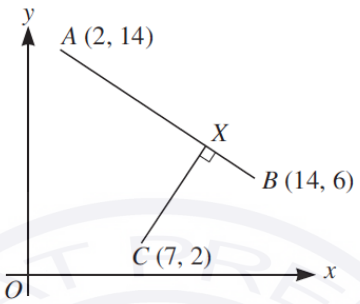
AS-Level

Pure Mathematics P1

Topic :Coordinate Geometry and Circles

May 2013- May 2023

Question 1



The diagram shows three points $A\,(2,\ 14)$, $B\,(14,\ 6)$ and $C\,(7,\ 2)$. The point X lies on AB, and CX is perpendicular to AB. Find, by calculation,

(i) the coordinates of X, [6]

(ii) the ratio $AX : XB$. [2]

Question 2

The point R is the reflection of the point $(-1,\ 3)$ in the line $3y + 2x = 33$. Find by calculation the coordinates of R. [7]

Question 3

A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

(i) For the case where $m = 1$, the curve and the line intersect at the points A and B. Find the coordinates of the mid-point of AB. [4]

(ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

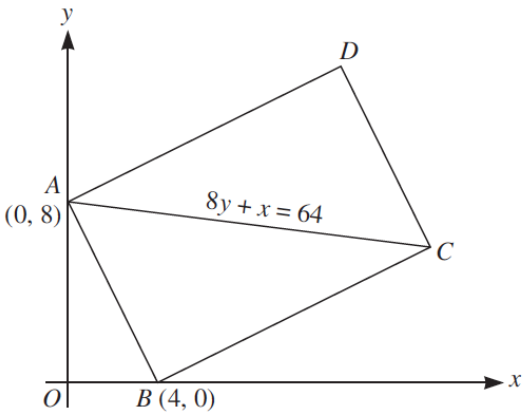
Question 4

The point A has coordinates $(3,\ 1)$ and the point B has coordinates $(-21,\ 11)$. The point C is the mid-point of AB.

(i) Find the equation of the line through A that is perpendicular to $y = 2x - 7$. [2]

(ii) Find the distance AC. [3]

Question 5



The diagram shows a rectangle $ABCD$ in which point A is $(0, 8)$ and point B is $(4, 0)$. The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D. [7]

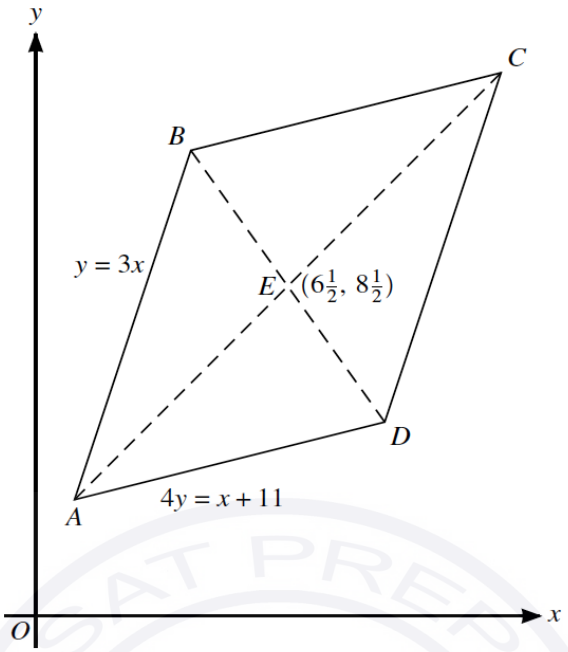
Question 6

The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

(i) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

(ii) A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]

Question 7



The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$. Find, by calculation, the coordinates of A, B, C and D. [9]

Question 8

Find the coordinates of the point at which the perpendicular bisector of the line joining (2, 7) to (10, 3) meets the x-axis. [5]

Question 9

The coordinates of points A and B are $(a, 2)$ and $(3, b)$ respectively, where a and b are constants. The distance AB is $\sqrt{(125)}$ units and the gradient of the line AB is 2. Find the possible values of a and of b. [6]

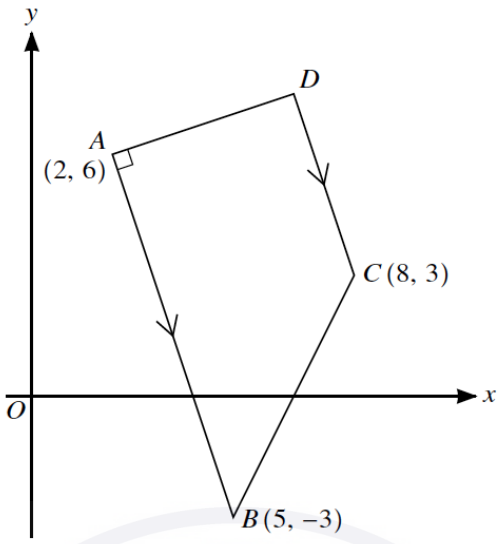
Question 10

A is the point $(a, 2a - 1)$ and B is the point $(2a + 4, 3a + 9)$, where a is a constant.

(i) Find, in terms of a, the gradient of a line perpendicular to AB. [3]

(ii) Given that the distance AB is $\sqrt{(260)}$, find the possible values of a. [4]

Question 11



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle BAD is 90°. The coordinates of A, B and C are (2, 6), (5, −3) and (8, 3) respectively.

(i) Find the equation of AD. [3]

(ii) Find, by calculation, the coordinates of D. [3]

The point E is such that $ABCE$ is a parallelogram.

(iii) Find the length of BE. [2]

Question 12

The line $4x + ky = 20$ passes through the points $A\,(8, -4)$ and $B\,(b, 2b)$, where k and b are constants.

(i) Find the values of k and b. [4]

(ii) Find the coordinates of the mid-point of AB. [1]

Question 13

The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

Question 14

The point C lies on the perpendicular bisector of the line joining the points $A\,(4, 6)$ and $B\,(10, 2)$. C also lies on the line parallel to AB through (3, 11).

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

Question 15

The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

Question 16

Points A, B and C have coordinates $A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$, where k is a constant.

(i) Given that $AB = BC$, calculate the possible values of k. [3]

The perpendicular bisector of AB intersects the x-axis at D.

(ii) Calculate the coordinates of D. [5]

Question 17

A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

Question 18

Two points have coordinates $A(5, 7)$ and $B(9, -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C(1, 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

(ii) Find, by calculation, the distance BX. [5]

Question 19

Triangle ABC has vertices at $A(-2, -1)$, $B(4, 6)$ and $C(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

Question 20

Three points have coordinates $A(0, 7)$, $B(8, 3)$ and $C(3k, k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

Question 21

A curve has equation $y = 3x - \frac{4}{x}$ and passes through the points $A\,(1, -1)$ and $B\,(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

Question 22

Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

Question 23

The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

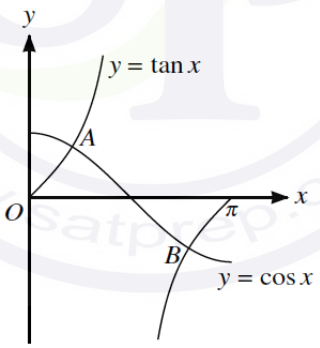
Question 24

C is the mid-point of the line joining $A\,(14, -7)$ to $B\,(-6, 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

(ii) Find the distance AD. [2]

Question 25



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

(ii) Find by calculation the coordinates of B. [3]

Question 26

$A\,(-1, 1)$ and $P\,(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B\,(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

Question 27

Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

Question 28

The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

(ii) Find the coordinates of B. [3]

Question 29

The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

The line through P and Q meets the x-axis at $H\,(h, 0)$ and the y-axis at $K\,(0, k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

Question 30

The points $A\,(1, 1)$ and $B\,(5, 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

Question 31

A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0, 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

Question 32

The coordinates of points A and B are $(-3k - 1, k + 3)$ and $(k + 3, 3k + 5)$ respectively, where k is a constant $(k \neq -1)$.

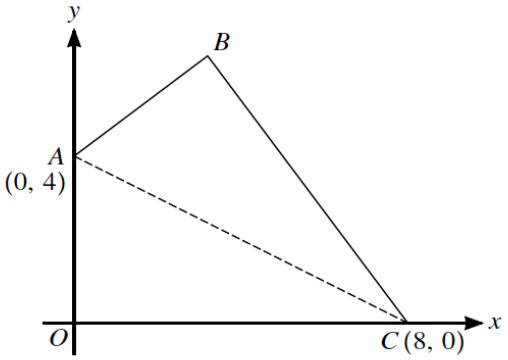
(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

Question 33

Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

Question 34



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are (0, 4) and (8, 0) respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

(ii) Find, by calculation, the coordinates of B. [3]

Question 35

Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

Question 36

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

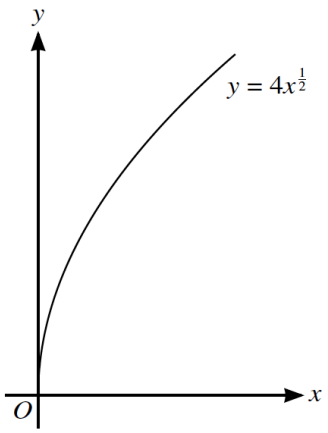
(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

Question 37



The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

Question 38

The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB. A point C has coordinates (x, y), where x and y are variables.

(i) State the coordinates of D. [1]

(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

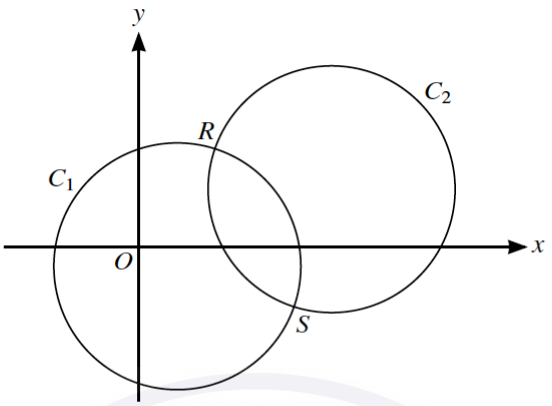
Question 39

The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

Question 40

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

Question 41

(a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

Question 42

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

Question 43

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

Question 44

A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

(d) Find the x-coordinates of A and B. [3]

Question 45

A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

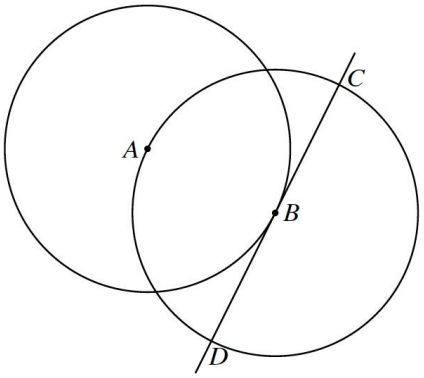
Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

Question 46



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

Question 47

The points $A\,(7, 1)$, $B\,(7, 9)$ and $C\,(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

Question 48

Points $A\,(-2, 3)$, $B\,(3, 0)$ and $C\,(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

(b) Hence state the coordinates of D. [1]

(c) Find an equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

Question 49

The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

Question 50

Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

Question 51

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

Question 52

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

Question 53

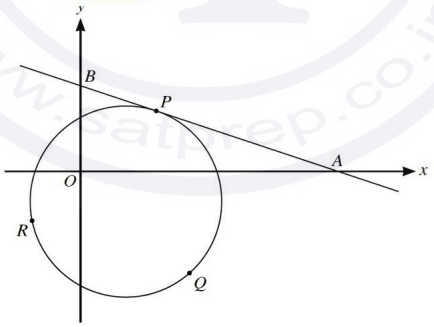
The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

Question 54



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P\,(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

Question 55

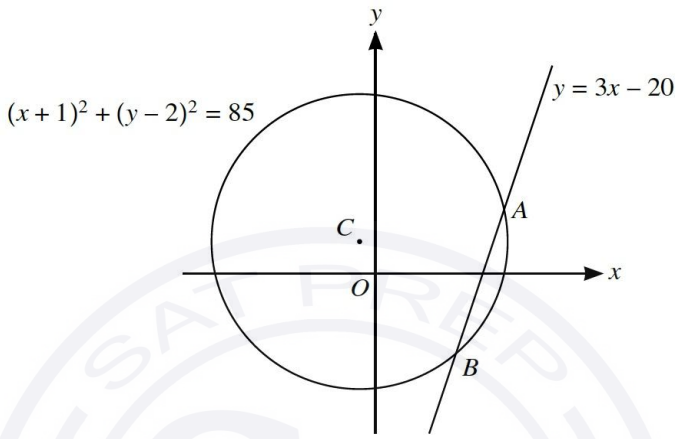
A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

Question 56

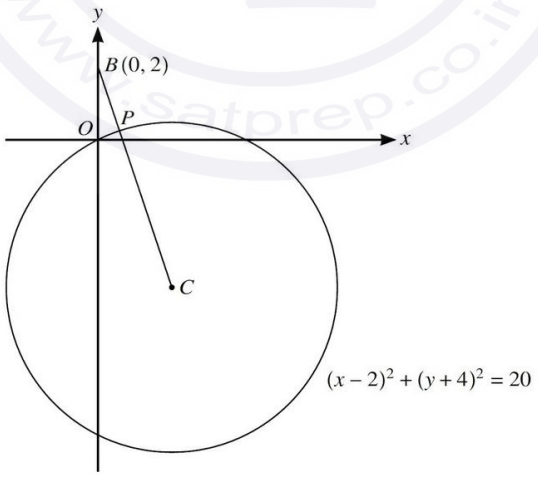


The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

Question 57



The diagram shows the circle with equation $(x - 2)^2 + (y + 4)^2 = 20$ and with centre C. The point B has coordinates (0, 2) and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

Question 58

The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

Question 59

The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

Question 60

The coordinates of points A, B and C are $A\,(5,\ -2)$, $B\,(10,\ 3)$ and $C\,(2p,\ p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]

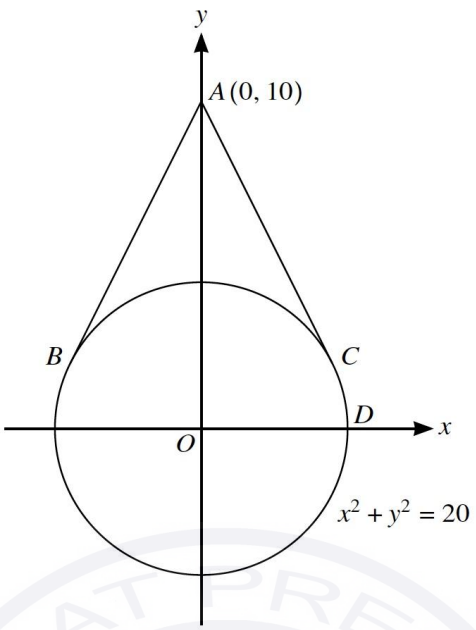
Question 61

Points A and B have coordinates $(5,\ 2)$ and $(10,\ -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle with centre A which passes through B. [3]

Question 62



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A\,(0,\ 10)$.

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

Question 63

Points $A\,(7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

Question 64

A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

(b) Find an equation of the circle with diameter AB. [3]

Question 65

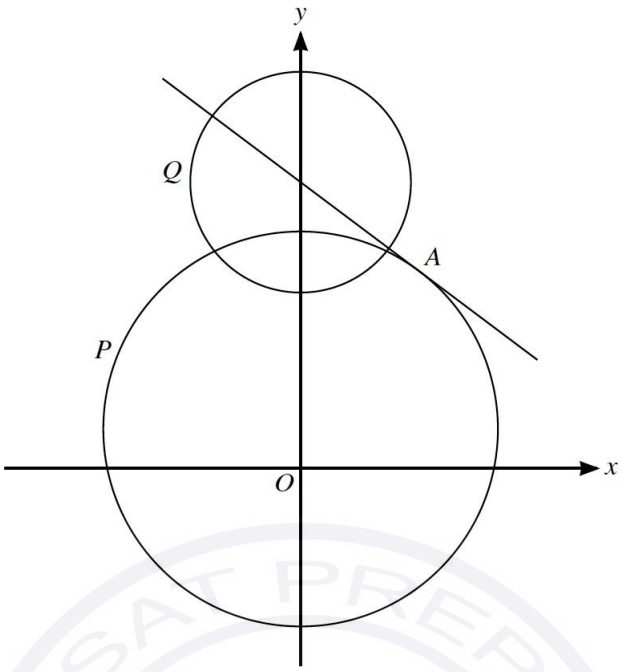
The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

Question 66



The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

(b) Find the equation of the tangent to the circle P at A. [2]

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]