**AS-Level**

**Topic :Trigonometry**

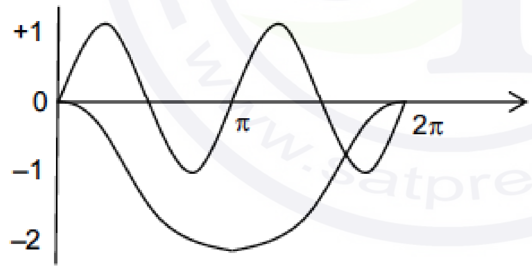
**May 2013-May 2023**

**Answer**

Question 1

| | | | | |
|---|---|---|---|---|
| **(i)** | $2\cos^2\theta = \tan^2\theta$ <br> $\rightarrow 2\cos^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$ | M1 | | Use of $t^2 = s^2 \div c^2$ or alternative. |
| | $\rightarrow$ Uses $c^2 + s^2 = 1 \rightarrow 2c^4 = 1 - c^2$ | A1 | [2] | Correct eqn. |
| **(ii)** | $(2c^2 - 1)(c^2 + 1) = 0 \rightarrow c = \pm\dfrac{1}{\sqrt{2}}$ | M1 | | Method of solving for 3-term quadratic. |
| | $\rightarrow \theta = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$. | A1 A1√ | [3] | (in terms of $\pi$). √ for $\pi - 1^{st}$ ans. Cannot gain A1√ if other answers given in the range. |

Question 2

| | | | | |
|---|---|---|---|---|
| **(i)** |   | B1 <br> DB1 <br> B1 <br> DB1 | [4] | y = sin2x has 2 cycles, starts and finishes on the $x$-axis, max comes first. <br> From +1 to −1. Smooth curves. <br> y = cosx − 1 has one cycle, starts and finishes on $x$-axis, with a minimum pt. <br> From 0 to −2, smooth curve, flattens. |
| **(ii)** | **(a)** $\sin 2x = -\frac{1}{2} \rightarrow$ 4 solutions | B1√ | [1] | √ for their curve. |
| | **(b)** $\sin 2x + \cos x + 1 = 0 \rightarrow$ 3 solutions. | B1√ | [1] | √ for intersections of their curves. |

Question 3

| | | |
|---|---|---|
| $a = \sin\theta - 3\cos\theta,\ \ b = 3\sin\theta + \cos\theta$ | | |
| **(i)** $a^2 + b^2 =$ $(s^2 + 9c^2 - 6sc) + (9s^2 + c^2 + 6sc)$ | B1 | Correct squaring |
| $10c^2 + 10s^2 = 10$ | M1 A1 [3] | Use of $s^2 + c^2 = 1$ to get constant. (can get 2/3 for missing $6sc$) |
| **(ii)** $2s - 6c = 3s + c \rightarrow s = -7c$ | M1 | Collecting and t = s÷c |
| $\rightarrow \tan\theta = -7$ | A1 | |
| $\rightarrow 98.1°$ | A1 | |
| and 278.1° | A1√ [4] | For $180^0$ + first answer, providing no extra answers in the range. |

Question 4

| | | |
|---|---|---|
| **(i)** $\dfrac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$ | **M1** | |
| $\dfrac{\sin^2\theta - \sin\theta\cos\theta + \cos\theta\sin\theta = \cos^2\theta}{\sin^2\theta - \cos^2\theta}$ | **A1** | |
| $\dfrac{1}{\sin^2\theta - \cos^2\theta}$ **AG** | **A1** **[3]** | www |
| **(ii)** $s^2 - (1 - s^2) = \frac{1}{3}$ or $1 - c^2 - c^2 = \frac{1}{3}$ or $3(s^2 - c^2) = c^2 + s^2$ | **M1** | Applying $c^2 + s^2 = 1$ |
| $\sin\theta = (\pm)\sqrt{\frac{2}{3}}$ or $\cos\theta = (\pm)\sqrt{\frac{1}{3}}$ or $\tan\theta = (\pm)\sqrt{2}$ | **A1** | Or $s = (\pm)\ 0.816$, $c = (\pm)\ 0.577$, $t = (\pm)\ 1.414$ |
| $\theta = 54.7°, 125.3°, 234.7°, 305.3°$ | **A1A1** **[4]** | any 2 solutions for 1st A1<br>>4 solutions in range max A1A0 |

Question 5

| | | |
|---|---|---|
| **(a)** $x^2 - 1 = \sin\frac{\pi}{3}$ | M1 | |
| $x = \pm 1.366$ | A1A1√ [3] | √ for negative of 1st answer |
| **(b)** $2\theta + \frac{\pi}{3} = \frac{5\pi}{6}\left(\text{or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6}\right)$ | B1 | 1 correct angle on RHS is sufficient |
| $2\theta = \frac{\pi}{2} = \left(\text{or } \frac{11\pi}{6}\right)$ | M1 | Isolating $2\theta$ |
| $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ | A1A1 [4] | SC decimals 0.785 & 2.88 scores M1B1 |

Question 6

| | | |
|---|---|---|
| **(i)** $\sin x = \sqrt{(1 - p^2)}$ | B1 [1] | Allow $1 - p$ if following $\sqrt{(1 - p^2)}$; ± is B0. |
| **(ii)** $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\sqrt{1-p^2}}{p}$ | B1√ [1] | √ for answer to **(i)** used. |
| **(iii)** $\tan(90 - x) = \dfrac{p}{\sqrt{1-p^2}}$ | B1√ [1] | √ for reciprocal of **(ii)** |

Question 7

| | | |
|---|---|---|
| **(i)** $4(1-\cos^2 x) + 8\cos x - 7 = 0$ | **M1** | Use $c^2 + s^2 = 1$ |
| $4c^2 - 8c + 3 = 0 \rightarrow (2\cos x - 1)(2\cos x - 3) = 0$ | **M1** | Attempt to solve |
| $x = 60°$ or $300°$ | **A1A1** **[4]** | |
| **(ii)** $\tfrac{1}{2}\theta = 60°$ (or $300°$) | **M1** | Allow 300° in addition |
| $\theta = 120°$ only | **A1** **[2]** | |

Question 8

$$\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$$

| | | |
|---|---|---|
| **(i)** LHS $\dfrac{\left(\frac{s}{c}\right)+1}{\left(\frac{s^2}{c}+c\right)} = \dfrac{s+c}{s^2+c^2}$ | M1 M1 | Use of t = s/c twice; Correct algebra and use of $s^2 + c^2 = 1$ |
| = RHS | A1 [3] | AG all ok |
| **(ii)** $s + c = 3s - 2c$ | | |
| $\rightarrow \tan x = \frac{3}{2}$ Allow $\cos^2 = \frac{4}{13}$, $\sin^2 = \frac{9}{13}$ | M1 | Uses (i) and $t = \frac{s}{c}$; $t = \frac{2}{3}$ or 0 is M0 |
| $\rightarrow x = 0.983$ and 4.12 or 4.13 | A1 A1√ [3] | co. √ 1st + π, providing no excess solns in range. Allow 0.313π, 1.31π |

Question 9

| | | |
|---|---|---|
| **(i)** $\frac{1}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}\equiv\tan\theta$. | M1 | Correct addition of fractions |
| LHS $=\frac{1+s-c^2}{c(1+s)}=\frac{s^2+s}{c(1+s)}=\frac{s}{c}$ | M1M1 | Use of $s^2+c^2=1$. $(1+s)$cancelled. |
| $=\tan\theta$ | A1 [4] | → answer given. |
| **(ii)** → $\tan\theta+2=0$ ie $\tan\theta=-2$ | M1 | Uses part **(i).** Allow $\tan\theta=\pm2$ |
| → $\theta=116.6°$ or $296.6°$ | A1 A1√ [3] | Co. √ for 180°+ and no other solutions in the range. |

Question 10

| | | |
|---|---|---|
| reflex angle $\theta$ is such that $\cos\theta=k$, | | |
| **(i)** **(a)** $\sin\theta=-\sqrt{(1-k^2)}$ | B1 B1 [2] | (−) B1 rest B1 |
| **(b)** Uses $t=s/c$ → $\frac{-\sqrt{1-k^2}}{k}$ | B1√ [1] | √ for **(i)** ÷ $k$. |
| **(ii)** $\theta$ is in 4th quadrant. | | |
| $2\theta$ lies between 540° and 720° | B1 | co |
| $\sin2\theta$ is negative in both these quadrants. | B1 [2] | co |

Question 11

| | | |
|---|---|---|
| **(i)** LHS $\equiv\frac{\sin^2\theta-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ cao | **B1** | Put over common denominator |
| $\equiv\frac{1-\cos^2\theta-1+\cos\theta}{(1-\cos\theta)\sin\theta}$ | **M1** | Use $s^2=1-c^2$ oe |
| $\equiv\frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ | **M1** | Correct factorisation from line 2 |
| $\equiv\frac{1}{\tan\theta}$ | **A1** **[4]** | **AG** |
| **(ii)** $\tan\theta=(\pm)\frac{1}{2}$ | **M1** | |
| 26.6°, 153.4° | **A1A1√** **[3]** | Ft for 180 – 1st answer |

Question 12

| | | |
|---|---|---|
| $a=1$, $b=2$ | **B1B1** **[2]** | Or $1+2\sin x$ |

Question 13

| | | | |
|---|---|---|---|
| **(i)** | $(s^2-c^2)(s^2+c^2)$ OR $s^2(1-c^2)-c^2(1-s^2)$ | **M1** | **OR** $\sin^4\theta-(1-\sin^2\theta)^2$ |
| | $\sin^2\theta-\cos^2\theta$ | **A1** | $\sin^4\theta-(1-2\sin^2\theta+\sin^4\theta)$ |
| | $2\sin^2\theta-1$ www **AG** | **A1** **[3]** | $=2\sin^2\theta-1$ **AG** |
| **(ii)** | $2\sin^2\theta-1=\frac{1}{2} \Rightarrow \sin\theta=(\pm)\frac{\sqrt{3}}{2}$ or $(\pm)0.866$ | **B1** | **OR** $\cos 2\theta=-\frac{1}{2}\to 2\theta=120, 240$ etc. |
| | $\theta=60°$ | **B1** | Ft for 180 – *their* 60 |
| | $\theta=120°$ | **B1√** | Ft for 180 + *their* 60, 360 – *their* 60 |
| | $\theta=240°, 300°$ | **B1√** **[4]** | Allow $\frac{\pi}{3}, \frac{2\pi}{3}$ etc. Extra sols in range −1 |

Question 14

$1 + \sin x\tan x = 5\cos x$

| | | | |
|---|---|---|---|
| **(i)** | Replaces t by s/c | M1 | Correct formula |
| | $1+\frac{s^2}{c}=5c$ | | |
| | Replace $s^2$ by $1-c^2$ | M1 | Correct formula used in appropriate place |
| | $\to 6c^2-c-1\ (=0)$ | A1 [3] | AG |
| **(ii)** | Soln of quadratic $\to$ ($c = -⅓$ or ½) | M1 | Correct method |
| | $\to x = 60°$ or $109.5°$ | A1 A1 [3] | co co |

Question 15

| | | |
|---|---|---|
| $\tan^{-1}(3) =$ 1.249 or 71.565° | **M1** | Attempt at $\tan^{-1}3$ or right angle triangle with attempt at hypotenuse = √10 |
| sin 1.25 or sin71.6 or 0.949 soi | **M1** | Attempt at sin $\tan^{-1}3$ |
| ($x =$) 1.95 cao, accept $1+\frac{3}{\sqrt{10}}$ oe | **A1** **[3]** | Answer only **B3** |

Question 16

| | | |
|---|---|---|
| $13\sin^2\theta+2\cos\theta+\cos^2\theta=4+2\cos\theta$ | **M1** | Attempt to multiply by $2+\cos\theta$ |
| $13\sin^2\theta+1-\sin^2\theta=4\to\sin^2\theta=\frac{1}{4}$ or $13-13\cos^2\theta+\cos^2\theta=4\to\cos^2\theta=\frac{3}{4}$ | **M1** | Use of $s^2+c^2$ appropriately |
| 30°, 150° | **A1A1√** **[4]** | **SC** both answers correct in radians, **A1** only<br>Ft on 180 – their first value of $\theta$ |

Question 17

| | | | |
|---|---|---|---|
| **(i)** | $\tan\theta = 1/3$<br>$\theta = 18.4°$ only | **M1**<br>**A1**<br>**[2]** | Ignore solns. outside range 0→180 |
| **(ii)** | $\mathbf{tan}2x = (\pm)1/\sqrt{3}$ Must be sq. root soi | **M1** | $\sin 2x = (\pm)1/2$ **or** $\cos 2x = (\pm)\sqrt{\mathbf{3/2}}$<br>using $c^2 + s^2 = 1$. Not $\tan x = (\pm)\frac{1}{\sqrt{3}}$ etc. |
| | $(x) = 15$<br>$(x) =$ any correct second value (75, 105, 165)<br>$(x) =$ cao | **A1**<br>**A1**√<br>**A1**<br>**[4]** | ft for (90 ± their 15) or (180 – their 15)<br>All four correct. Extra solns in range 1 |

Question 18

| | | | |
|---|---|---|---|
| **(i)** | $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$.<br>Divides top and bottom by cos $\theta$<br>$\rightarrow \frac{t-1}{t+1}$ | B1<br>[1] | Answer given. |
| **(ii)** | $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$<br>$\rightarrow \frac{t-1}{t+1} = \frac{t}{6}$<br>$\rightarrow t^2 - 5t + 6 = 0$<br>$\rightarrow t = 2$ or $t = 3$<br>$\rightarrow \theta = 63.4°$ or $71.6°$ | B1<br>M1<br>A1 A1<br>[4] | Using the identity.<br>Forms a 3 term quadratic with terms all on same side.<br>co co |

Question 19

| | | | |
|---|---|---|---|
| | $\theta$ is obtuse , $\sin\theta = k$ | | |
| **(i)** | $\cos\theta = -\sqrt{(1 - k^2)}$ | B1<br>[1] | cao |
| **(ii)** | $\tan\theta = \frac{\sin\theta}{\cos\theta}$ used | M1 | Used, attempt at cosine seen in (i) |
| | $\rightarrow \tan\theta = -\frac{k}{\sqrt{(1-k^2)}}$ aef | A1√<br>[2] | Ft for their cosine as a function of $k$ only, from part (i) |
| **(iii)** | $\sin(\theta + \pi) = -k$ | B1<br>[1] | cao |

Question 20

| | | | |
|---|---|---|---|
| **(a)** | $1+3\sin^2\theta+4\cos\theta=0$ | **M1** | Attempt to multiply by cos $\theta$ |
| | $1+3(1-\cos^2\theta)+4\cos\theta+0$ | **M1** | Use $c^2+s^2=1$ |
| | $3\cos^2\theta-4\cos\theta-4=0$ **AG** | **A1** | |
| | $\cos\theta=-2/3$ | **B1** | Ignore other solution |
| | $\theta=131.8$ *or* $228.2$ | **B1B1√** [6] | Ft for 360 – 1[st] soln. –1 extra solns in range |
| **(b)** | $c=b/a$ cao | **B1** | Radians 2.30 & 3.98 scores SCB1 |
| | $d=a-b$ | **B1** [2] | Allow $D=(0, a-b)$ |

Question 21

| | | | |
|---|---|---|---|
| **(i)** | $\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\left(\frac{1}{s}-\frac{c}{s}\right)^2$ | **M1** | Use of tan = sin/cos |
| | $\frac{(1-c)^2}{s^2}=\frac{(1-c)^2}{1-c^2}$ | **M1** | Use of $s^2=1-c^2$ |
| | $=\frac{(1-c)(1-c)}{(1-c)(1+c)}$ or $\frac{(1-c)^2}{(1-c)(1+c)}$ | **A1** | |
| | $\equiv\frac{1-\cos x}{1+\cos x}$ | **A1** [4] | ag |
| **(ii)** | $\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\frac{2}{5}$ | | |
| | $\frac{1-\cos x}{1+\cos x}=\frac{2}{5}\to\cos x\frac{3}{7}$ | **M1** | Making cos*x* the subject |
| | $\to$ $x=1.13$ or $5.16$ | **A1 A1√** [3] | $2\pi$ – 1[st] answer. |

Question 22

| | | |
|---|---|---|
| $4x^2+x^2=1/2$ soi | **B1** | |
| Solve as quadratic in $x^2$ | **M1** | E.g. $(4x^2-1)(2x^2+1)$ or $x^2=$ formula |
| $x^2=1/4$ | **A1** | Ignore other solution |
| $x=\pm1/2$ | **A1** [4] | |

Question 23

| | | | |
|---|---|---|---|
| **(i)** | $4\cos^2\theta + 15\sin\theta = 0$ | **M1** | Replace $\tan\theta$ by $\dfrac{\sin\theta}{\cos\theta}$ and multiply by $\sin\theta$ or equivalent |
| | $4(1-s^2)+15s=0 \rightarrow 4\sin^2\theta - 15\sin\theta - 4 = 0$ | **M1A1** **[3]** | Use $c^2 = 1 - s^2$ and rearrange to **AG** (www) |
| **(ii)** | $\sin\theta = -1/4$ | **B1** | |
| | $\theta = 194.5$ or $345.5$ | **B1B1**√ [3] | Ignore other solution<br>Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03) |

Question 24

| | | | |
|---|---|---|---|
| **(a)** | $3x = -\sqrt{3}/2$ | **M1** | Accept −0.866 at this stage |
| | $x = \dfrac{-\sqrt{3}}{6}$ oe | **A1** **[2]** | Or $\dfrac{-3}{6\sqrt{3}}$ or $\dfrac{-1}{2\sqrt{3}}$ |
| **(b)** | $(2\cos\theta - 1)(\sin\theta - 1) = 0$ | **M1** | Reasonable attempt to factorise and solve |
| | $\cos\theta = 1/2$ or $\sin\theta = 1$ | **A1** | Award B1B1 www |
| | $\theta = \pi/3$ *or* $\pi/2$ | **A1A1** **[4]** | Allow 1.05, 1.57. SCA1for both 60°, 90° |

Question 25

| | | | |
|---|---|---|---|
| **(i)** | $3\sin^2 x - \cos^2 x + \cos x = 0$ | **M1** | Multiply by $\cos x$ |
| | Use $s^2 = 1 - c^2$ and simplify to 3-term quad | **M1** | Expect $4c^2 - c - 3 = 0$ |
| | $\cos x = -3/4$ and 1 | **A1** | |
| | $x$ = 2.42 (allow $0.77\pi$) or 0 (extra in range max 1) | **A1A1** [5] | SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$ |
| **(ii)** | $2x$ = $2\pi - their\,2.42$ or 360 – 138.6 | **B1**√ | Expect $2x$ = 3.86 |
| | $x$ = 1.21 (0.385π), 1.93 (0.614/5π), 0, π (3.14) (extra max 1) | **B1B1** [3] | Any 2 correct B1. Remaining 2 correct B1. SCB1for all 69.3, 110.7, 0, 180 (degrees)<br>SCB1 for .361, π/2, 2.78 after $4c^2 + c - 3 = 0$ |

Question 26

| | | | |
|---|---|---|---|
| (i) | $\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}\equiv\frac{4}{\sin\theta\tan\theta}$ | | |
| | LHS $=\frac{1+2c+c^2-(1-2c+c^2)}{(1-c)(1+c)}$ | **M1** | Attempt at combining fractions. |
| | $=\frac{4c}{1-c^2}$ | **A1 A1** | A1 for numerator. A1 denominator |
| | $=\frac{4c}{s^2}$ | | Essential step for award of A1 |
| | $=\frac{4}{ts}$ **AG** | **A1** [4] | |
| (ii) | $\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}\right)=3.$ | | Uses part (i) to eliminate "$s$" correctly. |
| | $\to s\times\frac{4}{ts}=3\ (\to t=\frac{4}{3})$ | **M1** | |
| | $\theta=53.1°$ and $233.1°$ | **A1 A1√** [3] | √ for 180° + 1st answer. |

Question 27

| | | |
|---|---|---|
| $3\sin^2\theta=4\cos\theta-1$<br>Uses $s^2+c^2=1$<br>$\to 3c^2+4c-4\ (=0)$ | **M1 A1** | Equation in $\cos\theta$ only. All terms on one side of (=) |
| $(\to c=\frac{2}{3}$ or $-2)$<br>$\to\theta=48.2°$ or $311.8°$<br>0.841, 5.44 rads, **A1** only<br>$(0.268\pi,\ 1.73\pi)$ | **A1 A1√** [4] | For 360° − 1st answer. |

Question 28

| | | | |
|---|---|---|---|
| $4\sin^2x=6\cos^2x\Rightarrow\tan^2x=\frac{6}{4}$ or $4\sin^2x=6(1-\sin^2x)$ | **M1** | | Or $4(1-\cos^2x)=6\cos^2x$ |
| [$\tan x=(\pm)1.225$ or $\sin x=(\pm)0.7746$ or $\cos x=(\pm)0.6325$] | | | |
| $x=50.8$ (Allow 0.886 (rad)) | **A1** | | Or any other angle correct |
| Another angle correct | **A1√** | | Ft from 1st angle (Allow radians) |
| $x=50.8°,\ 129.2°,\ 230.8°,\ 309.2°$<br>[ 0.886, 2.25/6, 4.03, 5.40 (rad) ] | **A1** | [4] | All 4 angles correct in degrees |

## Question 29

| | | | | |
|---|---|---|---|---|
| (i) | $2\sin 2x = 6\cos 2x$<br>$\tan 2x = k$<br>$\rightarrow \tan 2x = 3$ or $k = 3$ | M1<br>A1 | [2] | Expand and collect as far as $\tan 2x$ = a constant from sin ÷ cos soi<br>cwo |
| (ii) | $x = (\tan^{-1}(\textit{their } k)) \div 2$<br>$(71.6^\circ$ or $-108.4^\circ) \div 2$<br>$x = 35.8^\circ, -54.2^\circ$<br>$x = 0.624^c, -0.946^c$<br>$x = 0.198\pi^c, -0.301\pi^c$ | M1<br>A1 A1√ | [3] | Inverse then ÷2. soi.<br>√ on 1st answer +/ − 90° if in given range but no extra solutions in the given range.<br>Both SR A1A0 |

## Question 30

| | | | | |
|---|---|---|---|---|
| (i) | $\cos^4 x = (1-\sin^2 x)^2 \quad = 1 - 2\sin^2 x + \sin^4 x$ AG | B1 | [1] | Could be LHS to RHS or vice versa |
| (ii) | $8\sin^4 x + 1 - 2\sin^2 x + \sin^4 x = 2(1-\sin^2 x)$<br>$9\sin^4 x = 1$<br>$x = 35.3^o$ (or any correct solution)<br>Any correct second solution from 144.7°, 215.3°, 324.7°<br>The remaining 2 solutions | M1<br>A1<br>A1<br>A1√<br>A1 | [5] | Substitute for $\cos^4 x$ and $\cos^2 x$ or<br>OR sub for $\sin^4 x \rightarrow 3\cos^2 x = 2$<br>$\rightarrow \cos x = (\pm)\sqrt{2/3}$<br>Allow the first 2 **A1** marks for radians<br>(0.616, 2.53, 3.76, 5.67) |

## Question 31

| | | | |
|---|---|---|---|
| (i) | $\frac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = \frac{2\sin\theta}{\cos\theta}$ | M1 | Replace $\tan\theta$ by $\sin\theta / \cos\theta$ |
| | $2\sin\theta\cos\theta + \cos^2\theta = 2\sin^2\theta + 2\sin\theta\cos\theta \Rightarrow c^2 = 2s^2$ | M1 A1 | Mult by c(s + c) or making this a common denom.. For **A1** simplification to **AG** without error or omission must be seen. |
| | **Total:** | 3 | |
| (ii) | $\tan^2\theta = 1/2$ or $\cos^2\theta = 2/3$ or $\sin^2\theta = 1/3$ | B1 | Use $\tan\theta = s/c$ or $c^2 + s^2 = 1$ and simplify to one of these results |
| | $\theta = 35.3°$ or $144.7°$ | B1 B1 FT | FT for 180 – other solution. SR **B1** for radians 0.615, 2.53 (0.196π, 0.804π) Extra solutions in range amongst solutions of which 2 are correct gets **B1B0** |
| | **Total:** | 3 | |

## Question 32

| | | | |
|---|---|---|---|
| (i) | LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2$ | M1 | Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only. |
| | $=\frac{(1-s)^2}{1-s^2}$ | M1 | Uses $s^2 + c^2 = 1$ leading to a function of sin only. |
| | $=\frac{(1-s)(1-s)}{(1-s)(1+s)}=\frac{1-\sin\theta}{1+\sin\theta}$ | A1 | AG. Must show use of factors for **A1**. |
| | **Total:** | **3** | |
| (ii) | Uses part (i) → $2 - 2s = 1 + s$ | | |
| | $\rightarrow s = ⅓$ | M1 | Uses part (i) to obtain $s = k$ |
| | $\theta = 19.5°$ or $160.5°$ | A1A1 FT | FT from error in 19.5°<br>Allow $0.340^c$ ($0.3398^c$) & 2.80(2) or $0.108\pi^c$ & $0.892\pi^c$ for **A1** only.<br>Extra answers in the range lose the second **A1** if gained for 160.5°. |
| | **Total:** | **3** | |

## Question 33

| | | | |
|---|---|---|---|
| (i) | $\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}\equiv\frac{2}{\sin\theta}$. | | |
| | $\frac{(1+c)^2+s^2}{s(1+c)}=\frac{1+2c+c^2+s^2}{s(1+c)}$ | M1 | Correct use of fractions |
| | $=\frac{2+2c}{s(1+c)}=\frac{2(1+c)}{s(1+c)}\rightarrow\frac{2}{s}$ | M1 A1 | Use of trig identity, **A1** needs evidence of cancelling |
| | **Total:** | **3** | |
| (ii) | $\frac{2}{s}=\frac{3}{c}\rightarrow t=\frac{2}{3}$ | M1 | Use part **(i)** and $t = s \div c$, may restart from given equation |
| | $\rightarrow\theta = 33.7°$ or $213.7°$ | A1 A1FT | FT for 180° + 1st answer. 2nd **A1** lost for extra solns in range |
| | **Total:** | **3** | |

## Question 34

| | | | |
|---|---|---|---|
| (i) | $\cos\theta+4+5\sin^2\theta+5\sin\theta-5\sin\theta-5\ (=0)$ | M1 | Multiply throughout by $\sin\theta+1$. Accept if $5\sin\theta-5\sin\theta$ is not seen |
| | $5(1-\cos^2\theta)+\cos\theta-1\ (=0)$ | M1 | Use $s^2=1-c^2$ |
| | $5\cos^2\theta-\cos\theta-4=0$ AG | A1 | Rearrange to AG |
| | | **3** | |
| (ii) | $\cos\theta=1$ and $-0.8$ | B1 | Both required |
| | $\theta=[0°,360°]$, $[143.1°]$, $[216.9°]$ | B1 B1 B1 FT | Both solutions required for 1st mark. For 3rd mark FT for $(360° - \mathit{their}\ 143.1°)$<br>Extra solution(s) in range (e.g. 180°) among 4 correct solutions scores $\frac{3}{4}$ |
| | | **4** | |

## Question 35

| | | | |
|---|---|---|---|
| (i) | *EITHER:* Uses $\tan^2 2x = \frac{\sin^2 2x}{\cos^2 2x}$ | (M1 | Replaces $\tan^2 2x$ by $\frac{\sin^2 2x}{\cos^2 2x}$ not $\frac{\sin^2}{\cos^2}2x$ |
| | Uses $\sin^2 2x = (1 - \cos^2 2x)$ | M1 | Replaces $\sin^2 2x$ by $(1 - \cos^2 2x)$ |
| | $\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$ | A1) | AG. All correct |
| | *OR:* $\tan^2 2x = \sec^2 2x - 1$ | (M1 | Replaces $\tan^2 2x$ by $\sec^2 2x - 1$ |
| | $\sec^2 2x = \frac{1}{\cos^2 2x}$ Multiply through by $\cos^2 2x$ and rearrange | M1 | Replaces $\sec^2 2x$ by $\frac{1}{\cos^2 2x}$ |
| | $\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$ | A1) | AG. All correct |
| | | 3 | |
| (ii) | $\cos 2x = -½ , -1$ | M1 | Uses **(i)** to get values for $\cos 2x$. Allow incorrect sign(s). |
| | $2x = 120°, 240°$ or $2x = 180°1$ $x = 60°$ or $120°$ | A1 A1 FT | **A1** for 60° or 120° FT for 180−1st answer |
| | or $x = 90°$ | A1 | Any extra answer(s) in given range only penalise fourth mark so max 3/4. |
| | | 4 | |

## Question 36

| | | | |
|---|---|---|---|
| 7(a) | $a = -2, \quad b = 3$ | B1B1 | |
| | | 2 | |
| (b)(i) | $s + s^2 - sc + 2c + 2sc - 2c^2 = s + sc \rightarrow s^2 - 2c^2 + 2c = 0$ | B1 | Expansion of brackets must be correct |
| | $1 - \cos^2\theta - 2\cos^2\theta + 2\cos\theta = 0$ | M1 | Uses $s^2 = 1 - c^2$ |
| | $3\cos^2\theta - 2\cos\theta - 1 = 0$ | A1 | AG |
| | | 3 | |
| (b)(ii) | $\cos\theta = 1$ or $-\frac{1}{3}$ | B1 | |
| | $\theta = 0°$ or $109.5°$ or $-109.5°$ | B1B1B1 FT | FT for − *their* 109.5° |
| | | 4 | |

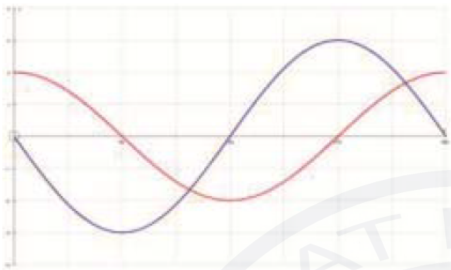
## Question 37

| | | | |
|---|---|---|---|
| (a) | $2\tan x+5=2\tan^2 x+5\tan x+3 \rightarrow 2\tan^2 x+3\tan x-2(=0)$ | **M1A1** | Multiply by denom., collect like terms to produce 3-term quad. in tan$x$ |
| | 0.464 (accept $0.148\pi$), 2.03 (accept $0.648\pi$) | **A1A1** | **SCA1** for both in degrees 26.6º, 116.6º only |
| | | **4** | |
| (b) | $\alpha=30°$ $k=4$ | **B1B1** | Accept $\alpha=\pi/6$ |
| | | **2** | |

Question 38

| | | | |
|---|---|---|---|
| (a)(i) | $\dfrac{\tan^2\theta-1}{\tan^2\theta+1}=\dfrac{\dfrac{\sin\theta^2}{\cos\theta^2}-1}{\dfrac{\sin\theta^2}{\cos\theta^2}+1}$ | **M1** | |
| | $=\dfrac{\sin\theta^2-\cos\theta^2}{\sin\theta^2+\cos\theta^2}$ | **A1** | multiplying by $\cos\theta^2$ <br> Intermediate stage can be omitted by multiplying directly by $\cos\theta^2$ |
| | $=\sin\theta^2-\cos\theta^2=\sin\theta^2-(1-\sin\theta^2)=2\sin^2\theta-1$ | **A1** | Using $\sin\theta^2+\cos\theta^2=1$ twice. Accept $a=2,\ b=-1$ |
| | ALT 1 $\dfrac{\sec^2\theta-2}{\sec^2\theta}$ | **M1** | ALT 2 $\dfrac{\tan^2\theta-1}{\sec^2\theta}$ |
| | $1-\dfrac{2}{\sec^2\theta}=1-2\cos^2\theta$ | **A1** | $(\tan^2\theta-1)\cos^2\theta$ |
| | $1-2(1-\sin^2\theta)=2\sin^2\theta-1$ | **A1** | $\sin^2\theta-\cos^2\theta=\sin^2\theta-(1-\sin^2\theta)=2\sin^2\theta-1$ |
| | | **3** | |
| (a)(ii) | $2\sin^2\theta-1=\dfrac{1}{4}\rightarrow\sin\theta=(\pm)\sqrt{\dfrac{5}{8}}$ or $(\pm)0.7906$ | **M1** | OR $\dfrac{t^2-1}{t^2+1}=\dfrac{1}{4}\rightarrow 3t^2=5\rightarrow t=(\pm)\sqrt{\dfrac{5}{3}}$ or $t=(\pm)1.2910$ |
| | $\theta=-52.2$ | **A1** | |
| | | **2** | |
| (b)(i) | $\sin x=2\cos x\rightarrow\tan x=2$ | **M1** | Or sin$x$ = $\sqrt{\dfrac{4}{5}}$ or cos$x$ = $\sqrt{\dfrac{1}{5}}$ |
| | $x=1.11$ with no additional solutions | **A1** | Accept $0.352\pi$ or $0.353\pi$. Accept in co-ord form ignoring $y$ co-ord |
| | | **2** | |
| (b)(ii) | Negative answer in range $-1<y<-0.8$ | **B1** | |
| | $-0.894$ or $-0.895$ or $-0.896$ | **B1** | |
| | | **2** | |

Question 39

| | | | |
|---|---|---|---|
| )(i) | $2\cos x = -3\sin x \rightarrow \tan x = -\frac{2}{3}$ | **M1** | Use of tan=sin/cos to get tan =, or other valid method to find sin or cos =.<br>M0 for $\tan x = +/-\frac{3}{2}$ |
| | $\rightarrow x = 146.3°$ or $326.3°$awrt | **A1 A1FT** | FT for 180 added to an incorrect first answer in the given range. The second A1 is withheld if any further values in the range $0° \leqslant x \leqslant 360°$ are given. Answers in radians score A0, A0. |
| | | **3** | |
| )(ii) | No labels required on either axis. Assume that the diagram is 0° to 360° unless labelled otherwise. Ignore any part of the diagram outside this range. | | |
| |  | **B1** | Sketch of $y = 2\cos x$.<br>One complete cycle; start and finish at top of curve at roughly the same positive $y$ value and go below the $x$ axis by roughly the same distance. (Can be a poor curve but not straight lines.) |
| | | **B1** | Sketch of $y = -3\sin x$<br>One complete cycle; start and finish on the $x$ axis, must be inverted and go below and then above the $x$ axis by roughly the same distance. (Can be a poor curve but not straight lines.) |
| | | **B1** | Fully correct answer including the sine curve with clearly larger amplitude than cosine curve. Must now be reasonable curves. |
| | | | Note: Separate diagrams can score 2/3 |
| | | **3** | |
| '(iii) | $x < 146.3°, x > 326.3°$ | **B1FT B1FT** | Does not need to include 0°, 360°. √ from their answers in (i) Allow combined statement as long as correct inequalities if taken separately. SC For two correct values including ft but with $\leqslant$ and $\geqslant$ B1 |
| | | **2** | |

## Question 40

| | | | |
|---|---|---|---|
| (i) | $a + \frac{1}{2}b = 5$ | **B1** | Alternatively these marks can be awarded when ½ and –1 appear after $a$ or $b$ has been eliminated. |
| | $a - b = 11$ | **B1** | |
| | $\rightarrow a = 7$ and $b = -4$ | **B1** | |
| | | **[3]** | |
| ii) | $a + b$ or *their* $a$ + *their* $b$ (3) | **B1** | Not enough to be seen in a table of values – must be selected. Graph from their values can get both marks.<br>**Note: Use of $b^2 - 4ac$ scores 0/3** |
| | $a - b$ or *their* $a$ – *their* $b$ (11). | **B1** | |
| | $\rightarrow k < 3$ , $k > 11$ | **B1** | Both inequalities correct. Allow combined statement as long as correct inequalities if taken separately.<br>Both answers correct from T & I or guesswork 3/3 otherwise 0/3 |
| | | **3** | |

## Question 41

| | | | |
|---|---|---|---|
| (i) | $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. | | Accept abbreviations s and c |
| | LHS = $\sin\theta + \cos\theta - \sin^2\theta\cos\theta - \sin\theta\cos^2\theta$ | **M1** | Expansion |
| | $= \sin\theta(1 - \cos^2\theta) + \cos\theta(1 - \sin^2\theta)$ or $(s + c - c(1 - c^2) - s(1 - s^2))$ | **M1A1** | Uses identity twice. Everything correct. AG |
| | Uses $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^3\theta + \cos^3\theta$ (RHS) | | or from RHS: M1 for use of trig ID twice |
| | **Or** | | |
| | LHS = $(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$ | **M1** | M1 for factorisation |
| | $= \sin^3\theta + \sin\theta\cos^2\theta - \sin^2\theta\cos\theta + \cos\theta\sin^2\theta + \cos^3\theta - \sin\theta\cos^2\theta = \sin^3\theta + \cos^3\theta$ | **M1A1** | |
| | | **3** | |
| (ii) | $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta \rightarrow \sin^3\theta = 2\cos^3\theta$ | **M1** | |
| | $\rightarrow \tan^3\theta = 2 \rightarrow \theta = 51.6°$ or $231.6°$ (only) | **A1A1FT** | Uses $\tan^3 = \sin^3 \div \cos^3$. A1 CAO. A1FT, 180 + their acute angle. $\tan^3\theta = 0$ gets M0 |
| | | **3** | |

## Question 42

| | | |
|---|---|---|
| $\dfrac{(\tan\theta+1)(1-\cos\theta)+(\tan\theta-1)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ soi | **M1** | |
| $\dfrac{\tan\theta-\tan\theta\cos\theta+1-\cos\theta+\tan\theta-1+\tan\theta\cos\theta-\cos\theta}{1-\cos^2\theta}$ www | **A1** | |
| $\dfrac{2(\tan\theta-\cos\theta)}{\sin^2\theta}$ www **AG** | **A1** | |
| | **3** | |
| $(2)(\tan\theta-\cos\theta)\ (=0) \rightarrow (2)\left(\dfrac{\sin\theta}{\cos\theta}-\cos\theta\right)\ (=0)$ soi | **M1** | Equate numerator to zero and replace $\tan\theta$ by $\sin\theta / \cos\theta$ |
| $(2)\left(\sin\theta-\left(1-\sin^2\theta\right)\right)\ (=0)$ | **DM1** | Multiply by $\cos\theta$ and replace $\cos^2\theta$ by $1-\sin^2\theta$ |
| $\sin\theta = 0.618(0)$ soi | **A1** | Allow $(\sqrt{5}-1)/2$ |
| $\theta = 38.2°$ | **A1** | Apply penalty –1 for extra solutions in range |
| | **4** | |

## Question 43

| | | | |
|---|---|---|---|
| (i) | $fg(x) = 2 - 3\cos(\frac{1}{2}x)$ | **B1** | Correct $fg$ |
| | $2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\ \frac{1}{3}\right)$ | **M1** | M1 for correct order of operations to solve their $fg(x) = 1$ as far as using inverse cos expect 1.23, ( or 70.5° ) condone $x$ =. |
| | $x$ **= 2.46 awrt or** $\dfrac{4.7\pi}{6}$ $(0.784\pi$ **awrt**$)$ | **A1** | One solution only in the given range, ignore answers outside the range.<br>Answer in degrees A0. |
| | | | Alternative:<br>Solve $f(y) = 1 \rightarrow y = 1.23 \rightarrow \frac{1}{2}x = 1.23$ **B1M1**<br>$\rightarrow x = 2.46$ **A1** |
| | | **3** | |

| | | | |
|---|---|---|---|
| (ii) |  | B1 | One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted "V". If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark 0 → $2\pi$. |
| | | B1 | Start and finish at roughly the same negative $y$ value. Significantly more above the $x$ axis than below or correct range implied by labels . |
| | | B1 | Fully correct. Curves not lines.<br>Must be a reasonable curve clearly turning at both ends.<br>Labels not required but must be appropriate if present. |
| | | 3 | |

## Question 44

| | | | |
|---|---|---|---|
| (i) | In $\Delta$ *ABD*, $\tan\theta = \frac{9}{BD} \rightarrow BD = \frac{9}{\tan\theta}$ or $9\tan(90 - \theta)$ or $9\cot\theta$<br>or $\sqrt{\left[(20\tan\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule) | B1 | Both marks can be gained for correct equated expressions. |
| | In $\Delta$ *DBC*, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ | B1 | |
| | $20\sin\theta = \frac{9}{\tan\theta}$ | M1 | Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary. |
| | → **$20\sin^2\theta = 9\cos\theta$ AG** | A1 | Correct manipulation of their expression to arrive at given answer. |
| | | | **SC:**<br>In $\Delta$ *DBC*, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ B1<br>In $\Delta$ *ABD*, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$<br>$\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \rightarrow \cos\theta = \frac{20\sin^2\theta}{9}$ M1<br>→ **$20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4 |
| | | 4 | |
| (ii) | Uses $s^2 + c^2 = 1 \rightarrow 20\cos^2\theta + 9\cos\theta - 20 (= 0)$ | M1 | Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$ |
| | $\rightarrow \cos\theta = 0.8$ | A1 | www |
| | → **$\theta = 36.9°$ awrt** | A1 | www. Allow $0.644^c$ awrt. Ignore 323.1° or $2.50^c$.<br>Note: correct answer without working scores 0/3. |
| | | 3 | |

## Question 45

| | | | |
|---|---|---|---|
| (i) | $\frac{(\cos\theta - 4)(5\cos\theta - 2) - 4\sin^2\theta}{\sin\theta(5\cos\theta - 2)} \ (= 0)$ | M1 | Accept numerator only |
| | $\frac{5\cos^2\theta - 22\cos\theta + 8 - 4\left(1 - \cos^2\theta\right)}{\sin\theta(5\cos\theta - 2)} \ (= 0)$ | M1 | Simplify numerator and use $s^2 = 1 - c^2$. Accept numerator only |
| | $9\cos^2\theta - 22\cos\theta + 4 = 0$ www **AG** | A1 | |
| | | 3 | |
| (ii) | Attempt to solve for $\cos\theta$, (formula, completing square expected) | M1 | Expect $\cos\theta = 0.1978$. Allow 2.247 in addition |
| | $\theta = 78.6°$, $281.4°$ (only, second solution in the range) | A1A1FT | Ft for (360° – 1st solution) |
| | | 3 | |

## Question 46

| | | | |
|---|---|---|---|
| '(a) | $3\left(1-\cos^2 2\theta\right)+8\cos 2\theta=0 \rightarrow 3\cos^2 2\theta-8\cos 2\theta-3\ (=0)$ | M1 | Use $s^2=1-c^2$ and simplify to 3-term quadratic in $2\theta$ |
| | $\cos 2\theta=-\frac{1}{3}$ soi | A1 | Ignore other solution |
| | $2\theta = 109.(47)^{\circ}$ or $250.(53)^{\circ}$ | A1 | One solution is sufficient, may be implied by either of the next solns |
| | $\theta = 54.7^{\circ}$ or $125.3^{\circ}$ | A1A1ft | Ft for 180° – other solution<br>Use of double angles leads to $3c^4-7c^2+2=0 \Rightarrow c=\pm1/\sqrt{3}$ for M1A1A1 then A1A1 for each angle<br>Similar marking if $3\sin^2 2\theta=-8\cos 2\theta$ is squared leading to $9\sin^4 2\theta+64\sin^2 2\theta-64=0$ |
| | | 5 | |
| '(b) | $\sqrt{3}=a+\tan 0 \rightarrow a=\sqrt{3}$ | B1 | $b=8$ or $-4$ (or $-10$, 14 etc) scores M1A0 |
| | $0=\tan(-b\pi/6)+\sqrt{3}$ taken as far as $\tan^{-1}$, angle units consistent | M1 | A0 if $\tan^{-1}\left(-\sqrt{3}\right)$ is not exact; (b=2 no working scores B2) |
| | $b=2$ | A1 | |
| | | 3 | |

## Question 47

| | | | |
|---|---|---|---|
| (i) | $q \leqslant f(x) \leqslant p+q$ | B1B1 | B1 each inequality – allow two separate statements<br>Accept <, $(q, p+q)$, $[q, p+q]$<br>Condone $y$ or $x$ or f in place of f($x$) |
| | | 2 | |
| (ii) | **(a)** 2 | B1 | Allow $\frac{\pi}{4}, \frac{3\pi}{4}$ |
| | **(b)** 3 | B1 | Allow $0, \frac{\pi}{2}, \pi$ |
| | **(c)** 4 | B1 | Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ |
| | | 3 | |
| (iii) | $3\sin^2 2x+2=4 \rightarrow \sin^2 2x=\frac{2}{3}$ soi | M1 | |
| | Sin2$x$ = (±)0.816(5). Allow $\sin 2x=(\pm)\sqrt{\frac{2}{3}}$ or $2x=\sin^{-1}(\pm)\sqrt{\frac{2}{3}}$ | A1 | OR Implied by at least one correct value for $x$. Allow $\sin^{-1}$ form |
| | ($2x$ =) **at least two** of 0.955(3), 2.18(6), 4.09(7) , 5.32(8) | A1 | Can be implied by corresponding values of $x$ below<br>Allow for at least two of 0.304π, 0.696π, 1.30(4)π, 1.69(6)π<br>**OR** at least <u>two</u> of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)° |
| | ($x$ =) 0.478, 1.09, 2.05, 2.66. | A1A1 | Allow 0.152π, 0.348π, 0.652π, 0.848π<br>SC A1 for 2 or 3 correct.<br>SC A1 for all of 27.4°, 62.6°, 117.4°, 152.6°<br>Sin2$x=\pm\frac{2}{3} \rightarrow x=0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1 |
| | | 5 | |

## Question 48

| | | | |
|---|---|---|---|
| 4(i) | $a^2+2ab+b^2, a^2-2ab+b^2$ | B1 | Correct expansions. |
| | $\sin^2x+\cos^2x=1$ used $\rightarrow (a+b)^2+(a-b)^2=1$ | M1 | Appropriate use of $\sin^2x+\cos^2x=1$ with $(a+b)^2$ and $(a-b)^2$ |
| | $a^2+b^2=½$ | A1 | No evidence of $\pm 2ab$, scores 2/3 |
| | **Alternative method for question 4(i)** | | |
| | $2a=(s+c)$ & $2b=(s-c)$ or $a=½(s+c)$ & $b=½(s-c)$ | B1 | |
| | $a^2+b^2=\frac{1}{4}(s+c)^2+\frac{1}{4}(s-c)^2=½(s^2+c^2)$ | M1 | Appropriate use of $\sin^2x+\cos^2x=1$ |
| | $a^2+b^2=½$ | A1 | Method using only $(\sin x-b)^2$ and $(a-\cos x)^2$ scores 0/3. |
| | | 3 | SC B1 for assuming $\theta$ is acute giving $a=\frac{1}{\sqrt{5}}+b$ or $2\sqrt{5}-b$ |
| 4(ii) | $\tan x=\frac{\sin x}{\cos x} \rightarrow \frac{a+b}{a-b}=2$ | M1 | Use of $\tan x=\frac{\sin x}{\cos x}$ to form an equation in a and b only |
| | $a=3b$ | A1 | |
| | | 2 | |

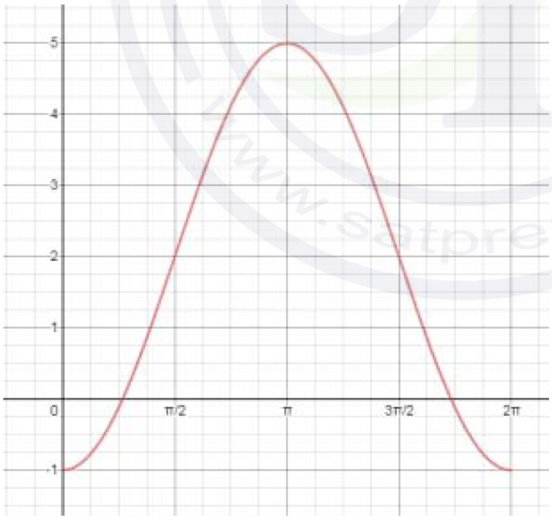
## Question 49

| | | | |
|---|---|---|---|
| (i) | 3, –3 | B1 | Accept ± 3 |
| | –½ | B1 | |
| | 2½ | B1 | |
| | | 3 | Condone misuse of inequality signs. |
| (ii) | | | Only mark the curve from $0 \rightarrow 2\pi$. If the $x$ axis is not labelled assume that $0 \rightarrow 2\pi$ is the range shown. Labels on axes are not required. |
| | 2 complete oscillations of a cosine curve starting with a maximum at (0,a), a>0 | B1 | |
| | Fully correct curve which must appear to level off at 0 and/or $2\pi$. | B1 | |
| | Line starting on positive $y$ axis and finishing below the $x$ axis at $2\pi$. Must be straight. | B1 | |
| | | 3 | |
| (iii) | 4 | B1 | |
| | | 1 | |

## Question 50

| | | | |
|---|---|---|---|
| (i) | LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2 = \frac{(1-s)(1-s)}{c^2} = \frac{(1-s)(1-s)}{1-s^2}$ | **B1** | Expresses tan in terms of sin and cos |
| | | **B1** | correctly $1-s^2$ as the denominator |
| | $= \frac{(1-s)(1-s)}{(1-s)(1+s)}$ | **M1** | Factors and correct cancelling www |
| | $\frac{1-\sin x}{1+\sin x}$ AG | **A1** | |
| | | **4** | |
| (ii) | Uses part **(i)** to obtain $\frac{1-\sin 2x}{1+\sin 2x} = \frac{1}{3} \rightarrow \sin 2x = ½$ | **M1** | Realises use of $2x$ and makes $\sin 2x$ the subject |
| | $x = \frac{\pi}{12}$ | **A1** | Allow decimal (0.262) |
| | (or) $x = \frac{5\pi}{12}$ | **A1** | **FT** for ½$\pi$ – 1st answer.<br>Allow decimal (1.31)<br>$\frac{\pi}{12}$ and $\frac{5\pi}{12}$ only, and no others in range.<br>SC $\sin x = ½ \rightarrow \frac{\pi}{6}\ \frac{5\pi}{6}$ B1 |
| | | **3** | |

## Question 51

| | | | |
|---|---|---|---|
| (i) | $-1 \leqslant f(x) \leqslant 5$ or [–1, 5] (may use y or f instead of f(x)) | **B1 B1** | $-1 < f(x) \leqslant 5$ or $-1 \leqslant x \leqslant 5$ or (–1,5) or [5,–1] B1 only |
| | | **2** | |
| (ii) |  | ***B1** | Start and end at –ve $y$, symmetrical, centre +ve. |
| | $g(x) = 2-3\cos x$ for $0 \leqslant x \leqslant p$ | **DB1** | Shape all ok. Curves not lines. One cycle [0,2$\pi$]<br>Flattens at each end. |
| | | **2** | |

| | | | |
|---|---|---|---|
| (iii) | (greatest value of $p$ =) $\pi$ | **B1** | |
| | | **1** | |
| (iv) | $x = 2 - 3\cos x \rightarrow \cos x = ⅓(2 - x)$ | **M1** | Attempt at cos*x* the subject. Use of $cos^{-1}$ |
| | $g^{-1}(x) = \cos^{-1}\frac{2-x}{3}$ (may use 'y =') | **A1** | Must be a function of x, |
| | | **2** | |

## Question 52

| | | | |
|---|---|---|---|
| (i) | $3\cos^4\theta + 4(1-\cos^2\theta) - 3(=0)$ | **M1** | Use $s^2 = 1 - c^2$ |
| | $3x^2 + 4(1-x) - 3(=0) \rightarrow 3x^2 - 4x + 1(=0)$ | **A1** | AG |
| | | **2** | |
| ii) | Attempt to solve for $x$ | **M1** | Expect $x = 1$, 1/3 |
| | $\cos\theta = (\pm)1,\ (\pm)0.5774$ | **A1** | Accept $(\pm)\left(\frac{1}{\sqrt{3}}\right)$ SOI |
| | $(\theta =)$ 0°, 180°, 54.7°, 125.3° | **A3,2,1,0** | A2,1,0 if more than 4 solutions in range |
| | | **5** | |

## Question 53

| | | | |
|---|---|---|---|
| (a) | $(2x + 1) = \tan^{-1}(⅓)$ (= 0.322 or 18.4 OR −0.339 rad or 8.7°) | ***M1** | Correct order of operations. Allow degrees. |
| | Either *their* 0.322 + $\pi$ or $2\pi$<br>Or *their* $-0.339 + \frac{\pi}{2}$ or $\pi$ | **DM1** | Must be in radians |
| | $x = 1.23$ **or** $x = 2.80$ | **A1** | AWRT for either correct answer, accept 0.39π or 0.89π |
| | | **A1** | For the second answer with no other answers between 0 and 2.8<br>**SC1** For both 1.2 and 2.8 |
| | | **4** | |
| b)(i) | $5\cos^2 x - 2$ | **B1** | Allow $a = 5$, $b = -2$ |
| | | **1** | |
| b)(ii) | −2 | **B1FT** | FT for sight of *their b* |
| | 3 | **B1FT** | FT for sight of *their a* + *b* |
| | | **2** | |

## Question 54

| | | | |
|---|---|---|---|
| (i) | $4\tan x + 3\cos x + \frac{1}{\cos x} = 0 \rightarrow 4\sin x + 3\cos^2 x + 1 = 0$ | **M1** | Multiply by $\cos x$ or common denominator of $\cos x$ |
| | $4\sin x + 3\left(1-\sin^2 x\right) + 1 = 0 \rightarrow 3\sin^2 x - 4\sin x - 4 = 0$ | **M1** | Use $\cos^2 x = 1 - \sin^2 x$ and simplify to 3-term quadratic in $\sin x$ |
| | $\sin x = -\frac{2}{3}$ | **A1** | AG |
| | | **3** | |
| (ii) | $2x - 20° = 221.8°, 318.2°$ | **M1A1** | Attempt to solve $\sin(2x-20) = -2/3$ (M1). At least 1 correct (A1) |
| | $x = 120.9°, 169.1°$ | **A1** **A1FT** | FT for 290° – other solution. **SC** A1 both answers in radians |
| | | **4** | |

## Question 55

| | | |
|---|---|---|
| $2\tan\theta - 6\sin\theta + 2 = \tan\theta + 3\sin\theta + 2 \rightarrow \tan\theta - 9\sin\theta\ (=0)$ | **M1** | Multiply by denominator and simplify |
| $\sin\theta - 9\sin\theta\cos\theta\ (=0)$ | **M1** | Multiply by $\cos\theta$ |
| $\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta = 0, \quad \cos\theta = \frac{1}{9}$ | **M1** | Factorise and attempt to solve at least one of the factors = 0 |
| $\theta = 0$ or $83.6°$ (only answers in the given range) | **A1A1** | |
| | **5** | |

## Question 56

| | | | |
|---|---|---|---|
| (a) | $(\tan x - 2)(3\tan x + 1)\ (=0)$. **or** formula **or** completing square | **M1** | Allow reversal of signs in the factors. Must see a method |
| | $\tan x = 2$ or $-\frac{1}{3}$ | **A1** | |
| | $x = 63.4°$(only value in range) or $161.6°$(only value in range) | **B1FT** **B1FT** | |
| | | **4** | |
| (b) | Apply $b^2 - 4ac < 0$ | **M1** | SOI. Expect $25 - 4(3)(k) < 0$, $\tan x$ must not be in coefficients |
| | $k > \frac{25}{12}$ | **A1** | Allow $b^2 - 4ac = 0$ leading to correct $k > \frac{25}{12}$ for M1A1 |
| | | **2** | |
| (c) | $k = 0$ | **M1** | SOI |
| | $\tan x = 0$ or $\frac{5}{3}$ | **A1** | |
| | $x = 0°$ or $180°$ or $59.0°$ | **A1** | All three required |
| | | **3** | |

## Question 57

| | | |
|---|---|---|
| (a) | $\dfrac{\tan\theta}{1+\cos\theta}+\dfrac{\tan\theta}{1-\cos\theta}=\dfrac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$ | **M1** |
| | $=\dfrac{2\tan\theta}{\sin^2\theta}$ | **M1** |
| | $=\dfrac{2\sin\theta}{\cos\theta\sin^2\theta}$ | **M1** |
| | $=\dfrac{2}{\sin\theta\cos\theta}$ **AG** | **A1** |
| | | **4** |
| (b) | $\dfrac{2}{\sin\theta\cos\theta}=\dfrac{6\cos\theta}{\sin\theta}$ | **M1** |
| | $\cos^2\theta=\dfrac{1}{3}\rightarrow\quad\cos\theta=(\pm)0.5774$ | **A1** |
| | 54.7º, 125.3º<br>(**FT** for 180º – 1st solution) | **A1**<br>**A1FT** |
| | | **4** |

## Question 58

| | | |
|---|---|---|
| (a) | $3\cos\theta=8\tan\theta\rightarrow\ 3\cos\theta=\dfrac{8\sin\theta}{\cos\theta}$ | **M1** |
| | $3(1-\sin^2\theta)=8\sin\theta$ | **M1** |
| | $3\sin^2\theta+8\sin\theta-3=0$ | **A1** |
| | | **3** |
| (b) | $(3\sin\theta-1)(\sin\theta+3)=0\rightarrow\sin\theta=⅓$ | **M1** |
| | $\theta=19.5°$ | **A1** |
| | | **2** |

## Question 59

| | | |
|---|---|---|
| (a) | $-1\leqslant f(x)\leqslant 2$ | **B1 B1** |
| | | **2** |
| (b) | $k=1$ | **B1** |
| | Translation by 1 unit upwards parallel to the y-axis | **B1** |
| | | **2** |
| (c) | $y=-\dfrac{3}{2}\cos 2x-\dfrac{1}{2}$ | **B1** |
| | | **1** |

## Question 60

| | | |
|---|---|---|
| (a) | $\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$ | M1 |
| | Use of $\sin^2\theta+\cos^2\theta=1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$. | M1A1 |
| | | 3 |
| (b) | $\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta} \rightarrow \tan\theta=1.5$ | M1 |
| | $\theta=0.983$ or $4.12$<br>(**FT** on second value for 1st value + $\pi$) | A1<br>A1FT |
| | | 3 |

## Question 61

| | | |
|---|---|---|
| $3\tan^4\theta+\tan^2\theta-2\ (=0)$ | M1 | SOI 3-term quartic, condone sign errors for this mark only |
| $(3\tan^2\theta-2)(\tan^2\theta+1)\ (=0)$ | M1 | Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$. |
| $\tan\theta=(\pm)\sqrt{\dfrac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$ | A1 | SOI Implied by final answer = 39.2° after 1st M1 scored |
| 39.2°, 140.8° | A1<br>A1 FT | FT for 2nd solution =180° – 1st solution |
| | 5 | |

## Question 62

| | | | |
|---|---|---|---|
| (a) | $\left(\dfrac{1}{\cos x}-\dfrac{\sin x}{\cos x}\right)\left(\dfrac{1}{\sin x}+1\right)$ | B1 | Uses "$\tan x = \sin x \div \cos x$" throughout |
| | $\left(\dfrac{1-\sin x}{\cos x}\right)\left(\dfrac{1+\sin x}{\sin x}\right)$ or $\left(\dfrac{1-\sin^2 x}{\cos x\sin x}\right)$ | M1 | Correct algebra leading to two or four terms |
| | $\left(\dfrac{\cos^2 x}{\cos x\sin x}\right)$ | A1 | OE. A correct expression which can be cancelled directly to $\dfrac{\cos x}{\sin x}$ e.g. $\dfrac{\cos x(1-\sin^2 x)}{\sin x(1-\sin^2 x)}$ |
| | $\left(\dfrac{\cos^2 x}{\cos x\sin x}\right)=\left(\dfrac{\cos x}{\sin x}\right)=\dfrac{1}{\tan x}$ | A1 | AG. Must show cancelling. If $x$ is missing throughout their working withhold this mark. |
| | | 4 | |
| (b) | Uses **(a)** $\rightarrow \dfrac{1}{\tan x}=2\tan^2 x \quad \tan^3 x=\dfrac{1}{2}$ | M1 | Reducing to $\tan^3 x = k$. |
| | $(x=)$ 38.4° | A1 | AWRT.<br>Ignore extra answers outside the range 0 to 180° but A0 if within. |
| | | 2 | |

## Question 63

| | | | |
|---|---|---|---|
| (a) | $\left(\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}=\right)\frac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$ | *M1 | Put over a single common denominator |
| | $\frac{2\sin^2\theta}{\cos^2\theta}$ | DM1 | Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator |
| | $2\tan^2\theta$ | A1 | AG |
| | | 3 | |
| (b) | $2\tan^2\theta=8\ \rightarrow\ \tan\theta=(\pm)2$ | B1 | SOI |
| | $(\theta=)\ 63.4°,\ \ 116.6°$ | B1<br>B1 FT | FT on 180 – 1st solution (with justification) |
| | | 3 | |

## Question 64

| | | |
|---|---|---|
| $\tan\theta+2\sin\theta=3\tan\theta-6\sin\theta$ leading to $2\tan\theta-8\sin\theta\ [=0]$ | M1 | OE |
| $2\sin\theta-8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$ | M1 | |
| $\cos\theta=\frac{1}{4}$ | A1 | Ignore $\sin\theta=0$ |
| $\theta=75.5°$ **only** | A1 | |
| | 4 | |

## Question 65

| | | | |
|---|---|---|---|
| (a) | $\frac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\frac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$<br>or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\frac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k] | M1 | Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by $\tan x$ or $\sin x$ |
| | $\frac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}\cdot\frac{\cos x}{\cos x}$ or $\frac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\frac{1+\cos x}{1-\cos x}[=k]$ | A1 | AG, WWW |
| | | 2 | |
| (b) | $k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$ | M1 | Gather like terms on LHS and RHS |
| | $k-1=(k+1)\cos x$ leading to $\cos x=\frac{k-1}{k+1}$ | A1 | WWW, OE |
| | | 2 | |
| (c) | Obtaining $\cos x$ from *their* **(b)** or **(a)** | M1 | Expect $\cos x=\frac{3}{5}$ |
| | ±0.927 (only solutions in the given range) | A1 | AWRT. Accept $\pm0.295\pi$ |
| | | 2 | |

## Question 66

| | | | |
|---|---|---|---|
| (a) | $\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$ | *M1 | For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s). |
| | $\equiv\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$ | DM1 | For multiplying out the numerators correctly. Condone sign errors for this mark. |
| | $\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$ | DM1 | For simplifying denominator to $\cos^2 x$. |
| | $\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$ | A1 | AG. Do not award A1 if undefined notation such as s, c, t or missing $x$'s used throughout or brackets are missing. |
| | **Alternative method for Question 10(a)** | | |
| | $\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$ | *M1 | Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x= 1-\sin^2 x$ |
| | $\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$ | DM1 | Separating into partial fractions. |
| | $\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$ | DM1 | Use of 1-1 or similar |
| | $\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$ | A1 | |
| | | 4 | |
| (b) | $\cos x=\frac{1}{2}$ | *B1 | OE. WWW. |
| | $x=\frac{\pi}{3}$ | DB1 | Or AWRT 1.05 |
| | $x=0$ from $\tan x=0$ or $\sin x=0$ | B1 | WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside. |
| | | 3 | |

## Question 67

| | | | |
|---|---|---|---|
| (a) | Reach $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}-2\tan^2\theta$ **or** $\sec^2\theta-\frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2-\sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$ | **M1** | May start with $1-\tan^2\theta$ |
| | $1-\tan^2\theta$ | **A1** | AG, must show sufficient stages |
| | | **2** | |
| (b) | $1-\tan^2\theta=2\tan^4\theta \Rightarrow 2\tan^4\theta+\tan^2\theta-1$ [= 0] | **M1** | Forming a 3-term quadratic in $\tan^2\theta$ or e.g. $u$ |
| | $\tan^2\theta$ = 0.5 or −1 leading to $\tan\theta=[\pm]\sqrt{0.5}$ | **M1** | |
| | $\theta$ = 35.3° and 144.7° (AWRT) | **A1** | Both correct. Radians 0.615, 2.53 scores A0. |
| | | **3** | |

## Question 68

| | | | |
|---|---|---|---|
| (a) | $\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$ | M1 | Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction. |
| | $\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$ | *M1 | Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine. |
| | $k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$ | DM1 | Gather like terms on one side of the equation. |
| | $(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$ | A1 | AG. Factorise to obtain answer. |
| | | **4** | |
| (b) | $5\sin^2 x + 3\sin x - 5 = 0$ | B1 | |
| | $\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$ | M1 | Use formula or complete the square. |
| | $x = 48.1°, 131.9°$ | A1<br>A1 FT | AWRT. Maximum A1 if extra solutions in range.<br>FT for 180 – *their* answer or 540 – *their* answer if $\sin x$ is negative<br>If M0 given and correct answers only **SCB1B1** available.<br>If answers in radians; 0.839, 2.30 can score **SCB1** for both. |
| | | **4** | |

## Question 69

| | | | |
|---|---|---|---|
| (a) | [(3rd term – 1st term) = (5th term – 3rd term) leading to…]<br>$-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$<br>$\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$<br>OR<br>[(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$ | *M1 | OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving $\sin x$ and $\cos x$. |
| | Elimination of $\sin x$ and $\cos x$ to give an expression in $\tan x$<br>$\left[\tan x = -\frac{1}{\sqrt{3}}\right]$ | DM1 | For use of $\frac{\sin x}{\cos x} = \tan x$ |
| | $[x =]\frac{5\pi}{6}$ only | A1 | CAO. Must be exact. |
| | | **3** | |
| (b) | $d = 2\cos x$ or $d = 2\cos(\textit{their } x)$ | B1 FT | Or an equivalent expression involving $\sin x$ and $\cos x$ e.g. $-3\sqrt{3}\sin(\textit{their } x) - \cos(\textit{their } x)$ $\left[= -\sqrt{3}\right]$<br>FT for *their* $x$ from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form. |
| | $S_{25} = \frac{25}{2}\left(2\times(2\cos(\textit{their } x)) + (25-1)\times(\textit{their } d)\right)$<br>$\left[= 12.5\left(2\times(-\sqrt{3}) + 24(-\sqrt{3})\right)\right]$ | M1 | Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with<br>$a$ replaced by either 2(cos(*their* $x$)) or $\pm\sqrt{3}$ and<br>$d$ replaced by either 2(cos(*their* $x$)) or $\pm\sqrt{3}$. |
| | $-325\sqrt{3}$ | A1 | Must be exact. |
| | | **3** | |

## Question 70

| | | |
|---|---|---|
| $2\cos^2\theta - 7\cos\theta + 3[=0]$ | **M1** | Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors. |
| $(2\cos\theta - 1)(\cos\theta - 3) = 0$ | **DM1** | Solving *their* 3-term quadratic using factorisation, formula or completing the square. |
| [$\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ leading to] $\theta = -60°$ or $\theta = 60°$ | **A1** | |
| $\theta = -60°$ **and** $\theta = 60°$ | **A1 FT** | FT for ± same answer between 0° and 90° or 0 and $\frac{\pi}{2}$.<br>$\pm\frac{\pi}{3}$ or $\pm 1.05$ AWRT scores maximum M1M1A0A1FT.<br>**Special case:** If M1 DM0 scored then SC B1 for $\theta = -60°$ **or** $\theta = 60°$, and SC B1 FT can be awarded for $\pm$(*their* 60°). |
| | **4** | |

## Question 71

| | | | |
|---|---|---|---|
| (a) | $a = 5$ | **B1** | |
| | $b = 2$ | **B1** | |
| | $c = 3$ | **B1** | |
| | | **3** | |
| (b)(i) | 3 | **B1** | |
| | | **1** | |
| (b)(ii) | 2 | **B1** | |
| | | **1** | |

## Question 72

| | | |
|---|---|---|
| $3\cos\theta(2\tan\theta - 1) + 2(2\tan\theta - 1)\ [=0]$ | **M1** | Or similar partial factorisation; condone sign errors. |
| $(2\tan\theta - 1)(3\cos\theta + 2)\ [=0]$<br>[leading to $\tan\theta = \frac{1}{2}$, $\cos\theta = -\frac{2}{3}$] | **M1** | OE. At least 2 out of 4 products correct. |
| 26.6°, 131.8° | **A1 A1** | WWW. Must be 1 d.p. or better.<br>Final A0 if extra solution within the interval.<br>**SC B1** No factorisation: Division by $2\tan\theta - 1$ leading to 131.8° or division by $3\cos\theta + 2$ or similar leading to 26.6°. |

## Question 73

| | | | |
|---|---|---|---|
| (a) | $\dfrac{(\sin\theta+2\cos\theta)(\cos\theta+2\sin\theta)-(\sin\theta-2\cos\theta)(\cos\theta-2\sin\theta)}{(\cos\theta-2\sin\theta)(\cos\theta+2\sin\theta)}$ | ***M1** | Obtain an expression with a common denominator |
| | $\dfrac{5\sin\theta\cos\theta+2\cos^2\theta+2\sin^2\theta-(5\sin\theta\cos\theta-2\sin^2\theta-2\cos^2\theta)}{\cos^2\theta-4\sin^2\theta}$ $=\dfrac{4(\cos^2\theta+\sin^2\theta)}{\cos^2\theta-4\sin^2\theta}$ | **A1** | |
| | $\dfrac{4}{\cos^2\theta-4(1-\cos^2\theta)}$ | **DM1** | Use $\cos^2\theta+\sin^2\theta=1$ twice |
| | $\dfrac{4}{5\cos^2\theta-4}$ | **A1** | **AG** |
| | | **4** | |
| (b) | $\dfrac{4}{5\cos^2\theta-4}=5$ leading to $25\cos^2\theta=24$ leading to $\cos\theta=\sqrt{\dfrac{24}{25}}\left[=(\pm)0.9798\right]$ | **M1** | Make $\cos\theta$ the subject |
| | $\theta=11.5°$ or $168.5°$ | **A1** **A1 FT** | FT on 180° – 1st solution |
| | | **3** | |

## Question 74

| | | | |
|---|---|---|---|
| (a) | $6y+2-7y^{1/2}$ [= 0] | ***M1** | OE Rearrange to a 3-term quadratic. |
| | $\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0] | **DM1** | Or use of formula or completing the square. |
| | $[y^{1/2}=]\dfrac{1}{2},\dfrac{2}{3}$ | **A1** | Answers only **SC B1** if DM1 not scored. |
| | $[y=]\dfrac{1}{4},\dfrac{4}{9}$ | **A1** | Answers only **SC B1** if DM1 not scored. |
| | | **4** | |
| (b) | Use of $\tan x$ = *their y* values | **M1** | Must have at least 2 values of y from part **(a).** |
| | $x=$ 14[.0], 24[.0], $x=$ 194[.0], 204[.0] | **A1** **A1 FT** | FT for 180 + angle (twice). AWRT |
| | | **3** | |

## Question 75

| | | | |
|---|---|---|---|
| (a) | $[p=]\ 3$ | **B1** | |
| | | **1** | |
| (b) | $[q=]\ \dfrac{1}{2}$ | **B1** | |
| | | **1** | |
| (c) | $[r=]\ -2$ | **B1** | |
| | | **1** | |

## Question 76

| | Answer | Mark | Guidance |
|---|---|---|---|
| (a) | $4\cos^4 x+\cos^2 x-3=0 \Rightarrow (4\cos^2 x-3)(\cos^2 x+1)=0$ | M1 | Attempt to solve 3 term quartic (or quadratic in another variable). |
| | $\Rightarrow [\cos^2 x=]\frac{3}{4}$ $[\cos^2 x=-1]$ | A1 | If M0 scored then **SC B1** is available for sight of $\frac{3}{4}$ [and $-1$]. |
| | $\Rightarrow \cos x=[\pm]\sqrt{their\frac{3}{4}}$ **OE** $\left[=\pm\frac{\sqrt{3}}{2}\right]$ | M1 | Square rooting '*their* $\cos^2 x$'. Allow without ±. May be implied by correct final answer(s). Ignore $\sqrt{-1}$. |
| | $[x=]\ \frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ | A1<br>A1 FT | Dependent on preceding M1 only. Exact answers needed. A1 for any 2 correct answers A1 A1 for 4 correct answers and no others inside the range $0\leqslant x\leqslant 2\pi$ A0 A1 FT can be awarded for two exact answers that are $2\pi-$'*their* $\frac{\pi}{6}$ and $\frac{5\pi}{6}$' within the range $0\leqslant x\leqslant 2\pi$. |
| | | | **SC**: If all 4 answers given in degrees (30, 150, 210, 330) or non-exact (AWRT 0.524, 2.62, 3.67, 5.76 or $0.167\pi$, $0.833\pi$, $1.17\pi$, 1.83) and no others then **SC B1**. |
| | | 5 | |
| (b) | $\cos^2 x=\frac{-1-\sqrt{1+16k}}{8}<0$ [$\therefore$ no solutions]. | B1 | State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k\geqslant 0$. |
| | $[\cos^2 x]=\frac{-1\pm\sqrt{1+16k}}{8}$ | *M1 | Must use quadratic formula. Allow any value of $k$ **but not** ±3. Condone + rather than ±. |
| | Substituting $k=5$ and obtain 1 from the formula | DM1 | Or argue logically if $k>5 \Rightarrow 1+16k>81 \Rightarrow >1$. |
| | $\cos^2 x=1$ or $\cos^2 x>$ or $\geqslant 1$ | A1 | Needs to be linked to $\cos^2 x$. |
| | Concluding statement having considered both ± cases. $\therefore$ no solutions | A1 | Dependent upon all previous marks having been scored. |
| | **Alternative method for question 11(b)** | | |
| | $\cos^2 x=\frac{-1-\sqrt{1+16k}}{8}<0$ [$\therefore$ no solutions]. | B1 | State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k\geqslant 0$. |
| | $[\cos^2 x]=\frac{-1\pm\sqrt{1+16k}}{8}$ | *M1 | Must use quadratic formula. Allow any value of $k$ **but not** ±3. Condone + rather than ±. |
| | $\frac{-1+\sqrt{1+16k}}{8}*1 \Rightarrow -1+\sqrt{1+16k}*8 \Rightarrow 1+16k*81$ | DM1 | $*$ represents any inequality or =. |
| | $k*5$ | A1 | $*$ represents any inequality or =. |
| | Concluding statement having considered both ± cases. $\therefore$ no solutions | A1 | Dependent upon all previous marks having been scored. |
| | | 5 | |

## Question 77

| | | | |
|---|---|---|---|
| (a) | **EITHER** **(1)**{Translation} $\begin{pmatrix}\{30°\}\\\{0\}\end{pmatrix}$ **OR** **(2)**{Translation} $\begin{pmatrix}\{60°\}\\\{0\}\end{pmatrix}$ | **B2,1,0** | B2 for fully correct, B1 with two elements correct.<br>{ } indicates different elements.<br>Accept angle in radians. |
| | **(3)**{Stretch} {factor 2} {in $x$-direction} | **B2,1,0** | B2 for fully correct, B1 with two elements correct.<br>{ } indicates different elements. |
| | **(4)** Stretch factor 4 in $y$-direction **and** correct order | **B1** | Stretch, $y$-direction and factor **and** correct order.<br>Correct order is **either** (1) then (3) **or** (3) then (2).<br>(4) can be anywhere in the sequence. |
| | | **5** | |
| (b) | $4\sin\left(\frac{1}{2}x-30°\right)=2\sqrt{2}\Rightarrow\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)[=45]$ | **M1** | SOI |
| | $\frac{1}{2}x-30=45$ or $135$ $\Rightarrow x=2(45+30)$ or $x=2(135+30)$ | **M1** | SOI. The M marks are independent. |
| | $x=150°, x=330°$ | **A1** | Both exact values, condone $\frac{5\pi}{6},\frac{11\pi}{6}$.<br>A0 if extra solutions in the interval.<br>Ignore other solutions outside $[0°,360°]$. |
| | | **3** | |

## Question 78

| | | | |
|---|---|---|---|
| (a) | $\frac{\sin^3\theta}{\sin\theta-1}-\frac{\sin^2\theta}{1+\sin\theta}=\frac{\sin^3\theta(1+\sin\theta)}{(\sin\theta-1)(1+\sin\theta)}-\frac{\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$<br>$\left[=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}\right]$ | ***M1** | Using a common denominator. |
| | $-\frac{\sin^2\theta+\sin^4\theta}{1-\sin^2\theta}$ | **DM1** | Reaching $\pm(1-\sin^2\theta)$ in denominator.<br>SOI by $\pm\cos^2\theta$. |
| | $-\frac{\sin^2\theta(1+\sin^2\theta)}{\cos^2\theta}$ | **DM1** | Using $\sin^2\theta+\cos^2\theta=1$ in denominator and isolating $\sin^2\theta$ in numerator. |
| | $-\tan^2\theta(1+\sin^2\theta)$ | **A1** | AG - Using/stating $\tan\theta=\frac{\sin\theta}{\cos\theta}$ is sufficient for A1.<br>May be working from both sides provided the argument is complete.<br>A0 if $\theta$ or brackets missing throughout, or sign errors.<br>Allow recovery if AG follows from *their* working. |
| | **Alternative method for Q4(a)** | | |
| | $-\tan^2\theta(1+\sin^2\theta)=-\frac{\sin^2\theta(1+\sin^2\theta)}{1-\sin^2\theta}$ | ***M1** | Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta+\cos^2\theta=1$. |
| | $\frac{-\sin^2\theta-\sin^4\theta}{(1-\sin\theta)(1+\sin\theta)}$ | **DM1** | Factorising denominator. |
| | $\frac{\sin^2\theta+\sin^3\theta-\sin^3\theta+\sin^4\theta}{(\sin\theta-1)(1+\sin\theta)}=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$ | **DM1** | Factorising numerator. |

| | | | |
|---|---|---|---|
| (a) | $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1+\sin\theta}$ | **A1** | AG<br>A0 if $\theta$ or brackets missing throughout, or sign errors.<br>Allow recovery if AG follows from *their* working. |
| | | **4** | |
| (b) | $-\tan^2\theta\left(1+\sin^2\theta\right) = \tan^2\theta\left(1-\sin^2\theta\right)$ leading to $[2]\tan^2\theta = 0$ | **M1** | Obtaining a (trig function)$^2 = 0$ WWW. |
| | $\tan\theta = 0$ leading to $[\theta =]\pi$ | **A1** | Ignore extra solutions outside the interval $(0, 2\pi)$. |
| | **Alternative method for Q4(b)** | | |
| | $-\dfrac{\sin^2\theta}{\cos^2\theta}(1+\sin^2\theta) = \dfrac{\sin^2\theta}{\cos^2\theta}(1-\sin^2\theta)$ leading to<br>$-\sin^2\theta - \sin^4\theta = \sin^2\theta - \sin^4\theta$ leading to $[2]\sin^2\theta = 0$ | **M1** | Obtaining a (trig function)$^2 = 0$ WWW. |
| | $\sin\theta = 0$ leading to $[\theta =]\pi$ | **A1** | Ignore extra solutions outside the interval $(0, 2\pi)$. |
| | | **2** | |

## Question 79

| | | |
|---|---|---|
| $8\left(1-\cos^2\theta\right) + 6\cos\theta + 1 \quad [=0]$ | **M1** | Expect $8\cos^2\theta - 6\cos\theta - 9 = 0$. |
| $(4\cos\theta + 3)(2\cos\theta - 3) \quad [=0]$ | **A1** | Factors or formula or completing square must be shown. |
| $[\rightarrow \cos\theta = -0.75 \rightarrow \theta =]\ 138.6°$ only, | **A1** | AWRT, ignore solutions outside the given range, answer in radians A0. |
| | **3** | |

## Question 80

| | | |
|---|---|---|
| Use of $\sin^2\alpha + \cos^2\alpha = 1$ eg $\sin\alpha = [\pm]\sqrt{1-\left(\dfrac{8}{17}\right)^2}$ | ***M1** | Or Pythagoras seen (may quote 8, 15, 17 triple). |
| $\sin\alpha = \dfrac{15}{17}$ | **A1** | |
| $\tan\alpha = \dfrac{15}{8}$ | **A1** | |
| $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha} = \dfrac{17}{15} + \dfrac{8}{15}$ | **DM1** | Dealing with reciprocals and addition of fractions correctly. |
| $= \dfrac{5}{3}$ oe | **A1** | Correct answer with no working shown scores 0.<br>Extra answers from $\sin\alpha = -\dfrac{15}{17}$ are allowed. |
| | **5** | |

## Question 81

| | | | |
|---|---|---|---|
| (a) | $k^2 - 4 \times 8 \times 2$ [< 0] | M1 | Use of $b^2 - 4ac$ but not just in the quadratic formula. |
| | $-8 < k < 8$ or $-8 < k$, $k < 8$ or $\lvert \boldsymbol{k} \rvert < 8$ or $(-8, 8)$ | A1 | Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$. |
| | | **2** | |
| (b) | $2(4\cos\theta - 1)(\cos\theta - 1)$ or $(4\cos\theta - 1)(\cos\theta - 1)$ | M1 | OE<br>Or use of formula or completing the square. Allow use of replacement variable. |
| | $\cos\theta = \frac{2}{8}$, $\cos\theta = 1$ | A1 | OE For both answers.<br>**SC:** If M0, **SC B1** available for sight of $\cos\theta = \frac{2}{8}$ and 1 |
| | [$\theta$ =] 0°, 75.5° | A1 | AWRT ISW rejection of 0°.<br>For both answers and no others in the range $0° \leqslant \theta \leqslant 180°$, must be in degrees.<br>**SC:** If M0 B1 scored, **SC B1** available for correct answers.<br>**SC**: If M1 A0 scored, **SC B1** available for $\cos\theta = \frac{2}{8}$ and $\theta$ = 75.5° only, WWW. |
| | | **3** | |

## Question 82

| | | | |
|---|---|---|---|
| (a) | $\frac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}\left[= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}\right]$ | *M1 | Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only. |
| | $\dfrac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}}$ | DM1 | Divide throughout by $\cos^2\theta$. |
| | $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ AG | A1 | |
| (b) | $\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 2 \Rightarrow \tan^2\theta + 1 = 2(\tan^2\theta - 1)$ | *M1 | Equate expression from **(a)** to 2 and clear fraction. |
| | $\tan\theta = [\pm]\sqrt{3}$ | DM1 | Simplify as far as $\tan\theta =$ . May be implied by a correct final answer in degrees or radians. |

**Alternative method for first two marks of Question 7(b)**

| | | |
|---|---|---|
| $\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = 2 \Rightarrow 1 = 2\sin^2\theta - 2(1 - \sin^2\theta)$ | *M1 | Equate expression to 2, clear fraction and use trig identities to form an equation in $sin\theta$ or $cos\theta$ only. |
| $sin\theta = [\pm]\sqrt{\frac{3}{4}}$ or $cos\theta = [\pm]\sqrt{\frac{1}{4}}$ | DM1 | Simplify as far as $sin\theta =$, or $cos\theta =$. |
| $\theta = \frac{1}{3}\boldsymbol{\pi}, \frac{2}{3}\boldsymbol{\pi}$ | A1<br>A1 FT | A1 for either correct answer then A1FT For their second value being $\pi -$ (their first) and no others in range $0 \leqslant \theta \leqslant \pi$, both values must be exact and in radians.<br>**SC:** B1 for $\theta = 60°, 120°$ or $0.333\pi, 0.667\pi$ AWRT. or 1.05, 2.09 AWRT. |
| | **4** | |

## Question 83

| | | | |
|---|---|---|---|
| (a) | $\dfrac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}\left[=\dfrac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{\sin^2\theta-\cos^2\theta}\right]=1$ | ***M1** | Use common denominator and equate to 1. |
| | $2\sin\theta\ [=\sin^2\theta-\cos^2\theta]=\sin^2\theta-(1-\sin^2\theta)$ | **DM1** | Multiply by common denominator and replace $\cos^2\theta$ by $1-\sin^2\theta$. |
| | $2\sin^2\theta-2\sin\theta-1=0$ | **A1** | OE In the given form. |
| | | **3** | |
| (b) | $[\sin\theta=]\dfrac{2\pm\sqrt{(-2)^2-4(2)(-1)}}{4}\left[=\dfrac{2\pm\sqrt{4+8}}{4}=\dfrac{1\pm\sqrt{3}}{2}\right]$ | **M1** | Use formula or complete the square to solve a quadratic equation of the correct form. |
| | 201.5° or 338.5° | **A1 A1 FT** | AWRT; A1 for either solution correct.<br>A1 FT for 540 – (first value).<br>If M0, allow **SC B1 B1FT** similarly. |
| | | **3** | |

## Question 84

| | | | |
|---|---|---|---|
| (a) | $\tan\theta\sin\theta=1$ leading to $\sin^2\theta=\cos\theta$ | **M1** | Use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and multiplication by $\cos\theta$. |
| | $1-\cos^2\theta=\cos\theta$ or $\cos^2\theta+\cos\theta-1\ [=0]$ | **M1** | Use of trig identity to form a 3-term quadratic. |
| | $[\cos\theta=]\ \dfrac{-1\pm\sqrt{5}}{2}$ | **M1** | Use of formula or completion of the square must be seen on a 3-term quadratic. Expect 0.6180 . |
| | 51.8º, | **A1** | Both A marks dependent on the 2nd M1. |
| | 308.2º | **A1 FT** | FT for (360º – 1st soln), A0 if extra solutions in range.<br>Radians 0.905 and 5.38, A1 only for both. |
| | | **5** | |
| (b) | $\dfrac{\tan\theta}{\sin\theta}-\dfrac{\sin\theta}{\tan\theta}=\dfrac{\sin\theta}{\sin\theta\cos\theta}-\dfrac{\sin\theta\cos\theta}{\sin\theta}=\dfrac{1}{\cos\theta}-\cos\theta$ | **M1** | Use $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ twice with correct use of fractions. |
| | $=\dfrac{1-\cos^2\theta}{\cos\theta}=\dfrac{\sin^2\theta}{\cos\theta}$ | **M1** | Use $1-\cos^2\theta=\sin^2\theta$ with correct use of fractions. |
| | $=\tan\theta\sin\theta$ | **A1** | WWW |
| | | **3** | |

## Question 85

| | | | |
|---|---|---|---|
| (a) | $3\sin^2x - 3\sin^2x\cos^2x - 4\cos^2x\ [=0]$ | **M1** | Replace $\tan^2x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2x$. |
| | $3(1-\cos^2x) - 3(1-\cos^2x)\cos^2x - 4\cos^2x\ [=0]$ | **M1** | Replace $\sin^2x$ by $1-\cos^2x$ twice. |
| | $3\cos^4x - 10\cos^2x + 3 = 0$ or $-3\cos^4x + 10\cos^2x - 3 = 0$ | **A1** | Or multiple of these equations. |
| | | **3** | |
| (b) | $(3\cos^2x - 1)(\cos^2x - 3)\ [=0]$ | **M1** | OE, using *their* equation in the given form. Allow unusual notation if meaning is clear. |
| | $\cos x = [\pm]\frac{1}{\sqrt{3}}$ | **A1** | SOI Answer only **SC B1**. |
| | 54.7º, | **A1** | |
| | 125.3º | **A1 FT** | Only other answer and must be from correct factorisation for A1. FT for 180° – *their* first answer . Answers only **SC B1**, **SC B1 FT**. |
| | | **4** | |

## Question 86

| | | | |
|---|---|---|---|
| (a)(i) | $\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$ | ***B1** | Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www. |
| | $[\theta =]\ 0, \frac{\pi}{2}, \pi$ | **DB 2,1,0** | B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. **SC DB1** for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents. |
| | | **3** | Verifying 3 answers rather than expanding and solving 0/3. |
| (a)(ii) | $\cos 0 + \sin 0 = [1+0=]1$ and $\cos\frac{\pi}{2} + \sin\frac{\pi}{2}[=0+1] = 1$ | **B1** | Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees. |
| | $\cos\pi + \sin\pi[=-1+0] = -1$ or $\neq 1$ | **B1** | www |
| | | **2** | |
| (b) | $\frac{(\cos\theta - \sin\theta)\sin\theta + (\cos\theta + \sin\theta)(1-\cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$ | **M1** | Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta - \sin^2\theta$. Condone brackets missing if recovered. |
| | $= \frac{\cos\theta\sin\theta - \sin^2\theta + \cos\theta - \cos^2\theta + \sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ | **A1** | |
| | $= \frac{\sin\theta + \cos\theta - \cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$ | **A1** | AG Clear evidence of using $sin^2\theta + cos^2\theta = 1$ in either the numerator or denominator. Condone c, s and/or omission of $\theta$. Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1. |
| | | **3** | |

| | | | |
|---|---|---|---|
| (c) | $\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$<br>leading to $1=2(1-2\sin^2\theta)$ | *M1 | Replacing LHS with the expression from **(b)** and attempting to simplify i.e. condone omission of $(\cos\theta+\sin\theta-1)=0$ at this stage.<br>M0 for $0=2(1-2\sin^2\theta)$ |
| | $k\sin^2\theta=1$ or 3 leading to $\sin\theta=[\pm]\sqrt{\dfrac{1 \text{ or } 3}{k}}$<br>$\left[4\sin^2\theta=1 \text{ leading to } \sin\theta=\pm\dfrac{1}{2}\right]$ | DM1 | Dividing by k and taking the square root of a positive value < 1.<br>This mark can be implied by the solutions $\dfrac{1}{6}\pi,\dfrac{5}{6}\pi$. |
| | Solutions $0,\dfrac{1}{6}\pi,\dfrac{1}{2}\pi,\dfrac{5}{6}\pi$ | A1 | Allow 0, 0.524, 1.57, 2.62 AWRT.<br>If M0 **SCB1** for $(\cos\theta+\sin\theta-1)=0 \Rightarrow 0, \dfrac{1}{2}\pi$.<br>If M0 **SCB1** for all four correct answers and no others.<br>Ignore answers outside of the range.<br>Answers in degrees A0. |
| | | **3** | |

## Question 87

| | | |
|---|---|---|
| $4\sin\theta+\tan\theta=0 \Rightarrow 4\sin\theta+\dfrac{\sin\theta}{\cos\theta}[=0]$ | M1 | For use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$. BOD if $\theta$ missing. |
| $\Rightarrow \sin\theta(4\cos\theta+1)[=0 \Rightarrow \sin\theta=0 \text{ or}]\ \cos\theta=-\dfrac{1}{4}$ | M1 | WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta+1)[=0]$ or $\sin\theta(4+\sec\theta)[=0]$. |
| $\theta = 104.5°$ | A1 | AWRT 1.82 rads A0. Ignore answers outside (0, 180°).<br>If M1 M0, **SC B1** for $\theta = 104.5°$ max 2/3. |
| | **3** | |