AS-Level

Trigonometry And Circular measure

2013-2018

Question 1

The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that PS = 6 cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, \[3\]

(ii) find the perimeter of the shaded region. \[2\]
Question 2

The diagram shows a square $ABCD$ of side 10 cm. The mid-point of $AD$ is $O$ and $BXC$ is an arc of a circle with centre $O$.

(i) Show that angle $BOC$ is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

Question 3

In the diagram, $OAB$ is a sector of a circle with centre $O$ and radius 8 cm. Angle $BOA$ is $\alpha$ radians. $OAC$ is a semicircle with diameter $OA$. The area of the semicircle $OAC$ is twice the area of the sector $OAB$.

(i) Find $\alpha$ in terms of $\pi$. [3]

(ii) Find the perimeter of the complete figure in terms of $\pi$. [2]
Question 4

The diagram shows sector $OAB$ with centre $O$ and radius 11 cm. Angle $AOB = \alpha$ radians. Points $C$ and $D$ lie on $OA$ and $OB$ respectively. Arc $CD$ has centre $O$ and radius 5 cm.

(i) The area of the shaded region $ABDC$ is equal to $k$ times the area of the unshaded region $OCD$. Find $k$. [3]

(ii) The perimeter of the shaded region $ABDC$ is equal to twice the perimeter of the unshaded region $OCD$. Find the exact value of $\alpha$. [4]

Question 5

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from $A$ to $O$ and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is $\theta$ radians. Calculate

(i) the value of $\theta$. [4]

(ii) the area of paper needed to make the cone. [2]
Question 6

The diagram shows a metal plate made by fixing together two pieces, $OABCD$ (shaded) and $OAED$ (unshaded). The piece $OABCD$ is a minor sector of a circle with centre $O$ and radius $2r$. The piece $OAED$ is a major sector of a circle with centre $O$ and radius $r$. Angle $AOD$ is $\alpha$ radians. Simplifying your answers where possible, find, in terms of $\alpha$, $\pi$ and $r$,

(i) the perimeter of the metal plate. [3]

(ii) the area of the metal plate. [3]

It is now given that the shaded and unshaded pieces are equal in area.

(iii) Find $\alpha$ in terms of $\pi$. [2]

Question 7

The diagram shows part of a circle with centre $O$ and radius 6 cm. The chord $AB$ is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region. [3]

(ii) the ratio of the area of the shaded region to the area of the triangle $AOB$, giving your answer in the form $k : 1$. [3]
Question 8

The diagram shows a sector of a circle with radius $r$ cm and centre $O$. The chord $AB$ divides the sector into a triangle $AOB$ and a segment $AXB$. Angle $AOB$ is $\theta$ radians.

(i) In the case where the areas of the triangle $AOB$ and the segment $AXB$ are equal, find the value of the constant $p$ for which $\theta = p \sin \theta$. [2]

(ii) In the case where $r = 8$ and $\theta = \frac{\pi}{4}$, find the perimeter of the segment $AXB$. [3]

Question 9

The diagram shows triangle $ABC$ in which $AB$ is perpendicular to $BC$. The length of $AB$ is 4 cm and angle $CAB$ is $\alpha$ radians. The arc $DE$ with centre $A$ and radius 2 cm meets $AC$ at $D$ and $AB$ at $E$. Find, in terms of $\alpha$,

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [3]
Question 10

In the diagram, $OADC$ is a sector of a circle with centre $O$ and radius 3 cm. $AB$ and $CB$ are tangents to the circle and angle $ABO = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and $\pi$.

(i) the perimeter of the shaded region.  \hspace{1cm} [3]

(ii) the area of the shaded region. \hspace{1cm} [3]

Question 11

The diagram shows a triangle $AOB$ in which $OA$ is 12 cm, $OB$ is 5 cm and angle $AOB$ is a right angle. Point $P$ lies on $AB$ and $OP$ is an arc of a circle with centre $A$. Point $Q$ lies on $AB$ and $OQ$ is an arc of a circle with centre $B$.

(i) Show that angle $BAO$ is 0.3948 radians, correct to 4 decimal places. \hspace{1cm} [1]

(ii) Calculate the area of the shaded region. \hspace{1cm} [5]
Question 12

In the diagram, $AB$ is an arc of a circle with centre $O$ and radius 4 cm. Angle $AOB$ is $\alpha$ radians. The point $D$ on $OB$ is such that $AD$ is perpendicular to $OB$. The arc $DC$, with centre $O$, meets $OA$ at $C$.

(i) Find an expression in terms of $\alpha$ for the perimeter of the shaded region $ABDC$. \hspace{1cm} [4]

(ii) For the case where $\alpha = \frac{1}{6}\pi$, find the area of the shaded region $ABDC$, giving your answer in the form $k\pi$, where $k$ is a constant to be determined. \hspace{1cm} [4]

Question 13

In the diagram, $OAB$ is a sector of a circle with centre $O$ and radius $r$. The point $C$ on $OB$ is such that angle $ACO$ is a right angle. Angle $AOB$ is $\alpha$ radians and is such that $AC$ divides the sector into two regions of equal area.

(i) Show that $\sin\alpha \cos\alpha = \frac{1}{2}\alpha$. \hspace{1cm} [4]

It is given that the solution of the equation in part (i) is $\alpha = 0.9477$, correct to 4 decimal places.

(ii) Find the ratio

\[
\text{perimeter of region } OAC : \text{perimeter of region } ACB,
\]

\hspace{1cm} giving your answer in the form $k : 1$, where $k$ is given correct to 1 decimal place. \hspace{1cm} [5]

(iii) Find angle $AOB$ in degrees. \hspace{1cm} [1]
Question 14

In the diagram, $AYB$ is a semicircle with $AB$ as diameter and $OAXB$ is a sector of a circle with centre $O$ and radius $r$. Angle $AOB = 2\theta$ radians. Find an expression, in terms of $r$ and $\theta$, for the area of the shaded region. [4]

Question 15

The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre $O$. The radius of sector $COD$ is $2r$ and angle $COD$ is $\theta$ radians. The radius of each of the sectors $BOA$ and $FOE$ is $r$, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of $\theta$. [4]
Question 16

The diagram shows a metal plate $OABC$, consisting of a right-angled triangle $OAB$ and a sector $OBC$ of a circle with centre $O$. Angle $AOB = 0.6$ radians, $OA = 6$ cm and $OA$ is perpendicular to $OC$.

(i) Show that the length of $OB$ is 7.270 cm, correct to 3 decimal places. [1]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Question 17

The diagram shows a circle with centre $A$ and radius $r$. Diameters $CAD$ and $BAE$ are perpendicular to each other. A larger circle has centre $B$ and passes through $C$ and $D$.

(i) Show that the radius of the larger circle is $r/2$. [1]

(ii) Find the area of the shaded region in terms of $r$. [6]
Question 18

(a)

In Fig. 1, $OAB$ is a sector of a circle with centre $O$ and radius $r$. $AX$ is the tangent at $A$ to the arc $AB$ and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. \hspace{1cm} [2]

(ii) Find the area of the shaded segment in terms of $r$ and $\alpha$. \hspace{1cm} [2]

(b)

In Fig. 2, $ABC$ is an equilateral triangle of side $4$ cm. The lines $AX$, $BX$ and $CX$ are tangents to the equal circular arcs $AB$, $BC$ and $CA$. Use the results in part (a) to find the area of the shaded region, giving your answer in terms of $\pi$ and $\sqrt{3}$. \hspace{1cm} [6]
Question 19

The diagram shows triangle $ABC$ where $AB = 5\text{ cm}$, $AC = 4\text{ cm}$ and $BC = 3\text{ cm}$. Three circles with centres at $A$, $B$ and $C$ have radii $3\text{ cm}$, $2\text{ cm}$ and $1\text{ cm}$ respectively. The circles touch each other at points $E$, $F$ and $G$, lying on $AB$, $AC$ and $BC$ respectively. Find the area of the shaded region $EFG$. 

[7]

Question 20

The diagram shows a circle with radius $r\text{ cm}$ and centre $O$. The line $PT$ is the tangent to the circle at $P$ and angle $POT = \alpha$ radians. The line $OT$ meets the circle at $Q$.

(i) Express the perimeter of the shaded region $PQT$ in terms of $r$ and $\alpha$. 

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. 

[3]
Question 21

In the diagram, triangle $ABC$ is right-angled at $C$ and $M$ is the mid-point of $BC$. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of $BM$ and $MC$ by $x$,

(i) find $AM$ in terms of $x$. \[3\]
(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. \[2\]

Question 22

The diagram shows a major arc $AB$ of a circle with centre $O$ and radius 6 cm. Points $C$ and $D$ on $OA$ and $OB$ respectively are such that the line $AB$ is a tangent at $E$ to the arc $CED$ of a smaller circle also with centre $O$. Angle $COD = 1.8$ radians.

(i) Show that the radius of the arc $CED$ is 3.73 cm, correct to 3 significant figures. \[2\]
(ii) Find the area of the shaded region. \[4\]
Question 23

The diagram shows a metal plate $ABCD$ made from two parts. The part $BCD$ is a semicircle. The part $DAB$ is a segment of a circle with centre $O$ and radius 10 cm. Angle $BOD$ is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Question 24

In the diagram $OCA$ and $ODB$ are radii of a circle with centre $O$ and radius $2r$ cm. Angle $AOB = \alpha$ radians. $CD$ and $AB$ are arcs of circles with centre $O$ and radii $r$ cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of $\alpha$. [2]

(ii) It is given that the area of the shaded region is $30\text{ cm}^2$. Find the value of $r$. [3]
Question 25

In the diagram, $AB = AC = 8 \text{ cm}$ and angle $CAB = \frac{2}{7} \pi$ radians. The circular arc $BC$ has centre $A$, the circular arc $CD$ has centre $B$ and $ABD$ is a straight line.

(i) Show that angle $CBD = \frac{9}{14} \pi$ radians. [1]

(ii) Find the perimeter of the shaded region. [5]

Question 26

The diagram shows two circles with centres $A$ and $B$ having radii $8 \text{ cm}$ and $10 \text{ cm}$ respectively. The two circles intersect at $C$ and $D$ where $CAD$ is a straight line and $AB$ is perpendicular to $CD$.

(i) Find angle $ABC$ in radians. [1]

(ii) Find the area of the shaded region. [6]
Question 27

The diagram shows a circle with radius $r$ cm and centre $O$. Points $A$ and $B$ lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of $r$ and $\theta$. \hspace{1cm} [3]

(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. \hspace{1cm} [4]

Question 28

In the diagram, $OAXB$ is a sector of a circle with centre $O$ and radius 10 cm. The length of the chord $AB$ is 12 cm. The line $OX$ passes through $M$, the mid-point of $AB$, and $OX$ is perpendicular to $AB$. The shaded region is bounded by the chord $AB$ and by the arc of a circle with centre $X$ and radius $XA$.

(i) Show that angle $AXB$ is 2.498 radians, correct to 3 decimal places. \hspace{1cm} [3]

(ii) Find the perimeter of the shaded region. \hspace{1cm} [3]

(iii) Find the area of the shaded region. \hspace{1cm} [3]
Question 29

The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point $P$ lies on $DC$ and $AP$ is an arc of a circle with centre $B$. Point $Q$ lies on $DC$ and $AQ$ is an arc of a circle with centre $D$.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Calculate the areas of the sectors $BAP$ and $DAQ$. [3]

(iii) Calculate the area of the shaded region. [3]

Question 30

The diagram shows a semicircle with centre $O$ and radius $6$ cm. The radius $OC$ is perpendicular to the diameter $AB$. The point $D$ lies on $AB$, and $DC$ is an arc of a circle with centre $B$.

(i) Calculate the length of the arc $DC$. [3]

(ii) Find the value of

\[
\frac{\text{area of region } P}{\text{area of region } Q'}
\]

giving your answer correct to 3 significant figures. [4]
Question 31

The diagram shows an isosceles triangle $ABC$ in which $AC = 16\text{ cm}$ and $AB = BC = 10\text{ cm}$. The circular arcs $BE$ and $BD$ have centres at $A$ and $C$ respectively, where $D$ and $E$ lie on $AC$.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Find the area of the shaded region. [5]

Question 32

The diagram shows a sector $POQ$ of a circle of radius $10\text{ cm}$ and centre $O$. Angle $POQ$ is $2.2$ radians. $QR$ is an arc of a circle with centre $P$ and $POR$ is a straight line.

(i) Show that the length of $PQ$ is $17.8\text{ cm}$, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

(ii) Find the y-coordinate of $B$. [2]
Question 33

The diagram shows a triangle $OAB$ in which angle $OAB = 90^\circ$ and $OA = 5\, \text{cm}$. The arc $AC$ is part of a circle with centre $O$. The arc has length $6\, \text{cm}$ and it meets $OB$ at $C$. Find the area of the shaded region. [5]

Question 34

The diagram shows points $A$ and $B$ on a circle with centre $O$ and radius $r$. The tangents to the circle at $A$ and $B$ meet at $T$. The shaded region is bounded by the minor arc $AB$ and the lines $AT$ and $BT$. Angle $AOB$ is $2\theta$ radians.

(i) In the case where the area of the sector $AOB$ is the same as the area of the shaded region, show that $\tan \theta = 2\theta$. [3]

(ii) In the case where $r = 8\, \text{cm}$ and the length of the minor arc $AB$ is $19.2\, \text{cm}$, find the area of the shaded region. [3]
Question 35

![Diagram of a circle with points A, B, O, and T, and a sector OAB with radius r cm and angle \( \theta \) radians.]

The diagram shows a circle with centre \( O \) and radius \( r \) cm. The points \( A \) and \( B \) lie on the circle and \( AT \) is a tangent to the circle. Angle \( AOB = \theta \) radians and \( OBT \) is a straight line.

(i) Express the area of the shaded region in terms of \( r \) and \( \theta \). \([3]\)

(ii) In the case where \( r = 3 \) and \( \theta = 1.2 \), find the perimeter of the shaded region. \([4]\)