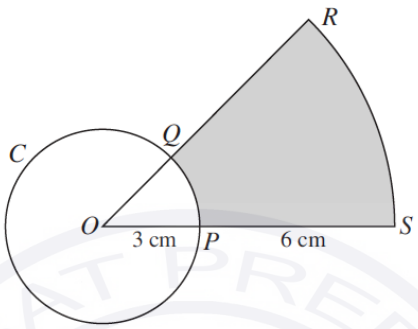
AS-Level

Pure Mathematics P1

Topic : Circular measure

May 2013- May 2025

Question 1

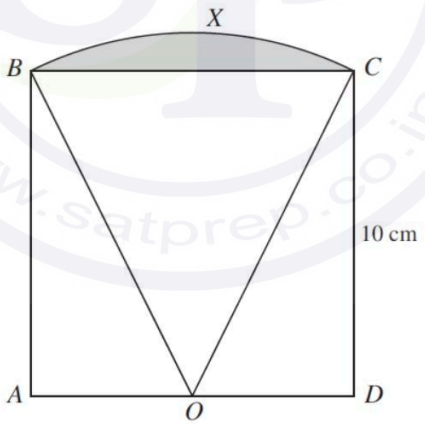


The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]

(ii) find the perimeter of the shaded region. [2]

Question 2



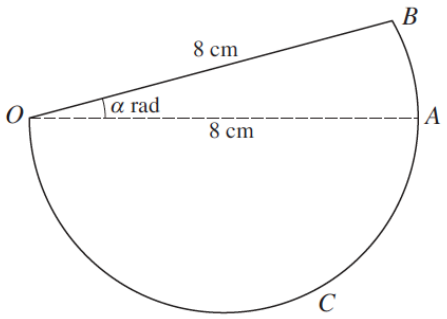
The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

Question 3

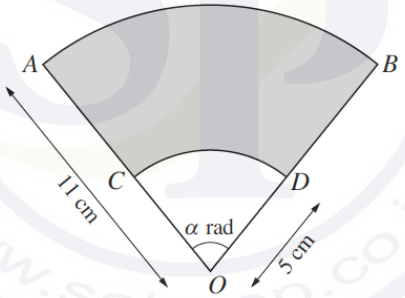


In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.

(i) Find α in terms of π. [3]

(ii) Find the perimeter of the complete figure in terms of π. [2]

Question 4



The diagram shows sector OAB with centre O and radius 11 cm. Angle $AOB = \alpha$ radians. Points C and D lie on OA and OB respectively. Arc CD has centre O and radius 5 cm.

(i) The area of the shaded region $ABDC$ is equal to k times the area of the unshaded region OCD. Find k. [3]

(ii) The perimeter of the shaded region $ABDC$ is equal to twice the perimeter of the unshaded region OCD. Find the exact value of α. [4]

Question 5

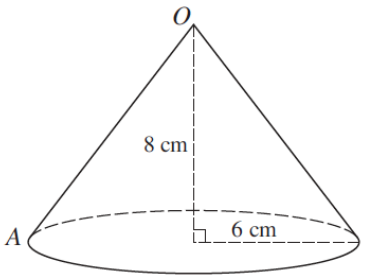


Fig. 1

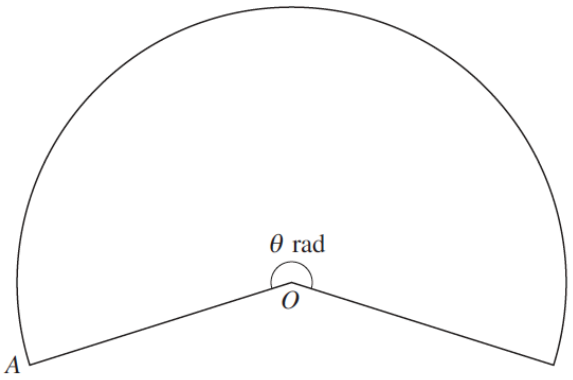


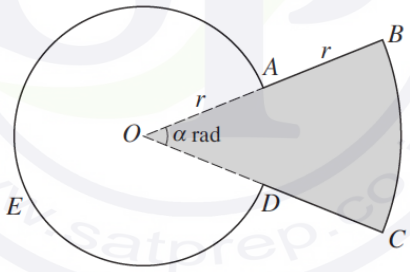
Fig. 2

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is θ radians. Calculate

(i) the value of θ, [4]

(ii) the area of paper needed to make the cone. [2]

Question 6



The diagram shows a metal plate made by fixing together two pieces, $OABCD$ (shaded) and $OAED$ (unshaded). The piece $OABCD$ is a minor sector of a circle with centre O and radius $2r$. The piece $OAED$ is a major sector of a circle with centre O and radius r. Angle AOD is α radians. Simplifying your answers where possible, find, in terms of α, π and r,

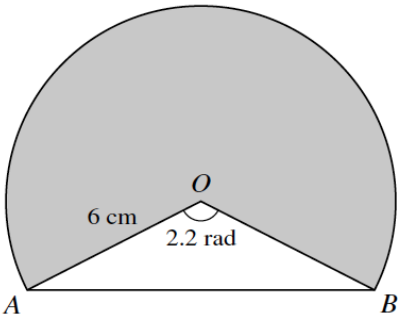
(i) the perimeter of the metal plate, [3]

(ii) the area of the metal plate. [3]

It is now given that the shaded and unshaded pieces are equal in area.

(iii) Find α in terms of π. [2]

Question 7

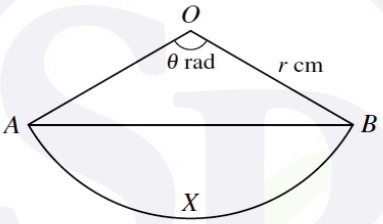


The diagram shows part of a circle with centre O and radius 6 cm. The chord AB is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region, [3]

(ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form $k : 1$. [3]

Question 8

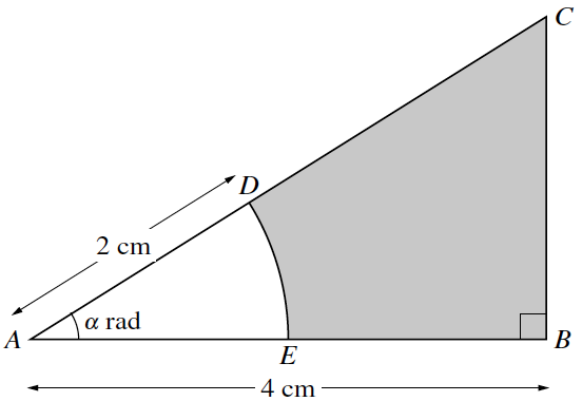


The diagram shows a sector of a circle with radius r cm and centre O. The chord AB divides the sector into a triangle AOB and a segment AXB. Angle AOB is θ radians.

(i) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant p for which $\theta = p \sin \theta$. [2]

(ii) In the case where $r = 8$ and $\theta = 2.4$, find the perimeter of the segment AXB. [3]

Question 9

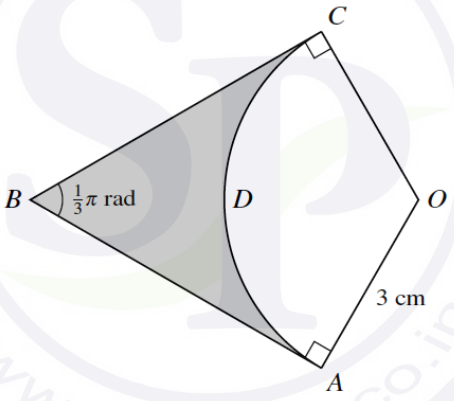


The diagram shows triangle ABC in which AB is perpendicular to BC. The length of AB is 4 cm and angle CAB is α radians. The arc DE with centre A and radius 2 cm meets AC at D and AB at E. Find, in terms of α,

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [3]

Question 10

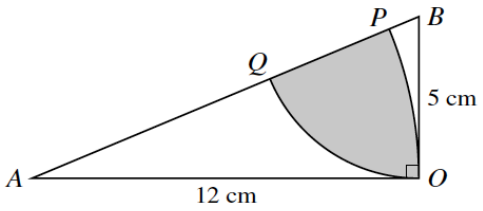


In the diagram, $OADC$ is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π,

(i) the perimeter of the shaded region, [3]

(ii) the area of the shaded region. [3]

Question 11

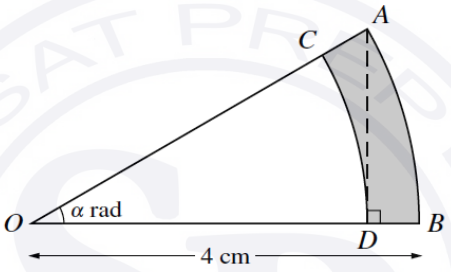


The diagram shows a triangle AOB in which OA is 12 cm, OB is 5 cm and angle AOB is a right angle. Point P lies on AB and OP is an arc of a circle with centre A. Point Q lies on AB and OQ is an arc of a circle with centre B.

(i) Show that angle BAO is 0.3948 radians, correct to 4 decimal places. [1]

(ii) Calculate the area of the shaded region. [5]

Question 12

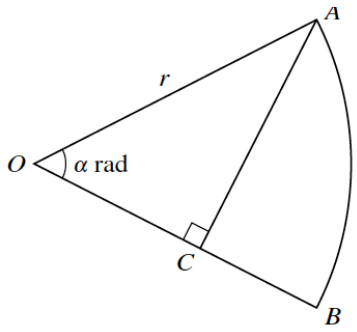


In the diagram, AB is an arc of a circle with centre O and radius 4 cm. Angle AOB is α radians. The point D on OB is such that AD is perpendicular to OB. The arc DC, with centre O, meets OA at C.

(i) Find an expression in terms of α for the perimeter of the shaded region $ABDC$. [4]

(ii) For the case where $\alpha = \frac{1}{6}\pi$, find the area of the shaded region $ABDC$, giving your answer in the form $k\pi$, where k is a constant to be determined. [4]

Question 13



In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

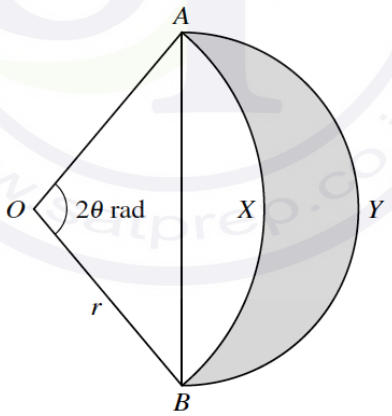
(ii) Find the ratio

perimeter of region OAC : perimeter of region ACB,

giving your answer in the form $k : 1$, where k is given correct to 1 decimal place. [5]

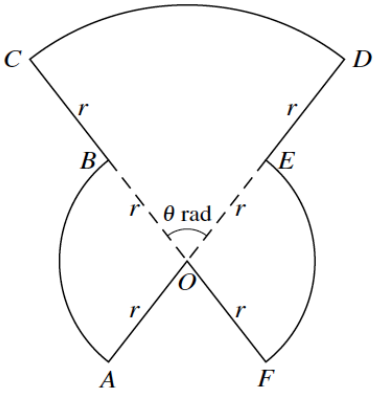
(iii) Find angle AOB in degrees. [1]

Question 14



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

Question 15

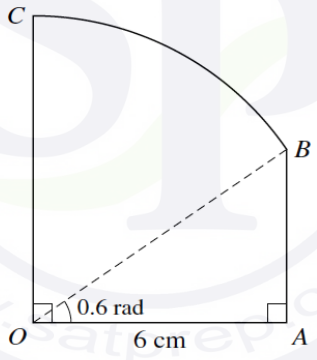


The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre O. The radius of sector COD is $2r$ and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of θ. [4]

Question 16



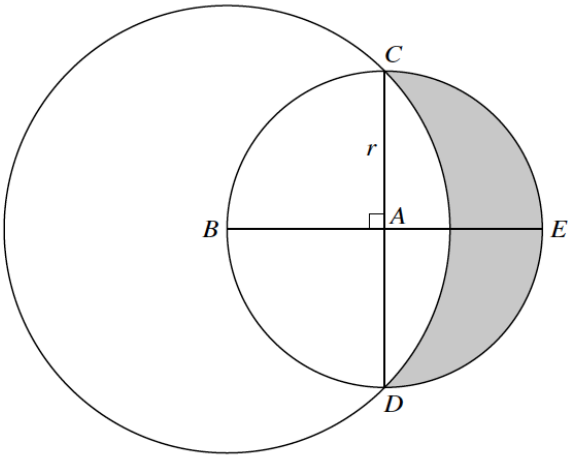
The diagram shows a metal plate $OABC$, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O. Angle $AOB = 0.6$ radians, $OA = 6$ cm and OA is perpendicular to OC.

(i) Show that the length of OB is 7.270 cm, correct to 3 decimal places. [1]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Question 17



The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

Question 18

(a)

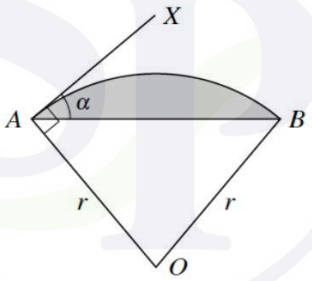


Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

Continue on the next pages…

(b)

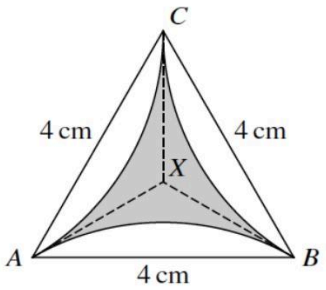
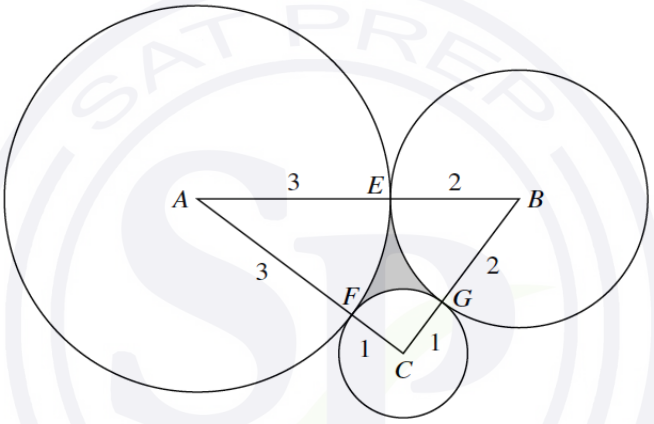


Fig. 2

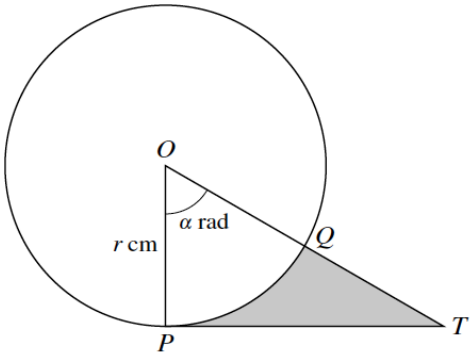
In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

Question 19



The diagram shows triangle ABC where $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG. [7]

Question 20

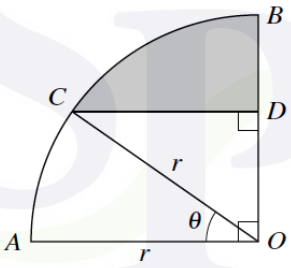


The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

Question 21

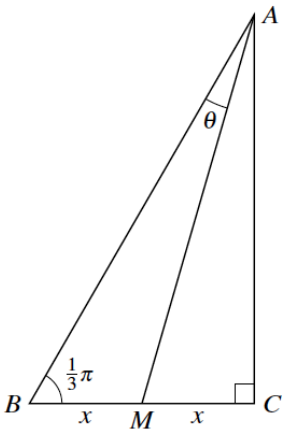


In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

Question 22

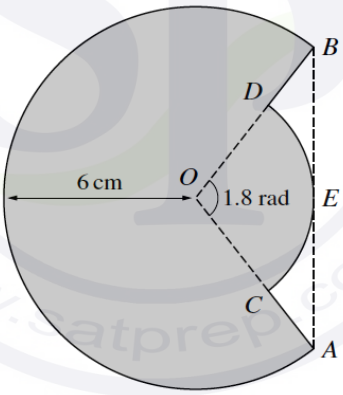


In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. [2]

Question 23

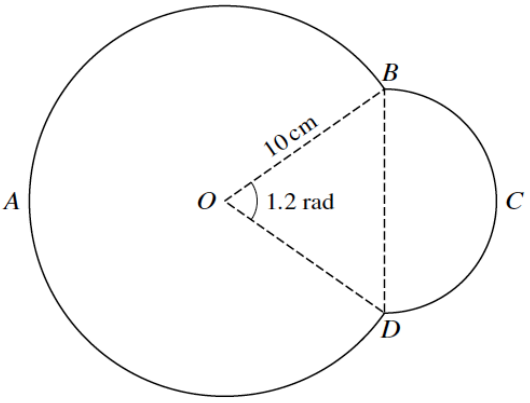


The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle $COD = 1.8$ radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

Question 24



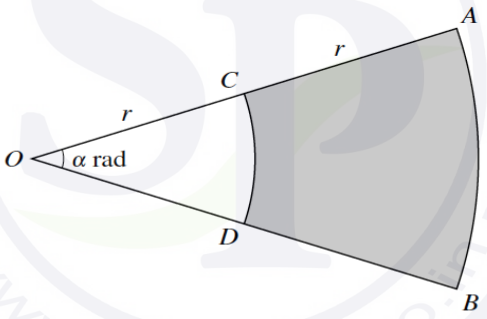
The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Question 25

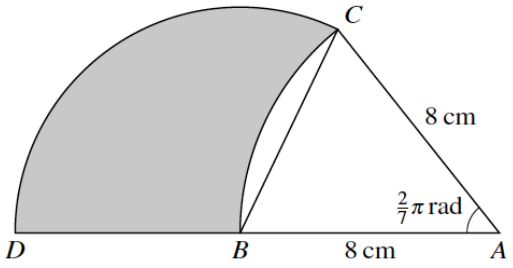


In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of α. [2]

(ii) It is given that the area of the shaded region is 30 cm^2. Find the value of r. [3]

Question 26

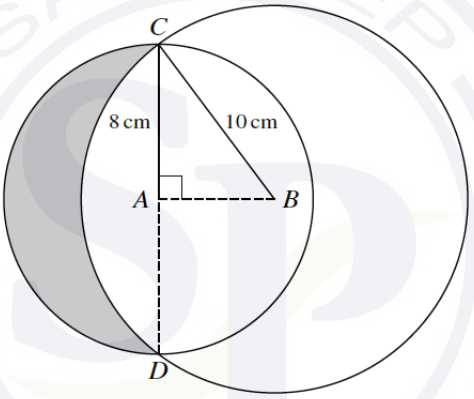


In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

(ii) Find the perimeter of the shaded region. [5]

Question 27

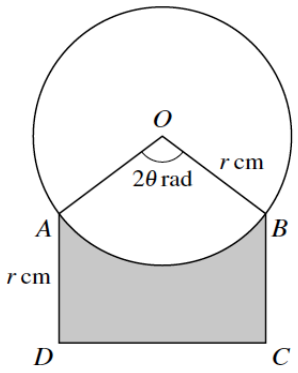


The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

(ii) Find the area of the shaded region. [6]

Question 28

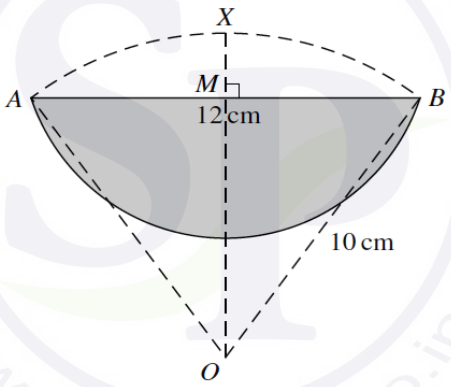


The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

Question 29



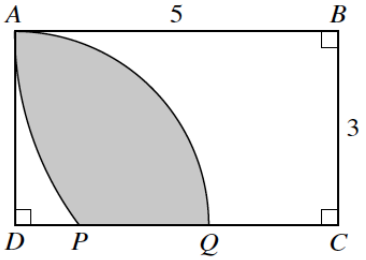
In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

Question 30



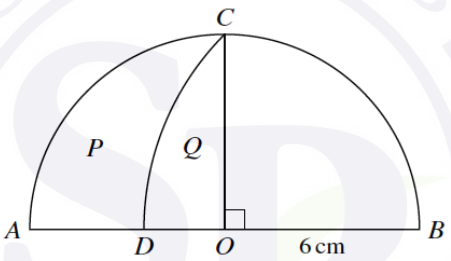
The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Calculate the areas of the sectors BAP and DAQ. [3]

(iii) Calculate the area of the shaded region. [3]

Question 31



The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

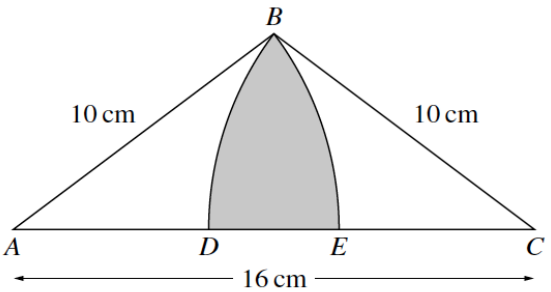
(i) Calculate the length of the arc DC. [3]

(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

Question 32

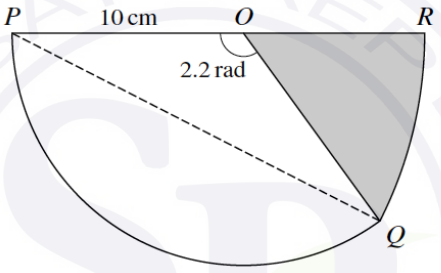


The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

(ii) Find the area of the shaded region. [5]

Question 33



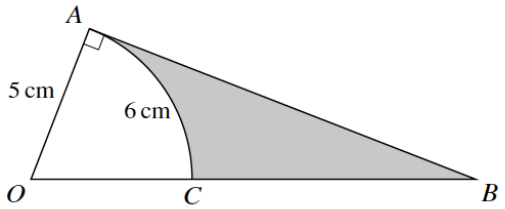
The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

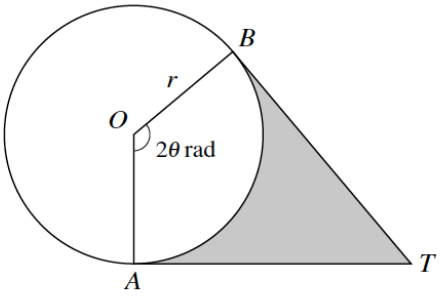
(ii) Find the y-coordinate of B. [2]

Question 34



The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

Question 35

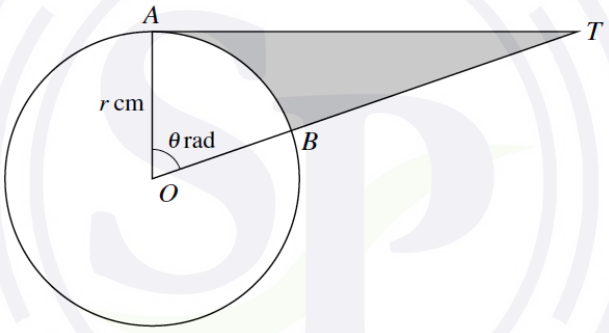


The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

Question 36

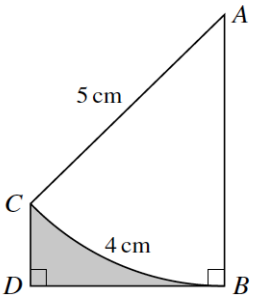


The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

Question 37

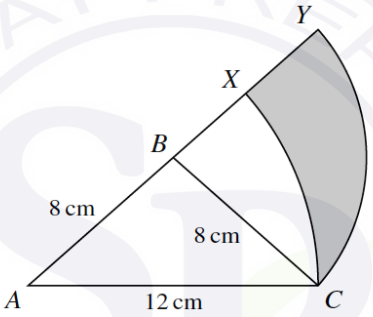


The diagram shows an arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.

(i) Find angle BAC in radians. [1]

(ii) Find the area of the shaded region BDC. [5]

Question 38

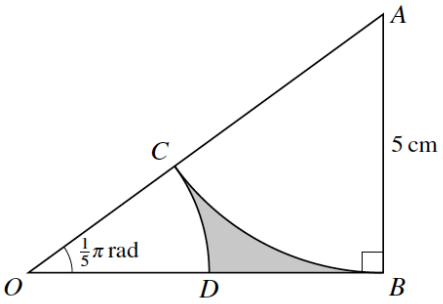


The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

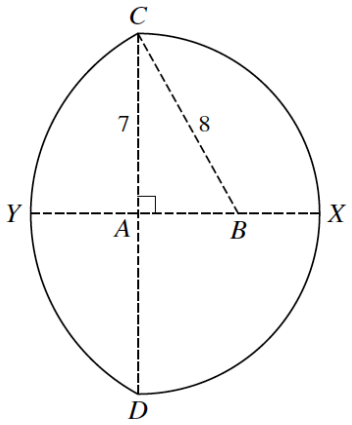
(ii) Find the perimeter of the shaded region. [4]

Question 39



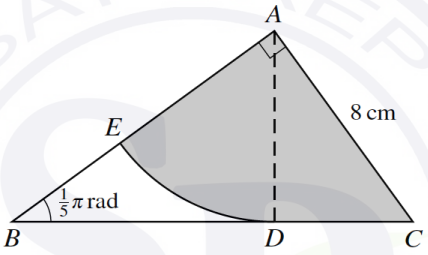
The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

Question 40



In the diagram, CXD is a semicircle of radius 7 cm with centre A and diameter CD. The straight line $YABX$ is perpendicular to CD, and the arc CYD is part of a circle with centre B and radius 8 cm. Find the total area of the region enclosed by the two arcs. [6]

Question 41

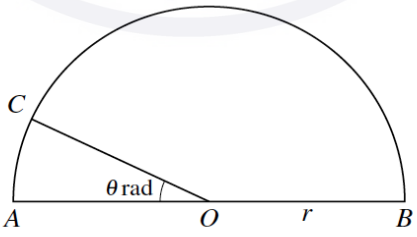


The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

(ii) Find the area of the shaded region. [3]

Question 42

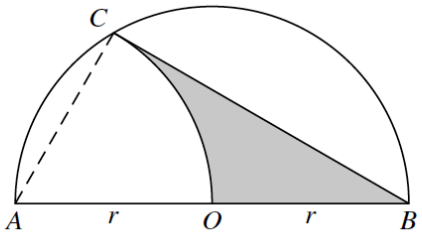


The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

Question 43

A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

Question 44

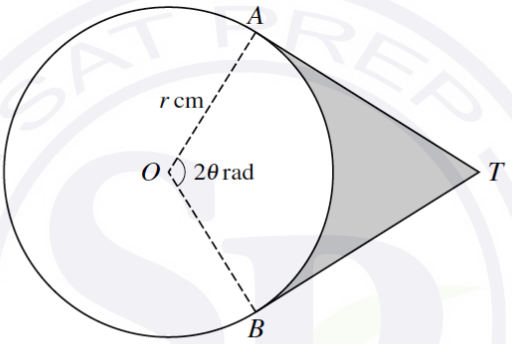


The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

Question 45

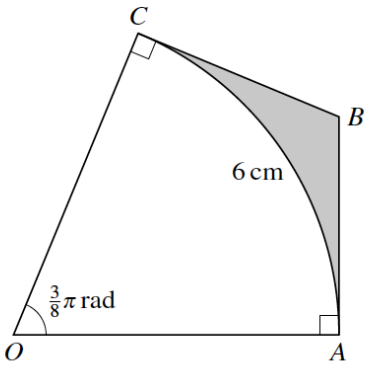


The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

Question 46



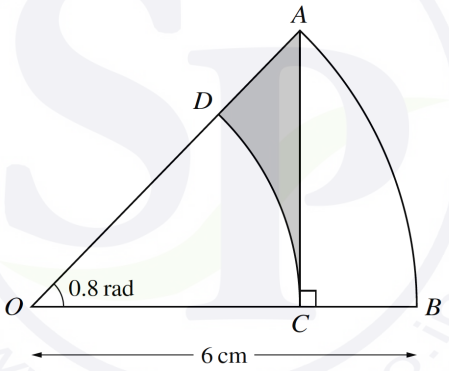
The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

(ii) Find the perimeter of the shaded region. [2]

(iii) Find the area of the shaded region. [4]

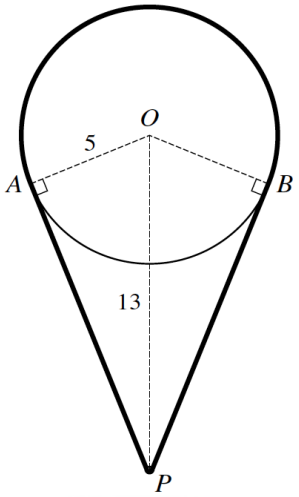
Question 47



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

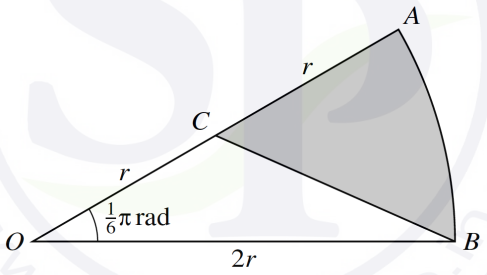
Question 48



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

Question 49



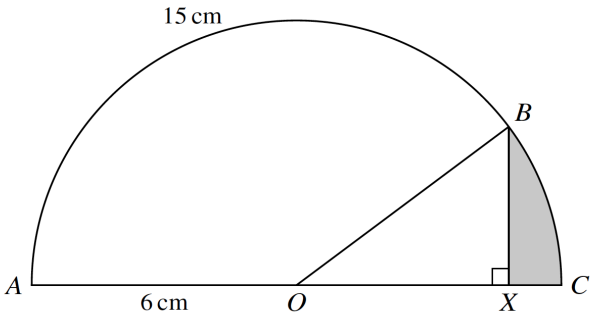
In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

(b) Find the exact perimeter of the shaded region. [2]

(c) Find the exact area of the shaded region. [3]

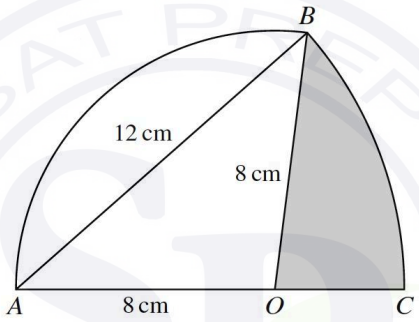
Question 50



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

Question 51



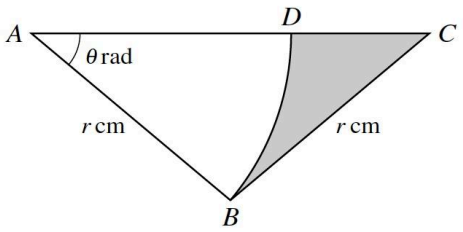
In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

(b) Find the area of the shaded region. [4]

(c) Find the perimeter of the shaded region. [3]

Question 52

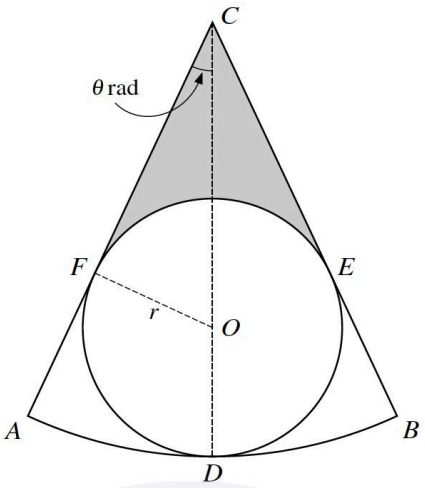


In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

Question 53



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

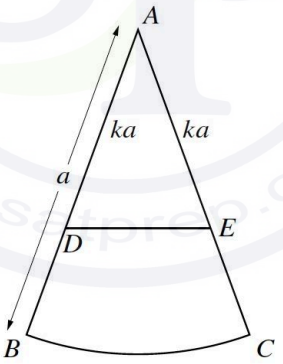
(a) Find CD in terms of r and $\sin\theta$. [3]

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

Question 54



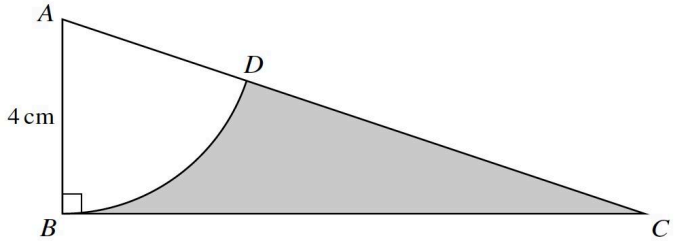
The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

Question 55

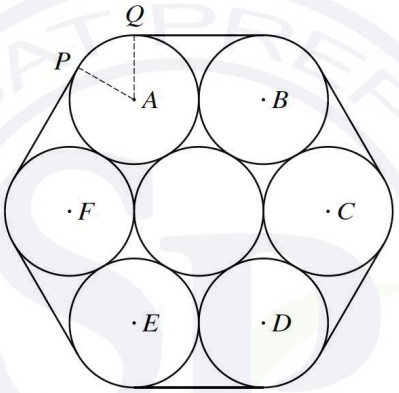


The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

(b) Find the perimeter of the shaded region. [4]

Question 56



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

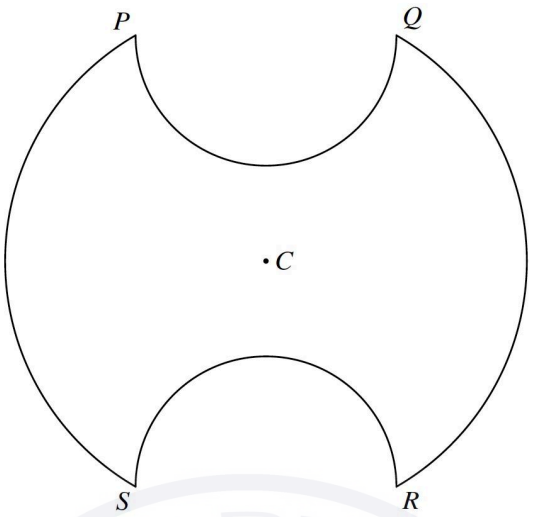
(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

(b) Find the length of the rope. [4]

(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]

Question 57



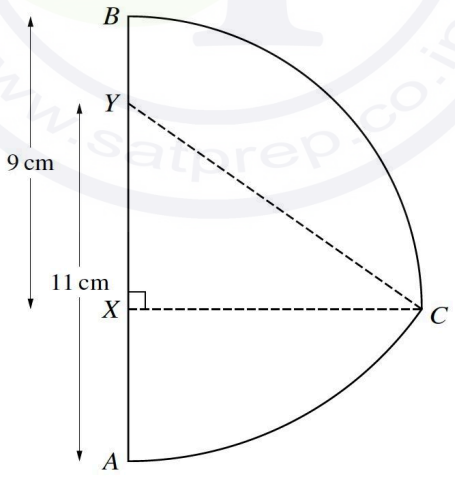
The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

(b) Find the exact perimeter of the plate. [3]

(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)$ cm^2. [5]

Question 58

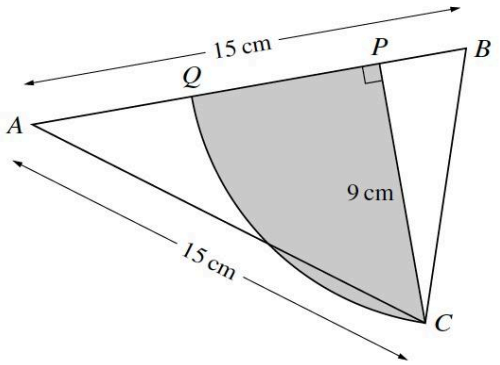


In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

(b) Find the perimeter of ABC. [6]

Question 59

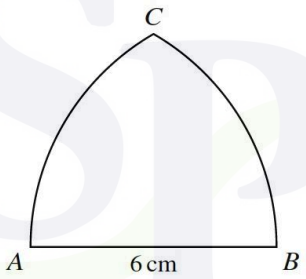


In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

Question 60

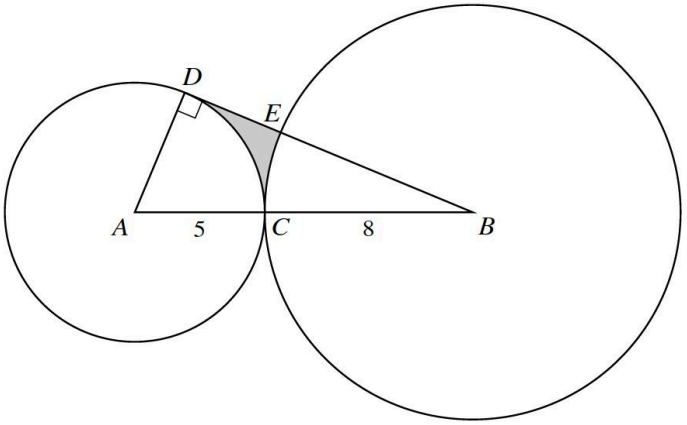


The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC. The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

Question 61

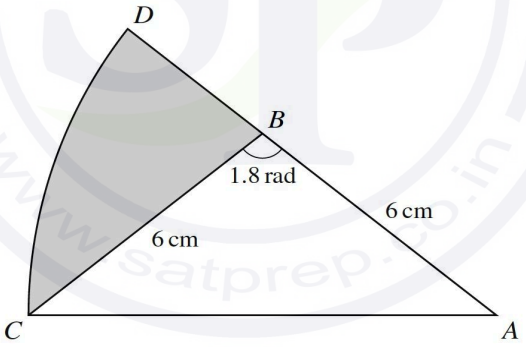


The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

Question 62

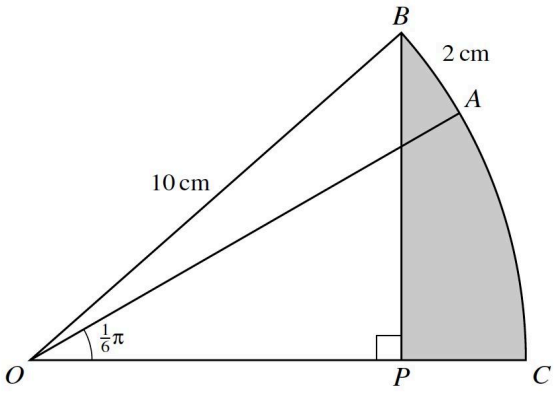


The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

Question 63

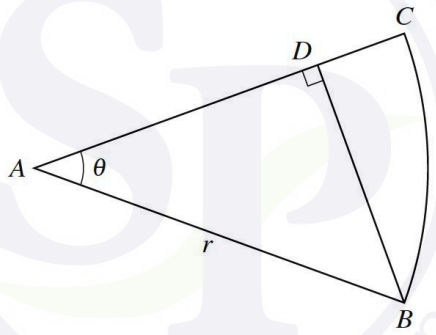


The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

(a) Find the angle BOC. [2]

(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

Question 64

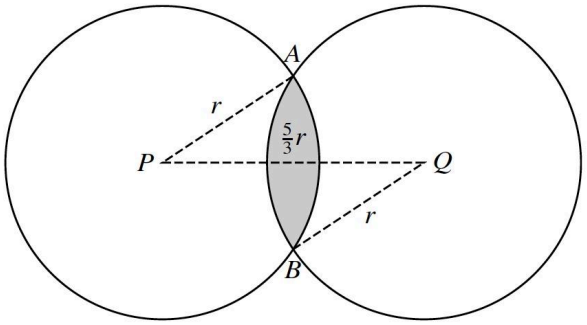


The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

Question 65

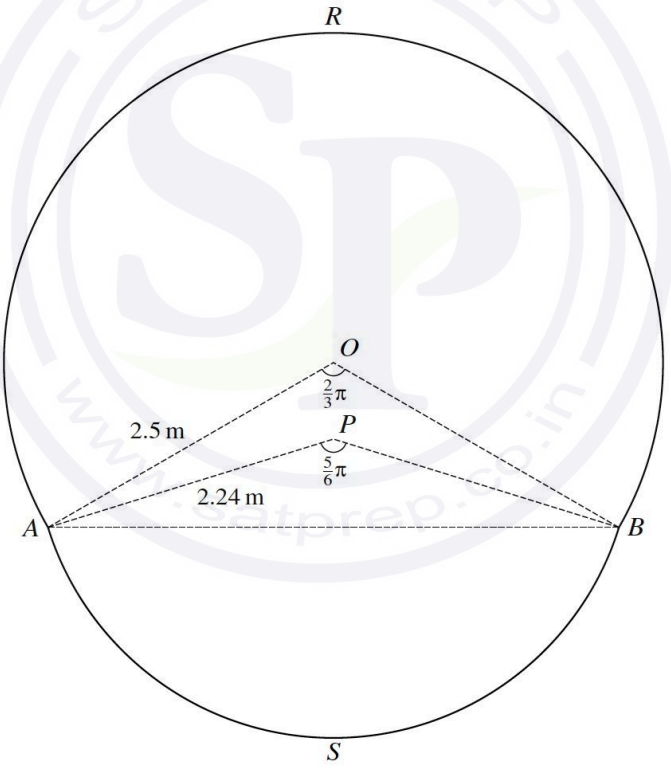


The diagram shows two identical circles intersecting at points A and B and with centres at P and Q. The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

(a) Find the perimeter of the shaded region in terms of r. [4]

(b) Find the area of the shaded region in terms of r. [3]

Question 66



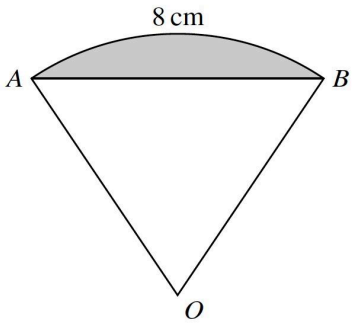
The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O, a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

Question 66

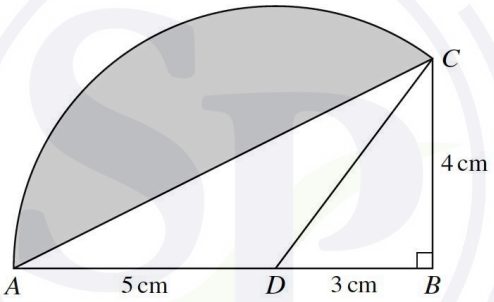


The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

(b) Find the area of the shaded segment. [2]

Question 67

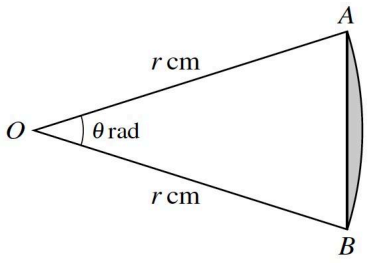


The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

Question 68

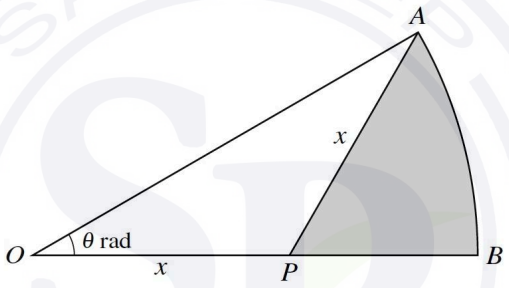


The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

(b) Find the perimeter of the shaded region. [2]

Question 69

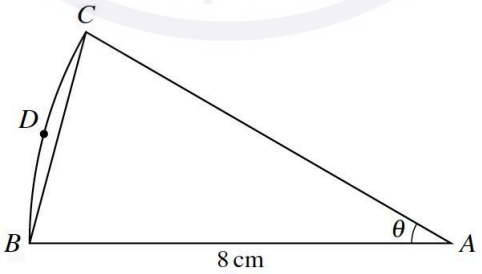


The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

(b) Find the area of the shaded region APB in terms of x and θ. [4]

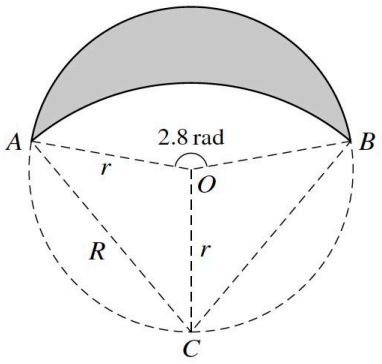
Question 70



The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi$ cm^2. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

Question 71



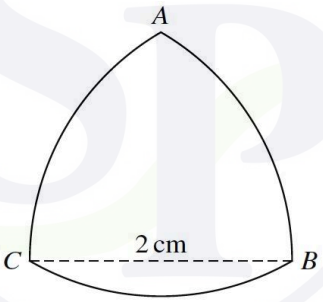
The diagram shows points A, B and C lying on a circle with centre O and radius r. Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r. The lower arc is part of a circle with centre C and radius R.

(a) State the size of angle ACO in radians. [1]

(b) Find R in terms of r. [1]

(c) Find the area of the shaded region in terms of r. [7]

Question 72

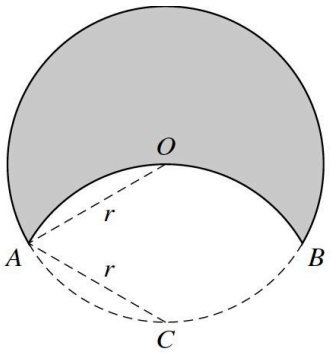


The diagram shows the shape of a coin. The three arcs AB, BC and CA are parts of circles with centres C, A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

(a) Find the perimeter of the coin. [2]

(b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

Question 73



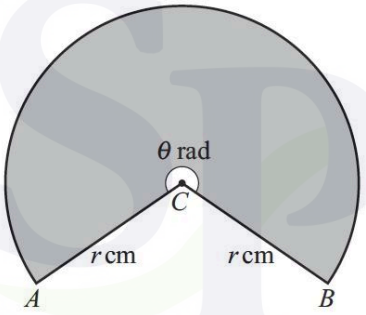
The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O, and the minor arc AOB of a circle, also with radius r but with centre C. The point C lies on the circle with centre O.

(a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k. [1]

(b) State the perimeter of the shaded motif in terms of π and r. [1]

(c) Find the area of the shaded motif, giving your answer in terms of π, r and $\sqrt{3}$. [5]

Question 74

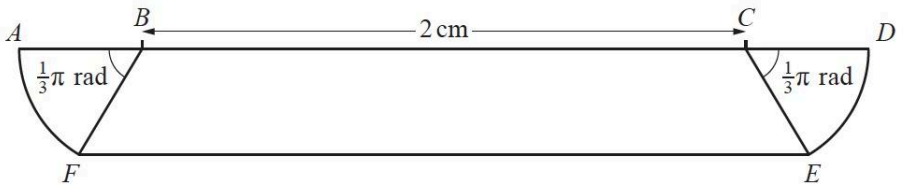


The diagram shows a sector of a circle with centre C. The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2.

(a) Find the values of r and θ. [4]

(b) Find the area of triangle ACB. [2]

Question 75



The diagram shows a symmetrical plate $ABCDEF$. The line $ABCD$ is straight and the length of BC is 2 cm. Each of the two sectors ABF and DCE is of radius r cm and each of the angles ABF and DCE is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

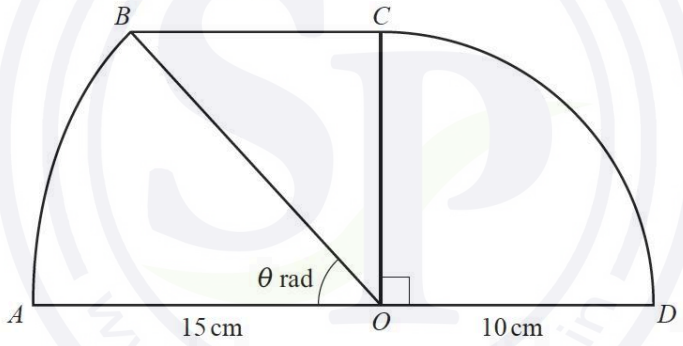
(i) Show that the length $EF = 2.4$ cm. [2]

(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r. Give your answer correct to 3 significant figures. [4]

Question 76

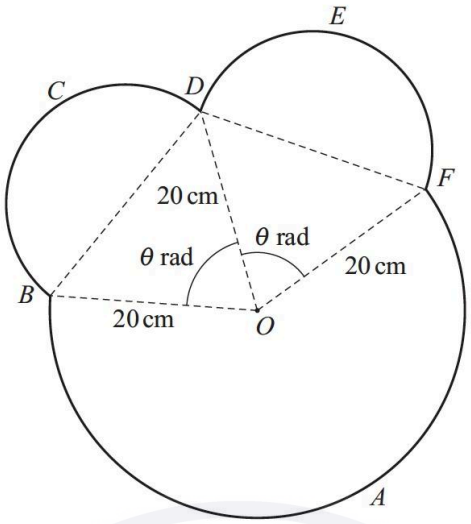


In the diagram, AOD and BC are two parallel straight lines. Arc AB is part of a circle with centre O and radius 15 cm. Angle $BOA = \theta$ radians. Arc CD is part of a circle with centre O and radius 10 cm. Angle $COD = \frac{1}{2}\pi$ radians.

(a) Show that $\theta = 0.7297$, correct to 4 decimal places. [1]

(b) Find the perimeter and the area of the shape $ABCD$. Give your answers correct to 3 significant figures. [7]

Question 77

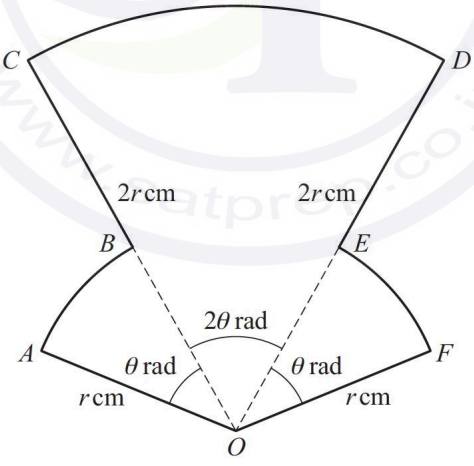


The diagram shows a metal plate $ABCDEF$ consisting of five parts. The parts BCD and DEF are semicircles. The part $BAFO$ is a sector of a circle with centre O and radius 20 cm, and D lies on this circle. The parts OBD and ODF are triangles. Angles BOD and DOF are both θ radians.

(a) Given that $\theta = 1.2$, find the area of the metal plate. Give your answer correct to 3 significant figures. [5]

(b) Given instead that the area of each semicircle is $50\pi\,\text{cm}^2$, find the exact perimeter of the metal plate. [5]

Question 78

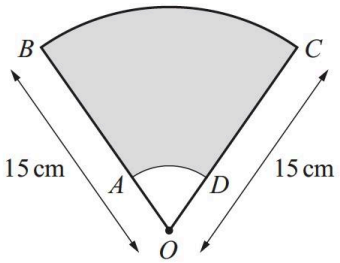


The diagram shows a metal plate $OABCDEF$ consisting of sectors of two circles, each with centre O. The radii of sectors AOB and EOF are r cm and the radius of sector COD is $2r$ cm.
Angle AOB = angle $EOF = \theta$ radians and angle $COD = 2\theta$ radians.

It is given that the perimeter of the plate is 14 cm and the area of the plate is $10\,\text{cm}^2$.

Given that $r > \frac{3}{2}$ and $\theta < \frac{3}{4}$, find the values of r and θ. [6]

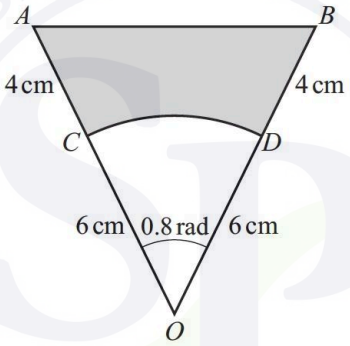
Question 79



The diagram shows a sector of a circle, centre O, where $OB = OC = 15\,\text{cm}$. The size of angle BOC is $\frac{2}{5}\pi$ radians. Points A and D on the lines OB and OC respectively are joined by an arc AD of a circle with centre O. The shaded region is bounded by the arcs AD and BC and by the straight lines AB and DC. It is given that the area of the shaded region is $\frac{209}{5}\pi\,\text{cm}^2$.

Find the perimeter of the shaded region. Give your answer in terms of π. [5]

Question 80

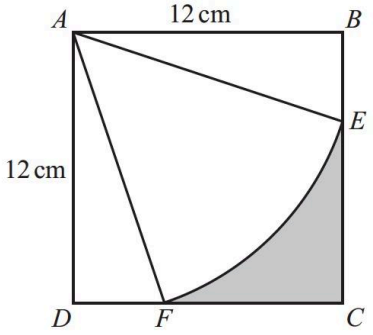


The diagram shows a triangle OAB where $OA = OB = 10\,\text{cm}$ and angle $AOB = 0.8$ radians.
Points C and D on OA and OB respectively are such that the arc CD is part of a circle with centre O and radius 6 cm. The shaded region is bounded by the arc CD and the line segments CA, AB and BD.

(a) Find the perimeter of the shaded region. [3]

(b) Find the area of the shaded region. [3]

Question 81



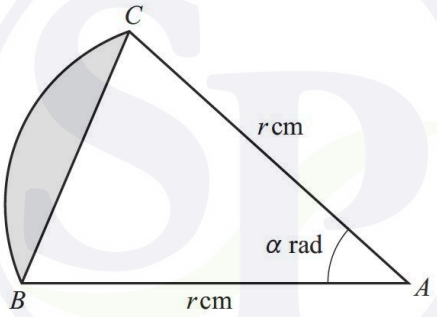
The diagram shows a square $ABCD$ where each side has length 12 cm. Points E and F lie on the sides BC and CD respectively and are such that $BE = \frac{1}{3}BC$ and $DF = \frac{1}{3}DC$. The arc EF is part of a circle with centre A. The shaded region is bounded by the arc EF and the line segments EC and FC.

(a) Show that the size of angle EAF is 0.9273 radians, correct to 4 significant figures. [2]

(b) Find the perimeter of the shaded region. [3]

(c) Find the area of the shaded region. [3]

Question 82



The diagram shows a sector ABC of a circle with centre A and radius r cm. The angle BAC is α radians, where $0 < \alpha < \frac{1}{2}\pi$.

(a) It is given that the area of the triangle ABC is $4\,\text{cm}^2$ and the area of the sector ABC is $8\alpha\,\text{cm}^2$.

Find the exact area of the shaded segment. [4]

(b) It is given instead that the length of the chord BC is $\frac{1}{\sqrt{2}}r$ cm but the area of the triangle ABC is still $4\,\text{cm}^2$.

Find the area of the shaded segment. Give your answer correct to 3 significant figures. [4]