AS-Level

Topic : Circular measure

May 2013-May 2025

Answer

Question 1

(i)	$\frac{1}{2}.3^2\pi = \frac{1}{2}9^2\theta - \frac{1}{2}3^2\theta$	M1 A1		M1 needs $\frac{1}{2}r^2\theta$ once. A1 all correct.
	$\rightarrow \theta = \frac{1}{4}\pi$	A1	[3]	Answer given
(ii)	$P = 6+6+3\times\frac{1}{4}\pi + 9\times\frac{1}{4}\pi = 21.4$ cm.	M1		M1 is for use of $s=r\theta$ once.
	or $12 + 3\pi$	A1	[2]	

Question 2

(i) $BOC = 2\tan^{-1}\frac{1}{2} = 0.9273$	M1 A1 [2]	Correct trigonometry. (ans given)
(ii) $OB = \sqrt{(10^2 + 5^2)}$ or $11.2 = r$	B1	Use of trig (or Pyth) for the $OB = \sqrt{125}$.
Arc $BXC = \sqrt{125} \times 0.9273$	M1	Use of $s = r\theta$ with θ in rads , $r \neq 10$
$\rightarrow$ Perimeter = 20.4 cm	A1 [3]	
(iii) Area = $\frac{1}{2}r^2\theta$	M1	Correct formula used with rads, $r \neq 10$.
$- \frac{1}{2}.10.10 \rightarrow 7.96$ cm².	A1 [2]	Allow 7.95 or 7.96

Question 3

(i)	$(OAB) = \frac{1}{2}\times 8^2\alpha$, $(OAC) = \frac{1}{2}\times\pi\times 4^2$	**B1B1**		Accept 25.1 (for OAC)
	$\alpha = \frac{\pi}{8}$	**B1**	**[3]**	
(ii)	$8 + 8 \times$ their $\alpha + \frac{1}{2}\times 8\times\pi$	**B1** √		23.7 gets B1B0
	$8 + 5\pi$	**B1**	**[2]**	SC B1 for e.g. 5π (omitted OB)

Question 4

(i) sector areas are $\frac{1}{2}11^2\alpha, \frac{1}{2}5^2\alpha$	B1	Sight of 11^2, 5^2
$k = \dfrac{\frac{1}{2}\times 11^2\alpha - \frac{1}{2}\times 5^2\alpha}{\frac{1}{2}\times 5^2\alpha}$	M1	Or $\dfrac{11^2 - 5^2}{5^2}$
$k = \dfrac{96}{25}$ or 3.84	A1 [3]	

(ii) perimeter shaded region= $11\alpha + 5\alpha + 6 + 6 = 16\alpha + 12$	B1	
perimeter unshaded region = $5\alpha + 5 + 5 = 5\alpha + 10$	B1	
$16\alpha + 12 = 2(5\alpha + 10)$	M1	
$\alpha = 4/3$ or 1.33	A1 [4]	

Question 5

(i) slant length = 10 cm.	B1	
circumference of base = 12π	B1	
arc length = 10θ ($= 12\pi$)	B1√	Use of $r\theta$, θ calculated, not 6 or 8.
$\to \theta = 1.2\pi$ or 3.77 radians.	B1 [4]	
(ii) $\frac{1}{2}r^2\theta = 188.5$ cm ^{2} or 60π.	M1 A1√ [2]	Use of $\frac{1}{2}r^2\theta$ with radians and r = calculated '10', not 6 or 8.

Question 6

(i) $r(2\pi - \alpha)$ $\quad + 2r\alpha + 2r$	**B1B1**	
$2\pi r + r\alpha + 2r$	**B1√** **[3]**	ft for $r\alpha$ instead of $2r\alpha$ or omission $2r$. SC1 for $2r\alpha + 4r$. (Plate = shaded part)
(ii) $\frac{1}{2}(2r)^2\alpha + \pi r^2 - \frac{1}{2}r^2\alpha$	**B1B1**	Either B1 can be scored in (iii)
$\dfrac{3r^2\alpha}{2} + \pi r^2$	**B1** **[3]**	
(iii) $\pi r^2 - \frac{1}{2}r^2\alpha = 2r^2\alpha$	**M1**	For equating *their* 2 parts from (ii)
$\alpha = \frac{2}{5}\pi$	**A1** **[2]**	

Question 7

(i)	$s = r\theta$	M1		Used with major or minor arc
	Angle of major arc = $2\pi - 2.2 = (4.083)$	B1		Could be gained in **(ii)**.
	Perimeter = 12 + 24.5 = 36.5 or $12\pi - 1.2$	A1		co
	(or full circle − minor arc B1)			
			[3]	
(ii)	Area of major sector = $\frac{1}{2}r^2\theta$ = (73.49)	M1		Used with major/minor sector.
	Area of triangle = $\frac{1}{2}.6^2\sin 2.2 = (14.55)$	M1		Correct formula or method.
				$(2\pi - 2.2)/\sin 2.2$ gets M1M1
	Ratio = 5.05 : 1 (Allow 5.03 → 5.06)	A1		co
			[3]	

Question 8

(i)				
	$\frac{1}{2}r^2\theta = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$	B1		Correct equation.
	→ $2\sin\theta = \theta$ → p = 2.	B1		All ok – answer given.
			[2]	
(ii)	Chord length = 8sin1.2 × 2 (14.9)	M1		Needs ×2. Any method ok.
	(or from cosine rule)			
	Arc length = 2.4 × 8 (19.2)	B1		
	Perimeter = sum of these = 34.1	A1		co
			[3]	

Question 9

(i)	area Δ $= \frac{1}{2} \times 4 \times 4\tan\alpha$ oe soi	**B1**		$4\tan\alpha = \sqrt{16/\cos^2\alpha - 16}$. (Can also score in
	Area sector $= \frac{1}{2} \times 2^2\alpha$ oe soi	**B1**		answer) Accept θ throughout
	Shaded area $= 8\tan\alpha - 2\alpha$ cao	**B1**		Little/no working – accept terms in answer
			[3]	
(ii)	$DC = \frac{4}{\cos\alpha} - 2$ oe soi	**B1**		
	Arc $DE = 2\alpha$ soi anywhere provided clear	**B1**		$\frac{4}{\cos\alpha} = \sqrt{16 + 16\tan^2\alpha}$. Can score in answer
	Perimeter $= \frac{4}{\cos\alpha} + 4\tan\alpha + 2\alpha$ cao	**B1**		Little/no working – accept terms in answer
			[3]	

Question 10

(i)	CB or $AB = \dfrac{3}{\tan\frac{\pi}{6}}$ or $3\tan\frac{\pi}{3}$	**B1**	Allow throughout for e.g. $3\sqrt{3}$,
		B1	$\sqrt{27}, \sqrt{3^3}, \left(\sqrt{3}\right)^3, \dfrac{9}{\sqrt{3}}$
	Arc or $AC = 3\times\left[\dfrac{2\pi}{3} \text{ or } \dfrac{\pi}{3}\right]$ $(= 2\pi \text{ or } \pi)$	**B1** **[3]**	After B0B0 SCB1 for 16.7
	Perimeter $= 6\sqrt{3} + 2\pi$ oe	**B1√**	*Their AB* in form $k\sqrt{3}$
(ii)	Area $OABC$ $(2)\times\dfrac{1}{2}\times 3\times$ *their AB* $(=9\sqrt{3}$ or $\dfrac{9\sqrt{3}}{2})$	**B1**	
	Area $OADC$ $\dfrac{1}{2}\times 3^2\times\left(\dfrac{2\pi}{2} \text{ or } \dfrac{\pi}{3}\right)$ $\left(= 3\pi \text{ or } \dfrac{3\pi}{2}\right)$	**B1**	After B0B0 SCB1 for 6.16 or 6.17. Allow $\left(\sqrt{3}\right)^5 - 3\pi$
	Shaded area $9\sqrt{3} - 3\pi$ oe	**[3]**	

Question 11

(i)	$\tan\theta = \dfrac{5}{12}$ $\rightarrow (\theta = 0.3948)$	M1 [1]	Any valid trig method ag
(ii)	Other angle in triangle $= \frac{1}{2}\pi - 0.3948$	B1	Unsimplified OK
	Area of triangle $AOB = \frac{1}{2}\times12\times5\ (= 30)$	B1	co
	Use of $\frac{1}{2}r^2\theta$ once	M1	With θ in radians and $r = 5$ or 12
	Shaded area = sector + sector – triangle $= \frac{1}{2}\times12^2\times0.3948 + \frac{1}{2}5^2\theta - 30$	DM1	Sum of 2 sectors – triangle or any other valid method using the given angle and a different one.
	$= 28.43 + 14.70 - 30 = 13.1$	A1 [5]	co

Question 12

(i)	Arc $AB = 4\alpha$	**B1**	
	Arc $DC = (4\cos\alpha)\alpha$	**B1**	
	AC (or DB) $= 4 - 4\cos\alpha$	**B1**	
	Perimeter $= 4\alpha\cos\alpha + 4\alpha + 8 - 8\cos\alpha$	**B1** **[4]**	
(ii)	$OD = 4\cos\frac{\pi}{6}\left(= 2\sqrt{3}\right)$	**B1**	
	Shaded area $= \left[\ \frac{1}{2}\times4^2\times\frac{\pi}{6}\ \right]\ \left[\ -\frac{1}{2}\left(2\sqrt{3}\right)^2\times\frac{\pi}{6}\ \right]$	**B1B1**	
	$\dfrac{\pi}{3}$	**B1** **[4]**	Or $k = \frac{1}{3}$

Question 13

(i)	$OC = r\cos\alpha$ or $AC = r\sin\alpha$ or oe soi (Area $\Delta OAC =$) $\frac{1}{2}r^2 \sin\alpha \cos\alpha$ $\frac{1}{2}r^2 \sin\alpha\cos\alpha = \frac{1}{2}\times\frac{1}{2}r^2\alpha$ oe $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$	**M1** **A1** **M1** **A1** **[4]**	 Or e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{4}r^2\alpha$ $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{2}r^2\cos\alpha\sin\alpha$ AG
(ii)	Perimeter $\Delta OAC = r + r\sin\alpha + r\cos\alpha = 2.4(0)r$ Perim. $ACB = r\alpha + r\sin\alpha + r - r\cos\alpha = 2.18r$ or $2.17r$ Ratio $= \dfrac{2.4(0)}{2.18 \text{ or } 2.17} : 1 = 1.1 : 1$	**M1A1** **M1A1** **A1** **[5]**	Allow with r a number. 2.0164 gets M1A0 Allow with r a number. 0.9644 gets M1A0 Allow 2.2 www. Use of cos = 0.6, sin = 0.8, $\alpha = 0.9$ is PA 1
(iii)	54.3° cao	**B1** **[1]**	

Question 14

Radius of semicircle = $\frac{1}{2}AB = r\sin\theta$ Area of semicircle = $\frac{1}{2}\pi r^2\sin^2\theta = A_1$ Shaded area = semicircle – segment $= A_1 - \frac{1}{2}r^2 2\theta + \frac{1}{2}r^2\sin 2\theta$	B1 B1√ B1B1 [4]	aef Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$ B1 (–sector), B1 for + (triangle)

Question 15

(i)	Sector $OCD = \frac{1}{2}(2r)^2\theta \quad (= 2r^2\theta)$ Sector(s) $OAB/OEF = (2)\frac{1}{2}r^2(\pi - \theta)$ Total $= r^2(\pi + \theta)$	**B1** **B1** **B1** [3]	$2r^2\theta$ seen somewhere Accept with/without factor (2) **AG www**
(ii)	Arc $CD = 2r\theta$ Arc(s) AB/EF $(2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	**B1** **B1** **B1** **B1** [4]	Accept with/without factor (2) Must be simplified

Question16

(i)	Length of $OB = \dfrac{6}{\cos 0.6} = 7.270$	**M1** [1]	ag Any valid method
(ii)	$AB = 6\tan 0.6$ or 4.1 Arc length = 7.27 × (½π – 0.6) = (7.06) Perimeter = 6 + 7.27 + 7.06 + 6tan0.6 = 24.4	**B1** **M1** **A1** [3]	Sight of in **(ii)** Use of $s = r\theta$ with sector angle
(iii)	Area of AOB = ½ × 6 × 7.27 × sin0.6 Area of OBC = ½ × 7.27² × (½π – 0.6) → area = 12.31 + 25.65 = 38.0	**M1** **M1** **A1** [3]	Use of any correct area method Use of $\frac{1}{2}r^2\theta$.

Question 17

(i)	$BC^2 = r^2 + r^2 = 2r^2 \to BC = r\sqrt{2}$	**B1** [1]	**AG**
(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi	**M1**	Expect $\frac{1}{2}\pi r^2$
	Area $\Delta BCAD = \frac{1}{2}(2r)r$	**M1**	Expect r^2 (could be embedded)
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$ .oe	**A1**	
	Area semi-circle $CADE = \frac{1}{2}\pi r^2$	**B1**	
	Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$		
	or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$	**DM1**	Depends on the area ΔBCD
	$= r^2$	**A1** [6]	

Question 18

(a) (i)	$BAO = OBA = \frac{\pi}{2} - \alpha$ $AOB = \pi - \left(\frac{\pi}{2} - \alpha\right) - \left(\frac{\pi}{2} - \alpha\right) = 2\alpha$ **AG**	**M1A1** **[2]**	Allow use of 90º or 180º Or other valid reasoning
(ii)	$\frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2 \sin 2\alpha$ oe	**B2,1,0** **[2]**	SCB1 for reversed subtraction
(b)	Use of $\alpha = \frac{\pi}{6}$, $r = 4$ 1 segment $S = \left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ Area ABC $T = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $\left(= 4\sqrt{3}\right)$ $T - 3S = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3} - 3$ $\left[\left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}\right]$ $16\sqrt{3} - 8\pi$ cao	**B1B1** **M1** **B1** **M1** **A1** **[6]**	 Ft *their* **(ii)**, α, r OR $AXB = \frac{T}{3} = 4\tan\frac{\pi}{6}$ or $\frac{1}{2}\left(\frac{4}{\sqrt{3}}\right)^2 \sin\frac{2\pi}{3}$ $\left(= \frac{4\sqrt{3}}{3}\right)$ OR $3\left[\frac{T}{3} - S\right] = 3\left[\frac{4\sqrt{3}}{3} - \left(\frac{8\pi}{3} - 4\sqrt{3}\right)\right]$

Question 19

$BAC = \sin^{-1}(3/5)$ or $\cos^{-1}(4/5)$ or $\tan^{-1}(3/4)$ $ABC = \sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$ or $\tan^{-1}(4/3)$ $ACB = \pi/2$ (Allow 90º) Shaded area = ΔABC – sectors ($AEF + BEG + CFG$) $\Delta ABC = \frac{1}{2} \times 4 \times 3$ oe Sum sectors $= \frac{1}{2}\left[3^2 0.6435\right)$ + $2^2 0.9273 + 1^2 1.5708]$ **OR** $\frac{\pi}{360}\left[3^2 36.8(7) + 2^2 53.1(3) + 1^2 90\right]$ 6 – 5.536 = 0.464	**B1** **B1** **B1** **M1** **B1** **M1** **A1** [7]	Accept 36.8(7)º Accept 53.1(3)º

Question 20

(i)	$PT = r\tan\alpha$	B1	
	$QT = OT - OQ = \dfrac{r}{\cos\alpha} - r$ or $\sqrt{r^2 + r^2\tan^2\alpha} - r$	B1	
	Perimeter = sum of the 3 parts including $r\alpha$	B1 [3]	
(ii)	Area of triangle = $\frac{1}{2}\times 10\times 10\tan\dfrac{\pi}{3}$	M1	Correct formula used, $50\sqrt{3}, 86.6$
	Area of sector = $\frac{1}{2}\times 10^2 \times \frac{1}{3}\pi$	M1	Correct formula used, $\dfrac{50\pi}{3}$, 52.36
	Shaded region has area 34 (2sf)	A1 [3]	

Question 21

(i)	$CD = r\cos\theta,\ BD = r - r\sin\theta$ oe	B1 B1	allow degrees but not for last B1
	Arc $CB = r\,(\frac{1}{2}\pi - \theta)$ oe	B1	
	$\rightarrow P = r\cos\theta + r - r\sin\theta + r\,(\frac{1}{2}\pi - \theta)$ oe	B1√ [4]	√ sum – assuming trig used
(ii)	Sector $= \frac{1}{2}.5^2.(\frac{1}{2}\pi - 0.6)$ (12.135)	M1	Uses $\frac{1}{2}r^2\theta$
	Triangle = $\frac{1}{2}$.5cos0.6.5sin0.6 (5.825)	M1	Uses $\frac{1}{2}bh$ with some use of trig.
	→ Area = 6.31	A1	
	(or $\frac{1}{4}$ circle – triangle – sector)	[3]	

Question 22

(i)	$\tan\left(\dfrac{\pi}{3}\right) = \dfrac{AC}{2x}$ or $\cos\left(\dfrac{\pi}{3}\right)\left(= \sin\dfrac{\pi}{6}\right) = \dfrac{2x}{AB}$ $\rightarrow AC = 2\sqrt{3}x$ or $AB = 4x$	B1	Either trig ratio
	$AM = \sqrt{13x^2}, \sqrt{13}x, 3.61x$	M1A1 [3]	Complete method.
(ii)	$\tan(M\hat{A}C) = \dfrac{x}{\text{Their } AC}$	M1	"Their AC" must be f(x), $\left(M\hat{A}C\right) \neq \theta$.
	$\theta = \dfrac{1}{6}\pi - \tan^{-1}\dfrac{1}{2\sqrt{3}}$ **AG**	A1 [2]	Justifies $\dfrac{\pi}{6}$ and links MAC & θ

Question 23

(i)	$\cos 0.9 = OE/6$ or $= \sin\left(\frac{\pi}{2} - 0.9\right)$ oe	**M1**		Other methods possible
	$OE = 6\cos 0.9 = 3.73$ oe AG	**A1**	[2]	
(ii)	Use of $(2\pi - 1.8)$ or equivalent method	**M1**		Expect 4.48
	Area of large sector $= \frac{1}{2} \times 6^2 \times (2\pi - 1.8)$ oe	**M1**		Or $\pi 6^2 - \frac{1}{2}6^2 1.8$. Expect 80.70
	Area of small sector $\frac{1}{2} \times 3.73^2 \times 1.8$	**M1**		Expect 12.52 Other methods possible
	Total area = 80.7(0) + 12.5(2) = 93.2	**A1**	[4]	

Question 24

(i)	$\frac{r}{10} = \sin 0.6$ or $\frac{r}{10} = \cos 0.97$ or $BD = \sqrt{200 - 200\cos 1.2}\,(= 11.3)$	**M1**		Or other valid alternative.
	$r = 10 \times 0.5646$, $r = 10 \times \sin 0.6$, $r = 10 \times \cos 0.971$ or $r = \frac{1}{2} BD$ $\rightarrow r = 5.646$ AG	**A1**	[2]	
(ii)	Major arc $= 10(\theta)$ $(= 50.832)$	**M1**		$\theta = 2\pi - 1.2$ or $\pi - 1.2$
	$\theta = 2\pi - 1.2$ $(= 5.083)$ or $C = 2\pi \times 10$, Minor arc $= 1.2 \times 10$ Semicircle $= 5.646\pi$ $(= 17.737)$ Major arc + semicircle	**B1**		Implied by 5.1
	$= 68.6$	**A1**	[3]	
(iii)	Area of major sector $= \frac{1}{2}10^2(\theta)$ $(= 254.159)$	**M1**		$\theta = 2\pi - 1.2$ or $\pi - 1.2$
	Area of triangle OBD $= \frac{1}{2}10^2 \sin 1.2$ $(= 46.602)$	**M1**		Use of $\frac{1}{2}ab\sin C$ or other complete method
	Area = semicircle + sector + triangle $(= 50.1 + 254.2 + 46.6)$ $= 351$	**A1**	[3]	

Question 25

(i)	$2r\alpha + r\alpha + 2r = 4.4r$	**M1**		At least 3 of the 4 terms required
	$\alpha = 0.8$	**A1**	[2]	
(ii)	$\frac{1}{2}(2r)^2 0.8 - \frac{1}{2}(r^2)0.8 = 30$	**M1A1**√		Ft through on *their* α
	$(3/2)r^2 \times 0.8 = 30 \rightarrow r = 5$	**A1**	[3]	

Question 26

(i)	$ABC = \pi/2 - \pi/7 = 5\pi/14$. $CBD = \pi - 5\pi/14 = 9\pi/14$	B1	AG Or other valid exact method.
	Total:	1	
(ii)	$\sin\frac{\pi}{7} = \frac{½BC}{8}$ or $\frac{BC}{\sin\frac{2\pi}{7}} = \frac{8}{\sin\frac{5\pi}{14}}$ or $BC^2 = 8^2 + 8^2 - 2(8)(8)\cos\frac{2\pi}{7}$	M1	
	$BC = 6.94(2)$	A1	
	arc CD = *their* $6.94 \times 9\pi/14$	M1	Expect 14.02(0)
	arc $CB = 8 \times 2\pi/7$	M1	Expect 7.18(1)
	perimeter = 6.94 + 14.02 + 7.18 = 28.1	A1	
	Total:	5	

Question 27

(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or cos = 6/10 or tan = 8/6. Accept 0.295π.
	Total:	1	
(ii)	$AB = 6$(Pythagoras) $\rightarrow$ $\Delta BCD = 8 \times 6 = 48.0$	M1A1	**OR** $8\times10\sin0.6435$ or $½\times10\times10\sin((2)\times0.927)=48$. 24or 40or80 gets **M1A0**
	Area sector $BCD = ½ \times 10^2 \times (2) \times$ *their* 0.9273	*M1	Expect 92.7(3). 46.4 gets **M1**
	Area segment = 92.7(3) – 48	*A1	Expect 44.7(3). Might not appear until final calculation.
	Area semi-circle – segment = $½ \times \pi \times 8^2 -$ *their* $(92.7 - 48)$	DM1	Dep. on previous **M1A1** OR $\pi \times 8^2 - (½ \times \pi \times 8^2 +$ *their* $44.7)$.
	Shaded area = 55.8 – 56.0	A1	
	Total:	6	

Question 28

(i)	$(AB) = 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	Allow unsimplifed throughout eg r + r, $\frac{2\theta}{2}$ etc
	(Arc AB) = $2r\theta$	B1	
	$(P =)\ 2r + 2r\theta + 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	
	Total:	3	
(ii)	Area sector AOB = ($½ r^2 2\theta$) $\frac{25\pi}{6}$ or13.1	B1	Use of segment formula gives 2.26 **B1B1**
	Area triangle AOB = ($½\times2r\sin\theta\times r\cos\theta$ or $½ \times r^2 \sin2\theta$) $\frac{25\sqrt{3}}{4}$ or10.8	B1	
	Area rectangle $ABCD$ = ($r \times 2r\sin\theta$) 25	B1	
	(Area =) Either 25 – ($25\pi/6 - 25\sqrt{3}/4$) or 22.7	B1	Correct final answer gets **B4**.
	Total:	4	

Question 29

(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) $\rightarrow XM = 2$	**B1**	(could find $\sqrt{40}$ and use $\sin^{-1}$ or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	**M1 A1**	AG Needs × 2 and correct trig for **M1**
	(Alternative 1: $\sin AOM = \frac{6}{10}, AOM = 0.6435, AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, **B1** for AOM, **M1**,**A1** for completion) (Alternative 2: Use of circle theorem, **B1** for AOB, **M1**,**A1**for completion)
	Total:	**3**	
(ii)	$AX = \sqrt{(6^2 + 2^2)} = \sqrt{40}$	**B1**	CAO, could be gained in part (i) or part (iii)
	Arc $AYB = r\theta = \sqrt{40} \times 2.498$	**M1**	Allow for incorrect $\sqrt{40}$ (not $r = 6 or 12 or 10$)
	Perimeter = 12 + arc = 27.8 cm	**A1**	
	Total:	**3**	
(iii)	area of sector $AXBY = ½ \times (\sqrt{40})^2 \times 2.498$	**M1**	Use of $½r^2\theta$ with their r, (not $r = 6 or r = 10$)
	Area of triangle $AXB = ½ \times 12 \times 2$, Subtract these → 38.0 cm²	**M1 A1**	Use of $½bh$ and subtraction. Could gain **M1** with $r = 10$.
	Total:	**3**	

Question 30

(i)	$\sin^{-1}\left(\frac{3}{5}\right) = 0.6435$ AG	**M1**	OR $(PBC =)\cos^{-1}\left(\frac{3}{5}\right) = 0.9273 \Rightarrow (ABP =)\frac{\pi}{2} - 0.9273 = 0.6435$ Or other valid method. Check working and diagram for evidence of incorrect method
(ii)	Use (once) of sector area $= ½r^2\theta$	**M1**	
	Area sector $BAP = ½ \times 5^2 \times 0.6435 = 8.04$	**A1**	
	Area sector $DAQ = ½ \times ½\pi \times 3^2 = 7.07$, Allow $\frac{9\pi}{4}$	**A1**	
		3	
(iii)	*EITHER:* Region = sect + sect – (rect – Δ) or sect – [rect – (sect + Δ)]	**(M1**	Use of correct strategy
	(Area Δ BPC =) ½ × 3 × 4 = 6 Seen	**A1**	
	8.04 + 7.07 – (15 – 6) = 6.11	**A1)**	
	OR1: Region = sector ADQ – (trap $ABPD$ – sector ABP).	**(M1**	Use of correct strategy
	(Area trap $ABPD$ =) ½ (5 + 1) × 3 = 9 Seen	**A1**	
	7.07 – (9 – 8.04) = 7.07 – 0.96 = 6.11	**A1)**	
	OR2: Area segment AP= 2.5686 Area segment AQ = 0.5438 Region = segment AP + segment AQ + ΔAPQ.	**(M1**	Use of correct strategy
	(Area ΔAPQ =) ½ × 2 × 3 = 3 Seen	**A1**	
	2.57 + 0.54 + 3 = 6.11	**A1)**	
		3	

Question 31

(i)	Pythagoras $\rightarrow r = \sqrt{72}$ OE or $\cos 45 = \frac{6}{r} \rightarrow r = \frac{6}{\cos 45} = 6\sqrt{2}$	**M1**	Correct method leading to $r =$
	Arc $DC = \sqrt{72} \times \frac{1}{4}\pi = \frac{3\sqrt{2}}{2}\pi$, 2.12π, 6.66	**M1 A1**	Use of $s=r\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
		3	
(ii)	Area of sector-BDC is $\frac{1}{2} \times 72 \times \frac{1}{4}\pi$ ($= 9\pi$ or 28.274…)	***M1**	Use of $\frac{1}{2}r^2\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
	Area $Q = 9\pi - 18$ (10.274…)	**DM1**	Subtracts their $\frac{1}{2} \times 6 \times 6$ from their $\frac{1}{2}r^2\theta$
	Area P is ($\frac{1}{4}\pi 6^2$ – area Q) = 18	**M1**	Uses {$\frac{1}{4}\pi 6^2$ – (their area Q using $\sqrt{72}$)}
	Ratio is $\frac{18}{9\pi - 18}\left(\frac{18}{10.274}\right) \rightarrow 1.75$	**A1**	
		4	

Question 32

(i)	$\cos A = 8/10 \rightarrow A = 0.6435$	**B1**	AG Allow other valid methods e.g. $\sin A = 6/10$
		1	
(ii)	*EITHER:* Area $\Delta ABC = \frac{1}{2} \times 16 \times 6$ or $\frac{1}{2} \times 10 \times 16 \sin 0.6435 = 48$	**(M1A1**	
	Area 1 sector $\frac{1}{2} \times 10^2 \times 0.6435$	**M1**	
	Shaded area $= 2 \times$ *their* sector $-$ *their* ΔABC	**M1)**	
	OR: $\Delta BDE = 12$, $\Delta BDC = 30$	**(B1 B1**	
	Sector = 32.18	**M1**	
	2×segment + ΔBDE	**M1)**	
	=16.4	**A1**	
		5	

Question 33

5(i)	$\frac{PQ}{2} = 10 \times \sin 1.1$	**M1**	Correct use of sin/cos rule
	($PQ =$) 17.8 (17.82…implies **M1, A1**) AG	**A1**	OR $PQ = \frac{10 \sin 2.2}{\sin\left(\frac{\pi}{2} - 1.1\right)}$ *or* $\frac{10 \sin 2.2}{\sin 0.4708}$ *or* $\sqrt{200 - 200\cos 2.2} = 17.8$
		2	
5(ii)	Angle $OPQ = (\pi/2 - 1.1)$ [accept 27°]	**B1**	OE Expect 0.4708 or 0.471. Can be scored in part **(i)**
	Arc QR =17.8 × *their* ($\pi/2$ – 1.1)	**M1**	Expect 8.39. (or 8.38).
	Perimeter $= 17.8 - 10 + 10 +$ *their* arc QR	**M1**	
	26.2	**A1**	For both parts allow correct methods in degrees
		4	

Question 34

Angle $AOC = \frac{6}{5}$ or 1.2		M1	Allow 68.8°. Allow $\frac{5}{6}$
AB = 5 × tan(*their* 1.2) OR by e.g. Sine Rule	Expect 12.86	DM1	OR $OB = \frac{5}{\cos their 1.2}$. Expect 13.80
Area $\Delta OAB = \frac{1}{2} \times 5 \times their\ 12.86$	Expect 32.15	DM1	OR $\frac{1}{2} \times 5 \times their\ OB \times \sin their\ 1.2$
Area sector $\frac{1}{2} \times 5^2 \times their\ 1.2$	Expect 15	DM1	All DM marks are dependent on the first M1
Shaded region = 32.15 – 15 = 17.2		A1	Allow degrees used appropriately throughout. 17.25 scores A0
		5	

Question 35

(i)	AT or $BT = r\tan\theta$ or $OT = \frac{r}{\cos\theta}$	B1	May be seen on diagram.
	$\frac{1}{2}r^2 2\theta$, & $\frac{1}{2} \times r \times (r\tan\theta$ or $AT)$ or $\frac{1}{2} \times r \times (\frac{r}{\cos\theta}$ or $OT) \sin\theta$	M1	Both formulae, ($\frac{1}{2}r^2\theta$, $\frac{1}{2}bh$ or $\frac{1}{2}ab\sin\theta$), seen with 2θ used when needed.
	$\frac{1}{2}r^2 2\theta = 2 \times \frac{1}{2} \times r \times r\tan\theta - \frac{1}{2}r^2 2\theta$ oe $\rightarrow 2\theta = \tan\theta$ **AG**	A1	Fully correct working from a correct statement. Note: $\frac{1}{2}r^2 2\theta = \frac{1}{2} r^2\tan\theta$ is a valid statement.
		3	
(ii)	$\theta = 1.2$ or sector area = 76.8	B1	
	Area of kite = 165 awrt	B1	
	164.6 – 76.8 = 87.8 awrt	B1	awrt 87.8 with little or no working can be awarded 3/3. SC Final answers that round to 88 with little or no working can be awarded 2/3.
		3	

Question 36

(i)	$(\tan\theta = \frac{AT}{r}) \rightarrow AT = r\tan\theta$ or $OT = \frac{r}{\cos\theta}$ SOI	B1	CAO
	$\rightarrow A = \frac{1}{2}r^2\tan\theta \quad - \frac{1}{2}r^2\theta$	B1 B1	B1 for $\frac{1}{2}r^2\tan\theta$. B1 for "$-\frac{1}{2}r^2\theta$" If Pythagoras used may see area of triangle as $\frac{1}{2}r\sqrt{r^2 + r^2\tan^2\theta}$ or $\frac{1}{2}r\left(\frac{r}{\cos\theta}\right)\sin\theta$
		3	
(ii)	$\tan\theta = \frac{AT}{3} \rightarrow AT = 7.716$	M1	Correct use of trigonometry and radians in rt angle triangle
	Arc length $= r\theta = 3.6$	B1	Accept 3×1.2
	OT by Pythagoras or $\cos 1.2 = \frac{3}{OT}$ $(= 8.279)$	M1	Correct method for OT
	Perimeter $= AT +$ arc $+ OT -$ radius $= 16.6$	A1	CAO, www
		4	

Question 37

(i)	0.8 oe	**B1**	
		1	
(ii)	$BD = 5\sin their\ 0.8$	**M1**	Expect 3.58(7). Methods using degrees are acceptable
	$DC = 5 - 5\cos their\ 0.8$	**M1**	Expect 1.51(6)
	Sector = $\frac{1}{2} \times 5^2 \times their 0.8$ OR Seg = $\frac{1}{2} \times 5^2 \times [their\ 0.8 - \sin their\ 0.8]$	**M1**	Expect 10 for sector. Expect 1.03(3) for segment
	Trap = $\frac{1}{2}(5 + theirDC) \times theirBD$ oe OR $\Delta BDC = \frac{1}{2} theirBD \times theirCD$	**M1**	OR (for last 2 marks) if X is on AB and XC is parallel to BD:
	Shaded area = 11.69 – 10 OR 2.71(9) – 1.03(3) = 1.69 cao	**A1**	$BDCX$ –(sector – ΔAXC) = 5.43(8) – [10 – 6.24(9)] = 1.69 cao M1A1
		5	

Question 38

(i)	$A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(\frac{3}{4})$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ **or** $B\hat{A}C = \cos^{-1}(\frac{3}{4})$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$	**M1**	Correct method for $A\hat{B}C$, expect 1.696^cawrt Or for $B\hat{A}C$, expect 0.723^cawrt
	$C\hat{B}Y = \pi - A\hat{B}C$ or $2\times C\hat{A}B$	**M1**	For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2 \times C\hat{A}B$
	OR		
	Find CY from $\Delta\ ACY$ using Pythagoras or similar Δs	**M1**	Expect $4\sqrt{7}$
	$C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(their\ CY)^2}{2\times 8\times 8}\right)$	**M1**	Correct use of cosine rule
	$\mathbf{C\hat{B}Y = 1.445^c}$ AG	**A1**	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
(ii)	Arc $CY = 8 \times 1.445$	**B1**	Use of $s=8\theta$ for arc CY, Expect 11.56
	$B\hat{A}C = \frac{1}{2}(\pi - A\hat{B}C)$ or $\cos^{-1}(\frac{3}{4})$	***M1**	For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect 0.7227^c
	Arc $XC = 12 \times$ (their $B\hat{A}C$)	**DM1**	Expect 8.673
	Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www**	**A1**	Omission of '+4' only penalised here.
		4	

Question 39

Angle $OAB = \pi/2 - \pi/5 = 3\pi/10$ soi	**B1**	Allow 54° *or* 0.9425 *rads*
Sector $CAB = \frac{1}{2} \times \left(\textit{their}\,\frac{3\pi}{10}\right) \times 5^2$	**M1**	Expect 11.78
$OA = \dfrac{5}{\sin\frac{\pi}{5}} = 8.507$	**M1A1**	May be implied by $OC = 3.507$
Sector $COD = \frac{1}{2} \times (\textit{their}\,3.507)^2 \times \frac{\pi}{5}$	**M1**	Expect 3.86
$\Delta OAB = \frac{1}{2} \times 5 \times (\textit{their}\,8.507)\sin\frac{3\pi}{10}$	**M1**	Or $\frac{1}{2} \times 5 \times \dfrac{5}{\tan\frac{\pi}{5}}$ or $2.5 \times \sqrt{(\textit{their}\,8.507)^2 - 25}$
$= 17.20$ or 17.21	**A1**	
Shaded area $17.20(\textit{or}\,17.21) - 11.78 - 3.86 = 1.56$ or 1.57	**A1**	
	8	

Question 40

Angle $CBA = \sin^{-1}\left(\frac{7}{8}\right) = 1.0654$ or $CBD = \cos^{-1}\left(\frac{-17}{32}\right) = 2.13$	**B1**	Accept 61.0°, 66° or 122°
Sector $BCYD = \frac{1}{2} \times 8^2 \times 2 \times \textit{their}\,1.0654(\text{rad})$ soi or sector CBY $= \frac{1}{2} \times 8^2 \times \textit{their}\,1.0654(\text{rad})$	**M1**	Expect 68.1(9). Angle must be in radians (or *their* $61/360 \times 2 \times 8^2$) Or sector DBY
$\Delta BCD = 7 \times \sqrt{8^2 - 7^2}$ or $\frac{1}{2} \times 8^2 \times \sin(2 \times \textit{their}\,1.0654)$ soi	**M1**	Expect 27.1(1). Award M1 for ABC or ABD
Semi-circle $CXD = \frac{1}{2}\pi \times 7^2 = 76.9(7)$	**M1**	M1M1 for segment area formula used correctly
Total area = *their*68.19 – *their*27.11 + *their*76.97 = 118.0–118.1	**M1A1**	Cannot gain M1 without attempt to find angle CBA or CBD
	6	

Question 41

(i)	Angle EAD = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi **or** Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi	**B1**	
	$AD = 8\sin(\frac{3\pi}{10})$ **or** $8\cos(\frac{\pi}{5})$	**M1**	Angles used must be correct
	(AD =) 6.47	**A1**	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)}$ or $AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)}$ or 11.(01)	**B1**	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ **oe**	**M1**	
	(AD =) 6.47	**A1**	
		3	
(ii)	Area sector $= \frac{1}{2}(theirAD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	**M1**	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times theirAD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	**M1**	Or e.g. $\frac{1}{2}\, theirAD \times \sqrt{8^2 - theirAD^2}$. 15.2(2)
	(Shaded area =) 35.0 or 34.9	**A1**	
		3	

Question 42

Perimeter of $AOC = 2r + r\theta$	**B1**	
Angle $COB = \pi - \theta$	**B1**	Could be on the diagram. Condone $180 - \theta$.
Perimeter of $BOC = 2r + r(\pi - \theta)$	**B1**	**FT** on angle COB if of form $(k\pi - \theta)$, $k > 0$.
$(2r +)\ \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \rightarrow \theta = \frac{\pi - 2}{3}$)	**M1**	Sets up equation using $r(k\pi - \theta)$ and ×2 on correct side. Condone any omissions of OA, OB and/or OC.
$\theta = 0.38$	**A1**	Equivalent answer in degrees scores A0.
	5	

Question 43

Uses $A = \frac{1}{2}r^2\theta$	**M1**	Uses area formula.
$\theta = \frac{2A}{r^2}$	**A1**	
$P = r + r + r\theta$	**B1**	
$P = 2r + \frac{2A}{r}$	**A1**	Correct simplified expression for P.
	4	

Question 44

(i)	Angle $CAO = \frac{\pi}{3}$	B1	
		1	
(ii)	(Sector AOC) $= \frac{1}{2}r^2 \times their \frac{\pi}{3}$	M1	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(their\frac{\pi}{3}\right)$ or $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ or $\frac{1}{2}(r)(r)\sqrt{3}$	M1	For M1M1, $their\frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < ½$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ or $\frac{1}{2}(r)(r)\sqrt{3}$	A1	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	A1	
		4	

Question 45

(i)	Arc length $AB = 2r\theta$	B1	
	$\text{Tan}\,\theta = \frac{AT}{r}$ or $\frac{BT}{r} \to AT$ or $BT = r\tan\theta$	B1	Accept or $\sqrt{\left(\left(\frac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\frac{r\sin\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$ NOT $(90 - \theta)$
	$P = 2r\theta + 2r\tan\theta$	B1FT	OE, FT for *their* arc length + 2 × *their AT*
		3	
4(ii)	Area ΔAOT = ½ × 5 × 5 tan 1.2 **or** Area $AOBT$ = 2 × ½ × 5 × 5 tan 1.2	B1	
	Sector area = ½ × 25 × 2.4 (or 1.2)	*M1	Use of $½r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	DM1	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = 34.3 (cm^2)	A1	AWRT
	Alternative method for question 4(ii)		
	Area of ΔABT = ½ × (5 × tan 1.2)^2 × sin(π – 2.4) (= 55.86)	B1	
	Segment area = ½ × 25 × (2.4 – sin 2.4) (= 21.56)	*M1	Use of $½r^2(\theta - \sin\theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	DM1	Subtraction of segment from ΔABT, using 2.4 where appropriate.
	Area = 34.3 (cm^2)	A1	AWRT
		4	

Question 46

3(i)	$OA\times\frac{3}{8}\pi=6$	**M1**	
	$OA=\frac{16}{\pi}=5.093(0)$	**A1**	
3(ii)	$AB=their\,5.0930\times\tan\frac{3}{16}\pi$	**M1**	
	Perimeter $=2\times3.4030+6=12.8$	**A1**	
(iii)	Area $OABC=(2\times½)\times their\,5.0930\times their\,3.4030$	**M1**	
	Area sector $=½\times(their\,5.0930)^2\times\frac{3}{8}\pi$	**M1**	
	Shaded area $=their\,17.331-their\,15.279=2.05$	**M1A1**	

Question 47

$OC=6\cos0.8=4.18(0)$	**M1A1**	SOI
Area sector $OCD=\frac{1}{2}(their\,4.18)^2\times0.8$	***M1**	OE
$\Delta OCA=\frac{1}{2}\times6\times their\,4.18\times\sin0.8$	**M1**	OE
Required area = *their* ΔOCA – *their* sector OCD	**DM1**	SOI. If not seen *their* areas of sector and triangle must be seen
2.01	**A1**	CWO. Allow or better e.g. 2.0064
	6	

Question 48

$\cos POA=\frac{5}{13}\rightarrow POA=1.17(6)$ Allow 67.4° **or** $\sin=\frac{12}{13}$ or $\tan=\frac{12}{5}$	**M1 A1**
Reflex $AOB=2\pi-2\times their\,1.17(6)$ OE in degrees **or** minor arc AB $=5\times2\times their\,1.17(6)$	**M1**
Major arc $=5\times their\,3.93(1)$ **or** $2\pi\times5$ - *their* 11.7(6)	**M1**
AP (or BP) $=\sqrt{13^2-5^2}=12$	**B1**
Cord length = 43.7	**A1**
	6

Question 49

(a)	$BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$	M1
	$BC = r\sqrt{\left(5-2\sqrt{3}\right)}$	A1
		2
(b)	Perimeter = $\frac{2\pi r}{6} + r + r\sqrt{\left(5-2\sqrt{3}\right)}$	M1 A1
		2
(c)	Area = sector – triangle	
	Sector area = $\frac{1}{2}4r^2\frac{\pi}{6}$	M1
	Triangle area = ½ r. $2r$ sin $\frac{\pi}{6}$	M1
	Shaded area = $r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$	A1
		3

Question 50

Angle AOB = 15 ÷ 6 = 2.5 radians	B1
Angle BOC = π – 2.5 (**FT** on angle AOB)	B1FT
BC = 6(π – 2.5) (BC = 3.850)	M1
sin(π – 2.5) = BX ÷6 (BX = 3.59)	M1
Either OX = 6cos(π – 2.5) **or** Pythagoras (OX = 4.807)	M1
XC = 6 – OX (XC = 1.193) → P = 8.63	A1
	6

Question 51

(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2+12^2-8^2}{2\times8\times12}$	M1	Or other correct method
	BAO = 0.723	A1	
		2	
(b)	Sector ABC = ½×12²×*their* 0.7227	*M1	Accept 52.1
	Triangle AOB = ½×8×12 sin(*their* 0.7227) or ½×12×√28	*M1	or ½×8×8 sin(π – 2×*their* 0.7227). Expect 31.7 or 31.8
	Shaded area = *their* 52.0 – *their* 31.7 = 20.3	DM1 A1	M1 dependent on both previous M marks
		4	
(c)	Arc BC = 12×*their* 0.7227	*M1	Expect 8.67
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	DM1 A1	
		3	

Question 52

(a)	Use of correct formula for the area of triangle *ABC*	**M1**	Use of 180–2θ scores M0. Condone 2π–2θ
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	**A1**	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\ \frac{1}{2}r^2\theta$	**B1 FT**	FT for *their* triangle area $-\ \frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
(b)	Arc $BD = r\theta = 6$ cm	**B1**	SOI
	$AC = 2r\cos\theta = (2\times10\cos0.6 = 20\cos0.6 = 16.506)$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	***M1**	Finding *AC* or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))} - r$ (= 6.506)	**DM1**	Subtracting *r* from *their AC* or *r-rcosθ* from *their* half *AC* (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	**A1**	AWRT
		4	

Question 53

(a)	$\left(\sin\theta = \frac{r}{OC}\rightarrow\right) OC = \frac{r}{\sin\theta}$	**M1 A1**	
	$CD = r + \frac{r}{\sin\theta}$	**A1**	
		3	
(b)	Radius of arc *AB* = $4 + \frac{4}{\sin\frac{\pi}{6}} = 4+8=12$	**B1**	SOI
	(Arc *AB* =) *their* $12\times\frac{2\pi}{6}$ or $\left(\frac{1}{2}AB=\right)\left(\textit{their}\ 12\times\frac{\pi}{6}\right)$	**M1**	Expect 4π, must use *their* CD, not 4
	Perimeter = $24+4\pi$	**A1**	
		3	

(c)	Area $FOC = \frac{1}{2} \times 4 \times their\ OC \times \sin\frac{\pi}{3}$	M1	
	$8\sqrt{3}$	A1	
	Area sector $FOE = \frac{1}{2} \times \frac{2\pi}{3} \times 4^2 = \frac{16\pi}{3}$	B1	
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1	
	Alternative method for question 10(c)		
	$FC = \sqrt{(their\ OC)^2 - 4^2}$	M1	$\sqrt{48}$ or $4\sqrt{3}$
	Area $FOC = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$	A1	
	Area of half sector $FOE = \frac{1}{2} \times \frac{\pi}{3} \times 4^2 = \frac{8\pi}{3}$	B1	
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1	
		4	

Question 54

(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	M1	Attempt to find the area of ΔADE.
	$\frac{1}{4}k^2a^2$	A1	OE.
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	B1	
	$2 \times \frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	M1	OE. For $2 \times \Delta ADE = \text{sector}ABC$ with at least one correct area.
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	A1	
		5	
(b)	$2 \times \frac{1}{2}(ka)^2 \sin\theta = \frac{1}{2}a^2\theta$	M1	Condone omission of '2' or '1/2' on LHS for M1 only.
	$k^2 = \frac{\theta}{2\sin\theta}$	A1	
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	A1	OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707(\text{AWRT}) < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE
		3	

Question 55

(a)	$\frac{1}{2}\times4^2\times\text{angle BAD}=10$	**M1**	Use of sector area formula
	Angle BAD $=1.25$	**A1**	OE. Accept $0.398\pi,\ 71.6^o$ for **SC B1** only
		2	
(b)	Arc $BD=4\times their\,1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC=4\tan(their\,1.25)$	**M1**	Expect 12.0(4). May use ACB=0.321 or 18.4°
	$CD=\dfrac{4}{\cos(their\,1.25)}-4$ or $\sqrt{4^2+(their\,BC)^2}-4$	**M1**	Expect 12.69 – 4 = 8.69. May use ACB.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

Question 56

(a)	[By symmetry] [6 × $P\hat{A}Q$ = 2π], [$P\hat{A}Q$ =] 2π÷6,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre A ÷ 6	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why ΔPAQ is an equilateral triangle	M1	Assumption of this scores M0
	Using ΔPAQ is an equilateral triangle $\therefore\ P\hat{A}Q=\frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O+O\hat{A}B=\frac{2\pi}{3}$, equilateral triangles	M1	
	$P\hat{A}Q=2\pi-\left(\frac{\pi}{2}+\frac{2\pi}{3}+\frac{\pi}{2}\right)=\frac{\pi}{3}$	A1	AG

(a)	**Alternative method for Question 12(a)**		
	$Sin\theta=\frac{20}{40}$, with θ clearly identified	M1	
	$\theta=\frac{\pi}{6}, 2\theta=\frac{\pi}{3}=F\hat{A}O$ and by similar triangles = $P\hat{A}Q$	A1	AG
		2	
(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta=20\times\frac{\pi}{3}$	*M1	Use of $r\theta$ with $r=20$ and θ in radians
	Total length = $6\times((their\,40)+k\pi)$	DM1	$6\times$(*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240+40\pi$ or 366 (AWRT) (cm)	A1	Or directly: (6× diameter) + circumference
		4	

(c)	[Triangle area =] $\frac{1}{2}\times40\times40\times\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times40\times20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = 6 × $400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2}\times(40+80)\times40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = 2 × $1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle *ABC* = $400\sqrt{3}$ or 693 (AWRT) or 4 × Area of half of triangle *ABC* = $4\times200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle *ABDE* = $1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2\times400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times200\sqrt{3}+1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer.
		2	
(d)	Each rectangle area = 40 × 20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta=\frac{1}{2}\times20^2\times\frac{\pi}{3}\left[=\frac{200\pi}{3}\right]$	B1	SOI.
	Total area = $2400\sqrt{3}+4800+400\pi$ or 10 200 (cm²) (AWRT)	B1	Or directly: part **(c)** + 6800 + area circle radius 20.
		3	

Question 56

(a)	**Either** Let midpoint of *PQ* be *H*: sin *HCP* = $\frac{2}{4}$ ⇒ Angle *HCP* = $\frac{\pi}{6}$ **Or** sin *PSQ* = $\frac{4}{8}$ ⇒ Angle *PSQ* = $\frac{\pi}{6}$ **Or** using cosine rule: angle *PCQ* = $\frac{\pi}{3}$ **Or** by inspection: triangle *PCQ* or *PCT* is equilateral so angle *PCQ* = $\frac{\pi}{3}$	**M1**	
	Angle *PCS* = $\pi-\frac{\pi}{6}-\frac{\pi}{6}=\frac{2}{3}\pi$	**A1**	AG
		2	
(b)	Perimeter = $2\times4\times\frac{2\pi}{3}$ or $8\pi-\frac{8\pi}{3}$	**M1**	Length of two arcs *PS* and *QR*
	$+2\pi\times2$	**M1**	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	**A1**	Must be a single term
		3	

(c)	Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	M1	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	M1	Attempts to find area of segment
	Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$	M1	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	M1	
	$\pi \times 4^2 - \pi \times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	M1	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	M1	Uses correct formula for triangle
	Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$	M1	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	M1	
	$2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi \times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
		5	

Question 57

(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	B1	AG. OE using cosine rule.
		1	
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1	
	$AB = 9 + 11 - theirXY$	B1 FT	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc $AC = 11 \times 0.9582$	M1	
	Arc $BC = 9\times\frac{\pi}{2}$	M1	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	A1	AWRT. Answer must be evaluated as a single decimal.
		6	

Question 58

(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	B1	AG. OE using cosine rule.
		1	
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1	
	$AB = 9 + 11 - their XY$	B1 FT	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc $AC = 11 \times 0.9582$	M1	
	Arc $BC = 9 \times \frac{\pi}{2}$	M1	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	A1	AWRT. Answer must be evaluated as a single decimal.
		6	

Question 59

(a)	**EITHER** By using trigonometry: $B\hat{A}C = 0.6435\ldots$ and $A\hat{B}C = \frac{\pi - 0.6435}{2}$ **OR** By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \frac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \frac{\sqrt{10}}{10}$	M1	$\frac{3}{\sqrt{10}} = 0.9486\ldots$ $\frac{\sqrt{10}}{10} = 0.3162\ldots$
	$A\hat{B}C = \frac{\pi - 0.6435}{2}$ or $\tan^{-1}\frac{9}{3}$ or $\sin^{-1}\frac{3}{\sqrt{10}}$ or $\cos^{-1}\frac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ = 1.25 radians (3 sf)	A1	AG. Final answer must be 1.25, more accurate value 1.24904… with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
(b)	$BC = \sqrt{(their\,3)^2 + 9^2}$ or $\frac{9}{\sin 1.25}$ [= $\sqrt{90}$, $3\sqrt{10}$ or 9.48697…]	M1	Using correct method(s) to find BC.
	Area of sector = $\frac{1}{2} \times (their\,BC)^2 \times \tan^{-1}3$ [= 56.207 *or* 56.25]	M1	Using $\tan^{-1}3$ or 1.25 and *their BC*, but not 9 or 15, in correct area of sector formula.
	Area of triangle PBC = 13.4 to 13.6 or $\frac{1}{2} \times 9 \times 3$	B1	
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	A1	AWRT
		4	

Question 60

(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times \textit{their}\ \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \textit{their}\ \frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	**B1**	
	One arc $\frac{1}{6}\times 12\pi$ leading to two arcs $2\times\frac{1}{6}\times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
		3	
(b)	Sector = $\frac{1}{2}\times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2}\times(6^2)\times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right) + 6\pi\ \left[= 6\pi - 9\sqrt{3} + 6\pi\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times \textit{their}\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times(6^2)\times \textit{their}\left(\frac{\pi}{3}\right) - \frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\right) +$ $\frac{1}{2}\times(6^2)\times\sin\left(\textit{their}\left(\frac{\pi}{3}\right)\right)\ \left[= 12\pi - 18\sqrt{3} + 9\sqrt{3}\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × segment + triangle** A1 if correct (unsimplified).
	Area $\left[= 6\pi - 9\sqrt{3} + 6\pi\right] = 12\pi - 9\sqrt{3}$	**A1**	Must be simplified exact value.
		4	

Question 61

(a)	$\tan A = \frac{12}{5}$ or $\cos A = \frac{5}{13}$ or $\sin A = \frac{12}{13}$	**M1**	OR $\tan B = \frac{5}{12}$ or $\cos B = \frac{12}{13}$ or $\sin B = \frac{5}{13}$
	$A = 1.176$ $B = 0.3948$	**A1**	Allow 1.18 or 67.4°, Allow 0.395 or 22.6°. May be implied by $\frac{\pi}{2} - 1.176$
	$DE = 4$	**B1**	If trigonometry used accept AWRT 4.00
	Arcs = $5 \times their 1.176$ *and* $8 \times their 0.3948$	**M1**	Or corresponding calculations in degrees.
	[Perimeter = 5.880 + 3.158 + 4 =] 13.0	**A1**	Accept 13. If *DE* is outside the given range this mark cannot be awarded.
		5	
(b)	Area of triangle = $\frac{1}{2} \times 5 \times their 12$ [= 30]	**B1 FT**	
	Area of sectors = $\frac{1}{2} \times 5^2 \times their\ 1.176 + \frac{1}{2} \times 8^2 \times their\ 0.3948$	**M1**	Or corresponding calculations in degrees
	[Area = 30 – 14.70 –12.63 =] 2.67	**A1**	Allow 2.66 to 2.67
		3	

Question 62

(a)	$6\sin 0.9 = \frac{AC}{2}$ or $AC^2 = 6^2 + 6^2 - 2 \times 6 \times 6\cos 1.8$	**M1**	OE Correct working in degrees is acceptable throughout.
	$AC = 9.40$	**A1**	SOI Accept 9.39 – 9.41, may be used but not seen for A1.
	Angle *CAB* = ½(π – 1.8)	**M1**	SOI Expect 0.6708 (or 0.671).
	Arc *CD* = *their* 9.40 × *their* 0.6708	**M1**	Expect 6.306 (or 6.31), do not accept 6 for *their AC* or 1.8 for *CAB*.
	[Perimeter = 6 + 3.40 + 6.306 =] 15.7	**A1**	Accept 15.69 – 15.72.
		5	
(b)	Sector *ADC* – Δ*ABC* = ½ × *their* 9.40^2 × *their* 0.6708 – ½ × 6^2 × sin 1.8	**M1 M1**	Accept correct use of their answers from part **(a).**
	[29.64 – 17.53 =] 12.1	**A1**	AWRT
		3	

Question 63

(a)	$\left[A\hat{O}B=\right]\ \frac{2}{10}$	**B1**	OE Sight of 0.2 from $s=r\theta$ but $10\theta=2$ is not enough. ISW if $\frac{2}{10}=\frac{\pi}{5}$.
	$[B\hat{O}C=]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	**B1**	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
(b)	$[BP]=10\sin\left(\frac{5\pi+6}{30}\right)$ and $[OP]=10\cos\left(\frac{5\pi+6}{30}\right)$ [= 6.6208…] and [= 7.494…] **OR** [BP] = $10\sin\left(\frac{5\pi+6}{30}\right)$ and [O$\hat{B}$P] = $\left(\frac{5\pi-3}{15}\right)$ [= 6.6208…] and [= 0.84719…]	**M1**	OE Any correct method for **both** lengths, for *their* angle BOC (which may have been incorrectly 'simplified' but not 0.2) or length BP and O$\hat{B}$P. May be seen as part of $\frac{1}{2}ab\sin C$. Sight of correct method enough. Can be implied by the next A1.
	Area of $\Delta OBP=\frac{1}{2}\times\ 10\sin\left(\frac{5\pi+6}{30}\right)\times 10\cos\left(\frac{5\pi+6}{30}\right)$ or $\frac{1}{2}\times 10\times 10\sin\left(\frac{5\pi+6}{30}\right)\times\sin\left(\left(\frac{5\pi-3}{15}\right)\right)$ [=24.809]	**A1**	OE Can be implied by any answer in range (24.7, 24.9) or a final answer in the range (11.3, 11.5) WWW.
	[Sector BOC] $=\frac{1}{2}\times 10^2\times$ *their* $\left(\frac{5\pi+6}{30}\right)$ $\left[=50\left(\frac{5\pi+6}{30}\right)=36.1799...\right]$	**M1**	Use of $\frac{1}{2}r^2\theta$ with *their* angle BOC (may have been incorrectly 'simplified' but not 0.2).
	Area of region BPC = 11.4	**A1**	CAO
		4	

Question 64

(a)	Sector area = $\frac{1}{2}r^2\left(\frac{\pi}{6}\right)\left[=\frac{\pi}{12}r^2\right]$	B1	Using $\frac{1}{2}r^2\theta$ with θ in radians SOI. B0 if using a value for r.
	$BD = \sin\frac{\pi}{6}r\left[=\frac{1}{2}r\right]$ and $AD = \cos\frac{\pi}{6}r\left[=\frac{\sqrt{3}}{2}r\right]$ so triangle area = $\frac{1}{2}\left(\sin\frac{\pi}{6}r\right)\left(\cos\frac{\pi}{6}r\right)\left[=\frac{1}{2}\times\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\right]$ or $\frac{1}{2}r\left(\cos\frac{\pi}{6}r\right)\left(\sin\frac{\pi}{6}\right)\left[=\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\times\frac{1}{2}\right]$	B1	SOI Finding triangle area. Decimals B0 unless exact values seen in working.
	Area of $BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$	B1	OE e.g. $\frac{r^2}{4}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ with $\cos\frac{\pi}{6}$ and $\sin\frac{\pi}{6}$ evaluated. Must be exact, in terms of r^2. ISW
		3	
(b)	Angle $BAC = \sin^{-1}\left(\frac{\frac{\sqrt{3}}{2}r}{r}\right)\left[=\frac{\pi}{3}\right]$	B1	SOI by length of AD, CD or arc, or by perimeter.
	Length $AD = \cos\frac{\pi}{3}r\left[=\frac{1}{2}r\right]$ [so length $CD = \frac{1}{2}r$]	M1	SOI Finding length by Pythagoras, or by trigonometry with *their* angle BAC, provided $BAC \neq \frac{\pi}{6}$.
	Length of arc $BC = r\times\frac{\pi}{3}$	M1	SOI Using $r\theta$ with θ in radians. Condone $\theta = \frac{\pi}{6}$.
	Perimeter of $BCD = \frac{\sqrt{3}}{2}r + \frac{1}{2}r + \frac{\pi}{3}r$	A1	OE e.g. $r\left(\frac{\sqrt{3}+1}{2}+\frac{\pi}{3}\right)$ with e.g. $\cos\frac{\pi}{3}$ evaluated. Must be exact, in terms of r. ISW
		4	

Question 65

(a)	$APQ = \cos^{-1}\frac{\frac{5}{6}r}{r}\left[=\cos^{-1}\frac{5}{6}\right]$	*M1	May use cosine rule to find APB. Stating APQ or APB as an incorrect multiple of π is M0.
	$= 0.5857$	A1	Accept 0.586 or 33.6° or APB (1.171 or 67.1°).
	Perimeter $= 4\times r\times \textit{their}\,0.5857 = 2.34r$ or $0.745\pi r$ or $(293/125)r$	DM1 A1	Must use a numerical value of *their* angle.
		4	
(b)	Use of sector formula: Sector APB $= ½r^2\times(2\times \textit{their}\,0.5857)$ or Sector APC (C is on PQ so PC = r) $= ½r^2\times(\textit{their}\,0.5857)$	M1	Any sector with *their* appropriate angle. It must be clear the appropriate numerical angle is being used.
	Use of appropriate formula for area of triangle and correct combination with the sector to find the area of a half segment, one segment or both segments	M1	e.g. Area APB = $½r^2\times\sin(2\times \textit{their}\,0.5857)$.
	Shaded area [$= 2\times 0.1250r^2$] $= 0.250r^2$	A1	or $0.0796\pi r^2$, allow $\frac{1}{4}r^2$ or $0.25r^2$.
		3	

Question 66

(a)	$2.5\times\frac{4\pi}{3}+2.24\times\frac{5\pi}{6}$ [= 10.47[2] + 5.86[4] or $\frac{10\pi}{3}+\frac{28\pi}{15}$]	B1	For either arc correct. Arc ARB could be AR+RB.
		M1	For adding two (or three) arc lengths using different radii and angles and nothing else. SOI
	16.34 or $\frac{26\pi}{5}$	A1	AWRT Condone 16.33 only.
		3	
(b)	Area $AOB=\frac{1}{2}\times 2.5^2\sin\frac{2\pi}{3}$ [=2.706] Area $APB=\frac{1}{2}\times 2.24^2\sin\frac{5\pi}{6}$ [=1.254]	M1	For either ΔAOB or ΔAPB (AB = 4.33, h= 1.25, 0.58) or any other valid method.
	[Difference =] 1.45	A1	AWRT Condone 1.46 only.
		2	
(c)	Area $AOB=\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}$ [=13.09] Area $APB=\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}$ [=6.57]	B1	For either sector area correct
	[Area of cross section =] $\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}+\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}+$ "*their* 10(*b*)" [=13.09+6.57+ "*their* 10(*b*)"]	M1	Adding two **sector areas** from different sectors and '*their* 10**(b)**' and nothing else. SOI
	21.1	A1	CAO Condone slight inaccuracies in intermediate working if the correct answer is arrived at.
		3	

Question 67

(a)	$[2r+8=20\Rightarrow]\ r=6$	B1	
	Angle $AOB=\frac{8}{their\,6}$	*M1	Expect $\frac{4}{3}$ OE $(76.4°)$. M0 Assume triangle is equilateral.
	$AB=2\times 6\sin their\frac{2}{3}$ **or** $\sqrt{6^2+6^2-2\times 6^2\cos their\frac{4}{3}}$ **or** $AB=\frac{6}{\sin\left(\frac{\pi}{2}-their\frac{2}{3}\right)}\times\sin their\frac{4}{3}$	DM1	For 6 read *their* 6.
	Perimeter $=[7.42+8=]\ 15.4$	A1	AWRT
		4	
(b)	Area $=\frac{1}{2}\times 6^2\times their\frac{4}{3}-\frac{1}{2}\times 6^2\times\sin their\frac{4}{3}$ **or** Area $=\frac{1}{2}\times 6^2\times their\frac{4}{3}-2\times\frac{1}{2}\left(6\sin their\frac{2}{3}\right)\left(6\cos their\frac{2}{3}\right)$	M1	Sector area – whole triangle area. For 6 read *their* 6. Sector area – 2(half triangle area).
	$=[24-17.49=]\ 6.51$	A1	AWRT
		2	

Question 67

(a)	$\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ **used** to find ADC	M1	May use cosine rule or $CAD = \tan^{-1}\frac{4}{8}$.
	$BDC = 0.927[3] \quad \rightarrow ADC = \pi - 0.927[3] \quad [= 2.214 \text{ to } 2.215]$	A1	Allow degrees, 126.87, and 0.7048π or $0.705\ \pi$.
	$Arc\ AC = 5 \times their\ 2.214$	M1	Use of $r\theta$ or $\frac{\theta}{360}.2\pi r$ Expect 11.07 .
	$AC = \sqrt{8^2+4^2}$ *or* $2\times5\times\sin 1.107$	M1	Expect 8.94 .
	[Perimeter = 11.07 + 8.94 =] 20.0	A1	Accept AWRT [20.01, 20.02].
		5	
(b)	Sector $ACD = ½ \times 5^2 \times their\ 2.214$	M1	See use of $\frac{1}{2}r^2\theta$ or $\frac{\theta}{360}.\pi r^2$. Expect 27.7 .
	Subtracting the area of $\Delta ADC = ½ \times 5 \times 4$ or $\frac{1}{2}5^2 \sin their\ 2.214$ or $\frac{1}{2}\times8\times4 - \frac{1}{2}\times3\times4$	M1	Subtracting the area of ΔADC, expect -10.
	Shaded area $= 27.7 - 10 = 17.7$	A1	Accept AWRT [17.67, 17.68]. Correct answer cannot come from an angle of 2.215 .
		3	

Question 68

(a)	$\frac{½r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate θ *or* r using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	$\Delta OAB = ½ \times their\ 16^2 \times \sin their\ 0.6$	M1	Allow Segment = 76.8 – ½ × *their* 16^2 × sin *their* 0.6. Expect 72.27 .
	[Area = 76.8 – 72.27 =] 4.53	A1	AWRT
		5	
(b)	$AB = 2\times16\times\sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2\times16^2\cos 0.6$	M1	Any valid method with *their* r, θ. Expect $AB = 9.46$.
	Perimeter = 9.6 + 9.46 = 19.1	A1	AWRT
		2	

Question 69

(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi-2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi-2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	*B1	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow p, a or r for OA) containing only terms with x and θ but not just $OA = 2x\cos\theta$. Do not condone $\sin\pi - 2\theta$ until missing brackets recovered or $\cos(180 - 2\theta)$ until it becomes $-cos2\theta$ etc.
	$OA = 2x\cos\theta$ leading to Arc length $= 2x\theta\cos\theta$	DB1	AG Complete correct method showing all necessary working. Condone $2x\cos\theta\times\theta$.
		2	If B0 but www then **SCB1** for $OA = 2x\cos\theta$ leading to Arc length $= 2x\theta\cos\theta$.
(b)	Sector area $= \frac{1}{2}(2x\cos\theta)^2\times\theta$	M1	OE Using sector formula with a correct OA. Condone $cos\theta^2$ for $cos^2\theta$ and missing brackets.
	Triangle area $= \frac{1}{2}\times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi-2\theta)$	M1	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π.
	[Area APB =] *Their* sector area – *their* triangle area	M1	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area APB =] $\frac{1}{2}(2x\cos\theta)^2\times\theta - \frac{1}{2}x^2\sin(\pi-2\theta)$ [$= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)$]	A1	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect ‘simplifications’.
		4	

Question 70

$\frac{1}{2}\times 8^2\times\theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	B1	SOI OE e.g. $\frac{2\pi}{12}$, $0.524(3\text{s.f.})$ Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
Arc length $= 8\times their\frac{\pi}{6}$ [= 4.1887…]	M1	OE FT *their* θ. Look for $\frac{4\pi}{3}$.
[BC =] $2\times 8\sin\left(\frac{1}{2}\times their\frac{\pi}{6}\right)$ [= 4.1411…]	M1	Attempt to find BC or BC^2 (see alt. methods below) FT *their* θ. Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6} - 4\sqrt{2}$.
Perimeter = 8.33	A1	AWRT Must be combined into one term.

Question 71

(a)	Angle ACO = 0.7	B1	Don’t allow AWRT 0.7 .
		1	
(b)	[R =] 1.53 r	B1	Allow AWRT 1.53r.
		1	

(c)	Sector $OAB = \frac{1}{2}r^2 \times 2.8 \quad \left[= 1.4r^2\right]$	B1	
	Sector $CAB = \frac{1}{2}(their\ R)^2 \times 2 \times their\ 0.7$	*M1	
	$1.638\,r^2$	A1	Allow AWRT $1.64\,r^2$.
	$[2] \times \frac{1}{2}r^2 \sin(\pi - 1.4)$ OR $[2] \times \frac{1}{2} r \times their R \sin 0.7$	*M1	
	$2 \times 0.4927r^2$	A1	Allow AWRT $0.98\,r^2$ to $0.99\,r^2$.
	$1.4r^2 - \left(their\ 1.638r^2 - their\ 0.985r^2\right)$	DM1	
	$0.747r^2$ to $0.748r^2$	A1	
		7	

(c)	**General guidance for alternative methods**		
	Finding any useful sector area of the circle radius, r	B1	May be 'nested' in a segment.
	Finding the area of sector CAB	*M1A1	May be 'nested' in a segment.
	Finding the area of one useful triangle	*M1	May be 'nested' in a segment.
	Finding the total area of useful triangles	A1	May be 'nested' in a segment.
	A correct plan for the shaded area	DM1	
	$0.747r^2$ to $0.748r^2$	A1	
		7	

Question 72

(a)	[Arc length =] $2 \times \frac{\pi}{3}$ or $\frac{60}{360} \times 2\pi \times 2$	B1	Finding one correct arc length – may be implied by correct final answer.
	[Perimeter =] 2π or 6.28	B1	AWRT
		2	
(b)	[Area of one sector =] $\frac{1}{2} \times 2^2 \times \frac{\pi}{3}$ or $\frac{60}{360} \times \pi \times 2^2 \left[= \frac{2\pi}{3} \text{ or } 2.09\right]$	B1	SOI AWRT
	[Area of triangle =] $\frac{1}{2} \times 2^2 \times \sin\left(\frac{\pi}{3}\right)$ or other valid method $\left[= \sqrt{3} \text{ or } 1.73\right]$	B1	AWRT Allow use of 60°
	[Area of coin = 3 segments + triangle ⇒] $3\left(\frac{2\pi}{3} - \sqrt{3}\right) + \sqrt{3}$ [= 2.82]	M1	OE Or 3 sectors – 2 triangles $\left(3 \times \frac{2\pi}{3} - 2 \times \sqrt{3}\right)$ or Sector + 2 segments $\left(\frac{2\pi}{3} + 2\left(\frac{2\pi}{3} - \sqrt{3}\right)\right)$
	$2\pi - 2\sqrt{3}$ or $2(\pi - \sqrt{3})$	A1	Must be one of these simplified versions but equivalent decimal answers can score B1B1M1
		4	

Question 73

(a)	$k = \frac{2}{3}$	**B1**	Allow ACB = $\frac{2\pi}{3}$.
		1	
(b)	Perimeter of shaded area $= 2\pi r$	**B1**	
		1	

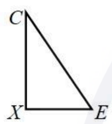
(c)	Major sector OAB $= \frac{1}{2}r^2 \times \frac{4\pi}{3}$	***M1**	Expect $\frac{2}{3}\pi r^2$. Finds area of any relevant sector or triangle. Can be embedded in segment formula.
	One or both segments $= [2] \times \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin\frac{\pi}{3}\right)$	***M1**	
	$= [2]\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area $= \frac{2}{3}\pi r^2 - 2\left(\frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4}\right)$	**DM1**	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	**A1**	

(c)	**Alternative method for Question 6(c)**		
	Sector $CAOB$ $= [2] \times \frac{1}{2}r^2 \textit{their} \frac{1}{3}\pi$	***M1**	Expect $[2] \times \frac{1}{6}\pi r^2$. Can be embedded in segment formula.
	One or both segments $= [2] \times \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin\frac{\pi}{3}\right)$	***M1**	
	$= [2]\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area $= \pi r^2 - \left\{\frac{1}{3}\pi r^2 + 2\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	**A1**	

Question 74

(a)	State $2r + r\theta = 65$ and $\frac{1}{2}r^2\theta = 225$	B1	
	Form a 3-term quadratic or cubic in r or θ or $r\theta$ from correct arc and sector formula	*M1	Condone sign errors.
	Solve *their* 3 term quadratic or cubic to obtain values of r or θ	DM1	Expect $2r^2 - 65r + 450 = (2r-45)(r-10)$ or $18\theta^2 - 97\theta + 72 = (9\theta - 8)(2\theta - 9)$.
	$r = 10$ and $\theta = 4.5$ ignore $r = 22.5$ and $\theta = \frac{8}{9}$, do not ignore $r = 0$	A1	**B1 SC** if no quadratic or cubic solution. If $r = 0$ included A0 or B0 SC.
		4	
(b)	Use correct formula for area of triangle with clear use of angle being $2\pi -$ their θ	M1	Expect 1.783 or 102.2°, *their* θ must be reflex.
	48.9	A1	AWRT, WWW or a second answer. Or greater accuracy; condone absence of units.
		2	

Question 75

(a)(i)	 $X\hat{C}E = \frac{\pi}{6}$ $C\hat{E}X = \frac{\pi}{3}$		
	$\frac{XE}{0.4} = \sin\frac{\pi}{6}$ or $\frac{XE}{0.4} = \cos\frac{\pi}{3}$ $[XE = 0.2]$	M1	A correct trig expression involving XE. Do not condone a mixture of degrees and radians.
	Length $EF = 2 + 2 \times 0.2 = 2.4$	A1	AG
		2	

(a)(ii)	$[CX =]0.4\cos\frac{\pi}{6}$ or $0.4\sin\frac{\pi}{3}$ or $\sqrt{0.4^2 - 0.2^2}$	B1	OE, SOI Expect $\frac{\sqrt{3}}{5}$ or 0.3464.
	$[\text{Sector} =]\frac{1}{2} \times (0.4)^2 \times \frac{\pi}{3}$	B1	SOI Expect 0.0838 or $\frac{2\pi}{75}$. Allow use of $\frac{60}{360}\pi(0.4)^2$.
	Either Area of *their* (rectangle + two triangles + two sectors) Or Area of *their* (trapezium + two sectors)	M1	Either implied by a correct answer or areas clearly labelled. Expect 0.6928 + 0.06928 + 0.1676 or $\frac{2\sqrt{3}}{5} + \frac{\sqrt{3}}{25} + \frac{4\pi}{75}$. Or 0.7621 + 0.1676 or $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$.
	0.930	A1	AWRT Condone $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$.
		4	

(b)	[Length AD =] $2+2r$	B1	Must be seen alone or part of a list and not part of a product.
	[Arc length =] $r\times\frac{\pi}{3}$	B1	May be implied by $r\times\frac{\pi}{3}\times 2$. Must be seen alone or part of a list.
	$[\text{EF} =] 2+2r\sin\frac{\pi}{6}$ or $2+2r\cos\frac{\pi}{3}$ or $2+r$	B1	Must be seen alone or part of a list and not part of a product.
	[$4+3r+\frac{2\pi r}{3}=6$ leading to] 0.393	B1	AWRT Condone $\frac{6}{2\pi+9}$. NB: Using $EF = 2.4$ gives 0.391.
		4	

Question 76

(a)	Angle $\theta=\frac{\pi}{2}-\cos^{-1}\frac{10}{15}$ or $\sin^{-1}\frac{10}{15}=0.7297$	B1	Condone working in degrees if converted to radians at the end. AG
		1	

(b)	$BC=\sqrt{15^2-10^2}\ [=11.18\ldots \text{ or } 5\sqrt{5}]$	B1	
	Arc $AB = 15\times 0.7297[=10.9455]$	B1	
	Perimeter = *their* BC + *their* arc AB + 25 + 5π	M1	
	Perimeter = 62.8	A1	AWRT
	Area sector $AOB=\frac{1}{2}\times 15^2\times 0.7297[=82.09]$	B1	
	Area = $\frac{1}{2}\times 10\times$ *their* BC + *their* sector AOB + $\frac{\pi}{4}\times 10^2$	M1	
	Area = 217	A1	AWRT
		7	

Question 77

'(a)	Area of sector $BOF = \frac{1}{2} \times 20^2 \times (2\pi - 2.4)$ $[=776.63\ldots]$	M1	Or combination of large semi-circle and small sector: $\frac{1}{2} \times 20^2 \times \pi + \frac{1}{2} \times 20^2 \times (\pi - 2.4)$.
	Length $BD = DF = 2 \times 20 \sin 0.6$ or $\sqrt{20^2 + 20^2 - 2 \times 20 \times 20 \cos 1.2}$ $[=22.58\ldots]$	M1*	Length of radius of small circles is acceptable for M1.
	Area of two semicircles $= \pi \times (20 \sin 0.6)^2$ $[=400.64\ldots]$	DM1	
	Area of triangles $= 2 \times \frac{1}{2} \times 20 \times 20 \sin 1.2$ $[=372.81\ldots]$	M1	
	Total area $= 1550\,[\text{cm}^2]$	A1	Expect 1550.09 but accept AWRT to 3sf.
		5	
'(b)	$\frac{1}{2}\pi r^2 = 50\pi \Rightarrow r = 10$	B1	May be seen as $20 \sin \frac{\theta}{2}$, where $\theta = \frac{\pi}{3}$.
	$\Rightarrow \theta = \frac{\pi}{3}$	M1*	OE Finding θ using *their* r. Allow working in degrees.
	Arc length of sector $BOF = 20 \times \left(2\pi - \textit{their}\,\frac{2\pi}{3}\right)$	DM1	
	Total perimeter $= 20 \times \left(2\pi - \textit{their}\,\frac{2\pi}{3}\right) + 2\pi \times \textit{their}\,10$	DM1	Dependent on the first dM1.
	$\frac{140\pi}{3}$ or $46\frac{2}{3}\pi$	A1	Must be a single exact term.
		5	

Question 78

[Perimeter =] $r + r\theta + r + 2r \times 2\theta + r + r\theta + r$ $\quad [=4r + 6r\theta]$	B1	
[Area =] $\frac{1}{2}r^2\theta + \frac{1}{2}(2r)^2 \times 2\theta + \frac{1}{2}r^2\theta$ $\quad [=5r^2\theta]$	B1	
$4r + 6r\theta = 14$ and $5r^2\theta = 10$	M1*	$ar + br\theta = 14$ and $cr^2\theta = 10$ where a, b and c are constants $\neq 0$. Terms may be uncollected.
EITHER		
$5r^2 \frac{14 - 4r}{6r} = 10$ or $4r + 6r\left(\frac{10}{5r^2}\right) = 14$	DM1	Eliminate θ to get an equation in r.
$[\Rightarrow 2r^2 - 7r + 6 = 0 \Rightarrow]$ $(r-2)(2r-3) = 0$	DM1	Factorise or other accepted method for solving their 3-term quadratic.
OR		
$5\left(\frac{14}{4 + 6\theta}\right)^2 \theta = 10$ or $4\left(\sqrt{\frac{10}{5\theta}}\right) + 6\left(\sqrt{\frac{10}{5\theta}}\right)\theta = 14$	DM1	Eliminate r to get an equation in θ.
$[\Rightarrow 18\theta^2 - 25\theta + 8 = 0 \Rightarrow]$ $(9\theta - 8)(2\theta - 1) = 0$	DM1	Factorise or other accepted method for solving their 3-term quadratic.
Then		
$r = 2$ and $\theta = 0.5$	B1	Condone extra answers $r = \frac{3}{2}$ and $\theta = \frac{8}{9}$.
	6	

Question 79

Use correct sector area formula	M1	
Obtain $\frac{1}{2}\times 15^2\times\frac{2}{5}\pi - \frac{1}{2}\times x^2\times\frac{2}{5}\pi = \frac{209}{5}\pi$ or equivalent	A1	
Obtain $[x]=4$	A1	AWRT 4.00.
Use correct arc length formula twice	M1	
Obtain $22+\frac{38}{5}\pi$	A1	OE. Must be in terms of π. Like terms must be collected. Not from a rounded value of x.
	5	

Question 80

(a)	6×0.8	B1	Accept $\frac{45.8}{360}\times 12\pi$.
	$AB^2=10^2+10^2-2\times 10\times 10\cos 0.8$ or $AB=2(10\sin 0.4)$ or $AB=\frac{10\sin 0.8}{\sin\left(\frac{\pi-0.8}{2}\right)}$	M1	Allow angles correctly converted to degrees for this mark. $0.8\,\text{rad}=45.8°$, $O\hat{A}B=O\hat{B}A=67.1°$. $\left(\frac{\pi-0.8}{2}\right)=1.17$ This mark can be implied by AWRT 7.8.
	20.6	A1	AWRT
		3	
(b)	[Area of sector =] $\frac{1}{2}\times 6^2\times 0.8$	B1	
	[Area of triangle =] $\frac{1}{2}\times 10^2\times\sin 0.8$ or $10\sin 0.4\times 10\cos 0.4$ or other complete method.	M1	OE Allow use of *their* value of θ or $\frac{1}{2}\theta$ in degrees.
	21.5	A1	AWRT
		3	

Question 81

(a)	$\frac{1}{2}\pi-2\left(\tan^{-1}\frac{4}{12}\text{ or }\sin^{-1}\frac{4}{\sqrt{160}}\text{ or }\cos^{-1}\frac{12}{\sqrt{160}}\right)$ Or $2\sin^{-1}\left(\frac{\frac{1}{2}EF}{AE}\left[=\frac{\sqrt{32}}{\sqrt{160}}=\frac{\sqrt{2}}{\sqrt{10}}\right]\right)$ or other valid method.	M1	Attempt a complete valid method for finding angle *EAF*. $\sqrt{160}$ may be replaced by 12.65 AWRT and $\sqrt{32}$ by 5.657 AWRT. Working in degrees should give $E\hat{A}F=53.13°$ which converts to the correct value in radians.
	$[E\hat{A}F=0.927295\ldots=]\ 0.9273$	A1	AG If decimals are used, at least one of 0.32175.., 0.6435 or 0.92729.. must be seen in the first method and 5.6569 and 12.649 or 0.46364 seen in the second method for the correct level of accuracy. **Note:** $E\hat{A}F=53.14°$ does not convert to the correct answer, so A0.
	Alternative Method for Question 8(a)		
	$12^2-(24+24+32)=\frac{1}{2}(\textit{theirAE}\times\textit{their AF})\sin E\hat{A}F$ $\sin E\hat{A}F=\frac{2\{12^2-(24+24+32)\}}{(\textit{theirAE}\times\textit{their AF})}$ $E\hat{A}F=\sin^{-1}\frac{2\{12^2-(24+24+32)\}}{(\textit{theirAE}\times\textit{their AF})}$	M1	Attempt a complete valid method for finding angle *EAF*. Area of ABCD – (Area Δs ADF, ABE &CEF) = Area ΔAEF. $E\hat{A}F=\sin^{-1}\left(\textit{their }\frac{4}{5}\right)$. Using the cosine rule should lead to $\cos^{-1}\frac{3}{5}$.
	$\left[E\hat{A}F=\sin^{-1}\frac{4}{5}=\right]\frac{\frac{1}{2}EF}{AE}$	A1	AG
		2	

(b)	$[AE \text{ or } AF =] \sqrt{160}$ or $\dfrac{4}{\sin E\hat{A}B}$	B1	OE Expect AWRT 12.65.
	$[r\theta =]\ (\textit{their}\ 12.65) \times 0.9273$	M1	Use of $r\theta$ with $(\textit{their}\ \sqrt{160})$. Note: using $r = 12$ scores M0.
	$[11.729... + 8 + 8 =]$ 27.7 AWRT	A1	
		3	
(c)	$\frac{1}{2}(\textit{their}\ 12.65)^2 \times 0.9273$ [= 74.184]	M1	Use $\frac{1}{2}r^2\theta$ for area of sector with $(\textit{their}\ 12.65)$. Note: using $r = 12$ scores M0.
	$144 - 24 - 24 - \left\{\frac{1}{2}(\textit{their}12.65)^2 \times 0.9273\right\}$	M1	Attempt a complete method for finding area of shaded region. Condone use of $r = 12$.
	[Area =] 21.8	A1	AWRT
	Alternative Method for Question 8(c)		
	$\frac{1}{2}(\textit{their}\ 12.65)^2 (0.9273 - \sin 0.9273)$ [=10.183]	M1	Area of the segment with $(\textit{their}\ 12.65)$. Note: using r =12 scores M0.
	$32 - \frac{1}{2}(\textit{their}12.65)^2 (0.9273 - \sin 0.9273)$	M1	Attempt a complete method for finding area of shaded region, condone use of $r = 12$.
	[Area =] 21.8	A1	AWRT
		3	

Question 82

(a)	$\frac{1}{2}r^2\alpha = 8\alpha \Rightarrow r = 4$	B1	
	$\frac{1}{2}r^2 \sin\alpha = 4 \Rightarrow \alpha = \frac{\pi}{6}$	B1	
	Area of segment = $\frac{1}{2} \times 4^2 \times \frac{\pi}{6} - 4$	M1	Using *their r* and *their* α. Condone $4 - \frac{1}{2} \times 4^2 \times \frac{\pi}{6}$. Allow use of 8 × *their* α – 4
	$= \frac{4}{3}\pi - 4$	A1	Fraction must be simplified. Allow $\frac{4\pi - 12}{3}$.
		4	
(b)	$r^2 + r^2 - 2r^2\cos\alpha = \frac{1}{2}r^2 \Rightarrow 2\cos\alpha = \frac{3}{2}$	*M1	Or $\alpha = 2\sin^{-1}\left(\frac{\sqrt{2}}{4}\right)$ or $2\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ Using $\alpha = \frac{\pi}{6}$ or $r = 4$ implies 0/4.
	$\alpha = 0.723[0.72273\ldots] \Rightarrow \frac{1}{2}r^2\sin(\textit{their}\ 0.723)[=4]$	DM1	Or $\alpha = 41.4 \Rightarrow \frac{1}{2}r^2\sin(\textit{their}\ 41.4)[=4]$
	$r = 3.48\ [3.4777\ldots]$	A1	Accept r^2 = 12.1 AWRT.
	Area of segment = $\frac{1}{2} \times 3.48^2 \times 0.723 - 4 = 0.371$ [0.37068…]	A1	Or $\frac{1}{2} \times 3.48^2 \times \frac{\pi}{180} \times 41.4 - 4 = 0.371$ AWRT.
		4	