AS-Level

Pure Mathematics P1

Topic : Quadratics

May 2013- May 2023

Question 1

The straight line $y = mx + 14$ is a tangent to the curve $y = \frac{12}{x} + 2$ at the point P. Find the value of the constant m and the coordinates of P. [5]

Question 2

A curve has equation $y = 2x^2 - 3x$.

(i) Find the set of values of x for which $y > 9$. [3]

(ii) Express $2x^2 - 3x$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and state the coordinates of the vertex of the curve. [4]

The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 - 3x \quad \text{and} \quad g(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $gf(x) = 0$ has equal roots. [3]

Question 3

(i) Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

Question 4

A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.

(i) For the case where the line is a tangent to the curve, find the value of the constant c. [3]

(ii) For the case where $c = 11$, find the x-coordinates of the points of intersection of the line and the curve. Find also, by integration, the area of the region between the line and the curve. [7]

Question 5

(i) Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

Question 6

Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]

Question 7

Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

Question 8

A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

Question 9

(a) Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

Question 10

Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

Question 11

A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

Question 12

(i) Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

Question 13

Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

Question 14

Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

Question 15

A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

(i) Find the set of values of c for which the curve and the line meet. [4]

(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

Question 16

Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

Question 17

The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

Question 18

A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet. [4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. [4]

Question 19

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

Question 20

Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

Question 21

A line has equation $y = x + 1$ and a curve has equation $y = x^2 + bx + 5$. Find the set of values of the constant b for which the line meets the curve. [4]

Question 22

The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii) Find the coordinates of P. [2]

Question 23

A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

Question 24

A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

Question 25

Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

Question 26

The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

Question 27

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

Question 28

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

Question 29

The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

Question 30

Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

Question 31

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

Question 32

By using a suitable substitution, solve the equation

$$(2x - 3)^2 - \frac{4}{(2x - 3)^2} - 3 = 0.$$ [4]

Question 33

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

Question 34

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

Question 35

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

Question 36

Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

Question 37

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

Question 38

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

Question 39

The point P lies on the line with equation $y = mx + c$, where m and c are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point P on the curve such that the straight line is a tangent to the curve at P.

(a) Find the coordinates of P, giving the y-coordinate in terms of m. [6]

The normal to the curve at P intersects the curve again at the point Q.

(b) Find the coordinates of Q in terms of m. [4]

Question 40

The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

(a) Given that the curve and the line intersect at the points with x-coordinates 0 and $\frac{3}{4}$, find the values of k and a. [4]

(b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

Question 41

(a) Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

Question 42

Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

Question 43

The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

Question 44

Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

Question 45

A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

Question 46

The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

Question 47

(a) Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

Question 48

Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

Question 49

The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]