	B1	Either trig ratio
	$AM = \sqrt{13x^2}, \sqrt{13}x, 3.61x$	M1A1 [3]	Complete method.
(ii)	$\tan(M\hat{A}C) = \dfrac{x}{\text{Their } AC}$	M1	"Their AC" must be f(x), $(M\hat{A}C) \neq \theta$.
	$\theta = \frac{1}{6}\pi - \tan^{-1}\dfrac{1}{2\sqrt{3}}$ **AG**	A1 [2]	Justifies $\dfrac{\pi}{6}$ and links MAC & θ

Question 23

(i)	$\cos 0.9 = OE/6$ or $= \sin\left(\frac{\pi}{2} - 0.9\right)$ oe $OE = 6\cos 0.9 = 3.73$ oe AG	M1 A1	[2]	Other methods possible
(ii)	Use of $(2\pi - 1.8)$ or equivalent method Area of large sector $= ½ \times 6^2 \times (2\pi - 1.8)$ oe Area of small sector $½ \times 3.73^2 \times 1.8$ Total area = 80.7(0) + 12.5(2) = 93.2	M1 M1 M1 A1	[4]	Expect 4.48 Or $\pi 6^2 - ½ 6^2 1.8$. Expect 80.70 Expect 12.52 Other methods possible

Question 24

(i)	$\frac{r}{10} = \sin 0.6$ or $\frac{r}{10} = \cos 0.97$ or $BD = \sqrt{200 - 200\cos 1.2}\ (= 11.3)$ $r = 10 \times 0.5646$, $r = 10 \times \sin 0.6$, $r = 10 \times \cos 0.971$ or $r = ½\ BD$ $\rightarrow r = 5.646$ AG	M1 A1	[2]	Or other valid alternative.
(ii)	Major arc $= 10(\theta)\ (= 50.832)$ $\theta = 2\pi - 1.2\ (= 5.083)$ or C = $2\pi \times 10$, Minor arc = 1.2×10 Semicircle = $5.646\pi\ (= 17.737)$ Major arc + semicircle = 68.6	M1 B1 A1	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Implied by 5.1
(iii)	Area of major sector $= ½ 10^2(\theta)\ (= 254.159)$ Area of triangle OBD $= ½ 10^2 \sin 1.2\ (= 46.602)$ Area = semicircle + sector + triangle (= 50.1 + 254.2 + 46.6) = 351	M1 M1 A1	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Use of $½ ab\sin C$ or other complete method

Question 25

(i)	$2r\alpha + r\alpha + 2r = 4.4r$ $\alpha = 0.8$	M1 A1	[2]	At least 3 of the 4 terms required
(ii)	$½(2r)^2 0.8 - ½(r^2)0.8 = 30$ $(3/2)r^2 \times 0.8 = 30 \rightarrow r = 5$	M1A1√ A1	[3]	Ft through on *their* α

Question 26

(i)	$ABC = \pi/2 - \pi/7 = 5\pi/14$. $CBD = \pi - 5\pi/14 = 9\pi/14$	B1	**AG** Or other valid exact method.
	Total:	1	
(ii)	$\sin\frac{\pi}{7} = \frac{½BC}{8}$ **or** $\frac{BC}{\sin\frac{2\pi}{7}} = \frac{8}{\sin\frac{5\pi}{14}}$ **or** $BC^2 = 8^2 + 8^2 - 2(8)(8)\cos\frac{2\pi}{7}$	M1	
	$BC = 6.94(2)$	A1	
	arc CD = *their* $6.94 \times 9\pi/14$	M1	Expect 14.02(0)
	arc $CB = 8 \times 2\pi/7$	M1	Expect 7.18(1)
	perimeter = 6.94 + 14.02 + 7.18 = 28.1	A1	
	Total:	5	

Question 27

'(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or cos = 6/10 or tan = 8/6. Accept 0.295π.
	Total:	1	
(ii)	$AB = 6$(Pythagoras) $\rightarrow$ $\Delta BCD = 8 \times 6 = 48.0$	M1A1	**OR** $8\times10\sin0.6435$ or $½\times10\times10\sin((2)\times0.927)=48$. 24or 40or80 gets **M1A0**
	Area sector $BCD = ½ \times 10^2 \times (2) \times$ *their* 0.9273	*M1	Expect 92.7(3). 46.4 gets **M1**
	Area segment = 92.7(3) – 48	*A1	Expect 44.7(3). Might not appear until final calculation.
	Area semi-circle – segment = $½ \times \pi \times 8^2$ – *their*$(92.7 - 48)$	DM1	Dep. on previous **M1A1** OR $\pi \times 8^2 - (½ \times \pi \times 8^2 +$ *their*$44.7)$.
	Shaded area = 55.8 – 56.0	A1	
	Total:	6	

Question 28

(i)	$(AB) = 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	Allow unsimplifed throughout eg r + r, $\frac{2\theta}{2}$ etc
	(Arc AB) = $2r\theta$	B1	
	$(P =)\ 2r + 2r\theta + 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	
	Total:	3	
(ii)	Area sector AOB = (½ r^2 2θ) $\frac{25\pi}{6}$ or13.1	B1	Use of segment formula gives 2.26 **B1B1**
	Area triangle AOB = (½×2$r$sinθ×$r$cosθ or ½ × r^2 sin2θ) $\frac{25\sqrt{3}}{4}$ or10.8	B1	
	Area rectangle $ABCD$ = (r × 2$r$sinθ) 25	B1	
	(Area =) Either 25 – (25π/6 – 25$\sqrt{3}$/4) or 22.7	B1	Correct final answer gets **B4**.
	Total:	4	

Question 29

(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) $\rightarrow XM = 2$	**B1**	(could find $\sqrt{40}$ and use $\sin^{-1}$or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	**M1 A1**	AG Needs × 2 and correct trig for **M1**
	(Alternative 1: $\sin AOM = \frac{6}{10}, AOM = 0.6435, AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, **B1** for AOM, **M1**,**A1** for completion) (Alternative 2: Use of circle theorem, **B1** for AOB, **M1**,**A1**for completion)
	Total:	**3**	
(ii)	$AX = \sqrt{(6^2 + 2^2)} = \sqrt{40}$	**B1**	CAO, could be gained in part (i) or part (iii)
	Arc $AYB = r\theta = \sqrt{40} \times 2.498$	**M1**	Allow for incorrect $\sqrt{40}$ (not $r = 6 \text{ or } 12 \text{ or } 10$)
	Perimeter = 12 + arc = 27.8 cm	**A1**	
	Total:	**3**	
(iii)	area of sector $AXBY = ½ \times (\sqrt{40})^2 \times 2.498$	**M1**	Use of $½r^2\theta$ with their r, (not $r = 6 \text{ or } r = 10$)
	Area of triangle $AXB = ½ \times 12 \times 2$, Subtract these $\rightarrow$ 38.0 cm²	**M1 A1**	Use of $½bh$ and subtraction. Could gain **M1** with $r = 10$.
	Total:	**3**	

Question 30

(i)	$\sin^{-1}\left(\frac{3}{5}\right) = 0.6435$ AG	**M1**	OR $(PBC =)\cos^{-1}\left(\frac{3}{5}\right) = 0.9273 \Rightarrow (ABP =)\frac{\pi}{2} - 0.9273 = 0.6435$ Or other valid method. Check working and diagram for evidence of incorrect method
(ii)	Use (once) of sector area $= ½r^2\theta$	**M1**	
	Area sector $BAP = ½ \times 5^2 \times 0.6435 = 8.04$	**A1**	
	Area sector $DAQ = ½ \times ½\pi \times 3^2 = 7.07$, Allow $\frac{9\pi}{4}$	**A1**	
		3	
(iii)	*EITHER:* Region = sect + sect – (rect – Δ) or sect – [rect – (sect + Δ)]	**(M1**	Use of correct strategy
	(Area Δ BPC =) ½ × 3 × 4 = 6 Seen	**A1**	
	8.04 + 7.07 – (15 – 6) = 6.11	**A1)**	
	OR1: Region = sector ADQ – (trap $ABPD$ – sector ABP).	**(M1**	Use of correct strategy
	(Area trap $ABPD$ =) ½ (5 + 1) × 3 = 9 Seen	**A1**	
	7.07 – (9 – 8.04) = 7.07 – 0.96 = 6.11	**A1)**	
	OR2: Area segment AP= 2.5686 Area segment AQ = 0.5438 Region = segment AP + segment AQ + ΔAPQ.	**(M1**	Use of correct strategy
	(Area ΔAPQ =) ½ × 2 × 3 = 3 Seen	**A1**	
	2.57 + 0.54 + 3 = 6.11	**A1)**	
		3	

Question 31

(i)	Pythagoras $\rightarrow r = \sqrt{72}$ OE or $\cos 45 = \frac{6}{r} \rightarrow r = \frac{6}{\cos 45} = 6\sqrt{2}$	**M1**	Correct method leading to $r =$
	Arc DC = $\sqrt{72} \times \frac{1}{4}\pi = \frac{3\sqrt{2}}{2}\pi$, 2.12π, 6.66	**M1 A1**	Use of $s=r\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
		3	
(ii)	Area of sector-BDC is $\frac{1}{2} \times 72 \times \frac{1}{4}\pi$ (= 9π or 28.274…)	***M1**	Use of $\frac{1}{2}r^2\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
	Area $Q = 9\pi - 18$ (10.274…)	**DM1**	Subtracts their $\frac{1}{2} \times 6 \times 6$ from their $\frac{1}{2}r^2\theta$
	Area P is ($\frac{1}{4}\pi 6^2$ – area Q) = 18	**M1**	Uses {$\frac{1}{4}\pi 6^2$ – (their area Q using $\sqrt{72}$)}
	Ratio is $\frac{18}{9\pi - 18}$ $\left(\frac{18}{10.274}\right) \rightarrow 1.75$	**A1**	
		4	

Question 32

(i)	$\cos A = 8/10 \rightarrow A = 0.6435$	**B1**	AG Allow other valid methods e.g. $\sin A = 6/10$
		1	
(ii)	*EITHER:* Area $\Delta ABC = \frac{1}{2} \times 16 \times 6$ or $\frac{1}{2} \times 10 \times 16 \sin 0.6435 = 48$	**(M1A1**	
	Area 1 sector $\frac{1}{2} \times 10^2 \times 0.6435$	**M1**	
	Shaded area $= 2 \times$ *their* sector $-$ *their* ΔABC	**M1)**	
	OR: $\Delta BDE = 12$, $\Delta BDC = 30$	**(B1 B1**	
	Sector = 32.18	**M1**	
	2×segment + ΔBDE	**M1)**	
	=16.4	**A1**	
		5	

Question 33

5(i)	$\frac{PQ}{2} = 10 \times \sin 1.1$	**M1**	Correct use of sin/cos rule
	($PQ =$) 17.8 (17.82…implies **M1**, **A1**) AG	**A1**	OR $PQ = \frac{10 \sin 2.2}{\sin\left(\frac{\pi}{2} - 1.1\right)}$ *or* $\frac{10 \sin 2.2}{\sin 0.4708}$ *or* $\sqrt{200 - 200\cos 2.2} = 17.8$
		2	
5(ii)	Angle OPQ = (π/2 – 1.1) [accept 27°]	**B1**	OE Expect 0.4708 or 0.471. Can be scored in part **(i)**
	Arc QR =17.8 × *their* (π/2 – 1.1)	**M1**	Expect 8.39. (or 8.38).
	Perimeter $= 17.8 - 10 + 10 +$ *their* arc QR	**M1**	
	26.2	**A1**	For both parts allow correct methods in degrees
		4	

Question 34

Angle $AOC = \frac{6}{5}$ or 1.2		**M1**	Allow 68.8°. Allow $\frac{5}{6}$
AB = 5 × tan(*their* 1.2) OR by e.g. Sine Rule	Expect 12.86	**DM1**	OR $OB = \frac{5}{\cos their 1.2}$. Expect 13.80
Area $\Delta OAB = \frac{1}{2} \times 5 \times their\, 12.86$	Expect 32.15	**DM1**	OR $\frac{1}{2} \times 5 \times$ *their OB* × sin *their* 1.2
Area sector $\frac{1}{2} \times 5^2 \times their\, 1.2$	Expect 15	**DM1**	All DM marks are dependent on the first M1
Shaded region = 32.15 – 15 = 17.2		**A1**	Allow degrees used appropriately throughout. 17.25 scores A0
		5	

Question 35

(i)	*AT* or *BT* = *r*tanθ or $OT = \frac{r}{\cos\vartheta}$	**B1**	May be seen on diagram.
	½*r*²2*θ*, & ½×*r*×(*r*tan*θ* or *AT*) or ½×*r*×($\frac{r}{\cos\vartheta}$ or *OT*) sinθ	**M1**	Both formulae, (½*r*²*θ*, ½*bh or* ½*absinθ*), seen with 2*θ* used when needed.
	½*r*²2*θ* = 2×½×*r*×*r*tan*θ* – ½*r*²2*θ* oe → 2*θ* = tan*θ* **AG**	**A1**	Fully correct working from a correct statement. Note: ½*r*²2*θ* = ½ *r*²tan*θ* is a valid statement.
		3	
(ii)	*θ* = 1.2 or sector area = 76.8	**B1**	
	Area of kite = 165 awrt	**B1**	
	164.6 – 76.8 = 87.8 awrt	**B1**	awrt 87.8 with little or no working can be awarded 3/3. SC Final answers that round to 88 with little or no working can be awarded 2/3.
		3	

Question 36

(i)	$(\tan\theta = \frac{AT}{r}) \rightarrow AT = r\tan\theta$ or $OT = \frac{r}{\cos\theta}$ SOI	**B1**	CAO
	$\rightarrow A = \frac{1}{2}r^2\tan\theta \quad - \frac{1}{2}r^2\theta$	**B1 B1**	B1 for $\frac{1}{2}r^2\tan\theta$. B1 for "$-\frac{1}{2}r^2\theta$" If Pythagoras used may see area of triangle as $\frac{1}{2}r\sqrt{r^2 + r^2\tan^2\theta}$ or $\frac{1}{2}r\left(\frac{r}{\cos\theta}\right)\sin\theta$
		3	
(ii)	$\tan\theta = \frac{AT}{3} \rightarrow AT = 7.716$	**M1**	Correct use of trigonometry and radians in rt angle triangle
	Arc length = *rθ* = 3.6	**B1**	Accept 3×1.2
	OT by Pythagoras or $\cos 1.2 = \frac{3}{OT}$ (= 8.279)	**M1**	Correct method for *OT*
	Perimeter = *AT* + arc + *OT* – radius = 16.6	**A1**	CAO, www
		4	

Question 37

(i)	0.8 oe	B1	
		1	
(ii)	$BD = 5\sin \textit{their}\ 0.8$	M1	Expect 3.58(7). Methods using degrees are acceptable
	$DC = 5 - 5\cos \textit{their}\ 0.8$	M1	Expect 1.51(6)
	Sector = ½ × 5^2 × *their*0.8 OR Seg = ½ × 5^2 × [*their* 0.8 – sin*their* 0.8]	M1	Expect 10 for sector. Expect 1.03(3) for segment
	Trap = ½(5 + *theirDC*) × *theirBD* oe OR Δ*BDC* = ½*theirBD* × *theirCD*	M1	OR (for last 2 marks) if *X* is on *AB* and *XC* is parallel to *BD*:
	Shaded area = 11.69 – 10 OR 2.71(9) – 1.03(3) = 1.69 cao	A1	*BDCX* –(sector – Δ*AXC*) = 5.43(8) – [10 – 6.24(9)] = 1.69 cao M1A1
		5	

Question 38

(i)	$A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(¾)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ **or** $B\hat{A}C = \cos^{-1}(¾)$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$	M1	Correct method for $A\hat{B}C$, expect 1.696^cawrt Or for $B\hat{A}C$, expect 0.723^cawrt
	$C\hat{B}Y = \pi - A\hat{B}C$ or $2 \times C\hat{A}B$	M1	For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2 \times C\hat{A}B$
	OR		
	Find *CY* from Δ *ACY* using Pythagoras or similar Δs	M1	Expect $4\sqrt{7}$
	$C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(\textit{their}\ CY)^2}{2\times8\times8}\right)$	M1	Correct use of cosine rule
	$\mathbf{C\hat{B}Y = 1.445^c}$ AG	A1	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
(ii)	Arc *CY* = 8 × 1.445	B1	Use of $s=8\theta$ for arc *CY*, Expect 11.56
	$B\hat{A}C = ½(\pi - A\hat{B}C)$ or $\cos^{-1}(¾)$	*M1	For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect 0.7227^c
	Arc *XC* = 12 × (their $B\hat{A}C$)	DM1	Expect 8.673
	Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www**	A1	Omission of '+4' only penalised here.
		4	

Question 39

Angle $OAB = \pi/2 - \pi/5 = 3\pi/10$ soi	**B1**	Allow 54° *or* 0.9425 *rads*
Sector $CAB = \frac{1}{2}\times\left(their\frac{3\pi}{10}\right)\times 5^2$	**M1**	Expect 11.78
$OA = \frac{5}{\sin\frac{\pi}{5}} = 8.507$	**M1A1**	May be implied by $OC = 3.507$
Sector $COD = \frac{1}{2}\times(their\,3.507)^2\times\frac{\pi}{5}$	**M1**	Expect 3.86
$\Delta OAB = \frac{1}{2}\times 5\times(their\,8.507)\sin\frac{3\pi}{10}$	**M1**	Or $\frac{1}{2}\times 5\times\frac{5}{\tan\frac{\pi}{5}}$ or $2.5\times\sqrt{(their\,8.507)^2 - 25}$
= 17.20 or 17.21	**A1**	
Shaded area $17.20(or\,17.21) - 11.78 - 3.86 = 1.56$ or 1.57	**A1**	
	8	

Question 40

Angle $CBA = \sin^{-1}\left(\frac{7}{8}\right) = 1.0654$ or $CBD = \cos^{-1}\left(\frac{-17}{32}\right) = 2.13$	**B1**	Accept 61.0°, 66° or 122°
Sector $BCYD = ½\times 8^2\times 2\times their\,1.0654(\text{rad})$ soi or sector CBY $= ½\times 8^2\times their\,1.0654(\text{rad})$	**M1**	Expect 68.1(9). Angle must be in radians (or *their* 61/360 × 2 × 8^2) Or sector DBY
$\Delta BCD = 7\times\sqrt{8^2 - 7^2}$ or $½\times 8^2\times\sin(2\times their\,1.0654)$ soi	**M1**	Expect 27.1(1). Award M1 for ABC or ABD
Semi-circle $CXD = ½\pi\times 7^2 = 76.9(7)$	**M1**	M1M1 for segment area formula used correctly
Total area = *their*68.19 – *their*27.11 + *their*76.97 = 118.0–118.1	**M1A1**	Cannot gain M1 without attempt to find angle CBA or CBD
	6	

Question 41

3(i)	Angle EAD = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi or Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi	B1	
	$AD = 8\sin(\frac{3\pi}{10})$ or $8\cos(\frac{\pi}{5})$	M1	Angles used must be correct
	(AD =) 6.47	A1	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)}$ or $AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)}$ or 11.(01)	B1	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ **oe**	M1	
	(AD =) 6.47	A1	
		3	
(ii)	Area sector $= \frac{1}{2}(theirAD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	M1	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times theirAD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	M1	Or e.g. ½ $theirAD \times \sqrt{8^2 - theirAD^2}$. 15.2(2)
	(Shaded area =) 35.0 or 34.9	A1	
		3	

Question 42

Perimeter of $AOC = 2r + r\theta$	B1	
Angle $COB = \pi - \theta$	B1	Could be on the diagram. Condone $180 - \theta$.
Perimeter of $BOC = 2r + r(\pi - \theta)$	B1	**FT** on angle COB if of form $(k\pi - \theta)$, $k > 0$.
$(2r +)\ \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \rightarrow \theta = \frac{\pi - 2}{3}$)	M1	Sets up equation using $r(k\pi - \theta)$ and ×2 on correct side. Condone any omissions of OA, OB and/or OC.
$\theta = 0.38$	A1	Equivalent answer in degrees scores A0.
	5	

Question 43

Uses $A = ½r^2\theta$	M1	Uses area formula.
$\theta = \frac{2A}{r^2}$	A1	
$P = r + r + r\theta$	B1	
$P = 2r + \frac{2A}{r}$	A1	Correct simplified expression for P.
	4	

Question 44

(i)	Angle $CAO = \frac{\pi}{3}$	**B1**	
		1	
(ii)	(Sector AOC) = $\frac{1}{2}r^2 \times their \frac{\pi}{3}$	**M1**	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(their \frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**M1**	For M1M1, $their \frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < ½$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**A1**	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	**A1**	
		4	

Question 45

(i)	Arc length $AB = 2r\theta$	**B1**	
	$\text{Tan}\,\theta = \frac{AT}{r}$ or $\frac{BT}{r} \rightarrow AT \text{ or } BT = r\tan\theta$	**B1**	Accept or $\sqrt{\left(\left(\frac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\frac{r\sin\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$ NOT $(90 - \theta)$
	$P = 2r\theta + 2r\tan\theta$	**B1FT**	OE, FT for *their* arc length + 2 × *their AT*
		3	
4(ii)	Area ΔAOT = ½ × 5 × 5 tan 1.2 **or** Area $AOBT$ = 2 × ½ × 5 × 5 tan 1.2	**B1**	
	Sector area = ½ × 25 × 2.4 (or 1.2)	***M1**	Use of $½r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	**DM1**	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = 34.3 (cm^2)	**A1**	AWRT
	Alternative method for question 4(ii)		
	Area of ΔABT = ½ × (5 × tan 1.2)^2 × sin(π – 2.4) (= 55.86)	**B1**	
	Segment area = ½ × 25 × (2.4 – sin 2.4) (= 21.56)	***M1**	Use of $½r^2(\theta - \sin\theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	**DM1**	Subtraction of segment from ΔABT, using 2.4 where appropriate.
	Area = 34.3 (cm^2)	**A1**	AWRT
		4	

Question 46

3(i)	$OA \times \frac{3}{8}\pi = 6$	**M1**	
	$OA = \frac{16}{\pi} = 5.093(0)$	**A1**	
3(ii)	$AB = their\,5.0930 \times \tan\frac{3}{16}\pi$	**M1**	
	Perimeter $= 2 \times 3.4030 + 6 = 12.8$	**A1**	
(iii)	Area $OABC = (2 \times ½) \times their\,5.0930 \times their\,3.4030$	**M1**	
	Area sector $= ½ \times (their\,5.0930)^2 \times \frac{3}{8}\pi$	**M1**	
	Shaded area $= their\,17.331 - their\,15.279 = 2.05$	**M1A1**	

Question 47

$OC = 6\cos 0.8 = 4.18(0)$	**M1A1**	SOI
Area sector $OCD = \frac{1}{2}(their\,4.18)^2 \times 0.8$	***M1**	OE
$\Delta OCA = \frac{1}{2} \times 6 \times their\,4.18 \times \sin 0.8$	**M1**	OE
Required area = *their* ΔOCA – *their* sectorOCD	**DM1**	SOI. If not seen *their* areas of sector and triangle must be seen
2.01	**A1**	CWO. Allow or better e.g. 2.0064
	6	

Question 48

$\cos POA = \frac{5}{13} \rightarrow POA = 1.17(6)$ Allow 67.4° **or** $\sin = \frac{12}{13}$ or $\tan = \frac{12}{5}$	**M1 A1**
Reflex $AOB = 2\pi - 2 \times their\ 1.17(6)$ OE in degrees **or** minor arc AB = $5 \times 2 \times their\,1.17(6)$	**M1**
Major arc = $5 \times their\ 3.93(1)$ **or** $2\pi \times 5$ - *their* 11.7(6)	**M1**
AP (or BP) = $\sqrt{13^2 - 5^2} = 12$	**B1**
Cord length = 43.7	**A1**
	6

Question 49

(a)	$BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$	M1
	$BC = r\sqrt{\left(5-2\sqrt{3}\right)}$	A1
		2
(b)	Perimeter = $\frac{2\pi r}{6} + r + r\sqrt{\left(5-2\sqrt{3}\right)}$	M1 A1
		2
(c)	Area = sector – triangle	
	Sector area = $\frac{1}{2}4r^2\frac{\pi}{6}$	M1
	Triangle area = $\frac{1}{2} r. 2r \sin\frac{\pi}{6}$	M1
	Shaded area = $r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$	A1
		3

Question 50

Angle $AOB = 15 \div 6 = 2.5$ radians	B1
Angle $BOC = \pi - 2.5$ (**FT** on angle AOB)	B1FT
$BC = 6(\pi - 2.5)$ $(BC = 3.850)$	M1
$\sin(\pi - 2.5) = BX \div 6$ $(BX = 3.59)$	M1
Either $OX = 6\cos(\pi - 2.5)$ **or** Pythagoras $(OX = 4.807)$	M1
$XC = 6 - OX$ $(XC = 1.193) \rightarrow P = 8.63$	A1
	6

Question 51

(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2+12^2-8^2}{2\times8\times12}$	M1	Or other correct method
	$BAO = 0.723$	A1	
		2	
(b)	Sector $ABC = \frac{1}{2}\times12^2\times their\,0.7227$	*M1	Accept 52.1
	Triangle $AOB = \frac{1}{2}\times8\times12\sin(their\,0.7227)$ or $\frac{1}{2}\times12\times\sqrt{28}$	*M1	or $\frac{1}{2}\times8\times8\sin(\pi-2\times their\,0.7227)$. Expect 31.7 or 31.8
	Shaded area = *their* 52.0 – *their* 31.7 = 20.3	DM1 A1	M1 dependent on both previous M marks
		4	
(c)	Arc $BC = 12\times their\,0.7227$	*M1	Expect 8.67
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	DM1 A1	
		3	

Question 52

(a)	Use of correct formula for the area of triangle *ABC*	**M1**	Use of 180–2θ scores M0. Condone $2\pi - 2\theta$
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	**A1**	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	**B1 FT**	FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
(b)	Arc $BD = r\theta = 6$ cm	**B1**	SOI
	$AC = 2r\cos\theta =$ ($2\times10\cos0.6 = 20\cos0.6 = 16.506$) or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	***M1**	Finding *AC* or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))} - r$ (= 6.506)	**DM1**	Subtracting *r* from *their AC* or $r - r\cos\theta$ from *their* half *AC* (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	**A1**	AWRT
		4	

Question 53

(a)	$\left(\sin\theta = \frac{r}{OC} \rightarrow\right) OC = \frac{r}{\sin\theta}$	**M1 A1**	
	$CD = r + \frac{r}{\sin\theta}$	**A1**	
		3	
(b)	Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4 + 8 = 12$	**B1**	SOI
	(Arc *AB* =) *their* $12\times\frac{2\pi}{6}$ or $\left(\frac{1}{2}AB = \right)\left(\textit{their } 12\times\frac{\pi}{6}\right)$	**M1**	Expect 4π, must use *their* CD, not 4
	Perimeter = $24 + 4\pi$	**A1**	
		3	

(c)	Area $FOC = \frac{1}{2} \times 4 \times their\ OC \times \sin\frac{\pi}{3}$	M1	
	$8\sqrt{3}$	A1	
	Area sector $FOE = \frac{1}{2} \times \frac{2\pi}{3} \times 4^2 = \frac{16\pi}{3}$	B1	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	A1	
	Alternative method for question 10(c)		
	$FC = \sqrt{(their\ OC)^2 - 4^2}$	M1	$\sqrt{48}$ or $4\sqrt{3}$
	Area $FOC = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$	A1	
	Area of half sector $FOE = \frac{1}{2} \times \frac{\pi}{3} \times 4^2 = \frac{8\pi}{3}$	B1	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	A1	
		4	

Question 54

(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	M1	Attempt to find the area of ΔADE.
	$\frac{1}{4}k^2a^2$	A1	OE.
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	B1	
	$2 \times \frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	M1	OE. For $2 \times \Delta ADE = \text{sector}ABC$ with at least one correct area.
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	A1	
		5	
(b)	$2 \times \frac{1}{2}(ka)^2 \sin\theta = \frac{1}{2}a^2\theta$	M1	Condone omission of ‘2’ or ‘1/2’ on LHS for M1 only.
	$k^2 = \frac{\theta}{2\sin\theta}$	A1	
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	A1	OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707\text{(AWRT)} < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE
		3	

Question 55

(a)	$\frac{1}{2} \times 4^2 \times \text{angle BAD} = 10$	**M1**	Use of sector area formula
	$\text{Angle BAD} = 1.25$	**A1**	OE. Accept 0.398π, 71.6^o for **SC B1** only
		2	
(b)	$\text{Arc } BD = 4 \times \textit{their} 1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC = 4\tan(\textit{their} 1.25)$	**M1**	Expect 12.0(4). May use ACB=0.321 or 18.4^o
	$CD = \frac{4}{\cos(\textit{their} 1.25)} - 4$ or $\sqrt{4^2 + (\textit{their } BC)^2} - 4$	**M1**	Expect 12.69 – 4 = 8.69. May use ACB.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

Question 56

(a)	[By symmetry] [6 × $P\hat{A}Q$ = 2π], [$P\hat{A}Q$ =] 2π÷6,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre A ÷ 6	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why ΔPAQ is an equilateral triangle	M1	Assumption of this scores M0
	Using ΔPAQ is an equilateral triangle ∴ $P\hat{A}Q = \frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$, equilateral triangles	M1	
	$P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$	A1	AG

(a)	**Alternative method for Question 12(a)**		
	$Sin\theta = \frac{20}{40}$, with θ clearly identified	M1	
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles = $P\hat{A}Q$	A1	AG
		2	
(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta = 20 \times \frac{\pi}{3}$	*M1	Use of $r\theta$ with $r = 20$ and θ in radians
	Total length = $6 \times ((\textit{their } 40) + k\pi)$	DM1	6×(*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240 + 40\pi$ or 366 (AWRT) (cm)	A1	Or directly: (6× diameter) + circumference
		4	

(c)	[Triangle area =] $\frac{1}{2}\times40\times40\times\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times40\times20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = $6\times400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2}\times(40+80)\times40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = $2\times1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle *ABC* = $400\sqrt{3}$ or 693 (AWRT) or 4 × Area of half of triangle *ABC* = $4\times200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle *ABDE* = $1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2\times400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times200\sqrt{3}+1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer.
		2	
(d)	Each rectangle area = 40 × 20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta=\frac{1}{2}\times20^2\times\frac{\pi}{3}\left[=\frac{200\pi}{3}\right]$	B1	SOI.
	Total area = $2400\sqrt{3}+4800+400\pi$ or 10 200 (cm^2) (AWRT)	B1	Or directly: part **(c)** + 6800 + area circle radius 20.
		3	

Question 57

(a)	**Either** Let midpoint of *PQ* be *H*: sin *HCP* = $\frac{2}{4}$ ⇒ Angle *HCP* = $\frac{\pi}{6}$ **Or** sin *PSQ* = $\frac{4}{8}$ ⇒ Angle *PSQ* = $\frac{\pi}{6}$ **Or** using cosine rule: angle *PCQ* = $\frac{\pi}{3}$ **Or** by inspection: triangle *PCQ* or *PCT* is equilateral so angle *PCQ* = $\frac{\pi}{3}$	**M1**	
	Angle *PCS* = $\pi-\frac{\pi}{6}-\frac{\pi}{6}=\frac{2}{3}\pi$	**A1**	AG
		2	
(b)	Perimeter = $2\times4\times\frac{2\pi}{3}$ or $8\pi-\frac{8\pi}{3}$	**M1**	Length of two arcs *PS* and *QR*
	$+2\pi\times2$	**M1**	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	**A1**	Must be a single term
		3	

(c)	Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	M1	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	M1	Attempts to find area of segment
	Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$	M1	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	M1	
	$\pi\times 4^2 - \pi\times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	M1	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	M1	Uses correct formula for triangle
	Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$	M1	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	M1	
	$2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi\times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
		5	

Question 57

(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	B1	AG. OE using cosine rule.
		1	
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1	
	$AB = 9 + 11 - theirXY$	B1 FT	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc AC = 11 × 0.9582	M1	
	Arc $BC = 9\times\frac{\pi}{2}$	M1	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	A1	AWRT. Answer must be evaluated as a single decimal.
		6	

Question 58

(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	**B1**	AG. OE using cosine rule.
		1	
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	***M1 A1**	
	$AB = 9 + 11 - \mathit{their}XY$	**B1 FT**	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc $AC = 11 \times 0.9582$	**M1**	
	Arc $BC = 9 \times \frac{\pi}{2}$	**M1**	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	**A1**	AWRT. Answer must be evaluated as a single decimal.
		6	

Question 59

(a)	**EITHER** By using trigonometry: $B\hat{A}C = 0.6435\ldots$ and $A\hat{B}C = \frac{\pi - 0.6435}{2}$ **OR** By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \frac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \frac{\sqrt{10}}{10}$	**M1**	$\frac{3}{\sqrt{10}} = 0.9486\ldots$ $\frac{\sqrt{10}}{10} = 0.3162\ldots$
	$A\hat{B}C = \frac{\pi - 0.6435}{2}$ or $\tan^{-1}\frac{9}{3}$ or $\sin^{-1}\frac{3}{\sqrt{10}}$ or $\cos^{-1}\frac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ = 1.25 radians (3 sf)	**A1**	AG. Final answer must be 1.25, more accurate value 1.24904… with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
(b)	$BC = \sqrt{(\mathit{their}\,3)^2 + 9^2}$ or $\frac{9}{\sin 1.25}$ [= $\sqrt{90}$, $3\sqrt{10}$ or 9.48697…]	**M1**	Using correct method(s) to find *BC*.
	Area of sector = $\frac{1}{2} \times (\mathit{their}\,BC)^2 \times \tan^{-1} 3$ [= 56.207 *or* 56.25]	**M1**	Using $\tan^{-1} 3$ or 1.25 and *their BC*, but not 9 or 15, in correct area of sector formula.
	Area of triangle *PBC* = 13.4 to 13.6 or $\frac{1}{2} \times 9 \times 3$	**B1**	
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	**A1**	AWRT
		4	

Question 60

(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times \textit{their}\ \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \textit{their}\ \frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	**B1**	
	One arc $\frac{1}{6} \times 12\pi$ leading to two arcs $2 \times \frac{1}{6} \times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
		3	
(b)	Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2} \times (6^2) \times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2} \times (6^2) \times \sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right) + 6\pi\ \left[= 6\pi - 9\sqrt{3} + 6\pi\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2 \times \left(\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)\right) - \frac{1}{2} \times (6^2) \times \sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2} \times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2 \times \left(\frac{1}{2} \times (6^2) \times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2} \times (6^2) \times \sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\right) + \frac{1}{2} \times (6^2) \times \sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\ \left[= 12\pi - 18\sqrt{3} + 9\sqrt{3}\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × segment + triangle** A1 if correct (unsimplified).
	Area $\left[= 6\pi - 9\sqrt{3} + 6\pi\right] = 12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
		4	

Question 61

(a)	$\tan A = \frac{12}{5}$ or $\cos A = \frac{5}{13}$ or $\sin A = \frac{12}{13}$	**M1**	OR $\tan B = \frac{5}{12}$ or $\cos B = \frac{12}{13}$ or $\sin B = \frac{5}{13}$
	$A = 1.176$ $B = 0.3948$	**A1**	Allow 1.18 or 67.4°, Allow 0.395 or 22.6°. May be implied by $\frac{\pi}{2} - 1.176$
	$DE = 4$	**B1**	If trigonometry used accept AWRT 4.00
	Arcs = $5 \times their 1.176$ *and* $8 \times their 0.3948$	**M1**	Or corresponding calculations in degrees.
	[Perimeter = 5.880 + 3.158 + 4 =] 13.0	**A1**	Accept 13. If *DE* is outside the given range this mark cannot be awarded.
		5	
(b)	Area of triangle = $\frac{1}{2} \times 5 \times their 12$ [= 30]	**B1 FT**	
	Area of sectors = $\frac{1}{2} \times 5^2 \times their\ 1.176 + \frac{1}{2} \times 8^2 \times their\ 0.3948$	**M1**	Or corresponding calculations in degrees
	[Area = 30 – 14.70 –12.63 =] 2.67	**A1**	Allow 2.66 to 2.67
		3	

Question 62

(a)	$6 \sin 0.9 = \frac{AC}{2}$ or $AC^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \cos 1.8$	**M1**	OE Correct working in degrees is acceptable throughout.
	$AC = 9.40$	**A1**	SOI Accept 9.39 – 9.41, may be used but not seen for A1.
	Angle *CAB* = ½(π – 1.8)	**M1**	SOI Expect 0.6708 (or 0.671).
	Arc *CD* = *their* 9.40 × *their* 0.6708	**M1**	Expect 6.306 (or 6.31), do not accept 6 for *their AC* or 1.8 for *CAB*.
	[Perimeter = 6 + 3.40 + 6.306 =] 15.7	**A1**	Accept 15.69 – 15.72.
		5	
(b)	Sector *ADC* – Δ*ABC* = ½ × *their* 9.40^2 × *their* 0.6708 – ½ × 6^2 × sin 1.8	**M1 M1**	Accept correct use of their answers from part **(a).**
	[29.64 – 17.53 =] 12.1	**A1**	AWRT
		3	

Question 63

(a)	$\left[A\hat{O}B=\right]\ \frac{2}{10}$	**B1**	OE Sight of 0.2 from $s=r\theta$ but $10\theta=2$ is not enough. ISW if $\frac{2}{10}=\frac{\pi}{5}$.
	$[B\hat{O}C=]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	**B1**	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
(b)	$[BP]=10\sin\left(\frac{5\pi+6}{30}\right)$ and $[OP]=10\cos\left(\frac{5\pi+6}{30}\right)$ [= 6.6208…] and [= 7.494…] **OR** $[BP]=10\sin\left(\frac{5\pi+6}{30}\right)$ and $[O\hat{B}P]=\left(\frac{5\pi-3}{15}\right)$ [= 6.6208…] and [= 0.84719…]	**M1**	OE Any correct method for **both** lengths, for *their* angle BOC (which may have been incorrectly 'simplified' but not 0.2) or length BP and $O\hat{B}P$. May be seen as part of $\frac{1}{2}ab\sin C$. Sight of correct method enough. Can be implied by the next A1.
	Area of $\Delta OBP=\frac{1}{2}\times\ 10\sin\left(\frac{5\pi+6}{30}\right)\times 10\cos\left(\frac{5\pi+6}{30}\right)$ or $\frac{1}{2}\times 10\times 10\sin\left(\frac{5\pi+6}{30}\right)\times\sin\left(\left(\frac{5\pi-3}{15}\right)\right)$ [=24.809]	**A1**	OE Can be implied by any answer in range (24.7, 24.9) or a final answer in the range (11.3, 11.5) WWW.
	$[\text{Sector } BOC]=\frac{1}{2}\times 10^2\times \textit{their}\ \left(\frac{5\pi+6}{30}\right)$ $\left[=50\left(\frac{5\pi+6}{30}\right)=36.1799...\right]$	**M1**	Use of $\frac{1}{2}r^2\theta$ with *their* angle BOC (may have been incorrectly 'simplified' but not 0.2).
	Area of region $BPC=11.4$	**A1**	CAO
		4	

Question 64

(a)	Sector area = $\frac{1}{2}r^2\left(\frac{\pi}{6}\right)\left[=\frac{\pi}{12}r^2\right]$	B1	Using $\frac{1}{2}r^2\theta$ with θ in radians SOI. B0 if using a value for r.
	$BD = \sin\frac{\pi}{6}r\left[=\frac{1}{2}r\right]$ and $AD = \cos\frac{\pi}{6}r\left[=\frac{\sqrt{3}}{2}r\right]$ so triangle area = $\frac{1}{2}\left(\sin\frac{\pi}{6}r\right)\left(\cos\frac{\pi}{6}r\right)\left[=\frac{1}{2}\times\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\right]$ **or** $\frac{1}{2}r\left(\cos\frac{\pi}{6}r\right)\left(\sin\frac{\pi}{6}\right)\left[=\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\times\frac{1}{2}\right]$	B1	SOI Finding triangle area. Decimals B0 unless exact values seen in working.
	Area of $BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$	B1	OE e.g. $\frac{r^2}{4}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ with $\cos\frac{\pi}{6}$ and $\sin\frac{\pi}{6}$ evaluated. Must be exact, in terms of r^2. ISW
		3	
(b)	Angle $BAC = \sin^{-1}\left(\frac{\frac{\sqrt{3}}{2}r}{r}\right)\left[=\frac{\pi}{3}\right]$	B1	SOI by length of AD, CD or arc, or by perimeter.
	Length $AD = \cos\frac{\pi}{3}r\left[=\frac{1}{2}r\right]$ [so length $CD = \frac{1}{2}r$]	M1	SOI Finding length by Pythagoras, or by trigonometry with *their* angle BAC, provided $BAC \neq \frac{\pi}{6}$.
	Length of arc $BC = r\times\frac{\pi}{3}$	M1	SOI Using $r\theta$ with θ in radians. Condone $\theta = \frac{\pi}{6}$.
	Perimeter of $BCD = \frac{\sqrt{3}}{2}r + \frac{1}{2}r + \frac{\pi}{3}r$	A1	OE e.g. $r\left(\frac{\sqrt{3}+1}{2}+\frac{\pi}{3}\right)$ with e.g. $\cos\frac{\pi}{3}$ evaluated. Must be exact, in terms of r. ISW
		4	

Question 65

(a)	$APQ = \cos^{-1}\frac{\frac{5}{6}r}{r}\left[=\cos^{-1}\frac{5}{6}\right]$	*M1	May use cosine rule to find APB. Stating APQ or APB as an incorrect multiple of π is M0.
	$= 0.5857$	A1	Accept 0.586 or 33.6° or APB (1.171 or 67.1°).
	Perimeter $= 4\times r\times \textit{their}\,0.5857 = 2.34r$ or $0.745\pi r$ or $(293/125)r$	DM1 A1	Must use a numerical value of *their* angle.
		4	
(b)	Use of sector formula: Sector APB $= ½r^2\times(2\times \textit{their}\,0.5857)$ or Sector APC (C is on PQ so PC = r) $= ½r^2\times(\textit{their}\,0.5857)$	M1	Any sector with *their* appropriate angle. It must be clear the appropriate numerical angle is being used.
	Use of appropriate formula for area of triangle and correct combination with the sector to find the area of a half segment, one segment or both segments	M1	e.g. Area APB = $½r^2\times\sin(2\times \textit{their}\,0.5857)$.
	Shaded area [$= 2\times 0.1250r^2$] $= 0.250r^2$	A1	or $0.0796\pi r^2$, allow $\frac{1}{4}r^2$ or $0.25r^2$.
		3	

Question 66

(a)	$2.5\times\frac{4\pi}{3}+2.24\times\frac{5\pi}{6}$ [= 10.47[2] + 5.86[4] or $\frac{10\pi}{3}+\frac{28\pi}{15}$]	B1	For either arc correct. Arc ARB could be AR+RB.
		M1	For adding two (or three) arc lengths using different radii and angles and nothing else. SOI
	16.34 or $\frac{26\pi}{5}$	A1	AWRT Condone 16.33 only.
		3	
(b)	Area $AOB=\frac{1}{2}\times2.5^2\sin\frac{2\pi}{3}$ [=2.706] Area $APB=\frac{1}{2}\times2.24^2\sin\frac{5\pi}{6}$ [=1.254]	M1	For either ΔAOB or ΔAPB (AB = 4.33, h= 1.25, 0.58) or any other valid method.
	[Difference =] 1.45	A1	AWRT Condone 1.46 only.
		2	
(c)	Area $AOB=\frac{1}{2}\times2.5^2\times\frac{4\pi}{3}$ [=13.09] Area $APB=\frac{1}{2}\times2.24^2\times\frac{5\pi}{6}$ [=6.57]	B1	For either sector area correct
	[Area of cross section =] $\frac{1}{2}\times2.5^2\times\frac{4\pi}{3}+\frac{1}{2}\times2.24^2\times\frac{5\pi}{6}+$ "$their\,10(b)$" $[=13.09+6.57+$ "$their\,10(b)$"$]$	M1	Adding two **sector areas** from different sectors and '$their$ 10**(b)**' and nothing else. SOI
	21.1	A1	CAO Condone slight inaccuracies in intermediate working if the correct answer is arrived at.
		3	

Question 67

(a)	$[2r+8=20\Rightarrow]\ r=6$	B1	
	Angle $AOB=\frac{8}{their\,6}$	*M1	Expect $\frac{4}{3}$ OE $(76.4°)$. M0 Assume triangle is equilateral.
	$AB=2\times6\sin their\frac{2}{3}$ **or** $\sqrt{6^2+6^2-2\times6^2\cos their\frac{4}{3}}$ **or** $AB=\frac{6}{\sin\left(\frac{\pi}{2}-their\frac{2}{3}\right)}\times\sin their\frac{4}{3}$	DM1	For 6 read $their\,6$.
	Perimeter $=[7.42+8=]\ 15.4$	A1	AWRT
		4	
(b)	Area $=\frac{1}{2}\times6^2\times their\frac{4}{3}-\frac{1}{2}\times6^2\times\sin their\frac{4}{3}$ **or** Area $=\frac{1}{2}\times6^2\times their\frac{4}{3}-2\times\frac{1}{2}\left(6\sin their\frac{2}{3}\right)\left(6\cos their\frac{2}{3}\right)$	M1	Sector area **–** whole triangle area. For 6 read $their\,6$. Sector area **–** 2(half triangle area).
	$=[24-17.49=]\ 6.51$	A1	AWRT
		2	

Question 67

(a)	$\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ **used** to find ADC	M1	May use cosine rule or $CAD = \tan^{-1}\frac{4}{8}$.
	$BDC = 0.927[3] \rightarrow ADC = \pi - 0.927[3]$ $[= 2.214 \text{ to } 2.215]$	A1	Allow degrees, 126.87, and 0.7048π or 0.705π.
	$Arc\ AC = 5 \times their\ 2.214$	M1	Use of $r\theta$ or $\frac{\theta}{360}.2\pi r$ Expect 11.07 .
	$AC = \sqrt{8^2 + 4^2}$ *or* $2 \times 5 \times \sin 1.107$	M1	Expect 8.94 .
	[Perimeter = 11.07 + 8.94 =] 20.0	A1	Accept AWRT [20.01, 20.02].
		5	
(b)	Sector $ACD = \frac{1}{2} \times 5^2 \times their\ 2.214$	M1	See use of $\frac{1}{2}r^2\theta$ or $\frac{\theta}{360}.\pi r^2$. Expect 27.7 .
	Subtracting the area of $\Delta ADC = \frac{1}{2} \times 5 \times 4$ or $\frac{1}{2}5^2 \sin their\ 2.214$ or $\frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 3 \times 4$	M1	Subtracting the area of ΔADC, expect -10.
	Shaded area = 27.7 − 10 = 17.7	A1	Accept AWRT [17.67, 17.68]. Correct answer cannot come from an angle of 2.215 .
		3	

Question 68

(a)	$\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate θ *or* r using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	$\Delta OAB = \frac{1}{2} \times their\ 16^2 \times \sin their\ 0.6$	M1	Allow Segment = 76.8 − ½ × *their* 16^2 × sin *their* 0.6. Expect 72.27 .
	[Area = 76.8 − 72.27 =] 4.53	A1	AWRT
		5	
(b)	$AB = 2 \times 16 \times \sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 0.6$	M1	Any valid method with *their* r, θ. Expect $AB = 9.46$.
	Perimeter = 9.6 + 9.46 = 19.1	A1	AWRT
		2	

Question 69

(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi-2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi - 2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	***B1**	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow p, a or r for OA) containing only terms with x and θ but not just $OA = 2x\cos\theta$. Do not condone $\sin\pi - 2\theta$ until missing brackets recovered or $\cos(180 - 2\theta)$ until it becomes $-\cos 2\theta$ etc.
	$OA = 2x\cos\theta$ leading to Arc length $= 2x\theta\cos\theta$	**DB1**	AG Complete correct method showing all necessary working. Condone $2x\cos\theta \times \theta$.
		2	If B0 but www then **SCB1** for $OA = 2x\cos\theta$ leading to Arc length $= 2x\theta\cos\theta$.
(b)	Sector area $= \frac{1}{2}(2x\cos\theta)^2 \times \theta$	**M1**	OE Using sector formula with a correct OA. Condone $\cos\theta^2$ for $\cos^2\theta$ and missing brackets.
	Triangle area $= \frac{1}{2} \times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi - 2\theta)$	**M1**	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π.
	[Area *APB* =] *Their* sector area – *their* triangle area	**M1**	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area *APB* =] $\frac{1}{2}(2x\cos\theta)^2 \times \theta - \frac{1}{2}x^2\sin(\pi - 2\theta)$ [$= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)$]	**A1**	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect 'simplifications'.
		4	

Question 70

$\frac{1}{2} \times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	**B1**	SOI OE e.g. $\frac{2\pi}{12}$, 0.524(3 s.f.) Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
Arc length $= 8 \times their\frac{\pi}{6}$ [= 4.1887…]	**M1**	OE FT *their θ*. Look for $\frac{4\pi}{3}$.
[BC =] $2 \times 8\sin\left(\frac{1}{2} \times their\frac{\pi}{6}\right)$ [= 4.1411…]	**M1**	Attempt to find BC or BC^2 (see alt. methods below) FT *their θ*. Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6} - 4\sqrt{2}$.
Perimeter = 8.33	**A1**	AWRT Must be combined into one term.