AS-Level

Topic : Quadratics

May 2013-May 2023

Answer

Question 1

$mx+14=\frac{12}{x}+2 \rightarrow mx^2+12x-12=0$		
Uses $b^2=4ac \rightarrow m=-3$	M1	Eliminates x (or y)
$-3x^2+12x-12=0 \rightarrow P(2,8)$	M1	Any use of discriminant
	A1	
[Or $m=-12x^{-2}$ M1 Sub M1 $x=2$ A1]	DM1 A1	Any valid method.
[$\rightarrow m=-3$ and $y=8$ M1 A1]	[5]	

Question 2

$f: x \mapsto 2x^2-3x$, $g: x \mapsto 3x+k$,		
(i) $2x^2-3x-9>0$ $\rightarrow x=3$ or $-1\frac{1}{2}$	M1 A1	For solving quadratic. Ignore $>$ or $\geqslant$
Set of x $x>3$, or $x<-1\frac{1}{2}$	A1 [3]	condone $\geqslant$ or $\leqslant$
(ii) $2x^2-3x=2(x-\frac{3}{4})^2-\frac{9}{8}$	B3,2,1	$-x^2$ in bracket is an error.
Vertex $(\frac{3}{4},-\frac{9}{8})$	B1$\checkmark$ [4]	$\checkmark$ on 'c' and 'b'.
(iii) $gf(x)=6x^2-9x+k=0$	B1	
Use of $b^2-4ac \rightarrow k=\frac{27}{8}$ oe.	M1 A1 [3]	Used on a quadratic (even fg).

Question 3

$2x^2-10x+8 \rightarrow a(x+b)^2+c$		
(i) $a=2,\ b=-2\frac{1}{2},\ c=-4\frac{1}{2}$	$3\times$ B1	Or $2\left(x-2\frac{1}{2}\right)^2-4\frac{1}{2}$
$\rightarrow$ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2},-4\frac{1}{2})$	B1$\checkmark$ [4]	Can score by sub $x=2\frac{1}{2}$ into original but not by differentiation
(ii) $2x^2-10x+8-kx=0$		Sets equation to 0 and uses
Use of "b^2-4ac"	M1	discriminant correctly
$(-10-k)^2-64<0$ or $k^2+20k+36<0$	M1	Realises discriminant <0. Allow $\leqslant$
$\rightarrow k=-18$ or -2	A1	co Dep on 1st M1 only
$-18<k<-2$	A1 [4]	co

Question 4

(i) $x^2 + 4x + c - 8\ (= 0)$	**M1**	Attempt to simplify to 3-term quadratic
$16 - 4(c - 8) = 0$	**M1**	Apply $b^2 - 4ac = 0$. '= 0' soi
$c = 12$	**A1**	
$-2 - 2x = 2 \rightarrow x = (-2)$	**M1**	Equate derivs of curve and line. Expect $x = -2$
$-4 + c = 8 + 4 - 4$	**M1**	Sub *their* $x = -2$ into line and curve, and equate
$c = 12$	**A1** **[3]**	
(ii) $x^2 + 4x + 3 \rightarrow (x + 1)(x + 3)\ (= 0) \rightarrow$ $x = -1$ or -3	**B1**	
$\int(8 - 2x - x^2) - [\int(2x + 11)$ or area of trapezium$]$	**M1M1**	Attempt to integrate. At some stage subtract
$-x^2 - \left[\frac{x^3}{3}\right] - [x^2 + 11x]$ or $\left[8x - x^2 - \frac{x^3}{3}\right] - \frac{1}{2}(5 + 9) \times 2$	**A1B1**	A1 for curve, B1 for line **OR** $\left[-3x - 2x^2 - \frac{x^3}{3}\right]$ A2,1,0
Apply *their* limits to at least integral for curve	**M1**	For M marks allow reversed limits and/or subtraction of areas but then final A0
$1\frac{1}{3}$ oe	**A1** **[7]**	

Question 5

(i) $(2x - 3)^2 - 9$	**B1B1** **[2]**	For -3 and -9
(ii) $2x - 3 > 4 \quad 2x - 3 < -4$	**M1**	At least one of these statements
$x > 3\frac{1}{2}$ (*or*) $x < -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$	**A1**	Allow 'and' $3\frac{1}{2}$, $-\frac{1}{2}$ soi scores first M1
$4x^2 - 12x - 7 \rightarrow (2x - 7)(2x + 1)$	**M1**	Attempt to solve 3-term quadratic
$x > 3\frac{1}{2}$ (*or*) $< -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$	**A1** **[2]**	Allow 'and' $3\frac{1}{2}$, $-\frac{1}{2}$ soi scores first M1

Question 6

$x^2 + x(k - 2) + (k - 2)(= 0)$	**M1**	Equate and move terms to one side of equ.
$(k - 2)^2 - 4(k - 2)(> 0)$ soi	**M1**	Apply $b^2 - 4ac$ (>0). Allow $\geqslant$ at this stage.
$(k - 2)(k - 6)(> 0)$	**DM1**	
$k < 2$ or $k > 6$ (condone $\leqslant, \geqslant$) Allow $\{-\infty, 2\}$U$\{6, \infty\}$ etc.	**A2** **[5]**	Attempt to factorise or solve or find 2 solns. **SCA1** for 2, 6 seen with wrong inequalities

Question 7

$2(x-3)^2-11$	**B1B1B1** **[3]**	For 2, $(x-3)^2$, -11. Or a=2, b= 3, c= 11

Question 8

$x^2-4x+c=2x-7 \to x^2-6x+c+7(=0)$	**M1**	All terms on one side
$36-4(c+7)<0$	**DM1**	Apply $b^2-4ac<0$. Allow $\leqslant$.
$c>2$	**A1**	
	[3]	

Question 9

(a)	$y=2x^2-4x+8$	**M1**		
	Equates with $y=mx$ and selects a, b, c	**M1**		Equate + solution or use of dy/dx
	Uses $b^2=4ac$	**A1**		Use of discriminant
	$\to m=4$ or -12.		[3]	for both.
(b) (i)	$f(x)=x^2+ax+b$	**M1**		Any valid method allow
	Eqn of form $(x-1)(x-9)$			$(x+1)(x+9)$ for **M1**
	$\to a=-10$, $b=9$	**A1**		must be stated
	(or using 2 sim eqns **M1 A1**)		[2]	
(ii)	Calculus or $x=\frac{1}{2}(1+9)$ by symmetry	**M1**		Any valid method
	$\to(5,-16)$	**A1**		
			[2]	

Question 10

$kx^2-3x=x-k \Rightarrow kx^2-4x+k(=0)$	**M1**		Eliminate y and rearrange into 3-term quad
$(-4)^2-4(k)(k)$ soi	**M1**		b^2-4ac .
$k>2$, $k<-2$ cao Allow $(2,\infty)$ etc. Allow $2<k<-k$	**A1**		
		[3]	

Question 11

(i)	$2x^2-6x+5>13$ $2x^2-6x-8(>0)$ $(x=)-1$ and 4. $x>4,\ x<-1$	 **M1** **A1** **A1**	[3]	Sets to 0 + attempts to solve Both values required Allow all recognisable notation.
(ii)	$2x^2-6x+5=2x+k$ $\rightarrow 2x^2-8x+5-k(=0)$ Use of b^2-4ac $\rightarrow -3$	 **M1*** **DM1** **A1**	[3]	 Equates and sets to 0. Use of discriminant
	OR $\frac{dy}{dx}=4x-6$ $4x-6=2$ $x=2$ $x=2\rightarrow y=1$ Using *their* (2,1) in $y=2x+k$ or $y=2x^2-6x+5$ $\rightarrow k=-3$	 **M1*** **DM1** **A1**	[3]	 Sets (their $\frac{dy}{dx}$) = 2 Uses *their* $x=2$ and *their* $y=1$

Question 12

(i)	$(x+3)^2-7$	**B1B1**	[2]	For $a=3,\ b=-7$
(ii)	$1,-7$ seen $x>1,\quad x<-7$ oe	**B1** **B1**	[2]	$x>1$ or $x<-7$ Allow $x\leqslant-7, x\geqslant1$ oe

Question 13

$(3k)^2-4\times2\times k$	**M1**	Attempt b^2-4ac
$9k^2-8k>0$ soi Allow $9k^2-8k\geqslant 0$	**A1**	Must involve correct inequality. Can be implied by correct answers
0, 8/9 soi	**A1**	
$k<0$, $k>8/9$ (or 0.889)	**A1**	Allow $(-\infty, 0)$, $(8/9, \infty)$
Total:	**4**	

Question 14

$ax+3a=-\frac{2}{x}\rightarrow ax^2+3ax+2\ (=0)$	***M1**	Rearrange into a 3-term quadratic.
Apply $b^2-4ac>0$ SOI	**DM1**	Allow $\geqslant$. If no inequalities seen, **M1** is implied by 2 correct final answers in a or x.
$a<0$, $a>\frac{8}{9}$ (or 0.889) OE	**A1 A1**	For final answers accept $0>a>\frac{8}{9}$ but not $\leqslant$, $\geqslant$.
	4	

Question 15

(i)	$1+cx=cx^2-3x \to cx^2-x(c+3)-1\ (=0)$	M1	Multiply throughout by x and rearrange terms on one side of equality
	Use $b^2-4ac\left[=(c+3)^2+4c=c^2+10c+9 \text{ or } (c+5)^2-16\right]$	M1	Select their correct coefficients which must contain 'c' twice Ignore = 0, < 0, >0 etc. at this stage
	(Critical values) –1, –9	A1	SOI
	$c\leqslant-9,\quad c\geqslant-1$	A1	
		4	
(ii)	Sub their c to obtain a quadratic $\left[c=-1\to -x^2-2x-1(=0)\right]$	M1	
	$x=-1$	A1	
	Sub their c to obtain a quadratic $\left[c=(-9\to -9x^2+6x-1(=0)\right]$	M1	
	$x=1/3$	A1	[Alt 1: $dy/dx=-1/x^2=c$, when $c=-1, x=\pm1, c=-9, x=\pm\frac{1}{3}$ Give M1 for equating the gradients, A1 for all four answers and M1A1 for checking and eliminating] [Alt 2: $dy/dx=-1/x^2=c$ leading to $1/x-1/x^2=(-1/x^2)(x)-3$ Give M1 A1 at this stage and M1A1 for solving]
		4	

Question 16

[3] $\left[(x-2)^2\right]$ [–5]	B1B1B1	OR $a=3$, $b=-2$, $c=-5$. 1st mark is dependent on the form $(x+a)^2$ following 3
	3	

Question 17

(i)			A complete method as far as finding a set of values for k by:
	Either $(x-3)^2+k-9>0$, $k-9>0$		Either completing the square and using '*their* $k-9$' > or $\geqslant 0$ OR
	or $2x-6=0\to(3, k-9)$, $k-9>0$	M1	Differentiating and setting to 0, using '*their* $x=3$' to find y and using '*their* $k-9$' > or $\geqslant$0 OR
	or $b^2<4ac$ oe $\to 36<4k$		Use of discriminant < or $\leqslant$ 0. Beware use of > and incorrect algebra.
	$\to k>9$ Note: not $\geqslant$	A1	T&I leading to (or no working) correct answer 2/2 otherwise 0/2.
		2	
(ii)	**EITHER**		
	$x^2-6x+k=7-2x\to x^2-4x+k-7\ (=0)$	*M1	Equates and collects terms.
	Use of $b^2-4ac=0$ $(16-4(k-7)=0)$	DM1	Correct use of discriminant = 0, involving k from a 3 term quadratic.
	OR		
	$2x-6=-2\to x=2$ $(y=3)$	*M1	Equates their $\frac{dy}{dx}$ to ± 2, finds a value for x.
	(*their* 3) or 7– 2(*their* 2) = (*their* 2)^2 – 6(*their* 2) + k	DM1	Substitutes their value(s) into the appropriate equation.
	$\to k=11$	A1	
		3	

Question 18

(i)	For *their* 3-term quad a recognisable application of b^2-4ac	M1	Expect $2x^2-x(3+k)+1-k^2\ \ (=0)$ oe for the 3-term quad.
	$(b^2-4ac=)\ (3+k)^2-4(2)(1-k^2)$ oe	A1	Must be correct. Ignore any RHS
	$9k^2+6k+1$	A1	Ignore any RHS
	$(3k+1)^2\geqslant 0$ Do not allow > 0. Hence curve and line meet. **AG**	A1	Allow $(9)\left(k+\frac{1}{3}\right)^2\geqslant 0$. Conclusion required.
	ALT Attempt solution of 3-term quadratic	M1	
	Solutions $x=k+1,\quad ½(1-k)$	A1A1	
	Which exist for all values of k. Hence curve and line meet. **AG**	A1	
		4	
(ii)	$k=-1/3$	B1	**ALT** $dy/dx=4x-3\Rightarrow 4x-3=k$
	Sub (one of) *their* $k=-⅓$ into either line 1 $\rightarrow 2x^2-\frac{8}{3}x+\frac{8}{9}\ (=0)$ Or into the derivative of line 1 $\rightarrow 4x-(3+k)(=0)$	M1	Sub $k=4x-3$ into line 1 $\rightarrow 2x^2-x(4x)+1-(4x-3)^2(=0)$
	x = 2/3 Do not allow unsubstantiated $\left(\frac{2}{3},-\frac{1}{9}\right)$ following $k=-\frac{1}{3}$	A1	x = 2/3, y = –1/9 (both required) [from $-18x^2+24x-8\ \ (=0)$ oe]
	y = –1/9 Do not allow unsubstantiated $\left(\frac{2}{3},-\frac{1}{9}\right)$ following $k=-\frac{1}{3}$	A1	$k=-1/3$
		4	

Question 19

(i)	$2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ $\rightarrow$ 3 term quadratic.	*M1	Attempt to eliminate y (or x) to form a 3 term quadratic. Expect $3x^2-kx+12$ or $3y^2-5ky+(2k^2+12)\ (=0)$
	Use of $b^2-4ac\rightarrow k^2-144<0$	DM1	Using the discriminant, allow $\leqslant$, = 0; expect 12 and −12
	$-12<k<12$	A1	Do NOT accept $\leqslant$. Separate statements OK.
		3	
(ii)	Using $k=15$ in their 3 term quadratic	M1	From (i) or restart. Expect $3x^2-15x+12$ or $3y^2-75y+462\ (=0)$
	x = 1,4 or y = 11, 14	A1	Either pair of x or y values correct..
	(1, 14) and (4, 11)	A1	Both pairs of coordinates
		3	
(iii)	Gradient of $AB=-1\rightarrow$ Perpendicular gradient = +1	B1FT	Use of $m_1m_2=-1$ to give +1 or ft from their A and B.
	Finding their midpoint using their (1, 14) and (4, 11)	M1	Expect (2½, 12½)
	Equation: $y-12½=(x-2½)$ [$y=x+10$]	A1	Accept correct unsimplified and isw
		3	

Question 20

$\left(4x^{½}-3\right)\left(x^{½}-2\right)$ oe soi Alt: $4x+6=11\sqrt{x}\Rightarrow 16x^2-73x+36$	**M1**	Attempt solution for $x^{½}$ or sub $u=x^{½}$
$x^{½}=3/4$ or 2 $(16x-9)(x-4)$	**A1**	Reasonable solutions for $x^{½}$ implies M1 ($x=2, 3/4$, M1A0)
$x=9/16$ oe or 4	**A1**	Little or no working shown scores SCB3, spotting one solution, B0
	3	

Question 21

$x^2+bx+5=x+1\rightarrow x^2+x(b-1)+4\ (=0)$	**M1**	Eliminate x or y with all terms on side of an equation
$(b^2-4ac=)\ \ (b-1)^2-16$	**M1**	
b associated with -3 & $+5$ or $b-1$ associated with ± 4	**A1**	$(x-2)^2=0\, or\, (x+2)^2=0, x=\pm 2, b-1=\pm 4$ (M1A1) Association can be an equality or an inequality
$b\geqslant 5, b\leqslant -3$	**A1**	
	4	

Question 22

2(i)	Eliminates x or $y\rightarrow\ y^2-4y+c-3=0$ or $x^2+(2c-16)x+c^2-48=0$	**M1**	Eliminates x or y completely to a quadratic
	Uses $b^2=4ac\ \rightarrow 4c-28=0$	**M1**	Uses discriminant = 0. (c the only variable) Any valid method (may be seen in part **(i)**)
	$c=7$	**A1**	
	Alternative method for question 2(i)		
	$\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{(x+3)}}=\dfrac{1}{4}$	**M1**	
	Solving	**M1**	
	$c=7$	**A1**	
		3	
(ii)	Uses $c=7, y^2-4y+4=0$	**M1**	Ignore (1,–2), c=-9
	(1, 2)	**A1**	
		2	

Question 23

(i)	$3kx-2k=x^2-kx+2 \rightarrow x^2-4kx+2k+2\ (=0)$	**B1**	kx terms combined correctly-implied by correct b^2-4ac
	Attempt to find b^2-4ac	**M1**	Form a quadratic equation in k
	1 and $-\frac{1}{2}$	**A1**	SOI
	$k>1,\ k<-\frac{1}{2}$	**A1**	Allow $x>1,\ x<-1/2$
		4	
(ii)	$y=3x-2, \quad y=-\dfrac{3}{2}x+1$	**M1**	Use of *their* k values (twice) in $y=3kx-2k$
	$3x-2=-\dfrac{3}{2}x+1$ OR $y+2=2-2y$	**M1**	Equate *their* tangent equations OR substitute $y=0$ into both lines
	$x=\dfrac{2}{3}, \rightarrow y=0$ in one or both lines	**A1**	Substitute $x=\dfrac{2}{3}$ in one or both lines
		3	

Question 24

Equation of line is $y=mx-2$	**B1**	**OR**
$x^2-2x+7=mx-2 \rightarrow x^2-x(2+m)+9=0$	**M1**	
Apply $b^2-4ac(=0) \rightarrow (2+m)^2-4\times 9\ (=0)$	***M1**	
$m=4$ or -8	**A1**	
$m=4 \rightarrow x^2-6x+9=0 \rightarrow x=3$ $m=-8 \rightarrow x^2+6x+9=0 \rightarrow x=-3$	**DM1**	
(3, 10), (–3, 22)	**A1A1**	

Question 25

$3x^2+2x+4=mx+1 \rightarrow 3x^2+x(2-m)+3\ (=0)$	**B1**
$(2-m)^2-36$ SOI	**M1**
$(m+4)(m-8)$ (>/= 0) **or** $2-m$ >/= 6 and $2-m$ </= –6 OE	**A1**
$m<-4,\ m>8$ WWW	**A1**

Question 26

(a)	$2x^2 + kx + k - 1 = 2x + 3 \rightarrow 2x^2 + (k-2)x + k - 4 = 0$	M1
	Use of $b^2 - 4ac = 0 \rightarrow (k-2)^2 = 8(k-4)$	M1
	$k = 6$	A1
		3
(b)	$2x^2 + 2x + 1 = 2\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{2}$ $a = \frac{1}{2}, b = \frac{1}{2}$	B1 B1
	vertex $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **(FT on *a* and *b* values)**	B1FT
		3

Question 27

(a)	$x(mx + c) = 16 \rightarrow mx^2 + cx - 16 = 0$	B1
	Use of $b^2 - 4ac = c^2 + 64m$	M1
	Sets to 0 $\rightarrow m = \frac{-c^2}{64}$	A1
		3
(b)	$x(-4x + c) = 16$ Use of $b^2 - 4ac \rightarrow c^2 - 256$	M1
	$c > 16$ and $c < -16$	A1 A1
		3

Question 28

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

$3x^2 - 4x + 4 = mx + m - 1 \rightarrow 3x^2 - (4+m)x + (5-m)\ (=0)$	M1	3-term quadratic
$b^2 - 4ac = (4+m)^2 - 4 \times 3 \times (5-m)$	M1	Find $b^2 - 4ac$ for *their* quadratic
$m^2 + 20m - 44$	A1	
$(m + 22)(m - 2)$	A1	Or use of formula or completing square. This step must be seen
$m > 2$, $m < -22$	A1	Allow $x > 2$, $x < -22$
	5	

Question 29

$2x^2+m(2x+1)-6x-4(=0)$	*M1	y eliminated and all terms on one side with correct algebraic steps. Condone ± errors
Using b^2-4ac on $2x^2+x(2m-6)+m-4\ (=0)$	DM1	Any use of discriminant with their a, b and c identified correctly.
$4m^2-32m+68$ **or** $2m^2-16m+34$ **or** $m^2-8m+17$	A1	
$(2m-8)^2+k$ **or** $(m-4)^2+k$ **or** minimum point $(4,k)$ **or** finds $b^2-4ac\ (=-4,-16,-64)$	DM1	OE. Any valid method attempted on their 3-term quadratic
$(m-4)^2+1$ oe + always $>0 \rightarrow 2$ solutions for all values of m **or** Minimum point (4,1) + (fn) always $>0 \rightarrow 2$ solutions for all values of m **or** $b^2-4ac<0$ + no solutions $\rightarrow$ 2 solutions for the original equation for all values of m	A1	Clear and correct reasoning and conclusion without wrong working.
	5	

Question 30

$2x^2+5=mx-3 \rightarrow 2x^2-mx+8\ (=0)$	B1	Form 3-term quadratic
m^2-64	M1	Find b^2-4ac .
$-8<m<8$	A1	Accept (-8, 8) and equality included
	3	

Question 31

$x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)\ [=0]$	M1	Eliminate y and form 3-term quadratic.
$(k-3)^2-4(6-k)[>0]$	M1	OE. Apply b^2-4ac .
$k^2-2k-15[>0]$	A1	Form 3-term quadratic.
$(k+3)(k-5)[>0]$	A1	Or $k=-3,\ 5$ from use of formula or completing square.
$k<-3,\quad k>5$	A1 FT	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
	5	

Question 32

$u=2x-3$ leading to $u^4-3u^2-4\ [=0]$	M1	Or $u=(2x-3)^2$ leading to $u^2-3u-4\ [=0]$
$(u^2-4)(u^2+1)\ [=0]$	M1	Or $(u-4)(u+1)\ [=0]$
$2x-3=[\pm]2$	A1	
$x=\frac{1}{2}$, $\frac{5}{2}$ **only**	A1	
	4	

Question 33

$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.
b^2-4ac leading to $(4+m)^2-4\times 9$	**DM1**	SOI. Use of the discriminant with *their* a, b and c
$4+m=\pm 6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI
Substitute both *their* m values into *their* equation in line 1	**DM1**	
$m=2$ leading to $x=3$; $m=-10$ leading to $x=-3$	**A1**	
(3, 0), (–3, 24)	**A1**	Accept 'when $x=3$, $y=0$; when $x=-3$, $y=24$' If final A0A0 scored, **SC B1** for one point correct WWW

Question 34

(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k\neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
(b)	[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2-4\times 16\times(10-k)=0$ WWW
	[Root or $x=$] $\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

Question 35

$(2k-3)x^2-kx-(k-2)=3x-4$	***M1**	Equating curve and line
$(2k-3)x^2-(k+3)x-(k-6)[=0]$	**DM1**	Forming a 3-term quadratic
$(k+3)^2+4(2k-3)(k-6)[=0]$	**DM1**	Use of discriminant (dependent on **both** previous M marks)
$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	**M1**	Simplifying and solving *their* 3-term quadratic in k
$k=3$	**A1**	

Question 36

$\{5(y-3)^2\}$ $\{+5\}$	**B1 B1**	Accept $a=-3$, $b=5$

Question 37

$kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2\ [=0]$	***M1**	Eliminate y and form 3-term quadratic. Allow 1 error.
$(-k+2)^2-4k(-k+2)$	**DM1**	Apply b^2-4ac ; allow 1 error but a, b and c must be correct for *their* quadratic.
$5k^2-12k+4$ or $(-k+2)(-k+2-4k)$	**A1**	May be shown in quadratic formula.
$(-k+2)(-5k+2)$	**DM1**	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
$\frac{2}{5}<k<2$	**A1**	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
	5	

Question 38

$x^2+2cx+4=4x+c$ leading to $x^2+2cx-4x+4-c\ [=0]$	***M1**	Equate *y*s and move terms to one side of equation.
$b^2-4ac=(2c-4)^2-4(4-c)$	**DM1**	Use of discriminant with *their* correct coefficients.
$[4c^2-16c+16-16+4c=]\ 4c^2-12c$	**A1**	
$b^2-4ac>0$ leading to $(4)c(c-3)>0$	**M1**	Correctly apply '>0' considering both regions.
$c<0,\ c>3$	**A1**	Must be in terms of c. **SC B1** instead of M1A1 for $c \leqslant 0$, $c \geqslant 3$
	5	

Question 39

(a)	$mx+c=-\frac{m}{x}\ \Rightarrow\ mx^2+cx+m=0$	**M1**	All x terms in the numerator. OE e.g. $mx^2+cx=-m$.
	$b^2-4ac=0 \Rightarrow c^2-4m^2=0$	**M1**	OE $b^2-4ac=0$ is implied by $c^2-4m^2=0$.
	$c=[\pm]2m$	**A1**	SOI. Allow $\pm$ at this stage.
	$mx^2\ [\pm]2mx+m=0 \Rightarrow x^2[\pm]2x+1=0$	**M1**	Sub $c=+2m$ Ignore substitution of $-2m$.
	$(x+1)^2=0 \Rightarrow x=-1$ only	**A1**	
	$y=m$ only or $(-1, m)$ only	**A1**	
	Alternative method to question 11(a)		
	$\frac{dy}{dx}=\frac{m}{x^2}$	**M1**	As this is a method mark a sign error is allowed.
	$\frac{m}{x^2}=m\ \Rightarrow x^2=1$	**M1 A1**	Equating *their* $\frac{dy}{dx}$ and m and attempt to solve.
	$x=\pm1$ or $x=-1$	**A1**	If $x=-1$ and $y=m$ are the only answers offered here award the final M1 A1.
	Selecting $x=-1$ as the only answer and attempt to find y	**M1**	
	$y=m$ or $(-1, m)$	**A1**	
		6	
(b)	Equation of normal is $y-m=\frac{-1}{m}(x+1)$	***M1**	Through *their* P with gradient $\frac{-1}{m}$, OE e.g. $y=\frac{-1}{m}x+\frac{m^2-1}{m}$. Allow use of the gradient of the curve as $-\frac{1}{\left[\frac{m}{(their\ x)^2}\right]}$ with *their* P. Coordinates of P must be in terms of m only.
	$\frac{-x}{m}-\frac{1}{m}+m=\frac{-m}{x}\ \Rightarrow\ x^2+x(1-m^2)-m^2\ [=0]$	**DM1**	OE Equating *their* normal equation to the equation of the curve and removing x from the denominator.
	$(x+1)(x-m^2)\ [=0]\ \Rightarrow\ x=m^2$	**A1**	or $x=\frac{m^2-1\pm\sqrt{1-2m^2+m^4+4m^2}}{2}=\frac{m^2-1\pm(m^2+1)}{2}=m^2$
	$y=\frac{-m}{m^2}=\frac{-1}{m}$	**A1**	or $\left(m^2, \frac{-1}{m}\right)$, ignore the coordinates of P.
		4	

Question 40

5(a)	$4\times 0^2-0+\frac{1}{2}k^2=0-a$	M1	Equating the equations of curve and line and substituting $x=0$. Condone slight errors e.g. ± sign errors.
	$4\times\left(\frac{3}{4}\right)^2-\frac{3}{4}k+\frac{1}{2}k^2=\frac{3}{4}-a$	M1	Equating the equations of curve and line and substituting $x=\frac{3}{4}$. Condone slight errors e.g. ± sign errors.
	$k=2,\ a=-2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$(x-0)\left(x-\frac{3}{4}\right)=0$ or $x(4x-3)=0$ [$\Longrightarrow 4x^2-3x=0$]	*M1	Use 0, $\frac{3}{4}$ to form a quadratic equation. **Do not allow** $(x+0)\left(x+\frac{3}{4}\right)=0$.
	$4x^2-kx+\frac{1}{2}k^2=x-a$ leading to $4x^2-(k+1)x+\frac{1}{2}k^2+a[=0]$	DM1	Equating the equations of curve and line and rearranging so that terms are all on same side. Condone slight errors e.g. ± sign errors.
	$k=2,\ a=-2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$-\frac{b}{a}=\frac{3}{4}+0$ and $\frac{c}{a}=0\times\frac{3}{4}$	*M1	Using sum and product of roots. Condone ± sign errors.
	$\frac{k+1}{4}=\frac{3}{4}$ and $\frac{\frac{1}{2}k^2+a}{4}=0$	DM1	Equating the equations of curve and line and equating to $\frac{3}{4}$ and 0.
	$k=2,\ a=-2$	A1 A1	WWW
		4	
(b)	$4x^2-kx+\frac{1}{2}k^2=x+\frac{7}{2}\Rightarrow 4x^2-kx-x+\frac{1}{2}k^2-\frac{7}{2}[=0]$	*M1	OE Substitute $a=-\frac{7}{2}$ and rearrange so that terms are all on same side, condone ± sign errors. Watch for multiples.
	$(k+1)^2-4\times 4\left(\frac{1}{2}k^2-\frac{7}{2}\right)$	*DM1	Use of b^2-4ac with the coefficients from *their* 3-term quadratic. Both coefficients 'b' and 'c' must consist of two components.
	$\Rightarrow 7k^2-2k-57$	A1	OE
	$(k-3)(7k+19)$ or other valid method	DM1	Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.
	$k=3,\ k=-\frac{19}{7}$	A1	OE e.g. AWRT -2.71. No ISW if inequalities used. **SC:** If second DM1 not scored, **SC B1** available for correct final answers.
	Alternative method for question 5(b)		
	$8x-k=1$ **and** $4x^2-kx+\frac{1}{2}k^2=x+\frac{7}{2}$	*M1	Equating gradients and equating line and curve.
	$4x^2-(8x-1)x+\frac{1}{2}(8x-1)^2=x+\frac{7}{2}$ **or** $4\left(\frac{k+1}{8}\right)^2-k\left(\frac{k+1}{8}\right)+\frac{1}{2}k^2=\frac{k+1}{8}+\frac{7}{2}$	*DM1	Forming an equation in x or k only.
	$28x^2-8x-3$ or $7k^2-2k-57$	A1	OE A correct 3 term quadratic in x or k only.
	$(14x+3)(2x-1)$ or $(k-3)(7k+19)$ or other valid method	DM1	OE Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.

Question 41

(a)	$x^2 - 8x + 11 = (x-4)^2$ … **or** $p = -4$	**B1**	If p and q-values given after *their* completed square expression, mark the expression and ISW.
	… -5 **or** $q = -5$	**B1**	
		2	
(b)	$(x-4)^2 - 5 = 1$ so $(x-4)^2 = 6$ so $x - 4 = [\pm]\sqrt{6}$	**M1**	Using *their* p and q values or by quadratic formula
	$x = 4 \pm \sqrt{6}$ **or** $\frac{8 \pm \sqrt{24}}{2}$	**A1**	Or **exact** equivalent. No FT; must have $\pm$ for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, **SC B1** possible for correct answers.
		2	

Question 42

$k^2 - 4 \times 8 \times 2$ [< 0]	**M1**	Use of $b^2 - 4ac$ but not just in the quadratic formula.
$-8 < k < 8$ or $-8 < k$, $k < 8$ or $\lvert k \rvert < 8$ or $(-8, 8)$	**A1**	Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$.
	2	

Question 43

(a)	$\mathbf{y} = 4\left(x + \frac{5}{2}\right)^2 - 19$		There is no requirement for the candidate to list a, b and c. Look at values in their final expression, condone omission of ^{2}, and award marks as follows:
		B1	$a = 4$
		B1	$b = \frac{5}{2}$ OE
		B1	$c = -19$
		3	
(b)	$\left(\textit{Their}\ 4\left(x + \frac{5}{2}\right)^2 - 19\right) = 45$ $\left[\Rightarrow \left(x + \frac{5}{2}\right)^2 = 16\right]$	***M1**	Equate their quadratic completed square form from **6(a)** to 45 or re-start and use completing the square.
	Solve as far as $x =$	**DM1**	Any valid method leading to two answers.
	$[x =]\frac{3}{2}, -\frac{13}{2}$	**A1**	**SC:** If M0 or M1 DM0 awarded, B1 available for correct final answers.
		3	
(c)	Quadratic **curve** that is the right way up **(**must be seen either side of stationary point**)**	**B1**	No axes required, ignore any axes even if incorrect.
	Stationary point stated using any valid method or correctly labelled on their diagram.	**B1 FT** **B1 FT**	FT *their* values from **6(a)** as long as *their* expression is of the form $p(qx+r)^2 + s$. Expect $\left(-\frac{5}{2}, -19\right)$. Condone if stated correctly but plotted incorrectly.
		3	

Question 44

$(3x+2)(x-1)=2 \Rightarrow 3x^2-x-4\ [=0]$	**M1**	OE Multiply by denominator and obtain a quadratic.
$(3x-4)(x+1)[=0]$	**M1**	Solve by factorising, formula or completing the square.
$[x=]\ -1, \frac{4}{3}$	**A1**	Allow 1.33 If M1 M0, **SC B1** possible for two correct answers.
	3	

Question 45

$x^2-kx+2=3x-2k$ leading to $x^2-x(k+3)+(2+2k)\ [=0]$	**M1**	3-term quadratic, may be implied in the discriminant.
$b^2-4ac=(k+3)^2-8(1+k)$ (ignore '= 0' at this stage)	**DM1**	Cannot just be seen in the quadratic formula.
$=(k-1)^2$ accept $(k-1)(k-1)$	**A1**	Or use of calculus to show minimum of zero at $k=1$ or sketch of $f(k)=k^2-2k+1$.
$\geqslant 0$ Hence will meet for all values of k	**A1**	Clear conclusion.
	4	

Question 46

$x^2-6x+c>2$ leading to $(x-3)^2-9+c>2$	**M1 A1**	M1 for completion of the square with an equation or in equality with the '2'.
$c>11-(x-3)^2$ and $(x-3)^2\geq 0$	**M1**	SOI
$c>11$	**A1**	
Alternative Method 1		
$\frac{dy}{dx}=2x-6=0$	**M1**	M1 for differentiating and setting $\frac{dy}{dx}=0$.
$x=3$	**A1**	
When $x=3,\ y=9-18+c$	**M1**	
$[-9+c>2]\ c>11$	**A1**	
Alternative Method 2		
$x^2-6x+c>2$ leading to $x^2-6x+c-2[>0]$ then use of 'b^2-4ac'	**M1**	
$36-4(1)(c-2)<0$	**M1 A1**	OE Must be correct inequality for M1.
$c>11$	**A1**	
	4	

Question 47

(a)	$4(x-3)^2$ seen or $a=4$ and $b=-3$	**B1**	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.
	$-36+p$ or $p-36$ seen or $c=p-36$	**B1**	
		2	
(b)	$p-36>0$ leading to $p>36$ or $24^2-4\times 4p\langle 0\Rightarrow p\rangle 36$ or $36<p$	**B1**	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.
		1	

Question 48

$[8x^6 + 215x^3 - 27 = 0]$ leading to $(8x^3 - 1)(x^3 + 27)[=0]$ **OR** $\dfrac{-215 \pm \sqrt{215^2 - 4.8.-27}}{2.8}$ or $\dfrac{-215 \pm \sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x = x^3$.
$\dfrac{1}{8}$, -27	**A1**	Both correct values seen. **SC**: if M0 scored **SC B1** is available for sight of $\dfrac{1}{8}$ and -27 OE
$\dfrac{1}{2}$ or $0.5, -3$	**A1**	**SC**: if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
	3	

Question 49

$kx - k = -\dfrac{1}{2x} \Rightarrow 2kx^2 - 2kx + 1[=0]$ OR quadratic in y: $x = \dfrac{y+k}{k} \Rightarrow y = -\dfrac{1}{2\left(\dfrac{y+k}{k}\right)} \Rightarrow 2y^2 + 2ky + k = 0$	***M1**	OE e.g. $kx^2 - kx + \dfrac{1}{2}[=0]$, $x^2 - x + \dfrac{1}{2k}[=0]$ Equate line and curve to form 3-term quadratic (all terms on one side).
$b^2 - 4ac[=0] \Rightarrow ([-]2k)^2 - 4(2k)(1)[=0]$ or $4k^2 - 8k\ [=0 \Rightarrow 4k(k-2) = 0]$ OR using equation in y: $(2k)^2 - 4(2)(k) = 0$	**DM1**	Use discriminant correctly with their a, b, c not in quadratic formula. DM0 if x still present. May see $k^2 - 4(k)\left(\dfrac{1}{2}\right) = 0$ or $1 - 4\left(\dfrac{1}{2k}\right) = 0$.
$k = 2$ only	**A1**	If DM0 then $k = 2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k = 0$ also present but uses $k = 2$, award A1.
$4x^2 - 4x + 1 = 0\left[\Rightarrow (2x-1)^2 = 0\right] \Rightarrow x = \dfrac{1}{2}$	**B1**	
$y = 2 \times \dfrac{1}{2} - 2 = -1$	**B1**	