AS-Level

Topic :Trigonometry

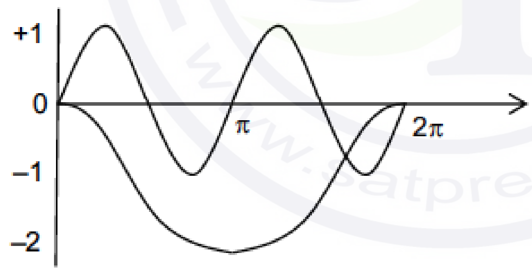
May 2013-May 2025

Answer

Question 1

(i)	$2\cos^2\theta = \tan^2\theta$ $\rightarrow 2\cos^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$	M1		Use of $t^2 = s^2 \div c^2$ or alternative.
	$\rightarrow$ Uses $c^2 + s^2 = 1 \rightarrow 2c^4 = 1 - c^2$	A1	[2]	Correct eqn.
(ii)	$(2c^2 - 1)(c^2 + 1) = 0 \rightarrow c = \pm\dfrac{1}{\sqrt{2}}$	M1		Method of solving for 3-term quadratic.
	$\rightarrow \theta = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$.	A1 A1√	[3]	(in terms of π). √ for π − 1st ans. Cannot gain A1√ if other answers given in the range.

Question 2

(i)	 	B1 DB1 B1 DB1	[4]	y =sin2x has 2 cycles, starts and finishes on the x-axis, max comes first. From +1 to −1. Smooth curves. y = cosx − 1 has one cycle, starts and finishes on x-axis, with a minimum pt. From 0 to −2, smooth curve, flattens.
(ii)	**(a)** $\sin 2x = -\frac{1}{2} \rightarrow$ 4 solutions	B1√	[1]	√ for their curve.
	(b) $\sin 2x + \cos x + 1 = 0 \rightarrow$ 3 solutions.	B1√	[1]	√ for intersections of their curves.

Question 3

$a = \sin\theta - 3\cos\theta$, $b = 3\sin\theta + \cos\theta$		
(i) $a^2 + b^2 =$		
$(s^2 + 9c^2 - 6sc) + (9s^2 + c^2 + 6sc)$	B1	Correct squaring
$10c^2 + 10s^2 = 10$	M1 A1	Use of $s^2 + c^2 = 1$ to get constant.
	[3]	(can get 2/3 for missing $6sc$)
(ii) $2s - 6c = 3s + c \rightarrow s = -7c$	M1	Collecting and $t = s \div c$
$\rightarrow \tan\theta = -7$	A1	
$\rightarrow 98.1°$	A1	
and $278.1°$	A1√	For 180^0 + first answer, providing no extra answers in the range.
	[4]	

Question 4

(i) $\dfrac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$	**M1**	
$\dfrac{\sin^2\theta - \sin\theta\cos\theta + \cos\theta\sin\theta = \cos^2\theta}{\sin^2\theta - \cos^2\theta}$	**A1**	
$\dfrac{1}{\sin^2\theta - \cos^2\theta}$ **AG**	**A1** **[3]**	www
(ii) $s^2 - (1 - s^2) = \dfrac{1}{3}$ or $1 - c^2 - c^2 = \dfrac{1}{3}$ or $3(s^2 - c^2) = c^2 + s^2$	**M1**	Applying $c^2 + s^2 = 1$
$\sin\theta = (\pm)\sqrt{\dfrac{2}{3}}$ or $\cos\theta = (\pm)\sqrt{\dfrac{1}{3}}$ or $\tan\theta = (\pm)\sqrt{2}$	**A1**	Or $s = (\pm)\ 0.816$, $c = (\pm)\ 0.577$, $t = (\pm)\ 1.414$
$\theta = 54.7°, 125.3°, 234.7°, 305.3°$	**A1A1** **[4]**	any 2 solutions for 1st A1; >4 solutions in range max A1A0

Question 5

(a) $x^2 - 1 = \sin\dfrac{\pi}{3}$	M1	
$x = \pm 1.366$	A1A1√ [3]	√ for negative of 1st answer
(b) $2\theta + \dfrac{\pi}{3} = \dfrac{5\pi}{6}\left(\text{or } \dfrac{13\pi}{6} \text{ or } \dfrac{\pi}{6}\right)$	B1	1 correct angle on RHS is sufficient
$2\theta = \dfrac{\pi}{2} = \left(\text{or } \dfrac{11\pi}{6}\right)$	M1	Isolating 2θ
$\theta = \dfrac{\pi}{4}, \dfrac{11\pi}{12}$	A1A1 [4]	SC decimals 0.785 & 2.88 scores M1B1

Question 6

(i) sin$x = \sqrt{(1 - p^2)}$	B1 [1]	Allow $1 - p$ if following $\sqrt{(1 - p^2)}$ ± is B0.
(ii) $\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{1-p^2}}{p}$	B1√ [1]	√ for answer to **(i)** used.
(iii) $\tan(90 - x) = \frac{p}{\sqrt{1-p^2}}$	B1√ [1]	√ for reciprocal of **(ii)**

Question 7

(i) $4(1-\cos^2 x) + 8\cos x - 7 = 0$	**M1**	Use $c^2 + s^2 = 1$
$4c^2 - 8c + 3 = 0 \to (2\cos x - 1)(2\cos x - 3) = 0$	**M1**	Attempt to solve
$x = 60°$ or $300°$	**A1A1** **[4]**	
(ii) $\frac{1}{2}\theta = 60°$ (or $300°$)	**M1**	Allow 300° in addition
$\theta = 120°$ only	**A1** **[2]**	

Question 8

$$\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$$

(i) LHS $\frac{\left(\frac{s}{c}\right)+1}{\left(\frac{s^2}{c}+c\right)} = \frac{s+c}{s^2+c^2}$	M1 M1	Use of t = s/c twice Correct algebra and use of $s^2 + c^2 = 1$
= RHS	A1 [3]	AG all ok
(ii) $s + c = 3s - 2c$		
$\to \tan x = \frac{3}{2}$ Allow $\cos^2 = \frac{4}{13}$, $\sin^2 = \frac{9}{13}$	M1	Uses (i) and $t = \frac{s}{c}$ $t = \frac{2}{3}$ or 0 is M0
$\to x = 0.983$ and 4.12 or 4.13	A1 A1√ [3]	co. √ 1st + π, providing no excess solns in range. Allow 0.313π, 1.31π

Question 9

(i) $\frac{1}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}\equiv\tan\theta$.	M1	Correct addition of fractions
$\text{LHS}=\frac{1+s-c^2}{c(1+s)}=\frac{s^2+s}{c(1+s)}=\frac{s}{c}$	M1M1	Use of $s^2+c^2=1$. $(1+s)$cancelled.
$=\tan\theta$	A1 [4]	→ answer given.
(ii) → $\tan\theta+2=0$ ie $\tan\theta=-2$	M1	Uses part **(i).** Allow $\tan\theta=\pm2$
→ $\theta=116.6°$ or $296.6°$	A1 A1√ [3]	Co. √ for 180°+ and no other solutions in the range.

Question 10

reflex angle θ is such that $\cos\theta=k$,		
(i) **(a)** $\sin\theta=-\sqrt{(1-k^2)}$	B1 B1 [2]	(−) B1 rest B1
(b) Uses $t=s/c$ → $\frac{-\sqrt{1-k^2}}{k}$	B1√ [1]	√ for **(i)** ÷ k.
(ii) θ is in 4th quadrant.		
2θ lies between 540° and 720°	B1	co
$\sin 2\theta$ is negative in both these quadrants.	B1 [2]	co

Question 11

(i) LHS $\equiv\frac{\sin^2\theta-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ cao	**B1**	Put over common denominator
$\equiv\frac{1-\cos^2\theta-1+\cos\theta}{(1-\cos\theta)\sin\theta}$	**M1**	Use $s^2=1-c^2$ oe
$\equiv\frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$	**M1**	Correct factorisation from line 2
$\equiv\frac{1}{\tan\theta}$	**A1** **[4]**	**AG**
(ii) $\tan\theta=(\pm)\frac{1}{2}$	**M1**	
26.6°, 153.4°	**A1A1√** **[3]**	Ft for 180 – 1st answer

Question 12

$a=1$, $b=2$	**B1B1** **[2]**	Or $1+2\sin x$

Question 13

(i) $(s^2-c^2)(s^2+c^2)$ OR $s^2(1-c^2)-c^2(1-s^2)$	**M1**	**OR** $\sin^4\theta-(1-\sin^2\theta)^2$
$\sin^2\theta-\cos^2\theta$	**A1**	$\sin^4\theta-(1-2\sin^2\theta+\sin^4\theta)$
$2\sin^2\theta-1$ www **AG**	**A1** **[3]**	$=2\sin^2\theta-1$ **AG**
(ii) $2\sin^2\theta-1=\frac{1}{2}\Rightarrow\sin\theta=(\pm)\frac{\sqrt{3}}{2}$ or $(\pm)0.866$	**B1**	**OR** $\cos 2\theta=-\frac{1}{2}\rightarrow 2\theta=120, 240$ etc.
$\theta=60°$	**B1**	Ft for 180 – *their* 60
$\theta=120°$	**B1**√	Ft for 180 + *their* 60, 360 – *their* 60
$\theta=240°, 300°$	**B1**√ **[4]**	Allow $\frac{\pi}{3}, \frac{2\pi}{3}$ etc. Extra sols in range −1

Question 14

$1+\sin x\tan x=5\cos x$

(i) Replaces t by s/c	M1	Correct formula
$1+\frac{s^2}{c}=5c$		
Replace s^2 by $1-c^2$	M1	Correct formula used in appropriate place
$\rightarrow 6c^2-c-1\ (=0)$	A1 [3]	AG
(ii) Soln of quadratic $\rightarrow$ ($c=-⅓$ or ½)	M1	Correct method
$\rightarrow x=60°$ or $109.5°$	A1 A1 [3]	co co

Question 15

$\tan^{-1}(3)=$ 1.249 or 71.565°	**M1**	Attempt at $\tan^{-1}3$ or right angle triangle with attempt at hypotenuse = $\sqrt{10}$
sin 1.25 or sin71.6 or 0.949 soi	**M1**	Attempt at $\sin\tan^{-1}3$
$(x=)$ 1.95 cao, accept $1+\frac{3}{\sqrt{10}}$ oe	**A1** **[3]**	Answer only **B3**

Question 16

$13\sin^2\theta+2\cos\theta+\cos^2\theta=4+2\cos\theta$	**M1**	Attempt to multiply by $2+\cos\theta$
$13\sin^2\theta+1-\sin^2\theta=4\rightarrow\sin^2\theta=\frac{1}{4}$	**M1**	Use of s^2+c^2 appropriately
or $13-13\cos^2\theta+\cos^2\theta=4\rightarrow\cos^2\theta=\frac{3}{4}$	**A1A1**√ **[4]**	**SC** both answers correct in radians, **A1** only
30°, 150°		Ft on 180 – their first value of θ

Question 17

(i)	$\tan\theta = 1/3$ $\theta = 18.4°$ only	**M1** **A1** **[2]**	Ignore solns. outside range 0→180
(ii)	$\mathbf{tan}2x = (\pm)1/\sqrt{3}$ Must be sq. root soi	**M1**	$\sin 2x = (\pm)1/2$ **or** $\cos 2x = (\pm)\sqrt{\mathbf{3/2}}$ using $c^2 + s^2 = 1$. Not $\tan x = (\pm)\frac{1}{\sqrt{3}}$ etc.
	$(x) = 15$ $(x) =$ any correct second value (75, 105, 165) $(x) =$ cao	**A1** **A1**√ **A1** **[4]**	ft for (90 ± their 15) or (180 – their 15) All four correct. Extra solns in range 1

Question 18

(i)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$. Divides top and bottom by cos θ → $\frac{t-1}{t+1}$	B1 [1]	Answer given.
(ii)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$ → $\frac{t-1}{t+1} = \frac{t}{6}$ → $t^2 - 5t + 6 = 0$ → $t = 2$ or $t = 3$ → $\theta = 63.4°$ or $71.6°$	B1 M1 A1 A1 [4]	Using the identity. Forms a 3 term quadratic with terms all on same side. co co

Question 19

	θ is obtuse , $\sin\theta = k$		
(i)	$\cos\theta = -\sqrt{(1-k^2)}$	B1 [1]	cao
(ii)	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ used	M1	Used, attempt at cosine seen in (i)
	→ $\tan\theta = -\frac{k}{\sqrt{(1-k^2)}}$ aef	A1√ [2]	Ft for their cosine as a function of k only, from part (i)
(iii)	$\sin(\theta + \pi) = -k$	B1 [1]	cao

Question 20

(a)	$1+3\sin^2\theta+4\cos\theta=0$	**M1**	Attempt to multiply by $\cos\theta$
	$1+3(1-\cos^2\theta)+4\cos\theta+0$	**M1**	Use $c^2+s^2=1$
	$3\cos^2\theta-4\cos\theta-4=0$ **AG**	**A1**	
	$\cos\theta=-2/3$	**B1**	Ignore other solution
	$\theta=131.8$ *or* 228.2	**B1B1√** [6]	Ft for 360 – 1[st] soln. –1 extra solns in range
(b)	$c=b/a$ cao	**B1**	Radians 2.30 & 3.98 scores SCB1
	$d=a-b$	**B1** [2]	Allow $D=(0, a-b)$

Question 21

(i)	$\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\left(\frac{1}{s}-\frac{c}{s}\right)^2$	**M1**	Use of tan = sin/cos
	$\frac{(1-c)^2}{s^2}=\frac{(1-c)^2}{1-c^2}$	**M1**	Use of $s^2=1-c^2$
	$=\frac{(1-c)(1-c)}{(1-c)(1+c)}$ or $\frac{(1-c)^2}{(1-c)(1+c)}$	**A1**	
	$\equiv\frac{1-\cos x}{1+\cos x}$	**A1** [4]	ag
(ii)	$\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\frac{2}{5}$		
	$\frac{1-\cos x}{1+\cos x}=\frac{2}{5}\rightarrow\cos x\frac{3}{7}$	**M1**	Making cos*x* the subject
	$\rightarrow x=1.13$ or 5.16	**A1 A1√** [3]	2π – 1[st] answer.

Question 22

$4x^2+x^2=1/2$ soi	**B1**	
Solve as quadratic in x^2	**M1**	E.g. $(4x^2-1)(2x^2+1)$ or $x^2=$ formula
$x^2=1/4$	**A1**	Ignore other solution
$x=\pm1/2$	**A1** [4]	

Question 23

(i)	$4\cos^2\theta + 15\sin\theta = 0$	**M1**	Replace $\tan\theta$ by $\frac{\sin\theta}{\cos\theta}$ and multiply by $\sin\theta$ or equivalent
	$4(1-s^2)+15s=0 \to 4\sin^2\theta - 15\sin\theta - 4 = 0$	**M1A1** **[3]**	Use $c^2 = 1-s^2$ and rearrange to **AG** (www)
(ii)	$\sin\theta = -1/4$ $\theta = 194.5$ or 345.5	**B1** **B1B1**√ [3]	Ignore other solution Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)

Question 24

(a)	$3x = -\sqrt{3}/2$ $x = \frac{-\sqrt{3}}{6}$ oe	**M1** **A1** **[2]**	Accept −0.866 at this stage Or $\frac{-3}{6\sqrt{3}}$ or $\frac{-1}{2\sqrt{3}}$
(b)	$(2\cos\theta - 1)(\sin\theta - 1) = 0$ $\cos\theta = 1/2$ or $\sin\theta = 1$ $\theta = \pi/3$ *or* $\pi/2$	**M1** **A1** **A1A1** **[4]**	Reasonable attempt to factorise and solve Award B1B1 www Allow 1.05, 1.57. SCA1for both 60°, 90°

Question 25

(i)	$3\sin^2 x - \cos^2 x + \cos x = 0$ Use $s^2 = 1-c^2$ and simplify to 3-term quad $\cos x = -3/4$ and 1	**M1** **M1** **A1**	Multiply by $\cos x$ Expect $4c^2 - c - 3 = 0$
	x = 2.42 (allow 0.77π) or 0 (extra in range max 1)	**A1A1** [5]	SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$
(ii)	$2x = 2\pi - their 2.42$ or 360 – 138.6	**B1**√	Expect $2x$ = 3.86
	x = 1.21 (0.385π), 1.93 (0.614/5π), 0, π (3.14) (extra max 1)	**B1B1** [3]	Any 2 correct B1. Remaining 2 correct B1. SCB1for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, π/2, 2.78 after $4c^2 + c - 3 = 0$

Question 26

(i)	$\dfrac{1+\cos\theta}{1-\cos\theta}-\dfrac{1-\cos\theta}{1+\cos\theta}\equiv\dfrac{4}{\sin\theta\tan\theta}$		
	LHS $=\dfrac{1+2c+c^2-(1-2c+c^2)}{(1-c)(1+c)}$	**M1**	Attempt at combining fractions.
	$=\dfrac{4c}{1-c^2}$	**A1 A1**	A1 for numerator. A1 denominator
	$=\dfrac{4c}{s^2}$		Essential step for award of A1
	$=\dfrac{4}{ts}$ **AG**	**A1** [4]	
(ii)	$\sin\theta\left(\dfrac{1+\cos\theta}{1-\cos\theta}-\dfrac{1-\cos\theta}{1+\cos\theta}\right)=3.$		Uses part (i) to eliminate "s" correctly.
	$\rightarrow s\times\dfrac{4}{ts}=3\ (\rightarrow t=\dfrac{4}{3})$	**M1**	
	$\theta=53.1°$ and $233.1°$	**A1 A1√** [3]	√ for 180° + 1st answer.

Question 27

$3\sin^2\theta=4\cos\theta-1$		
Uses $s^2+c^2=1$		
$\rightarrow 3c^2+4c-4\ (=0)$	**M1 A1**	Equation in $\cos\theta$ only. All terms on one side of (=)
$(\rightarrow c=\dfrac{2}{3}$ or $-2)$		
$\rightarrow\theta=48.2°$ or $311.8°$	**A1 A1√**	For 360° − 1st answer.
0.841, 5.44 rads, **A1** only		
$(0.268\pi, 1.73\pi)$	[4]	

Question 28

$4\sin^2x=6\cos^2x\Rightarrow\tan^2x=\dfrac{6}{4}$ or $4\sin^2x=6(1-\sin^2x)$	**M1**		Or $4(1-\cos^2x)=6\cos^2x$
[tan x = (±)1.225 or sin x = (±)0.7746 or cos x = (±)0.6325]			
x= 50.8 (Allow 0.886 (rad))	**A1**		Or any other angle correct
Another angle correct	**A1√**		Ft from 1st angle (Allow radians)
$x=50.8°, 129.2°, 230.8°, 309.2°$ [0.886, 2.25/6, 4.03, 5.40 (rad)]	**A1**	[4]	All 4 angles correct in degrees

Question 29

(i)	$2\sin 2x = 6\cos 2x$ $\tan 2x = k$ $\rightarrow \tan 2x = 3$ or $k = 3$	**M1** **A1**	[2]	Expand and collect as far as $\tan 2x$ = a constant from sin ÷ cos soi cwo
(ii)	$x = (\tan^{-1}(\textit{their}\, k)) \div 2$ $(71.6^\circ$ or $-108.4^\circ) \div 2$ $x = 35.8^\circ, -54.2^\circ$ $x = 0.624^c, -0.946^c$ $x = 0.198\pi^c, -0.301\pi^c$	**M1** **A1 A1**√	[3]	Inverse then ÷2. soi. √ on 1st answer +/ − 90° if in given range but no extra solutions in the given range. Both SR A1A0

Question 30

(i)	$\cos^4 x = (1-\sin^2 x)^2 \quad = 1 - 2\sin^2 x + \sin^4 x$ AG	**B1**	[1]	Could be LHS to RHS or vice versa
(ii)	$8\sin^4 x + 1 - 2\sin^2 x + \sin^4 x = 2(1-\sin^2 x)$ $9\sin^4 x = 1$ $x = 35.3^o$ (or any correct solution) Any correct second solution from 144.7°, 215.3°, 324.7° The remaining 2 solutions	**M1** **A1** **A1** **A1**√ **A1**	[5]	Substitute for $\cos^4 x$ and $\cos^2 x$ or OR sub for $\sin^4 x \rightarrow 3\cos^2 x = 2$ $\rightarrow \cos x = (\pm)\sqrt{2/3}$ Allow the first 2 **A1** marks for radians (0.616, 2.53, 3.76, 5.67)

Question 31

(i)	$\frac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = \frac{2\sin\theta}{\cos\theta}$	**M1**	Replace $\tan\theta$ by $\sin\theta / \cos\theta$
	$2\sin\theta\cos\theta + \cos^2\theta = 2\sin^2\theta + 2\sin\theta\cos\theta \Rightarrow c^2 = 2s^2$	**M1 A1**	Mult by c(s + c) or making this a common denom.. For **A1** simplification to **AG** without error or omission must be seen.
	Total:	**3**	
(ii)	$\tan^2\theta = 1/2$ or $\cos^2\theta = 2/3$ or $\sin^2\theta = 1/3$	**B1**	Use $\tan\theta = s/c$ or $c^2 + s^2 = 1$ and simplify to one of these results
	$\theta = 35.3^\circ$ or 144.7°	**B1 B1 FT**	FT for 180 – other solution. SR **B1** for radians 0.615, 2.53 (0.196π, 0.804π) Extra solutions in range amongst solutions of which 2 are correct gets **B1B0**
	Total:	**3**	

Question 32

(i)	LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2$	M1	Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only.
	$=\frac{(1-s)^2}{1-s^2}$	M1	Uses $s^2+c^2=1$ leading to a function of sin only.
	$=\frac{(1-s)(1-s)}{(1-s)(1+s)}=\frac{1-\sin\theta}{1+\sin\theta}$	A1	AG. Must show use of factors for **A1**.
	Total:	**3**	
(ii)	Uses part (i) → $2-2s=1+s$		
	→ $s=\frac{1}{3}$	M1	Uses part (i) to obtain $s=k$
	$\theta=19.5°$ or $160.5°$	A1A1 FT	FT from error in 19.5° Allow 0.340^c (0.3398^c) & 2.80(2) or $0.108\pi^c$ & $0.892\pi^c$ for **A1** only. Extra answers in the range lose the second **A1** if gained for 160.5°.
	Total:	**3**	

Question 33

(i)	$\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}\equiv\frac{2}{\sin\theta}$.		
	$\frac{(1+c)^2+s^2}{s(1+c)}=\frac{1+2c+c^2+s^2}{s(1+c)}$	M1	Correct use of fractions
	$=\frac{2+2c}{s(1+c)}=\frac{2(1+c)}{s(1+c)}\to\frac{2}{s}$	M1 A1	Use of trig identity, **A1** needs evidence of cancelling
	Total:	**3**	
(ii)	$\frac{2}{s}=\frac{3}{c}\to t=\frac{2}{3}$	M1	Use part **(i)** and $t=s\div c$, may restart from given equation
	→ $\theta=33.7°$ or $213.7°$	A1 A1FT	FT for 180° + 1st answer. 2nd **A1** lost for extra solns in range
	Total:	**3**	

Question 34

(i)	$\cos\theta+4+5\sin^2\theta+5\sin\theta-5\sin\theta-5\ (=0)$	M1	Multiply throughout by $\sin\theta+1$. Accept if $5\sin\theta-5\sin\theta$ is not seen
	$5(1-\cos^2\theta)+\cos\theta-1\ (=0)$	M1	Use $s^2=1-c^2$
	$5\cos^2\theta-\cos\theta-4=0$ AG	A1	Rearrange to AG
		3	
(ii)	$\cos\theta=1$ and -0.8	B1	Both required
	$\theta=[0°,360°]$, $[143.1°]$, $[216.9°]$	B1 B1 B1 FT	Both solutions required for 1st mark. For 3rd mark FT for $(360°-\mathit{their}\ 143.1°)$ Extra solution(s) in range (e.g. 180°) among 4 correct solutions scores $\frac{3}{4}$
		4	

Question 35

(i)	*EITHER:* Uses $\tan^2 2x = \frac{\sin^2 2x}{\cos^2 2x}$	(M1	Replaces tan²2x by $\frac{\sin^2 2x}{\cos^2 2x}$ not $\frac{\sin^2}{\cos^2}2x$
	Uses sin²2x = (1 – cos²2x)	M1	Replaces sin²2x by (1 – cos²2x)
	→ 2cos²2x + 3cos2x +1 = 0	A1)	AG. All correct
	OR: $\tan^2 2x = \sec^2 2x - 1$	(M1	Replaces tan²2x by $\sec^2 2x - 1$
	$\sec^2 2x = \frac{1}{\cos^2 2x}$ Multiply through by cos²2x and rearrange	M1	Replaces $\sec^2 2x$ by $\frac{1}{\cos^2 2x}$
	→ 2cos²2x + 3cos2x +1 = 0	A1)	AG. All correct
		3	
(ii)	cos 2x = −½ , −1	M1	Uses **(i)** to get values for cos 2x. Allow incorrect sign(s).
	2x = 120°, 240° or 2x = 180°1 x = 60° or 120°	A1 A1 FT	**A1** for 60° or 120° FT for 180−1st answer
	or x = 90°	A1	Any extra answer(s) in given range only penalise fourth mark so max 3/4.
		4	

Question 36

7(a)	$a = -2, \quad b = 3$	B1B1	
		2	
(b)(i)	$s + s^2 - sc + 2c + 2sc - 2c^2 = s + sc \rightarrow s^2 - 2c^2 + 2c = 0$	B1	Expansion of brackets must be correct
	$1 - \cos^2\theta - 2\cos^2\theta + 2\cos\theta = 0$	M1	Uses $s^2 = 1 - c^2$
	$3\cos^2\theta - 2\cos\theta - 1 = 0$	A1	AG
		3	
(b)(ii)	$\cos\theta = 1$ or $-\frac{1}{3}$	B1	
	$\theta = 0°$ or 109.5° or −109.5°	B1B1B1 FT	FT for – *their* 109.5°
		4	

Question 37

(a)	$2\tan x+5=2\tan^2 x+5\tan x+3 \rightarrow 2\tan^2 x+3\tan x-2(=0)$	M1A1	Multiply by denom., collect like terms to produce 3-term quad. in $\tan x$
	0.464 (accept 0.148π), 2.03 (accept 0.648π)	A1A1	**SCA1** for both in degrees 26.6°, 116.6° only
		4	
(b)	$\alpha=30°\quad k=4$	B1B1	Accept $\alpha=\pi/6$
		2	

Question 38

(a)(i)	$\frac{\tan^2\theta-1}{\tan^2\theta+1}=\frac{\frac{\sin\theta^2}{\cos\theta^2}-1}{\frac{\sin\theta^2}{\cos\theta^2}+1}$	M1	
	$=\frac{\sin\theta^2-\cos\theta^2}{\sin\theta^2+\cos\theta^2}$	A1	multiplying by $\cos\theta^2$ Intermediate stage can be omitted by multiplying directly by $\cos\theta^2$
	$=\sin\theta^2-\cos\theta^2=\sin\theta^2-(1-\sin\theta^2)=2\sin^2\theta-1$	A1	Using $\sin\theta^2+\cos\theta^2=1$ twice. Accept $a=2,\ b=-1$
	ALT 1 $\frac{\sec^2\theta-2}{\sec^2\theta}$	M1	ALT 2 $\frac{\tan^2\theta-1}{\sec^2\theta}$
	$1-\frac{2}{\sec^2\theta}=1-2\cos^2\theta$	A1	$(\tan^2\theta-1)\cos^2\theta$
	$1-2(1-\sin^2\theta)=2\sin^2\theta-1$	A1	$\sin^2\theta-\cos^2\theta=\sin^2\theta-(1-\sin^2\theta)=2\sin^2\theta-1$
		3	
(a)(ii)	$2\sin^2\theta-1=\frac{1}{4}\rightarrow\sin\theta=(\pm)\sqrt{\frac{5}{8}}$ or $(\pm)0.7906$	M1	OR $\frac{t^2-1}{t^2+1}=\frac{1}{4}\rightarrow 3t^2=5\rightarrow t=(\pm)\sqrt{\frac{5}{3}}$ or $t=(\pm)1.2910$
	$\theta=-52.2$	A1	
		2	
(b)(i)	$\sin x=2\cos x\rightarrow\tan x=2$	M1	Or $\sin x=\sqrt{\frac{4}{5}}$ or $\cos x=\sqrt{\frac{1}{5}}$
	$x=1.11$ with no additional solutions	A1	Accept 0.352π or 0.353π. Accept in co-ord form ignoring y co-ord
		2	
(b)(ii)	Negative answer in range $-1<y<-0.8$	B1	
	-0.894 or -0.895 or -0.896	B1	
		2	

Question 39

)(i)	$2\cos x = -3\sin x \rightarrow \tan x = -\frac{2}{3}$	M1	Use of tan=sin/cos to get tan =, or other valid method to find sin or cos =. M0 for $\tan x = +/-\frac{3}{2}$
	$\rightarrow x = 146.3°$ or $326.3°$awrt	A1 A1FT	FT for 180 added to an incorrect first answer in the given range. The second A1 is withheld if any further values in the range $0° \leqslant x \leqslant 360°$ are given. Answers in radians score A0, A0.
		3	
)(ii)	No labels required on either axis. Assume that the diagram is 0° to 360° unless labelled otherwise. Ignore any part of the diagram outside this range.		
		B1	Sketch of $y = 2\cos x$. One complete cycle; start and finish at top of curve at roughly the same positive y value and go below the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Sketch of $y = -3\sin x$ One complete cycle; start and finish on the x axis, must be inverted and go below and then above the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Fully correct answer including the sine curve with clearly larger amplitude than cosine curve. Must now be reasonable curves.
			Note: Separate diagrams can score 2/3
		3	
)(iii)	$x < 146.3°$, $x > 326.3°$	B1FT B1FT	Does not need to include 0°, 360°. √ from their answers in (i) Allow combined statement as long as correct inequalities if taken separately. SC For two correct values including ft but with $\leqslant$ and $\geqslant$ B1
		2	

Question 40

(i)	$a + \frac{1}{2}b = 5$	B1	Alternatively these marks can be awarded when ½ and –1 appear after a or b has been eliminated.
	$a - b = 11$	B1	
	$\rightarrow a = 7$ and $b = -4$	B1	
		[3]	
ii)	$a + b$ or *their* a + *their* b (3)	B1	Not enough to be seen in a table of values – must be selected. Graph from their values can get both marks. **Note: Use of $b^2 - 4ac$ scores 0/3**
	$a - b$ or *their* a – *their* b (11).	B1	
	$\rightarrow k < 3$, $k > 11$	B1	Both inequalities correct. Allow combined statement as long as correct inequalities if taken separately. Both answers correct from T & I or guesswork 3/3 otherwise 0/3
		3	

Question 41

(i)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$.		Accept abbreviations s and c
	LHS = $\sin\theta + \cos\theta - \sin^2\theta\cos\theta - \sin\theta\cos^2\theta$	**M1**	Expansion
	$= \sin\theta(1 - \cos^2\theta) + \cos\theta(1 - \sin^2\theta)$ or $(s + c - c(1 - c^2) - s(1 - s^2))$	**M1A1**	Uses identity twice. Everything correct. AG
	Uses $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^3\theta + \cos^3\theta$ (RHS)		or from RHS: M1 for use of trig ID twice
	Or		
	LHS = $(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)$	**M1**	M1 for factorisation
	$= \sin^3\theta + \sin\theta\cos^2\theta - \sin^2\theta\cos\theta + \cos\theta\sin^2\theta + \cos^3\theta - \sin\theta\cos^2\theta = \sin^3\theta + \cos^3\theta$	**M1A1**	
		3	
(ii)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta \rightarrow \sin^3\theta = 2\cos^3\theta$	**M1**	
	$\rightarrow \tan^3\theta = 2 \rightarrow \theta = 51.6°$ or $231.6°$ (only)	**A1A1FT**	Uses $\tan^3 = \sin^3 \div \cos^3$. A1 CAO. A1FT, 180 + their acute angle. $\tan^3\theta = 0$ gets M0
		3	

Question 42

$\dfrac{(\tan\theta+1)(1-\cos\theta)+(\tan\theta-1)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ soi	**M1**	
$\dfrac{\tan\theta-\tan\theta\cos\theta+1-\cos\theta+\tan\theta-1+\tan\theta\cos\theta-\cos\theta}{1-\cos^2\theta}$ www	**A1**	
$\dfrac{2(\tan\theta-\cos\theta)}{\sin^2\theta}$ www AG	**A1**	
	3	
$(2)(\tan\theta-\cos\theta)\ (=0)\rightarrow(2)\left(\dfrac{\sin\theta}{\cos\theta}-\cos\theta\right)\ (=0)$ soi	**M1**	Equate numerator to zero and replace $\tan\theta$ by $\sin\theta/\cos\theta$
$(2)\left(\sin\theta-\left(1-\sin^2\theta\right)\right)\ (=0)$	**DM1**	Multiply by $\cos\theta$ and replace $\cos^2\theta$ by $1-\sin^2\theta$
$\sin\theta = 0.618(0)$ soi	**A1**	Allow $(\sqrt{5}-1)/2$
$\theta = 38.2°$	**A1**	Apply penalty –1 for extra solutions in range
	4	

Question 43

(i)	$fg(x) = 2 - 3\cos(\frac{1}{2}x)$	**B1**	Correct fg
	$2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\ \frac{1}{3}\right)$	**M1**	M1 for correct order of operations to solve their $fg(x) = 1$ as far as using inverse cos expect 1.23, (or 70.5°) condone x =.
	x = **2.46 awrt or** $\frac{4.7\pi}{6}(0.784\pi$ **awrt**$)$	**A1**	One solution only in the given range, ignore answers outside the range. Answer in degrees A0.
			Alternative: Solve $f(y) = 1 \rightarrow y = 1.23 \rightarrow \frac{1}{2}x = 1.23$ **B1M1** $\rightarrow x = 2.46$ **A1**
		3	

(ii)		B1	One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted "V". If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark 0 → 2π.
		B1	Start and finish at roughly the same negative *y* value. Significantly more above the *x* axis than below or correct range implied by labels .
		B1	Fully correct. Curves not lines. Must be a reasonable curve clearly turning at both ends. Labels not required but must be appropriate if present.
		3	

Question 44

(i)	In Δ *ABD*, $\tan\theta = \frac{9}{BD} \rightarrow BD = \frac{9}{\tan\theta}$ or $9\tan(90-\theta)$ or $9\cot\theta$ or $\sqrt{\left[(20\tan\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule)	B1	Both marks can be gained for correct equated expressions.
	In Δ *DBC*, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$	B1	
	$20\sin\theta = \frac{9}{\tan\theta}$	M1	Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary.
	→ $20\sin^2\theta = 9\cos\theta$ AG	A1	Correct manipulation of their expression to arrive at given answer.
			SC: In Δ *DBC*, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ B1 In Δ *ABD*, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$ $\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \rightarrow \cos\theta = \frac{20\sin^2\theta}{9}$ M1 **→ $20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4
		4	
(ii)	Uses $s^2 + c^2 = 1 \rightarrow 20\cos^2\theta + 9\cos\theta - 20 \ (= 0)$	M1	Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$
	→ $\cos\theta = 0.8$	A1	www
	→ θ = 36.9° awrt	A1	www. Allow 0.644^c awrt. Ignore 323.1° or 2.50^c. Note: correct answer without working scores 0/3.
		3	

Question 45

(i)	$\frac{(\cos\theta-4)(5\cos\theta-2)-4\sin^2\theta}{\sin\theta(5\cos\theta-2)} \ (=0)$	M1	Accept numerator only
	$\frac{5\cos^2\theta-22\cos\theta+8-4(1-\cos^2\theta)}{\sin\theta(5\cos\theta-2)} \ (=0)$	M1	Simplify numerator and use $s^2 = 1 - c^2$. Accept numerator only
	$9\cos^2\theta - 22\cos\theta + 4 = 0$ www **AG**	A1	
		3	
(ii)	Attempt to solve for $\cos\theta$, (formula, completing square expected)	M1	Expect $\cos\theta = 0.1978$. Allow 2.247 in addition
	$\theta = 78.6°, 281.4°$ (only, second solution in the range)	A1A1FT	Ft for (360° – 1st solution)
		3	

Question 46

'(a)	$3\left(1-\cos^2 2\theta\right)+8\cos 2\theta=0 \ \rightarrow\ 3\cos^2 2\theta-8\cos 2\theta-3\ (=0)$	M1	Use $s^2=1-c^2$ and simplify to 3-term quadratic in 2θ
	$\cos 2\theta=-\frac{1}{3}$ soi	A1	Ignore other solution
	2θ = 109.(47)° or 250.(53)°	A1	One solution is sufficient, may be implied by either of the next solns
	θ = 54.7° or 125.3°	A1A1ft	Ft for 180° – other solution Use of double angles leads to $3c^4-7c^2+2=0 \Rightarrow c=\pm 1/\sqrt{3}$ for M1A1A1 then A1A1 for each angle Similar marking if $3\sin^2 2\theta=-8\cos 2\theta$ is squared leading to $9\sin^4 2\theta+64\sin^2 2\theta-64=0$
		5	
'(b)	$\sqrt{3}=a+\tan 0 \rightarrow\ a=\sqrt{3}$	B1	b = 8 or –4 (or –10, 14 etc) scores M1A0
	$0=\tan(-b\pi/6)+\sqrt{3}$ taken as far as $\tan^{-1}$, angle units consistent	M1	A0 if $\tan^{-1}\left(-\sqrt{3}\right)$ is not exact; (b=2 no working scores B2)
	$b=2$	A1	
		3	

Question 47

(i)	$q \leqslant \mathrm{f}(x) \leqslant p+q$	B1B1	B1 each inequality – allow two separate statements Accept <, $(q, p+q)$, $[q, p+q]$ Condone y or x or f in place of f(x)
		2	
(ii)	**(a)** 2	B1	Allow $\frac{\pi}{4}, \frac{3\pi}{4}$
	(b) 3	B1	Allow $0, \frac{\pi}{2}, \pi$
	(c) 4	B1	Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
		3	
(iii)	$3\sin^2 2x+2=4 \rightarrow \sin^2 2x=\frac{2}{3}$ soi	M1	
	Sin2x = (±)0.816(5). Allow $\sin 2x=(\pm)\sqrt{\frac{2}{3}}$ or $2x=\sin^{-1}(\pm)\sqrt{\frac{2}{3}}$	A1	OR Implied by at least one correct value for x. Allow $\sin^{-1}$ form
	($2x$ =) **at least two** of 0.955(3), 2.18(6), 4.09(7) , 5.32(8)	A1	Can be implied by corresponding values of x below Allow for at least two of 0.304π, 0.696π, 1.30(4)π, 1.69(6)π **OR** at least two of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)°
	(x =) 0.478, 1.09, 2.05, 2.66.	A1A1	Allow 0.152π, 0.348π, 0.652π, 0.848π SC A1 for 2 or 3 correct. SC A1 for all of 27.4°, 62.6°, 117.4°, 152.6° $\text{Sin}2x=\pm\frac{2}{3} \rightarrow x=0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1
		5	

Question 48

(i)	$a^2+2ab+b^2, a^2-2ab+b^2$	**B1**	Correct expansions.
	$\sin^2x + \cos^2x = 1$ used $\rightarrow (a+b)^2+(a-b)^2=1$	**M1**	Appropriate use of $\sin^2x + \cos^2x = 1$ with $(a+b)^2$ and $(a-b)^2$
	$a^2 + b^2 = ½$	**A1**	No evidence of $\pm 2ab$, scores 2/3
	Alternative method for question 4(i)		
	$2a = (s+c)$ & $2b = (s-c)$ or $a = ½(s+c)$ & $b = ½(s-c)$	**B1**	
	$a^2+b^2 = \frac{1}{4}(s+c)^2+\frac{1}{4}(s-c)^2 = ½(s^2+c^2)$	**M1**	Appropriate use of $\sin^2x + \cos^2x = 1$
	$a^2 + b^2 = ½$	**A1**	Method using only $(\sin x - b)^2$ and $(a-\cos x)^2$ scores 0/3.
		3	SC B1 for assuming θ is acute giving $a=\frac{1}{\sqrt{5}}+b$ or $2\sqrt{5}-b$
(ii)	$\tan x=\frac{\sin x}{\cos x} \rightarrow \frac{a+b}{a-b}=2$	**M1**	Use of $\tan x=\frac{\sin x}{\cos x}$ to form an equation in a and b only
	$a = 3b$	**A1**	
		2	

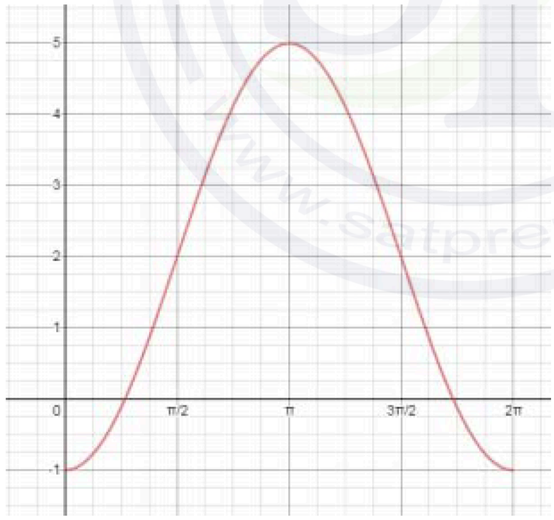
Question 49

(i)	3, –3	**B1**	Accept ± 3
	–½	**B1**	
	2½	**B1**	
		3	Condone misuse of inequality signs.
(ii)			Only mark the curve from $0 \rightarrow 2\pi$. If the x axis is not labelled assume that $0 \rightarrow 2\pi$ is the range shown. Labels on axes are not required.
	2 complete oscillations of a cosine curve starting with a maximum at (0,a), a>0	**B1**	
	Fully correct curve which must appear to level off at 0 and/or 2π.	**B1**	
	Line starting on positive y axis and finishing below the x axis at 2π. Must be straight.	**B1**	
		3	
(iii)	4	**B1**	
		1	

Question 50

(i)	LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2 = \frac{(1-s)(1-s)}{c^2} = \frac{(1-s)(1-s)}{1-s^2}$	B1	Expresses tan in terms of sin and cos
		B1	correctly $1-s^2$ as the denominator
	$= \frac{(1-s)(1-s)}{(1-s)(1+s)}$	M1	Factors and correct cancelling www
	$\frac{1-\sin x}{1+\sin x}$ AG	A1	
		4	
(ii)	Uses part **(i)** to obtain $\frac{1-\sin 2x}{1+\sin 2x} = \frac{1}{3} \rightarrow \sin 2x = ½$	M1	Realises use of $2x$ and makes $\sin 2x$ the subject
	$x = \frac{\pi}{12}$	A1	Allow decimal (0.262)
	(or) $x = \frac{5\pi}{12}$	A1	**FT** for $½\pi$ – 1st answer. Allow decimal (1.31) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ only, and no others in range. SC $\sin x = ½ \rightarrow \frac{\pi}{6}\ \frac{5\pi}{6}$ B1
		3	

Question 51

(i)	$-1 \leqslant f(x) \leqslant 5$ or $[-1, 5]$ (may use y or f instead of f(x))	B1 B1	$-1 < f(x) \leqslant 5$ or $-1 \leqslant x \leqslant 5$ or $(-1,5)$ or $[5,-1]$ B1 only
		2	
(ii)		*B1	Start and end at –ve y, symmetrical, centre +ve.
	$g(x) = 2-3\cos x$ for $0 \leqslant x \leqslant p$	DB1	Shape all ok. Curves not lines. One cycle $[0,2\pi]$ Flattens at each end.
		2	

(iii)	(greatest value of p =) π	B1	
		1	
(iv)	$x = 2 - 3\cos x \rightarrow \cos x = ⅓(2 - x)$	M1	Attempt at $\cos x$ the subject. Use of cos^{-1}
	$g^{-1}(x) = \cos^{-1}\dfrac{2-x}{3}$ (may use 'y =')	A1	Must be a function of x,
		2	

Question 52

(i)	$3\cos^4\theta + 4\left(1-\cos^2\theta\right) - 3(=0)$	M1	Use $s^2 = 1 - c^2$
	$3x^2 + 4(1-x) - 3(=0) \rightarrow 3x^2 - 4x + 1(=0)$	A1	AG
		2	
ii)	Attempt to solve for x	M1	Expect $x = 1$, 1/3
	$\cos\theta = (\pm)1,\ (\pm)0.5774$	A1	Accept $(\pm)\left(\dfrac{1}{\sqrt{3}}\right)$ SOI
	$(\theta =)$ 0°, 180°, 54.7°, 125.3°	A3,2,1,0	A2,1,0 if more than 4 solutions in range
		5	

Question 53

(a)	$(2x + 1) = \tan^{-1}(⅓)$ (= 0.322 or 18.4 OR −0.339 rad or 8.7°)	*M1	Correct order of operations. Allow degrees.
	Either *their* 0.322 + π or 2π Or *their* −0.339 + $\dfrac{\pi}{2}$ or π	DM1	Must be in radians
	$x = 1.23$ **or** $x = 2.80$	A1	AWRT for either correct answer, accept 0.39π or 0.89π
		A1	For the second answer with no other answers between 0 and 2.8 **SC1** For both 1.2 and 2.8
		4	
b)(i)	$5\cos^2 x - 2$	B1	Allow $a = 5$, $b = -2$
		1	
b)(ii)	−2	B1FT	FT for sight of *their* b
	3	B1FT	FT for sight of *their* $a + b$
		2	

Question 54

(i)	$4\tan x+3\cos x+\frac{1}{\cos x}=0 \rightarrow 4\sin x+3\cos^2 x+1=0$	**M1**	Multiply by $\cos x$ or common denominator of $\cos x$
	$4\sin x+3\left(1-\sin^2 x\right)+1=0 \rightarrow 3\sin^2 x-4\sin x-4=0$	**M1**	Use $\cos^2 x=1-\sin^2 x$ and simplify to 3-term quadratic in $\sin x$
	$\sin x=-\frac{2}{3}$	**A1**	AG
		3	
(ii)	$2x-20° = 221.8°, 318.2°$	**M1A1**	Attempt to solve $\sin(2x-20)=-2/3$ (M1). At least 1 correct (A1)
	$x = 120.9°, 169.1°$	**A1** **A1FT**	FT for 290° – other solution. **SC** A1 both answers in radians
		4	

Question 55

$2\tan\theta-6\sin\theta+2=\tan\theta+3\sin\theta+2 \rightarrow \tan\theta-9\sin\theta\ (=0)$	**M1**	Multiply by denominator and simplify
$\sin\theta-9\sin\theta\cos\theta\ (=0)$	**M1**	Multiply by $\cos\theta$
$\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta=0, \quad \cos\theta=\frac{1}{9}$	**M1**	Factorise and attempt to solve at least one of the factors = 0
$\theta=0$ or 83.6° (only answers in the given range)	**A1A1**	
	5	

Question 56

(a)	$(\tan x-2)(3\tan x+1)\ (=0)$. **or** formula **or** completing square	**M1**	Allow reversal of signs in the factors. Must see a method
	$\tan x=2$ or $-\frac{1}{3}$	**A1**	
	$x=63.4°$(only value in range) or $161.6°$(only value in range)	**B1FT** **B1FT**	
		4	
(b)	Apply $b^2-4ac<0$	**M1**	SOI. Expect $25-4(3)(k)<0$, $\tan x$ must not be in coefficients
	$k>\frac{25}{12}$	**A1**	Allow $b^2-4ac=0$ leading to correct $k>\frac{25}{12}$ for M1A1
		2	
(c)	$k=0$	**M1**	SOI
	$\tan x=0$ or $\frac{5}{3}$	**A1**	
	$x=0°$ or $180°$ or $59.0°$	**A1**	All three required
		3	

Question 57

(a)	$\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}=\frac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$	M1
	$=\frac{2\tan\theta}{\sin^2\theta}$	M1
	$=\frac{2\sin\theta}{\cos\theta\sin^2\theta}$	M1
	$=\frac{2}{\sin\theta\cos\theta}$ **AG**	A1
		4
(b)	$\frac{2}{\sin\theta\cos\theta}=\frac{6\cos\theta}{\sin\theta}$	M1
	$\cos^2\theta=\frac{1}{3}\to\ \cos\theta=(\pm)0.5774$	A1
	54.7°, 125.3° (**FT** for 180° – 1st solution)	A1 A1FT
		4

Question 58

(a)	$3\cos\theta=8\tan\theta\to\ 3\cos\theta=\frac{8\sin\theta}{\cos\theta}$	M1
	$3(1-\sin^2\theta)=8\sin\theta$	M1
	$3\sin^2\theta+8\sin\theta-3=0$	A1
		3
(b)	$(3\sin\theta-1)(\sin\theta+3)=0\to\sin\theta=⅓$	M1
	$\theta=19.5°$	A1
		2

Question 59

(a)	$-1\leqslant f(x)\leqslant 2$	B1 B1
		2
(b)	$k=1$	B1
	Translation by 1 unit upwards parallel to the y-axis	B1
		2
(c)	$y=-\frac{3}{2}\cos 2x-\frac{1}{2}$	B1
		1

Question 60

(a)	$\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	**M1**
	Use of $\sin^2\theta+\cos^2\theta=1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$.	**M1A1**
		3
(b)	$\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta} \rightarrow \tan\theta=1.5$	**M1**
	θ = 0.983 or 4.12 (**FT** on second value for 1st value + π)	**A1** **A1FT**
		3

Question 61

$3\tan^4\theta+\tan^2\theta-2\ (=0)$	**M1**	SOI 3-term quartic, condone sign errors for this mark only
$(3\tan^2\theta-2)(\tan^2\theta+1)\ (=0)$	**M1**	Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$.
$\tan\theta=(\pm)\sqrt{\dfrac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$	**A1**	SOI Implied by final answer = 39.2° after 1st M1 scored
39.2°, 140.8°	**A1** **A1 FT**	FT for 2nd solution =180° – 1st solution
	5	

Question 62

(a)	$\left(\dfrac{1}{\cos x}-\dfrac{\sin x}{\cos x}\right)\left(\dfrac{1}{\sin x}+1\right)$	**B1**	Uses "$\tan x=\sin x\div\cos x$" throughout
	$\left(\dfrac{1-\sin x}{\cos x}\right)\left(\dfrac{1+\sin x}{\sin x}\right)$ or $\left(\dfrac{1-\sin^2 x}{\cos x\sin x}\right)$	**M1**	Correct algebra leading to two or four terms
	$\left(\dfrac{\cos^2 x}{\cos x\sin x}\right)$	**A1**	OE. A correct expression which can be cancelled directly to $\dfrac{\cos x}{\sin x}$ e.g. $\dfrac{\cos x(1-\sin^2 x)}{\sin x(1-\sin^2 x)}$
	$\left(\dfrac{\cos^2 x}{\cos x\sin x}\right)=\left(\dfrac{\cos x}{\sin x}\right)=\dfrac{1}{\tan x}$	**A1**	AG. Must show cancelling. If x is missing throughout their working withhold this mark.
		4	
(b)	Uses **(a)** $\rightarrow \dfrac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\dfrac{1}{2}$	**M1**	Reducing to $\tan^3 x=k$.
	(x =) 38.4°	**A1**	AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within.
		2	

Question 63

(a)	$\left(\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}=\right)\frac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$	***M1**	Put over a single common denominator
	$\frac{2\sin^2\theta}{\cos^2\theta}$	**DM1**	Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator
	$2\tan^2\theta$	**A1**	AG
		3	
(b)	$2\tan^2\theta=8 \;\to\; \tan\theta=(\pm)2$	**B1**	SOI
	$(\theta=)\;63.4°,\;\;116.6°$	**B1** **B1 FT**	FT on 180 – 1st solution (with justification)
		3	

Question 64

$\tan\theta+2\sin\theta=3\tan\theta-6\sin\theta$ leading to $2\tan\theta-8\sin\theta\,[=0]$	**M1**	OE
$2\sin\theta-8\sin\theta\cos\theta\;(=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\,[=0]$	**M1**	
$\cos\theta=\frac{1}{4}$	**A1**	Ignore $\sin\theta=0$
$\theta=75.5°$ **only**	**A1**	
	4	

Question 65

(a)	$\frac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\frac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\frac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k]	**M1**	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by $\tan x$ or $\sin x$
	$\frac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}\cdot\frac{\cos x}{\cos x}$ or $\frac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\frac{1+\cos x}{1-\cos x}[=k]$	**A1**	AG, WWW
		2	
(b)	$k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$	**M1**	Gather like terms on LHS and RHS
	$k-1=(k+1)\cos x$ leading to $\cos x=\frac{k-1}{k+1}$	**A1**	WWW, OE
		2	
(c)	Obtaining $\cos x$ from *their* **(b)** or **(a)**	**M1**	Expect $\cos x=\frac{3}{5}$
	±0.927 (only solutions in the given range)	**A1**	AWRT. Accept ±0.295π
		2	

Question 66

(a)	$\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$	*M1	For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s).
	$\equiv\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$	DM1	For multiplying out the numerators correctly. Condone sign errors for this mark.
	$\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$	DM1	For simplifying denominator to $\cos^2 x$.
	$\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$	A1	AG. Do not award A1 if undefined notation such as s, c, t or missing x's used throughout or brackets are missing.
	Alternative method for Question 10(a)		
	$\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$	*M1	Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x = 1-\sin^2 x$
	$\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$	DM1	Separating into partial fractions.
	$\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$	DM1	Use of 1-1 or similar
	$\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$	A1	
		4	
(b)	$\cos x=\frac{1}{2}$	*B1	OE. WWW.
	$x=\frac{\pi}{3}$	DB1	Or AWRT 1.05
	$x=0$ from $\tan x=0$ or $\sin x=0$	B1	WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside.
		3	

Question 67

(a)	Reach $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}-2\tan^2\theta$ **or** $\sec^2\theta-\frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2-\sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$	**M1**	May start with $1-\tan^2\theta$
	$1-\tan^2\theta$	**A1**	AG, must show sufficient stages
		2	
(b)	$1-\tan^2\theta=2\tan^4\theta \Rightarrow 2\tan^4\theta+\tan^2\theta-1$ [= 0]	**M1**	Forming a 3-term quadratic in $\tan^2\theta$ or e.g. u
	$\tan^2\theta$ = 0.5 or −1 leading to $\tan\theta=[\pm]\sqrt{0.5}$	**M1**	
	θ = 35.3° and 144.7° (AWRT)	**A1**	Both correct. Radians 0.615, 2.53 scores A0.
		3	

Question 68

(a)	$\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction.
	$\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$	*M1	Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine.
	$k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$	DM1	Gather like terms on one side of the equation.
	$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$	A1	AG. Factorise to obtain answer.
		4	
(b)	$5\sin^2 x + 3\sin x - 5 = 0$	B1	
	$\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$	M1	Use formula or complete the square.
	$x = 48.1°, 131.9°$	A1 A1 FT	AWRT. Maximum A1 if extra solutions in range. FT for 180 – *their* answer or 540 – *their* answer if sin*x* is negative If M0 given and correct answers only **SCB1B1** available. If answers in radians; 0.839, 2.30 can score **SCB1** for both.
		4	

Question 69

(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$ OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	*M1	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sin*x* and cos*x*.
	Elimination of sin*x* and cos*x* to give an expression in tan*x* $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	DM1	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x=]\frac{5\pi}{6}$ only	A1	CAO. Must be exact.
		3	
(b)	$d = 2\cos x$ or $d = 2\cos(\textit{their } x)$	B1 FT	Or an equivalent expression involving sin*x* and cos*x* e.g. $-3\sqrt{3}\sin(\textit{their } x) - \cos(\textit{their } x)$ $\left[= -\sqrt{3}\right]$ FT for *their x* from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}\left(2\times\left(2\cos(\textit{their } x)\right) + (25-1)\times(\textit{their } d)\right)$ $\left[=12.5\left(2\times\left(-\sqrt{3}\right)+24\left(-\sqrt{3}\right)\right)\right]$	M1	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with *a* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$ and *d* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	A1	Must be exact.
		3	

Question 70

$2\cos^2\theta - 7\cos\theta + 3[=0]$	**M1**	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors.
$(2\cos\theta - 1)(\cos\theta - 3) = 0$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square.
[$\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ leading to] $\theta = -60°$ or $\theta = 60°$	**A1**	
$\theta = -60°$ **and** $\theta = 60°$	**A1 FT**	FT for ± same answer between 0° and 90° or 0 and $\frac{\pi}{2}$. $\pm\frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. **Special case:** If M1 DM0 scored then SC B1 for $\theta = -60°$ **or** $\theta = 60°$, and SC B1 FT can be awarded for $\pm($*their* $60°)$.
	4	

Question 71

(a)	$a = 5$	**B1**	
	$b = 2$	**B1**	
	$c = 3$	**B1**	
		3	
(b)(i)	3	**B1**	
		1	
(b)(ii)	2	**B1**	
		1	

Question 72

$3\cos\theta(2\tan\theta - 1) + 2(2\tan\theta - 1)\ [=0]$	**M1**	Or similar partial factorisation; condone sign errors.
$(2\tan\theta - 1)(3\cos\theta + 2)\ [=0]$ [leading to $\tan\theta = \frac{1}{2}$, $\cos\theta = -\frac{2}{3}$]	**M1**	OE. At least 2 out of 4 products correct.
26.6°, 131.8°	**A1 A1**	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta - 1$ leading to 131.8° or division by $3\cos\theta + 2$ or similar leading to 26.6°.

Question 73

(a)	$\frac{(\sin\theta+2\cos\theta)(\cos\theta+2\sin\theta)-(\sin\theta-2\cos\theta)(\cos\theta-2\sin\theta)}{(\cos\theta-2\sin\theta)(\cos\theta+2\sin\theta)}$	*M1	Obtain an expression with a common denominator
	$\frac{5\sin\theta\cos\theta+2\cos^2\theta+2\sin^2\theta-(5\sin\theta\cos\theta-2\sin^2\theta-2\cos^2\theta)}{\cos^2\theta-4\sin^2\theta}$ $=\frac{4(\cos^2\theta+\sin^2\theta)}{\cos^2\theta-4\sin^2\theta}$	A1	
	$\frac{4}{\cos^2\theta-4(1-\cos^2\theta)}$	DM1	Use $\cos^2\theta+\sin^2\theta=1$ twice
	$\frac{4}{5\cos^2\theta-4}$	A1	**AG**
		4	
(b)	$\frac{4}{5\cos^2\theta-4}=5$ leading to $25\cos^2\theta=24$ leading to $\cos\theta=\sqrt{\frac{24}{25}}\left[=(\pm)0.9798\right]$	M1	Make $\cos\theta$ the subject
	$\theta=11.5°$ or $168.5°$	A1 A1 FT	 FT on 180° – 1st solution
		3	

Question 74

(a)	$6y+2-7y^{1/2}$ [= 0]	*M1	OE Rearrange to a 3-term quadratic.
	$\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0]	DM1	Or use of formula or completing the square.
	$[y^{1/2}=]\frac{1}{2},\frac{2}{3}$	A1	Answers only **SC B1** if DM1 not scored.
	$[y=]\frac{1}{4},\frac{4}{9}$	A1	Answers only **SC B1** if DM1 not scored.
		4	
(b)	Use of $\tan x$ = *their y* values	M1	Must have at least 2 values of y from part **(a).**
	x = 14[.0], 24[.0], x = 194[.0], 204[.0]	A1 A1 FT	FT for 180 + angle (twice). AWRT
		3	

Question 75

(a)	$[p=]$ 3	B1	
		1	
(b)	$[q=]$ $\frac{1}{2}$	B1	
		1	
(c)	$[r=]$ –2	B1	
		1	

Question 76

(a)	$4\cos^4 x + \cos^2 x - 3 = 0 \Rightarrow (4\cos^2 x - 3)(\cos^2 x + 1) = 0$	M1	Attempt to solve 3 term quartic (or quadratic in another variable).
	$\Rightarrow [\cos^2 x =] \frac{3}{4}$ $[\cos^2 x = -1]$	A1	If M0 scored then **SC B1** is available for sight of $\frac{3}{4}$ [and -1].
	$\Rightarrow \cos x = [\pm]\sqrt{their\frac{3}{4}}$ **OE** $\left[= \pm\frac{\sqrt{3}}{2}\right]$	M1	Square rooting '*their* $\cos^2 x$'. Allow without ±. May be implied by correct final answer(s). Ignore $\sqrt{-1}$.
	$[x =]\ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	A1 A1 FT	Dependent on preceding M1 only. Exact answers needed. A1 for any 2 correct answers A1 A1 for 4 correct answers and no others inside the range $0 \leqslant x \leqslant 2\pi$ A0 A1 FT can be awarded for two exact answers that are $2\pi -$ '*their* $\frac{\pi}{6}$ and $\frac{5\pi}{6}$' within the range $0 \leqslant x \leqslant 2\pi$.
			SC: If all 4 answers given in degrees (30, 150, 210, 330) or non-exact (AWRT 0.524, 2.62, 3.67, 5.76 or 0.167π, 0.833π, 1.17π, 1.83) and no others then **SC B1**.
		5	
(b)	$\cos^2 x = \frac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	B1	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \frac{-1 \pm \sqrt{1+16k}}{8}$	*M1	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	Substituting $k = 5$ and obtain 1 from the formula	DM1	Or argue logically if $k > 5 \Rightarrow 1+16k > 81 \Rightarrow >1$.
	$\cos^2 x = 1$ or $\cos^2 x >$ or $\geqslant 1$	A1	Needs to be linked to $\cos^2 x$.
	Concluding statement having considered both ± cases. ∴ no solutions	A1	Dependent upon all previous marks having been scored.
	Alternative method for question 11(b)		
	$\cos^2 x = \frac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	B1	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \frac{-1 \pm \sqrt{1+16k}}{8}$	*M1	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	$\frac{-1+\sqrt{1+16k}}{8} * 1 \Rightarrow -1+\sqrt{1+16k} * 8 \Rightarrow 1+16k * 81$	DM1	∗ represents any inequality or =.
	$k * 5$	A1	∗ represents any inequality or =.
	Concluding statement having considered both ± cases. ∴ no solutions	A1	Dependent upon all previous marks having been scored.
		5	

Question 77

(a)	**EITHER** **(1)**{Translation} $\begin{pmatrix}\{30°\}\\\{0\}\end{pmatrix}$ **OR** **(2)**{Translation} $\begin{pmatrix}\{60°\}\\\{0\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. { } indicates different elements. Accept angle in radians.
	(3){Stretch} {factor 2} {in x-direction}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. { } indicates different elements.
	(4) Stretch factor 4 in y-direction **and** correct order	**B1**	Stretch, y-direction and factor **and** correct order. Correct order is **either** (1) then (3) **or** (3) then (2). (4) can be anywhere in the sequence.
		5	
(b)	$4\sin\left(\frac{1}{2}x-30°\right)=2\sqrt{2}\Rightarrow\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)[=45]$	**M1**	SOI
	$\frac{1}{2}x-30=45$ or 135 $\Rightarrow x=2(45+30)$ or $x=2(135+30)$	**M1**	SOI. The M marks are independent.
	$x=150°, x=330°$	**A1**	Both exact values, condone $\frac{5\pi}{6},\frac{11\pi}{6}$. A0 if extra solutions in the interval. Ignore other solutions outside $[0°,360°]$.
		3	

Question 78

(a)	$\frac{\sin^3\theta}{\sin\theta-1}-\frac{\sin^2\theta}{1+\sin\theta}=\frac{\sin^3\theta(1+\sin\theta)}{(\sin\theta-1)(1+\sin\theta)}-\frac{\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$ $\left[=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}\right]$	***M1**	Using a common denominator.
	$-\frac{\sin^2\theta+\sin^4\theta}{1-\sin^2\theta}$	**DM1**	Reaching $\pm\left(1-\sin^2\theta\right)$ in denominator. SOI by $\pm\cos^2\theta$.
	$-\frac{\sin^2\theta\left(1+\sin^2\theta\right)}{\cos^2\theta}$	**DM1**	Using $\sin^2\theta+\cos^2\theta=1$ in denominator and isolating $\sin^2\theta$ in numerator.
	$-\tan^2\theta\left(1+\sin^2\theta\right)$	**A1**	AG - Using/stating $\tan\theta=\frac{\sin\theta}{\cos\theta}$ is sufficient for A1. May be working from both sides provided the argument is complete. A0 if θ or brackets missing throughout, or sign errors. Allow recovery if AG follows from *their* working.
	Alternative method for Q4(a)		
	$-\tan^2\theta(1+\sin^2\theta)=-\frac{\sin^2\theta\left(1+\sin^2\theta\right)}{1-\sin^2\theta}$	***M1**	Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta+\cos^2\theta=1$.
	$\frac{-\sin^2\theta-\sin^4\theta}{(1-\sin\theta)(1+\sin\theta)}$	**DM1**	Factorising denominator.
	$\frac{\sin^2\theta+\sin^3\theta-\sin^3\theta+\sin^4\theta}{(\sin\theta-1)(1+\sin\theta)}=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$	**DM1**	Factorising numerator.

(a)	$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta}$	**A1**	AG A0 if θ or brackets missing throughout, or sign errors. Allow recovery if AG follows from *their* working.
		4	
(b)	$-\tan^2\theta\left(1+\sin^2\theta\right) = \tan^2\theta\left(1-\sin^2\theta\right)$ leading to $[2]\tan^2\theta = 0$	**M1**	Obtaining a (trig function)$^2 = 0$ WWW.
	$\tan\theta = 0$ leading to $[\theta =]\pi$	**A1**	Ignore extra solutions outside the interval $(0, 2\pi)$.
	Alternative method for Q4(b)		
	$-\frac{\sin^2\theta}{\cos^2\theta}(1+\sin^2\theta) = \frac{\sin^2\theta}{\cos^2\theta}(1-\sin^2\theta)$ leading to $-\sin^2\theta - \sin^4\theta = \sin^2\theta - \sin^4\theta$ leading to $[2]\sin^2\theta = 0$	**M1**	Obtaining a (trig function)$^2 = 0$ WWW.
	$\sin\theta = 0$ leading to $[\theta =]\pi$	**A1**	Ignore extra solutions outside the interval $(0, 2\pi)$.
		2	

Question 79

$8\left(1-\cos^2\theta\right) + 6\cos\theta + 1 \quad [=0]$	**M1**	Expect $8\cos^2\theta - 6\cos\theta - 9 = 0$.
$(4\cos\theta + 3)(2\cos\theta - 3) \quad [=0]$	**A1**	Factors or formula or completing square must be shown.
$[\rightarrow \cos\theta = -0.75 \rightarrow \theta =]\ 138.6°$ only,	**A1**	AWRT, ignore solutions outside the given range, answer in radians A0.
	3	

Question 80

Use of $\sin^2\alpha + \cos^2\alpha = 1$ eg $\sin\alpha = [\pm]\sqrt{1-\left(\frac{8}{17}\right)^2}$	***M1**	Or Pythagoras seen (may quote 8, 15, 17 triple).
$\sin\alpha = \frac{15}{17}$	**A1**	
$\tan\alpha = \frac{15}{8}$	**A1**	
$\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha} = \frac{17}{15} + \frac{8}{15}$	**DM1**	Dealing with reciprocals and addition of fractions correctly.
$= \frac{5}{3}$ oe	**A1**	Correct answer with no working shown scores 0. Extra answers from $\sin\alpha = -\frac{15}{17}$ are allowed.
	5	

Question 81

(a)	$k^2 - 4 \times 8 \times 2$ [< 0]	**M1**	Use of $b^2 - 4ac$ but not just in the quadratic formula.
	$-8 < k < 8$ or $-8 < k$, $k < 8$ or $\|\boldsymbol{k}\| < 8$ or $(-8, 8)$	**A1**	Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$.
		2	
(b)	$2(4\cos\theta - 1)(\cos\theta - 1)$ or $(4\cos\theta - 1)(\cos\theta - 1)$	**M1**	OE Or use of formula or completing the square. Allow use of replacement variable.
	$\cos\theta = \frac{2}{8}$, $\cos\theta = 1$	**A1**	OE For both answers. **SC:** If M0, **SC B1** available for sight of $\cos\theta = \frac{2}{8}$ and 1
	[$\theta =$] 0°, 75.5°	**A1**	AWRT ISW rejection of 0°. For both answers and no others in the range $0° \leqslant \theta \leqslant 180°$, must be in degrees. **SC:** If M0 B1 scored, **SC B1** available for correct answers. **SC**: If M1 A0 scored, **SC B1** available for $\cos\theta = \frac{2}{8}$ and $\theta =$ 75.5° only, WWW.
		3	

Question 82

(a)	$\frac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)} \left[= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}\right]$	***M1**	Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only.
	$\frac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}}$	**DM1**	Divide throughout by $\cos^2\theta$.
	$\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ AG	**A1**	
(b)	$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 2 \Rightarrow \tan^2\theta + 1 = 2(\tan^2\theta - 1)$	***M1**	Equate expression from **(a)** to 2 and clear fraction.
	$\tan\theta = [\pm]\sqrt{3}$	**DM1**	Simplify as far as $\tan\theta =$. May be implied by a correct final answer in degrees or radians.

Alternative method for first two marks of Question 7(b)

$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = 2 \Rightarrow 1 = 2\sin^2\theta - 2(1 - \sin^2\theta)$	***M1**	Equate expression to 2, clear fraction and use trig identities to form an equation in $sin\theta$ or $cos\theta$ only.
$sin\theta = [\pm]\sqrt{\frac{3}{4}}$ or $cos\theta = [\pm]\sqrt{\frac{1}{4}}$	**DM1**	Simplify as far as $sin\theta =$, or $cos\theta =$.
$\theta = \frac{1}{3}\boldsymbol{\pi}, \frac{2}{3}\boldsymbol{\pi}$	**A1** **A1 FT**	A1 for either correct answer then A1FT For their second value being $\pi -$ (their first) and no others in range $0 \leqslant \theta \leqslant \pi$, both values must be exact and in radians. **SC:** B1 for $\theta = 60°, 120°$ or $0.333\pi, 0.667\pi$ AWRT. or 1.05, 2.09 AWRT.
	4	

Question 83

(a)	$\frac{\sin\theta - \cos\theta + \sin\theta + \cos\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}\left[= \frac{\sin\theta - \cos\theta + \sin\theta + \cos\theta}{\sin^2\theta - \cos^2\theta}\right] = 1$	***M1**	Use common denominator and equate to 1.
	$2\sin\theta\ [= \sin^2\theta - \cos^2\theta] = \sin^2\theta - (1 - \sin^2\theta)$	**DM1**	Multiply by common denominator and replace $\cos^2\theta$ by $1 - \sin^2\theta$.
	$2\sin^2\theta - 2\sin\theta - 1 = 0$	**A1**	OE In the given form.
		3	
(b)	$[\sin\theta =]\frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{4}\left[= \frac{2 \pm \sqrt{4+8}}{4} = \frac{1 \pm \sqrt{3}}{2}\right]$	**M1**	Use formula or complete the square to solve a quadratic equation of the correct form.
	201.5° or 338.5°	**A1 A1 FT**	AWRT; A1 for either solution correct. A1 FT for 540 – (first value). If M0, allow **SC B1 B1FT** similarly.
		3	

Question 84

(a)	$\tan\theta\sin\theta = 1$ leading to $\sin^2\theta = \cos\theta$	**M1**	Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and multiplication by $\cos\theta$.
	$1 - \cos^2\theta = \cos\theta$ or $\cos^2\theta + \cos\theta - 1\ [= 0]$	**M1**	Use of trig identity to form a 3-term quadratic.
	$[\cos\theta =]\ \frac{-1 \pm \sqrt{5}}{2}$	**M1**	Use of formula or completion of the square must be seen on a 3-term quadratic. Expect 0.6180 .
	51.8º,	**A1**	Both A marks dependent on the 2nd M1.
	308.2º	**A1 FT**	FT for (360º – 1st soln), A0 if extra solutions in range. Radians 0.905 and 5.38, A1 only for both.
		5	
(b)	$\frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta} = \frac{\sin\theta}{\sin\theta\cos\theta} - \frac{\sin\theta\cos\theta}{\sin\theta} = \frac{1}{\cos\theta} - \cos\theta$	**M1**	Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ twice with correct use of fractions.
	$= \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$	**M1**	Use $1 - \cos^2\theta = \sin^2\theta$ with correct use of fractions.
	$= \tan\theta\sin\theta$	**A1**	WWW
		3	

Question 85

(a)	$3\sin^2 x - 3\sin^2 x\cos^2 x - 4\cos^2 x\ [=0]$	**M1**	Replace $\tan^2 x$ with $\dfrac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$.
	$3(1-\cos^2 x) - 3(1-\cos^2 x)\cos^2 x - 4\cos^2 x\ [=0]$	**M1**	Replace $\sin^2 x$ by $1-\cos^2 x$ twice.
	$3\cos^4 x - 10\cos^2 x + 3 = 0$ or $-3\cos^4 x + 10\cos^2 x - 3 = 0$	**A1**	Or multiple of these equations.
		3	
(b)	$(3\cos^2 x - 1)(\cos^2 x - 3)\ [=0]$	**M1**	OE, using *their* equation in the given form. Allow unusual notation if meaning is clear.
	$\cos x = [\pm]\dfrac{1}{\sqrt{3}}$	**A1**	SOI Answer only **SC B1**.
	54.7°,	**A1**	
	125.3°	**A1 FT**	Only other answer and must be from correct factorisation for A1. FT for 180° – *their* first answer. Answers only **SC B1**, **SC B1 FT**.
		4	

Question 86

(a)(i)	$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$	***B1**	Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www.
	$[\theta =]\ 0, \dfrac{\pi}{2}, \pi$	**DB 2,1,0**	B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. **SC DB1** for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents.
		3	Verifying 3 answers rather than expanding and solving 0/3.
(a)(ii)	$\cos 0 + \sin 0 = [1+0=]1$ and $\cos\dfrac{\pi}{2} + \sin\dfrac{\pi}{2}[=0+1] = 1$	**B1**	Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees.
	$\cos\pi + \sin\pi[=-1+0] = -1$ or $\neq 1$	**B1**	www
		2	
(b)	$\dfrac{(\cos\theta - \sin\theta)\sin\theta + (\cos\theta + \sin\theta)(1-\cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$	**M1**	Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta - \sin^2\theta$. Condone brackets missing if recovered.
	$= \dfrac{\cos\theta\sin\theta - \sin^2\theta + \cos\theta - \cos^2\theta + \sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$	**A1**	
	$= \dfrac{\sin\theta + \cos\theta - \cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$	**A1**	AG. Clear evidence of using $\sin^2\theta + \cos^2\theta = 1$ in either the numerator or denominator. Condone c, s and/or omission of θ. Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1.
		3	

(c)	$\frac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$ leading to $1=2(1-2\sin^2\theta)$	*M1	Replacing LHS with the expression from **(b)** and attempting to simplify i.e. condone omission of $(\cos\theta+\sin\theta-1)=0$ at this stage. M0 for $0=2(1-2\sin^2\theta)$
	$k\sin^2\theta=1$ or 3 leading to $\sin\theta=[\pm]\sqrt{\frac{1\text{ or }3}{k}}$ $\left[4sin^2\theta=1 \text{ leading to } \sin\theta=\pm\frac{1}{2}\right]$	DM1	Dividing by k and taking the square root of a positive value < 1. This mark can be implied by the solutions $\frac{1}{6}\pi,\frac{5}{6}\pi$.
	Solutions $0,\frac{1}{6}\pi,\frac{1}{2}\pi,\frac{5}{6}\pi$	A1	Allow 0, 0.524, 1.57, 2.62 AWRT. If M0 **SCB1** for $(\cos\theta+\sin\theta-1)=0 \Rightarrow 0, \frac{1}{2}\pi$. If M0 **SCB1** for all four correct answers and no others. Ignore answers outside of the range. Answers in degrees A0.
		3	

Question 87

$4\sin\theta+\tan\theta=0 \Rightarrow 4\sin\theta+\frac{\sin\theta}{\cos\theta}[=0]$	M1	For use of $\tan\theta=\frac{\sin\theta}{\cos\theta}$. BOD if θ missing.
$\Rightarrow \sin\theta(4\cos\theta+1)[=0 \Rightarrow \sin\theta=0 \text{ or}]\ \cos\theta=-\frac{1}{4}$	M1	WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta+1)[=0]$ or $\sin\theta(4+\sec\theta)[=0]$.
$\theta=104.5°$	A1	AWRT 1.82 rads A0. Ignore answers outside (0, 180°). If M1 M0, **SC B1** for $\theta=104.5°$ max 2/3.
	3	

Question 88

(a)	$5\cos^2\theta-\sin^2\theta+\cos\theta\ [=0]$	M1	Multiply by $\cos\theta$ and replace $\tan\theta$ by $\frac{\sin\theta}{\cos\theta}$.
	$5\cos^2\theta-(1-\cos^2\theta)+\cos\theta\ [=0]$	M1	
	$6\cos^2\theta+\cos\theta-1=0$	A1	Missing '= 0' can be condoned if '= 0' appears earlier.
		3	
(b)	$(3\cos\theta-1)(2\cos\theta+1)=0$	M1	Must have 3 term quadratic, expect $\cos\theta=\frac{1}{3},-\frac{1}{2}$. Factors (OE) must be shown.
	$\theta=\{1.23\}$; $\{2.09 \text{ or } \frac{2\pi}{3}\}$; $\{5.05 \text{ and } 4.19 \text{ (allow } \frac{4\pi}{3})\}$	A1 A1 A1 FT	For A1 FT is for both 2π – 1st solutions.
		4	

Question 89

(a)	$(2x-1)(4x^2+2x-1)=8x^3+4x^2-2x-4x^2-2x+1=8x^3-4x+1$	B1	AG Six correct terms leading to the correct answer.
		1	
(b)	Starting with the LHS $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the numerator and denominator.
	$=\dfrac{1}{1-\cos^2\theta-\cos^2\theta}$ need to see clear evidence of this step	DM1	For use of $\sin^2\theta+\cos^2\theta=1$ twice, in a correct expression, resulting in an expression in $\cos^2\theta$.
	$=\dfrac{1}{1-2\cos^2\theta}$	A1	AG
	Alternative method 1 for Question 7(b)		
	Starting with the RHS $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta-2\cos^2\theta}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\sin^2\theta+\cos^2\theta=1$ twice.
	$=\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ need to see clear evidence of this step	DM1	Dividing throughout by $\cos^2\theta$.
	$=\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$	A1	AG
(b)	**Alternative method 2 for Question 7(b)**		
	Starting with the LHS $\dfrac{\sec^2\theta}{\sec^2\theta-2}$	*M1	For use of $1+\tan^2\theta=\sec^2\theta$ twice.
	Clear statement $\Rightarrow$ $\dfrac{1}{1-2\cos^2\theta}$	DM1	AG For multiplying throughout by $\cos^2\theta$ to give the RHS.
		A1	
		3	
(c)	$\dfrac{1}{1-2\cos^2\theta}=4\cos\theta$ leading to $1=4\cos\theta(1-2\cos^2\theta)$ $[8\cos^3\theta-4\cos\theta+1=0]$	B1	Replace LHS with RHS from **(b)** and clear fractions.
	$(2\cos\theta-1)(4\cos^2\theta+2\cos\theta-1)[=0]$	*B1	Use of the expression from **(a)** with $x=\cos\theta$.
	$[x \text{ or } \cos\theta=]\dfrac{1}{2}$ and $\dfrac{-2\pm\sqrt{4+16}}{8}$ OR $0.31, -0.81$ AWRT	DB1	OE For all three values.
	$[\theta=]60°, 72°, 144°$	B2,1,0	B2 for three correct answers only, B1 for two correct answers and no others (but allow $36°$ instead of $144°$) in the given range or 3 correct answers plus other values in the given range. Ignore answers outside of the given range. Accept AWRT 72.0, 144.0 . **SC B1** for all 3 correct answers in radians and no others: $\dfrac{\pi}{3}, \dfrac{2\pi}{5}$ and $\dfrac{4\pi}{5}$.
		5	

Question 90

$[\tan^{-1}4x=]\left(\textit{their}-\dfrac{\pi}{6}\right)\pm\dfrac{\pi}{6}$ $\left[\tan^{-1}4x=\pm\dfrac{\pi}{3}, \pm1.047 \text{ or } 0\right]$	M1	OE Evaluating $\left(-\cos^{-1}\dfrac{\sqrt{3}}{2}\right)$ in rad and adding or subtracting $\dfrac{\pi}{6}$. Allow working with **both** angles in degrees.
$\left[4x=-\sqrt{3},\ x=\right]-\dfrac{\sqrt{3}}{4}$	A1	**Note:** answer of -0.43 or $\dfrac{\sqrt{3}}{4}$ implies M1
	2	

Question 91

(a)	$4\sin^2 x + 5\cos x + 2 \quad [=0]$	*M1	Multiply by $\sin x$ (or writing as a single fraction) and using $\tan x = \frac{\sin x}{\cos x}$.
	$4(1-\cos^2 x) + 5\cos x + 2 \quad [=0]$	DM1	Correctly obtaining a quadratic in $\cos x$ (allow sign errors).
	$4\cos^2 x - 5\cos x - 6 = 0$	A1	Condone missing x. Must be = 0 unless 0 appears on RHS earlier.
		3	
(b)	$(4\cos x + 3)(\cos x - 2) \ [=0]$	M1	Or use of formula or completing square.
	138.6°, 221.4°	A1 B1 FT	FT on 360° – 1st solution from quadratic in $\cos x$. Use of radians (2.42) A0 but allow B1 FT for 2π: 1st solution if use of radians is clear. **SC** If M0 scored **SC B1 B1** for correct final answer(s). If extra incorrect solutions in the range 0 → 360° are given award A1 B0.
		3	

Question 92

(a)	Expand bracket to obtain 3 terms and use correct identity	M1	θ may be missing or another symbol used.
	Use identity $\frac{\sin\theta}{\cos\theta} = \tan\theta$	M1	Does not require any further explanation. θ may be missing or another symbol used.
	Conclude with $2\tan\theta$	A1	WWW AG
		3	
(b)	Attempt solution of $5\tan^3\theta = 2\tan\theta$ to obtain at least one value of $\tan\theta$	M1	SOI Can be awarded if $\tan\theta$ is cancelled and ignored.
	Obtain at least two of $0, \pm 32.3$	A1	Or greater accuracy. **SC B1** if no method shown.
	Obtain all three values	A1	Or greater accuracy; and no others in $-90° < \theta < 90°$ range. Other units **SC B1** only for all 3 angles. **SC B1** if no method shown.
		3	

Question 93

(a)	Use identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$	M1	
	Use identity $\cos^2\theta = 1 - \sin^2\theta$	M1	
	$\pm(5\sin^2\theta + 7\sin\theta - 6 = 0)$	A1	
		3	
(b)	Attempt solution of *their* 3 term equation and correct process to find at least 1 value of $\sin x$ or $\sin 2x$ or $\sin\theta$	M1	Expect $(5s-3)(s+2) = 0,\ s = 3/5$.
	$x = 18.4$	A1	Or greater accuracy. **B1 SC** if no solution to the quadratic.
	$x = 71.6$ or (90 – *their* 18.4) or greater accuracy; and no other solutions for $0° < x < 180°$	A1FT	WWW **B1 SC** FT if no solution to the quadratic. **B1 SC** both correct in radians, 0.322, 1.25.
		3	

Question 94

(a)	$7\frac{\sin\theta}{\cos\theta} \div \cos\theta + 12[=0]$ $\left[\text{leading to } 7\frac{\sin\theta}{\cos\theta} + 12\cos\theta = 0\right]$	M1*	OE Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
	$7\sin\theta + 12(1-\sin^2\theta)[=0]$	DM1	Use of $s^2 + c^2 = 1$.
	$\Rightarrow 12\sin^2\theta - 7\sin\theta - 12 = 0$	A1	AG, WWW Condone use of s, c and t and/or omission of θ throughout working but the A1 is for cao.
		3	
(b)	$\left[12\sin^2\theta - 7\sin\theta - 12 = 0 \text{ leading to}\right](4\sin\theta + 3)(3\sin\theta - 4)$	M1	
	$\sin\theta = -\frac{3}{4}$ $\left[\text{or } \frac{4}{3}\right]$	B1	OE, WWW Can be implied by a correct value for $\sin^{-1}\left(-\frac{3}{4}\right)$ e.g. $-48.6°$.
	$[\theta =] 228.6°, 311.4°$	B1	AWRT, WWW No others in the range $0° \le \theta \le 360°$. Ignore any answers outside this range. Condone 229°, 311°.
		3	

Question 95

(a)	$\frac{\sin^2 x - \cos x - 1}{1+\cos x} = \frac{1-\cos^2 x - \cos x - 1}{1+\cos x}$ or $\frac{-\cos^2 x - \cos x}{1+\cos x}$	M1	For use of $\sin^2 x + \cos^2 x = 1$. Allow use of s, c, t or omission of x throughout.
	$= \frac{-\cos x(1+\cos x)}{1+\cos x}$	M1	For factorising.
	$= -\cos x$	A1	
		3	
(b)	$-\frac{1}{2}\cos x = \frac{1}{4} \Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$	M1	
	$x = 120°$ or $x = 240°$	A1	
		A1 FT	FT for 360 – *their* answer. A1 A0 if extra solution(s) in range. **SC B1** if answer in radians for both $\frac{2\pi}{3}, \frac{4\pi}{3}$.
		3	

Question 96

Let $x = \sin^2\theta$ $(2x+7)(2x-1) = 0$ or $(2\sin^2\theta + 7)(2\sin^2\theta - 1)$	M1	Or equivalent method.
$\Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin\theta = [\pm]\frac{1}{\sqrt{2}}$	M1	Finding $\sin^2\theta$ and then $\sin\theta$ (may be implied).
$\theta = 45°, 135°, 225°, 315°$	A1 A1	A1 for any two correct values. A1 for all correct and no others within the range. For answers in radians, A1 only for all 4 angles. If no (correct) working, then **SC B1** for all 4 solutions.
	4	

Question 97

$\cos\left(\frac{\pi}{6}\right) + \tan 2x + \frac{\sqrt{3}}{2} = 0 \Rightarrow \tan 2x = -\sqrt{3}$	M1	Making $\tan 2x$ the subject. $\tan 2x = 0$ is M0. Accept decimals and one sign error.
$\Rightarrow 2x = -\frac{\pi}{3} \Rightarrow x = -\frac{\pi}{6}$	A1	May come from non-exact working. Ignore answers outside the given range.
	2	

Question 98

(a)	Use $\tan^2\beta = \frac{\sin^2\beta}{\cos^2\beta}$	**B1**	E.g. $\tan^2\beta = \frac{\sin^2\beta}{\cos^2\beta}$ and then replaces $\sin^2\beta$ with a^2 or $\cos^2\beta$ with $1 - a^2$.
	$\cos\beta = -\sqrt{1-a^2}$	**B1**	
	Obtain $\frac{a^2}{1-a^2} + 3a\sqrt{1-a^2}$	**B1**	
		3	
(b)	Use correct identity to obtain 3-term quadratic equation in $\sin\theta$	***M1**	
	Obtain $\sin^2\theta + 4\sin\theta + 1[=0]$	**A1**	
	Attempt to solve quadratic	**DM1**	At least as far as $\frac{-4\pm\sqrt{12}}{2}$. $-15.5°$ implies attempt at solving quadratic.
	Obtain 195.5	**A1**	
	Obtain 344.5	**A1FT**	Following first answer; and no others for $0° < \theta < 360°$ but must be in 4th quadrant. **SC B1** for 3.41^c and 6.01^c.
		5	

Question 99

(a)	Use of $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$	**M1**	
	Relevant use of $\cos^2\theta = 1 - \sin^2\theta$ at least once	**M1**	
	$\frac{8\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta}$	**A1**	**AG** All necessary detail needed.
		3	
(b)	Attempt to solve *their* $5\sin^4\theta - 17\sin^2\theta + 9 = 0$ using a "correct" method	***M1**	Allow ± errors in arriving at *their* quadratic in $\sin^2\theta$. This can be implied by either $[\sin^2\theta =]$ 2.744 or 0.6559.
	$\sin\theta = [\pm]0.81[0]$ or $\sqrt{\frac{17-\sqrt{109}}{10}}$	**A1**	Condone inclusion of $\sin\theta = 1.65$ for this mark. Allow $\sqrt{\frac{17\pm\sqrt{109}}{10}}$. This mark can be implied by correct values.
	Their 54.1, and 180 − *their* 54.1 or 180 + *their* 54.1	**DM1**	A correct method for obtaining a second angle within the range $0 <$ *their* $54.1 < 90$.
	54.1, 125.9, 234.1	**A1**	AWRT A0 for additional values between 0° and 270°.
		4	

Question 100

Replace $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$	**M1**	
Replace $\sin^2\theta$ with $1-\cos^2\theta$ leading to a 3-term quadratic	***M1**	To obtain a 3-term quadratic in $\cos\theta$. Accept a cubic that has a common factor of $\cos\theta$. Condone +/–sign errors during simplification.
$9\cos^2\theta+\cos\theta-4\ [=0]$	**A1**	OE
Attempt to solve (*their* quadratic in $\cos\theta$) using a valid method	**DM1**	Only available for solution of a three-term quadratic. Correct values of $\cos\theta$ are 0.6134 and -0.7245.
Any two of $\pm 52.2,\ \pm 136.4$	**A1**	AWRT as final answers. Any two correct answers between $-180°$ and $180°$ as *their* final answers.
All four values	**A1**	Condone other answers outside the range but no others between $-180°$ and $180°$. **SC** after M1 *M1 A1 DM0, correct answers can score B1 B1.
	6	

Question 101

(a)	For appropriate use of $\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $\tan\theta\cos\theta=\sin\theta$ at least once	**M1**	**The first two marks can be applied in the reverse order but as soon as an error occurs, no further marks are awarded.** Candidates can work from LHS to RHS or RHS to LHS for full marks. If they work on both sides simultaneously, maximum of M1M1 only if a common correct expression is reached. Condone missing brackets. If the numerator and denominator are worked on separately, they need to be brought together for the second mark.
	For appropriate use of $\sin^2\theta+\cos^2\theta=1$ or $1+\tan^2\theta=\sec^2\theta$	**M1**	
	Fully correct proof	**A1**	WWW Condone missing brackets if recovered.
		3	
(b)	$\frac{\tan\theta+7}{\tan^2\theta-3}=\frac{5}{\tan\theta}\ \Rightarrow\ \tan\theta(\tan\theta+7)=5(\tan^2\theta-3)$	***B1**	OE Using part **(a)** and eliminating fractions.
	$\Rightarrow\ 4\tan^2\theta-7\tan\theta-15=0\ \Rightarrow (4\tan\theta+5)(\tan\theta-3)=0$	**DM1**	Factorising or other accepted method for solving *their* 3-term quadratic in $\tan\theta$. Condone errors made in forming 3-term quadratic.
	$\tan\theta=-1.25$ and $\tan\theta=3$	**B1**	OE Independent of factorisation of the quadratic. And no extra solutions. Allow e.g. $x=\ldots$ provided $x=\tan\theta$ is seen. Can be implied by a correct final answer.
	$\theta=71.6°$ and $\theta=128.7°$	**B1**	AWRT No others in range $0\leqslant\theta\leqslant 180°$. Ignore any answers outside this range. Maximum 3/4 if incorrect factorisation or formula.
		4	

Question 102

$6\sin^2\theta-\sin\theta-2[=0]\quad\Rightarrow\quad[(2\sin\theta+1)(3\sin\theta-2)=0]$	**M1**	For expressing as a 3-term quadratic. Terms need not all be on the same side.
$\sin\theta=-\frac{1}{2}$ or $\sin\theta=\frac{2}{3}$	**A1**	For both. Allow AWRT 0.667.
$\theta=-150°,\ -30°,\ 41.8°,\ 138.2°$	**A1 A1**	AWRT A1 for any correct angle from a correct value of $\sin\theta$, A1 for all 4 and no others in the interval $-180°<\theta<180°$. **SC B1** for $\frac{-5\pi}{6}$, $\frac{-\pi}{6}$, 0.730^c, 2.41^c if use of radians is clear.
	4	