AS-Level

Coordinate Geometry

May 2013-May 2023

Answer

Question 1

	A (2, 14), B (14, 6) and C (7, 2).			
(i)	m of $AB = -⅔$	B1		
	m of perpendicular $= \frac{3}{2}$	M1		For use of $m_1m_2 = -1$
	eqn of AB $y-14 = -\frac{2}{3}(x-2)$	M1		Allow M1 for unsimplified eqn
	eqn of CX $y-2 = \frac{3}{2}(x-7)$	M1		Allow M1 for unsimplified eqn
	Sim Eqns $\rightarrow$ $X(11, 8)$	M1 A1	[6]	For solution of sim eqns.
(ii)	$AX : XB = 14-8 : 8-6 = 3 : 1$ Or $\sqrt{(9^2+6^2)} : \sqrt{(3^2+2^2)} = 3 : 1$	M1 A1	[2]	Vector steps or Pythagoras.

Question 2

$3y + 2x = 33.$		
Gradient of line $= -⅔$	B1	
Gradient of perpendicular $= 3/2$	M1	Use of $m_1m_2 = -1$ with gradient of line
Eqn of perp $y - 3 = \frac{3}{2}(x+1)$	M1	Correct form of perpendicular eqn.
Sim Eqns $\rightarrow$ (3, 9)	M1 A1	Sim eqns.
$(-1, 3) \rightarrow (3, 9) \rightarrow (7, 15)$	M1 A1 [7]	Vectors or other method.

Question 3

(i)	$x^2-4x+4=x \Rightarrow x^2-5x+4=0$	M1	Eliminate y to reach 3-term quadratic
	$(x-1)(x-4)(=0)$ or other valid method	M1	Attempt solution
	(1, 1), (4, 4)	A1	
	Mid-point = (2½, 2½)	A1 √	ft dependent on 1st M1
		[4]	
(ii)	$x^2-(4+m)x+4=0 \to (4+m)^2-4(4)=0$	M1	Applying $b^2-4ac=0$
	$4+m=\pm 4$ or $m(8+m)=0$	DM1	Attempt solution
	$m=-8$	A1	Ignore $m=0$ in addition
	$x^2+4x+4=0$	M1	Sub non-zero m and attempt to solve
	$x=-2,\ y=16$	A1	Ignore (2, 0) solution from $m=0$
		[5]	
t (ii)	$2x-4=m$	M1	**OR** $2x-4=m$
	$x^2-4x+4=(2x-4)x$	DM1	Sub $x=\dfrac{m+4}{2},\ y=\dfrac{m(m+4)}{2}$ into quad
	$x=-2$ (ignore +2)	A1	$m=-8$ from resulting quad $m(m+8)=0$
	$m=-8$ (ignore 0)	A1	$x=-2$
	$y=16$	A1	$y=16$

Question 4

(i)	gradient of perpendicular = −½ soi	B1	
	$y-1=-½(x-3)$	B1	
		[2]	
(ii)	$C=(-9, 6)$	B1	soi in **(i)** or **(ii)**
	$AC^2=[3-(-9)]^2+[1-6]^2$ (ft on *their* C)	M1	**OR** $AB^2=[3-(-21)]^2+[1-11]^2$ M1
	$AC=13$	A1	$AB=26$ A1
		[3]	$AC=13$ A1

Question 5

A (0, 8) B (4, 0) $8y+x=33$		
m of $AB=-2$	B1	
m of $BC=½$	M1	Use of $m_1m_2=-1$ for BC or AD
Eqn $BC \to y-0=½(x-4)$	M1	Correct method for equation of BC
Sim eqns $\to C$ (16, 6)	M1 A1	Sim Eqns for BC, AC.
Vector step method $\to D$ (12, 14)	M1 A1	M1 valid method.
(or AD $y=½x+8$, CD $y=-2x+38$)	[7]	
(or $M=(8, 7) \to D=(12, 14)$)		

Question 6

(i) mid-point = (3, 4)	**B1**	soi
Grad. $AB = -\frac{1}{2} \rightarrow$ grad. of perp., = 2	**M1**	For use of $-1/m$ soi
$y - 4 = 2(x - 3)$	**M1**	ft on *their* (3, 4) and 2
$y = 2x - 2$	**A1**	
	[4]	
(ii) $q = 2p - 2$ √ $\qquad p^2 + q^2 = 4$ oe	**B1**√ **B1**	ft for 1st eqn.
$p^2 + (2p-2)^2 = 4 \rightarrow 5p^2 - 8p = 0$ $\{\text{OR } \frac{1}{4}(q+2)^2 + q^2 = 4 \rightarrow 5q^2 + 4q - 12 = 0\}$	**M1**	Attempt substn (linear into quadratic) & simplify
$(0, -2)$ and $\left(\frac{8}{5}, \frac{6}{5}\right)$	**A1A1**	
	[5]	

Question 7

Sim eqns $\rightarrow$ A (1, 3)	M1 A1	co Allow answer only B2
Vectors or mid-point $\rightarrow$ C (12, 14)	M1 A1√	Allow answer only B2√
Eqn of BC $4y = x + 44$ or CD $y = 3x - 22$	M1	equation ok – unsimplified
Sim eqns $\rightarrow$ B(4, 12) or D (9, 5)	DM1A1	Sim eqns. co
Vectors or mid-point $\rightarrow$ B(4, 12) or D (9, 5)	DM1A1	Valid method (or sim eqns) co
	[9]	

Question 8

(2, 7) to (10, 3)		
Mid-point (6, 5)	B1	co
Gradient = $-\frac{1}{2}$	B1	co
Perp gradient = 2	B1√	co
Eqn $y - 5 = 2(x - 6)$	M1	Must be correct form of Perp
Sets y to 0, $\rightarrow$ ($3\frac{1}{2}$, 0)	A1	co $x = 3\frac{1}{2}$ only is ok.
	[5]	

Question 9

$(a-3)^2 + (2-b)^2 = 125$ oe	**B1**	
$\frac{2-b}{a-3} = 2$ oe	**B1**	
$(a-3)^2 + (2a-6)^2 = 125$ (sub for a or b)	**M1**	Or $1/4(2-b)^2 + (2-b)^2 = 125$
$(5)(a+2)(a-8)$ (= 0) Attempt factorise/solve	**M1**	Or $(5)(b-12)(b+8)$ (= 0)
$a = -2$ or 8, $b = 12$ or -8	**A1A1**	Answers (no working) after 2 correct eqns score SCB1B1 for each correct pair (a, b)
	[6]	

Question 10

(i)	$m = \dfrac{3a+9-(2a-1)}{2a+4-a} = \dfrac{a+10}{a+4}$ oe e.g. $\dfrac{-a-10}{-a-4}$	**M1A1**	cao Allow omission of brackets for M1
	Gradient of perpendicular $= \dfrac{-(a+4)}{a+10}$ oe but not $\dfrac{-1}{\left(\dfrac{a+10}{a+4}\right)}$	**A1**√	Do not ISW. Max penalty for erroneous cancellation 1 mark
		[3]	
(ii)	$(\sqrt{})[(a+4)^2 + (a+10)^2] = (\sqrt{})260$	**M1**	Allow *their* $(a+4)$, $(a+10)$ from (i). Allow $(-a-4)^2$ etc. Allow omission of brackets
	$(\sqrt{})[(a+4)^2 + (a+10)^2]$ cao	**A1**	
	$(2)(a^2+14a-72)\ (=0)$	**A1**	
	$a = 4$ or -18 cao	**A1**	
		[4]	

Question 11

(i)	$m_{AB} = -3$ or $\dfrac{-9}{3}$	B1	oe
	$m_{AD} = \dfrac{1}{3}$	M1	use of $m_1m_2 = -1$ with grad AB
	Eqn AD $y-6 = \dfrac{1}{3}(x-2)$ or $3y = x+16$	A1	co – OK unsimplified
		[3]	
(ii)	Eqn CD $y-3 = -3(x-8)$ or $y = -3x+27$	B1√	OK unsimplified. √on m of AB.
	Sim Eqns	M1	Reasonable algebra leading to $x =$ or $y =$ with AD and CD
	$\rightarrow$ D (6½, 7½)	A1	
		[3]	
(iii)	Use of vectors or mid-point $\rightarrow$ E (5, 12) or mid-point (5,4.5)	B1	May be implied
	Length of $BE = 15$	B1	co
		[2]	

Question 12

(i)	$32-4k = 20 \Rightarrow k = 3$	**M1A1**	Sub (8, –4)	[alt: $(2b+4)/(b-8) = -4/k$
	$4b + 3\times 2b = 20$	**M1**	Sub (b, $2b$),	$4b + 2bk = 20$
	$b = 2$	**A1**		**M1** both **M1** solving **A1**, **A1**]
		[4]		
(ii)	Mid-point = (5, 0)	**B1**√	Ft on *their b*	
		[1]		

Question 13

(i)	$(9-p)^2+(3p)^2=169$	**M1**	Or $\sqrt{} = 13$
	$10p^2-18p-88\,(=0)$ oe	**A1**	3-term quad
	$p=4$ ***or*** $-11/5$ oe	**A1**	
		[3]	
(ii)	Gradient of given line $= -\dfrac{2}{3}$	**B1**	
	Hence gradient of AB $= \dfrac{3}{2}$	**M1**	Attempt using $m_1m_2=-1$
	$\dfrac{3}{2}=\dfrac{3p}{9-p}$ oe eg $\left(\dfrac{-2}{3}\right)\left(\dfrac{3p}{9-p}\right)=1$	**M1**	Or vectors $\begin{pmatrix}9-p\\3p\end{pmatrix}.\begin{pmatrix}3\\-2\end{pmatrix}$
	(includes previous M1)		
	$p=3$	**A1**	
		[4]	

Question 14

	$A(4, 6)$, $B\,(10, 2)$.		
(i)	$M=(7, 4)$	B1	co
	m of $AB = -\frac{2}{3}$	B1	co
	m of perpendicular $= \frac{3}{2}$		
	$\rightarrow\ y-4=\frac{3}{2}(x-7)$	M1 A1	Use of $m_1m_2=-1$ & their midpoint
		[4]	in the equation of a line. co
(ii)	Eqn of line parallel to AB through (3, 11)		
	$\rightarrow\ y-11=-\frac{2}{3}(x-3)$	M1	Needs to use m of AB
	Sim eqns $\rightarrow$ $C\,(9, 7)$	DM1A1	Must be using their correct lines.
		[3]	Co

Question 15

(i)	$y-2t=-2(x-3t)(y+2x=8t)$	M1	Unsimplified or equivalent forms
	Set x to 0 $\rightarrow$ $B(0, 8t)$		
	Set y to 0 $\rightarrow$ $A(4t, 0)$	M1	Attempt at both A and B, then using
	$\rightarrow$ Area $= 16t^2$	A1	cao
		[3]	
(ii)			cao
	$m=\dfrac{1}{2}$	B1	Unsimplified or equivalent forms
			co
	$\rightarrow\ y-2t=\dfrac{1}{2}(x-3t)(2y=x+t)$	M1	
		A1	correctly shown.
	Set y to 0 $\rightarrow$ $C\,(-t, 0)$		
	Midpoint of CP is (t, t)		
	This lies on the line $y=x$.	A1	
		[4]	

Question 16

	$A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$		
(i)	$AB = 10$	B1	
	$6^2 + (k-1)^2 = 10^2$	M1	Use of Pythagoras
	$k = -7$ and 9	A1 [3]	
(ii)	m of $AB = -\frac{3}{4}$ m perp $= \frac{4}{3}$	B1 M1	B1 M1 Use of $m_1m_2 = -1$
	$M = (1, 4)$		
	Eqn $y - 4 = \frac{4}{3}(x-1)$	B1	
	Set y to 0, $\rightarrow$ $x = -2$	M1 A1 [5]	Complete method leading to D.

Question 17

(i)	$x^2 - x + 3 = 3x + a \rightarrow x^2 - 4x + (3-a) = 0$	B1 [1]	AG
(ii)	$5 + (3-a) = 0 \rightarrow a = 8$	B1	Sub $x = -1$ into (i)
	$x^2 - 4x - 5 = 0 \rightarrow x = 5$	B1 [2]	OR B2 for $x = 5$ www
(iii)	$16 - 4(3-a) = 0$ (applying $b^2 - 4ac = 0$)	M1	OR $dy/dx = 2x - 1 \rightarrow 2x - 1 = 3$
	$a = -1$	A1	$x = 2$
	$(x-2)^2 = 0 \rightarrow x = 2$	A1	$y = 2^2 - 2 + 3 \rightarrow y = 5$
	$y = 5$	A1 [4]	$5 = 6 + a \rightarrow a = -1$

Question 18

(a)	$3x = -\sqrt{3}/2$	M1	Accept -0.866 at this stage
	$x = \frac{-\sqrt{3}}{6}$ oe	A1 [2]	Or $\frac{-3}{6\sqrt{3}}$ or $\frac{-1}{2\sqrt{3}}$
(b)	$(2\cos\theta - 1)(\sin\theta - 1) = 0$	M1	Reasonable attempt to factorise and solve
	$\cos\theta = 1/2$ or $\sin\theta = 1$	A1	Award B1B1 www
	$\theta = \pi/3$ or $\pi/2$	A1A1 [4]	Allow 1.05, 1.57. SCA1for both 60°, 90°

Question 19

(i)	$AB^2 = 6^2 + 7^2 = 85,\ BC^2 = 2^2 + 9^2 = 85$ ($\rightarrow$ isosceles)	**B1B1**	Or $AB = BC = \sqrt{85}$ etc
	$AC^2 = 8^2 + 2^2 = 68$	**B1**	
	$M = (2, -2)$ or $BM^2 = (\sqrt{85})^2 - (½\sqrt{68})^2$	**B1**	Where M is mid-point of AC
	$BM = \sqrt{2^2 + 8^2} = \sqrt{68}$ or $\sqrt{85 - 17} = \sqrt{68}$	**B1**	
	Area $\Delta ABC = \frac{1}{2}\sqrt{68}\sqrt{68} = 34$	**B1** [6]	
(ii)	Gradient of $AB = 7/6$	**B1**	
	Equation of AB is $y + 1 = \frac{7}{6}(x + 2)$	**M1**	Or $y - 6 = \frac{7}{6}(x - 4)$
	Gradient of $CD = -6/7$	**M1**	
	Equation of CD is $y + 3 = \frac{-6}{7}(x - 6)$	**M1**	
	Sim Eqns $2 = \frac{-6}{7}x + \frac{36}{7} - \frac{7}{6}x - \frac{14}{6}$	**M1**	
	$x = \frac{34}{85} = \frac{2}{5}$ oe	**A1** [6]	

Question 20

	A (0, 7), B (8, 3) and C ($3k$, k)		
(i)	m of AB is $-½$ oe.	**B1**	Using A,B or C to get an equation
	Eqn of AB is $y = -½x + 7$	**M1**	Using C or A,B in the equation
	Let $x = 3k$, $y = k$	**M1**	
	$\boldsymbol{k = 2.8}$ **oe**	**A1**	
	OR		
	$\frac{7-k}{0-3k} = \frac{3-k}{8-3k}$	**M1A1**	Using A,B & C to equate gradients
	$\rightarrow 20k = 56 \rightarrow k = 2.8$	**DM1A1**	Simplifies to a linear or 3 term quadratic = 0.
	OR		
	$\frac{7-k}{0-3k} = \frac{7-3}{0-8}$	**M1A1**	Using A,B and C to equate gradients
	$\rightarrow 20k = 56 \rightarrow k = 2.8$	**DM1A1** [4]	Simplifies to a linear or 3 term quadratic = 0.

(ii)	M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y-5=2(x-4)$ Let $x = 3k$, $y = k$ $k=\frac{3}{5}$ oe **OR** $(0-3k)^2+(7-k)^2=(8-3k)^2+(3-k)^2$ $-14k+49=73-54k \rightarrow 40k=24 \rightarrow k=0.6$	**B1** **M1** **M1** **A1** **M1A1** **DM1A1** [4]	anywhere in (ii) Use of $m_1m_2=-1$ soi Forming eqn using their M and their "*perpendicular m*" Use of Pythagoras. Simplifies to a linear or 3 term quadratic = 0.

Question 21

$y=3x-\frac{4}{x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3+\frac{4}{x^2}$ m of $AB=4$ Equate $\rightarrow x=\pm 2$ $\rightarrow C\ (2, 4)$ and $D\ (-2, -4)$ $\rightarrow M\ (0, 0)$ or stating M is the origin m of $CD=2$ Perpendicular gradient $(=-\frac{1}{2})$ $\rightarrow y=-\frac{1}{2}x$	**B1** **B1** **M1 A1** **B1**√ **M1** **A1** [7]	 Equating + solution. √ on their C and D Use of $m_1m_2=-1$, must use m_{CD} (not m = 4)

Question 22

(i)	$\frac{2+x}{2}=n \Rightarrow x=2n-2$ $\frac{m+y}{2}=-6 \Rightarrow y=-12-m$	**B1** **B1**	[2]	No MR for (½(2+n), ½(m – 6)) Expect $(2n-2, -12-m)$
(ii)	Sub *their* x, y into $y=x+1 \rightarrow -12-m=2n-2+1$ $\frac{m+6}{2-n}=-1$ oe Not nested in an equation Eliminate a variable $m=-9,\ n=-1$	**M1*** **B1** **DM1** **A1A1**	[5]	Expect $m+2n=-11$ Expect $m-n=-8$ Note: other methods possible

Question 23

$A(a, 0)$ and $B(0, b)$	**B1**		soi
$a^2 + b^2 = 100$	**M1***		Uses Pythagoras with their A & B.
M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$	**B1**$\surd$		$\surd$ on their A and B.
M lies on $2x + y = 10$			
$\rightarrow \quad a + \frac{b}{2} = 10$	**M1***		Subs into given line, using their M, to link a and b.
Sub $\rightarrow \quad a^2 + (20 - 2a)^2 = 100$	**DM1**		Forms quadratic in a or in b.
or $\left(10 - \frac{b}{2}\right)^2 + b^2 = 100$			
$\rightarrow a = 6, b = 8.$	**A1**	[6]	cao

Question 24

(i)	$C = (4, -2)$	**B1**		
	$m_{AB} = -1/2 \rightarrow m_{CD} = 2$	**M1**		Use of $m_1 m_2 = -1$ on their m_{AB}
	Equation of CD is $y + 2 = 2(x - 4)$ oe	**M1**		Use of *their* C and m_{CD} in a line equation
	$y = 2x - 10$	**A1**	[4]	
(ii)	$AD^2 = (14 - 0)^2 + (-7 - (-10))^2$	**M1**		Use *their* D in a correct method
	$AD = 14.3$ or $\sqrt{205}$	**A1**	[2]	

Question 25

(i)	$\tan x = \cos x \rightarrow \sin x = \cos^2 x$	**M1**	Use tan = sin/cos and multiply by cos
	$\sin x = 1 - \sin^2 x$	**M1**	Use $\cos^2 x = 1 - \sin^2 x$
	$\sin x = 0.6180$. Allow $(-1 + \sqrt{5})/2$	**M1**	Attempt soln of quadratic in $\sin x$. Ignore solution –1.618. Allow x = 0.618
	x-coord of $A = \sin^{-1} 0.618 = 0.666$ cao	**A1**	Must be radians. Accept 0.212π
	Total:	**4**	
(ii)	***EITHER*** x-coord of B is $\pi - \textit{their}\, 0.666$	**(M1**	Expect 2.475(3). Must be radians throughout
	y-coord of B is $\tan(\textit{their}\ 2.475)$ or $\cos(\textit{their}\ 2.475)$	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787 cao	**A1)**	Accept $x = 0.788\pi$
	OR y-coord of B is – (cos or tan (*their* 0.666))	**(M1**	
	x-coord of B is $\cos^{-1}(\textit{their}\ y)$ or $\pi + \tan^{-1}(\textit{their}\ y)$	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787	**A1)**	Accept $x = 0.788\pi$
	Total:	**3**	

Question 26

(i)	$(b-1)/(a+1)=2$	M1	OR Equation of AP is $y-1=2(x+1)\to y=2x+3$
	$b=2a+3$ CAO	A1	Sub $x=a$,$y=b$ $\to b=2a+3$
	Total:	2	
(ii)	$AB^2=11^2+2^2=125$ oe	B1	Accept $AB=\sqrt{125}$
	$(a+1)^2+(b-1)^2=125$	B1 FT	FT on *their* 125.
	$(a+1)^2+(2a+2)^2=125$	M1	Sub from part (i) → quadratic eqn in a (or possibly in b → $b^2-2b-99=0$)
	$(5)(a^2+2a-24)=0\to$ eg $(a-4)(a+6)=0$	M1	Simplify and attempt to solve
	$a=4$ or -6	A1	
	$b=11$ or -9	A1	OR (4, 11), (–6, –9) If **A0A0**, SR1 for either (4, 11) or (–6, –9)
	Total:	6	

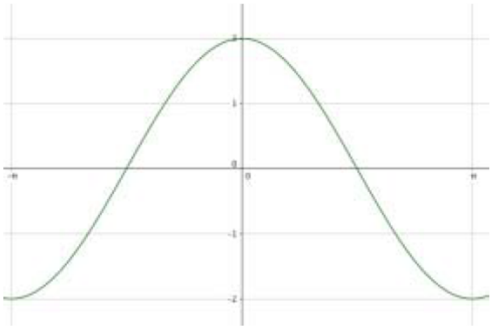
Question 27

EITHER Elim y to form 3-term quad eqn in $x^{1/3}$ (or u or y or even x)	(M1	Expect $x^{2/3}-x^{1/3}-2(=0)$ or $u^2-u-2\ (=0)$ etc.
$x^{1/3}$ (or u or y or x) = 2, –1	*A1	Both required. But $\underline{x}$ = 2,–1 and not then cubed or cube rooted scores **A0**
Cube solution(s)	DM1	Expect x = 8, – 1. Both required
(8, 3), (–1,0)	A1)	
OR Elim x to form quadratic equation in y	(M1	Expect $y+1=(y-1)^2$
$y^2-3y=0$	*A1	
Attempt solution	DM1	Expect y = 3, 0
(8, 3), (–1,0)	A1)	
Total:	4	

Question 28

(i)	Gradient = 1.5 Gradient of perpendicular = $-⅔$	B1	
	Equation of AB is $y-6=-⅔(x+2)$ Or $3y+2x=14$ oe	M1 A1	Correct use of straight line equation with a changed gradient and (– 2, 6), the (– (– 2)) must be resolved for the **A1** ISW.
			Using $y=mx+c$ gets **A1** as soon as c is evaluated.
	Total:	3	
(ii)	Simultaneous equations → Midpoint (1, 4)	M1	Attempt at solution of simultaneous equations as far as x =, or y =.
	Use of midpoint or vectors → B (4, 2)	M1A1	Any valid method leading to x, or to y.
	Total:	3	

Question 29

(i)		B1	One whole cycle – starts and finishes at –ve value
		DB1	Smooth curve, flattens at ends and middle. Shows (0, 2).
	Total:	**2**	
(ii)	$P(\frac{\pi}{3}, 1)\ Q(\pi, -2)$		
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	M1 A1	Pythagoras (on their coordinates) must be correct, OE.
	Total:	**2**	
(iii)	Eqn of PQ $y-1=-\frac{9}{2\pi}\left(x-\frac{\pi}{3}\right)$	M1	Correct form of line equation or sim equations from their P & Q
	If $y=0 \rightarrow h=\frac{5\pi}{9}$	A1	AG, condone $x=\frac{5\pi}{9}$
	If $x=0 \rightarrow k=\frac{5}{2}$,	A1	SR: non-exact solutions **A1** for both
	Total:	**3**	

Question 30

(i)	Mid-point of AB = (3, 5)	B1	Answers may be derived from simultaneous equations
	Gradient of AB = 2	B1	
	Eqn of perp. bisector is $y-5=-½(x-3) \rightarrow 2y=13-x$	M1A1	AG For **M1** FT from mid-point and gradient of AB
		4	
(ii)	$-3x+39=5x^2-18x+19 \rightarrow (5)(x^2-3x-4)(=0)$	M1	Equate equations and form 3-term quadratic
	$x=4$ or -1	A1	
	$y=4½$ or 7	A1	
	$CD^2=5^2+2½^2 \rightarrow CD=\sqrt{\frac{125}{4}}$	M1A1	Or equivalent integer fractions ISW
		5	

Question 31

(i)	$\frac{1}{\sqrt{3}}=\frac{2}{x}$ or $y-2=\frac{-1}{\sqrt{3}}x$	M1	OE, Allow $y-2=\frac{+1}{\sqrt{3}}x$. Attempt to express $\tan\frac{\pi}{6}$ *or* $\tan\frac{\pi}{3}$ <u>exactly</u> is required or the use of $1/\sqrt{3}$ *or* $\sqrt{3}$
	$(x=)2\sqrt{3}$	A1	OE
		2	
(ii)	Mid-point (*a*, *b*) = (½ *their* **(i)**, 1)	B1FT	Expect ($\sqrt{3}$, 1)
	Gradient of AB leading to gradient of bisector, m	M1	Expect $-1/\sqrt{3}$ leading to $m=\sqrt{3}$
	Equation is $y-their\,b=m(x-their\,a)$ OE	DM1	Expect $y-1=\sqrt{3}\left(x-\sqrt{3}\right)$
	$y=\sqrt{3}x-2$ OE	A1	
		4	

Question 32

(i)	Gradient, m, of $AB=\frac{3k+5-(k+3)}{k+3-(-3k-1)}$ OE $\left(=\frac{2k+2}{4k+4}\right)$ $=\frac{1}{2}$	M1A1	Condone omission of brackets for M mark
		2	
(ii)	Mid-pt = $[\frac{1}{2}(-3k-1+k+3),\ \frac{1}{2}(3k+5+k+3)]=$ $\left(\frac{-2k+2}{2},\frac{4k+8}{2}\right)$ SOI	B1B1	B1 for $\frac{-2k+2}{2}$, B1 for $\frac{4k+8}{2}$ (ISW) or better, i.e. $(-k+1,\ 2k+4)$
	Gradient of perpendicular bisector is $\frac{-1}{their\ m}$ SOI Expect -2	M1	Could appear in subsequent equation and/or could be in terms of k
	Equation: $y-(2k+4)=-2\left[x-(-k+1)\right]$ OE	DM1	Through *their* mid-point and with *their* $\frac{-1}{m}$ (now numerical)
	$y+2x=6$	A1	Use of numerical k in **(ii)** throughout scores SC2/5 for correct answer
		5	

Question 33

EITHER		
Gradient of bisector $= -\frac{3}{2}$	**B1**	
gradient $AB = \frac{5h-h}{4h+6-h}$	***M1**	Attempt at $\frac{y-step}{x-step}$
Either $\frac{5h-h}{4h+6-h} = \frac{2}{3}$ or $-\frac{4h+6-h}{5h-h} = -\frac{3}{2}$	***M1**	Using $m_1m_2 = -1$ appropriately to form an equation.
OR		
Gradient of bisector $= -\frac{3}{2}$	**B1**	
Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x + \frac{h}{3} = y$ oe	***M1**	Obtain equation of AB using gradient from $m_1m_2 = -1$ and a point.
Substitute co-ordinates of one of the other points	***M1**	Arrive at an equation in h.
h = 2	**A1**	
Midpoint is $\left(\frac{5h+6}{2}, 3h\right)$ or (8, 6)	**B1FT**	Algebraic expression or FT for numerical answer from '*their h*'
Uses midpoint and '*their h*' with $3x + 2y = k$	**DM1**	Substitutes '*their midpoint*' into $3x + 2y = k$. If $y = -\frac{3}{2}x + c$ is used (expect $c = 18$) the method mark should be withheld until they $\times 2$.
$\rightarrow k = 36$ soi	**A1**	
	7	

Question 34

(i)	Eqn of AC $y = -\frac{1}{2}x + 4$ (gradient must be $\Delta y / \Delta x$)	**M1A1**	Uses gradient and a given point for equa. CAO
	Gradient of $OB = 2$ $\rightarrow y = 2x$ (If y missing only penalise once)	**M1 A1**	Use of $m_1m_2 = -1$, answers only ok.
		4	
(ii)	Simultaneous equations $\rightarrow$ ((1.6, 3.2))	**M1**	Equate and solve for M1 and reach $\geqslant 1$ solution
	This is mid-point of OB. $\rightarrow B$ (3.2, 6.4)	**M1 A1**	Uses mid-point. CAO
	or		
	Let coordinates of B (h, k) $OA = AB \rightarrow h^2 = 8k - k^2$ $OC = BC \rightarrow k^2 = 16h - h^2 \rightarrow (3.2, 6.4)$		M1 for both equations, M1 for solving with $y = 2x$
	or		
	gradients ($\frac{k-4}{h} \times \frac{k}{h-8} = -1$)		M1 for gradient product as –1, M1 solving with $y = 2x$
	or		
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$		M1 for complete equation, M1 solving with $y = 2x$
		3	

Question 35

(i)	Gradient, m, of $AB = 3/4$	B1	
	Equation of BC is $y-4=\frac{-4}{3}(x-3)$	M1A1	Line through (3, 4) with gradient $\frac{-1}{m}$ (M1). (Expect $y=\frac{-4}{3}x+8$)
	$x=6$	A1	Ignore any y coordinate given.
		4	
(ii)	$(AC)^2=7^2+1^2 \rightarrow AC=7.071$	M1A1	M mark for $\sqrt{(their 6+/-1)^2+1}$.
		2	

Question 36

(i)	$2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ $\rightarrow$ 3 term quadratic.	*M1
	Use of $b^2-4ac \rightarrow k^2-144<0$	DM1
	$-12<k<12$	A1
		3
(ii)	Using $k=15$ in their 3 term quadratic	M1
	$x=1,4$ or $y=11, 14$	A1
	(1, 14) and (4, 11)	A1
		3
(iii)	Gradient of $AB=-1 \rightarrow$ Perpendicular gradient $=+1$	B1FT
	Finding their midpoint using their (1, 14) and (4, 11)	M1
	Equation: $y-12\frac{1}{2}=(x-2\frac{1}{2})$ [$y=x+10$]	A1
		3

Question 37

)(i)	$4x^{1/2} = x + 3 \rightarrow$ $(x^{1/2})^2 - 4x^{1/2} + 3\ (= 0)$ **OR** $16x = x^2 + 6x + 9$	**M1**	Eliminate y from the 2 equations and then: **Either** treat as quad in $x^{1/2}$ **OR** square both sides and RHS is 3-term
	$x^{1/2} = 1$ or 3 $\ x^2 - 10x + 9\ (= 0)$	**A1**	If in 1st method $x^{1/2}$ becomes x, allow only M1 unless subsequently squared
	$x = 1$ or 9	**A1**	
	$y = 4$ or 12	**A1ft**	Ft from *their* x values If the 2 solutions are found by trial substitution B1 for the first coordinate and B3 for the second coordinate
	$AB^2 = (9-1)^2 + (12-4)^2$	**M1**	
	$AB = \sqrt{128}$ or $8\sqrt{2}$ oe or 11.3	**A1**	
		6	
)(ii)	$dy/dx = 2x^{-1/2}$	**B1**	
	$2x^{-1/2} = 1$	**M1**	Set *their* derivative = *their* gradient of AB and attempt to solve
	(4, 8)	**A1**	Alternative method without calculus: $M_{AB} = 1$, tangent is $y = mx + c$ where m = 1 and meets $y = 4x^{1/2}$ when $4x^{1/2} = x + c$. This is a quadratic with $b^2 = 4ac$, so $16 - 4 \times 1 \times c = 0$ so c = 4 B1 Solving $4x^{1/2} = x + 4$ gives $x = 4$ and $y = 8$ M1A1
		3	
)(iii)	Equation of normal is $y - 8 = -1(x - 4)$	**M1**	Equation through *their* T and with gradient $-1/$*their* gradient of AB. Expect $y = -x + 12$,
	Eliminate y (or x) $\rightarrow$ $-x + 12 = x + 3$ or $y - 3 = 12 - y$	**M1**	May use *their* equation of AB
	(4½, 7½)	**A1**	
		3	

Question 38

'(i)	$D=(5, 1)$	B1	
		1	
(ii)	$(x-5)^2+(y-1)^2=20$ **oe**	B1	**FT** on *their D*. Apply ISW, oe but not to contain square roots
		1	
(iii)	$(x-1)^2+(y-3)^2=(9-x)^2+(y+1)^2$ **soi**	M1	Allow 1 sign slip For M1 allow with √ signs round both sides but sides must be equated
	$x^2-2x+1+y^2-6y+9=x^2-18x+81+y^2+2y+1$	A1	
	$y=2x-9$ www **AG**	A1	
	Alternative method for question 7(iii)		
	grad. of $AB=-\frac{1}{2}\rightarrow$ grad of perp bisector $=\frac{-1}{-\frac{1}{2}}$	M1	
	Equation of perp. bisector is $y-1=2(x-5)$	A1	
	$y=2x-9$ www **AG**	A1	
		3	
(iv)	Eliminate y (or x) using equations in **(ii)** and **(iii)**	*M1	To give an (unsimplified) quadratic equation
	$5x^2-50x+105\ (=0)$ **or** $5(x-5)^2=20$ **or** $5y^2-10y-75\ (=0)$ **or** $5(y-1)^2=80$	DM1	Simplify to one of the forms shown on the right (allow arithmetic slips)
	$x=3$ and 7, **or** $y=-3$ and 5	A1	
	$(3, -3), (7, 5)$	A1	Both pairs of x & y correct implies A1A1. SC B2 for no working
		4	

Question 39

Attempt to find the midpoint M	M1	
(1, 4)	A1	
Use a gradient of $\pm\frac{2}{3}$ and *their M* to find the equation of the line.	M1	
Equation is $y-4=-\frac{2}{3}(x-1)$	A1	AEF
Alternative method for question 2		
Attempt to find the midpoint M	M1	
(1, 4)	A1	
Replace 1 in the given equation by c and substitute *their M*	M1	
Equation is $y-4=-\frac{2}{3}(x-1)$	A1	AEF
	4	

Question 40

(a)	Centre = (2, –1)	**B1**	
	$r^2 = [2-(-3)]^2 + [-1-(-5)]^2$ or $[2-7]^2 + [-1-3]^2$ OE	**M1**	OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE
	$(x-2)^2 + (y+1)^2 = 41$	**A1**	Must not involve surd form **SCB3** $(x+3)(x-7)+(y+5)(y-3)=0$
		3	
(b)	Centre = *their* (2, –1) + $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ = (10, 3)	**B1FT**	SOI FT on *their* (2, –1)
	$(x-10)^2 + (y-3)^2 =$ *their* 41	**B1FT**	FT on *their* 41 even if in surd form **SCB2** $(x-5)(x-15)+(y+1)(y-7)=0$
		2	
(c)	Gradient m of line joining centres = $\frac{4}{8}$ OE	**B1**	
	Attempt to find mid-point of line.	**M1**	Expect (6, 1)
	Equation of RS is $y-1=-2(x-6)$	**M1**	Through *their* (6, 1) with gradient $\frac{-1}{m}$
	$y=-2x+13$	**A1**	AG
	Alternative method for question 12(c)		
	$(x-2)^2+(y+1)^2-41=(x-10)^2+(y-3)^2-41$ OE	**M1**	
	$x^2-4x+4+y^2+2y+1=x^2-20x+100+y^2-6y+9$ OE	**A1**	Condone 1 error or errors caused by 1 error in the first line
	$16x+8y=104$	**A1**	
	$y=-2x+13$	**A1**	AG
		4	
(d)	$(x-10)^2+(-2x+13-3)^2=41$	**M1**	Or eliminate y between C_1 and C_2
	$x^2-20x+100+4x^2-40x+100=41 \rightarrow 5x^2-60x+159=0$	**A1**	AG
		2	

Question 41

)(a)	Mid-point is (–1, 7)	**B1**
	Gradient, *m*, of *AB* is 8/12 OE	**B1**
	$y-7=-\frac{12}{8}(x+1)$	**M1**
	$3x+2y=11$ **AG**	**A1**
		4
)(b)	Solve simultaneously $12x-5y=70$ and *their* $3x+2y=11$	**M1**
	$x=5,\ y=-2$	**A1**
	Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11)	**M1**
	$(r)=\sqrt{12^2+5^2}$ or $\sqrt{13^2+0}=13$	**A1**
	Equation of circle is $(x-5)^2+(y+2)^2=169$	**A1**
		5

Question 42

(a)	Express as $(x-4)^2+(y+2)^2=16+4+5$	**M1**
	Centre *C*(4, –2)	**A1**
	Radius = $\sqrt{25}$ = 5	**A1**
		3
(b)	*P*(1,2) to *C*(4, – 2) has gradient $-\frac{4}{3}$ (**FT** on coordinates of *C*)	**B1FT**
	Tangent at *P* has gradient = $\frac{3}{4}$	**M1**
	Equation is $y-2=\frac{3}{4}(x-1)$ or $4y = 3x + 5$	**A1**
		3
(c)	*Q* has the same coordinate as *P* $y = 2$	**B1**
	Q is as far to the right of *C* as *P* $x = 3 + 3 + 1 = 7$ *Q* (7, 2)	**B1**
		2
(d)	Gradient of tangent at *Q* = $-\frac{3}{4}$ by symmetry (**FT** from part (**b**))	**B1FT**
	Eqn of tangent at *Q* is $y-2=-\frac{3}{4}(x-7)$ or $4y + 3x = 29$	**M1**
	$T\left(4, \frac{17}{4}\right)$	**A1**
		3

Question 43

)(a)	Centre is (3, 1)	**B1**
	Radius = 5 (Pythagoras)	**B1**
	Equation of C is $(x-3)^2+(y-1)^2=25$ (**FT** on *their* centre)	**M1** **A1FT**
		4
)(b)	Gradient from (3, 1) to (7, 4) = ¾ (this is the normal)	**B1**
	Gradient of tangent $=-\frac{4}{3}$	**M1**
	Equation is $y-4=-\frac{4}{3}(x-7)$ or $3y+4x=40$	**M1A1**
		4
)(c)	*B* is centre of line joining centres → (11, 7)	**B1**
	Radius = 5 New equation is $(x-11)^2+(y-7)^2=25$ (**FT** on coordinates of B)	**M1** **A1FT**
		3

Question 44

(a)	$(-6-8)^2+(6-4)^2$	**M1**	OE
	$=200$	**A1**	
	$\sqrt{200}>10$, hence outside circle	**A1**	AG ('Shown' not sufficient). Accept equivalents of $\sqrt{200}>10$
	Alternative method for question 11(a)		
	Radius = 10 and C = (8, 4)	**B1**	
	Min(x) on circle = 8 – 10 = –2	**M1**	
	Hence outside circle	**A1**	AG
		3	
.(b)	angle $=\sin^{-1}\left(\frac{their\,10}{their\,10\sqrt{2}}\right)$	**M1**	Allow decimals for 10√2 at this stage. If cosine used, angle *ACT* or *BCT* must be identified, or implied by use of 90°– 45°.
	angle = $\sin^{-1}(\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or $\frac{10}{10\sqrt{2}}$ or $\frac{10}{\sqrt{200}}) = 45°$	**A1**	AG Do not allow decimals
	Alternative method for question 11(b)		
	$(10\sqrt{2})^2=10^2+TA^2$	**M1**	
	TA = 10 → 45°	**A1**	AG
		2	

(c)	Gradient, m, of $CT = -\frac{1}{7}$	B1	OE
	Attempt to find mid-point (M) of CT	*M1	Expect (1, 5)
	Equation of AB is $y-5=7(x-1)$	DM1	Through *their* (1, 5) with gradient $-\frac{1}{m}$
	$y=7x-2$	A1	
		4	
(d)	$(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of y	M1	Substitute *their* equation of AB into equation of circle.
	$50x^2-100x\ (=0)$	A1	
	$x = 0$ and 2	A1	WWW
	Alternative method for question 11(d)		
	MC $= \begin{pmatrix}7\\-1\end{pmatrix}$	M1	
	$\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}-1\\-7\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$, $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}1\\7\end{pmatrix}=\begin{pmatrix}2\\12\end{pmatrix}$	A1	
	$x = 0$ and 2	A1	
		3	

Question 45

(a)	$r=\sqrt{(6^2+3^2)}$ or $r^2 = 45$	B1	Sight of r = 6.7 implies B1
	$(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$	M1	Using centre given and *their* radius or r in correct formula
	$(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$	A1	Do not allow $\left(\sqrt{45}\right)^2$ for r^2
		3	
(b)	C has coordinates (11, 4)	B1	
	0.5	B1	OE, Gradient of AB, BC or AC.
	Grad of CD $=-2$	M1	Calculation of gradient needs to be shown for this M1.
	($\frac{1}{2}\times -2=-1$) then states ∴ perpendicular → hence shown or tangent	A1	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	B1	
	0.5	B1	OE, Gradient of AB, BC or AC.
	Gradient of the perpendicular is −2 → Equation of the perpendicular is $y-4=-2(x-11)$	M1	Use of $m_1m_2=-1$ with *their* gradient of AB, BC or AC and correct method for the equation of the perpendicular. Could use $D(5, 16)$ instead of $C(11,4)$.
	Checks $D(5, 16)$ **or** checks gradient of CD and then states D lies on the line **or** CD has gradient −2→ hence shown or tangent	A1	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.

(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of AB, BC or $AC = 0.5$	**B1**	Only one of AB, BC or AC needed.
	Equation of the perpendicular is $y-4=-2(x-11)$	**B1**	Finding equation of CD.
	$(x-5)^2+(-2x+26-1)^2=45 \rightarrow (x^2-22x+121=0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x-11)^2=0$ or $b^2-4ac=0 \rightarrow$ repeated root $\rightarrow$ hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of B, C & D without using r.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	$\therefore$ Pythagoras valid $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	Triangle ACD could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$ $\left(=\begin{pmatrix}6\\3\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix} \textbf{ or } \begin{pmatrix}12\\6\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right)$	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$
	Scalar product = 0 then states $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	
		4	
(c)	$E\,(-1, 4)$	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y=2x+6$ may be used to find E
		2	

Question 46

(a)	$m_{AB}=\dfrac{4-2}{-1-3}=-\dfrac{1}{2}$	**B1**	
	Equation of tangent is $y-2=2(x-3)$	**B1 FT**	(3, 2) with *their* gradient $-\dfrac{1}{m_{AB}}$
		2	
(b)	$AB^2=4^2+2^2=20$ or $r^2=20$ or $r=\sqrt{20}$ or $AB=\sqrt{20}$	**B1**	
	Equation of circle centre B is $(x-3)^2+(y-2)^2=20$	**M1 A1**	FT *their* 20 for M1
		3	
(c)	$(x-3)^2+(2x-6)^2=$ *their* 20	**M1**	Substitute *their* $y-2=2x-6$ into *their* circle, centre B
	$5x^2-30x+25=0$ or $5(x-3)^2=20$	**A1**	
	$[(5)(x-5)(x-1)$ or $x-3=\pm 2]$ $\quad x=5, 1$	**A1**	
		3	

Question 47

(a)	Centre of circle is (4, 5)	B1 B1	
	$r^2 = (7-4)^2 + (1-5)^2$	M1	OE. Either using *their* centre and A or C **or** using A and C and dividing by 2.
	$r = 5$	A1 FT	FT on *their* (4, 5) if used.
	Equation is $(x-4)^2 + (y-5)^2 = 25$	A1	OE. Allow 5^2 for 25.
		5	
(b)	Gradient of radius = $\frac{9-5}{7-4} = \frac{4}{3}$	B1 FT	FT for use of *their* centre.
	Equation of tangent is $y-9 = -\frac{3}{4}(x-7)$	B1	or $y = \frac{-3x}{4} + \frac{57}{4}$
		2	

Question 48

(a)	Gradient of $AB = -\frac{3}{5}$, gradient of $BC = \frac{5}{3}$ or lengths of all 3 sides or vectors	M1	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc} = -1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC} = 0$ or $\cos ABC = 0$ from cosine rule	A1	WWW
		2	
(b)	Centre = mid-point of AC = (2,4)	B1	
		1	
(c)	$(x - their\ x_c)^2 + (y - their\ y_c)^2 [= r^2]\ or\ (their\ x_c - x)^2 + (their\ y_c - y)^2 = [r^2]$	M1	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	A1	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of BE is $y = -4(x-3)$ or $y-4 = -4(x-2)$ leading to $y = -4x+12$ Substitute equation of BE into circle and form a 3-term quadratic.	M1	Use of mid-point formula, vectors, steps on a diagram May be seen to find x coordinate at E
	$(x, y) = (1, 8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	A1	E = (1, 8) Accept without working for both marks **SC B2**
	Gradient of BD, m, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent	B1	Or gradient of BE = -4
	Equation of tangent is $y - 8 = ¼(x-1)$ OE	M1 A1	For M1, equation through *their* E or (1, 8) (not, A, B or C) and with gradient $\frac{-1}{their - 4}$
		5	

Question 49

(a)	$(5-1)^2 + (11-5)^2 = 52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient $= \frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that A lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x-5)^2 + (y-11)^2 = 52$ and $y - 5 = -\frac{2}{3}(x-1)$	M1	Both equations seen and attempt to solve. May see $y = -\frac{2}{3}x + \frac{17}{3}$
	Solving simultaneously to obtain $(y-5)^2 = 0$ or $(x-1)^2 = 0 \Rightarrow 1$ root or tangent or discriminant $= 0 \Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx} = \frac{10-2x}{2y-22} = \frac{10-2}{10-22}$	M1	Attempting implicit differentiation of circle equation and substitute $x = 1$ and $y = 5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
		2	
(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x+3)^2 + (y+1)^2 = 52$	B1 FT	FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

Question 50

Gradient AB $= \frac{1}{2}$	B1	SOI
Lines meet when $-2x + 4 = \frac{1}{2}(x-8) + 3$ Solving as far as $x =$	*M1	Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of AB (but not -2) and solving. Expect $x = 2$, $y = 0$.
Using mid-point to get as far as $p =$ or $q =$	DM1	Expect $\frac{8+p}{2} = 2$ or $\frac{3+q}{2} = 0$
$p = -4$, $q = -3$	A1	Allow coordinates of B are $(-4, -3)$.
Alternative method for Question 6		
Gradient AB $= \frac{1}{2}$	B1	SOI
$\frac{q-3}{p-8} = \frac{1}{2}$ [leading to $2q = p - 2$], $\frac{q+3}{2} = -2\left(\frac{8+p}{2}\right) + 4$ [leading to $q = -11 - 2p$]	*M1	Equating gradient of AB with *their* gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
Solving simultaneously *their* 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p =$ or $q =$.
$p = -4$, $q = -3$	A1	Allow coordinates of B are $(-4, -3)$.

Question 51

(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3 - \sqrt{3}$ scores B0
		1	
(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at E is the limit of the gradients of the chords as the x-value tends to 3 or δx tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

Question 52

(a)	When $y = 0$ $\quad x^2 - 4x - 77 = 0$ [$\Rightarrow (x+7)(x-11) = 0$ or $(x-2)^2 = 81$]	**M1**	Substituting $y = 0$
	So x-coordinates are -7 and 11	**A1**	
		2	
(b)	Centre of circle C is $(2, -3)$	**B1**	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	**M1**	For either gradient (M1 sign error, M0 if x-coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	**M1**	For either perpendicular gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	**A1**	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y\frac{dy}{dx}$ seen	**B1**	
	$2x - 4 + 2y\frac{dy}{dx} + 6\frac{dy}{dx} = 0$	**M1**	Fully differentiated $= 0$ with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	**M1**	For either gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	**A1**	
		6	

Question 53

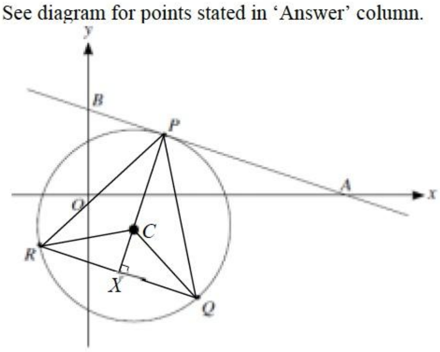
(a)	$x^2+(2x+5)^2=20$ leading to $x^2+4x^2+20x+25=20$	M1	Substitute $y=2x+5$ and expand bracket.
	$(5)(x^2+4x+1)[=0]$	A1	3-term quadratic.
	$x=\dfrac{-4\pm\sqrt{16-4}}{2}$	M1	OE. Apply formula or complete the square.
	$A=(-2+\sqrt{3},1+2\sqrt{3})$	A1	Or 2 correct x values.
	$B=(-2-\sqrt{3},1-2\sqrt{3})$	A1	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2=their(x_2-x_1)^2+their(y_2-y_1)^2$	M1	Using *their* coordinates in a correct distance formula. Condone one sign error in x_2-x_1 or y_2-y_1
	$\left[AB^2=48+12 \text{ leading to}\right] AB=\sqrt{60}$	A1	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	
(b)	$x^2+m^2(x-10)^2=20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1)-20m^2x+20(5m^2-1)$ $[=0]$	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2-4ac=]400m^4-80(m^2+1)(5m^2-1)$	M1	Using correct coefficients from *their* quadratic equation.
	$400m^4-80(5m^4+4m^2-1)=0 \rightarrow (-80)(4m^2-1)=0$	A1	OE. Must have '=0' for A1.
	$m=\pm\dfrac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2=10^2-20$	M1	
	$l=\sqrt{80}$	A1	
	$\tan\alpha=\dfrac{\sqrt{20}}{\sqrt{80}}=\dfrac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x-axis.
	$m=\pm\dfrac{1}{2}$	A1	
		5	

Question 54

(a)

Centre is $(3, -2)$	**B1**	
Gradient of radius $= \dfrac{(their\ {-2}) - 4}{(their\ 3) - 5}$ [= 3]	***M1**	Finding gradient using *their* centre (not (0, 0)) and P (5,4).
Equation of tangent $y - 4 = -\dfrac{1}{3}(x - 5)$	**DM1**	Using P and the negative reciprocal of *their* gradient to find the equation of AB.
Sight of $[x =]17$ and $[y =]\ \dfrac{17}{3}$	**A1**	
$\left[\text{Area} = \dfrac{1}{2} \times \dfrac{17}{3} \times 17 = \right]\ \dfrac{289}{6}$	**A1**	Or $48\dfrac{1}{6}$ or AWRT 48.2.
Alternative method for question 12(a)		
$2x + 2y\dfrac{dy}{dx} - 6 + 4\dfrac{dy}{dx} = 0$	**B1**	
At P: $10 + 8\dfrac{dy}{dx} - 6 + 4\dfrac{dy}{dx} = 0\left[\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{3}\right]$	***M1**	Find the gradient using P (5,4) in *their* implicit differential (with at least one correctly differentiated y term).
Equation of tangent $y - 4 = -\dfrac{1}{3}(x - 5)$	**DM1**	Using P and *their* value for the gradient to find the equation of AB.
Sight of $[x =]17$ and $[y =]\ \dfrac{17}{3}$	**A1**	
$\left[\text{Area} = \dfrac{1}{2} \times \dfrac{17}{3} \times 17 = \right]\ \dfrac{289}{6}$	**A1**	Or $48\dfrac{1}{6}$ or AWRT 48.2.

(b)

Radius of circle $= \sqrt{40}$,	**B1**	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
Area of $\Delta CRQ = \dfrac{1}{2} \times (their\ r)^2 \sin 120 \left[= \dfrac{1}{2} \times 40 \times \dfrac{\sqrt{3}}{2}\right]$ OR Area of $\Delta CQX = \dfrac{1}{2} \times \sqrt{40}\cos 30 \times \sqrt{40}\cos 60$ OE $\left[= \dfrac{1}{2} \times \sqrt{30} \times \sqrt{10}\right]$ OR Area of circle – 3× Area of segment = $40\pi - 3 \times (40\dfrac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area $= \dfrac{1}{2}QR^2\sin 60$	**M1**	Using $\dfrac{1}{2}r^2\sin\theta$ with *their* r and 120 or 60 [×3] Using $\dfrac{1}{2}$ ×base×height in a correct right-angled triangle [×6]. Use of cosine rule and area of large triangle
$30\sqrt{3}$	**A1**	AWRT 52[.0] implies B1M1A0.
	3	See diagram for points stated in 'Answer' column. 

Question 55

(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2=13$ or $r=\sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
(b)	$(x-5)^2+(5x-10-2)^2=13$	**M1**	Substitute $y=5x-10$ into *their* equation.
	$26x^2-130x+156\ [=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ [=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

Question 56

(a)	$(x+1)^2+(3x-22)^2=85$	**M1**	OE. Substitute equation of line into equation of circle.
	$10x^2-130x+400\ [=0]$	**A1**	Correct 3-term quadratic
	$[10](x-8)(x-5)$ leading to $x=8$ or 5	**A1**	Dependent on factors or formula or completing of square seen.
	(8, 4), (5, –5)	**A1**	If M1A1A0A0 scored, then **SC B1** for correct final answer only.
		4	
(b)	Mid-point of $AB=\left(6\frac{1}{2},-\frac{1}{2}\right)$	**M1**	Any valid method
	Use of $C=(-1, 2)$	**B1**	SOI
	$r^2=\left(-1-6\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$	**M1**	Attempt to find r^2. Expect $r^2=62\frac{1}{2}$.
	Equation of circle is $(x+1)^2+(y-2)^2=62\frac{1}{2}$	**A1**	OE.
		4	

Question 57

(a)	Equation of BC is $\{y=\}\{2\}\{-3x\}$	**B2, 1, 0**	OE forms $y+4=-3(x-2)$ or $y-2=-3(x-0)$.
		2	
(b)	$(x-2)^2+(2-3x+4)^2=20$	***M1**	OE Sub line equation into equation of circle to eliminate y.
	$10(x-2)^2=20$ or $[10](x^2-4x+2)[=0]$	**A1**	OE Accept $(10x^2-40x+20)$.
	$x-2=[\pm]\sqrt{2}$ or $x=\dfrac{4[\pm]\sqrt{16-8}}{2}$	**DM1**	Correctly solving *their* quadratic.
	$x=2-\sqrt{2}$	**A1**	OE only solution. Answer only **SC B1** If DM1 not scored.
	$y=3\sqrt{2}-4$	**A1**	OE only solution. Answer only **SC B1** If DM1 not scored.
		5	

Question 58

(a)	$1+1+a+b-12=0[\Rightarrow a+b=10]$ $4+36+2a-6b-12=0[\Rightarrow 2a-6b=-28]$	**B1 B1**	B1 for each equation. Allow unsimplified. Can be implied by correct values for a and b.
	$a=4, b=6$	**B1**	
	Centre is $\left(-\dfrac{their\ a}{2},-\dfrac{their\ b}{2}\right)$ $[-2,-3]$	**B1 FT**	Or $x=-2, y=-3$
		4	
(b)	Gradient of AC is $\dfrac{1-their\ y}{1-their\ x}$ $[=\dfrac{1--3}{1--2}=\dfrac{1+3}{1+2}=\dfrac{4}{3}]$	***M1**	Using *their* centre correctly.
	Gradient of tangent is $=\dfrac{-1}{their\ \frac{4}{3}}\left[=-\dfrac{3}{4}\right]$	**A1 FT**	Use of $m_1m_2=-1$ to obtain the gradient of the tangent.
	Equation: $y-1=$ '$their\ -\dfrac{3}{4}$'$(x-1)$ or $y=-\dfrac{3}{4}x+\dfrac{7}{4}$	**DM1**	Using $(1,1)$ with *their* gradient of the tangent at A.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	**A1**	

Question 59

(a)	Express as $(x+3)^2+(y-1)^2=26+9+1[=36]$	M1	Completing the square on x and y or using the form $x^2+y^2+2gx+2fy+c=0$, centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$. SOI by correct answer.
	Centre $(-3, 1)$	B1	
	Radius 6	B1	
	So lowest point is $(-3, -5)$	A1 FT	FT on *their* centre and *their* radius.
		4	
(b)	Intersects when $x^2+(kx-5)^2+6x-2(kx-5)-26=0$ **or** $(x+3)^2+(kx-5-1)^2=36$	*M1	Substituting $y=kx-5$ into *their* circle equation or rearranging and equating y.
	$x^2+k^2x^2-10kx+25+6x-2kx+10-26=0$ **or** $x^2+6x+9+k^2x^2-12kx+36=36$ leading to $k^2x^2+x^2+6x-12kx+9[=0]$ **or** $(k^2+1)x^2+(6-12k)x+9[=0]$	DM1 A1	Rearranging to 3-term quadratic (terms grouped, all on one side). Allow 1 error. Correct quadratic (need to see 9 as constant term).
	$(6-12k)^2-4(k^2+1)\times 9\ [>0]$ $[\text{leading to } 144k^2-144k+36-36k^2-36>0]$	DM1	Using discriminant $b^2-4ac[>0]$ with *their* values. Allow if in square root.
	$[108k^2-144k=0$ leading to$]$ $k=0$ or $k=\frac{4}{3}$	A1	Need not see method for solving.
	$k<0, k>\frac{4}{3}$	A1	Do not accept $\frac{4}{3}<k<0$.
		6	

Question 60

(a)	$(5-2p)^2+(p+2)^2=(10-2p)^2+(3-p)^2$	M1 A1	Allow one sign error for M mark only.
	$25-20p+4p^2+p^2+4p+4=100-40p+4p^2+9-6p+p^2$ $30p=80 \rightarrow p=\frac{8}{3}$ oe	A1	Allow 2.67 AWRT.
		3	
(b)(i)	$m_{AC}=\frac{p+2}{2p-5}$ $\quad m_{BC}=\frac{p-3}{2p-10}$	M1	Allow a sign error.
	$\frac{p+2}{2p-5}\times\frac{p-3}{2p-10}=-1$	M1	Use of $m_1m_2=-1$ with their m_{AC} and m_{BC}.
	$p^2-p-6=-(4p^2-30p+50) \rightarrow 5p^2-31p+44\ (=0)$	A1	
	$p=4$ (Ignore $p=\frac{11}{5}$)	A1	Factors $(p-4)(5p-11)$, or formula or completing square must be seen.
		4	
(b)(ii)	Mid-point of $AB=(7½, ½)$	B1	SOI
	$r^2=2½^2+2½^2$ $\left[=\frac{50}{4}\right]$ or $r=\sqrt{(2½^2+2½^2)}$ $\left[=\frac{5\sqrt{2}}{2}\right]$	*M1	Or $r^2=\frac{1}{4}(5^2+5^2)$ $\left[=\frac{50}{4}\right]$ etc.
	Equation of circle is $(x-their\,7½)^2+(y-their\,½)^2=their\,\frac{50}{4}$	DM1	Must use r^2 not r or d or d^2
	$x^2+y^2-15x-y+44=0$	A1	CAO
		4	

Question 61

(a)	Mid-point AB is $\left(\frac{10+5}{2},\frac{2-1}{2}\right)\left[=\left(\frac{15}{2},\frac{1}{2}\right)\right]$	B1	Accept unsimplified.
	Gradient of $AB = \frac{-1-2}{10-5}=\frac{-3}{5}$ Gradient perpendicular $= \frac{5}{3}$	M1	For use of $\frac{\text{Change in y}}{\text{Change in } x}$, condone inconsistent order of x and y, and $m_1m_2 = -1$.
	$\frac{y-\frac{1}{2}}{x-\frac{15}{2}}=\frac{5}{3}$ $\left[y-\frac{1}{2}=\frac{5}{3}\left(x-\frac{15}{2}\right)\right]$	A1	OE ISW Any correct version e.g. $y=\frac{5}{3}x-12$ or $5x-3y=36$.
		3	
(b)	[Radius =] $\sqrt{34}$ or 5.8 AWRT or [(radius)^2 =] 34	B1	Sight of $\sqrt{34}$ or 34. Condone confusion of r and r^2.
	$(x-5)^2+(y-2)^2$	B1	Sight of $(x-5)^2+(y-2)^2$
	$(x-5)^2+(y-2)^2=34$	B1	CAO ISW
	Alternative method for Question 1(b)		
	$x^2+y^2-10x-4y$	B1	
	$[c=]5$ or $[c=]-5$	B1	Substitution of (10, –1) into $x^2+y^2-10x-4y+c=0$.
	$x^2+y^2-10x-4y-5=0$	B1	
		3	

Question 62

(a)	$x^2+(mx+10)^2=20$ **or** $y^2+\left(\frac{y-10}{m}\right)^2=20$ **or** $mx+10=\sqrt{20-x^2}$	*M1	Substitute equation of line into equation of circle.
	$x^2(1+m^2)+20mx+80$ $[=0]$ **or** $y^2(m^2+1)-20y+(100-20m^2)[=0]$	A1	Collect terms into a 3 term quadratic.
	$(20m)^2-4(1+m^2)\times 80[=0 \Rightarrow 80m^2-320=0 \Rightarrow [80](m^2-4)=0]$ **or** $(-20)^2-4(m^2+1)(100-20m^2)[=0 \Rightarrow [80](m^4-4m^2)=0]$	DM1	Use $b^2-4ac[=0]$.
	$m=\pm 2$	A1	Two values for m.
		4	
(b)	**Method 1: Use of quadratic**		
	$(1+2^2)x^2\pm 20(2)x+80\left[=0 \Rightarrow 5x^2\pm 40x+80=0\right]$ **or** $y^2(2^2+1)-20y+(100-20(2^2))[=0 \Rightarrow [5](y^2-4y+4)=0]$	M1	Sub *their* m into *their* quadratic in x or y **or** restart with *their* tangent equation and equation of circle.
	$[5](x\pm 4)^2=0 \Rightarrow x=\pm 4$ **or** $y=2$	A1	Correct solutions or one correct pair (x, y).
	$(-4,2)$, $(4,2)$	A1	Two correct points with x and y paired correctly.
	Method 2: Using equation of normal		
	$2x+10=-\frac{1}{2}x$ **or** $-2x+10=\frac{1}{2}x$	M1	Equate tangent and normal and solve for x.
	$x=\pm 4$	A1	Two correct x-values or one correct pair (x, y).
	$(-4,2)$, $(4,2)$	A1	Two correct points with x and y paired correctly.
		3	

(c)	**Method 1: Using angle at circumference**		
	$\cos BOA = \frac{\sqrt{20}}{10}$ or $\sin BOA = \frac{\sqrt{80}}{10}$ or $\tan BOA = \frac{\sqrt{80}}{\sqrt{20}}[=2]$	***M1**	Use a trig function in triangle *AOB*.
	$BOA = 63.4° \Rightarrow BOC = 126.8°$ or $126.9°$	**DM1**	Strategy involving doubling
	$[BDC =]63.4°$	**A1**	AWRT
	Metho 2: Using cosine rule		
	$BC = 8$, $BD = \sqrt{\left(\sqrt{20}+4\right)^2 + 2^2}$, $CD = \sqrt{\left(\sqrt{20}-4\right)^2 + 2^2}$	***M1**	Calculate two lengths in triangle *BCD*.
	$64 = 80 - 16\sqrt{5}\cos BDC$	**DM1**	Use cosine rule with *their* lengths
	$\cos BDC = \frac{\sqrt{5}}{5} \Rightarrow [BDC =]63.4°$	**A1**	AWRT
	Method 3: Subtract angles from 90 °		
	Calculate one angle at D $[=13.28]$	***M1**	*ODB* or angle between *CD* and the vertical from *D*
	Calculate a second angle at D $[=13.28]$ and subtract both from $90°$	**DM1**	
	$[BDC =]63.4°$	**A1**	AWRT
		3	

Question 63

$r^2 = (7+2)^2 + (12-5)^2$	**B1**	Expect 130, may use *AC* rather than *r*.
Equation of circle is $(x+2)^2 + (y-5)^2 = 130$	**B1 FT**	OE FT *their* 130, may use distance *BC* rather than circle.
$(x+2)^2 + (-2x+21)^2 = 130$	**M1**	Substitute $y = -2x+26$ into a circle equation.
$5x^2 - 80x + 315\ [=0]$ leading to $[5](x-9)(x-7)$	**M1**	Factorisation OE must be seen.
$x = 9$	**A1**	With or without $x = 7$.
$y = 8$ OR (9, 8)	**A1**	$y = 8$ or $(9,8)$ only. Both A1's dependent on the first M1.
	6	

Question 64

(a)	$(x-1)^2 + (x-9+4)^2 = 40$	**M1**	Substitute line into circle.
	$x^2 - 6x - 7\ [=0]$ leading to $(x+1)(x-7)\ [=0]$	**M1**	Simplify to 3-term quadratic and factorise OE.
	(–1, –10), (7, –2) or $x = -1$ and 7, $y = -10$ and –2	**A1 A1**	Answers only **SC B1**, **SC B1** but must see a correct quadratic equation.
		4	
(b)	[C is mid-point =] $\left(\frac{their\ x_1 + their\ x_2}{2}, \frac{their\ y_1 + their\ y_2}{2}\right)$	**M1**	Expect (3, –6).
	Radius = $\sqrt{(their\ x - their\ 3)^2 + (their\ y - their\ (-6))^2}$ OR $their\ \sqrt{\left((7-(-1))^2 + (-2-(-10))^2\right)}/2$	**M1**	Expect $\sqrt{32}$.
	$(x-3)^2 + (y+6)^2 = 32$	**A1**	OE
		3	

Question65

)(a)	$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$ or using $x=2y-12$	*M1	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2+(3-2a)x+a^2-11[=0]$	A1	OE e.g. $5x^2+4(3-2a)x+4a^2-44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2-y(54+4a)+133+a^2+24$.
	$(3-2a)^2-4\times\frac{5}{4}(a^2-11)[=0]$	DM1	OE Using b^2-4ac on *their* 3 term quadratic $[=0]$.
	Method 1 for final 2 marks		
	Using $a=4$: $(3-8)^2-5(5)=0$	A1	Clearly substituting $a=4$.
	$a=-16$	B1	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2-12a+64=0 \Rightarrow (a-4)(a+16)=0 \Rightarrow a=4$	A1	AG Full method clearly shown.
	$a=-16$	B1	Condone no method shown for this value.
		5	If M0, **SCB1** available for substituting $a=4$, finding P(2, 7) and verifying that $CP^2=20$.
(b)	Centre (4, 3) identified or used or the point P is (2, 7)	B1	
	∴ gradient of normal $=-2$	B1	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	M1	Condone use of $(\pm4,\pm3)$.
	$\frac{y-3}{(x-4)}=-2$ or $y-7=-2(x-2)$	A1	OE Condone $f(x)=$.
		4	
(c)	**Method 1 for Question 10(c)**		
	Diameter: $y-3=\frac{1}{2}(x-4)$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$ Or $2(x-4)+2(y-3)\frac{dy}{dx}=0$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$	*M1	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$ $\left[\frac{5}{4}x^2-10x=0\right]$	DM1	Obtaining an unsimplified equation in x or y only. $[y^2-6y+5=0]$.
	$x=0$ or 8, $y=1$ or 5 [(0, 1) and (8, 5)]	A1	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y-1=-2x$ and $y-5=-2(x-8)$	A1	$2x+y=1$ and $2x+y=21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are (4+4, 3+2) = (8, 5) and (4 – 4, 3 – 2) = (0, 1)	*M1 A1	Vector approach using their centre and gradient = 0.5. Condone answers only with no working.
	Equations are $y-5=-2(x-8)$ and $y-1=-2x$	DM1 A1	Forming equations of tangents using *their* (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x-4)^2+(-2x+c-3)^2=20$ $\left[5x^2+(4-4c)x+(c-3)^2-4=0\right]$	*M1	Obtaining an unsimplified equation in x only using equation of circle with $y=-2x+c$.
	$(4-4c)^2-20\left((c-3)^2-4\right)[=0]$ [leading to $-4c^2-32c+120c+16-100=0$]	DM1	Using $b^2-4ac[=0]$.

(c)	**Method 1 for Question 10(c)**		
	Diameter: $y-3=\frac{1}{2}(x-4)$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$ Or $2(x-4)+2(y-3)\frac{dy}{dx}=0$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$	*M1	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$ $\left[\frac{5}{4}x^2-10x=0\right]$	DM1	Obtaining an unsimplified equation in x or y only. $[y^2-6y+5=0]$.
	$x=0$ or 8, $y=1$ or 5 [(0, 1) and (8, 5)]	A1	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y-1=-2x$ and $y-5=-2(x-8)$	A1	$2x+y=1$ and $2x+y=21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are (4+4, 3+2) = (8, 5) and (4 – 4, 3 – 2) = (0, 1)	*M1 A1	Vector approach using their centre and gradient = 0.5 . Condone answers only with no working.
	Equations are $y-5=-2(x-8)$ and $y-1=-2x$	DM1 A1	Forming equations of tangents using *their* (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x-4)^2+(-2x+c-3)^2=20$ $\left[5x^2+(4-4c)x+(c-3)^2-4=0\right]$	*M1	Obtaining an unsimplified equation in x only using equation of circle with $y=-2x+c$.
	$(4-4c)^2-20\left((c-3)^2-4\right)[=0]$ [leading to $-4c^2-32c+120c+16-100=0$]	DM1	Using $b^2-4ac[=0]$.
(c)	$4c^2-88c+84[=0]$ [leading to $c^2-22c+21=0$]	A1	
	$c=21$ and $c=1$ or $y=-2x+21$ **and** $y=-2x+1$	A1	Condone no method shown for solution.
		4	

Question 66

(a)	$x^2+(y-2)^2=100$	B1	OE e.g. $(x-0)^2+(y-2)^2=10^2$ ISW.
		1	
(b)	Gradient of radius $=\left[\frac{10-2}{6-0}=\right]\frac{4}{3}$ or gradient of tangent $=\frac{-3}{4}$	M1	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x+2(y-2)\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3}{4}$ at (6, 10) $y=2+\sqrt{100-x^2} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(100-x^2\right)^{-\frac{1}{2}}(-2x)=-\frac{3}{4}$.
	Equation of tangent is $y-10=-\frac{3}{4}(x-6)$ $\left[\Rightarrow y=-\frac{3}{4}x+\frac{29}{2}\right]$	A1	OE ISW Allow e.g. $\frac{58}{4}$.
		2	
(c)	Coordinates of centre of circle Q are $\left(0, \textit{their}\ \frac{29}{2}\right)$	M1	SOI From a linear equation in **(b)**.
	Equation of circle Q is $x^2+\left(y-\textit{their}\ \frac{29}{2}\right)^2=\left(\frac{5\sqrt{5}}{2}\right)^2\left[=\frac{125}{4}\right]$	A1FT	OE e.g. $(x-0)^2+(y-14.5)^2=31.25$ ISW.
	$x^2+(11-2)^2=100 \Rightarrow x^2=19$ and $x^2+\left(11-\frac{29}{2}\right)^2=\frac{125}{4} \Rightarrow x^2=19$ OR e.g. $\frac{125}{4}-\left(y-\frac{29}{2}\right)^2+(y-2)^2=100 \Rightarrow 25y=275 \Rightarrow y=11$	B1	OE e.g. $x=[\pm]\sqrt{19}$, $x^2-19=x^2-19$ Correct argument to verify both y-coords are 11 ISW.
		3	

(d)	$x^2+\left(-\frac{3}{4}x+\frac{29}{2}-\frac{29}{2}\right)^2=\frac{125}{4}\left[\Rightarrow\frac{25}{16}x^2=\frac{125}{4}\Rightarrow x^2=20\right]$ or $y^2-29y+199[=0]$	**M1**	Substitute equation of *their* tangent into equation of *their* circle. May see $y=\sqrt{31.25-x^2}+14.5$.
	$x=\pm2\sqrt{5}$ or $y=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $x=\pm\sqrt{20}$ For 2 x-values or 2 y-values or correct (x,y) pair.
	$y\left[=\left(-\frac{3}{4}\times\pm\sqrt{20}\right)+\frac{29}{2}\right]=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $\frac{58}{4}+\frac{3\sqrt{20}}{4}$, $\frac{58}{4}-\frac{3\sqrt{20}}{4}$ Correct (x,y) pairs.
		3	