AS-Level

Topic : Sequence and Series

May 2013-May 2023

Answer

Question 1

(a)	$S_n = 2n^2 + 8n$			
	$S_1 = 10 \quad = a$	B1		
	$S_2 = 24 = a + (a + d)\ d = 4$	M1 A1	[3]	correct use of S_n formula.
b)	GP $a = 64\ ar = 48 \rightarrow r = ¾$	B1		
	$\rightarrow$3rd term is $ar^2 = 36$	M1		ar^2 numerical – for their r
	AP $a = 64,\ a + 8d = 48 \rightarrow d = -2$	B1		
	$36 = 64 + (n - 1)(-2)$	M1		correct use of $a+(n-1)d$
	$\rightarrow n = 15.$	A1	[5]	

Question 2

(a) $57 = 2(24 + 3d) \rightarrow d = 1.5$			
$48 = 12 + (n - 1)1.5 \rightarrow n = 25$	M1 A1		Use of correct S_n formula.
	M1 A1		Use of correct T_n formula.
(b) $ar^2 = 4a \quad r = \pm 2$		[4]	
$\frac{a(r^6 - 1)}{r - 1} = ka$	B1		(allow for $r = 2$)
	B1		
$\rightarrow k = 63$ or $k = -21$	B1 B1		
		[4]	

Question 3

(i) $ar^2 = -108,\ ar^5 = 32$	B1		
$r^3 = \frac{32}{-108} = \left(-\frac{8}{27}\right)$	M1		Eliminating a
$r = \left(-\frac{2}{3}\right)$ or –0.666 or –0.667	A1	[3]	$-\frac{2}{3}$ from little or no working $\rightarrow \frac{3}{3}$ www
(ii) $a = -243$	B1 √	[1]	ft on *their* r $\left(-\frac{108}{r^2} \text{ or } \frac{32}{r^5}\right)$
(iii) $S_\infty = \frac{-243}{1 + \frac{2}{3}} = -\frac{729}{5}$ or –145.8	M1A1	[2]	Accept –146. For M1 $\lvert r \rvert$ must be < 1

Question 4

(a) $\frac{a}{1-r} = 8a \Rightarrow 1(a) = 8(a)(1-r)$	B1	
$r = \frac{7}{8}$ oe	B1	
	[2]	
(b) $a + 4d = 197$	B1	Or $2a + 9d = 408$
$\frac{10}{2}[2a + 9d] = 2040$	B1	Attempt to solve simultaneously
$d = 14$	M1A1	
	[4]	

Question 5

(a) (i) $a = 300, d = 12$		
$\rightarrow 540 = 300 + (n-1)12 \rightarrow n = 21$	M1 A1	Use of nth term. Ans 20 gets 0.
	[2]	Ignore incorrect units
(ii) $S_{26} = 13\ (600 + 25\times12) = 11700$	M1	Correct use of s_n formula.
$\rightarrow$ 3 hours 15 minutes.	A1	
	[2]	
(b) $ar = 48$ and $ar^2 = 32 \rightarrow r = ⅔$	M1	Needs ar and ar^2 + attempt at a and r.
$\rightarrow a = 72.$	A1	
$S_\infty = 72 \div ⅓ = 216.$	M1	Correct S_∞ formula with $\lvert r \rvert < 1$
	A1√	
	[4]	

Question 6

(a) $\frac{10}{2}(2a+9d) = 400$ oe	**B1**	$\rightarrow 2a + 9d = 80$
$\frac{20}{2}(2a+19d) = 1400$ OR		
$\frac{10}{2}[2(a+10d)+9d] = 1000$	**B1**	$\rightarrow 2a + 19d = 140$ or $2a + 29d = 200$
$d = 6 \quad a = 13$	**M1A1A1**	Solve sim. eqns both from S_n formulae
	[5]	
(b) $\frac{a}{1-r} = 6 \qquad \frac{2a}{1-r^2} = 7$	**B1B1**	
$\frac{12(1-r)}{1-r^2} = 7$ or $\frac{1-r^2}{1-r} = \frac{12}{7}$	**M1**	Substitute or divide
$r = \frac{5}{7}$ or 0.714	**A1**	
$a = \frac{12}{7}$ or 1.71(4)	**A1√**	Ignore any other solns for r and a
	[5]	

Question 7

36, 32, ...		
(i) $r = \frac{8}{9}$ $S_\infty = (\textit{their } a) \div (1 - \textit{their } r)$	M1	Method for r and S_∞ ok. ($\lvert r \rvert < 1$)
$S_\infty = 36 \div \frac{1}{9} = 324$	A1	co
	[2]	
(ii) $d = -4$	B1	co
$0 = \frac{n}{2}(72 + (n-1)(-4))$	M1	S_n formula ok and a value for d $\left(\neq \frac{8}{9}\right)$
$\rightarrow n = 19$	A1	Condone $n = 0$ but no other soln
	[3]	

Question 8

(i) GP 8 $8r$ $8r^2$ AP 8 $8 + 8d$ $8 + 20d$		
$8r = 8 + 8d$ and $8r^2 = 8 + 20d$	B1 B1	B1 for each equation.
Eliminates d $\rightarrow$ $2r^2 - 5r + 3 = 0$	M1	Correct elimination.
$\rightarrow r = 1.5$ (or 1)	A1	co (no penalty for including $r = 1$)
	[4]	
(ii) 4th term of GP $= ar^3 = 8 \times 27/8 = 27$	B1√	co
If $r = 1.5$, $d = 0.5$ 4th term of AP $= a + 3d = 9\frac{1}{2}$	M1A1	needs $a + 3d$ and correct method for d
	[3]	

Question 9

(i) $200/2(2a + 199d) = 4 \times 100/2(2a + 99d)$	**M1A1**	Correct formula used (once) M1, correct eqn A1
$d = 2a$ cao	**A1**	
	[3]	
(ii) $a + 99d = a + 99 \times 2a$	**M1**	Sub. *their* part(i) into correct formula
$199a$ cao	**A1**	
	[2]	

Question 10

(i) $S_P = \dfrac{2}{1-\dfrac{1}{2}}, S_P = \dfrac{3}{1-\dfrac{1}{3}}$	**M1**	At least one correct
$S_P = 4, S_Q = \dfrac{9}{2}$	**A1**	At least one correct
$S_R = 5$ cao	**A1** **[3]**	
(ii) $\dfrac{4}{1-r} = their\ S_R$	**M1**	
$r = \dfrac{1}{5}$	**A1**	

Question 11

(a) $S_n = 32n - n^2$.		
Set n to 1, a or $S_1 = 31$	B1	co
Set n to 2 or other value $S_2 = 60$		
$\rightarrow$ 2nd term = 29 $\rightarrow$ $d = -2$	M1 A1	Correct method.
(or equates formulae – compares		co
coeffs n^2, n)	[3]	
[M1 comparing, A1 d A1 a]		[M1 only when coeffs compared]
(b) $\dfrac{a}{1-r} = 20$, $\dfrac{a(1-r)^2}{1-r}$, or $a + ar = 12.8$	B1 B1	co co
Elimination of $\dfrac{a}{1-r}$ or a or r	M1	'Correct' elimination to form equation in a or r
$\rightarrow$ $(r = 0.6)$ $\rightarrow$ $a = 8$	DM1 A1 [5]	Complete method leading to $a =$ Condone $a = 8$ and 32

Question 12

(i) $S = \frac{a}{1-r}$, $3S = \frac{a}{1-2r}$	**B1**	At least $3S = \frac{a}{1-2r}$
$1 - r = 3 - 6r$	**M1**	Eliminate S
$r = \frac{2}{5}$	**A1** **[3]**	
(ii) $7 + (n-1)d = 84$ and/or $7 + (3n-1)d = 245$	**B1**	At least one of these equations seen
$[(n-1)d = 77,\ (3n-1)d = 238,\ 2nd = 161]$	**B1**	Two different seen – unsimplified ok
$\frac{n-1}{3n-1} = \frac{77}{238}$ (must be from the correct u_n formula)	**M1**	Or other attempt to elim d. E.g. sub $d = \frac{161}{2n}$ (if n is eliminated d must be found)
$n = 23$ $(d = \frac{77}{22} = 3.5)$	**A1** **[4]**	

Question 13

(a)	2222/17 (=131 or 130.7) 131 × 17 (=2227) −2222 + 2227 = 5	**M1** **M1** **A1** **[3]**	Ignore signs. Allow 2239/17→131.7 or 132 Ignore signs. Use 131. 5 www gets 3/3
(b)	$r = \frac{2\cos\theta}{\sqrt{3}}$ soi oe $(-1<)\frac{2\cos\theta}{\sqrt{3}}<1$ or $(0<)\frac{2\cos\theta}{\sqrt{3}}<1$ soi $\pi/6,\ 5\pi/6$ soi (but dep. on M1) $\pi/6<\theta<5\pi/6$ cao	**B1** **M1**√ **A1A1** **A1** **[5]**	 Ft on *their r*. Ignore a 2nd inequality on LHS Allow 30°, 150°. Accept ≤

Question 14

(a)	1st, 2nd, *n*th are 56, 53 and −22 $a = 56,\ d = -3$ $-22 = 56 + (n-1)(-3)$ $\to\ n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\to$ 459	 M1 A1 M1 A1 [4]	 Uses correct u_n formula. co Needs positive integer *n* Co
(b)	1st, 2nd, 3rd are $2k+6$, $2k$ and $k+2$.		
(i)	Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses *a, r* and eliminates $\to 2k^2 - 10k - 12 = 0$ $\to k = 6$	 M1 DM1 A1 [3]	 Correct method for equation in *k*. Forms quad. or cubic equation with no brackets or fractions. Co
(ii)	$S_\infty = \frac{a}{1-r}$ with $r = \frac{2k}{2k+6}$ or $\frac{k+2}{2k}$ $(=\frac{2}{3})$ $\to$ 54	M1 A1 [2]	Needs attempt at *a* and *r* and S_∞ Co

Question 15

(a)	$ar^2 = \frac{1}{3}$, $ar^3 = \frac{2}{9}$		
	$\rightarrow r = \frac{2}{3}$ aef	M1	Any valid method, seen or implied. Could be answers only.
	Substituting $\rightarrow a = \frac{3}{4}$	A1	Both a and r
	$\rightarrow S_\infty = \frac{\frac{3}{4}}{\frac{1}{3}} = 2\frac{1}{4}$ aef	M1 A1 [4]	Correct formula with $\lvert r\rvert < 1$, cao
(b)	$4a = a + 4d \rightarrow 3a = 4d$	B1	May be implied in $360 = 5/2(a + 4a)$
	$360 = S_5 = \frac{5}{2}(2a + 4d)$ or $12.5a$	M1	Correct S_n formula or sum of 5 terms
	$\rightarrow a = 28.8^\circ$ aef	A1	cao, may be implied
	Largest = $a + 4d$ or $4a = 115.2^\circ$ aef	B1 [4]	(may use degrees or radians)

Question 16

(i) **(a)**	$1.92 + 1.84 + 1.76 + ...$ oe	**B1**	OR a=0.96, d= –.04 & ans
	$\frac{20}{2}[2\times1.92 + 19\times(-0.08)]$ oe	**M1**	doubled/adjusted
	23.2 cao	**A1** [3]	Corr formula used with corr d & *their* a, n $a = 1, n = 21 \rightarrow 12.6$ (25.2), $a = 0.96, n = 21 \rightarrow 11.76$ (23.52)
(b)	$1.92 + 1.92(.96) + 1.92(.96)^2 + ...$	**B1**	
	$\frac{1.92(1-.96^{20}}{1-.96}$	**M1**	OR a=.96, r =.96 & ans /doubled/adjusted
	26.8 cao	**A1** [3]	Corr formula used with r =.96 & *their* a, n $a = .96, n = 21 \rightarrow 13.82$ (27.63) $a = 1, n = 21 \rightarrow 14.39$ (28.78)
(ii)	$\frac{1.92}{1-.96} = 48$ or $\frac{0.96}{1-0.96} = 24$ & then Double **AG**	**M1A1** [2]	$a = 1 \rightarrow 25$ (50) but must be doubled for M1 $1.92\frac{(1-0.96^n)}{1-0.96} < 48 \rightarrow 0.96^n > 0$ (www) 'which is true' scores SCB1

Question 17

(i)	$x^2 - 4x = 12$	**M1**	$4x - x^2 = 12$ scores M1A0
	$x = -2$ or 6	**A1**	
	3^{rd} term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	**A1A1** [4]	SC1 for 16, 48 after $x = 2, -6$
(ii)	$r^2 = \frac{x^2}{4x}\left(= \frac{x}{4}\right)$ soi	**M1**	
	$\frac{4x}{1 - \frac{x}{4}} = 8$	**M1**	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
	$x = \frac{4}{3}$ or $r = \frac{1}{3}$	**A1**	
	3^{rd} term $= \frac{16}{27}$ (or 0.593)	**A1** [4]	
	ALT		
	$\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ **or** $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$	**M1**	
	$x^2 = 4x\left(1 - \frac{1}{2}x\right)$ $\quad r = \frac{2(1-r)}{4}$	**M1**	
	$x = \frac{4}{3}$ $\quad r = \frac{1}{3}$	**A1**	

Question 18

$a + 11d = 17$	**B1**	
$\frac{31}{2}(2a + 30d) = 1023$	**B1**	
Solve simultaneous equations	**M1**	
$d = 4,\ a = -27$	**A1**	At least one correct
31st term = 93	**A1** **[5]**	

Question 19

$r=\frac{3+2d}{3}$ or $\frac{3+12d}{3+2d}$ or $r^2=\frac{3+12d}{3}$	B1	1 correct equation in r and d only is sufficient
$(3+2d)^2=3(3+12d)$ oe OR sub $2d=3r-3$	M1	Eliminate r or d using valid method
$(4)d(d-6)=0$ OR $3r^2=18r-15\rightarrow(r-1)(r-5)$	DM1	Attempt to simplify and solve quadratic
$d=6$ $r=5$	A1 A1 [5]	Ignore $d=0$ or $r=1$ Do not allow -5 or ±5

Question 20

(i)	(a)	$a+(n-1)d$ = 10 + 29×2	M1	Use of nth term of an AP with a=±10, d=±2, n=30 or 29
		= 68	A1 [2]	Condone – 68 → 68
	(b)	$\frac{1}{2}n(20+2(n-1))=2000$ or 0	M1	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000.
		$\rightarrow 2n^2+18n-4000=0$ oe (n=) 41	A1 A1 [3]	Correct 3 term quadratic = 0.
(ii)		$r=1.1$, oe	B1	e.g. $\frac{11}{10}$, 110%
		Uses $S_{30}=\frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645)	M1	Use of S_n formula for a GP, a=±10, n=30.
		Percentage lost = $\frac{2000-1645}{2000}\times100$	DM1	Fully correct method for % left with "their 1645"
		= 17.75	A1 [4]	allow 17.7 or 17.8.

Question 21

(a)	$a = 50$, $ar^2 = 32$	**B1**	seen or implied
	$\rightarrow r = \frac{4}{5}$ (allow $-\frac{4}{5}$ for M mark)	**M1**	Finding r and use of correct S_∞ formula
	$\rightarrow S_\infty = 250$	**A1** [3]	Only if $\lvert r \rvert < 1$
(b) (i)	$2\sin x$, $3\cos x$, $(\sin x + 2\cos x)$. $3c - 2s = (s + 2c) - 3c$ (or uses a, $a + d$, $a + 2d$)	**M1**	Links terms up with AP, needs one expression for d.
	$\rightarrow 4c = 3s \rightarrow t = \frac{4}{3}$	**M1 A1** [3]	Arrives at $t = k$. ag
	SC uses $t = \frac{4}{3}$ to show $u_1 = \frac{8}{5}, u_2 = \frac{9}{5}, u_3 = \frac{10}{5}$, **B1** only		
(ii)	$\rightarrow c = \frac{3}{5}$, $s = \frac{4}{5}$ or calculator $x = 53.1°$	**M1**	
	$\rightarrow a = 1.6$, $d = 0.2$	**M1**	Correct method for both a and d.
	$\rightarrow S_{20} = 70$	**A1** [3]	(Uses S_n formula)

Question 22

(a)	$\frac{6}{1-r} = \frac{12}{1+r}$	**M1**		
	$r = \frac{1}{3}$	**A1**		
	$S = 9$	**A1**	[3]	
(b)	$\frac{13}{2}\left[2\cos\theta + 12\sin^2\theta\right] = 52$	**M1***		Use of correct formula for sum of AP
	$2\cos\theta + 12(1-\cos^2\theta) = 8 \rightarrow 6\cos^2\theta - \cos\theta - 2(=0)$	**DM1**		Use $s^2 = 1 - c^2$ & simplify to 3-term quad
	$\cos\theta = 2/3$ or $-1/2$ soi	**A1**		
	$\theta = 0.841$, 2.09 Dep on previous A1	**A1A1**	[5]	Accept 0.268π, 2π/3. SRA1 for 48.2˚, 120˚ Extra solutions in range –1

Question 23

(a) (i)	$200+(15-1)(+/-5)$ $= 130$	**M1** **A1**	[2]	Use of nth term with $a = 200$, $n = 14$ or 15and $d = +/- 5$.
(ii)	$\frac{n}{2}\left[400+(n-1)(+/-5)\right]=(3050)$ $\rightarrow\ 5n^2-405n+6100\ (=0)$ $\rightarrow\ 20$	**M1** **A1** **A1**	[3]	Use of S_n a=200 and $d = +/- 5$.
(b) (i)	$ar^2, ar^5 \quad \rightarrow \quad r = ½$ $\frac{63}{2}=\frac{a\left(1-½^6\right)}{½} \quad \rightarrow \ a = 16$	**M1 A1** **M1 A1**	[4]	Both terms correct. Use of $S_n = 31.5$ with a numeric r.
(ii)	Sum to infinity $= \frac{16}{½} = 32$	**B1**√	[1]	√ for their a and r with $\lvert r \rvert < 1$.

Question 24

$a(1+r)=50$ or $\frac{a(1-r^2)}{1-r}=50$	**B1**		
$ar(1+r)=30$ or $\frac{a(1-r^3)}{1-r}=30+a$	**B1**		Or otherwise attempt to solve for r
Eliminating a or r	**M1**		Any correct method
$r=3/5$	**A1**		
$a=125/4$ oe	**A1**		
$S=625/8$ oe	**A1**√	[6]	Ft through on *their* r and a $(-1<r<1)$

Question 25

:(i)	$S=\frac{r^2-3r+2}{1-r}$	**M1**	
	$S=\frac{(r-1)(r-2)}{1-r}=\frac{-(1-r)(r-2)}{1-r}=2-r$ OR $\frac{(1-r)(2-r)}{1-r}=2-r$ OE	**A1**	**AG** Factors must be shown. Expressions requiring minus sign taken out mus be shown
	Total:	**2**	
(ii)	Single range $1<S<3$ or (1, 3)	**B2**	Accept $1<2-r<3$. Correct range but with $S=2$ omitted scores SR **B1** $1 \leqslant S \leqslant 3$ scores SR **B1**. [$S>1$ and $S<3$] scores SR **B1**.
	Total:	**2**	

Question 26

(a)	$(S_n =)\ \frac{n}{2}\left[32+(n-1)8\right]$ and 20000	M1	**M1** correct formula used with d from $16 + d = 24$
		A1	**A1** for correct expression linked to 20000.
	$\rightarrow n^2 + 3n - 5000\ (<,=,>0)$	DM1	Simplification to a three term quadratic.
	$\rightarrow (n = 69.2) \rightarrow 70$ terms needed.	A1	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.
	Total:	4	
(b)	$a = 6,\ \frac{a}{1-r} = 18 \rightarrow r = \frac{2}{3}$	M1A1	Correct S_∞ formula used to find r.
	New progression $a = 36,\ r = \frac{4}{9}$ oe	M1	Obtain new values for a and r by any valid method.
	New $S_\infty = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	A1	(Be aware that $r = -\frac{2}{3}$ leads to 64.8 but can only score M marks)

Question 27

(a)	$a = 32,\ a + 4d = 22,\ \rightarrow d = -2.5$	B1	
	$a + (n-1)d = -28 \rightarrow n = 25$	B1	
	$S_{25} = \frac{25}{2}(64 - 2.5 \times 24) = 50$	M1 A1	**M1** for correct formula with $n = 24$ or $n = 25$
	Total:	4	
(b)	$a = 2000,\ r = 1.025$	B1	$r = 1 + 2.5\%$ ok if used correctly in S_n formula
	$S_{10} = 2000\left(\frac{1.025^{10} - 1}{1.025 - 1}\right) = 22400$ or a value which rounds to this	M1 A1	**M1** for correct formula with $n = 9$ or $n = 10$ and their a and r
			SR: correct answer only for $n = 10$ **B3**, for $n = 9$, **B1** (£19 900)
	Total:	3	

Question 28

$\frac{1}{2}n\left[-24+(n-1)6\right]\ \sim 3000$ Note: $\sim$ denotes <u>any</u> inequality or equality	M1	Use correct formula with RHS ≈ 3000 (e.g. 3010).
$(3)(n^2 - 5n - 1000)\ (\sim 0)$	A1	Rearrange into a 3-term quadratic.
$n \sim 34.2\ (\& -29.2)$	A1	
35. Allow $n \geqslant 35$	A1	
	4	

Question 29

(a)	Uses r = (1.05 or 105%)$^{9,\ 10 \text{ or } 11}$	B1	Used to multiply repeatedly or in any GP formula.
	New value = 10000×1.05^{10} = ($)16 300	B1	
		2	

(b)	*EITHER:* $n = 1 \rightarrow 5$ $a = 5$	(B1	Uses $n = 1$ to find a
	$n = 2 \rightarrow 13$	B1	Correct S_n for any other value of n (e.g. $n = 2$)
	$a + (a + d) = 13 \quad \rightarrow d = 3$	M1 A1)	Correct method leading to $d =$
	OR: $\left(\frac{n}{2}\right)(2a+(n-1)d)=\left(\frac{n}{2}\right)(3n+7)$		$\left(\frac{n}{2}\right)$ maybe be ignored
	$\therefore dn + 2a - d = 3n + 7 \rightarrow dn = 3n \rightarrow d = 3$	(*M1A1	Method mark awarded for equating terms in n from correct S_n formula.
	$2a - (\textit{their}\ 3) = 7, \quad a = 5$	DM1 A1)	
		4	

Question 30

(i)	$\frac{3a}{1-r} = \frac{a}{1+2r}$	M1	Attempt to equate 2 sums to infinity. At least one correct
	$3 + 6r = 1 - r$	DM1	Elimination of 1 variable (a) at any stage and multiplication
	$r = -\frac{2}{7}$	A1	
		3	
(ii)	$½n[2\times15+(n-1)4]=½n[2\times420+(n-1)(-5)]$	M1A1	Attempt to equate 2 sum to n terms, at least one correct (**M1**). Both correct (**A1**)
	$n = 91$	A1	
		3	

Question 31

(i)	$40 + 60 \times 1.2 = 112$	M1A1	Allow 1.12 m. Allow **M1** for 40 + 59 × 1.2 OE
		2	
(ii)	Find rate of growth e.g. 41.2/40 or 1.2/40	*M1	SOI, Also implied by 3% , 0.03 or 1.03 seen
	$40 \times (1 + \textit{their}\ 0.03)^{60 \text{ or } 59}$	DM1	
	236	A1	Allow 2.36 m
		3	

Question 32

$\left[\frac{a(1-r^n)}{1-r}\right][\div]\left[\frac{a}{1-r}\right]$	M1M1	Correct formulae **used** with/without $r = 0.99$ or $n = 100$.
	DM1	Allow numerical a (M1M1). 3rd M1 is for division $\frac{S_n}{S_\infty}$ (or ratio) SOI
$1 - 0.99^{100}$ SOI OR $\frac{63(a)}{100(a)}$ SOI	A1	Could be shown multiplied by 100(%). Dep. on DM1
63(%) Allow 63.4 or 0.63 but not 2 infringements (e.g. 0.634, 0.63%)	A1	$n = 99$ used scores Max M3. Condone $a = 0.99$ throughout $S_n = S_\infty$ (without division shown) scores 2 / 5
	5	

Question 33

(i)	$r = 1.02$ or $\frac{102}{100}$ used in a GP in some way.	**B1**	Can be awarded here for use in S_n formula.
	Amount in 12th week = $8000\,(their\ r)^{11}$ or $(their\ a\ from\ \frac{8000}{their\ r.})\,(their\ r)^{12}$	**M1**	Use of ar^{n-1} with a = 8000 & n = 12 or with a = $\frac{8000}{1.02}$ and n = 13.
	= 9950 (kg) awrt	**A1**	Note: Final answer of either 9943 or 9940 implies M1. Full marks can be awarded for a correct answer from a list of terms.
		3	
(ii)	In 12 weeks, total is $\frac{8000\left((their\,r)^{12}-1\right)}{\left((their\,r)-1\right)}$	**M1**	Use of S_n with a = 8000 and n = 12 or addition of 12 terms.
	= 107000 (kg) awrt	**A1**	Correct answer but no working 2/2
		2	

Question 34

3(a)	$ar = 12$ and $\frac{a}{1-r} = 54$	**B1 B1**	CAO, OE CAO, OE
	Eliminates a or r $\rightarrow$ $9r^2 - 9r + 2 = 0$ or $a^2 - 54a + 648 = 0$	**M1**	Elimination leading to a 3-term quadratic in a or r
	$\rightarrow r = \frac{2}{3}$ or $\frac{1}{3}$ hence to a $\rightarrow a = 18$ or 36	**A1**	Needs both values.
		4	
3(b)	nth term of a progression is $p + qn$		
b)(i)	first term $= p + q$. Difference $= q$ or last term $= p + qn$	**B1**	Need first term and, last term or common difference
	$S_n = \frac{n}{2}\left(2(p+q)+(n-1)q\right)$ or $\frac{n}{2}(2p+q+nq)$	**M1A1**	Use of S_n formula with their a and d. ok unsimplified for A1.
		3	
b)(ii)	Hence $2(2p+q+4q)=40$ and $3(2p+q+6q)=72$	**DM1**	Uses their S_n formula from **(i)**
	Solution $\rightarrow p = 5$ and $q = 2$ [Could use S_n with a and d $\rightarrow a = 7, d = 2 \rightarrow p = 5, q = 2$.]	**A1**	Note: answers 7, 2 instead of 5, 2 gets M1A0 – must attempt to solve for M1
		2	

Question35

$a+(n-1)3=94$	**B1**	
$\frac{n}{2}\left[2a+(n-1)3\right]=1420$ OR $\frac{n}{2}[a+94]=1420$	**B1**	
Attempt elimination of a or n	**M1**	
$3n^2-191n+2840\ (=0)$ OR $a^2-3a-598\ (=0)$	**A1**	3-term quadratic (not necessarily all on the same side)
$n = 40$ (only)	**A1**	
$a = -23$ (only)	**A1**	Award 5/6 if a 2nd pair of solutions (71/3, 26) is given in addition or if given as the only answer.
	6	

Question36

From the AP: $x-4=y-x$	**B1**	Or equivalent statement e.g. $y = 2x - 4$ or $x=\frac{y+4}{2}$.
From the GP: $\frac{y}{x}=\frac{18}{y}$	**B1**	Or equivalent statement e.g. $y^2 = 18x$ or $x=\frac{y^2}{18}$.
Simultaneous equations: $y^2-9y-36=0$ or $2x^2-17x+8=0$	**M1**	Elimination of either x or y to give a three term quadratic (= 0)
OR		
$4+d=x$, $4+2d=y \rightarrow \frac{4+2d}{4+d}=r$ oe	**B1**	
$(4+d)\left(\frac{4+2d}{4+d}\right)^2=18 \rightarrow 2d^2-d-28=0$	**M1**	Uses $ar^2 = 18$ to give a three term quadratic (= 0)
$d = 4$	**B1**	Condone inclusion of $d=\frac{-7}{2}$ oe
OR		
From the GP $\frac{y}{x}=\frac{18}{y}$	**B1**	
$\rightarrow x=\frac{y^2}{18} \rightarrow 4+d=\frac{y^2}{18} \rightarrow d=\frac{y^2}{18}-4$	**B1**	
$4+2\left(\frac{y^2}{18}-4\right)=y \rightarrow y^2-9y-36=0$	**M1**	
$\boldsymbol{x=8, y=12}$.	**A1**	Needs both x and y. Condone $\left(\frac{1}{2},-3\right)$ included in final answer. Fully correct answer www 4/4.
	4	
AP 4th term = **16**	**B1**	Condone inclusion of $\frac{-13}{2}$ oe
GP 4th term = $8 \times \left(\frac{12}{8}\right)^3$	**M1**	A valid method using their x and y from **(i)**.
= **27**	**A1**	Condone inclusion of -108
		Note: Answers from fortuitous $x = 8$, $y = 12$ in **(i)** can only score M1. Unidentified correct answer(s) with no working seen after valid $x = 8$, $y = 12$ to be credited with appropriate marks.
	3	

Question37

(i)	$S_{80}=\frac{80}{2}[12+79\times(-4)]$ or $\frac{80}{2}[6+l], l=-310$	**M1A1**	Correct formula (M1). Correct a, d and n (A1) .
	$-12\,160$	**A1**	
		3	
(ii)	$S_\infty=\frac{6}{1-\frac{1}{3}}=9$	**M1A1**	Correct formula with $\lvert r\rvert<1$ for M1
		2	

Question 38

(i)	$S_n = \frac{p(2^n - 1)}{2-1}$ soi	M1	
	$p(2^n - 1) > 1000p \rightarrow \quad 2^n > 1001$ **AG**	A1	
		2	
(ii)	$p + (n-1)p = 336$	B1	Expect $np = 336$
	$\frac{n}{2}[2p + (n-1)p] = 7224$	B1	Expect $\frac{n}{2}(p + np) = 7224$
	Eliminate n or p to an equation in one variable	M1	Expect e.g. $168(1 + n) = 7224$ or $1 + 336/p = 43$ etc
	$n = 42$, $p = 8$	A1A1	
		5	

Question 39

(i)	$\frac{x}{2}[2 + (x-1)(-/+0.02)]$ or $1.01x - 0.01x^2$ or $0.99x + 0.01x^2$ **oe**	B1	Allow – or + 0.02. Allow n used
		1	
(ii)	Equate to 13 **then** *either* simplify to a 3-term quadratic equation **or** find at least 1 solution (need not be correct) to an unsimplified quadratic	M1	Expect $n^2 - 101n + 1300$ (=0) or $0.99x + 0.01x^2 = 13$. Allow x used
	16	A1	Ignore 85.8 or 86
		2	
(iii)	Use of $\frac{a(1-r^{n})}{1-r}$ with $a = 1$, $r = 0.92$, $n = 20$ soi	M1	
	(=) 10.1	A1	
	Use of $(S_\infty =)\frac{a}{1-r}$ with $a = 1$, $r = 0.92$	M1	OR $\frac{(1)(1-0.92^{n})}{1-0.92} = 13 \rightarrow 0.92^n = -0.04$ oe
	$S_\infty = 12.5$ so never reaches target or < 13	A1	Conclusion required – 'Shown' is insufficient No solution so never reaches target or < 13
		4	

Question 40

(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11 \text{ to } 15)}$	M1	Either statement seen or implied.
	$5(2a + 9d)$ oe	B1	
	$7.5(2a + 14d) - 5(2a + 9d)$ or $\frac{5}{2}[(a + 10d) + (a+14d)]$ oe	A1	
	$d = \frac{a}{3}$ **AG**	A1	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
(a)(ii)	$(a+9d) = 36 + (a+3d)$	M1	Correct use of $a + (n-1)d$ twice and addition of ±36
	$a = 18$	A1	
		2	Correct answer www scores 2/2
(b)	$S_\infty = 9 \times S_4$; $\frac{a}{1-r} = 9\frac{a(1-r^4)}{1-r}$ or $9(a + ar + ar^2 + ar^3)$	B1	May have 12 in place of a.
	$9(1 - r^n) = 1$ where n = 3,4 or 5	M1	Correctly deals with a and correctly eliminates '$1 - r$'
	$r^4 = \frac{8}{9}$ oe	A1	
	(5^{th} term =) $10\frac{2}{3}$ or 10.7	A1	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

Question 41

(a)	$ar^2 = 48$, $ar^3 = 32$, r = ⅔ **or** a = 108	M1	Solution of the 2 eqns to give r (or a). A1 (both)
	r = ⅔ **and** a = 108	A1	
	$S_\infty = \frac{108}{\frac{1}{3}} = 324$	A1	**FT** Needs correct formula and r between −1 and 1.
		3	
(b)	Scheme A $a = 2.50$, $d = 0.16$ $S_n = 12(5 + 23\times0.16)$	M1	Correct use of either AP S_n formula.
	S_n = 104 tonnes.	A1	
	Scheme B $a = 2.50$, $r = 1.06$	B1	Correct value of r used in GP.
	$= \frac{2.5(1.06^{24} - 1)}{1.06 - 1}$	M1	Correct use of either S_n formula.
	S_n = 127 tonnes.	A1	
		5	

Question 42

(i)	$\frac{5k-6}{3k}=\frac{6k-4}{5k-6} \quad \to \quad (5k-6)^2=3k(6k-4)$	M1	OR any valid relationship
	$25k^2-60k+36=18k^2-12k \quad \to \quad 7k^2-48k+36$	A1	AG
		2	
(ii)	$k=\frac{6}{7}$, 6	B1B1	Allow 0.857(1) for $\frac{6}{7}$
	When $k=\frac{6}{7}$, $r=-\frac{2}{3}$	B1	Must be exact
	When $k=6$, $r=\frac{4}{3}$	B1	
		4	
(iii)	Use of $S_\infty=\frac{a}{1-r}$ with $r=their\ -\frac{2}{3}$ and $a=3\times their\ \frac{6}{7}$	M1	Provided 0 < |*their* –2/3| < 1
	$\frac{18}{7}\div\left(1+\frac{2}{3}\right)=\frac{54}{35}$ or 1.54	A1	FT if 0.857(1) has been used in part (ii).
		2	

Question 43

(a)(i)	21st term = 13 + 20 × 1.2 = 37 (km)	B1	
		1	
(a)(ii)	$S_{21}=\frac{1}{2}\times21\times(26+20\times1.2)$ or $\frac{1}{2}\times21\times(13+their\ 37)$	M1	A correct sum formula used with correct values for *a*, *d* and *n*.
	525 (km)	A1	
		2	
(b)(i)	$\frac{x-3}{x}=\frac{x-5}{x-3}$ oe (or use of *a*, *ar* and ar^2)	M1	Any valid method to obtain an equation in one variable.
	(*a* = *or x* =) 9	A1	
		2	
(b)(ii)	$r=\left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}}=\frac{2}{3}$. Fourth term = $9\times(\frac{2}{3})^3$	M1	Any valid method to find *r* and the fourth term with *their a* & *r*.
	$2\frac{2}{3}$ **or** 2.67	A1	OE, AWRT
		2	
(b)(iii)	$S_\infty=\frac{a}{1-r}=\frac{9}{1-\frac{2}{3}}$	M1	Correct formula and using *their* '*r*' and '*a*', with $\|r\|<1$, to obtain a numerical answer.
	27 **or** 27.0	A1	AWRT
		2	

Question 44

(i)	Identifies common ratio as 1.1	**B1**	
	Use of $x(1.1)^{20} = 20$	**M1**	SOI
	$x\left(=\frac{20}{(1.1)^{20}}\right)=3.0$	**A1**	Accept 2.97
		3	
(ii)	$their 3.0\times\frac{\left[(1.1)^{21}-1\right]}{1.1-1}\rightarrow 192$	**M1** **A1**	Correct formula used for M mark. Allow 2.97 used from **(i)** Accept 190 from $x = 2.97\ldots$
		2	

Question 45

(a)	2%	**B1**	
		1	
(b)	Bonus = 600 + 23 × 100 = 2900	**B1**	
	Salary = 30000×1.03^{23}	**M1**	Allow 30000×1.03^{24} (60984)
	= 59207.60	**A1**	Allow answers of 3significant figure accuracy or better
	$\frac{their\,2900}{their\,59200}$	**M1**	SOI
	4.9(0)%	**A1**	
		5	

Question 46

(a)	$r=\cos^2\theta$ SOI	**M1**
	$S_\infty=\frac{\sin^2\theta}{1-\cos^2\theta}$	**M1**
	1	**A1**
		3
(b)(i)	$d=\sin^2\theta\cos^2\theta-\sin^2\theta$	**M1**
	$\sin^2\theta\left(\cos^2\theta-1\right)$	**M1**
	$-\sin^4\theta$	**A1**
		3
(b)(ii)	Use of $S_{16}=\frac{16}{2}[2a+15d]$	**M1**
	With <u>both</u> $a=\frac{3}{4}$ and $d=-\frac{9}{16}$	**A1**
	$S_{16}=-55\frac{1}{2}$	**A1**
		3

Question 47

1st term is −6, 2nd term is −4.5 (**M1** for using *k*th terms to find both *a* and *d*)	**M1**
$\rightarrow a = -6,\ d = 1.5$	**A1 A1**
$S_n = 84 \rightarrow 3n^2 - 27n - 336 = 0$	**M1**
Solution $n = 16$	**A1**
	5

Question 48

$117 = \frac{9}{2}(2a + 8d)$	**B1**
Either 91 = S_4 with '*a*' as $a + 4d$ **or** $117 + 91 = S_{13}$ (**M1** for overall approach. **M1** for S_n)	**M1M1**
Simultaneous Equations $\rightarrow a = 7,\ d = 1.5$	**A1**
	4

Question 49

(a)	$\$36\,000 \times (1.05)^n$ (**B1** for $r = 1.05$. **M1** method for *r*th term)	**B1M1**
	\$53 200 after 8 years.	**A1**
		3
(b)	$S_{10} = 36000\frac{(1.05^{10} - 1)}{(1.05 - 1)}$	**M1**
	\$453 000	**A1**
		2

Question 50

(a)	$(d =) -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$	**B1**	Allow sign error(s). Award only at form (*d* =)... stage
	$-\frac{\sin^2\theta}{\cos^4\theta} - \frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$	**M1**	Allow sign error(s). Can imply B1
	$\frac{-\sin^2\theta - \cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$	**M1**	
	$-\frac{1}{\cos^4\theta}$	**A1**	AG, WWW
		4	

(b)	$a = \frac{4}{3}$, $d = -\frac{16}{9}$	B1	SOI, both required. Allow $a = \frac{1}{\frac{3}{4}}$, $d = -\frac{1}{\frac{9}{16}}$
	$u_{13} = \frac{1}{\cos^2\theta} - \frac{12}{\cos^4\theta} = \frac{4}{3} + 12\left(\frac{-16}{9}\right)$	M1	Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed
	–20	A1	WWW
		3	

Question 51

S_x *and* S_{x+1}	M1	Using two values of n in the given formula
$a = 5$, $d = 2$	A1 A1	
$a + (n - 1)d > 200 \to 5 + 2(k - 1) > 200$	M1	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
$(k =)$ 99	A1	Condone $\geqslant 99$
Alternative method for question 4		
$\frac{n}{2}(2a + (n-1)d) \equiv n^2 + 4n \to \left(\frac{d}{2} = 1,\ a - \frac{1}{2}d = 4\right)$	M1	Equating two correct expressions of S_n and equating coefficients of n and n^2
$d = 2$, $a = 5$	A1 A1	
$a + (n - 1)d > 200 \to 5 + 2(k - 1) > 200$	M1	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
$(k =)$ 99	A1	Condone $\geqslant 99$

Question 52

$(-2p)^2 = (2p + 6) \times (p + 2)$ or $\frac{-2p}{2p+6} = \frac{p+2}{-2p}$	M1	OE. Using "*a, b, c* then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p + 2$ to form a correct relationship in terms of p only
$(2p^2 - 10p - 12 = 0)$ $p = 6$	A1	
$a = 18$ and $r = -\frac{2}{3}$	A1	
$(s_\infty) =$ *their a* $\div (1 -$ *their r*$)$ $\left(= 18 \div \frac{5}{3}\right)$	M1	Correct formula used with their values for a and r, $\lvert r \rvert < 1$ Both a & r from the same value of p.
$(s_\infty =)$10.8	A1	OE. A0 if an extra solution given
		SC B2 for $s_\infty = \frac{2p+6}{1 - \frac{-2p}{2p+6}}$ *or* $\frac{2p+6}{1 - \frac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
	5	

Question 53

(a)	$S = \frac{a}{1-r}$, $2S = \frac{a}{1-R}$	B1	SOI at least one correct
	$\frac{2a}{1-r} = \frac{a}{1-R}$	M1	SOI
	$2 - 2R = 1 - r \rightarrow r = 2R - 1$	A1	AG
		3	

(b)	$ar^2 = aR \rightarrow (a)(2R-1)^2 = R(a)$	*M1	
	$4R^2 - 5R + 1\ (=0) \rightarrow (4R-1)(R-1)\ (=0)$	DM1	Allow use of formula or completing square.
	$R = \frac{1}{4}$	A1	Allow $R = 1$ in addition
	$S = \frac{2a}{3}$	A1	

Question 54

(a)(i)	$\frac{\cos\theta}{1-r} = \frac{1}{\cos\theta}$	B1	
	$1 - r = \cos^2\theta$ leading to $r = 1 - \cos^2\theta$	M1	Eliminate fractions
	$r = \sin^2\theta$ leading to 2nd term = $\cos\theta\sin^2\theta$	A1	AG
		3	
(a)(ii)	$S_{12} = \frac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)} = \frac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$	M1	Evidence of correct substitution, use of S_n formula and attempt to evaluate
	1.937	A1	
		2	
(b)	$[d =]\cos\theta\sin^2\theta - \cos\theta$	M1	Use of $d = u_2 - u_1$
	$-\frac{1}{8}$	A1	
	[85th term =] $\frac{1}{2} + 84 \times -\frac{1}{8}$	M1	Use of $a + 84d$ with a calculated value of d
	-10	A1	
		4	

Question 55

(a)	$ar = \frac{24}{100} \times \frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2 - 100r + 24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0] \rightarrow r = \frac{2}{5}, \frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	
(b)	$3 \times \{(a+4d)\} = \{(2(a+1) + 11(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a + d = 13$	A1	
	$\left[\frac{5}{2}\right] \times 3\{(2a+4d)\} = \left[\frac{5}{2}\right] \times 2\{(4(a+1) + 4(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a + 2d = 8$	A1	
	Solve 2 linear equations simultaneously	DM1	Elimination or substitution expected
	$d = 7,\ a = 6$	A1	**SC B1** for a=6, d=7 without complete working
		6	

Question 56

(a)	$\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$	B1	SOI
	$18^2 = a(b+3)$ OE or 2 correct statements about r from the GP, e.g. $r=\frac{18}{a}$ and $b + 3 = 18r$ or $r^2=\frac{b+3}{a}$	B1	SOI
	$324 = a(2a+3) \Rightarrow 2a^2+3a-324[=0]$ or $b^2+3b-648[=0]$ or $6r^2-r-12[=0]$ or $4d^2+3d-162[=0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a-12)(2a+27)[=0]$ or $(b-24)(b+27)[=0]$ or $(2r-3)(3r+4)[=0]$ or $(d-6)(4d+27)[=0]$	M1	Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a, b, r* or *d*.
	$a = 12, b = 24$	A1	WWW. Condone extra 'solution' $a=-13.5, b=-27$ only.
		5	

(b)	Common difference $d = 6$	B1 FT	SOI. FT *their* $\frac{a}{2}$
	$S_{20}=\frac{20}{2}(2\times12+19\times6)$	M1	Using correct sum formula with *their a*, *their* calculated d and 20.
	1380	A1	
		3	

Question 57

$(-12)^2 = 8k \times 2k$	**M1**	Forming an equation in k
$k=-3$	**A1**	
Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	With $-1<r<1$
$S_\infty=-768$	**A1**	
Alternative method for Question 5		
$r^2=\frac{2k}{8k}$	**M1**	
$r=[\pm]0.5$	**A1**	
Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	$-1<r<1$
$S_\infty=-768$	**A1**	
	4	

Question 58

$10(2a + 19d) = 405$	**B1**	
$20(2a + 39d) = 1410$	**B1**	
Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6$, $d = 1.5$]	**M1**	Reach $a =$ or $d =$
Using the correct formula for 60th term with their a and d	**M1**	
60th term = 94.5	**A1**	OE, e.g. $\frac{189}{2}$
	5	

Question 59

$ar = 54$ and $\frac{a \text{ or } their\ a}{1-r} = 243$	**B1**	SOI
$\frac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54[=0]$ $[9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	***M1**	Forming a 3-term quadratic expression in r or a using *their* 2nd term and S_∞. Allow ± sign errors.
$k(3r-2)(3r-1)[=0]$ OR $(a-81)(a-162)[=0]$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give ±*their* coefficient of r^2.
$54 \div \left(their\ \frac{2}{3}\right) = a$ OR $54 \div (their\ 81) = r$	**DM1**	May be implied by final answer.
Tenth term $= \frac{512}{243}$ $\left[\text{OR } 81\times\left(\frac{2}{3}\right)^9 \text{ OR } 54\times\left(\frac{2}{3}\right)^8\right]$	**A1**	OE. Must be exact. **Special case:** If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
	5	

Question 60

(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$ OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	***M1**	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving $\sin x$ and $\cos x$.
	Elimination of $\sin x$ and $\cos x$ to give an expression in $\tan x$ $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	**DM1**	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x =]\frac{5\pi}{6}$ only	**A1**	CAO. Must be exact.
		3	
(b)	$d = 2\cos x$ or $d = 2\cos(their\ x)$	**B1 FT**	Or an equivalent expression involving $\sin x$ and $\cos x$ e.g. $-3\sqrt{3}\sin(their\ x) - \cos(their\ x)$ $\left[= -\sqrt{3}\right]$ FT for *their* x from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}\left(2\times(2\cos(their\ x)) + (25-1)\times(their\ d)\right)$ $\left[= 12.5\left(2\times(-\sqrt{3}) + 24(-\sqrt{3})\right)\right]$	**M1**	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with a replaced by either 2(cos(*their* x)) or $\pm\sqrt{3}$ and d replaced by either 2(cos(*their* x)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	**A1**	Must be exact.
		3	

Question 61

(a)	$\dfrac{5a}{1-\left(\pm\frac{1}{4}\right)}$	**B1**	Use of correct formula for sum to infinity.
	$\frac{8}{2}\left[2a+7(-4)\right]$	***M1**	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a=8a-112$ leading to $a=[28]$	**DM1**	Solve equation to reach a value of *a*.
	$a=28$	**A1**	Correct value.
		4	
(b)	*their* $28+(k-1)(-4)=0$	**M1**	Use of correct method with *their a*.
	$[k=]8$	**A1**	
		2	

Question 62

$ar^2=a+d$	**B1**	
$ar^4=a+5d$	**B1**	
$a^2r^4=a(a+5d)$ leading to $a^2+5ad=(a+d)^2$	***M1**	Eliminating *r* or complete elimination of *a* and *d*.
$\left[3ad-d^2=0 \text{ leading to}\right]$ $d=3a$ OR $\left[r=2 \text{ leading to}\right]$ $d=3a$	**A1**	
$S_{20}=\frac{20}{2}[2a+19\times 3a]$	**DM1**	Use of formula with *their d* in terms of *a*.
$590a$	**A1**	
	6	

Question 63

(a)	$\frac{n}{2}\left[8+(n-1)d\right]=5863$ leading to $n\left[8+(n-1)d\right]=11726$ leading to $(n-1)d=\frac{11726}{n}-8$	**B1**	Must show a useful intermediate step. WWW AG.
		1	
(b)	$4+(n-1)d=139$ leading to $\frac{11726}{n}-8=135$	***M1**	OE Use of correct u_n formula with expression from **(a)** or S_n formula to eliminate *d*.
	$n=\frac{11726}{143}=82$	**A1**	
	$81d=\frac{11726}{82}-8$	**DM1**	Substitute *their n* into a correct u_n or S_n formula
	$d=\frac{5}{3}$	**A1**	Accept $\frac{138}{81}$ OE fraction only If M0 DM0 scored them **SC B1 B1** for correct *n* and *d* values only.
		4	

Question 64

(a)	$2 \times 6k = k + k + 6$ or $6k - k = k + 6 - 6k$ or $2d = 6$ leading to $d = 3$, $\therefore 6k - 3 = k$	**B1**	OE A correct equation in k only. Can be implied by correct final answer.
	$k = \frac{6}{10}$ or 0.6	**B1**	OE
		2	
(b)	$d = 3$	**B1**	Correct value of d can be implied by a correct final answer. Working may be seen in part **(a)** but must be used in **(b)**.
	$S_{30} = \frac{30}{2}(2 \times \text{'their } k\text{'} + 29 \times \text{'their } d\text{'})$	**M1**	It needs to be clear that the candidate is using a correct sum formula. There is no requirement to check the candidates working for d but it must be clearly identified.
	$S_{30} = 1323$	**A1**	ISW if corrected to 1320.
		3	

Question 65

$r = 0.8$	**B1**	OE
$a = 12.5$	**B1**	OE
$S_\infty = 12.5 \div (1 - 0.8)$	**M1**	Using $\frac{a}{1-r}$ with '*their a*' and '*their r*' but $\lvert r \rvert$ must be < 1.
$S_\infty = \frac{125}{2}, 62\frac{1}{2}$ or 62.5	**A1**	$\frac{12\frac{1}{2}}{\frac{1}{5}}$ or similar **does not** get A1.
	4	

Question 66

$a + 12d = 12$	**B1**	For correct equation.
$\frac{30}{2}(2a + (30-1)d) = -15$	**B1**	For correct equation in a and d. If using $\frac{n}{2}(a + l)$, must replace l with an expression involving a and d.
$a = 72$, $d = -5$	**B1**	Both values correct SOI.
$S_{50} = \frac{50}{2}(2(\text{their } a) + 49(\text{their } d))$	**M1**	Using sum formula with *their a* and *d* values obtained via a valid method.
$S_{50} = -2525$	**A1**	
	5	

Question 67

(a)	$216r^3 = 64 \rightarrow r = 2/3$	**B1**	Allow decimal to 3sf (AWRT).
	$S_\infty = \dfrac{216}{1 - their\,⅔} = 648$ cao	**M1 A1**	M1 depends on *their* $\lvert r\rvert < 1$.
		3	
(b)	$216\left(\dfrac{2}{3}\right) = 144 \rightarrow 144 = a + d$	**B1 FT**	SOI, may be implied in the use of $96 = 144 + 3d$ and finding a. Mis-reads not condoned in **9(b)**.
	$216\left(\dfrac{2}{3}\right)^2 = 96 \rightarrow 96 = a + 4d$	**B1 FT**	SOI, may be implied in the use of $96 = 144 + 3d$ and finding a.
	Solve simultaneously	***M1**	No working may be seen.
	$d = -16, \quad a = 160$	**A1**	Both required.
	$S_{21} = \dfrac{21}{2}\{320 + 20(-16)\} = 0$	**DM1 A1**	Or use of $\dfrac{21}{2}(a+u_{21})$.
		6	

Question 68

$2a - a = a^2 - 2a$	**B1**	OE An unsimplified correct equation in a or d only, e.g. $a^2 + a = 4a$. Can be implied by correct values for a or d.
$a = 3$ or $d = 3$	**B1**	Condone 'extra' solution of $a = 0$ or $d = 0$.
$a = 3$ and $d = 3$	**B1**	SOI
$S_{50} = \dfrac{50}{2}(2 \times their\,a + 49 \times their\,d)$	**M1**	May be done using 50th term (=150). Their a and d must be numerical.
3825	**A1**	ISW **SC B2** for $1275a$ or $1275d$
	5	

Question 69

$ar^2 = 1764$ and $ar + ar^2 = 3444$ or $ar = 1680$ or $\dfrac{a(1-r^3)}{1-r} - a = 3444$	**B1**	Two correct algebraic statements.
Attempt to solve as far as $r =$ or $a =$	**M1**	Any valid method, e.g. $1764 \div 1680$ or from $20r^2 - 41r + 21$ OE (condone solving using a calculator).
$r = \dfrac{1764}{1680} = \dfrac{21}{20}$ or 1.05 [$a = 1600$]	**A1**	Note: $r = \dfrac{1764}{3444 - 1764}$ www implies B1 and M1.
17 500	**A1**	AWRT e.g. 17 474.1…..
	4	

Question 70

(a)	$r = 0.8$	**B1**	SOI
	$50 \times (their\, 0.8)^7 = 10.5$	**M1**	Evaluate 8th or 9th term in GP.
	$50 \times (their\, 0.8)^8 = 8.39$. Hence 9th impact required	**A1**	**AG** Two terms correct + conclusion (mention of 9th impact or u_9 somewhere in the solution). Statement that one is <10 (and the other >10) is insufficient unless it mentions 9th impact or u_9.
	Alternative method for final two marks: Logarithm method		
	$50 \times (their\, 0.8)^n < 10 \Rightarrow (their\, 0.8)^n < 0.5$ $n \log(their\, 0.8) < \log 0.5$ $n > \dfrac{\log 0.5}{\log(their\, 0.8)} \Rightarrow [n >] 7.2$	**M1**	
	$n = 8$ hence 9th impact required	**A1**	**AG** Need conclusion that mentions 9th impact or u_9.
		3	
(b)	$\dfrac{50\left(1 - (their\, 0.8)^{20}\right)}{1 - their\, 0.8}$	**M1**	OE Use of formula with *their* r SOI.
	$= 247$	**A1**	Must be to the nearest mm (not 247.1).
		2	
(c)	$\dfrac{50}{1 - their\, 0.8}$	**M1**	Use of sum to infinity formula with *their* r SOI. Substituting a value of n into the sum formula M0.
	$= 250$	**A1**	
		2	

Question 71

(a)	$5.00 + 20 \times 0.02$ or $5.02 + 19 \times 0.02$	**M1**	Allow for $a = 5$, $n = 20$ with $d = 0.02$ only. $a = 5$, $n = 21$(OE) with $d = 0.2$ gets M1 only.
	5.40	**A1**	
		2	
(b)	$r = \dfrac{5.02}{5} = 1.004$ or $\dfrac{251}{250}$	**B1**	
	$5.00 \times (their 1.004)^{20}$ or $5.02 \times (their 1.004)^{19}$	**M1**	Allow $a = 5$, $n = 20$.
	5.42	**A1**	Any correct rounding of 5.41557108 .
		3	

Question 72

(a)	$r = \frac{a}{a+2}$	**B1**	OE SOI
	$\frac{a}{1-\frac{a}{a+2}} = 264$	**M1**	Use of S∞ formula.
	$\frac{a(a+2)}{a+2-a} = 264$ leading to $\frac{a(a+2)}{2} = 264$ leading to $a^2 + 2a - 528$ $[=0]$	**M1***	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24)$ $[=0]$	**DM1**	Attempt to solve.
	$a = 22$ (only)	**A1**	22 without working **SC DB1** (dep on 2nd M1).
		5	
(b)	$d = \frac{6^2}{6+2} - 6 = -\frac{3}{2}$	**B1**	
	$\frac{n}{2}\left\{12 + (n-1)\left(\frac{-3}{2}\right)\right\}[<] - 480$	**M1***	Forming an inequation with *their* numerical *d*. May use an equality.
	$[3](n^2 - 9n - 640)$ $[>0]$	**A1**	OE May contain terms on LHS and RHS.
	$[n=]$ $\frac{9 \pm \sqrt{81+2560}}{2}$	**DM1**	OE. Expect 30.19 . Working for solution must be shown.
	31 only	**A1**	Must come from a correct first inequality (or an equality). 31 no working **SC DB1** (dep on correct quadratic and correct inequality/equality).
		5	

Question 73

(a)	$\left[ar = 16, \frac{a}{1-r} = 100\right]$ leading to $a = \frac{16}{r}$ and $a = 100(1-r)$	**B1**	Rearranging two algebraic statements to give $a =$. These can be implied by a correct equation in one variable.
	$100(1-r)r = 16$ leading to $100r^2 - 100r + 16[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1) = 0$ OR $\frac{25 \pm \sqrt{25^2 - 4.25.4}}{2.25}$ leading to $r = \left[\frac{4}{5} \text{ or } \frac{1}{5}\right]$	**DM1**	Condone $(5r-4)(5r-1)$ following $100r^2 - 100r + 16$.
	$a = 20$, $a = 80$	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar = 16, \frac{a}{1-r} = 100\right]$ leading to $r = \frac{16}{a}$ and $r = \frac{100-a}{100}$	**B1**	Rearranging two algebraic statements to give $r =$. These can be implied by a correct equation in one variable.
	$1600 = 100a - a^2$ leading to $a^2 - 100a + 1600[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80) = 0$ OR $\frac{100 \pm \sqrt{100^2 - 4.1600}}{2}$	**DM1**	
	$a = 20$, $a = 80$	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
		4	

(b)	$r = \frac{4}{5}$, $\frac{1}{5}$	**B1**	OE SOI
	$[u_n =]$ *their* $20 \times$ *their* $\left(\frac{4}{5}\right)^{n-1}$ $[v_n =]$ *their* $80 \times$ *their* $\left(\frac{1}{5}\right)^{n-1}$	**B1FT**	2 expressions for the nth term FT *their* values from part **(a)** if $\lvert r \rvert$ less than 1.
	Method 1 for final 2 marks		
	$20 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1}$	**M1**	Correctly separating the numerator and denominator of *their* $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n = 80 \times \frac{4^{n-2}}{5^{n-1}}$.
	$u_n = \frac{1}{4} \times 80 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1} = 4^{n-2} \times 80 \times \left(\frac{1}{5}\right)^{n-1} = 4^{n-2} \times v_n$	**A1**	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20 \times 0.8^{n-1}}{80 \times 0.2^{n-1}} = \frac{1}{4} \times 4^{n-1}$	**M1**	Dividing two nth terms of the correct format and simplifying their terms in r.
	$= 4^{-1} \times 4^{n-1} = 4^{n-2}$	**A1**	AG
		4	

Question 74

(a)	$2(2p-6) = p + \frac{p^2}{6} \Rightarrow \frac{p^2}{6} - 3p + 12[=0]$ OR $(2p-6) - \frac{p^2}{6} = p - (2p-6) \Rightarrow \frac{p^2}{6} - 3p + 12[=0]$ OR $\frac{1}{6}d^2 + d[=0]$	***M1**	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d. OE e.g. $p^2 - 18p + 72[=0]$, $\frac{1}{2}p^2 - 9p + 36[=0]$.
	$p^2 - 18p + 72[=0] \Rightarrow (p-6)(p-12)[=0]$ **or** $\frac{18 \pm \sqrt{(-18)^2 - 4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d + 1\right)[=0] \Rightarrow d = -6$	**DM1**	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p = 12$ only	**A1**	Since $p = 6$ gives $d = 0$. If *M1 DM0 then $p = 12$ only, award **SC B1**, max 2/3 marks. A0 XP if error in either factor and $p = 12$ only. $p = 12$ only by trial and improvement 3/3.
		3	
(b)	For GP $r = \left[\frac{2p-6}{\frac{p^2}{6}}\right] = \frac{18}{24}\left[=\frac{3}{4}\right]$	**B1**	OE SOI.
	Sum to infinity $= \frac{24}{1-\frac{3}{4}} = 96$	**B1 FT**	FT *their* <u>value</u> of p if used correctly to find r (B0 if 'p' used) provided $\lvert r \rvert < 1$. e.g. $p = 18 \Rightarrow [S_\infty =]\frac{54}{1-\frac{5}{9}} = 121.5$.
		2	