AS-Level

Topic :Function

May 2013-May 2025

Answer

Question 1

	$f : x \mapsto 2x + k,\ g : x \mapsto x^2 - 6x + 8,$			
(i)	$2(2x+3)+3=25$ $\rightarrow x = 4$ or $\{f(11) = 25,\ f(4) = 11\}$	M1 A1	[2]	ff(x) needs to be correctly formed
(ii)	$x^2 - 6x + 8 = 2x + k$ $x^2 - 8x + 8 - k = 0$ Uses $b^2 - 4ac < 0$ $\rightarrow k < -8$	M1 M1 A1	[3]	Eliminates y to form eqn in x. Uses the discriminant – even if =0.>0
(iii)	$x^2 - 6x + 8 = (x-3)^2 - 1$ $y = (x-3)^2 - 1$ Makes x the subject $\rightarrow \pm\sqrt{(x+1)} + 3$ Needs specifically to lose the "–".	B1 B1 M1 A1√	[4]	For "–3" and "–1" Makes x the subject, in terms of x and without – or ±.

Question 2

(i) $2(x-3)^2 - 5$ or $a = 2$, $b = -3$, $c = -5$	**B1B1B1** **[3]**	
(ii) 3	**B1** √ **[1]**	ft on – *their b*. Allow $k \geq 3$ or $x \geq 3$
(iii) $(y) \geq 27$	**B1** **[1]**	Allow >. Allow $27 \leq y \leq \infty$ etc. **OR** (x/y interchange as 1st operation)
(iv) $2(x-3)^2 = (y+5)$	**M1**	$x = 2(y-3)^2 - 5$
$x - 3 = (\pm)\sqrt{\frac{1}{2}(y+5)}$	**M1**	$(y-3)^2 = \frac{1}{2}(x+5)$
$x = 3 + /\pm\sqrt{\frac{1}{2}(y+5)}$	**A1** √	$y - 3 = (\pm)\sqrt{\frac{1}{2}(x+5)}$
$(f^{-1}(x)) = 3 + \sqrt{\frac{1}{2}(x+5)}$ for $x \geq 27$	**A1B1** √ **[5]**	ft on *their* 27 from **(iii)**

Question 3

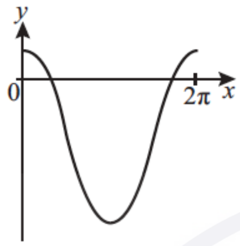
(i) Range is (y) $\geq c^2 + 4c$	B1	Allow >
$x^2 + 4x = (x+2)^2 - 4$	M1	**OR** $\frac{dy}{dx} = 2x + 4 = 0$
(Smallest value of c is) -2	A1	-2 with no (wrong) working gets B2
	[3]	
(ii) $5a + b = 11$	B1	
$(a+b)^2 + 4(a+b) = 21$	B1	
$(11 - 5a + a)^2 + 4(11 - 5a + a) = 21$	M1	**OR** corresponding equation in b
$(8)(2a^2 - 13a + 18) = (8)(2a - 9)(a - 2) = 0$	M1	**OR** $(8)(2b + 23)(b - 1) = 0$
$a = \frac{9}{2}, 2$ OR $b = \left(-\frac{23}{2}\right), 1$	A1 A1	A1 for either a or b correct. Condone 2nd value. Spotted solution scores only B marks.
	[6]	
lt. **(ii)** Last 5 marks $f^{-1}(x) = \sqrt{x+4} - 2$ B1 $g(1) = f^{-1} = (21)$ used M1 $a + b = \sqrt{25} - 2 = 3$ A1 Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2$, $b = 1$ A1		**Alt. (ii)** Last 4 marks $(a + b + 7)(a + b - 3) = 0$ M1A1 (Ignore solution involving $a + b = -7$) Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2$, $b = 1$ A1

Question 4

$(x+1)(x-2)$ or other valid method	M1	Attempt soln of eqn or other method
$-1, 2$	A1	
$x < -1$, $x > 2$	A1	Penalise $\leq, \geq$
	[3]	

Question 5

$f : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.

(i) $3\cos x - 2 = 0 \rightarrow \cos x = ⅔$ $\rightarrow x = 0.841$ or 5.44	M1 A1 A1√ [3]	Makes cos subject, then $\cos^{-1}$ √ for 2π − 1st answer.
(ii) range is $-5 \leqslant f(x) \leqslant 1$	B2,1 [2]	B1 for $\geqslant -5$. B1 for $\leqslant 1$.
(iii) 	B1,B1 [2]	B1 starts and ends at same point. Starts decreasing. One cycle only. B1 for shape, not 'V' or 'U'.
(iv) max value of $k = \pi$ or 180°.	B1 [1]	
(iv) $g^{-1}(x) = \cos^{-1}\left(\frac{x+2}{3}\right)$	M1 A1 [2]	Make x the subject, copes with 'cos'. Needs to be in terms of x.

Question 6

(i) $x = (\pm)\sqrt{y-1}$ $f^{-1} : x \mapsto \sqrt{x-1}$ for $x > 1$		**B1** **B1B1** **[3]**	**OR** $y^2 = x - 1$ (x/y interchange 1st)
(ii) $ff(x) = (x^2+1)^2 + 1$ $x^2 + 1 = (\pm)13/4$ $x = 3/2$		**B1** **M1** **A1** **[3]**	Or $x^4 + 2x^2 - (153/16) = 0$ Or $x^2 = 9/4, (-17/4)$ www. Condone $\pm 3/2$
.(ii) $f(x) = f^{-1}(185/16) = 13/4$ $x = f^{-1}(13/4)$ $x = 3/2$	**M1** **M1** **A1**		**Alt.**(ii) f(3/2) = 13/4 **B1** f(13/4) = 185/16 **B1** $x = 3/2$ **B1** **SC.B2** answer 1.5 with no working

Question 7

f : $x \mapsto 2x - 3,\ x \in \mathbb{R}$, g : $x \mapsto x^2 + 4x, x \in \mathbb{R}$.		
(i) ff = 2(2x − 3) −3 Solves = 11 → x = 5 (or 2x−3 =11, x = 7. 2x−3=7 → x = 5)	M1 A1 [2]	Either forms ff correctly, or solves 2 equations co
(ii) min at $x = -2$ → Range ⩾ −4	M1 A1 [2]	Any valid method – could be guesswork.
(iii) $x^2 + 4x - 12 \quad (> 0)$ → $x = 2$ or -6 → $x < -6$, $x > 2$.	M1 A1 A1 [3]	Makes quadratic = 0 + 2 solutions Correct limits – even if >,<,⩾,⩽,= co
(iv) gf(x) = $(2x - 3)^2 + 4(2x - 3) = p$ → $4x^2 - 4x - 3 - p = 0$ Uses "$b^2 - 4ac$" $16 = 16(-3 - p)$ → $p = -4$	B1 M1 A1 [3]	co unsimplified Use of discriminant co
(v) − 2	B1 [1]	co
(vi) $y = (x+2)^2 - 4$ $\sqrt{y+4} = x+2$ $h^{-1}(x) = \sqrt{x+4} - 2$	B2,1 M1 A1 [4]	−1 for each error Correct order of operations co with x, not y. ± left A0.

Question 8

(i)	$-5 \leqslant f(x) \leqslant 4$ For f(x) allow x or y; allow <, [–5, 4], (–5,4)	**B1** **[1]**	Allow less explicit answers (eg $-5 \to 4$)
(ii)	$f^{-1}(x)$ approximately correct (independent of f)	**B1**	
	Closed region between (1, 1) and (4, 4); line reaches x-axis	**DB1** **[2]**	Ignore line $y = x$
(iii)	LINE: $f^{-1}(x) = \frac{1}{3}(x+2)$	**B1**	Allow $y =$ ..... but must be a function of x
	for $-5 \leqslant x \leqslant 1$	**B1B1**	cao but allow <
	CURVE: $5 - y = \frac{4}{x}$ OR $x = 5 - \frac{4}{y}$	**M1**	
	$f^{-1}(x) = 5 - \frac{4}{x}$ oe	**A1**	cao
	for $1 < x \leqslant 4$	**B1** **[6]**	cao but allow < or <

Question 9

(a) (i)	$(a+b)^{\frac{1}{3}} = 2,\ (9a+b)^{\frac{2}{3}} = 16$	**B1B1**	Ignore 2nd soln (–9, 17) throughout
	$a+b=8,\ 9a+b=64$	**M1**	Cube etc. & attempt to solve
	$a=7,\ b=1$	**A1** **[4]**	Correct answers without any working 0/4
(ii)	$x = (7y+1)^{\frac{1}{3}}$ (x/y interchange as first or last step)	**B1**√	ft on from *their a, b* or in terms of *a, b*
	$x^3 = 7y+1$ or $y^3 = 7x+1$	**B1**√	ft on from *their a, b* or in terms of *a, b*
	$f^{-1}(x) = \frac{1}{7}(x^3 - 1)$ cao	**B1**	A function of x required
	Domain of f^{-1} is $x \geqslant 1$ cao	**B1** **[4]**	Accept >. Must be x
(b)	$\frac{dy}{dx} = \left[\frac{1}{3}(7x^2+1)^{-\frac{2}{3}}\right] \times [14x]$	**B1B1**	
	When $x = 3$, $\frac{dy}{dx} = \frac{1}{3} \times (64)^{-\frac{2}{3}} \times 42$ $\left(= \frac{7}{8}\right)$	**M1**	
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{7}{8} \times 8$	**DM1**	Use chain rule
	7	**A1** **[5]**	

Question 10

$f : x \mapsto 6 - 4\cos\left(\frac{1}{2}x\right)$

(i)	$6 - 4\cos\left(\frac{1}{2}x\right) = 4 \rightarrow 4\cos\left(\frac{1}{2}x\right) = 2$	M1	Makes $\cos\left(\frac{1}{2}x\right)$ the subject.
	$\frac{1}{2}x = \frac{1}{3}\pi \quad x = \frac{2}{3}\pi$	M1	Looks up "$\frac{1}{2}x$" before ×2
		A1 [3]	co (120° gets A0 – decimals A0)
(ii)	Range is $2 \leqslant f(x) \leqslant 10$	B1 B1 [2]	condone <
(iii)		B1	Point of inflexion at π
		B1 [2]	Fully correct
(iv)	$\cos\left(\frac{1}{2}x\right) = \frac{1}{4}(6 - y)$	M1	Makes $\cos\left(\frac{1}{2}x\right)$ the subject
	$\frac{1}{2}x = \cos^{-1}\left(\frac{1}{4}(6 - y)\right)$	M1	Order of operations correct (M marks allowed if + for –)
	$f^{-1}(x) = 2\cos^{-1}\left(\frac{6 - x}{4}\right)$	A1 [3]	oe – needs to be a function of x not y

Question 11

(i)	$(x-1)^2 - 16$		**B1B1** **[2]**	
(ii)	-16		**B1√** **[1]**	Ft from **(i)**
(iii)	$9 \leqslant (x-1)^2 - 16 \leqslant 65$ **OR**	$x^2 - 2x - 15 = 9 \rightarrow 6, -4$	**M1**	**OR** $x^2 - 2x - 24 \geqslant 0, x^2 - 2x - 80 \leqslant 0,$
	$25 \leqslant (x-1)^2 \leqslant 81$	$x^2 - 2x - 15 = 65 \rightarrow 10, -8$	**M1**	$(x-6)(x+4) \geqslant 0 \quad (x-10)(x+8) \leqslant 0$
	$5 \leqslant x - 1 \leqslant 9$	$p = 6$	**A1**	$x \geqslant 6$
	$6 \leqslant x \leqslant 10$	$q = 10$	**A1**	$x \leqslant 10$
			[4]	**SC B2, B2** for trial/improvement
(iv)	$x = (y-1)^2 - 16$	[interchange x/y]	**M1**	**OR** $(x-1)^2 = y + 16$
	$y - 1 = (\pm)\sqrt{x+16}$		**M1**	$x = 1 + (\pm)\sqrt{y+16}$
	$f^{-1}(x) = 1 + \sqrt{x+16}$		**A1** **[3]**	$f^{-1}(x) = 1 + \sqrt{x+16}$

Question 12

(i)	Attempt to find $(f^{-1})^{-1}$	**M1**	
	$2xy = 1-5x$ or $\frac{1}{2x} = y + \frac{5}{2}$ Allow 1 sign error	**A1**	Or with *x*/*y* transposed.
	$x = \frac{1}{2y+5}$ oe Allow 1 sign error (total)	**A1**	Or with *x*/*y* transposed. Allow $x = \frac{\frac{1}{2}}{y+\frac{5}{2}}$.
	$(f(x)) = \frac{1}{2x+5}$ for $x \geqslant -\frac{9}{4}$	**A1 B1**	Allow $\frac{\frac{1}{2}}{x+\frac{5}{2}}$. Condone $x > \frac{-9}{4}$, $(\frac{-9}{4}, \infty)$
	(Allow $-\frac{9}{4} \leqslant x \leqslant \infty$)	**[5]**	(etc.)
(ii)	$f^{-1}\left(\frac{1}{x}\right) = \frac{1-\frac{5}{x}}{\frac{2}{x}}$	**M1**	Reasonable attempt to find $f^{-1}\left(\frac{1}{x}\right)$.
	$\frac{x-5}{2}$ or $\frac{1}{2}x - \frac{5}{2}$	**A1** **[2]**	

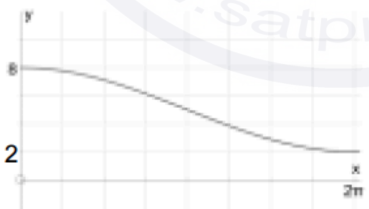
Question 13

	$f : x \mapsto 2x^2 - 6x + 5$		
(i)	$2x^2 - 6x + 5 - p = 0$ has no real roots	M1	Sets to 0 with *p* on LHS.
	Uses $b^2 - 4ac \rightarrow 36 - 8(5-p)$	DM1	Uses discriminant.
	Sets to 0 $\rightarrow p < \frac{1}{2}$	A1 [3]	co – must be "<", not "≤".
(ii)	$2x^2 - 6x + 5 = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	3 × B1 [3]	co
(iii)	Range of g $\frac{1}{2} \leqslant g(x) \leqslant 13$	B1√ B1 [2]	√ on **(ii)** co from sub of $x = 4$
	$h : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$		
(iv)	Smallest $k = \frac{3}{2}$	B1√ [1]	√ on **(ii)**
(v)	$h(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	M1	Using comp square form to try and get *x* as subject or *y* if transposed.
	Order of operations $\pm\frac{1}{2}$, $\div 2$, √, $\pm\frac{3}{2}$	DM1	Order must be correct
	$\rightarrow$ Inverse $= \frac{3}{2} + \sqrt{\left(\frac{x}{2} - \frac{1}{4}\right)}$	A1 [3]	co (without ±)

Question 14

	$h = 60(1 - \cos kt)$		
(i)	Max h when $\cos = -1 \rightarrow 120$	B1 [1]	Co
(ii)	$h = 0$ and $t = 30$, or $h = 120$ and $t = 15$ $\rightarrow \cos 30k = 1$ or $\cos 15k = -1$ $\rightarrow 30k = 2\pi$ or $15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$	M1 A1 [2]	Substituting a correct pair of values into the equation. co ag
(iii)	$90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3}$ or $\rightarrow kt = \frac{4\pi}{3}$ $\rightarrow$ Either $t = 10$ or 20 or both $\rightarrow t = 10$ minutes	 B1 B1 B1 [3]	 co – but there must be evidence of correct subtraction.

Question 15

	$f : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ for $0 \leqslant x \leqslant 2\pi$.		
(i)	$5 + 3\cos\left(\frac{1}{2}x\right) = 7$ $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$ $\frac{1}{2}x = 0.84$ $x = 1.68$ only, aef (in given range)	 B1 M1A1 [3]	 Makes $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$ Looks up $\cos^{-1}$ first, then $\times 2$
(ii)		B1 B1 [2]	y always +ve, m always –ve. from (0, 8) to $(2\pi, 2)$ (may be implied)
(iii)	No turning point on graph or 1:1	B1 [1]	cao, independent of graph in (ii)
(iv)	$y = 5 + 3\cos\left(\frac{1}{2}x\right)$ Order; -5, $\div 3$, $\cos^{-1}$, $\times 2$ $x = 2\cos^{-1}\left(\frac{x-5}{3}\right)$	M1 M1 A1 [3]	Tries to make x subject. Correct order of operations cao

Question 16

(i)	$3x+1 \leqslant -1$ (Accept $3x+1=-1, 3a+1=-1$) $x \leqslant -2/3 \Rightarrow$ largest value of a is $-2/3$ (in terms of a)	**M1** **A1** [2]	Do not allow gf in (i) to score in (iii) Accept $a \leqslant -2/3$ and $a=-2/3$
(ii)	$\text{fg}(x)=3(-1-x^2)+1$ $\text{fg}(x)+14=0 \Rightarrow 3x^2=12$ oe (2 terms) $x=-2$ only	**B1** **B1** **B1** [3]	No marks in this part for gf used
(iii)	$\text{gf}(x)=-1-(3x+1)^2$ oe $\text{gf}(x) \leqslant -50 \Rightarrow (3x+1)^2 \geqslant 49$ (Allow $\leqslant$ *or* $=$ $3x+1 \geqslant 7$ *or* $3x+1 \leq -7$ (one sufficient) www $x \leqslant -8/3$ only www	**B1** **M1** **A1** **A1** [4]	No marks in this part for fg used OR attempt soln of $9x^2+6x-48+/ \leqslant/\geqslant 0$ OR $x-2 \geqslant$ *or* $3x+8 \leqslant 0$(one suffic)

Question 17

(i)	$\text{f}: x \to x^2+ax+b$, $x^2+6x-8=(x+3)^2-17$ or $2x+6=0 \to x=-3 \to y=-17$ $\to$ Range $\text{f}(x) \geqslant -17$	**B1 B1** **B1**√ [3]	B1 for $(x+3)^2$. B1 for -17 or B1 for $x=-3$, B1 $y=-17$ Following through visible method.
(ii)	$(x-k)(x+2k)=0$ $\equiv x^2+5x+b=0$ $\to k=5$ $\to b=-2k^2=-50$	**M1** **A1** **A1** [3]	Realises the link between roots and the equation comparing coefficients of x
(iii)	$(x+a)^2+a(x+a)+b=a$ Uses $b^2-4ac \to 9a^2-4(2a^2+b-a)$ $\to a^2<4(b-a)$	**M1** **DM1** **A1** [3]	Replaces "x" by "$x+a$" in 2 terms Any use of discriminant

Question 18

$\text{f}: x \mapsto 3x+2$, $\text{g}: x \mapsto 4x-12$	**B1**	
$\text{f}^{-1}(x)=\dfrac{x-2}{3}$	**B1**	
$\text{gf}(x)=4(3x+2)-12$	**M1**	Equates, collects terms, +soln
Equate $\to x=\dfrac{2}{7}$	**A1** [4]	

Question 19

(i)	$-(1)(x-3)^2+4$	**B1B1B1** [3]	
(ii)	Smallest (m) is 3	**B1**√ [1]	Accept $m \geqslant 3$, $m = 3$. **Not** $x \geqslant 3$. Ft *their b*
(iii)	$(x-3)^2 = 4-y$	**M1**	Or x/y transposed. Ft *their a, b, c*
	Correct order of operations	**M1**	
	$f^{-1}(x) = 3+\sqrt{4-x}$ cao	**A1**	Accept $y=$ if clear
	Domain is $x \leqslant 0$	**B1** [4]	

Question 20

(i)	$2a+4b=8$	**M1**	Substitute in –2 and –3
	$2a^2+3a+4b=14$	**A1**	
	$2a^2+3a+(8-2a)=14 \rightarrow (a+2)(2a-3)=0$	**M1**	Sub linear into quadratic & attempt solution
	$a=-2$ or $3/2$	**A1**	If A0A0 scored allow SCA1 for either
	$b=3$ or $5/4$	**A1** **[5]**	$(-2,3)$ or (3/2, 5/4)
(ii)	$y=\left(x-\frac{1}{2}\right)^2-\frac{13}{4}$ Attempt completing of square	**M1A1**	Allow with *x*/*y* transposed
	$x-\frac{1}{2}=(\pm)\sqrt{y+\frac{13}{4}}$ oe	**DM1**	Allow with *x*/*y* transposed
	$f^{-1}(x)=\frac{1}{2}-\sqrt{x+\frac{13}{4}}$ oe	**A1**	Allow y =..... Must be a function of x
	Domain of f^{-1} is $(x) \geqslant -13/4$	**B1**√ **[5]**	Allow >, $-13/4 \leqslant x \leqslant \infty$, $\left[-\frac{13}{4},\infty\right]$ etc

Question 21

(i)	$2(ax^2+b)+3=6x^2-21$	**M1**	
	$a=3,\ b=-12$	**A1A1** [3]	
(ii)	$3x^2-12 \geqslant 0$ or $6x^2-21 \geqslant 3$	**M1**	Allow = or $\leqslant$ or > or <. Ft from *their a, b*
	$x \leqslant -2$ i.e. (max) $q=-2$	**A1** [2]	Must be in terms of q (eg $q \leqslant -2$)
(iii)	$y \geqslant 6(-3)^2-21 \Rightarrow$ range is $(y) \geqslant 33$	**B1** [1]	Do not allow $y > 33$. Accept all other notations e.g. [33, ∞) or [33, ∞]

(iv)	$y=6x^2-21 \Rightarrow x=(\pm)\sqrt{\frac{y+21}{6}}$	**M1**	
	$(\mathrm{fg})^{-1}(x)=-\sqrt{\frac{x+21}{6}}$	**A1**	Allow $y=$..... . Must be a function of x
	Domain is $x \geqslant 33$	**B1**√ [3]	ft from *their* part **(iii)** but x essential

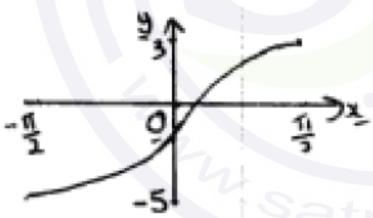
Question 22

	$\mathrm{f}: x \mapsto 6x-x^2-5$		
(i)	$6x-x^2-5 \leqslant 3$		
	$\rightarrow x^2-6x+8 \geqslant 0$	**M1**	$\pm(6x-x^2-8)=,\leqslant,\geqslant 0$ and attempts to solve
	$\rightarrow x=2, x=4$	**A1**	Needs both values whether =2, <2, >2
	$x \leqslant 2, x \geqslant 4$ condone < and/or >	**A1** [3]	Accept all recognisable notation.
(ii)	Equate $mx+c$ and $6x-x^2-5$ Use of "b^2-4ac"	**M1** **DM1**	Equates, sets to 0. Use of discriminant with values of $a.b.c$ independent of x.
	$4c=m^2-12m+16$. **AG**	**A1**	= (0) must appear before last line.
	OR		
	$\frac{dy}{dx}=6-2x=m \rightarrow x=\left(\frac{6-m}{2}\right)$	**M1**	Equates $\frac{dy}{dx}$ to m and rearrange
	$m\left(\frac{6-m}{2}\right)+c=6\left(\frac{6-m}{2}\right)-\left(\frac{6-m}{2}\right)^2-5$	**M1**	Equates $mx+c$ and $6x-x^2-5$ and substitutes for x
	$4c=m^2-12m+16$. **AG**	**A1** [3]	
(iii)	$6x-x^2-5=4-(x-3)^2$	**B1 B1** [2]	4 B1 – $(x-3)^2$ B1
(iv)	$k=3$.	**B1**√ [1]	√ for "b".
(v)	$g^{-1}(x)=\sqrt{4-x}+3$	**M1 A1** [2]	Correct order of operations. $\pm\sqrt{4-x}+3$ M1A0 $\sqrt{x-4}+3$ M1A0 $\sqrt{4-y}+3$ M1A0

Question 23

$\mathrm{f}: x \mapsto 10-3x,\ \mathrm{g}: x \mapsto \dfrac{10}{3-2x}$,		
$\mathrm{ff}(x) = 10-3(10-3x)$	**B1**	Correct unsimplified expression
$\mathrm{gf}(2) = \dfrac{10}{3-2(10-3(2))}\ (=-2)$	**B1**	Correct unsimplified expression with 2 in for x
$x = 2$	**B1** [3]	

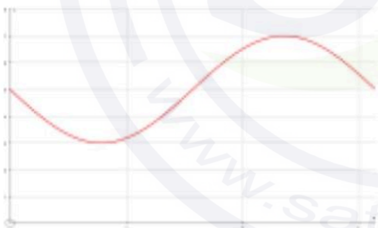
Question 24

(i)	$\mathrm{f}: x \to 4\sin x - 1$ for $-\dfrac{\pi}{2} \leqslant x \leqslant \dfrac{\pi}{2}$	**B1**	–5 and 3
	Range $-5 \leqslant \mathrm{f}(x) \leqslant 3$	**B1** [2]	Correct range
(ii)	$4s-1=0 \to s=\dfrac{1}{4} \to x=0.253$	**M1 A1**	Makes sin$x$ subject. Degrees **M1 A0**, (14.5°)
	$x=0 \to y=-1$	**B1** [3]	
(iii)		**B1**√ **B1** [2]	Shape from their range in (i) Flattens, curve.
(iv)	range $-\dfrac{1}{2}\pi \leqslant \mathrm{f}^{-1}(x) \leqslant \dfrac{1}{2}\pi$	**B1**	
	domain $-5 \leqslant x \leqslant 3$	**B1**√	√ on part (i) (only for 2 numerical values)
	Inverse $\mathrm{f}^{-1}(x) = \sin^{-1}\left(\dfrac{x+1}{4}\right)$	**M1 A1** [4]	Correct order of operations

Question 25

(i)	$(2x+3)^2+1$ Cannot score retrospectively in **(iii)**	**B1B1B1**	[3]	For $a=2,\ b=3,\ c=1$
(ii)	$g(x)=2x+3$ cao	**B1**	[1]	In **(ii)**,**(iii)** Allow if from $4\left(x+\frac{3}{2}\right)^2+1$
(iii)	$y=(2x+3)^2+1 \Rightarrow 2x+3=(\pm)\sqrt{y-1}$ or ft from **(i)**	**M1**		Or with x/y transposed.
	$x=(\pm)\frac{1}{2}\sqrt{y-1}-\frac{3}{2}$ or ft from **(i)**	**M1**		Or with x/y transposed Allow sign errors.
	$(\text{fg})^{-1}(x)=\frac{1}{2}\sqrt{x-1}-\frac{3}{2}$ cao Note alt. method $\text{g}^{-1}\text{f}^{-1}$	**A1**		Must be a function of x. Allow y = ...
	Domain is $(x)>10$	**B1**	[4]	Allow $(10, \infty)$, $10<x<\infty$ etc. but not with y or f or g involved. Not $\geqslant 10$
	ALT. method for first 3 marks:			
	Trying to obtain $g^{-1}\left[f^{-1}(x)\right]$	***M1**		
	$\text{g}^{-1}=½(x-3),\ \text{f}^{-1}=\sqrt{x-1}$	**DM1**		Both required
	A1 for $\frac{1}{2}\sqrt{x-1}-\frac{3}{2}$	**A1**		

Question 26

(i)	$3 \leqslant \text{f}(x) \leqslant 7$	**B1**		Identifying both 3 and 7 or correctly stating one inequality.
		B1	[2]	Completely correct statement. **NB** $3 \leqslant x \leqslant 7$ scores B1B0
(ii)		**B1***		One complete oscillation of a sinusoidal curve between 0 and π.
		DB1	[2]	All correct, initially going downwards, all above $\text{f}(x)=0$
(iii)	5-2sin2x = 6 → sin2x = −½	**M1**		Make sin2x the subject.
	→ $2x=\frac{7\pi}{6}$ or $\frac{11\pi}{6}$			
	→ $x=\frac{7\pi}{12}$ or $\frac{11\pi}{12}$			√ for $\frac{3\pi}{2}$ − 1st answer from sin2x = −½ only, if in given range
	0.583π or 0.917π			
	$\frac{\pi+0.524}{2}$ or $\frac{2\pi-0.524}{2}$	**A1 A1√**		
	1.83^c or 2.88^c		[3]	SR A1A0 for both.
(iv)	$k=\frac{\pi}{4}$	**B1**	[1]	
(v)	2sin2x = 5 − y → sin2x = ½(5 − y)	**M1**		Makes ±sin2x the subject soi by final answer.
		M1		Correct order of operations including correctly dealing with " – ".
	$(\text{g}^{-1}(x)) = ½\sin^{-1}\frac{(5-x)}{2}$	**A1**	[3]	Must be a function of x

Question 27

(i)	fg(x) = $5x$ Range of fg is $y \geqslant 0$ oe	**M1A1** **B1**	[3]	only Accept $y > 0$
(ii)	$y = 4/(5x+2) \Rightarrow x = (4-2y)/5y$ oe $g^{-1}(x) = (4-2x)/5x$ oe 0, 2 with no incorrect inequality $0 < x \leqslant 2$ oe, c.a.o.	**M1** **A1** **B1,B1** **B1**	[5]	Must be a function of x

Question 28

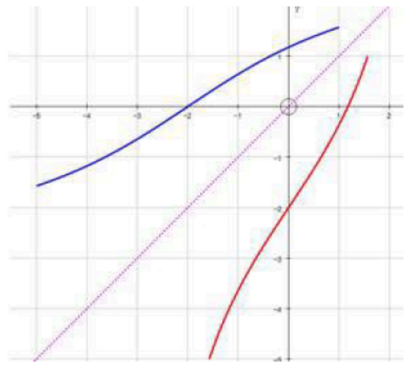
3(i)	gf(x) = $3(2x^2+3)+2 = 6x^2+11$	B1	AG
	fg(x) $= 2(3x+2)^2+3$ Allow $18x^2+24x+11$	B1	ISW if simplified incorrectly. Not retrospectively from **(ii)**
	Total:	**2**	
(ii)	$y = 2(3x+2)^2+3 \quad \Rightarrow 3x+2 = (\pm)\sqrt{(y-3)/2}$ oe	M1	Subtract 3;divide by 2;square root. Or x/y interchanged. Allow $\frac{\sqrt{y-3}}{2}$ for 1st M
	$\Rightarrow x = (\pm)\frac{1}{3}\sqrt{(y-3)/2} - \frac{2}{3}$ oe	M1	Subtract 2; divide by 3; Indep. of 1st M1. Or x/y interchanged.
	$\Rightarrow (\text{fg})^{-1}(x) = \frac{1}{3}\sqrt{(x-3)/2} - \frac{2}{3}$ oe	A1	Must be a function of x. Allow alt. method $g^{-1}f^{-1}(x)$ OR $18\left(x+\frac{2}{3}\right)^2+3 \Rightarrow (\text{fg})^{-1}(x) = \sqrt{\frac{x-3}{18}} - \frac{2}{3}$
	Solve *their* $(\text{fg})^{-1}(x) \geqslant 0$ or attempt range of fg	M1	Allow <u>range</u> $\geqslant 3$ for M only. Can be implied by correct answer or $x > 11$
	Domain is $x \geqslant 11$	A1	
	Total:	**5**	
3(iii)	$6(2x)^2+11 = 2(3x+2)^2+3$	M1	Replace x with $2x$ in gf and equate to *their* fg(x) from (i). Allow <u>12</u> $x^2+11=$
	$6x^2-24x=0$ oe	A1	Collect terms to obtain correct quadratic expression.
	$x = 0$, 4	A1	Both required
	Total:	**3**	

Question 29

(i)	$(3x-1)^2+5$	B1B1B1	First 2 marks dependent on correct $(ax+b)^2$ form. OR $a=3,\ b=-1,\ c=5$ e.g. from equating coefs
	Total:	**3**	
(ii)	Smallest value of p is 1/3 seen. (Independent of (i))	**B1**	Allow $p\geqslant 1/3$ or $p=1/3$ or 1/3 seen. But not in terms of x.
	Total:	**1**	
(iii)	$y=(3x-1)^2+5 \Rightarrow 3x-1=(\pm)\sqrt{y-5}$	**B1 FT**	OR $y=9\left(x-\frac{1}{3}\right)^2+5 \Rightarrow (y-5)/9=\left(x-\frac{1}{3}\right)^2$ (Fresh start)
	$x=(\pm)\frac{1}{3}\sqrt{y-5}+\frac{1}{3}$ OE	**B1 FT**	Both starts require 2 operations for each mark. FT for *their* values from part (i)
	$f^{-1}(x)=\frac{1}{3}\sqrt{x-5}+\frac{1}{3}$ OE domain is $x\geq$ *their*5	**B1B1 FT**	Must be a function of x and $\pm$ removed. Domain must be in terms of x. Note: $\sqrt{y-5}$ expressed as $\sqrt{y}-\sqrt{5}$ scores Max **B0B0B0B1** [See below for general instructions for different starts]
	Total:	**4**	
(iv)	$q<5$ CAO	**B1**	
	Total:	**1**	

9(iii) For start $(ax-b)^2+c$ or $a(x-b)^2+c$ $(a\neq 0)$ ft for their a, b, c
For start $(x-b)^2+c$ ft but award only **B1** for 3 correct operations
For start $a(bx-c)^2+d$ ft but award **B1** for first2 operations correct and **B1** for the next 3 operations correct

Question 30

(i)	$3\tan\left(\frac{1}{2}x\right)=-2 \rightarrow \tan\left(\frac{1}{2}x\right)=-\frac{2}{3}$	**M1**	Attempt to obtain $\tan\left(\frac{1}{2}x\right)=k$ from $3\tan\left(\frac{1}{2}x\right)+2=0$
	$\frac{1}{2}x=-0.6\ (-0.588) \rightarrow x=-1.2$	**M1 A1**	$\tan^{-1}k$. Seeing $\frac{1}{2}x=-33.69°$ or $x=-67.4°$ implies **M1M1**.
			Extra answers between -1.57 &1.57 lose the **A1**. Multiples of π are acceptable (eg -0.374π)
	Total:	**3**	
(ii)	$\frac{y+2}{3}=\tan\left(\frac{1}{2}x\right)$	**M1**	Attempt at isolating $\tan(\frac{1}{2}x)$
	$\rightarrow f^{-1}(x)=2\tan^{-1}\left(\frac{x+2}{3}\right)$	**M1 A1**	Inverse tan followed by × 2. Must be function of x for **A1**.
	$-5,1$	**B1 B1**	Values stated **B1** for -5, **B1** for 1.
	Total:	**5**	
(iii)		**B1 B1 B1**	A tan graph through the first, third and fourth quadrants. (**B1**) An invtan graph through the first, second and third quadrants.(**B1**) Two curves clearly symmetrical about $y=x$ either by sight or by exact end points. Line not required. Approximately in correct domain and range. (Not intersecting.) (**B1**) Labels on axes not required.

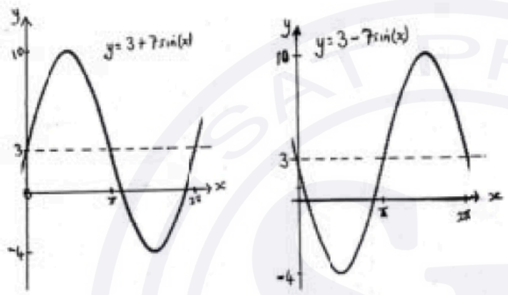
Question 31

9	$f: x \mapsto \frac{2}{3-2x}$ $g: x \mapsto 4x + a,$		
(i)	$y = \frac{2}{3-2x} \to y(3-2x) = 2 \to 3-2x = \frac{2}{y}$	M1	Correct first 2 steps
	$\to 2x = 3 - \frac{2}{y} \to f^{-1}(x) = \frac{3}{2} - \frac{1}{x}$	M1 A1	Correct order of operations, any correct form with $f(x)$ or $y =$
	Total:	3	
(ii)	$gf(-1) = 3$ $f(-1) = \frac{2}{5}$	M1	Correct first step
	$\frac{8}{5} + a = 3 \to a = \frac{7}{5}$	M1 A1	Forms an equation in a and finds a, OE
			(or $\frac{8}{3-2x} + a = 3$, **M1** Sub and solves **M1**, **A1**)
	Total:	3	
(iii)	$g^{-1}(x) = \frac{x-a}{4} = f^{-1}(x)$	M1	Finding $g^{-1}(x)$ and equating to their $f^{-1}(x)$ even if $a = 7/5$
	$\to x^2 - x(a+6) + 4(=0)$	M1	Use of $b^2 - 4ac$ on a quadratic with a in a coefficient
	Solving $(a+6)^2 = 16$ or $a^2 + 12a + 20(=0)$	M1	Solution of a 3 term quadratic
	$\to a = -2$ or -10	A1	
	Total:	4	

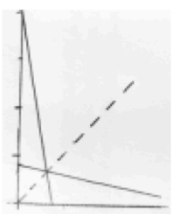
Question 32

5(i)	$y = \frac{2}{x^2-1} \Rightarrow x^2 = \frac{2}{y} + 1$ OE	M1	
	$x = (\pm)\sqrt{\frac{2}{y} + 1}$ OE	A1	With or without x/y interchanged.
	$f^{-1}(x) = -\sqrt{\frac{2}{x} + 1}$ OE	A1	Minus sign obligatory. Must be a function of x.
		3	
5(ii)	$\left(\frac{2}{x^2-1}\right)^2 + 1 = 5$	B1	
	$\frac{2}{x^2-1} = (\pm)2$ OE OR $x^4 - 2x^2 = 0$ OE $x^2 - 1 = (\pm)1 \Rightarrow x^2 = 2$ (or 0) $x = -\sqrt{2}$ or -1.41 only	B1	Condone $x^2 = 0$ as an additional solution
		4	

Question 34

(a)(i)	$4 = a + ½b$ $3 = a + b$	**M1**	Forming simultaneous equations and eliminating one of the variables – probably a. May still include $\sin\frac{\pi}{2}$ and / or $\sin\frac{\pi}{6}$
	$\rightarrow a = 5,\ \ b = -2$	**A1 A1**	
		3	
(a)(ii)	$ff(x) = a + b\sin(a + b\sin x)$	**M1**	Valid method for ff. Could be f(0) = N followed by f(N) = M.
	$ff(0) = 5 - 2\sin 5\ = 6.92$	**A1**	
6(b)	*EITHER:* $10 = c + d$ *and* $-4 = c - d$ $10 = c - d$ *and* $-4 = c + d$	**(M1**	Either pair of equations stated.
	$c = 3, d = 7$, -7 or ±7	**A1 A1)**	Either pair solved ISW **Alternately** c=3 **B1**, range = 14 **M1**→ $d = 7$, –7 or ±7 **A1**
	OR:  	**(M1 A1 A1)**	Either of these diagrams can be awarded M1.Correct values of c and/or d can be awarded the A1, A1
		3	

Question 35

(i)	$\frac{4-x}{5}$	**B1**	OE
	Equate a valid attempt at f^{-1} with f, or with x, or f with x $\rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$ **or** (0.667, 0.667)	**M1, A1**	Equating and an attempt to solve as far x =. Both coordinates.
		3	
(ii)		**B1**	Line $y = 4 - 5x$ – must be straight, through approximately (0,4) and intersecting the positive x axis near (1,0) as shown.
		B1	Line $y = \frac{4-x}{5}$ – must be straight and through approximately (0, 0.8). No need to see intersection with x axis.
		B1	A line through (0,0) and the point of intersection of a pair of straight lines with negative gradients. This line must be at 45° unless scales are different in which case the line must be labelled y=x.
		3	

Question 36

(i)	gg(x) = g($2x-3$) = $2(2x-3)-3 = 4x-9$	**M1A1**	
		2	
(ii)	$y=\frac{1}{x^2-9} \to x^2=\frac{1}{y}+9$ OE	**M1**	Invert; add 9 to both sides or with x/y interchanged
	$f^{-1}(x)=\sqrt{\frac{1}{x}+9}$	**A1**	
	Attempt soln of $\sqrt{\frac{1}{x}+9}>3$ or attempt to find range of f. $(y>0)$	**M1**	
	Domain is $x>0$ CAO	**A1**	May simply be stated for **B2**
		4	
(iii)	*EITHER:* $\frac{1}{(2x-3)^2-9}=\frac{1}{7}$	**(M1**	
	$(2x-3)^2=16$ or $4x^2-12x-7=0$	**A1**	
	x = 7/2 or –1/2	**A1**	
	x = 7/2 only	**A1)**	
	OR: g(x) = $f^{-1}\left(\frac{1}{7}\right)$	**(M1**	
	g(x) = 4	**A1**	
	$2x-3=4$	**A1**	
	x = 7/2	**A1)**	
		4	

Question 37

(i)(a)	f(x) > 2	**B1**	Accept y > 2, (2, ∞), (2, ∞], *range* > 2
		1	
(i)(b)	g(x) > 6	**B1**	Accept y > 6, (6, ∞), (6, ∞], *range* > 6
		1	
(i)(c)	2 < fg(x) < 4	**B1**	Accept 2 < y <4, (2, 4), 2 < *range* < 4
		1	
(ii)	The range of f is (partly) outside the domain of g	**B1**	
		1	

Question 38

)(i)	Smallest value of c is 2. Accept 2, $c = 2$, $c \geqslant 2$. Not in terms of x	B1	Ignore superfluous working, e.g. $\frac{d^2y}{dx^2} = 2$
		1	
)(ii)	$y = (x-2)^2 + 2 \to x - 2 = (\pm)\sqrt{y-2} \to x = (\pm)\sqrt{y-2} + 2$	M1	Order of operations correct. Allow sign errors
	$f^{-1}(x) = \sqrt{x-2} + 2$	A1	Accept $y = \sqrt{x-2} + 2$
	Domain of f^{-1} is $x \geqslant 6$. Allow $\geqslant 6$.	B1	Not $f^{-1}(x) \geqslant 6$. Not $f(x) \geqslant 6$. Not $y \geqslant 6$
		3	
(iii)	$\left[(x-2)^2 + 2 - 2\right]^2 + 2 = 51$ SOI Allow 1 term missing for M mark Or $(x^2 - 4x + 6)^2 - 4(x^2 - 4x + 6) + 6 = 51$	M1A1	ALT. $f(x) = f^{-1}(51)$ (M1)$= \sqrt{51-2} + 2$ (A1)
	$(x-2)^4 = 49$ or $(x^2 - 4x + 4)^2 = 49$ OR $x^4 - 8x^3 + 24x^2 - 32x - 33 = 0$ often implied by next line	A1	$(x-2)^2 + 2 = \sqrt{49} + 2$ OR $f(x) = 9$
	$(x-2)^2 = (\pm)7$ OR $x^2 - 4x - 3 = 0$. Ignore $x^2 - 4x + 11 = 0$	A1	$(x-2)^2 = 7$ OR $x = f^{-1}(9)$
	$x = 2 + \sqrt{7}$ only CAO $x = 2 + \sqrt[4]{49}$ scores 3/5	A1	$x = 2 + \sqrt{7}$
		5	

Question 39

'(i)	$25 - 2(x+3)^2$	B1 B1	Mark expression if present: B1 for 25 and B1 for $-2(x+3)^2$. If no expression award $a = 25$ B1 and $b = 3$ B1.
		2	
(ii)	$(-3, 25)$	B1FT	FT from answers to (i) or by calculus
		1	
(iii)	$(k) = -3$ also allow x or $k \geqslant -3$	B1FT	FT from answer to (i) or (ii) **NOT** $x = -3$
		1	
(iv)	**EITHER**		
	$y = 25 - 2(x+3)^2 \to 2(x+3)^2 = 25 - y$	*M1	Makes their squared term containing x the subject or equivalent with x/y interchanged first. Condone errors with +/- signs.
	$x + 3 = (\pm)\sqrt{½(25-y)}$	DM1	Divide by ±2 and then square root allow ±.
	OR		
	$y = 7 - 2x^2 - 12x \to 2x^2 + 12x + y - 7 (= 0)$	*M1	Rearranging equation of the curve.
	$x = \frac{-12 \pm \sqrt{12^2 - 8(y-7)}}{4}$	DM1	Correct use of their 'a, b and c' in quadratic formula. Allow just + in place of ±.
	$g^{-1}(x) = \sqrt{\left(\frac{25-x}{2}\right)} - 3$ oe isw if substituting $x = -3$	A1	± gets A0. Must now be a function of x. Allow $y =$
		3	

Question 40

9	$f : x \mapsto \frac{x}{2} - 2, \quad g : x \mapsto 4 + x - \frac{x^2}{2}$		
9(i)	$4 + x - \frac{x^2}{2} = \frac{x}{2} - 2 \rightarrow x^2 - x - 12 = 0$	M1	Equates and forms 3 term quadratic
	$\rightarrow$ (4, 0) and (−3, −3.5) Trial and improvement, B3 all correct or B0	A1 A1	A1 For both x values or a correct pair. A1 all.
		3	
9(ii)	$f(x) > g(x)$ for $x > 4, x < -3$	B1, B1	B1 for each part. Loses a mark for $\leqslant$ or $\geqslant$.
		2	
9(iii)	$fg(x) = 2 + \frac{x}{4} - \frac{x^2}{4} - 2 \; (= \frac{x}{2} - \frac{x^2}{4})$	B1	CAO, any correct form
	i.e. $-\frac{1}{4}((x-1)^2 - 1)$ or $\frac{dy}{dx} = \frac{1}{2} - \frac{2x}{4} = 0 \rightarrow x = 1$	M1 A1	Completes the square or uses calculus. First A1 is for $x = 1$ or completed square form
	$\rightarrow y = \frac{1}{4} \rightarrow$ Range of fg $\leqslant \frac{1}{4}$,	A1	CAO, OE e.g. $y \leqslant \frac{1}{4}$, $[-\infty, \frac{1}{4})$ etc.
		4	
9(iv)	Calculus or completing square on 'h' $\rightarrow x = 1$	M1	May use a sketch or $-\frac{b}{2a}$
	$k = 1$ (accept $k \geqslant 1$)	A1	Complete method. CAO
		2	

Question 41

(i)	$[2]\left[(x-3)^2\right][-7]$	B1B1B1	
		3	
(ii)	Largest value of k is 3. Allow ($k =$) 3.	B1	Allow $k \leqslant 3$ but not $x \leqslant 3$ as final answer.
		1	
1(iii)	$y = 2(x-3)^2 - 7 \rightarrow (x-3)^2 = ½(y+7)$ or with x/y transposed	M1	Ft *their* a, b, c. Order of operations correct. Allow sign errors
	$x = 3 \pm \sqrt{½(y+7)}$ Allow 3 + √ or 3 −√ or with x/y transposed	DM1	Ft *their* a, b, c. Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{½(x+7)}$	A1	
	(Domain is x) $\geqslant$ *their* −7	B1FT	Allow other forms for interval but if variable appears must be x
		4	
1(iv)	$x + 3 \leqslant 1$. Allow $x + 3 = 1$	M1	Allow $x + 3 \leqslant k$
	largest p is −2 . Allow ($p =$) −2	A1	Allow $p \leqslant -2$ but not $x \leqslant -2$ as final answer.
	$fg(x) = f(x+3) = 2x^2 - 7$ cao	B1	
		3	

Question 42

(i)	$2x^2 - 12x + 7 = 2(x-3)^2 - 11$	B1 B1	Mark full expression if present: B1 for $2(x-3)^2$ and B1 for – 11. If no clear expression award $a = -3$ and $b = -11$.
		2	
(ii)	Range (of f or y) $\geqslant$ 'their – 11'	B1FT	FT for their 'b' or start again. Condone $>$. Do NOT accept $x >$ or $\geqslant$
		1	
(iii)	(k =) –"their a" also allow x or $k \leqslant 3$	B1FT	FT for their "a" or start again using $\frac{dy}{dx} = 0$. Do NOT accept $x = 3$.
		1	
(iv)	$y = 2(x-3)^2 - 11 \rightarrow y + 11 = 2(x-3)^2$ $\frac{y+11}{2} = (x-3)^2$	*M1	Isolating their $(x-3)^2$, condone – 11.
	$x = 3 + \sqrt{\left(\frac{y+11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y+11}{2}\right)}$	DM1	Other operations in correct order, allow ± at this stage. Condone – 3.
	($g^{-1}(x)$ or y) = $3 - \sqrt{\left(\frac{x+11}{2}\right)}$	A1	needs '–'. x and y could be interchanged at the start.
		3	

Question 43

(a)(i)	[Greatest value of a is] 3	B1	Must be in terms of a. Allow $a < 3$. Allow $a \leqslant 3$
		1	
(a)(ii)	Range is $y > -1$	B1	Ft on *their a*. Accept any equivalent notation
	$y = (x-3)^2 - 1 \rightarrow (x-3)^2 = 1 + y \rightarrow x = 3(\pm)\sqrt{1+y}$	M1	Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{1+x}$ cao	A1	
		3	
(b)(i)	$gg(2x) = \left[(2x-3)^2 - 3\right]^2$	B1	
	$(2x-3)^4 - 6(2x-3)^2 + 9$	B1	
		2	
(b)(ii)	$\left[16x^4 - 96x^3 + 216x^2 - 216x + 81\right] + \left[\left(-24x^2 + 72x - 54\right) + 9\right]$ $16x^4 - 96x^3 + 192x^2 - 144x + 36$	B4,3,2,1,0	
		4	

Question 44

(i)	$\left[(x-2)^2\right]+[3]$	B1 DB1	2nd B1 dependent on ±2 in 1st bracket
		2	
(ii)	Largest k is 2 Accept $k \leqslant 2$	B1	Must be in terms of k
		1	
(iii)	$y=(x-2)^2+3 \Rightarrow x-2=(\pm)\sqrt{y-3}$	M1	
	$\Rightarrow f^{-1}(x)=2-\sqrt{x-3}$ for $x>4$	A1B1	
		3	
(iv)	$gf(x)=\dfrac{2}{x^2-4x+7-1}=\dfrac{2}{(x-2)^2+2}$	B1	Either form
	Since $f(x)>4 \Rightarrow gf(x)<2/3$ (or since $x<1$etc)	M1A1	2/3 in answer implies M1 www
	range of gf(x) is $0<gf(x)(<2/3)$	B1	Accept $0<y<2/3$, (0, 2/3) but $0<x<2/3$ is SCM1A1B0
		4	

Question 45

(i)	$\left[(x-2)^2\right]$ [+4]	B1 DB1	2nd B1 dependent on 2 inside bracket
		2	
(ii)	$(x-2)^2<5 \rightarrow -\sqrt{5}<x-2$ and/or $x-2<\sqrt{5}$	M1	Allow e.g. $x-2<\pm\sqrt{5}$, $x-2=\pm\sqrt{5}$ and decimal equivalents for √5 For M1, ft from *their*(i). Also allow √13 instead of √5 for clear slip
	$2-\sqrt{5}<x<2+\sqrt{5}$	A1A1	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x. Non-hence methods, if completely correct, score SC 1/3. Condone $\leqslant$
		[3]	

Question 46

(i)	Max(*a*) is 8	B1	Allow $a=8$ or $a\leqslant 8$
	Min(*b*) is 24	B1	Allow $b=24$ or $b\geqslant 24$
		2	SCB1 for 8 and 24 seen
(ii)	$gf(x)=\dfrac{96}{x-1}-4$ or $gf(x)=\dfrac{100-4x}{x-1}$	B1	$2\left(\dfrac{48}{x-1}\right)-4$ is insufficient Apply ISW
		1	
(iii)	$y=\dfrac{96}{x-1}-4 \rightarrow y+4=\dfrac{96}{x-1} \rightarrow x-1=\dfrac{96}{y+4}$	M1	FT from *their*(ii) provided (ii) involves algebraic fraction. Allow sign errors
	$(gf)^{-1}(x)=\dfrac{96}{x+4}+1$	A1	OR $\dfrac{100+x}{x+4}$. Must be a function of x. Apply ISW
		2	

Question 47

(i)	$(f^{-1}(x)) = \frac{x+2}{3}$ oe	B1	
	$y = \frac{2x+3}{x-1} \to (x-1)y = 2x+3 \to x(y-2) = y+3$	M1	Correct method to obtain $x =$, (or $y =$, if interchanged) but condone $+/-$ sign errors
	$(g^{-1}(x)$ or $y) = \frac{x+3}{x-2}$ oe $\left(eg \frac{5}{x-2}+1\right)$	A1	Must be in terms of x
	$x \neq 2$ only	B1	**FT** for value of x from their denominator = 0
		4	
(ii)	$(fg(x) =) \frac{3(2x+3)}{x-1} - 2 (= \frac{7}{3})$	B1	
	$18x + 27 = 13x - 13$ or $3(4x+11) = 7(x-1)$ $(5x = -40)$	M1	Correct method from their $fg = \frac{7}{3}$ leading to a linear equation and collect like terms. Condone omission of $2(x-1)$.
	Alternative method for question 7(ii)		
	$(f^{-1}(\frac{7}{3})) = \frac{13}{9}$	B1	
	$\frac{2x+3}{x-1} = \frac{13}{9} \to 9(2x+3) = 13(x-1)$ $(\to 5x = -40)$	M1	Correct method from $g(x) =$ their $\frac{13}{9}$ leading to a linear equation and collect like terms.
	$x = -8$	A1	
		3	

Question 48

(i)	$-2(x-3)^2 + 15$ $(a = -3, b = 15)$	B1 B1	Or seen as $a = -3$, $b = 15$ B1 for each value
		2	
(ii)	$(f(x) \leqslant)$ 15	B1	**FT** for $(\leqslant)$ their "b" Don't accept (3,15) alone
		1	
(iii)	$gf(x) = 2(-2x^2 + 12x - 3) + 5 = -4x^2 + 24x - 6 + 5$	B1	
	$gf(x) + 1 = 0 \to -4x^2 + 24x = 0$	M1	
	$x = 0$ or 6	A1	Forms and attempts to solve a quadratic Both answers given.
		3	

Question 49

$(y =)\left[(x-3)^2\right][-2]$	*B1 DB1	DB1 dependent on 3 in 1st bracket
$x - 3 = (\pm)\sqrt{y+2}$ **or** $y - 3 = (\pm)\sqrt{x+2}$	M1	Correct order of operations
$(g^{-1}(x)) = 3 + \sqrt{x+2}$	A1	Must be in terms of x
Domain (of g^{-1}) is $(x) > -1$	B1	Allow $(-1, \infty)$. Do not allow $y > -1$ or $g(x) > -1$ or $g^{-1}(x) > -1$
	5	

Question 50

(i)	$f(x) = g(x) \rightarrow 2x^2 + 6x + 1 + k \ (= 0)$	*M1	Forms a quadratic with all terms on same side.
	Use of $b^2 = 4ac$	DM1	Uses the discriminant = 0.
	$(k =)\ 3\frac{1}{2}$	A1	OE, WWW
	Alternative method for question 9(i)		
	$4x + 8 = 2\ (\rightarrow x = -1\frac{1}{2})$	*M1	Differentiating, equating gradients and solving to give $x =$
	Substitutes *their* x value into either $2x^2 + 6x + 1 + k = 0$ OR into the curve to find $y\left(= \frac{-13}{2}\right)$ then both values into the line.	DM1	Substituting appropriately for *their* x and proceeding to find a value of k.
	$(k =)\ 3\frac{1}{2}$	A1	OE, WWW
		3	
(ii)	$2x^2 + 6x - 8\ (< 0)$	M1	Forms a quadratic with all terms on same side
	-4 and 1	A1	
	$-4 < x < 1$	A1	CAO
		3	
(iii)	$(g^{-1}(x)) = \frac{x-1}{2}$	B1	Needs to be in terms of x.
	$(g^{-1}f(x)) = \frac{2x^2 + 8x + 1 - 1}{2} = 0 \rightarrow (2x^2 + 8x = 0) \rightarrow x =$	M1	Substitutes f into g^{-1} and attempts to solve it = 0 as far as $x =$
	$0, -4$	A1	CAO
		3	

(iv)	$2(x + 2)^2 - 7$	B1 B1	or $a = +2$, $b = -7$
	(Least value of $f(x)$ or $y =$) -7 or $\geqslant -7$	B1FT	FT for *their* b from a correct form of the expression.
		3	

Question 51

(i)	Range of f is $0 < f(x) < 3$	B1B1	OE. Range cannot be defined using x
	Range of g is $g(x) > 2$	B1	OE
		3	
(ii)	$(fg(x) =) \frac{3}{2\left(\frac{1}{x} + 2\right) + 1} = \frac{3x}{2 + 5x}$	B1B1	Second B mark implies first B mark
		2	
(iii)	$y = \frac{3x}{2 + 5x} \rightarrow 2y + 5xy = 3x \rightarrow 3x - 5xy = 2y$	M1	Correct order of operations
	$x(3 - 5y) = 2y \rightarrow x = \frac{2y}{3 - 5y}$	M1	Correct order of operations
	$\left((fg)^{-1}(x)\right) = \frac{2x}{3 - 5x}$	A1	
		3	

Question 52

[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
[Translation or Shift] [1 unit in y direction] or [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
	4	

Question 53

9(a)	$\left[2(x+3)^2\right][-7]$	B1B1	Stating $a=3, b=-7$ gets B1B1
		2	
9(b)	$y=2(x+3)^2-7 \to 2(x+3)^2=y+7 \to (x+3)^2=\frac{y+7}{2}$	M1	First 2 operations correct. Condone sign error or with x/y interchange
	$x+3=(\pm)\sqrt{\frac{y+7}{2}} \to x=(\pm)\sqrt{\frac{y+7}{2}}-3 \to \mathrm{f}^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$	A1FT	FT on *their* a and b. Allow $y=\ldots$
	Domain: $x \geqslant -5$ or $\geqslant$ -5 or $[-5, \infty)$	B1	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
		3	
9(c)	fg(x) = $8x^2-7$	B1FT	SOI. FT on *their* –7 from part **(a)**
	$8x^2-7=193 \to x^2=25 \to x=-5$ only	B1	
	Alternative method for question 9(c)		
	g(x) = $\mathrm{f}^{-1}(193) \to 2x-3=-\sqrt{100}-3$	M1	FT on *their* $\mathrm{f}^{-1}(x)$
	$x=-5$ only	A1	
		2	
9(d)	(Largest k is) $-\frac{1}{2}$	B1	Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$
		1	

Question 54

3(a)	$(y)=\mathrm{f}(-x)$	B1
		1
3(b)	$(y)=2\mathrm{f}(x)$	B1
		1
3(c)	$(y)=\mathrm{f}(x+4)-3$	B1 B1
		2

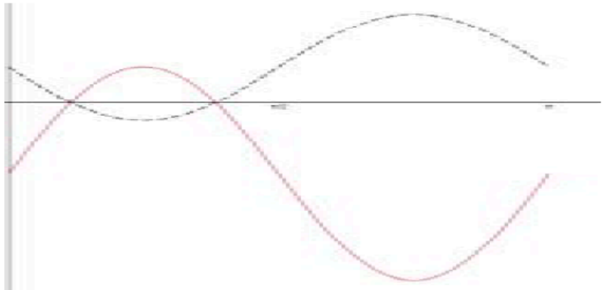
Question 55

(a)	$\left[(x-2)^2\right]$ $[-1]$	B1 B1
		2
(b)	Smallest $c = 2$ (**FT** on *their* part **(a)**)	B1FT
		1
(c)	$y=(x-2)^2-1 \rightarrow (x-2)^2 = y+1$	*M1
	$x = 2(\pm)\sqrt{y+1}$	DM1
	$(\mathrm{f}^{-1}(x)) = 2+\sqrt{x+1}$ for $x > 8$	A1
		3
(d)	$\mathrm{gf}(x) = \dfrac{1}{(x-2)^2-1+1} = \dfrac{1}{(x-2)^2}$ OE	B1
	Range of gf is $0 < \mathrm{gf}(x) < \dfrac{1}{9}$	B1 B1
		3

Question 56

(a)	$\mathrm{ff}(x) = a-2(a-2x)$	M1
	$\mathrm{ff}(x) = 4x-a$	A1
	$\mathrm{f}^{-1}(x) = \dfrac{a-x}{2}$	M1 A1
		4
(b)	$4x-a = \dfrac{a-x}{2} \rightarrow 9x = 3a$	M1
	$x = \dfrac{a}{3}$	A1
		2

Question 57

(a)	f(x) from −1 to 5	**B1B1**
	g(x) from −10 to 2 (**FT** from part (**a**))	**B1FT**
		3
(b)		**B2, 1**
		2
(c)	Reflect in x-axis	**B1**
	Stretch by factor 2 in the y direction	**B1**
	Translation by $-\pi$ in the x direction OR translation by $\begin{pmatrix} 0 \\ -\pi \end{pmatrix}$.	**B1**
		3

Question 58

(a)	$3(3x+b)+b = 9x+4b \rightarrow 10 = 18+4b$	**M1**
	$b = -2$	**A1**
	Either f(14) = 2 **or** $f^{-1}(x) = 2(x+a)$ etc.	**M1**
	$a = 5$	**A1**
		4
(b)	$gf(x) = 3\left(\frac{1}{2}x-5\right)-2$	**M1**
	$gf(x) = \frac{3}{2}x-17$	**A1**
		2

Question 59

(a)	$-1 \leqslant f(x) \leqslant 2$	**B1 B1**
		2
(b)	$k = 1$	**B1**
	Translation by 1 unit upwards parallel to the y-axis	**B1**
		2
(c)	$y = -\frac{3}{2}\cos 2x - \frac{1}{2}$	**B1**
		1

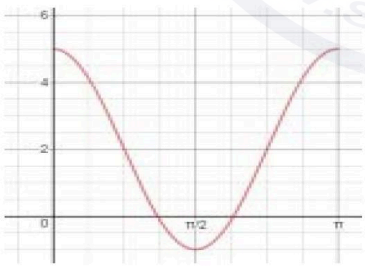
Question 60

(a)	$y = \frac{2x}{3x-1} \rightarrow 3xy - y = 2x \rightarrow 3xy - 2x = y$ (or $-y = 2x - 3xy$)	*M1	For 1st two operations. Condone a sign error
	$x(3y-2) = y \rightarrow x = \frac{y}{3y-2}$ (or $x = \frac{-y}{2-3y}$)	DM1	For 2nd two operations. Condone a sign error
	$\left(f^{-1}(x)\right) = \frac{x}{3x-2}$	A1	Allow $\left(f^{-1}(x)\right) = \frac{-x}{2-3x}$
		3	
(b)	$\left[\frac{2(3x-1)+2}{3(3x-1)}\right] = \left[\frac{6x}{3(3x-1)} = \frac{2x}{3x-1}\right]$	B1 B1	AG, WWW First B1 is for a correct single unsimplified fraction. An intermediate step needs to be shown. Equivalent methods accepted.
		2	
(c)	$(f(x)) > \frac{2}{3}$	B1	Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$
		1	

Question 61

(a)	$\left[(x+3)^2\right]$ $[-4]$	B1 B1	
		2	
(b)	[Translation or shift] $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$	B1 B1 FT	Accept [translation/shift] $\begin{pmatrix} -their\ a \\ their\ b \end{pmatrix}$ OR translation –3 units in x-direction and (translation) –4 units in y-direction.
		2	

Question 62

(a)	5, –1	B1 B1	Sight of each value
		2	
(b)		*B1	Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value.
		DB1	One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end.
		2	

(c)(i)	0 solution	B1	
		1	
(c)(ii)	2 solutions	B1	
		1	
(c)(iii)	1 solution	B1	
		1	
(d)	Stretch by (scale factor) $\frac{1}{2}$, parallel to x-axis or in x direction (or horizontally)	B1	
	Translation of $\begin{pmatrix}0\\4\end{pmatrix}$	B1	Accept translation/shift Accept translation 4 units in positive y-direction.
		2	
(e)	Translation of $\begin{pmatrix}-\frac{\pi}{2}\\0\end{pmatrix}$	B1	Accept translation/shift Accept translation $-\frac{\pi}{2}$ units in x-direction.
	Stretch by (scale factor) 2 parallel to y-axis (or vertically).	B1	
		2	

Question 63

(a)	0	B1	
		1	
(b)	$(f^{-1}(x)) = \frac{x+2}{4}$, $(g^{-1}(x)) = \frac{4-x}{x}$ or $\frac{4}{x}-1$	B1 B1	OE. Sight of correct inverses.
	$x^2 + 6x - 16 (= 0)$	B1	Equating inverses and simplifying.
	$(x+8)$ and $(x-2)$	M1	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
	$(x =)$ 2 or -8	A1	Do not accept answers obtained with no method shown.
		5	

Question 64

(a)	$fg(x) = (2x+1)^2 + 3$	B1	OE
		1	
(b)	$y = (2x+1)^2 + 3 \rightarrow 2x+1 = (\pm)\sqrt{y-3}$	M1	1st two operations. Allow one sign error or x/y interchanged
	$x = (\pm)\frac{1}{2}\left(\sqrt{y-3} \ -1\right)$	M1	OE 2nd two operations. Allow one sign error or x/y interchanged
	$\left(fg^{-1}(x) =\right) \frac{1}{2}\left(\sqrt{x-3} \ -1\right)$ for $(x) > 3$	A1 B1	Allow $(3, \infty)$
		4	
(c)	$gf(x) = 2(x^2+3)+1$	B1	SOI
	$(2x+1)^2 + 3 - 3 = 2(x^2+3)+1 \rightarrow 2x^2 + 4x - 6 \ (= 0)$	*M1	Express as 3-term quadratic
	$(2)(x+3)(x-1) \ (= 0)$	DM1	Or quadratic formula or completing the square
	$x = 1$	A1	
		4	

Question 65

$(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$	**B1 B1 B1**	
	3	

Question 66

(a)	$\left[\mathrm{f}(x)=\right](x+1)^2+2$	**B1 B1**	Accept $a=1, b=2$.
	Range [of f is (y)] $\geqslant 2$	**B1FT**	OE. Do not allow $x \geqslant 2$, FT on *their b*.
		3	
(b)	$y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$	**M1**	Or by using the formula. Allow one sign error.
	$\mathrm{f}^{-1}(x)=-\sqrt{x-2}-1$	**A1**	
		2	
(c)	$2\left(x^2+2x+3\right)+1=13$	**B1**	Or using a correct completed square form of f(x).
	$2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$	**B1**	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
	$x=-3$ **only**	**B1**	
		3	

Question 67

(a)	(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
	(Translation) $\begin{pmatrix}4\\0\end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
		4	
(b)	[y =] 3f(x – 4)	**B1 B1**	B1 for 3 , B1 for (x – 4) with no extra terms.
		2	

Question 68

(a)	$[\text{fg}(x)=]1/(2x+1)^2-1$	B1	SOI
	$1/(2x+1)^2-1=3$ leading to $4(2x+1)^2=1$ or $\frac{1}{(2x+1)}=[\pm]2$ or $16x^2+16x+3=0$	M1	Setting $\text{fg}(x)=3$ and reaching a stage before $2x+1=\pm½$ or reaching a 3 term quadratic in x
	$2x+1=\pm½$ or $2x+1=-½$ or $(4x+1)(4x+3)[=0]$	A1	Or formula or completing square on quadratic
	$x=-\frac{3}{4}$ only	A1	
	Alternative method for Question 8(a)		
	$x^2-1=3$	M1	
	$\text{g}(x)=-2$	A1	
	$\frac{1}{(2x+1)}=-2$	M1	
	$x=-\frac{3}{4}$ only	A1	
		4	
(b)	$y=\frac{1}{(2x+1)^2}-1$ leading to $(2x+1)^2=\frac{1}{y+1}$ leading to $2x+1=[\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x=[\pm]\frac{1}{2\sqrt{y+1}}-\frac{1}{2}$	DM1	Make $x(\text{or } y)$ the subject
	$-\frac{1}{2\sqrt{x+1}}-\frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2}-\frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}+\frac{1}{4}}+\frac{1}{2}\right)$
		3	

Question 69

(a)	$\text{f}(x)=(x-1)^2+4$	B1	
	$\text{g}(x)=(x+2)^2+9$	B1	
	$g(x)=f(x+3)+5$	B1 B1	B1 for each correct element. Accept $p=3, q=5$
		4	
(b)	Translation or Shift	B1	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	B1 FT	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $\text{f}(x+p)+q$ *or their* $\text{f}(x)\to\text{g}(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

Question 70

(a)	$ff(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone ± sign errors.
	$8x^4 - 10x^2 + 2$[= 0]	A1	
	$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

Question 71

(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
(b)	[$-\sin 6x$][+ $15x$] or [$\sin(-6x)$][+ $15x$] OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right) + \frac{5}{3}x$
		2	

Question 72

(a)	Range of f is $f(x) \geqslant -4$	**B1**	Allow y, f or 'range' or $[-4, \infty)$
		1	
(b)	$y = (x - 2)^2 - 4 \Rightarrow (x - 2)^2 = y + 4 \Rightarrow x - 2 = +\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
	$\left[f^{-1}(x)\right] = \sqrt{(x+4)} + 2$	**A1**	
		2	
(c)	$(x - 2)^2 - 4 = -\frac{5}{3}x + 2 \Rightarrow x^2 - 4x + 4 - 4 = -\frac{5}{3}x + 2$ [$\Rightarrow x^2 - \frac{7}{3}x - 2 = 0$]	**M1**	Equating and simplifying to a 3-term quadratic
	$(3x+2)(x-3)[=0]$ or $\frac{7 \pm \sqrt{7^2 - 4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
	$x = 3$ only	**A1**	
		3	

(d)	$f^{-1}(12) = 6$	**M1**	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	**M1**	Substitute *their* '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a\left(ax+2\right)+2$	**M1**	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	**M1**	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute 12 and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
		5	

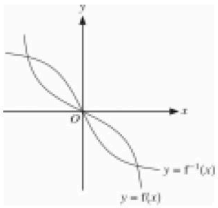
Question 73

$a = 2$	**B1**	
$b = \frac{\pi}{4}$	**B1**	or $\frac{2\pi}{8}$
$c = 1$	**B1**	
	3	

Question 74

{Reflection} {[in the] *x*-axis} or {Stretch of scale factor -1} {parallel to y-axis}	***B1 DB1**	{} indicate how the B1 marks should be awarded throughout.
Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the positive *y*-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
Alternative method for question 1		
{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the negative *y*-direction.
Then {Reflection} {in the *x*-axis} or {Stretch of scale factor -1} {parallel to *y*-axis}	***B1 DB1**	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	4	

Question 75

(a)		B1	A reflection of the given curve in $y = x$ (the line $y = x$ can be implied by position of curve).
		1	
(b)	$y = \frac{-x}{\sqrt{4-x^2}}$ leading to $x^2 = y^2(4-x^2)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2(1+y^2) = 4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x = (\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x = (\pm)\sqrt{\left(\frac{4y^2}{(1+y^2)}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$f^{-1}(x) = \frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
(c)	1 or $a = 1$	B1	Do not allow $x = 1$ or $-1 < x < 1$
		1	
(d)	$[fg(x) = f(2x) =]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$fg(x) = \frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
		2	

Question 76

{Reflection} {[in the] x-axis} or {Stretch of scale factor -1} {parallel to y-axis}	*B1 DB1	{} indicate how the B1 marks should be awarded throughout.
Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the positive y-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.

Question 77

(a)	$f(5) = [2]$ and $f(\textit{their}\ 2) = [5]$ OR $ff(5) = \left[\frac{2+3}{2-1}\right]$ OR $\dfrac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute $x = 5$.	M1	Clear evidence of applying f twice with $x = 5$.
	5	A1	
		2	

(b)	$\frac{x+3}{x-1}=y \Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=xy-x$	*M1	Setting f(*x*) = *y* or swapping *x* and *y*, clearing of fractions and expanding brackets. Allow ± sign errors.
	$xy-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding *x* or *y* = . Allow ± sign errors.
	$[\mathrm{f}^{-1}(x) \text{ or } y]=\frac{x+3}{x-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of *x*, cannot be *x* = .
		3	

Question 78

(a)	Stretch with [scale factor] either ± 2 or $\pm\frac{1}{2}$	B1	
	Scale factor $\frac{1}{2}$ in the *x*-direction	B1	
	Translation $\begin{pmatrix}0\\-3\end{pmatrix}$ or translation of 3 units in negative *y*-direction	B1	
		3	
(b)	(10 , 9)	B1 B1	B1 for each correct co-ordinate.
		2	

Question 79

(a)	$\{-3(x-2)^2\}$ $\{+14\}$	B1 B1	B1 for each correct term; condone $a = 2$, $b = 14$.
		2	
(b)	$[k=]\ 2$	B1	Allow $[x] \leqslant 2$.
		1	
(c)	[Range is] $[y] \leqslant -13$	B1	Allow $[\mathrm{f}(x)] \leqslant -13$, $[\mathrm{f}\,] \leqslant -13$ but NOT $x \leqslant -13$.
		1	
(d)	$y=-3(x-2)^2+14$ leading to $(x-2)^2=\frac{14-y}{3}$	M1	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if *x*, *y* on opposite sides.
	$x=2(\pm)\sqrt{\frac{14-y}{3}}$	A1	Allow $\frac{y-14}{-3}$.
	$[\mathrm{f}^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be *x* on RHS; must be negative square root only.
(e)	$[\mathrm{g}(x)=]\ \{-3(x+3-2)^2\}+\{14+1\}$	B2, 1, 0	OR $\{-3(x+3)^2\}+\{12(x+3)\}+\{3\}$
	$\mathrm{g}(x)=-3x^2-6x+12$	B1	
		3	

Question 80

(a)	$\left[x^{\frac{1}{2}}=\right]\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$	**M1 A1**	OE. Answer must come from formula or completing square. If M0A0 scored then **SC B1** for $2\pm\sqrt{3}$ only.
	$[x=](2\pm\sqrt{3})^2$	**M1**	Attempt to square *their* $2\pm\sqrt{3}$
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	Accept $7\pm4\sqrt{3}$ or $a=7, b=\pm4, c=3$ **SC B1** instead of second M1A1 for correct final answer only.
	Alternative method for question 9(a)		
	$-4x^{\frac{1}{2}}+1=0$ leading to $(x+1)^2=16x$ leading to $x^2-14x+1=0$	***M1 A1**	OE
	$x=\frac{14\pm\sqrt{196-4}}{2}$	**DM1**	Attempt to solve for x
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	**SC B1** instead of second M1A1 for correct final answer only.
		4	
(b)	$[\mathrm{gh}(x)=]\ m\left(x^{\frac{1}{2}}-2\right)^2+n$	**M1**	SOI
	$[\mathrm{gh}(x)=]\ m\left(x-4x^{\frac{1}{2}}+4\right)+n \equiv x-4x^{\frac{1}{2}}+1$	**A1**	SOI
	$m=1,\ n=-3$	**A1 A1**	WWW
		4	

Question 81

(a)	$2[\{(x-2)^2\}\ \{+3\}]$	**B1 B1**	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0
		2	
(b)	{Translation} $\begin{pmatrix}\{2\}\\\{3\}\end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\\{6\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		4	

Question 82

(a)	$\{2(x-4)^2\}$ $\{-9\}$	**B1 B1**	OE When *a* and *b* stated give priority to marking algebraic expression.
		2	
(b)	$y > -7$	**B1**	Allow f(x) > –7 or (–7, ∞) Don't allow x > –7.
		1	
(c)	$(x-4)^2 = \frac{y+9}{2}$	**M1**	2 operations correct. Allow a sign error.
	$x = 4\ [\pm]\sqrt{\frac{y+9}{2}}$	**M1**	2 operations correct. Allow a sign error.
	$[f^{-1}(x) =]4 - \sqrt{\frac{x+9}{2}}$	**A1 FT**	OE FT on *their* answer to **(a)** i.e. $-a-\sqrt{\left(\frac{x-b}{2}\right)}$.
		3	
(d)	fg(x) = f(2x + 4) = $2(2x+4-4)^2 - 9$	**M1**	Allow $2(2x+4)^2 - 16(2x+4) + 23$.
	$8x^2 - 9$ only	**A1**	
		2	

Question 83

(a)	$\{(x+1)^2 + 2(x+1) - 5\} + \{3\}$, or $\{(x+1+1)^2\} + \{-6+3\}$	**M1 M1**	M1 for dealing with $\begin{pmatrix}-1\\0\end{pmatrix}$ and M1 for dealing with $\begin{pmatrix}0\\3\end{pmatrix}$.
	$[y =]x^2 + 4x + 1$	**A1**	Answer only given full marks.
		3	
(b)	{Stretch} {x direction or horizontally or y-axis invariant} { factor ½}	**B2, 1, 0**	Additional transformation B0.
		2	

Question 84

(a)	$x \neq 1$ or $x < 1$, $x > 1$ or $(-\infty,1),(1,\infty)$ $[x \in \mathbb{R}]$	B1	Must be x not $f^{-1}(x)$ or y. Do not accept $1 < x < 1$.
		1	
(b)	$y = \frac{2x+1}{2x-1}$ leading to $(2x-1)y = 2x+1$ leading to $2xy - y = 2x+1$	*M1	Setting $y =$, removing fraction and expanding brackets.
	$2xy - 2x = y+1$ leading to $2x(y-1) = y+1$ leading to $x = \frac{y+1}{2(y-1)}$	DM1	Reorganising to get $x =$. Condone ± sign errors only.
	$[f^{-1}(x)] = \frac{x+1}{2(x-1)}, \frac{x+1}{x-1} \times \frac{1}{2}$ or $\frac{1}{x-1} + \frac{1}{2}$	A1	OE. Must be in terms of x. Do not allow $\frac{x+1}{x-1} \div 2$.
		3	
(c)	$(\textit{their } f^{-1}(3))$ leading to $(\textit{their } f^{-1}(3))^2 + 4$ $[f^{-1}(3) = 1, 1+4 =]$	M1	Correct order of operations and substitution of $x = 3$ needed.
	5	A1	
		2	
(d)	Sight of 'not one to one' or 'many to one' or 'one to many'	B1	Any reason mentioning 2 values, or + and —, such as: square root gives 2 values or horizontal line test crosses curve twice or 2 values because of turning point or 2 values because it is a quadratic.
		1	
(e)	$f(x) = 1 + \frac{2}{2x-1} = \frac{2x-1}{2x-1} + \frac{2}{2x-1} = \frac{2x+1}{2x-1}$	B1	AG Do not condone equating expressions and verification.
	$f'(x) = -4(2x-1)^{-2}$ or $2(2x-1)^{-1} + \{-(2x+1)2(2x-1)^{-2}\}$ or $\frac{(2\boldsymbol{x}-1)2 - 2(2\boldsymbol{x}+1)}{(2\boldsymbol{x}-1)^2}$	*M1	For $k(2x-1)^{-2}$ and no other terms or correct use of the product or quotient rule then ISW.
	Gradient $m = -4$	A1	Differentiation must have clearly taken place.
	Equation of tangent is $y - 3 = -4(x-1)$ $[\Rightarrow y = -4x+7]$	DM1	Using (1, 3) in the equation of a line with *their* gradient.
	Crosses axes at $\left(\frac{7}{4}, 0\right)$ and $(0, 7)$	A1 FT	SOI from *their* straight line or by integration from 0 to '*their* 7/4'.
	[Area =] $\frac{49}{8}$	A1	OE e.g. 6.13 AWRT. If M0 A0 DM0, **SC B2** available for correct answer.
		6	

Question 85

(a)	$y=\dfrac{x^2-4}{x^2+4}$ leading to $(x^2+4)y=(x^2-4)$ leading to $x^2y+4y=x^2-4$	***M1**	For clearing denominator and expanding brackets. If swap variables first, look for $y^2x+4x=y^2-4$.
	$x^2y-x^2=-4y-4$ leading to $x^2(1-y)=4y+4$ leading to $x^2=\ldots$	**DM1**	For making x^2 the subject. If swap variables first, look for $y^2(1-x)=4x+4\Rightarrow y^2=\ldots$
	$x^2=\dfrac{4y+4}{1-y}$ leading to $x=\sqrt{\dfrac{4y+4}{1-y}}$ leading to $[\mathrm{f}^{-1}(x)]=\sqrt{\dfrac{4x+4}{1-x}}$	**A1**	OE e.g. $\sqrt{\dfrac{-4x-4}{x-1}}$ without ± in final answer.
	Alternative method for Q6(a)		
	$x=\dfrac{y^2-4}{y^2+4}$ leading to $x=1-\dfrac{8}{y^2+4}$ leading to $x-1=\dfrac{-8}{y^2+4}$	***M1**	For division and reaching $x-1=\ldots$ (or $y-1=\ldots$)
	$y^2+4=\dfrac{-8}{x-1}$ leading to $y^2=\dfrac{-8}{x-1}-4$	**DM1**	For making y^2 (or x^2) the subject.
	$[y=][\mathrm{f}^{-1}(x)]=\sqrt{\dfrac{-8}{x-1}-4}$	**A1**	OE without ± in final answer.
		3	
(b)	$1-\dfrac{8}{x^2+4}=\dfrac{x^2+4}{x^2+4}-\dfrac{8}{x^2+4}\left[=\dfrac{x^2+4-8}{x^2+4}\right]=\dfrac{x^2-4}{x^2+4}$	**M1 A1**	Using common denominator or division to reach 1. Remainder -8. WWW
	$0<\mathrm{f}(x)<1$	**B1 B1**	B1 for each correct inequality. B0 if contradictory statement seen. Accept $\mathrm{f}(x)>0$, $\mathrm{f}(x)<1$; $1>\mathrm{f}(x)>0$; $(0,1)$ **SC B1** for $0\leqslant \mathrm{f}(x)\leqslant 1$.
		4	
(c)	Because the range of f does not include the whole of the domain of f (or any of it)	**B1**	Accept an answer that includes an example outside the domain of f, e.g. $\mathrm{f}(4)=\frac{12}{20}$. Must refer to the domain or > 2. Need not explicitly use the term 'domain' but must not refer just to the range.
		1	

Question 86

(a)	$[\mathrm{f}(x)]=\{-2(x+2)^2\}-\{5\}$	**B1 B1**	
		2	
(b)	$[\mathrm{f}(x)]<-7$	**B1**	Allow $y<-7$, <-7, $(-\infty,-7)$ or less than -7 $-\infty\langle f(x)\langle -7, -7\rangle f(x)\rangle -\infty$, $f<-7$
		1	
(c)	$y=-2(x+2)^2-5 \rightarrow (x+2)^2=\dfrac{-(y+5)}{2}$	**M1**	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
	$x=[\pm]\sqrt{\dfrac{-(y+5)}{2}}-2$	**M1**	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
	$[\mathrm{f}^{-1}(x)]=-2-\sqrt{\dfrac{-(x+5)}{2}}$ or $-2-\sqrt{-\dfrac{(x+5)}{2}}$	**A1**	Allow $[\mathrm{f}^{-1}(x)]=-2-\sqrt{\dfrac{x+5}{-2}}$.
		3	

Question 87

(a)	3	**B1**	Ignore any description.
		1	
(b)	2	**B1**	Ignore any description.
		1	
(c)	(8, 2)	**B1 B1**	Ignore any description. Allow vector notation and absence of brackets.
		2	
(d)	(1, 5)	**B1 FT**	FT each coordinate, (*their*8 – 7, *their*2 + 3) Allow vector notation and absence of brackets.
		B1 FT	
		2	

Question 88

(a)	Three points at the bottom of their transformed graph plotted at $y = 2$	**B1**	All 5 points of the graph must be connected.
	Bottom three points of /\/\ at $x = 0$, $x = 1$ & $x = 2$	**B1**	Must be this shape.
	All correct	**B1**	Condone extra cycles outside $0 \leqslant x \leqslant 2$.
		3	**SC:** If B0 B0 scored, B1 available for /\ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.
(b)	[g(x) =] f(2x) + 1	**B1 B1**	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y =$ or $f(x) =$.
		2	

Question 89

(a)	$a\left(x+\frac{1}{x}\right)+1$	B1	ISW
		1	
(b)	$a\left(2+\frac{1}{2}\right)+1=11$	M1	Substitute $x=2$ into *their* expression from **(a)** and equate to 11. This may be done in 2 stages: f(2)= 2.5, $g(2.5)=11$.
	[a =] 4	A1	
		2	
(c)	No,[because it is] not one-one	B1	Or other suitable explanation that may include one to many or many to one.
		1	
(d)	$[g^{-1}(x)]=\frac{x-1}{5}$ WWW	B1	Condone use of a instead of 5.
	$[g^{-1}f(x)=]\frac{x+\frac{1}{x}-1}{5}$ OE	M1	Correct combination of their $g^{-1}(x)$ with given $f(x)$ Condone use of a instead of 5.
	$\frac{x^2-x+1}{5x}$ or $\frac{1}{5}\left(x+\frac{1}{x}-1\right)$ or $\frac{1}{5}\left(x+x^{-1}-1\right)$ OE ISW	A1	Must not contain unresolved fractions e.g. $\frac{x+x^{-1}-1}{5}$.
		3	
(e)	The domain of f does not include the whole of the range of g. **Or** The range of g does not lie in the domain of f.	B1	Accept an answer that includes an example outside the domain of f, e.g. g(—1) = — 4 but for f, $x>0$.
		1	

Question 90

(a)	$(x-2)^2+5$	B1	
		1	
(b)	$2\left(\left\{(x+1)^2\right\}+\{5\}\right)$	B2, 1, 0	
		2	
(c)	$[g(x)=]$ $2f(x+3)$ or $k=2,\ h=3$	B1	In correct form. B0 if contradiction.
		1	
(d)	{Translation} $\left\{\begin{pmatrix}-3\\0\end{pmatrix}\right\}$	B2, 1, 0 FT	FT on *their* $x+3$ or $h=3$.
	{Stretch} {y direction, factor 2}	B2, 1, 0 FT	FT on *their* 2 or $k=2$.
		4	

Question 91

Stretch: $(2x)^2-2(2x)+5$ or $(x-1)^2+4$ leading to $(2x-1)^2+4$	M1	Replacing x by $2x$.
Reflection: $(-2x)^2-2(-2x)+5$ or $(-2x-1)^2+4$	M1	Replacing x by $-x$. FT on *their* stretch.
Stretch: $3\left\{(-2x)^2-2(-2x)+5\right\}$ or $3\left\{(-2x-1)^2+4\right\}$	M1	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).
$12x^2+12x+15$	A1	
	4	

Question 92

(a)	$[y] \leqslant -1$	B1	Accept f or $f(x) \leqslant -1$, $-\infty < y \leqslant -1$, $(-\infty, -1]$. Do not accept $x \leqslant -1$.
		1	
(b)	$y = -3x^2 + 2$ rearranged to $3x^2 = 2 - y$, leading to $x^2 = \frac{2-y}{3}$ or $y^2 = \frac{2-x}{3}$	M1	
	$x = [\pm]\sqrt{\frac{2-y}{3}} \rightarrow [f^{-1}(x)] = \{-\}\left\{\sqrt{\frac{2-x}{3}}\right\}$	A1 A1	A1 for minus, A1 for $\sqrt{\frac{2-x}{3}}$, allow $-\sqrt{\frac{x-2}{-3}}$.
		3	
(c)	$fg(x) = -3(-x^2 - 1)^2 + 2$	M1	SOI expect $-3x^4 - 6x^2 - 1$.
	$gf(x) = -(-3x^2 + 2)^2 - 1$	M1	SOI expect $-9x^4 + 12x^2 - 5$.
	$fg(x) - gf(x) + 8 = 0$ leading to $6x^4 - 18x^2 + 12$ $[= 0]$	A1	OE
	$[6](x^2 - 1)(x^2 - 2)$ $[= 0]$ or formula or completion of the square	M1	Solving a 3-term quadratic equation in x^2 must be seen.
	$x = -1$, $-\sqrt{2}$ only these **two** solutions	A1	Allow $-\sqrt{1}$, $-1.41[4]$ Answers only **SC B1**.
		5	

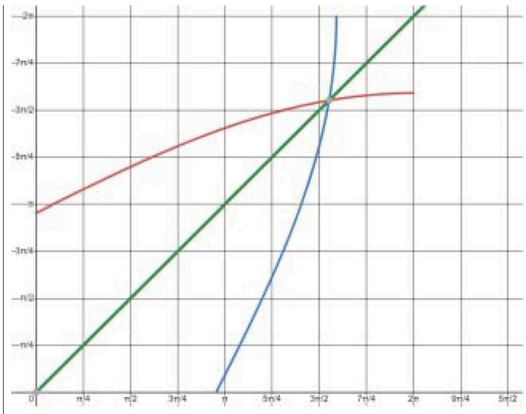
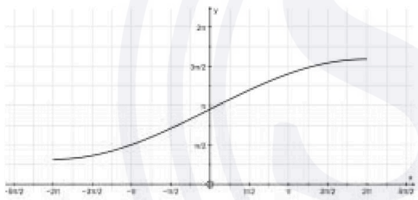
Question 93

{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
{Stretch} {[scale] factor 2} {parallel to *x*-axis}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct.
	4	Transformations can be in either order.

Question 94

(a)	$[y] < 2$ OR $[f(x)] < 2$	B1	OE e.g. $f < 2, (-\infty, 2), -\infty < f[x] < 2$. Do not accept $x < 2$ or $f(x) \leqslant 2$.
		1	
(b)	$y = 2 - \frac{5}{x+2}$ leading to $y(x+2) = 2(x+2) - 5$ leading to $xy + 2y = 2x - 1$	M1	or $\frac{5}{x+2} = 2 - y$ (allow sign errors).
	$2y + 1 = 2x - xy$ leading to $2y + 1 = x(2 - y)$	DM1	or $\frac{5}{2-y} = x + 2$ (allow sign errors).
	$x = \frac{2y+1}{2-y} \rightarrow f^{-1}(x) = \frac{2x+1}{2-x}$	A1	OE or $y = \frac{5}{2-x} - 2$.
	Domain is $x < 2$	B1 FT	FT on the numerical part of *their* range from part **(a),** including $x \neq 2$ not penalized. No FT for $x \in \mathcal{R}, x = k, x \neq k$.
		4	
(c)	$fg(x) = 2 - \frac{5}{x+3+2}$	B1	
	$= \frac{2(x+5)-5}{x+5}$ or $\frac{2(x+5)}{x+5} - \frac{5}{x+5}$	M1	Use of *their* common denominator.
	$= \frac{2x+5}{x+5}$	A1	
		3	

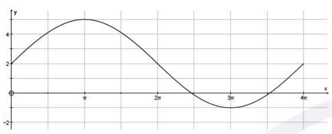
Question 95

	Answer	Marks	Guidance
(a)		*B1	The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $f^{-1}(x)$.
		DB1	Fully correct (needs to reach $y = 2\pi$ and x-axis and cross the line $y = x$ in the correct squares).
		2	
(b)	$y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$	M1	Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone $\pm$ sign errors. Variables may be interchanged initially. M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$.
	$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to [$f^{-1}(x)$ or y =] $4\sin^{-1}\left(\frac{x-3}{2}\right)$	A1	ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$. Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$.
		2	
(c)		B1	Continuing given graph from y intercept to -2π. The correct shape needed between 0 and -2π, including starting to level off (gradient in the final two squares needs to be reducing) as -2π is approached. The y co-ordinate at -2π must be in the correct square.
	Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function].	B1 FT	If there is no graph to the left of the y axis, no mark is available. FT an incorrect graph and if the answer is now 'No' provide an appropriate reason.
		2	
(d)	**{ } indicates different elements throughout.**		
	{Stretch} {factor 4} {in x-direction}	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of x-axis, horizontally or y-axis invariant. Wavelength or period increased by a factor of 4 for B2 or by 4 for B1.
	{Stretch} {factor 2} {in y-direction}	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of y-axis, vertically or x-axis invariant. Allow y 'co-ordinates' doubled or amplitude doubled for B2.
	{Translation} $\begin{pmatrix} \{0\} \\ \{3\} \end{pmatrix}$	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Allow shift. Any mention of y axis, y-direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1.
		6	After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1.

Question 96

{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
{Translation} $\begin{pmatrix} \{-6\} \\ \{0\} \end{pmatrix}$	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
	4	Transformations may be in either order.

Question 97

(a)	[Greatest =] 5	**B1**	No inequality required.
	[Least =] -1	**B1**	No inequality required.
			Condone $(-1,5)$ or equivalent.
		2	
(b)		**B1**	One complete cycle starting and finishing at $y = 2$. Maximum and minimum in correct quadrants. Shape and curvature approximately correct.
		B1 FT	Maximum and minimum (indicated on y-axis with numbers or lines, or labelled on graph). FT *their* greatest and least values. Award **B1** for 5 and -1 even if *their* values were incorrect in **(a)**.
		2	
(c)	1	**B1**	WWW
		1	

Question 98

(a)	$1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow \frac{2a}{7-a}=\frac{3}{2} \Rightarrow 7a=21\right] \Rightarrow a=\ldots$ OR $1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow (7-a)+2a=\frac{5}{2}(7-a)\ [\Rightarrow 7a=21\right] \Rightarrow a=\ldots$	**M1**	OE Substitute $x=7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
	$a=3$	**A1**	If M0, **SC B1** for $a=3$ with no working.
	$\mathrm{f}(5)=1+\frac{2(their3)}{5-their3}=4\left[\Rightarrow 4b-2=4\right] \Rightarrow b=\ldots$ OR $\mathrm{gf}(5)=b\left(1+\frac{2(their3)}{5-their3}\right)-2\ \left[\Rightarrow 4b-2=4\right] \Rightarrow b=\ldots$	**M1**	Evaluate f(5), either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT *their* a value.
	$b=\frac{3}{2}$	**A1**	OE e.g. $\frac{6}{4}$, 1.5 .
		4	
(b)	$x>1$	**B1**	Accept $(1,\infty)$ or $\{*: * > 1\}$ where * is any variable. B0 for $\mathrm{f}^{-1}(x)>1$ or $\mathrm{f}(x)>1$ or $y>1$.
		1	
(c)	EITHER $x-1=\frac{6}{y-3}\ \left[\Rightarrow (y-3)(x-1)=6\right]$ OR $x=1+\frac{6}{y-3} \Rightarrow x(y-3)=(y-3)+6$	***M1**	OE $y-1=\frac{6}{x-3} \Rightarrow (x-3)(y-1)=6$. OE $y=1+\frac{6}{x-3} \Rightarrow y(x-3)=(x-3)+6$. Allow *M1 for use of *their* 3 from **(a)**.
	$y-3=\frac{6}{x-1}$ or $y(x-1)=3x+3$	**DM1**	OE $x-3=\frac{6}{y-1}$ or $x(y-1)=3y+3$. Allow DM1 for use of *their* 3 from **(a)**.
	$\left[\mathrm{f}^{-1}(x)\right]=3+\frac{6}{x-1}$	**A1**	OE Correct answer e.g. $\frac{3x+3}{x-1}$ ISW. Must be in terms of x.
			*M1 DM1 possible for 'a' used, but A0 so max 2/3.
		3	

Question 99

(a)	$a=\frac{1}{2}$	B1	
	$b=\frac{\pi}{3}$	B1	$b=\frac{\pi}{3}+4n\pi, n\geqslant 0$.
		2	
(b)	x-coordinate = {4p} {–8}	B1 B1	OE, e.g. $4(p-2)$.
	y-coordinate = $-3q$	B1	
		3	

Question 100

(a)	Range is [y] > 1	B1	Allow f, f(x), (1,∞), etc.
		1	
(b)	$y=\frac{3}{x-2}+1$ leading to $y-1=\frac{3}{x-2}$ leading to $(x-2)(y-1)=3$	M1	Clearing the fraction.
	$x-2=\frac{3}{y-1}$	DM1	Reaching a stage which only requires one further operation.
	$x=\frac{3}{y-1}+2$ leading to $f^{-1}(x)=\frac{3}{x-1}+2$	A1	OE. Slightly longer routes lead to $f^{-1}(x)=\frac{2x+1}{x-1}$.
	[Domain is] $x>1$	B1FT	Must use x FT *their* **(a)**, must be a range.
		4	
(c)	$gf(x)=2\left(\frac{3}{x-2}+1\right)-2$ or $2\left(\frac{x+1}{x-2}\right)-2$	M1	Substitute f(x) into g(x).
	$\frac{6}{x-2}$	A1	
		2	

Question 101

(a)	$\frac{d}{dx}(x^2-8x+5)=0$ $[2x-8=0]$	M1	Correct differentiation of x^2 and equating their $\frac{dy}{dx}$ to 0.
	Alternative method 1 for first mark of Question 6(a)		
	$y=(x-4)^2-11$	M1	Attempt to complete the square as far as $y=(x-4)^2\pm k$.
	Alternative method 2 for first mark of Question 6(a)		
	$x=\frac{-b}{2a}=\frac{\pm 8}{2}$	M1	
	$x=4, y=-11$	A1	Answers from $x=\frac{8\pm\sqrt{64-20}}{2}$ leading to $x=4\pm\sqrt{11}$ scores M0A0
		2	
(b)	$x=($*their* x value from $a)+4$ [=8]	B1 FT	Can be from finding the equation of the transformed curve, differentiating and putting $\frac{dy}{dx}=0$.
	$y=\{($*their* y value from $a)\times 2\}+1$ [–21]	B1 FT	Can be from putting x = 8 in the equation of the transformed curve.
		2	If B0B0 scored, **SC B1** for sight of $(4,-22)$.

(c)	$2\left(x^2-8x+5\right)$ or $2\{(x-4)^2-11\}$	B1	Can be implied if both transformations done together: $2\left((x-4)^2-8(x-4)+5\right)+1$ OE.
	$\left((x-4)^2-8(x-4)+5\right)+1$ or $\left\{(x-4-4)^2-their11\right\}+1$	M1	For the x translation, each x becomes $(x-4)$.
		M1	For the y translation of +1.
	$y=2x^2-32x+107$ or $a=2, b=-32, c=107$	A1	Evidence to support *their* answer may be in **(b)** but answer must be seen in **(c).**
		4	

Question 102

8(a)	$y=(x+a)^2-a$ leading to $(x+a)^2=y\pm a$	*M1	x and y may be interchanged initially. Allow ± errors for these method marks.
	$x=[\pm]\sqrt{y\pm a}\pm a$	DM1	
	Alternative method for first 2 marks of Question 8(a)		
	$x=(y+a)^2-a$ leading to $y^2+2ay+a^2-a-x[=0]$	*M1	Allow ± errors for this method mark.
	$y=\dfrac{-2a\pm\sqrt{4a^2-4\left(a^2-a-x\right)}}{2}$	DM1	
	[y or $f^{-1}(x)$ =] $-\sqrt{x+a}-a$	A1	OE Must choose negative root.
		3	
(b)(i)	$x\geqslant -a$	B1	Ignore infinity limit if included.
		1	
(b)(ii)	y or $f^{-1}[(x)]\leqslant -a$	B1	Ignore negative infinity limit if included.
		1	
(c)	$\left[gf\left(\frac{7}{2}\right)=\right]2\left(\left(x+\frac{7}{2}\right)^2-\frac{7}{2}\right)-1$ or $2x^2+4\left(\frac{7}{2}\right)x+2\left(\frac{7}{2}\right)^2-2\left(\frac{7}{2}\right)-1[=0]$	B1	OE Alternatively, $\left[gf(x)=0\Rightarrow\right]f(x)=\frac{1}{2}$.
	$[x=]-\frac{7}{2}\pm 2$ or $\dfrac{-14\pm\sqrt{14^2-4\times 2\times\frac{33}{2}}}{4}$ $\left[\dfrac{-14\pm\sqrt{64}}{4}\right]$ or factorising	M1	OE Solving their three term quadratic equation as far as two solutions or correctly selecting the negative root only. Alternatively, $\pm\sqrt{\frac{1}{2}+\frac{7}{2}}-\frac{7}{2}$.
	$[x=]-\frac{11}{2}$	A1	If B1M0 scored then award **SCB1** for the correct final answer.
		3	

Question 103

	Answer	Marks	Guidance
(a)	$(2x-3)^2+4$	B1 B1	Or $a=-3$, $b=4$.
		2	
(b)	*their* $(2x-3)^2+4[<]8$ OR $4x^2-12x+13[<]8$	*M1	Linking quadratic with 8.
	$(2x-3)^2<4$ leading to $-2<2x-3<2$ OR $4x^2-12x+5<0$ leading to $(2x-1)(2x-5)<0$	DM1	Simplify to 3-term quadratic and solve. Condone no method shown.
	$\frac{1}{2}<x<2\frac{1}{2}$ leading to [LEAST] $p=\frac{1}{2}$, [GREATEST] $q=2\frac{1}{2}$	A1	
		3	
(c)	$\mathrm{gf}(x)=12x^2-36x+40$	B1	OE $\mathrm{gf}(x)=3(2x-3)^2+13$.
		1	
(d)	$y=(2x-3)^2+4$ leading to $(2x-3)^2=y-4$ leading to $2x-3=[\pm]\sqrt{y-4}$	*M1	
	$2x=3[\pm]\sqrt{y-4}$ leading to $x=\frac{3}{2}[\pm]\frac{\sqrt{y-4}}{2}$	DM1	
	$\mathrm{h}^{-1}(x)=\frac{3}{2}-\frac{\sqrt{x-4}}{2}$	A1	
		3	

Question 104

	Answer	Marks	Guidance
(a)	$\{-(x-3)^2\}$ $\{-1\}$	B1 B1	OE. Must be a quadratic e.g. $3x-1$ B0 B0. **SC B1** for correct use of generalised function notation.
		2	
(b)	$\{-(x-3)^2\}$ $\{+1\}$	B1 B1	OE. Must be a quadratic. **SC B1** for correct use of generalised function notation.
		2	
(c)	{Translation} $\begin{pmatrix}\{0\}\\\{2\}\end{pmatrix}$	B2, 1, 0	FT from **(a)** and **(b)** if a translation parallel to the y axis. B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		2	

Question 105

	Answer	Marks	Guidance
(a)	Attempt to form expression for $\mathrm{gf}(x)$	*M1	Expect $5\left((3x-2)^2+k\right)-1$; fg(x) is M0. Do not allow algebraic errors.
	Obtain $5(3x-2)^2+5k-1$	A1	OE e.g. $45x^2-60x+5k+19$.
	Their $5k-1=39$ or $5k-1\geqslant 39$	DM1	Or use $b^2-4ac=0$ (must be '= 0', could be implied later) on $45x^2-60x+5k+19-39\geqslant 0$ OE.
	Obtain $k=8$	A1	Do not accept $k\geqslant 8$.
		4	
(b)	Obtaining $(3(5x-1)-2)^2+$ *their k*	M1	May simplify and/or use k at this stage; k may have come from an inequality in **(a)**.
	Conclude $[\mathrm{fg}(x)]\geqslant 8$ allow $[y]\geqslant 8$	A1 FT	OE Following *their* value of k; must be $\geqslant$, not $>$. Allow an accurate written description.
		2	
(c)	State $\mathrm{g}^{-1}(x)=\frac{1}{5}(x+1)$	B1	OE $\frac{1}{5}(x+1)$ must be indicated as the inverse.
	$[\mathrm{h}(x)=]7x+4$	B1B1	If $7x+4$ only, it must be clear that this is $\mathrm{h}(x)$.
		3	

Question 106

(a)	State $(3\pi,\ -k)$	B1	
		1	
(b)	Obtain equation of form $[y=]c \pm k\sin\frac{1}{2}x$	M1	Any non-zero c.
	Obtain correct equation $[y=]2-k\sin\frac{1}{2}x$	A1	OE
	State $(3\pi,\ 2+k)$	B1 FT	Following part **(a)**, i.e. (*their* x, 2 – *their* y).
		3	

Question 107

(a)	Express $f(x)$ as: $a-(x-3)^2$ or $a-(3-x)^2$ where $a=\pm 19$ or ± 1	M1	OE If the form $-f(x)=(x^2-6x-10)$ is used the form must be returned to $f(x)=\ldots$ Completed square form must give $-x^2$. Answers must come from completion of the square (not calculus or graphs).
	$19-(3-x)^2$ or $19-(x-3)^2$	A1	OE
	$f(x)\leqslant 19$ or $y\leqslant 19$ with $\leqslant$, not $<$ or $-\infty < f(x) \leqslant 19$ or $-\infty \leqslant f(x) \leqslant 19$ or $(-\infty, 19]$ or $[-\infty, 19]$	A1 FT	Using *their* constant following the award of M1. **SC B1** answer only or answer from a method not involving completion of the square.
		3	
(b)	$g^{-1}(x)=\frac{1}{4}(x-k)$	B1	
	$g^{-1}f(x)=\frac{1}{4}(10+6x-x^2-k)=4x+k$	M1	OE May use *their* completed square form for f(x).
	Simplify the quadratic equation obtained from $g^{-1}f(x)=g(x)$ provided k is present and apply $b^2-4ac=0$ to this quadratic equation	*M1	Expect $x^2+10x-10+5k=0$.
	Obtain $100-4(5k-10)=0$ and hence $k=7$	A1	
	Use *their* k to form and solve a quadratic in x	DM1	Allow if *their* quadratic has two solutions.
	$(-5,-13)$ only	A1	**SC B1** if no method seen.
	Alternative Method for first 4 marks		
	State $f(x)=gg(x)$	(B1)	
	$gg(x)=16x+5k$	(M1)	
	Apply $b^2-4ac=0$ to quadratic equation obtained from $f(x)=gg(x)$	(*M1)	Provided k is present.
	$100-4(5k-10)=0$ and hence $k=7$	(A1)	
		6	

Question108

(a)	State $(\frac{5}{3}\pi,\ 0)$ for point A	B1	Or exact equivalent. Allow $x=\frac{5}{3}\pi$ or exact equivalent.
	$x=\frac{19}{6}\pi$ for point B	B1	Or exact equivalent. May be implied in coordinate or vector form.
	$y=-k$ for point B	B1	May be implied in coordinate or vector form.
		3	
(b)	Solve at least as far as $\sin^{-1}3t=k\pi$ with correct value for $\cos^{-1}\left(\frac{1}{2}\sqrt{2}\right)$	M1	Allow use of $\pi=3.14\ldots$. Allow $\sin^{-1}3t=30$.
	$\sin^{-1}3t=\frac{1}{6}\pi$ and hence $t=\frac{1}{6}$	A1	Or exact equivalent. Can use degrees if consistent.
		2	

Question 109

(a)	$\left[f^{-1}(x)=\right](x+1)^2$	**B1**	ISW Condone '$y =$'.
		1	
(b)	$0 < g(x) \leqslant \frac{1}{2}$ or $g(x) > 0$ and $g(x) \leqslant \frac{1}{2}$ or $\left(0, \frac{1}{2}\right]$	**B1**	Do not allow $g(x) > 0, g(x) \leqslant \frac{1}{2}$. Do not allow $g(x) > 0$ or $g(x) \leqslant \frac{1}{2}$. Condone g or y in place of $g(x)$.
	g^{-1} does not exist because it is one to many or g^{-1} does not exist because it is not one to one. Or g^{-1} does not exist because g is not one to one or g^{-1} does not exist because g is many to one or g^{-1} does not exist because g fails the horizontal line test.	**B1**	g^{-1} can be replaced by 'It' throughout. A correct statement followed by any further incorrect explanation can be awarded B1.
		2	
(c)	$f\left(\frac{25}{16}\right)=\frac{1}{4}$	**B1**	SOI
	$\frac{1}{\left(\sqrt{x}-1\right)^2+2}=\frac{1}{4}$	**M1**	Equating $\frac{1}{\left(\sqrt{x}-1\right)^2+2}$, or *their* 'simplified' version, to *their* $f\left(\frac{25}{16}\right)$.
	$\left[\left(\sqrt{x}-1\right)^2+2=4 \text{ leading to}\right]$ $\quad \sqrt{x}-1=\sqrt{2}$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-2\sqrt{x}+1+2=4 \text{ leading to}\right]$ $\quad x-2\sqrt{x}-1=0$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-1=2\sqrt{x} \text{ leading to}\right]$ $\quad x^2-6x+1=0$ leading to $x=\frac{6\pm\sqrt{36-4}}{2}$	**A1**	Simplification as far as $x = \ldots$ Allow just + in the results because $-$ can be disregarded at this stage. Can be implied by the final answer. Note: $x=1\pm\sqrt{2}$ scores A0.
	$3+2\sqrt{2}$	**A1**	Must discount the solution $3-2\sqrt{2}$.
		4	

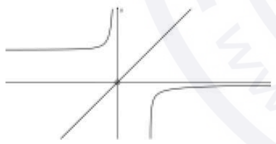
Question 110

Stretch **factor** 4 in y-direction/parallel to the y axis/vertically.	**B1**	Allow use of SF in place of factor. Allow in/on/along the y axis or 'the x axis is invariant.'
Translation $\begin{pmatrix}3\\0\end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix}0\\-8\end{pmatrix}$ or -8 parallel to the y axis or in the y direction, allow vertically.	**B2**	Condone 'Shift'. These translations can be combined as $\begin{pmatrix}3\\-8\end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3 B1 for 2 out of 3
Two translations, one in each direction, and a stretch only.	**M1**	Condone inaccurate terminology, such as up, down, left and right, if the intention is clear.
Correct order of operations. The stretch which must be in the in the y direction must come before the translation in the y direction.	**A1**	Condone inaccurate terminology if the intention is clear but numerical values must be correct.

Alternative Method for Question 2

Translation $\begin{pmatrix}3\\0\end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix}0\\-2\end{pmatrix}$ or -2 parallel to the y axis or in the y direction, allow vertically.	**(B2)**	Condone 'Shift'. These translations can be combined as $\begin{pmatrix}3\\-2\end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3. B1 for 2 out of 3.
Stretch **factor** 4 in y-direction/parallel to the y axis/vertically.	**(B1)**	Allow use of SF in place of factor. Allow in/on/along the y axis or "the x axis is invariant."
Two translations, one in each direction, and a stretch only.	**(M1)**	Condone inaccurate terminology, such as transform, move, up, down, left and right, if the intention is clear.
Correct order of operations. The stretch which must be in the in the y direction must come after the translation in the y direction.	**(A1)**	Condone inaccurate terminology if the intention is clear but numerical values must be correct.
	5	

Question 111

(a)		**B1**	For curve in correct quadrant.
		B1	Fully correct including line $y = x$. Horizontal asymptote closer to x axis than vertical asymptote is to y axis.
		2	
(b)	$x = \frac{2}{y^2} + 4$ leading to $y^2(x-4) = 2$ or $y^2 = \frac{2}{(x-4)}$	**M1**	Allow x and y swapped around.
	$y^2 = \frac{2}{(x-4)}$ leading to $y = [\pm]\sqrt{\frac{2}{x-4}}$ or $x = [\pm]\sqrt{\frac{2}{y-4}}$	**M1**	
	$\left[f^{-1}(x)\right] = -\sqrt{\frac{2}{x-4}}$	**A1**	
		3	
(c)	$[x] = -2$	**B1**	
		1	
(d)	Because f^{-1} is always negative and f is always positive or curves do not intersect	**B1**	Accept other correct answers e.g. 'f is only defined for positive values of x and f^{-1} is only defined for negative values of x' or 'domains do not overlap' or 'the y values cannot be the same' or 'the x values cannot be the same'.
		1	

Question 112

(a)	{Stretch} {factor 3} { in y-direction}	**B2,1,0**	2 out of 3 scores B1.
	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2,1,0**	Accept shift.
	Alternative Method for Question 2(a)		
	{Translation} $\begin{pmatrix} \{0\} \\ \left\{-\frac{2}{3}\right\} \end{pmatrix}$	**(B2,1,0)**	2 out of 3 scores B1. Accept shift.
	{Stretch} {factor 3} { in y-direction}	**(B2,1,0)**	
		4	
(b)	$[\mathrm{f}(x)] = \{-3\sin x\}\{-2\}$	**B1 B1**	No marks awarded if extra terms seen.
		2	

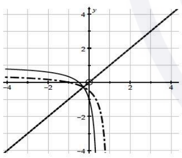
Question 113

(a)	$3(x-2)^2+2$ or $a=-2,\ b=2$	**B1 B1**	
		2	
(b)	2 or $k=2$ or $k \geq 2$	**B1FT**	FT on *their* a. Do not accept $x = 2$ or $x > 2$.
		1	
(c)	$3(x-2)^2+14-12=y \Rightarrow (x-2)^2=\frac{y-2}{3}$	**M1**	Using *their* completed square form.
	$x=[\pm]\sqrt{\frac{y-2}{3}}+2$	**DM1**	
	$\mathrm{f}^{-1}(x)=\sqrt{\frac{x-2}{3}}+2$	**A1**	OE, e.g. $y=\frac{\sqrt{3x-6}}{3}+2$.
		3	
(d)	Finding $\mathrm{f}^{-1}(29)$ [= 5]	**M1**	Or solving f(x) = 29 [using *their* completed square form, OE].
	Finding f^{-1}(*their* 5)	**M1**	Or solving f(x) = *their* 5.
	$x=3$	**A1**	If using f(x) method, $x = 1$ must be discarded.
	Alternative solution for Question 8(d)		
	$3\left(3(x-2)^2+2\right)-2)^2+2=29$ using *their* completed square form	**M1**	Or $3\left(3x^2-12x+14\right)^2-12\left(3x^2-12x+14\right)+14=29$. Allow if the '= 29' appears later in the working.
	Solving as far as $9(x-2)^4=9$ or $x^2-4x+3=0$	**DM1**	OE Or $[27]\left(x^4-8x^3+24x^2-32x+15\right)=0$.
	$x=3$ only	**A1**	WWW Only dependent on the first M1.
		3	

Question 114

(a)	Reflection [in] y-axis	**B1 B1**	B1 for reflection B1 mention of y-axis, OE. **SC B2** for stretch, SF –1, parallel to x-axis.
	Translation or shift $\begin{pmatrix}-1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and a correct vector/description. Do not accept 'left'/'right'. If two translations then B0 and B0 for the order.
	Stretch, factor 2, parallel to y-axis	**B2,1,0**	B2 all correct OE. B1 any 2 parts correct. This can be at any point in the sequence.
	Correct order and three correctly named transformations only	**DB1**	If a fourth transformation is given this mark is not awarded and no marks are given for the two transformations of the same type, except where the reflection is described as a stretch. If any transformation is incorrectly named this cannot be given. If translation is not $\begin{pmatrix}-1\\0\end{pmatrix}$ or $\begin{pmatrix}1\\0\end{pmatrix}$ then DB0 is given.
	Alternative Solution for first 3 marks		
	Translation or shift $\begin{pmatrix}1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and correct vector/description.
	Reflection [in] y-axis	**B1 B1**	B1 for 'reflection', B1 for 'in y-axis'.
	Alternative solutions		
	There are alternative solutions which can be marked in the same way e.g. the given stretch, translation $\begin{pmatrix}-4\\0\end{pmatrix}$, reflect in $x = -2.5$		
		6	
(b)	$g(x) = 2f(-x-1)$ or $a = 2,\ b = -1,\ c = -1$	**B1**	First B1 for $a = 2$ and no additional terms added to the function. $a = -2$ is B0.
		B1	Second B1 for $b = -1$ and $c = -1$.
		2	

Question 115

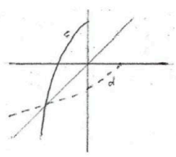
(a)(i)	$[f(-1) =]\ \frac{1}{3}$	**B1**	Condone 0.333.
		1	
(a)(ii)		**B1**	For showing the correct mirror line.
		B1	For correct shape: the curves should intersect in the first square in the third quadrant. To the left of the point of intersection, the reflection is below the original and crosses the x-axis. To the right of the point of intersection, the reflection is to the right the original.
		2	
(a)(iii)	$\frac{2x+1}{2x-1} = y \Rightarrow 2x+1 = y(2x-1)$	**M1***	Equating y to the given function and clearing of fractions. x and y may be interchanged at this stage.
	$2xy - 2x = y + 1$	**DM1**	Condone ± errors during simplification.
	$\frac{x+1}{2(x-1)},\ \frac{-x-1}{2-2x}$	**A1**	Allow 'f^{-1}' or '$y =$' but NOT '$x =$', nor fractions within fractions.
	[Domain of f^{-1} is] $x < 1$	**B1**	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
	Alternative Method for Question 5(a)(iii)		
	$y = 1 + \frac{2}{2x-1} \Rightarrow y - 1 = \frac{2}{2x-1}$	**M1***	Equating y to the given function after division by $2x-1$. Isolating the term in x. x and y may be interchanged at this stage.
	$2x = \frac{2}{y-1} + 1$	**DM1**	Condone ± errors during simplification.
	$\frac{1}{x-1} + \frac{1}{2}$	**A1**	OE Allow 'f^{-1}' or '$y =$' but NOT '$x =$', nor fractions within fractions.
	[Domain of f^{-1} is] $x < 1$	**B1**	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
		4	

(b)	$gf\left(\frac{1}{4}\right) = -7$	B1	
	$\frac{2x+1}{2x-1} = -7$	M1	Equating $\frac{2x+1}{2x-1}$ to *their* $gf\left(\frac{1}{4}\right)$.
	$[x =]\ \frac{3}{8}$	A1	OE
	Alternative solution for Question 5(b)		
	$gf\left(\frac{1}{4}\right) = -7$	B1	
	$x = f^{-1}(-7)$	M1	$x = f^{-1}\left(\textit{their } gf\left(\frac{1}{4}\right)\right)$
	$[x =]\ \frac{3}{8}$	A1	OE
		3	

Question 116

(a)	$a = 4$	B1	Allow $4\sin(2x) + 3$ if values of a, b and c are not stated.
	$b = 2$	B1	
	$c = 3$	B1	
		3	
(b)(i)	5	B1	Ignore attempts at finding solutions.
		1	
(b)(ii)	1	B1	Ignore attempts at finding solutions.
		1	

Question 117

(a)	Obtain $b = 2$ and $c = \frac{3}{2}$	B1	
	Obtain $\frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2$	B1	
	State range is $y \leqslant \frac{15}{2}$ or $f(x) \leqslant \frac{15}{2}$ with $\leqslant$ given or clearly implied (not $<$)	B1 FT	Following *their* value of a.
		3	
(b)	State that reflection is in x-axis	B1	Accept transformations in any order.
	State or imply that translation is by $\begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ or equivalent	B1 FT	Following *their* values of a and c in part **(a)**. Accept transformations in any order.
		2	
(c)	Sketch the correct graph appearing in second and third quadrants only	B1	
	State that each y-value is associated with a single x-value or equivalent	B1	Accept passes the horizontal line test. Ignore passes the vertical line test.
		2	
(d)	Sketch the correct graph with suitable labelling to distinguish the two curves	B1	Appearing in third and fourth quadrants only.
	Draw the line $y = x$	B1	See above; no need to label the line.
	Attempt correct process for finding the inverse function	M1	Allowing use of $\pm$ and y so far.
	Obtain $\frac{3}{2} - \sqrt{\frac{15}{4} - \frac{1}{2}x}$ or equivalent	A1	Must involve x at the conclusion.
		4	

Question 118

(a)	State or imply $g^{-1}(x)=\frac{1}{2}(x-k)$ or equivalent	B1	
	Obtain $\frac{1}{2}(3k+1-k)=c$ and hence $2k+1=2c$	B1	OE
	Or:		
	$g(c)=3k+1$	B1	
	$[2c+k=3k+1\Rightarrow]\ 2c=2k+1$	B1	OE
	Then:		
	$[gf(x)=]\ 2(4x^2-c)+k$	M1	Allow ± errors only.
	$[8x^2-2c+k\Rightarrow 8x^2-(2k+1)+k\Rightarrow]\ gf(x)=8x^2-k-1$	A1	**AG** All necessary detail needed.
		4	
(b)	$8(x-2)^2-k-1+3$	B1	OE Translation coming before the stretch
	$k\left(their\left(8(x-2)^2-k-1+3\right)\right)$	B1 FT	OE Stretch
	$[h(x)=]-\left(k\left(their\left(8(x-2)^2-k-1+3\right)\right)\right)$	B1 FT	OE Reflection
		3	
(c)	$k^2-2k=15$ or $k^2-2k\leqslant 15$	B1	OE
	$k=5$ only	B1	
	$c=\frac{11}{2}$ only	B1	OE
		3	

Question 119

(a)	$[f(x)=](x+2a)^2-4a^2+a$	B1	
	$\left(their\left(-4a^2+a\right)\right)=-33$	M1	Condone $\geqslant$ or $>$.
	$4a^2-a-33\ [=0]$	B1	Condone $\leqslant$ or $<$.
	$-\frac{11}{4}$, 3	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.
	Alternative Method for Question 11(a)		
	$x=-2a$, $y=-4a^2+a$	B1	The co-ordinates of the minimum point (likely to be either from differentiation or completing the square).
	$\left(their-4a^2+a\right)=-33$	M1	*Their* y-coordinate equated to -33. Condone $\geqslant$ or $>$.
	$-4a^2+a+33[=0]$	B1	OE Condone $\geqslant$ or $>$.
	$-\frac{11}{4}$, 3	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.

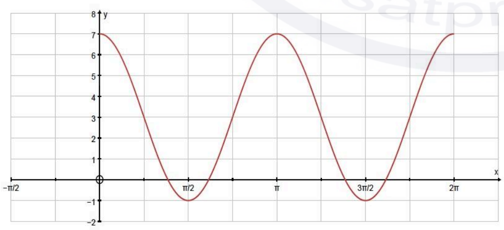
(a)	**Alternative Method 2 for Question 11(a)**		
	$x^2+4ax+a+33\ [=0]$	B1	Condone $\geqslant$ or $>$.
	$(4a)^2-4[1](a+33)\ [=0]$	M1	Condone $\geqslant$ or $>$.
	$16a^2-4a-132\ [=0]$	B1	OE Condone $\geqslant$ or $>$. Accept $4a^2-a-33$ or multiples thereof.
	$-\frac{11}{4}$, 3	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.
		4	

	Answer	Marks	Guidance
(b)	$\left[g(x)=\right]\ \frac{1}{2}\left(x^3+4\right)$	B1	Expression for $g(x)$. SOI
	Either		
	$\left[g(0)=\right]\ \frac{1}{2}\left((0)^3\pm 4\right)\quad \left[g(0)=2\right]$	M1	Replacing x with 0 in *their* $g(x)$.
	$gg(0)=6$	A1	
	$f(\textit{their}\ 6)\ \left[=36+24a+a\right]$	M1	
	Or		
	$fg(x)=\left(\frac{x^3+4}{2}\right)^2+4a\left(\frac{x^3+4}{2}\right)+a$ or $gg(x)=\dfrac{\left(\frac{x^3+4}{2}\right)^3+4}{2}\quad \left[=\frac{\left(x^3+4\right)^3}{16}+2\right]$	M1	Either composite function using *their* $g(x)$.
	$fgg(x)=\left\{\frac{\left(x^3+4\right)^3}{16}+2\right\}^2+4a\left\{\frac{\left(x^3+4\right)^3}{16}+2\right\}+a$	A1	Complete algebraic expression for $fgg(x)$.
	$fgg(0)=\left\{\frac{(0+4)^3}{16}+2\right\}^2+4a\left\{\frac{(0+4)^3}{16}+2\right\}+a$	M1	Replacing x with 0 in *their* $fgg(x)$.
(b)	**Then**		
	$36+24a+a=96$	A1	OE
	$\left[a=\right]\ \frac{12}{5}$	A1	OE
		6	

Question 120

(a)	$\{(x+2)^2\}\{-2\}$	**B1B1**	B1 for each correct { }. Allow $a=2$, $b=-2$. If contradictory, give preference to the expression.
		2	
(b)(i)	$y=(x+2)^2-2 \Rightarrow y+2=(x+2)^2$	***M1**	Equating *y* or $f^{-1}(x)$ or f^{-1} or $f(x)$ to *their* completed square form and first step. *x* and *y* may be interchanged at this stage. Condone ± errors.
	$x=[\pm]\sqrt{y+2}-2$	**DM1**	Condone ± errors during simplification.
	$[f^{-1}(x)=]-\sqrt{x+2}-2$	**A1**	Do not condone $x=$ or $f(x)=$.
		3	
(b)(ii)	$[gf(x)=]-\{$*their* completed square form$\}-4$ or $-(x^2+4x+2)-4$	***M1**	Using $fg(x)=\{(-x-4)+2\}^2-2$ scores 0/4.
	$[gf(x)=]-(x+2)^2-2$	**A1**	
	$x=[\pm]\sqrt{-y-2}-2$	**DM1**	Finding *x* from *their* completed square form, which must contain a $-(x+k)^2$ term. Condone ± errors only during simplification. $x=-\sqrt{y+2}-2$ is DM0 (Square rooting then ÷ or ×−1) *x* and *y* may be interchanged at this stage.
	$[(gf)^{-1}(x)=]-\sqrt{-x-2}-2$	**A1**	Do not condone $x=$.
	Alternative Method for Question 11(b)(ii)		
	$(gf)^{-1}=f^{-1}g^{-1}$	***M1**	SOI. Allow with *their* f^{-1} and *their* g^{-1}. Using $g^{-1}f^{-1}(x)$ scores 0/4.
	$g^{-1}(x)=-x-4$	**A1**	
	$(gf)^{-1}(x)=-\sqrt{-x-4+2}-2$	**DM1**	Allow with *their* f^{-1} and *their* g^{-1}.
	$[(gf)^{-1}(x)=]-\sqrt{-x-2}-2$	**A1**	Do not condone $x=$
		4	

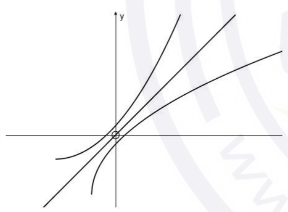
Question 121

(a)	[Greatest] 7, [least] −1	**B1B1**	B2 for answers of −1 and 7 only. B1 for one correct value. Ignore incorrect identification/inequality.
		2	
(b)		**B2,1,0**	Ignore any graph outside domain $(0, 2\pi)$. B1 for two complete cycles, one from 0 to approximately π and the other finishing at approximately 2π. Starting at *their* greatest value and initially decreasing. Condone straight lines and minimum above the *x*-axis or joining points with straight lines. B2 for correct curve; condone any incorrect *x*-axis intercepts. Graph must start to level off at both 0 and 2π. Ignore any *y*-labels, but the curve should be more above the *x*-axis than below.
		2	
(c)	3 [solutions]	**B1**	Ignore any graphs drawn
		1	

Question 122

{Stretch} {factor 2} {'parallel to y-axis' or 'in y-direction' or 'vertically'}	**B2,1,0**	B2 for 3 correct components. B1 for 2.
{Translation} $\begin{pmatrix}\{0\}\\ \{-14\}\end{pmatrix}$ or $\{-14\}$ {'parallel to the y-axis' or 'in the y-direction' or 'vertically'.}	**B2,1,0**	B2 for 3 correct components. B1 for 2.
Alternative Method for Question 1		
{Translation by} $\begin{pmatrix}\{0\}\\ \{-7\}\end{pmatrix}$ or $\{-7\}$ {'parallel to the y-axis' or 'in the y-direction' or 'vertically'}	**B2,1,0**	B2 for 3 correct components. B1 for 2.
{Stretch} {factor 2} {'parallel to y-axis' or 'in y-direction' or 'vertically'}	**B2,1,0**	B2 for 3 correct components. B1 for 2.
	4	

Question 123

(a)	{Stretch} {factor 3} {parallel to y-axis/in y-direction/vertically}	**B2,1,0**	
	Translation $\begin{pmatrix}-2\\ -5\end{pmatrix}$	**B2,1,0**	
		B1	Transformations correct and in the correct order.
	Alternative solution for Question 10(a)		
	Translation $\begin{pmatrix}-2\\ -\frac{5}{3}\end{pmatrix}$	**B2,1,0**	
	{Stretch} {factor 3} {parallel to y-axis/in y-direction/vertically}	**B2,1,0**	
		B1	Transformations correct and in the correct order.
		5	The translation parallel to the x-axis can be made anywhere in the sequence. Note: If 3 or more transformations are given then maximum 2/5 for any correct one.
(b)		**B1**	For line or curve in correct quadrants only.
		B1	Must not pass through (0, 0). Fully correct including the line $y = x$. No label needed. Approximate reflection of $y = f(x)$. Curve must not come back on itself.
		2	
(c)	$y = 3\sqrt{x+2} - 5 \Rightarrow \frac{y+5}{3} = \sqrt{x+2}$	**M1**	Allow x/y swap.
	$[g^{-1}(x)] = \left(\frac{x+5}{3}\right)^2 - 2$	**A1**	Must be in terms of x. Not '$x = \ldots$'.
		2	
(d)	[Range of g^{-1} is $g^{-1}(x)$] $\geqslant -2$	**B1 FT**	Following *their* $g^{-1}(x) = \left(\frac{x+a}{b}\right)^2 - c$ where a, b, c are non-zero. Not $x \geqslant -2$. Not > -2. Accept other notations, e.g. $[-2, \infty]$.
		1	
(e)	$[g^{-1}h(4) = g^{-1}(2)] = \frac{31}{9}$	**B1**	AWRT 3.44.
		1	
(f)	hg^{-1} is impossible since the **range** of g^{-1} is $x \geqslant -2$ is **not within** the **domain** of **h**, which is $x \geqslant 0$	**B1**	Minimum acceptable: 'The range of g^{-1} is not within the domain of h'.
		1	