

AS-Level
Pure Mathematics P1
Topic : Functions and Transformation
May 2013- May 2025

Question 1

The function f is defined by $f : x \mapsto 2x + k, x \in \mathbb{R}$, where k is a constant.

- (i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

The function g is defined by $g : x \mapsto x^2 - 6x + 8, x \in \mathbb{R}$.

- (ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8, x > 3$.

- (iii) Find an expression for $h^{-1}(x)$. [4]

Question 2

- (i) Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

- (ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geq k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k . [1]

The value of k is now given to be 7.

- (iii) Find the range of f . [1]

- (iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [5]

Question 3

The function f is defined by $f : x \mapsto x^2 + 4x$ for $x \geq c$, where c is a constant. It is given that f is a one-one function.

- (i) State the range of f in terms of c and find the smallest possible value of c . [3]

The function g is defined by $g : x \mapsto ax + b$ for $x \geq 0$, where a and b are positive constants. It is given that, when $c = 0$, $gf(1) = 11$ and $fg(1) = 21$.

- (ii) Write down two equations in a and b and solve them to find the values of a and b . [6]

Question 4

Solve the inequality $x^2 - x - 2 > 0$. [3]

Question 5

A function f is defined by $f : x \mapsto 3 \cos x - 2$ for $0 \leq x \leq 2\pi$.

- (i) Solve the equation $f(x) = 0$. [3]
- (ii) Find the range of f . [2]
- (iii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3 \cos x - 2$ for $0 \leq x \leq k$.

- (iv) State the maximum value of k for which g has an inverse. [1]
- (v) Obtain an expression for $g^{-1}(x)$. [2]

Question 6

The function f is defined by

$$f : x \mapsto x^2 + 1 \text{ for } x \geq 0.$$

- (i) Define in a similar way the inverse function f^{-1} . [3]
- (ii) Solve the equation $ff(x) = \frac{185}{16}$. [3]

Question 7

Functions f and g are defined by

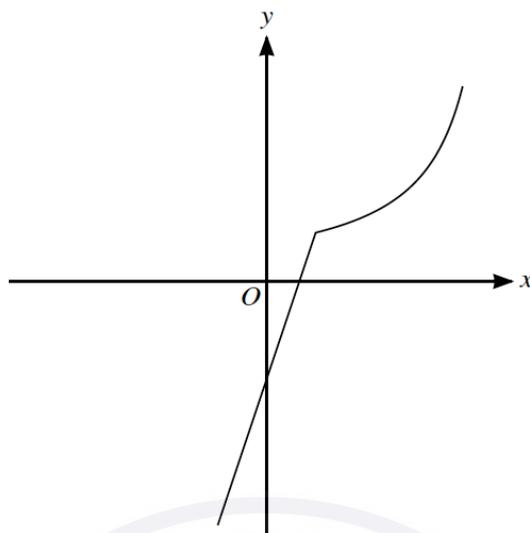
$$\begin{aligned} f : x &\mapsto 2x - 3, & x \in \mathbb{R}, \\ g : x &\mapsto x^2 + 4x, & x \in \mathbb{R}. \end{aligned}$$

- (i) Solve the equation $ff(x) = 11$. [2]
- (ii) Find the range of g . [2]
- (iii) Find the set of values of x for which $g(x) > 12$. [3]
- (iv) Find the value of the constant p for which the equation $gf(x) = p$ has two equal roots. [3]

Function h is defined by $h : x \mapsto x^2 + 4x$ for $x \geq k$, and it is given that h has an inverse.

- (v) State the smallest possible value of k . [1]
- (vi) Find an expression for $h^{-1}(x)$. [4]

Question 8



The diagram shows the function f defined for $-1 \leq x \leq 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leq x \leq 1, \\ \frac{4}{5-x} & \text{for } 1 < x \leq 4. \end{cases}$$

- (i) State the range of f . [1]
- (ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]
- (iii) Obtain expressions to define the function f^{-1} , giving also the set of values for which each expression is valid. [6]

Question 9

- (a) The functions f and g are defined for $x \geq 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$

$$g : x \mapsto x^2.$$

Given that $fg(1) = 2$ and $gf(9) = 16$,

- (i) calculate the values of a and b , [4]
 - (ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]
- (b) A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x -coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y -coordinate of P at the instant when P is at the point $(3, 4)$. [5]

Question 10

The function $f : x \mapsto 6 - 4 \cos\left(\frac{1}{2}x\right)$ is defined for $0 \leq x \leq 2\pi$.

- (i) Find the exact value of x for which $f(x) = 4$. [3]
- (ii) State the range of f . [2]
- (iii) Sketch the graph of $y = f(x)$. [2]
- (iv) Find an expression for $f^{-1}(x)$. [3]

Question 11

- (i) Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. [2]

The function f is defined for $p \leq x \leq q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

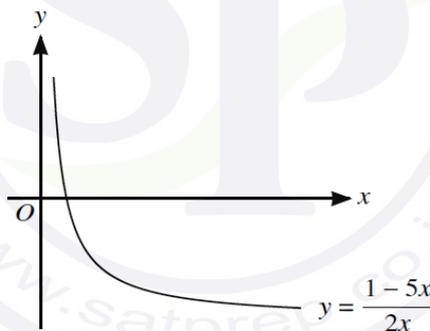
The range of f is given by $c \leq f(x) \leq d$, where c and d are constants.

- (ii) State the smallest possible value of c . [1]

For the case where $c = 9$ and $d = 65$,

- (iii) find p and q , [4]
- (iv) find an expression for $f^{-1}(x)$. [3]

Question 12



The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \frac{1 - 5x}{2x}$ for $0 < x \leq 2$.

- (i) Find an expression for $f(x)$ and state the domain of f . [5]
- (ii) The function g is defined by $g(x) = \frac{1}{x}$ for $x \geq 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

Question 13

The function f is defined by $f : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

- (i) Find the set of values of p for which the equation $f(x) = p$ has no real roots. [3]

The function g is defined by $g : x \mapsto 2x^2 - 6x + 5$ for $0 \leq x \leq 4$.

- (ii) Express $g(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]
(iii) Find the range of g . [2]

The function h is defined by $h : x \mapsto 2x^2 - 6x + 5$ for $k \leq x \leq 4$, where k is a constant.

- (iv) State the smallest value of k for which h has an inverse. [1]
(v) For this value of k , find an expression for $h^{-1}(x)$. [3]

Question 14

A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

- (i) Find the greatest height of the passenger above the ground. [1]

One complete revolution of the wheel takes 30 minutes.

- (ii) Show that $k = \frac{1}{15}\pi$. [2]
(iii) Find the time for which the passenger is above a height of 90 m. [3]

Question 15

The function $f : x \mapsto 5 + 3 \cos(\frac{1}{2}x)$ is defined for $0 \leq x \leq 2\pi$.

- (i) Solve the equation $f(x) = 7$, giving your answer correct to 2 decimal places. [3]
(ii) Sketch the graph of $y = f(x)$. [2]
(iii) Explain why f has an inverse. [1]
(iv) Obtain an expression for $f^{-1}(x)$. [3]

Question 16

The function f is defined by $f(x) = 3x + 1$ for $x \leq a$, where a is a constant. The function g is defined by $g(x) = -1 - x^2$ for $x \leq -1$.

- (i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

- (ii) solve the equation $fg(x) + 14 = 0$, [3]
(iii) find the set of values of x which satisfy the inequality $gf(x) \leq -50$. [4]

Question 17

The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

- (i) In the case where $a = 6$ and $b = -8$, find the range of f . [3]
- (ii) In the case where $a = 5$, the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k . [3]
- (iii) Show that if the equation $f(x + a) = a$ has no real roots, then $a^2 < 4(b - a)$. [3]

Question 18

Functions f and g are defined by

$$\begin{aligned} f : x &\mapsto 3x + 2, & x \in \mathbb{R}, \\ g : x &\mapsto 4x - 12, & x \in \mathbb{R}. \end{aligned}$$

Solve the equation $f^{-1}(x) = gf(x)$. [4]

Question 19

- (i) Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

The function $f : x \mapsto -x^2 + 6x - 5$ is defined for $x \geq m$, where m is a constant.

- (ii) State the smallest value of m for which f is one-one. [1]
- (iii) For the case where $m = 5$, find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

Question 20

The function f is such that $f(x) = a^2x^2 - ax + 3b$ for $x \leq \frac{1}{2a}$, where a and b are constants.

- (i) For the case where $f(-2) = 4a^2 - b + 8$ and $f(-3) = 7a^2 - b + 14$, find the possible values of a and b . [5]
- (ii) For the case where $a = 1$ and $b = -1$, find an expression for $f^{-1}(x)$ and give the domain of f^{-1} . [5]

Question 21

The function f is such that $f(x) = 2x + 3$ for $x \geq 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leq q$, where a , b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leq q$.

- (i) Find the values of a and b . [3]
 - (ii) Find the greatest possible value of q . [2]
- It is now given that $q = -3$.
- (iii) Find the range of fg . [1]
 - (iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

Question 22

The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

- (i) Find the set of values of x for which $f(x) \leq 3$. [3]
- (ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geq k$, where k is a constant.

- (iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]
- (iv) State the smallest value of k for which g has an inverse. [1]
- (v) For this value of k , find an expression for $g^{-1}(x)$. [2]

Question 23

Functions f and g are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R}, x \neq \frac{3}{2}.$$

Solve the equation $ff(x) = gf(2)$. [3]

Question 24

The function f is defined by $f : x \mapsto 4 \sin x - 1$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) State the range of f . [2]
- (ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]
- (iii) Sketch the graph of $y = f(x)$. [2]
- (iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1} . [4]

Question 25

- (i) Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a , b and c are constants. [3]
- (ii) Functions f and g are both defined for $x > 0$. It is given that $f(x) = x^2 + 1$ and $fg(x) = 4x^2 + 12x + 10$. Find $g(x)$. [1]
- (iii) Find $(fg)^{-1}(x)$ and give the domain of $(fg)^{-1}$. [4]

Question 26

A function f is defined by $f : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$.

- (i) Find the range of f . [2]
- (ii) Sketch the graph of $y = f(x)$. [2]
- (iii) Solve the equation $f(x) = 6$, giving answers in terms of π . [3]

The function g is defined by $g : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq k$, where k is a constant.

- (iv) State the largest value of k for which g has an inverse. [1]
- (v) For this value of k , find an expression for $g^{-1}(x)$. [3]

Question 27

The functions f and g are defined by

$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$
$$g(x) = \frac{4}{5x + 2} \quad \text{for } x \geq 0.$$

- (i) Find and simplify an expression for $fg(x)$ and state the range of fg . [3]
- (ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1} . [5]

Question 28

The functions f and g are defined for $x \geq 0$ by

$$f : x \mapsto 2x^2 + 3,$$
$$g : x \mapsto 3x + 2.$$

- (i) Show that $gf(x) = 6x^2 + 11$ and obtain an unsimplified expression for $fg(x)$. [2]
- (ii) Find an expression for $(fg)^{-1}(x)$ and determine the domain of $(fg)^{-1}$. [5]
- (iii) Solve the equation $gf(2x) = fg(x)$. [3]

Question 29

- (i) Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a , b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geq p$, where p is a constant.

- (ii) State the smallest value of p for which f is a one-one function. [1]
- (iii) For this value of p , obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1} . [4]
- (iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

Question 30

The function f is defined by $f(x) = 3 \tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]
- (ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1} . [5]
- (iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

Question 31

The function f is defined by $f : x \mapsto \frac{2}{3-2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$.

- (i) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

- (ii) Find the value of a for which $gf(-1) = 3$. [3]
- (iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

Question 32

The functions f and g are defined by

$$f(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$g(x) = x^2 + 1 \text{ for } x > 0.$$

- (i) Find an expression for $f^{-1}(x)$. [3]
- (ii) Solve the equation $gf(x) = 5$. [4]

Question 33

(a) The function f , defined by $f : x \mapsto a + b \sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

- (i) Find the values of the constants a and b . [3]
 - (ii) Evaluate $ff(0)$. [2]
- (b) The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leq g(x) \leq 10$. Find the values of the constants c and d . [3]

Question 35

A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

- (i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]
- (ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

Question 36

Functions f and g are defined for $x > 3$ by

$$f : x \mapsto \frac{1}{x^2 - 9},$$
$$g : x \mapsto 2x - 3.$$

- (i) Find and simplify an expression for $gg(x)$. [2]
- (ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]
- (iii) Solve the equation $fg(x) = \frac{1}{7}$. [4]

Question 37

Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

- (i) (a) State the range of the function f . [1]
- (b) State the range of the function g . [1]
- (c) State the range of the function fg . [1]
- (ii) Explain why the function gf cannot be formed. [1]

Question 38

The one-one function f is defined by $f(x) = (x-2)^2 + 2$ for $x \geq c$, where c is a constant.

- (i) State the smallest possible value of c . [1]
- In parts (ii) and (iii) the value of c is 4.
- (ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]
- (iii) Solve the equation $ff(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

Question 39

The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

- (i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x+b)^2$, where a and b are constants. [2]
- (ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geq k$.

- (iii) State the smallest value of k for which g has an inverse. [1]
- (iv) For this value of k , find $g^{-1}(x)$. [3]

Question 40

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \frac{1}{2}x - 2,$$

$$g : x \mapsto 4 + x - \frac{1}{2}x^2.$$

- (i) Find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. [3]
- (ii) Find the set of values of x for which $f(x) > g(x)$. [2]
- (iii) Find an expression for $fg(x)$ and deduce the range of fg . [4]

The function h is defined by $h : x \mapsto 4 + x - \frac{1}{2}x^2$ for $x \geq k$.

- (iv) Find the smallest value of k for which h has an inverse. [2]

Question 41

- (i) Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

The function f is defined by $f(x) = 2x^2 - 12x + 11$ for $x \leq k$.

- (ii) State the largest value of the constant k for which f is a one-one function. [1]
- (iii) For this value of k find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

The function g is defined by $g(x) = x + 3$ for $x \leq p$.

- (iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

Question 42

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

- (i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]
- (ii) State the range of f . [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leq k$.

- (iii) State the largest value of k for which g has an inverse. [1]
- (iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

Question 43

- (a) The one-one function f is defined by $f(x) = (x - 3)^2 - 1$ for $x < a$, where a is a constant.
- (i) State the greatest possible value of a . [1]
- (ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]
- (b) The function g is defined by $g(x) = (x - 3)^2$ for $x \geq 0$.
- (i) Show that $gg(2x)$ can be expressed in the form $(2x - 3)^4 + b(2x - 3)^2 + c$, where b and c are constants to be found. [2]
- (ii) Hence expand $gg(2x)$ completely, simplifying your answer. [4]

Question 44

- (i) Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]
- The function f is defined by $f(x) = x^2 - 4x + 7$ for $x < k$, where k is a constant.
- (ii) State the largest value of k for which f is a decreasing function. [1]
- The value of k is now given to be 1.
- (iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]
- (iv) The function g is defined by $g(x) = \frac{2}{x - 1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf . [4]

Question 45

- The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.
- (i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]
- (ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

Question 46

The function f is defined by $f(x) = \frac{48}{x - 1}$ for $3 \leq x \leq 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leq x \leq b$, where a and b are constants.

- (i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf . [2]

It is now given that the conditions for the formation of gf are satisfied.

- (ii) Find an expression for $gf(x)$. [1]
- (iii) Find an expression for $(gf)^{-1}(x)$. [2]

Question 47

Functions f and g are defined by

$$f : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$
$$g : x \mapsto \frac{2x + 3}{x - 1}, \quad x \in \mathbb{R}, x \neq 1.$$

- (i) Obtain expressions for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]
- (ii) Solve the equation $fg(x) = \frac{7}{3}$. [3]

Question 48

The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

- (i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]
- (ii) State the greatest value of $f(x)$. [1]

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

- (iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

Question 49

The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1} . [5]

Question 50

Functions f and g are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

- (i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$. [3]
- (ii) In the case where $k = -9$, find the set of values of x for which $f(x) < g(x)$. [3]
- (iii) In the case where $k = -1$, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$. [3]
- (iv) Express $f(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $f(x)$. [3]

Question 51

Functions f and g are defined by

$$f : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$

$$g : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

- (i) Find the range of f and the range of g . [3]
- (ii) Find an expression for $fg(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a , b and c are integers. [2]
- (iii) Find an expression for $(fg)^{-1}(x)$, giving your answer in the same form as for part (ii). [3]

Question 52

The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

Question 53

- (a) Express $2x^2 + 12x + 11$ in the form $2(x+a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leq -4$.

- (b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

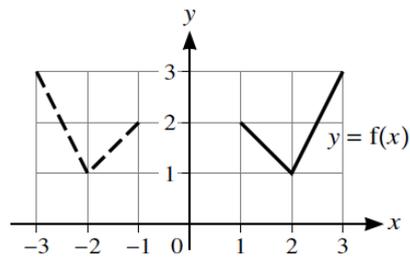
The function g is defined by $g(x) = 2x - 3$ for $x \leq k$.

- (c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]
- (d) State the largest value of k possible for the composition fg to be defined. [1]

Question 54

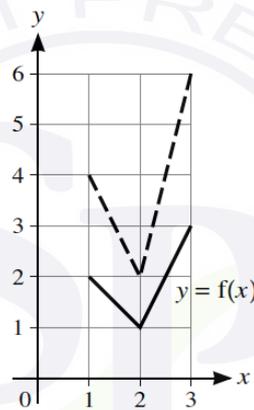
In each of parts (a), (b) and (c), the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



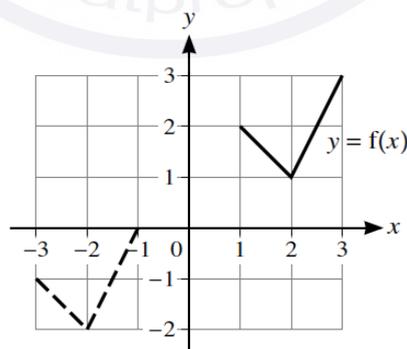
State, in terms of f , the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f , the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f , the equation of the graph shown with broken lines. [2]

Question 55

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

- (a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

It is given that f is a one-one function.

- (b) State the smallest possible value of c . [1]

It is now given that $c = 5$.

- (c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

- (d) Find an expression for $gf(x)$ and state the range of gf . [3]

Question 56

The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

- (a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x . [4]

- (b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a . [2]

Question 57

Functions f and g are such that

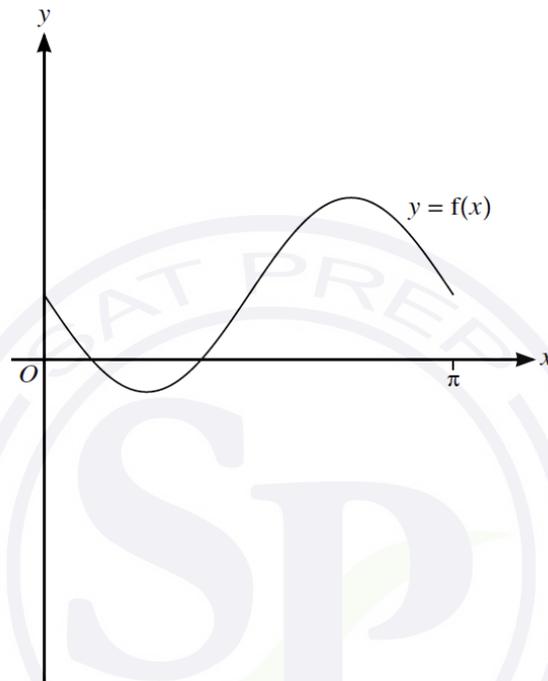
$$f(x) = 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi,$$

$$g(x) = -2f(x) \quad \text{for } 0 \leq x \leq \pi.$$

- (a) State the ranges of f and g .

[3]

The diagram below shows the graph of $y = f(x)$.



- (b) Sketch, on this diagram, the graph of $y = g(x)$.

[2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

- (c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$.

[3]

Question 58

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \frac{1}{2}x - a,$$

$$g : x \mapsto 3x + b,$$

where a and b are constants.

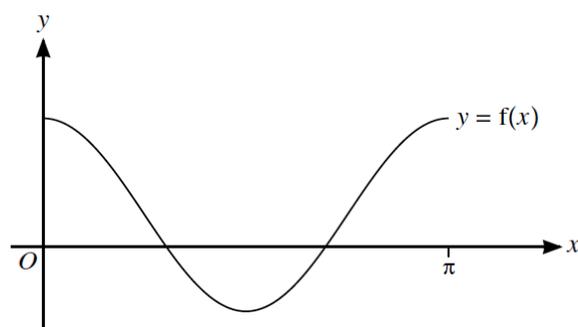
- (a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b .

[4]

- (b) Using these values of a and b , find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants.

[2]

Question 59



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2} \cos 2x + \frac{1}{2}$ for $0 \leq x \leq \pi$.

- (a) State the range of f . [2]

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x -axis is a tangent to the curve $y = g(x)$.

- (b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]
- (c) State the equation of the curve which is the reflection of $y = f(x)$ in the x -axis. Give your answer in the form $y = a \cos 2x + b$, where a and b are constants. [1]

Question 60

The function f is defined by $f(x) = \frac{2x}{3x-1}$ for $x > \frac{1}{3}$.

- (a) Find an expression for $f^{-1}(x)$. [3]
- (b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]
- (c) State the range of f . [1]

Question 61

- (a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]
- (b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$. Describe fully the transformation(s) involved. [2]

Question 62

A curve has equation $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$.

- (a) State the greatest and least values of y . [2]

.....
.....
.....
.....

- (b) Sketch the graph of $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$. [2]

- (c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3 \cos 2x + 2 = kx$ for $0 \leq x \leq \pi$ in each of the following cases.

- (i) $k = -3$ [1]

.....
.....

- (ii) $k = 1$ [1]

.....
.....

- (iii) $k = 3$ [1]

.....
.....

Functions f , g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3 \cos 2x + 2,$$

$$g(x) = f(2x) + 4,$$

$$h(x) = 2f\left(x + \frac{1}{2}\pi\right).$$

- (d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]
(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

Question 63

Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R}, x \neq -1.$$

(a) Find the value of $fg(7)$. [1]

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

Question 64

The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$

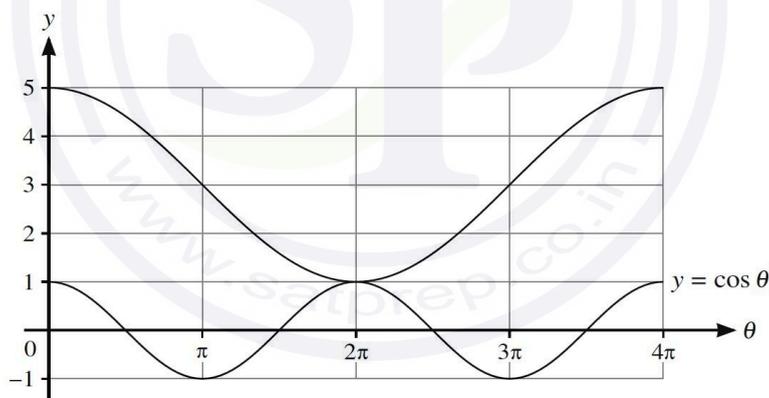
$$g(x) = 2x + 1 \quad \text{for } x > -\frac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

Question 65



In the diagram, the lower curve has equation $y = \cos \theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos \theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

Question 66

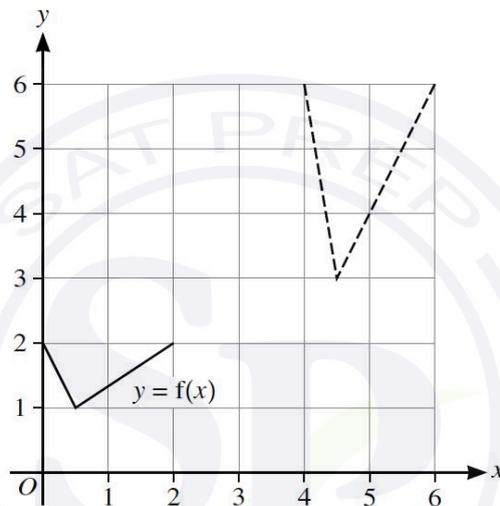
Functions f and g are defined as follows:

$$f : x \mapsto x^2 + 2x + 3 \text{ for } x \leq -1,$$

$$g : x \mapsto 2x + 1 \text{ for } x \geq -1.$$

- (a) Express $f(x)$ in the form $(x + a)^2 + b$ and state the range of f . [3]
- (b) Find an expression for $f^{-1}(x)$. [2]
- (c) Solve the equation $gf(x) = 13$. [3]

Question 67



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

- (a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]
- (b) State in terms of y , f and x , the equation of the graph shown with broken lines. [2]

Question 68

Functions f and g are defined as follows:

$$f : x \mapsto x^2 - 1 \text{ for } x < 0,$$

$$g : x \mapsto \frac{1}{2x + 1} \text{ for } x < -\frac{1}{2}.$$

- (a) Solve the equation $fg(x) = 3$. [4]
- (b) Find an expression for $(fg)^{-1}(x)$. [3]

Question 69

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

- (a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x+p) + q$, where p and q are constants. [4]
- (b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

Question 70

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geq 0$.

- (a) Find and simplify an expression for $ff(x)$. [2]
- (b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

Question 71

- (a) The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x-1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

- (b) The curve $y = \sin 2x - 5x$ is reflected in the y -axis and then stretched by scale factor $\frac{1}{3}$ in the x -direction.

Write down the equation of the transformed curve. [2]

Question 72

Functions f and g are defined as follows:

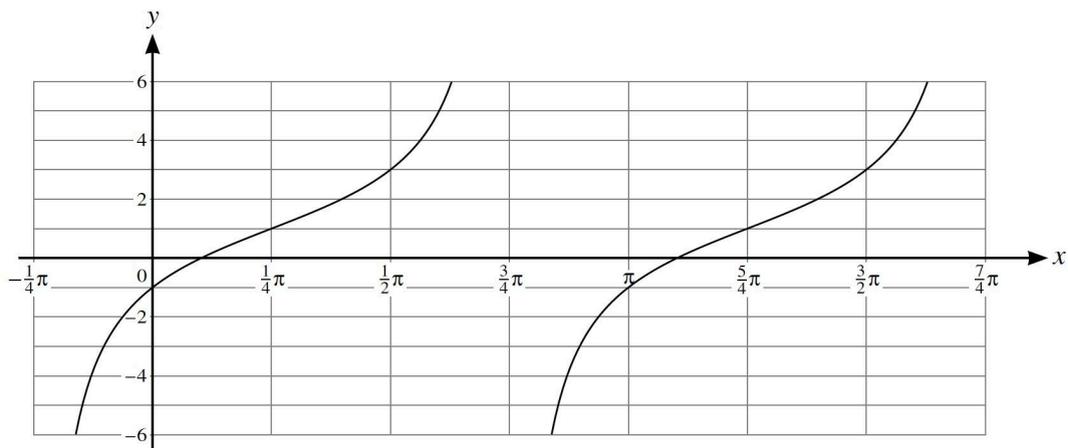
$$f(x) = (x-2)^2 - 4 \text{ for } x \geq 2,$$

$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

- (a) State the range of f . [1]
- (b) Find $f^{-1}(x)$. [2]
- (c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]
- (d) Given instead that $ggf^{-1}(12) = 62$, find the possible values of a . [5]

Question 73



The diagram shows part of the graph of $y = a \tan(x - b) + c$.

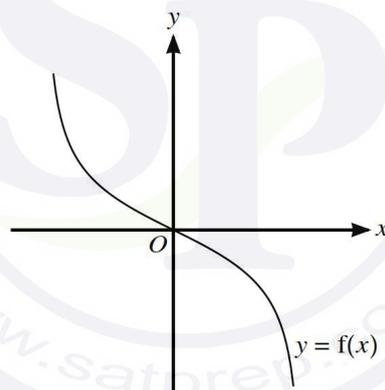
Given that $0 < b < \pi$, state the values of the constants a , b and c . [3]

Question 74

The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

Question 75



The diagram shows the graph of $y = f(x)$.

(a) On this diagram sketch the graph of $y = f^{-1}(x)$. [1]

It is now given that $f(x) = -\frac{x}{\sqrt{4-x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $f^{-1}(x)$. [4]

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

Question 76

The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

Question 77

The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of $ff(5)$. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

Question 78

The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

Question 79

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leq k$.

(b) State the largest possible value of the constant k . [1]

It is now given that $k = -1$.

(c) State the range of f . [1]

(d) Find an expression for $f^{-1}(x)$. [3]

The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p , q and r are constants. [3]

Question 80

Functions f , g and h are defined as follows:

$$f : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geq 0,$$

$$g : x \mapsto mx^2 + n \quad \text{for } x \geq -2, \text{ where } m \text{ and } n \text{ are constants,}$$

$$h : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geq 0.$$

(a) Solve the equation $f(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a , b and c are integers. [4]

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n . [4]

Question 81

(a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$

$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

Question 82

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f . [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

Question 83

(a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

(b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

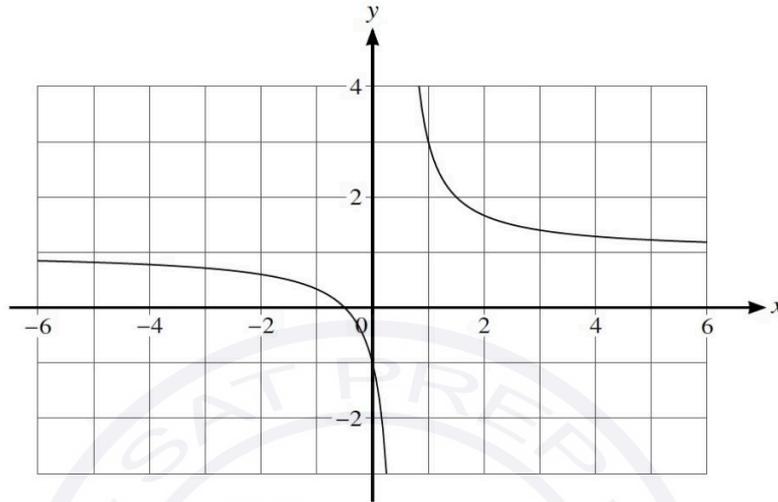
Question 84

Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \frac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1} . [1]

(b) Find an expression for $f^{-1}(x)$. [3]

(c) Find $gf^{-1}(3)$. [2]

(d) Explain why $g^{-1}(x)$ cannot be found. [1]

(e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x - and y -axes. [6]

Question 85

The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f . [4]

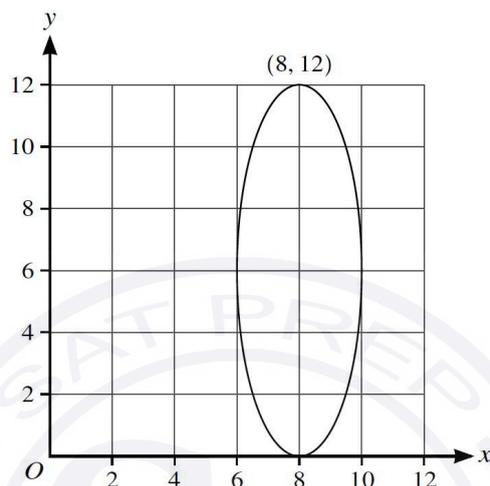
(c) Explain why the composite function ff cannot be formed. [1]

Question 86

The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

- (a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]
- (b) Find the range of f . [1]
- (c) Find an expression for $f^{-1}(x)$. [3]

Question 87



The diagram shows a curve which has a maximum point at $(8, 12)$ and a minimum point at $(8, 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y -direction.

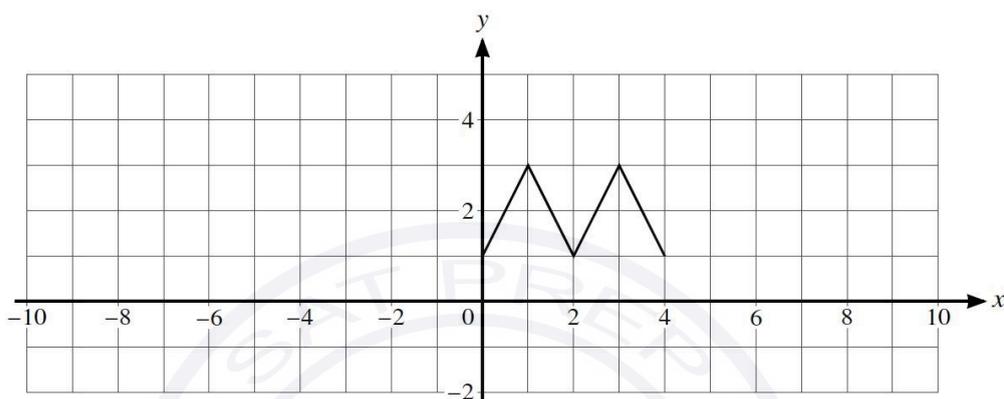
- (a) State the scale factor of the stretch. [1]
- (b) State the radius of the original circle. [1]
- (c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]
- (d) State the coordinates of the centre of the original circle. [2]

Question 88

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x -direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- (a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



- (b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

Question 89

Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$

$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

- (a) Find an expression for $gf(x)$. [1]
- (b) Given that $gf(2) = 11$, find the value of a . [2]
- (c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

It is given instead that $a = 5$.

- (d) Find and simplify an expression for $g^{-1}f(x)$. [3]
- (e) Explain why the composite function fg cannot be formed. [1]

Question 90

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$

$$g(x) = 2x^2 + 4x + 12.$$

- (a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]
- (b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]
- (c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]
- (d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

Question 91

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x -axis with scale factor $\frac{1}{2}$

Reflection in the y -axis

Stretch parallel to the y -axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants. [4]

Question 92

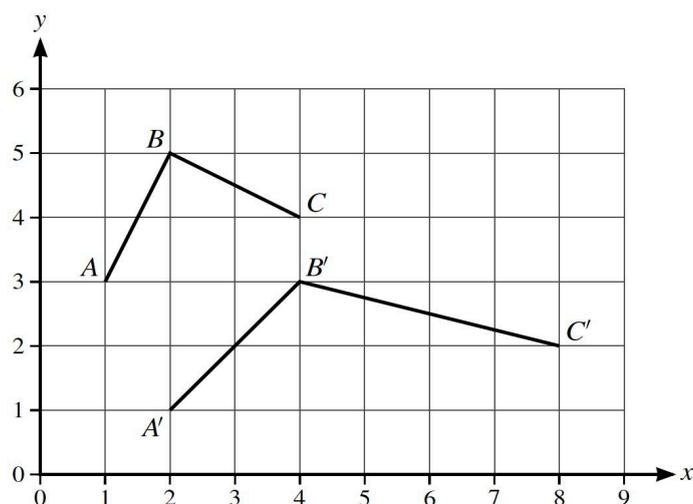
The function f is defined by $f(x) = -3x^2 + 2$ for $x \leq -1$.

- (a) State the range of f . [1]
- (b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leq -1$.

- (c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

Question 93



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC . The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations. [4]

Question 94

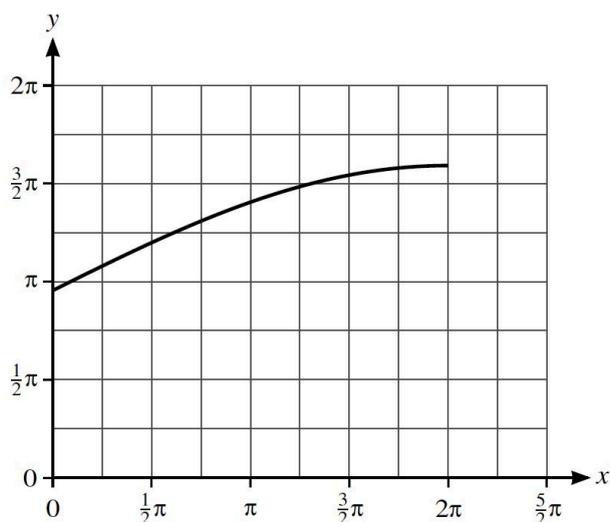
The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

- (a) State the range of f . [1]
- (b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

- (c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a, b, c and d are integers. [3]

Question 95



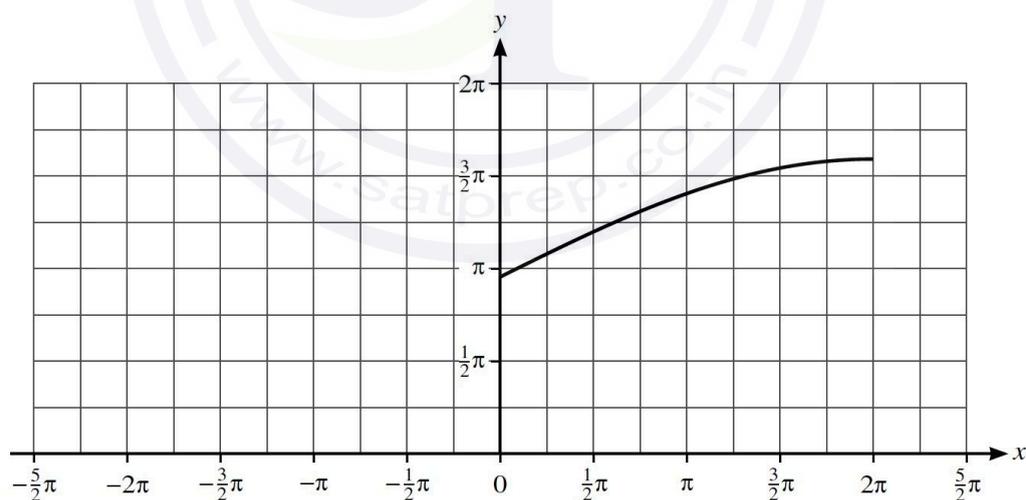
The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

(a) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) Find an expression for $f^{-1}(x)$. [2]

(c)

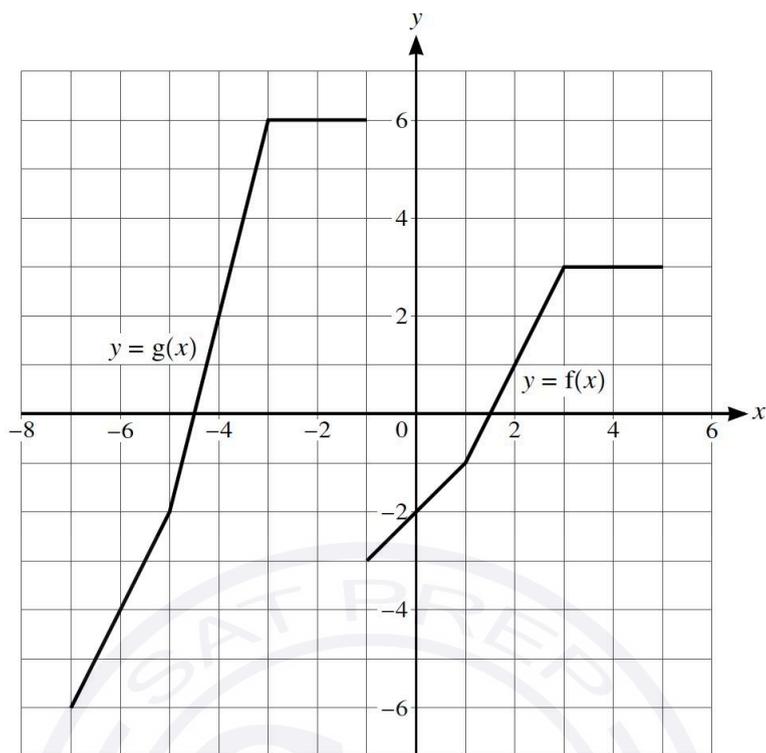


The diagram above shows part of the graph of the function $g(x) = 3 + 2 \sin \frac{1}{4}x$ for $-2\pi \leq x \leq 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

Question 96



The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

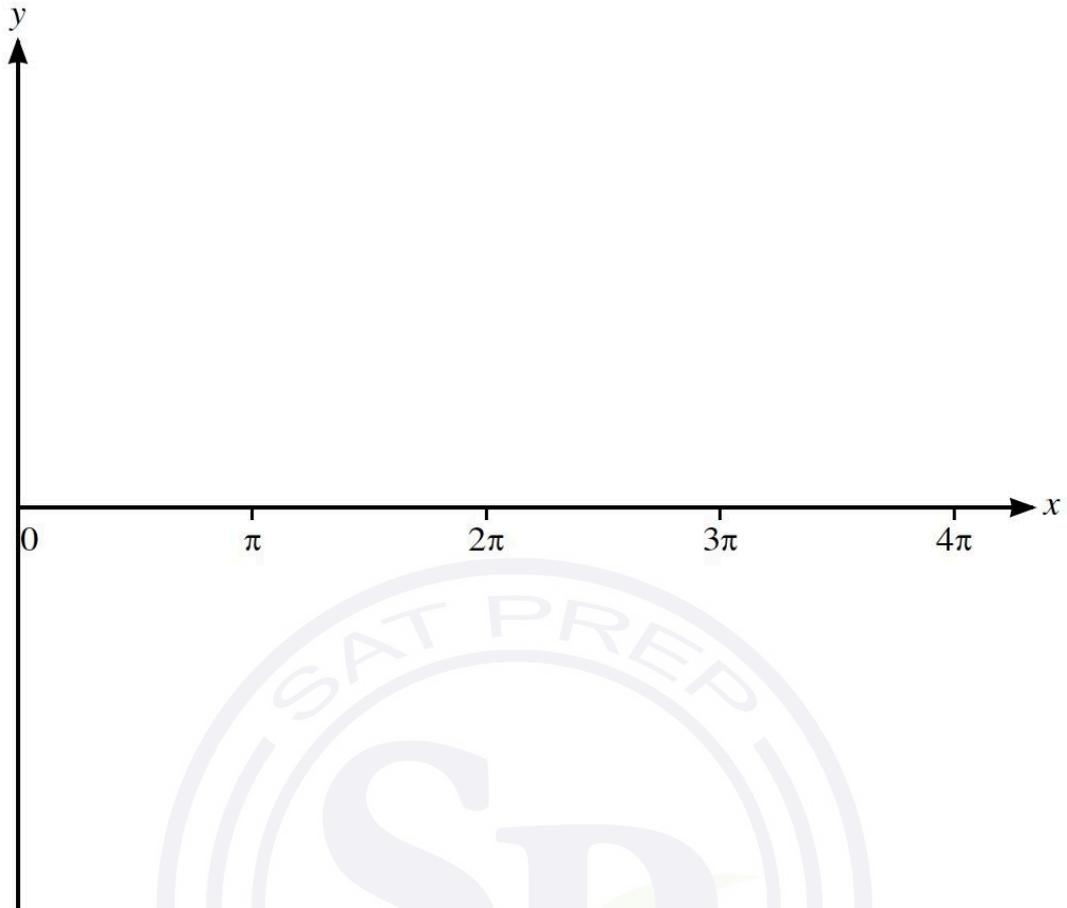
Question 97

A curve has equation $y = 2 + 3 \sin \frac{1}{2}x$ for $0 \leq x \leq 4\pi$.

(a) State greatest and least values of y . [2]

(b) Sketch the curve. [2]

Continue on the next page...



(c) State the number of solutions of the equation

$$2 + 3 \sin \frac{1}{2}x = 5 - 2x$$

for $0 \leq x \leq 4\pi$.

[1]

Question 98

The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b .

[4]

For the rest of this question, you should use the value of a which you found in (a).

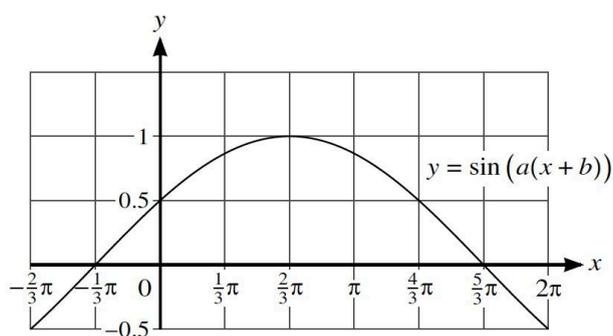
(b) Find the domain of f^{-1} .

[1]

(c) Find an expression for $f^{-1}(x)$.

[3]

Question 99



The diagram shows part of the graph of $y = \sin(a(x + b))$, where a and b are positive constants.

- (a) State the value of a and one possible value of b . [2]

Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x + 8)\right).$$

- (b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q . [3]

Question 100

The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

- (a) State the range of f . [1]
 (b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

- (c) Obtain a simplified expression for $gf(x)$. [2]

Question 101

Functions f and g are defined by

$$f(x) = (x + a)^2 - a \text{ for } x \leq -a,$$

$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

- (a) Find an expression for $f^{-1}(x)$. [3]
 (b) (i) State the domain of the function f^{-1} . [1]
 (ii) State the range of the function f^{-1} . [1]
 (c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$. [3]

Question 102

The equation of a curve is $y = x^2 - 8x + 5$.

- (a) Find the coordinates of the minimum point of the curve. [2]

The curve is stretched by a factor of 2 parallel to the y -axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

- (b) Find the coordinates of the minimum point of the transformed curve. [2]
- (c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a , b and c are integers to be found. [4]

Question 103

- (a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

- (b) Given that it is possible to form the composite function gf , find the least possible value of p and the greatest possible value of q . [3]
- (c) Find an expression for $gf(x)$. [1]

The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

- (d) Find an expression for $h^{-1}(x)$. [3]

Question 104

The transformation R denotes a reflection in the x -axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

- (a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T . [2]
- (b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R . [2]
- (c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

Question 105

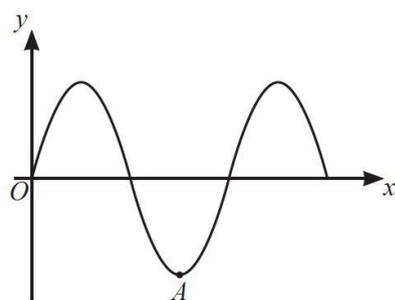
The functions f and g are defined for all real values of x by

$$f(x) = (3x - 2)^2 + k \quad \text{and} \quad g(x) = 5x - 1,$$

where k is a constant.

- (a) Given that the range of the function gf is $gf(x) \geq 39$, find the value of k . [4]
- (b) For this value of k , determine the range of the function fg . [2]
- (c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.
Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]

Question 106



The diagram shows part of the curve with equation $y = k \sin \frac{1}{2}x$, where k is a positive constant and x is measured in radians. The curve has a minimum point A .

(a) State the coordinates of A . [1]

(b) A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative y -direction

Reflection in the x -axis

Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to A . [3]

Question 107

The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f . [3]

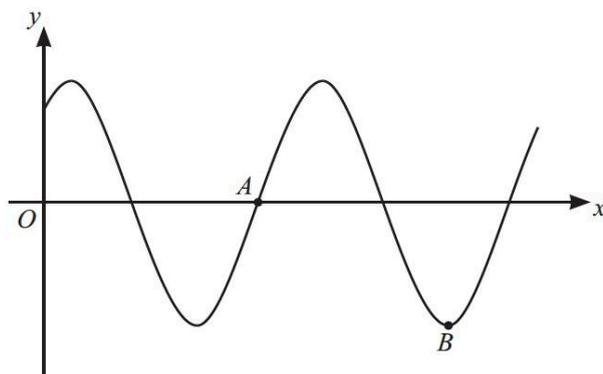
The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P .

Determine the coordinates of P . [6]

Question 108

(a)



The diagram shows the curve $y = k \cos(x - \frac{1}{6}\pi)$ where k is a positive constant and x is measured in radians. The curve crosses the x -axis at point A and B is a minimum point.

Find the coordinates of A and B . [3]

(b) Find the exact value of t that satisfies the equation

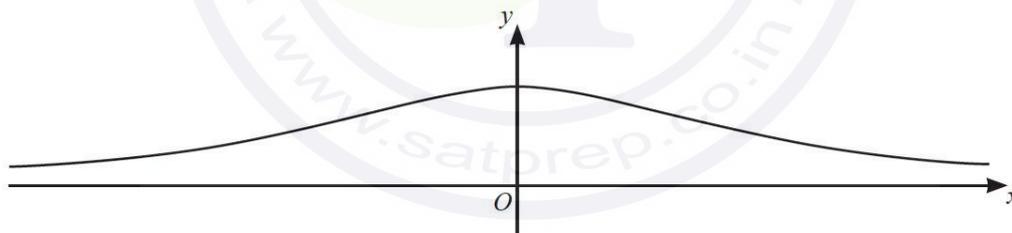
$$3 \sin^{-1}(3t) + 2 \cos^{-1}\left(\frac{1}{2}\sqrt{2}\right) = \pi. \quad [2]$$

Question 109

The function f is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$. [1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \frac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $h(x) = \frac{1}{x^2 + 2}$ for $x \geq 0$.

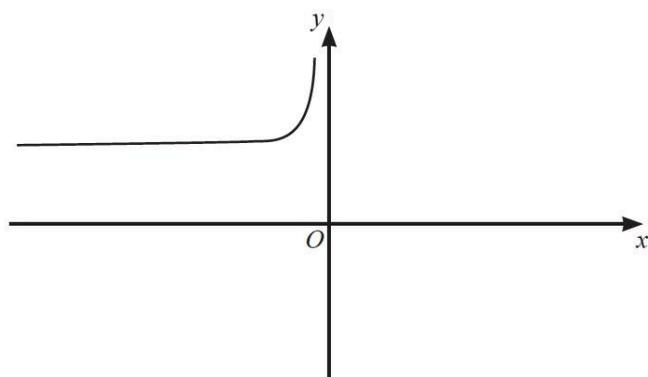
(c) Solve the equation $hf(x) = f\left(\frac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a , b and c are integers. [4]

Question 110

The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

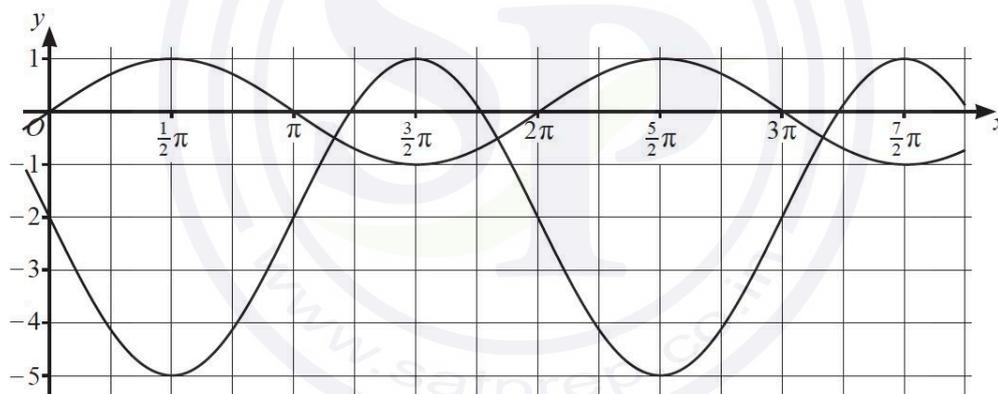
Question 111



The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

- (a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]
- (b) Find an expression for $f^{-1}(x)$. [3]
- (c) Solve the equation $f(x) = 4.5$. [1]
- (d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

Question 112



The diagram shows two curves. One curve has equation $y = \sin x$ and the other curve has equation $y = f(x)$.

- (a) In order to transform the curve $y = \sin x$ to the curve $y = f(x)$, the curve $y = \sin x$ is first reflected in the x -axis.
Describe fully a sequence of two further transformations which are required. [4]
- (b) Find $f(x)$ in terms of $\sin x$. [2]

Question 113

- (a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geq k$, where k is a constant.

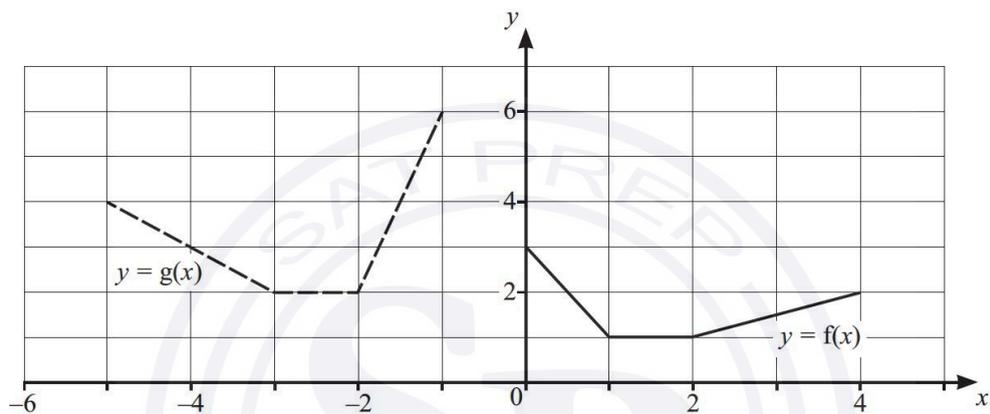
- (b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part (b).

- (c) Find an expression for $f^{-1}(x)$. [3]

- (d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

Question 114



In the diagram, the graph with equation $y = f(x)$ is shown with solid lines and the graph with equation $y = g(x)$ is shown with broken lines.

- (a) Describe fully a sequence of three transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [6]

- (b) Find an expression for $g(x)$ in the form $af(bx+c)$, where a , b and c are integers. [2]

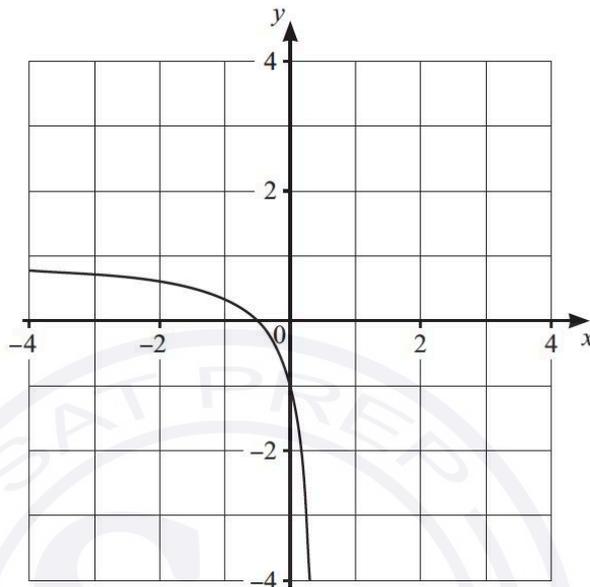
Question 115

The function f is defined by $f(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

(a) (i) State the value of $f(-1)$.

[1]

(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line.

[2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1} .

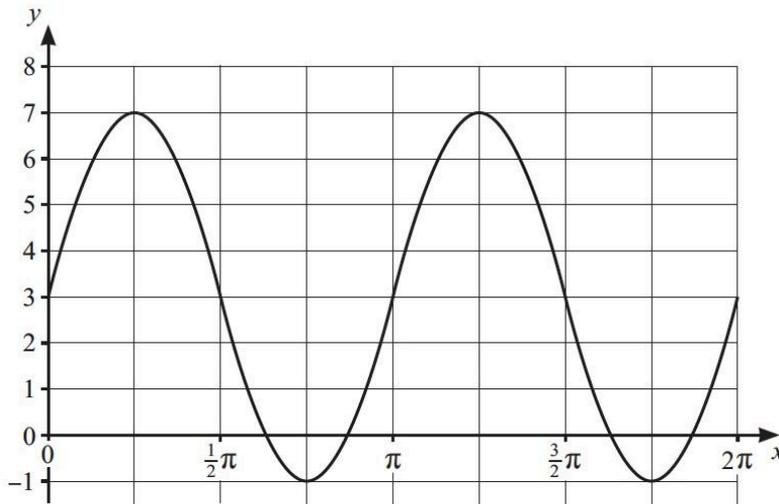
[4]

The function g is defined by $g(x) = 3x+2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$.

[3]

Question 116



The diagram shows the curve with equation $y = a \sin(bx) + c$ for $0 \leq x \leq 2\pi$, where a , b and c are positive constants.

- (a) State the values of a , b and c . [3]
- (b) For these values of a , b and c , determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for each of the following equations:
- (i) $a \sin(bx) + c = 7 - x$ [1]
- (ii) $a \sin(bx) + c = 2\pi(x - 1)$. [1]

Question 117

The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

- (a) Express $f(x)$ in the form $a - b(x - c)^2$, where a , b and c are constants, and state the range of f . [3]
- (b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin. Give details of the reflection and translation involved. [2]

The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leq 0$.

- (c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]
- (d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in (c), and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

Question 118

Functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - c \quad \text{and} \quad g(x) = 2x + k,$$

where c and k are positive constants. It is given that $g^{-1}(3k+1) = c$.

(a) Show that $gf(x) = 8x^2 - k - 1$. [4]

(b) The curve with equation $y = 8x^2 - k - 1$ is transformed to the curve with equation $y = h(x)$ by the following sequence of transformations.

Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Stretch in the y -direction by scale factor k

Reflection in the x -axis

Find an expression for $h(x)$ in terms of x and k . [3]

(c) The range of h is given by $h(x) \leq 15$.

Find the values of c and k . [3]

Question 119

The function f is defined by $f(x) = x^2 + 4ax + a$ for $x \in \mathbb{R}$, where a is a constant.

The function g is such that $g^{-1}(x) = \sqrt[3]{2x-4}$ for $x \in \mathbb{R}$.

(a) Given that the range of f is $f(x) \geq -33$, find the possible values of a . [4]

(b) Given instead that $fgg(0) = 96$, find the value of a . [6]

Question 120

(a) Express $x^2 + 4x + 2$ in the form $(x+a)^2 + b$, where a and b are integers. [2]

The functions f and g are defined as follows.

$$f(x) = x^2 + 4x + 2 \quad \text{for } x \leq -2$$

$$g(x) = -x - 4 \quad \text{for } x \geq -2$$

(b) (i) Find an expression for $f^{-1}(x)$. [3]

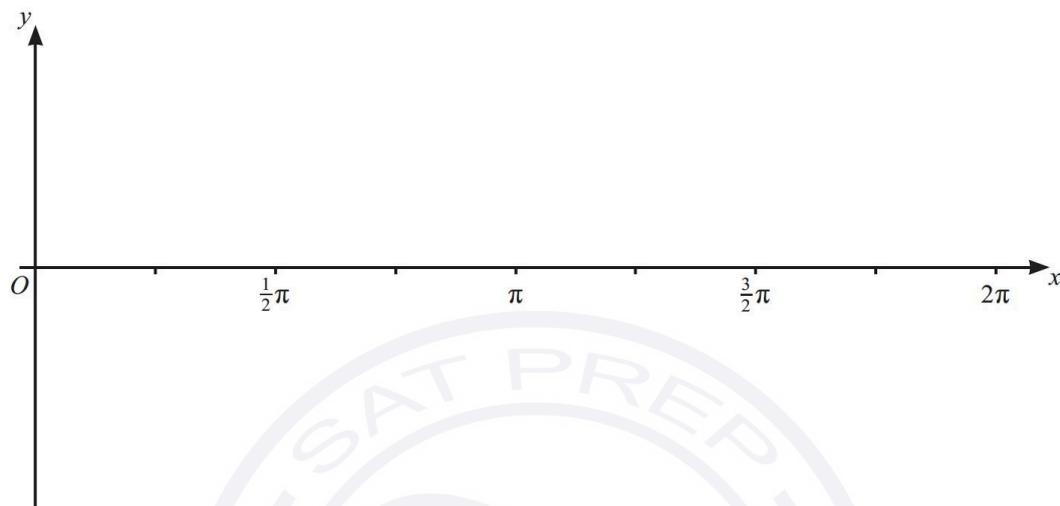
(ii) Find an expression for $(gf)^{-1}(x)$. [4]

Question 121

The equation of a curve is $y = 4 \cos 2x + 3$ for $0 \leq x \leq 2\pi$.

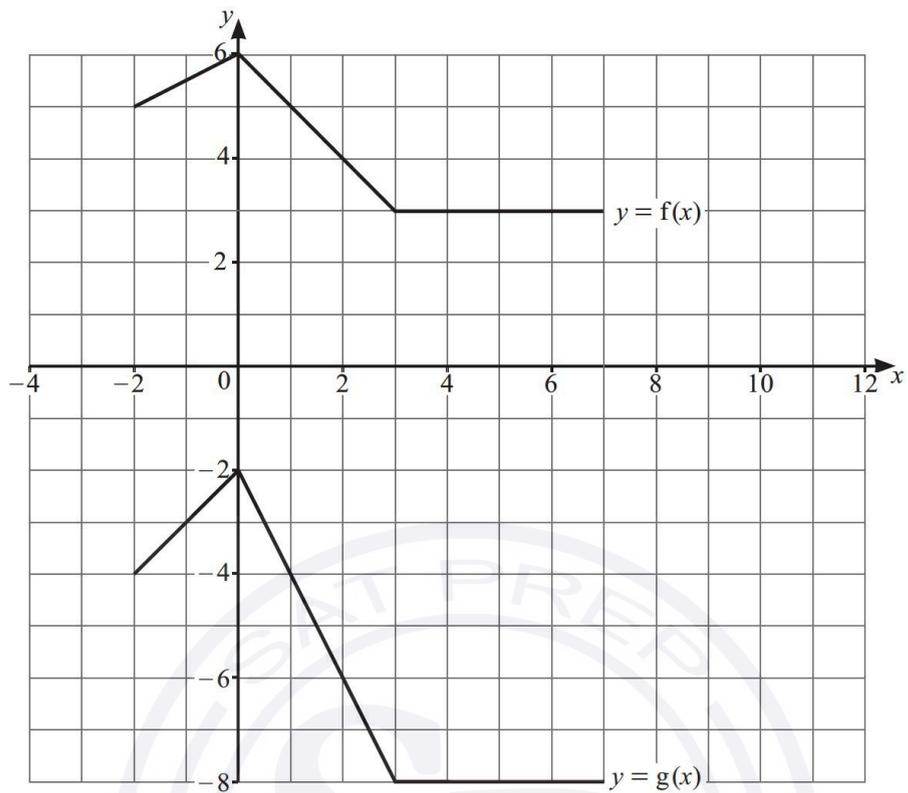
(a) State the greatest and least possible values of y . [2]

(b) Sketch the curve. [2]



(c) Hence determine the number of solutions of the equation $4 \cos 2x + 3 = 2x - 1$ for $0 \leq x \leq 2\pi$. [1]

Question 122



The diagram shows the graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. Make clear the order in which the transformations should be applied. [4]

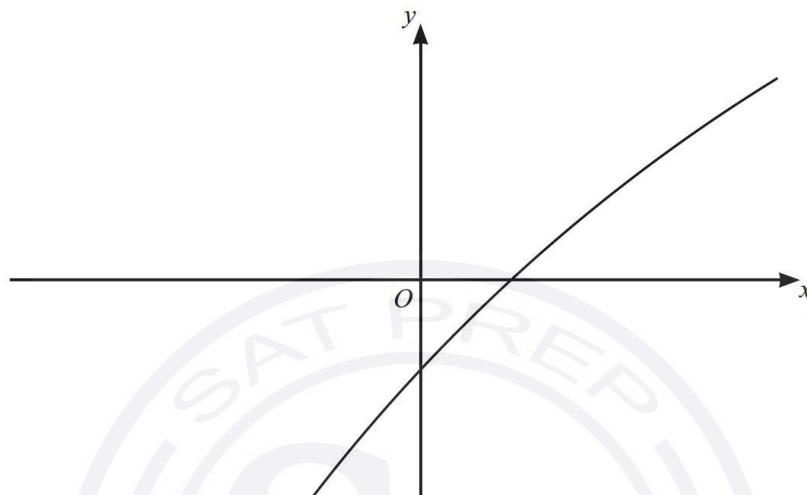
Question 123

The functions f and g are defined by

$$f(x) = \sqrt{x} \quad \text{for } x \geq 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \quad \text{for } x \geq -2.$$

- (a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. [5]



The diagram shows the graph of $y = g(x)$.

- (b) On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. [2]
 (c) Find an expression for $g^{-1}(x)$. [2]
 (d) State the range of g^{-1} . [1]

The function h is defined by

$$h(x) = x - 2 \quad \text{for } x \geq 0.$$

- (e) Find the value of $g^{-1}h(4)$. [1]
 (f) Explain why the composite function hg^{-1} cannot be formed. [1]