AS-Level

Pure Mathematics P1

Topic : Sequence and Series

May 2013- May 2025

Question 1

(a) In an arithmetic progression, the sum, S_n, of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]

(b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the nth term respectively of an arithmetic progression. Find the value of n. [5]

Question 2

(a) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression. [4]

(b) The third term of a geometric progression is four times the first term. The sum of the first six terms is k times the first term. Find the possible values of k. [4]

Question 3

The third term of a geometric progression is -108 and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

Question 4

(a) In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio. [2]

(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference. [4]

Question 5

(a) An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the nth mile takes 9 minutes, find the value of n. [2]

(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon. [2]

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [4]

Question 6

(a) In an arithmetic progression the sum of the first ten terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term. [5]

(b) A geometric progression has first term a, common ratio r and sum to infinity 6. A second geometric progression has first term $2a$, common ratio r^2 and sum to infinity 7. Find the values of a and r. [5]

Question 7

The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity. [2]

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

Question 8

The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is r, where $r \neq 1$. Find

(i) the value of r, [4]

(ii) the 4th term of each progression. [3]

Question 9

An arithmetic progression has first term a and common difference d. It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find d in terms of a. [3]

(ii) Find the 100th term in terms of a. [2]

Question 10

Three geometric progressions, P, Q and R, are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression P is $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$.

Progression Q is $3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$.

(i) Find the sum to infinity of progression R. [3]

(ii) Given that the first term of R is 4, find the sum of the first three terms of R. [3]

Question 11

(a) The sum, S_n, of the first n terms of an arithmetic progression is given by $S_n = 32n - n^2$. Find the first term and the common difference. [3]

(b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression. [5]

Question 12

(i) A geometric progression has first term a ($a \neq 0$), common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio $2r$ and sum to infinity $3S$. Find the value of r. [3]

(ii) An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245. Find the value of n. [4]

Question 13

(a) The first term of an arithmetic progression is -2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

Question 14

(a) The first, second and last terms in an arithmetic progression are 56, 53 and -22 respectively. Find the sum of all the terms in the progression. [4]

(b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.

(i) Find the value of k. [3]

(ii) Find the sum to infinity of the progression. [2]

Question 15

(a) The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

Question 16

A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

Question 17

The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

Question 18

The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

Question 19

The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

Question 20

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

Question 21

(a) The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

Question 22

(a) Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

Question 23

(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

Question 24

The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

Question 25

The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take. [2]

Question 26

(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

Question 27

(a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated $2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

Question 28

An arithmetic progression has first term −12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

Question 29

(a) Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

Question 30

(a) A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

Question 31

On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day.
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day.

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

Question 32

The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

Question 33

A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

Question 34

(a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

Question 35

In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a. [6]

Question 36

The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

(ii) Find the fourth term of each progression. [3]

Question 37

The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

Question 38

(i) The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

Question 39

Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

(i) Write down an expression for his total weight loss after x weeks. [1]

(ii) He reaches his 13 kg target during week n. Use your answer to part **(i)** to find the value of n. [2]

Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

Question 40

(a) In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

Question 41

(a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

Question 42

The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

Question 43

(a) Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

(ii) Find the total distance she runs in the 21-day period. [2]

(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

(ii) Find the fourth term of the progression. [2]

(iii) Find the sum to infinity of the progression. [2]

Question 44

A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

(ii) Find the total distance she runs over the 21 days. [2]

Question 45

A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

Question 46

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

Question 47

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

Question 48

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

Question 49

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

Question 50

The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

Question 51

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

Question 52

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

Question 53

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

Question 54

The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

Question 55

(a) A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

Question 56

The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

Question 57

The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

Question 58

The first term of an arithmetic progression is 84 and the common difference is −3.

(a) Find the smallest value of n for which the nth term is negative. [2]

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

Question 59

The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

Question 60

The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

(b) Hence find the exact sum of the first 25 terms of the progression. [3]

Question 61

The first term of an arithmetic progression is a and the common difference is −4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

Question 62

The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

Question 63

An arithmetic progression has first term 4 and common difference d. The sum of the first n terms of the progression is 5863.

(a) Show that $(n-1)d = \frac{11726}{n} - 8$. [1]

(b) Given that the nth term is 139, find the values of n and d, giving the value of d as a fraction. [4]

Question 64

The first, second and third terms of an arithmetic progression are k, $6k$ and $k+6$ respectively.

(a) Find the value of the constant k. [2]

(b) Find the sum of the first 30 terms of the progression. [3]

Question 65

The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity. [4]

Question 66

The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15.

Find the sum of the first 50 terms of the progression. [5]

Question 67

The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]

Question 68

The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

Question 69

A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

Question 70

A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

Question 71

The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

Question 72

A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

Question 73

The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

Question 74

The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

Question 75

The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

Question 76

The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

Question 77

The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

Question 78

(a) An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

(b) A geometric progression $a_1, a_2, a_3, \ldots$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, \ldots$) is denoted by S_E.

Find the values of S and S_E. [4]

Question 79

The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \ldots$. [3]

Question 80

The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

Question 81

The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) **(i)** Find the sum to infinity of the progression in terms of θ. [2]

(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

Question 82

(a) The first three terms of an arithmetic progression are 25, $4p-1$ and $13-p$, where p is a constant.

Find the value of the tenth term of the progression. [4]

(b) The first three terms of a geometric progression are 25, $4q-1$ and $13-q$, where q is a positive constant.

Find the sum to infinity of the progression. [4]

Question 83

The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

Question 84

An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression. [3]

Question 85

The first term of an arithmetic progression is -20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression. [2]

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k. [3]

Question 86

An arithmetic progression has first term 5 and common difference d, where $d > 0$. The second, fifth and eleventh terms of the arithmetic progression, in that order, are the first three terms of a geometric progression.

(a) Find the value of d. [3]

(b) The sum of the first 77 terms of the arithmetic progression is denoted by S_{77}. The sum of the first 10 terms of the geometric progression is denoted by G_{10}.

Find the value of $S_{77} - G_{10}$. [5]

Question 87

A geometric progression is such that its second term is -120 and its sum to infinity is 160.

(a) Find the common ratio. [4]

(b) The first nine terms of the progression are now removed.

Find the sum to infinity of the remaining terms of the progression. [3]

Question 88

An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400. [6]

Question 89

An arithmetic progression has first term a and common difference 2. The Nth term is 55 and the sum of the first $3N$ terms is 5760.

Find the values of N and a. [6]

Question 90

The first two terms of a geometric progression are

$$4\sin^2\theta, \quad 8\sin^3\theta,$$

where θ is an angle such that $0 < \theta < \frac{1}{6}\pi$.

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ. Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]

Question 91

(a) The first, second and third terms of an arithmetic progression are $4k$, k^2 and $8k$ respectively, where k is a non-zero constant.

(i) Find the value of k. [2]

(ii) Find the sum of the first 20 terms of the progression. [3]

(b) The fourth and sixth terms of a geometric progression are 36 and 6 respectively. The common ratio of the progression is positive.

Find the sum to infinity of the progression. Give your answer in the form $\frac{a}{\sqrt{b}-c}$, where a, b and c are integers. [5]

Question 92

The third term of a geometric progression is 18 and the sum of the first three terms is 26. It is given that the common ratio is negative.

(a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

(b) Find the exact value of the sum to infinity of the progression. [2]