AS-Level

Pure Mathematics P1

Topic : Trigonometry

May 2013- May 2023

Question 1

(i) Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \leqslant \theta \leqslant \pi$, giving solutions in terms of π. [3]

Question 2

(i) Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \leqslant x \leqslant 2\pi$. [4]

(ii) Hence state the number of solutions, in the interval $0 \leqslant x \leqslant 2\pi$, of the equations

(a) $2\sin 2x + 1 = 0$, [1]

(b) $\sin 2x - \cos x + 1 = 0$. [1]

Question 3

It is given that $a = \sin\theta - 3\cos\theta$ and $b = 3\sin\theta + \cos\theta$, where $0° \leqslant \theta \leqslant 360°$.

(i) Show that $a^2 + b^2$ has a constant value for all values of θ. [3]

(ii) Find the values of θ for which $2a = b$. [4]

Question 4

(i) Show that $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \leqslant \theta \leqslant 360°$. [4]

Question 5

(a) Find the possible values of x for which $\sin^{-1}(x^2 - 1) = \frac{1}{3}\pi$, giving your answers correct to 3 decimal places. [3]

(b) Solve the equation $\sin(2\theta + \frac{1}{3}\pi) = \frac{1}{2}$ for $0 \leqslant \theta \leqslant \pi$, giving θ in terms of π in your answers. [4]

Question 6

Given that $\cos x = p$, where x is an acute angle in degrees, find, in terms of p,

(i) $\sin x$, [1]

(ii) $\tan x$, [1]

(iii) $\tan(90° - x)$. [1]

Question 7

(i) Solve the equation $4\sin^2 x + 8\cos x - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Hence find the solution of the equation $4\sin^2\left(\frac{1}{2}\theta\right) + 8\cos\left(\frac{1}{2}\theta\right) - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [2]

Question 8

(i) Prove the identity $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

(ii) Hence solve the equation $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = 3\sin x - 2\cos x$ for $0 \leqslant x \leqslant 2\pi$. [3]

Question 9

(i) Prove the identity $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1 + \sin\theta} \equiv \tan\theta$. [4]

(ii) Solve the equation $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1 + \sin\theta} + 2 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

Question 10

The reflex angle θ is such that $\cos\theta = k$, where $0 < k < 1$.

(i) Find an expression, in terms of k, for

(a) $\sin\theta$, [2]

(b) $\tan\theta$. [1]

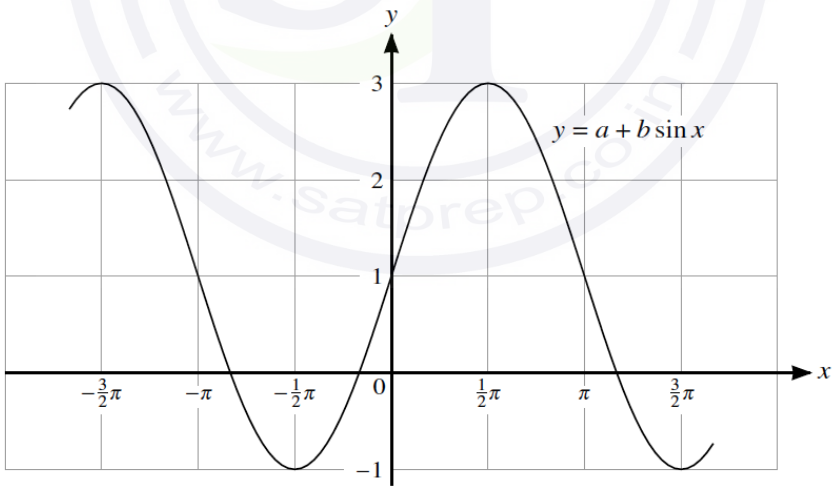
(ii) Explain why $\sin 2\theta$ is negative for $0 < k < 1$. [2]

Question 11

(i) Prove the identity $\dfrac{\sin\theta}{1 - \cos\theta} - \dfrac{1}{\sin\theta} \equiv \dfrac{1}{\tan\theta}$. [4]

(ii) Hence solve the equation $\dfrac{\sin\theta}{1 - \cos\theta} - \dfrac{1}{\sin\theta} = 4\tan\theta$ for $0° < \theta < 180°$. [3]

Question 12



The diagram shows part of the graph of $y = a + b\sin x$. State the values of the constants a and b. [2]

Question 13

(i) Show that $\sin^4\theta - \cos^4\theta \equiv 2\sin^2\theta - 1$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta = \frac{1}{2}$ for $0° \leqslant \theta \leqslant 360°$. [4]

Question 14

(i) Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0.$$ [3]

(ii) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0° \leqslant x \leqslant 180°$. [3]

Question 15

Find the value of x satisfying the equation $\sin^{-1}(x - 1) = \tan^{-1}(3)$. [3]

Question 16

Solve the equation $\dfrac{13 \sin^2 \theta}{2 + \cos \theta} + \cos \theta = 2$ for $0° \leqslant \theta \leqslant 180°$. [4]

Question 17

(i) Express the equation $3 \sin \theta = \cos \theta$ in the form $\tan \theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3 \sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

Question 18

(i) Prove the identity $\dfrac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \equiv \dfrac{\tan \theta - 1}{\tan \theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \dfrac{\tan \theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

Question 19

Given that θ is an obtuse angle measured in radians and that $\sin \theta = k$, find, in terms of k, an expression for

(i) $\cos \theta$, [1]

(ii) $\tan \theta$, [2]

(iii) $\sin(\theta + \pi)$. [1]

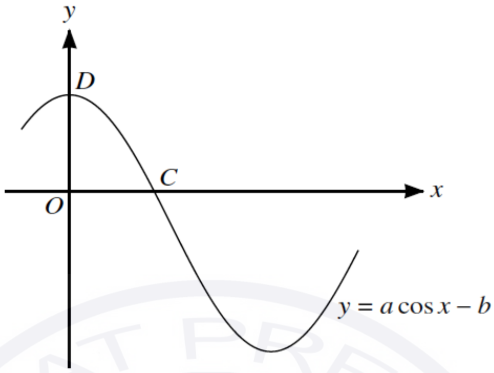
Question 20

(a) Show that the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ can be expressed as

$$3\cos^2\theta - 4\cos\theta - 4 = 0,$$

and hence solve the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ for $0° \leqslant \theta \leqslant 360°$. [6]

(b)



The diagram shows part of the graph of $y = a\cos x - b$, where a and b are constants. The graph crosses the x-axis at the point $C(\cos^{-1} c, 0)$ and the y-axis at the point $D(0, d)$. Find c and d in terms of a and b. [2]

Question 21

(i) Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 \equiv \frac{1-\cos x}{1+\cos x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

Question 22

Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

Question 23

(i) Show that the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

Question 24

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

Question 25

(i) Show that $3\sin x\tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x\tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x\tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

Question 26

(i) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

Question 27

Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

Question 28

Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

Question 29

(i) Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

Question 30

(i) Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

Question 31

(i) Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0° < \theta < 180°$. [3]

Question 32

(i) Prove the identity $\left(\dfrac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \dfrac{1-\sin\theta}{1+\sin\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\cos\theta} - \tan\theta\right)^2 = \dfrac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

Question 33

(i) Prove the identity $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

Question 34

(i) Show that the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ may be expressed as $5\cos^2\theta - \cos\theta - 4 = 0$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

Question 35

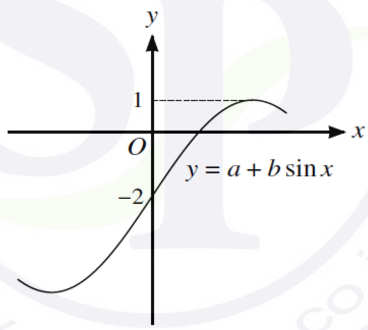
(i) Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

Question 36

(a)



The diagram shows part of the graph of $y = a + b\sin x$. Find the values of the constants a and b. [2]

(b) **(i)** Show that the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$. [3]

(ii) Hence solve the equation

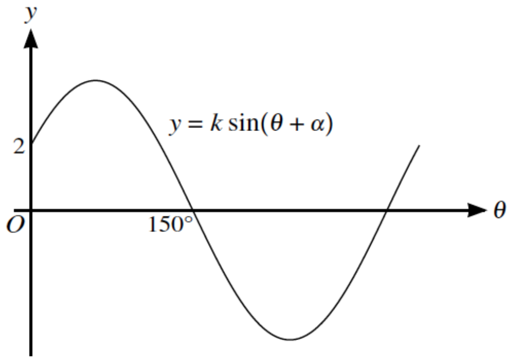
$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

for $-180° \leqslant \theta \leqslant 180°$. [4]

Question 37

(a) Express the equation $\dfrac{5+2\tan x}{3+2\tan x} = 1+\tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

(b)



The diagram shows part of the graph of $y = k\sin(\theta+\alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

Question 38

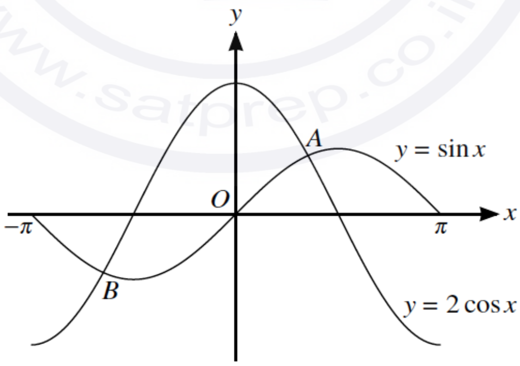
(a) **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

(ii) Find the y-coordinate of B. [2]

Question 39

(i) Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

Question 40

The function f is such that $f(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $f(\frac{1}{3}\pi) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

Question 41

(i) Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

Question 42

(i) Show that $\dfrac{\tan\theta + 1}{1 + \cos\theta} + \dfrac{\tan\theta - 1}{1 - \cos\theta} \equiv \dfrac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$. [4]

Question 43

Functions f and g are defined by

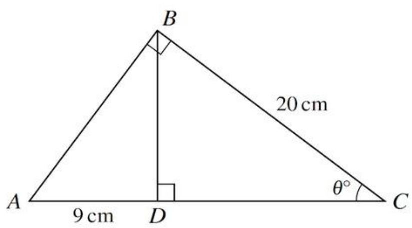
$$f : x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$g : x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $fg(x) = 1$. [3]

(ii) Sketch the graph of $y = f(x)$. [3]

Question 44



The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20\sin^2\theta = 9\cos\theta$. [4]

(ii) Hence, showing all necessary working, calculate θ. [3]

Question 45

(i) Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

(ii) Hence solve the equation

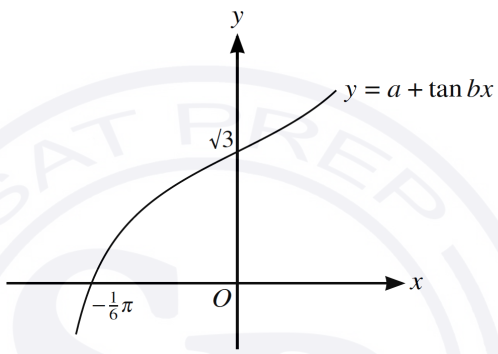
$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0° \leqslant \theta \leqslant 360°$. [3]

Question 46

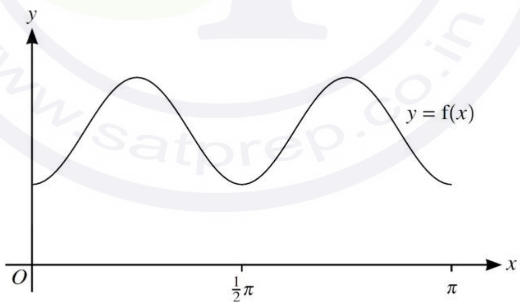
(a) Solve the equation $3\sin^2 2\theta + 8\cos 2\theta = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

(b)



The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x-axis at $\left(-\frac{1}{6}\pi, 0\right)$ and the y-axis at $(0, \sqrt{3})$. Find the values of a and b. [3]

Question 47



The function $f : x \mapsto p\sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

(i) In terms of p and q, state the range of f. [2]

(ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

(b) $f(x) = q$ [1]

(c) $f(x) = \frac{1}{2}p + q$ [1]

(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

Question 48

Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

Question 49

The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]

Question 50

(i) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

Question 51

The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

Question 52

(i) Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

Question 53

(a) Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

(b) The function $f : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

(ii) Find the range of f. [2]

Question 54

(i) Given that $4\tan x + 3\cos x + \frac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20°) + 3\cos(2x - 20°) + \frac{1}{\cos(2x - 20°)} = 0$$

for $0° \leqslant x \leqslant 180°$. [4]

Question 55

Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

Question 56

(a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

Question 57

(a) Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

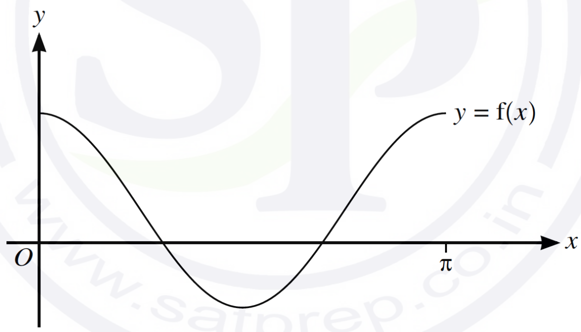
(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

Question 58

(a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

Question 59



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

Question 60

(a) Prove the identity $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} \equiv \dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

Question 61

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

Question 62

(a) Prove the identity $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) \equiv \dfrac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

Question 63

(a) Show that $\dfrac{\sin\theta}{1 - \sin\theta} - \dfrac{\sin\theta}{1 + \sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1 - \sin\theta} - \dfrac{\sin\theta}{1 + \sin\theta} = 8$, for $0° < \theta < 180°$. [3]

Question 64

Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

Question 65

(a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$

[2]

(b) Hence express $\cos x$ in terms of k. [2]

(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

Question 66

(a) Prove the identity $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

Question 67

(a) Prove the identity $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

(b) Hence solve the equation $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

Question 68

(a) Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k + 1)\sin^2 x + (k - 1)\sin x - (k + 1) = 0.$$ [4]

(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

Question 69

The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

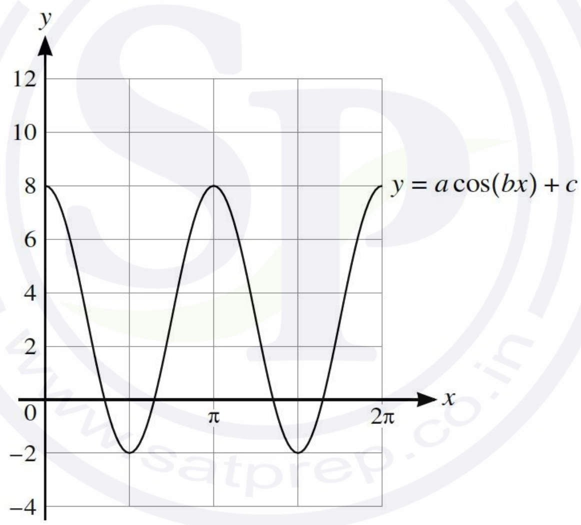
(a) Find the exact value of x. [3]

(b) Hence find the exact sum of the first 25 terms of the progression. [3]

Question 70

Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

Question 71



The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \dfrac{6}{\pi}x$ [1]

(ii) $a\cos(bx) + c = 6 - \dfrac{6}{\pi}x$ [1]

Question 72

Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

Question 73

(a) Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

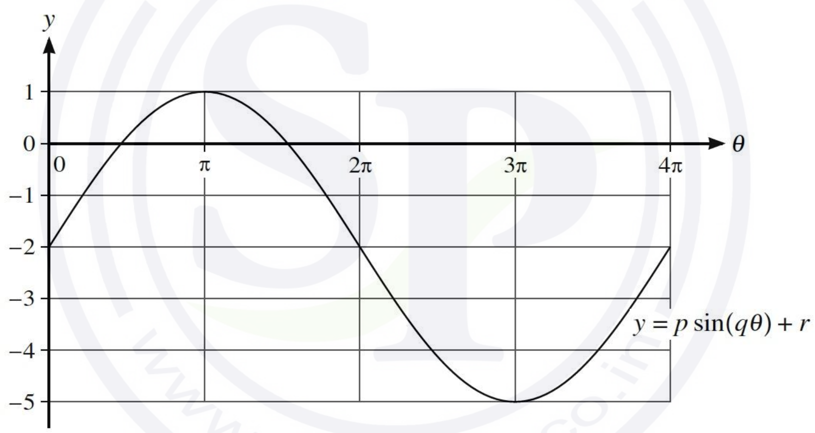
(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

Question 74

(a) Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

(b) Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

Question 75



The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

(b) State the value of q. [1]

(c) State the value of r. [1]

Question 76

The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

Question 77

(a) The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

Question 78

(a) Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1 + \sin\theta} = \tan^2\theta(1 - \sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

Question 79

Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

Question 80

It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha}$. [5]

Question 81

(a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

Question 82

(a) Prove the identity $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan^2\theta - 1}$. [3]

(b) Hence find the exact solutions of the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 2$ for $0 \leqslant \theta \leqslant \pi$. [4]

Question 83

(a) Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

Question 84

(a) By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

Question 85

(a) Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

Question 86

(a) **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. [3]

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta + \sin\theta} + \frac{1 - \cos\theta}{\cos\theta - \sin\theta} = 2(\cos\theta + \sin\theta - 1)$$

for $0 \leqslant \theta \leqslant \pi$. [3]

Question 87

Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]