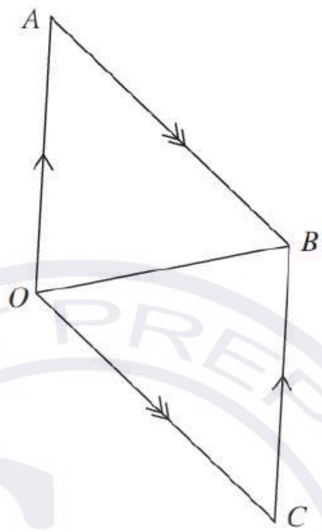
AS-Level

Vector

2013-2018

Question 1



The diagram shows a parallelogram $OABC$ in which

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

(i) Use a scalar product to find angle BOC. [6]

(ii) Find a vector which has magnitude 35 and is parallel to the vector $\overrightarrow{OC}$. [2]

Question 2

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) State the values of p and q for which $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$. [2]

(ii) In the case where $q = 2p$, find the value of p for which angle BOA is 90°. [2]

(iii) In the case where $p = 1$ and $q = 8$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

Question 3

Relative to an origin O, the position vectors of three points, A, B and C, are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

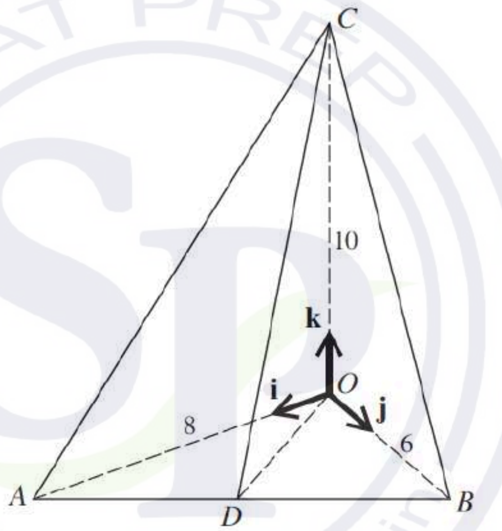
where p and q are constants.

(i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q. [2]

(ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q. [2]

(iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

Question 4



The diagram shows a pyramid $OABC$ in which the edge OC is vertical. The horizontal base OAB is a triangle, right-angled at O, and D is the mid-point of AB. The edges OA, OB and OC have lengths of 8 units, 6 units and 10 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

(i) Express each of the vectors $\overrightarrow{OD}$ and $\overrightarrow{CD}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to find angle ODC. [4]

Question 5

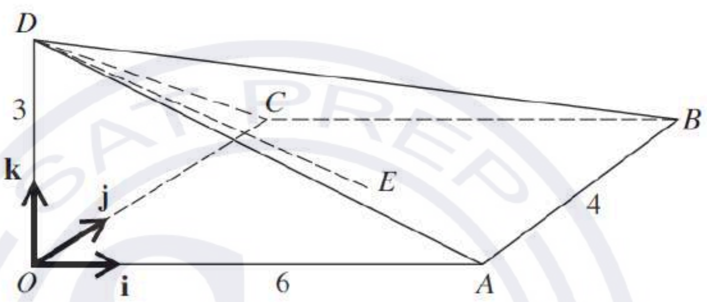
Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} + p\mathbf{k}.$$

(i) In the case where $p = 6$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(ii) Find the values of p for which angle $AOB = \cos^{-1}\left(\frac{1}{5}\right)$. [4]

Question 6



The diagram shows a pyramid $OABCD$ in which the vertical edge OD is 3 units in length. The point E is the centre of the horizontal rectangular base $OABC$. The sides OA and AB have lengths of 6 units and 4 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Express each of the vectors $\overrightarrow{DB}$ and $\overrightarrow{DE}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to find angle BDE. [4]

Question 7

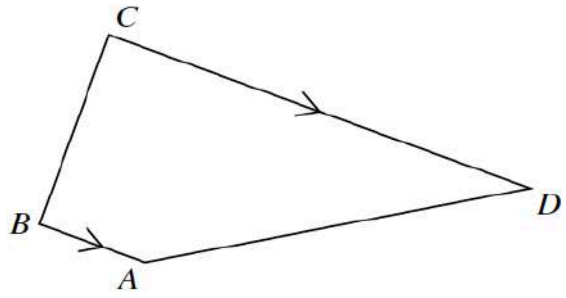
The position vectors of points A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}.$$

(i) Show that angle $BAC = \cos^{-1}\left(\frac{1}{3}\right)$. [5]

(ii) Use the result in part **(i)** to find the exact value of the area of triangle ABC. [3]

Question 8



The diagram shows a trapezium $ABCD$ in which BA is parallel to CD. The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

(i) Use a scalar product to show that AB is perpendicular to BC. [3]

(ii) Given that the length of CD is 12 units, find the position vector of D. [4]

Question 9

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3p \\ 4 \\ p^2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} -p \\ -1 \\ p^2 \end{pmatrix}.$$

(i) Find the values of p for which angle AOB is 90°. [3]

(ii) For the case where $p = 3$, find the unit vector in the direction of $\overrightarrow{BA}$. [3]

Question 10

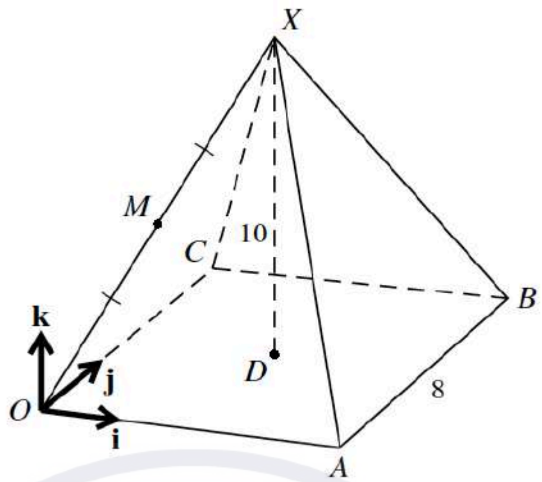
Three points, O, A and B, are such that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + p\mathbf{k}$ and $\overrightarrow{OB} = -7\mathbf{i} + (1-p)\mathbf{j} + p\mathbf{k}$, where p is a constant.

(i) Find the values of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [3]

(ii) The magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are a and b respectively. Find the value of p for which $b^2 = 2a^2$. [2]

(iii) Find the unit vector in the direction of $\overrightarrow{AB}$ when $p = -8$. [3]

Question 11



The diagram shows a pyramid $OABCX$. The horizontal square base $OABC$ has side 8 units and the centre of the base is D. The top of the pyramid, X, is vertically above D and $XD = 10$ units. The mid-point of OX is M. The unit vectors **i** and **j** are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector **k** is vertically upwards.

(i) Express the vectors $\overrightarrow{AM}$ and $\overrightarrow{AC}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle MAC. [4]

Question 12

Relative to an origin O, the position vector of A is $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and the position vector of B is $7\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

(i) Show that angle OAB is a right angle. [4]

(ii) Find the area of triangle OAB. [3]

Question 13

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Show that angle ABC is 90°. [4]

(ii) Find the area of triangle ABC, giving your answer correct to 1 decimal place. [3]

Question 14

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}.$$

(i) Use a vector method to find angle AOB. [4]

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$.

(ii) Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(iii) Show that triangle OAC is isosceles. [1]

Question 15

Relative to the origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}.$$

(i) Find the cosine of angle AOB. [3]

The position vector of C is given by $\overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k-3 \end{pmatrix}$.

(ii) Given that AB and OC have the same length, find the possible values of k. [4]

Question 16

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} p-6 \\ 2p-6 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4-2p \\ p \\ 2 \end{pmatrix},$$

where p is a constant.

(i) For the case where OA is perpendicular to OB, find the value of p. [3]

(ii) For the case where OAB is a straight line, find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Find also the length of the line OA. [4]

Question 17

Relative to an origin O, the position vectors of points A, B and C are given by

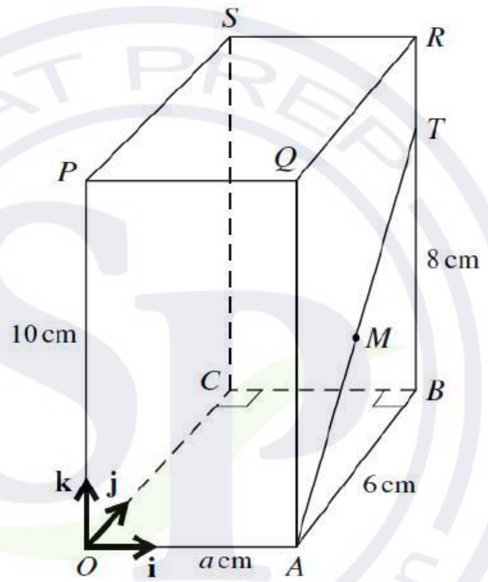
$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}.$$

(i) In the case where ABC is a straight line, find the values of p and q. [4]

(ii) In the case where angle BAC is 90°, express q in terms of p. [2]

(iii) In the case where $p = 3$ and the lengths of AB and AC are equal, find the possible values of q. [3]

Question 18

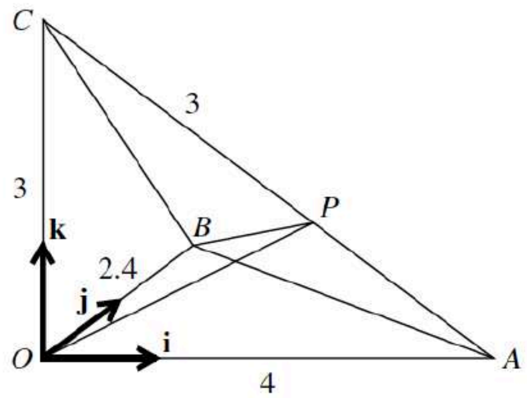


The diagram shows a cuboid $OABCPQRS$ with a horizontal base $OABC$ in which $AB = 6$ cm and $OA = a$ cm, where a is a constant. The height OP of the cuboid is 10 cm. The point T on BR is such that $BT = 8$ cm, and M is the mid-point of AT. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OP respectively.

(i) For the case where $a = 2$, find the unit vector in the direction of $\overrightarrow{PM}$. [4]

(ii) For the case where angle $ATP = \cos^{-1}\left(\frac{2}{7}\right)$, find the value of a. [5]

Question 19



The diagram shows a pyramid $OABC$ with a horizontal triangular base OAB and vertical height OC. Angles AOB, BOC and AOC are each right angles. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC respectively, with $OA = 4$ units, $OB = 2.4$ units and $OC = 3$ units. The point P on CA is such that $CP = 3$ units.

(i) Show that $\overrightarrow{CP} = 2.4\mathbf{i} - 1.8\mathbf{k}$. [2]

(ii) Express $\overrightarrow{OP}$ and $\overrightarrow{BP}$ in terms of **i**, **j** and **k**. [2]

(iii) Use a scalar product to find angle BPC. [4]

Question 20

The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where p is a constant.

(i) Find the value of p for which the lengths of AB and CB are equal. [4]

(ii) For the case where $p = 1$, use a scalar product to find angle ABC. [4]

Question 21

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

Question 22

Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where k is a constant.

(i) Find the value of k in the case where angle $AOB = 90°$. [2]

(ii) Find the possible values of k for which the lengths of AB and OC are equal. [4]

The point D is such that $\overrightarrow{OD}$ is in the same direction as $\overrightarrow{OA}$ and has magnitude 9 units. The point E is such that $\overrightarrow{OE}$ is in the same direction as $\overrightarrow{OC}$ and has magnitude 14 units.

(iii) Find the magnitude of $\overrightarrow{DE}$ in the form $\sqrt{n}$ where n is an integer. [4]

Question 23

The diagram shows a triangular pyramid $ABCD$. It is given that

$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB, DAC and CAB is 90°. [3]

(ii) Find the exact value of the area of the triangle ABC, and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

Question 24

Relative to an origin O, the position vectors of the points A, B and C are given by

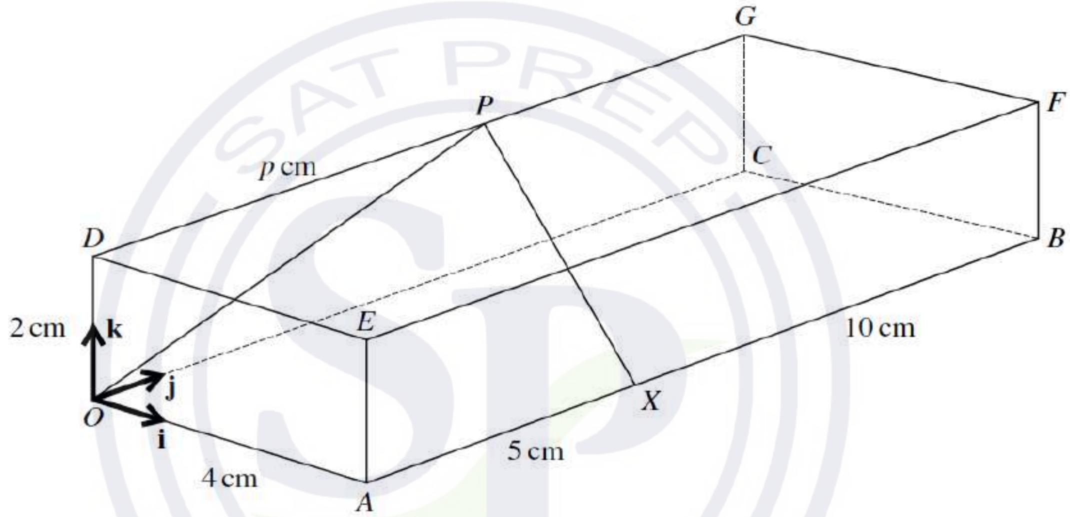
$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

Question 25



The diagram shows a cuboid $OABCDEFG$ with a horizontal base $OABC$ in which $OA = 4$ cm and $AB = 15$ cm. The height OD of the cuboid is 2 cm. The point X on AB is such that $AX = 5$ cm and the point P on DG is such that $DP = p$ cm, where p is a constant. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

(i) Find the possible values of p such that angle $OPX = 90°$. [4]

(ii) For the case where $p = 9$, find the unit vector in the direction of $\overrightarrow{XP}$. [2]

(iii) A point Q lies on the face $CBFG$ and is such that XQ is parallel to AG. Find $\overrightarrow{XQ}$. [3]

Question 26

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

(i) Use a scalar product to find angle OAB. [5]

(ii) Find the area of triangle OAB. [2]

Question 27

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

(i) Find the position vector of P. [3]

(ii) Find the distance OP. [1]

(iii) Determine whether OP is perpendicular to AB. Justify your answer. [2]

Question 28

Relative to an origin O, the position vectors of three points A, B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) In the case where $p = 2$, use a scalar product to find angle AOB. [4]

(ii) In the case where $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, find the values of p and q. [4]

Question 29

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90°$.

(i) Find the value of p. [2]

The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. [4]

Question 30

Relative to an origin O, the position vectors of the points A, B and C are given by

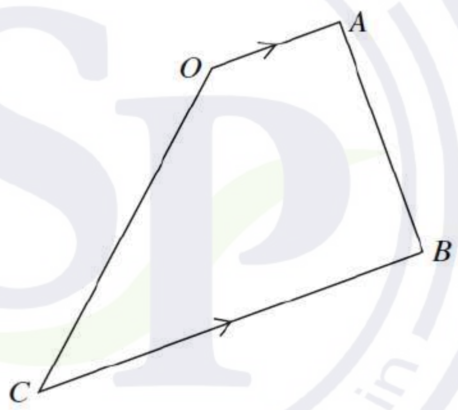
$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

A fourth point, D, is such that the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$ are the first, second and third terms respectively of a geometric progression.

(i) Find the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$. [5]

(ii) Given that D is a point lying on the line through B and C, find the two possible position vectors of the point D. [4]

Question 31



The diagram shows a trapezium $OABC$ in which OA is parallel to CB. The position vectors of A and B relative to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

(i) Show that angle OAB is 90°. [3]

The magnitude of $\overrightarrow{CB}$ is three times the magnitude of $\overrightarrow{OA}$.

(ii) Find the position vector of C. [3]

(iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]

Question 32

(a) Relative to an origin O, the position vectors of two points P and Q are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R is such that PQR is a straight line with Q the mid-point of PR. Find the position vector of R in terms of $\mathbf{p}$ and $\mathbf{q}$, simplifying your answer. [3]

(b) The vector $6\mathbf{i} + a\mathbf{j} + b\mathbf{k}$ has magnitude 21 and is perpendicular to $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the possible values of a and b, showing all necessary working. [6]

Question 33

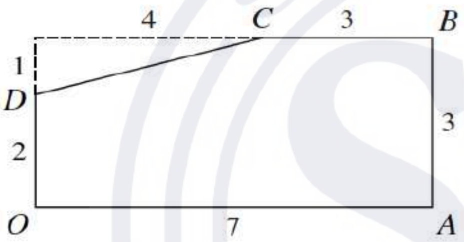


Fig. 1

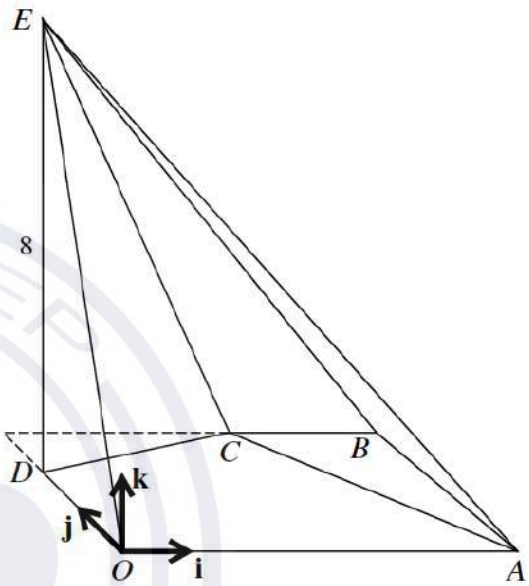


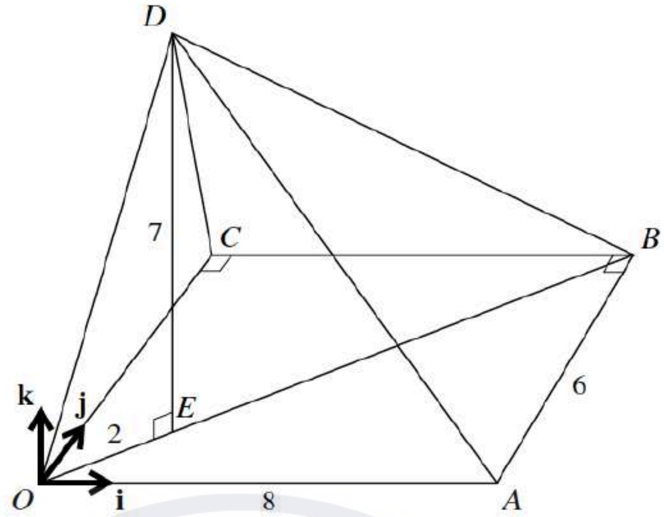
Fig. 2

Fig. 1 shows a rectangle with sides of 7 units and 3 units from which a triangular corner has been removed, leaving a 5-sided polygon $OABCD$. The sides OA, AB, BC and DO have lengths of 7 units, 3 units, 3 units and 2 units respectively. Fig. 2 shows the polygon $OABCD$ forming the horizontal base of a pyramid in which the point E is 8 units vertically above D. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OD and DE respectively.

(i) Find $\overrightarrow{CE}$ and the length of CE. [3]

(ii) Use a scalar product to find angle ECA, giving your answer in the form $\cos^{-1}\left(\frac{m}{\sqrt{n}}\right)$, where m and n are integers. [5]

Question 34

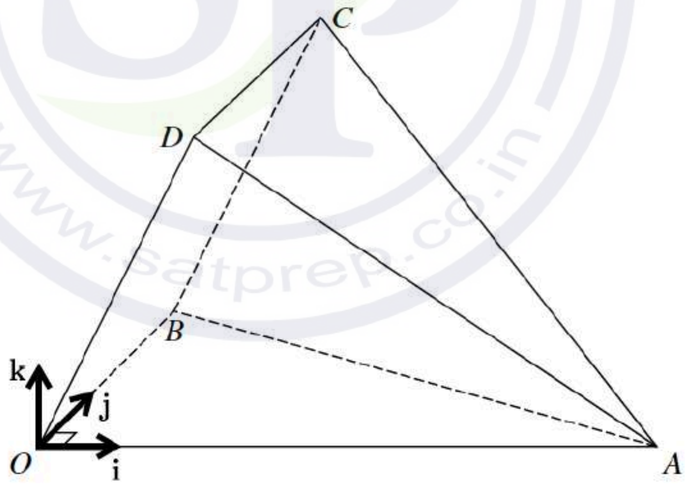


The diagram shows a pyramid $OABCD$ with a horizontal rectangular base $OABC$. The sides OA and AB have lengths of 8 units and 6 units respectively. The point E on OB is such that $OE = 2$ units. The point D of the pyramid is 7 units vertically above E. Unit vectors **i**, **j** and **k** are parallel to OA, OC and ED respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

(ii) Use a scalar product to find angle BDO. [7]

Question 35



The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors **i** and **j** are parallel to OA and OB respectively and the unit vector **k** is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to find angle CAD. [4]

Question 36

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.$$

(i) Find $\overrightarrow{AC}$. [1]

(ii) The point M is the mid-point of AC. Find the unit vector in the direction of $\overrightarrow{OM}$. [3]

(iii) Evaluate $\overrightarrow{AB}.\overrightarrow{AC}$ and hence find angle BAC. [4]