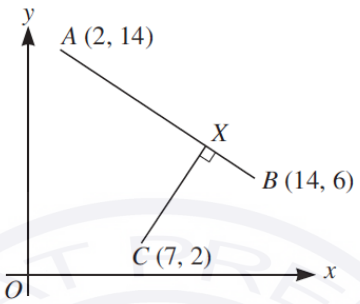
AS-Level

Pure Mathematics P1

Topic :Coordinate Geometry and Circles

May 2013- May 2025

Question 1



The diagram shows three points $A\,(2,\ 14)$, $B\,(14,\ 6)$ and $C\,(7,\ 2)$. The point X lies on AB, and CX is perpendicular to AB. Find, by calculation,

(i) the coordinates of X, [6]

(ii) the ratio $AX : XB$. [2]

Question 2

The point R is the reflection of the point $(-1,\ 3)$ in the line $3y + 2x = 33$. Find by calculation the coordinates of R. [7]

Question 3

A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

(i) For the case where $m = 1$, the curve and the line intersect at the points A and B. Find the coordinates of the mid-point of AB. [4]

(ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

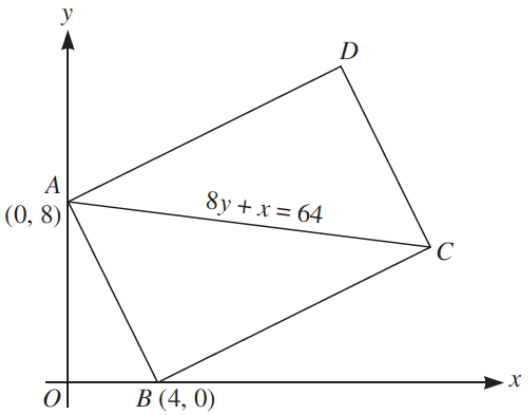
Question 4

The point A has coordinates $(3,\ 1)$ and the point B has coordinates $(-21,\ 11)$. The point C is the mid-point of AB.

(i) Find the equation of the line through A that is perpendicular to $y = 2x - 7$. [2]

(ii) Find the distance AC. [3]

Question 5



The diagram shows a rectangle $ABCD$ in which point A is $(0, 8)$ and point B is $(4, 0)$. The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D. [7]

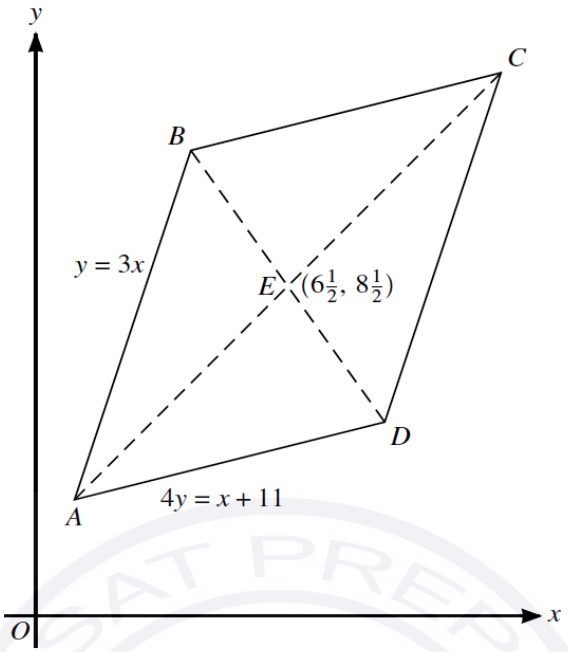
Question 6

The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

(i) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

(ii) A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]

Question 7



The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$. Find, by calculation, the coordinates of A, B, C and D. [9]

Question 8

Find the coordinates of the point at which the perpendicular bisector of the line joining (2, 7) to (10, 3) meets the x-axis. [5]

Question 9

The coordinates of points A and B are $(a, 2)$ and $(3, b)$ respectively, where a and b are constants. The distance AB is $\sqrt{(125)}$ units and the gradient of the line AB is 2. Find the possible values of a and of b. [6]

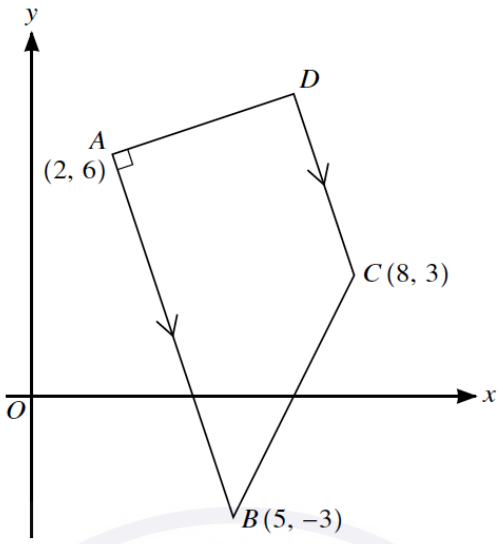
Question 10

A is the point $(a, 2a - 1)$ and B is the point $(2a + 4, 3a + 9)$, where a is a constant.

(i) Find, in terms of a, the gradient of a line perpendicular to AB. [3]

(ii) Given that the distance AB is $\sqrt{(260)}$, find the possible values of a. [4]

Question 11



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle BAD is 90°. The coordinates of A, B and C are (2, 6), (5, −3) and (8, 3) respectively.

(i) Find the equation of AD. [3]

(ii) Find, by calculation, the coordinates of D. [3]

The point E is such that $ABCE$ is a parallelogram.

(iii) Find the length of BE. [2]

Question 12

The line $4x + ky = 20$ passes through the points $A\,(8, -4)$ and $B\,(b, 2b)$, where k and b are constants.

(i) Find the values of k and b. [4]

(ii) Find the coordinates of the mid-point of AB. [1]

Question 13

The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

Question 14

The point C lies on the perpendicular bisector of the line joining the points $A\,(4, 6)$ and $B\,(10, 2)$. C also lies on the line parallel to AB through (3, 11).

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

Question 15

The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

Question 16

Points A, B and C have coordinates $A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$, where k is a constant.

(i) Given that $AB = BC$, calculate the possible values of k. [3]

The perpendicular bisector of AB intersects the x-axis at D.

(ii) Calculate the coordinates of D. [5]

Question 17

A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

Question 18

Two points have coordinates $A(5, 7)$ and $B(9, -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C(1, 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

(ii) Find, by calculation, the distance BX. [5]

Question 19

Triangle ABC has vertices at $A(-2, -1)$, $B(4, 6)$ and $C(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

Question 20

Three points have coordinates $A(0, 7)$, $B(8, 3)$ and $C(3k, k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

Question 21

A curve has equation $y = 3x - \frac{4}{x}$ and passes through the points $A\,(1, -1)$ and $B\,(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

Question 22

Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

Question 23

The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

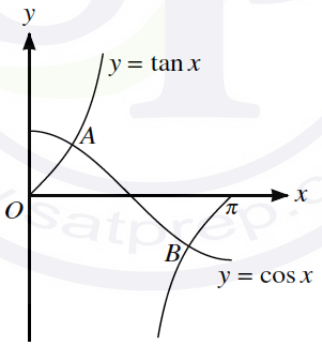
Question 24

C is the mid-point of the line joining $A\,(14, -7)$ to $B\,(-6, 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

(ii) Find the distance AD. [2]

Question 25



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

(ii) Find by calculation the coordinates of B. [3]

Question 26

$A\,(-1, 1)$ and $P\,(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B\,(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

Question 27

Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

Question 28

The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

(ii) Find the coordinates of B. [3]

Question 29

The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

The line through P and Q meets the x-axis at $H\,(h, 0)$ and the y-axis at $K\,(0, k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

Question 30

The points $A\,(1, 1)$ and $B\,(5, 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

Question 31

A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0, 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

Question 32

The coordinates of points A and B are $(-3k - 1, k + 3)$ and $(k + 3, 3k + 5)$ respectively, where k is a constant $(k \neq -1)$.

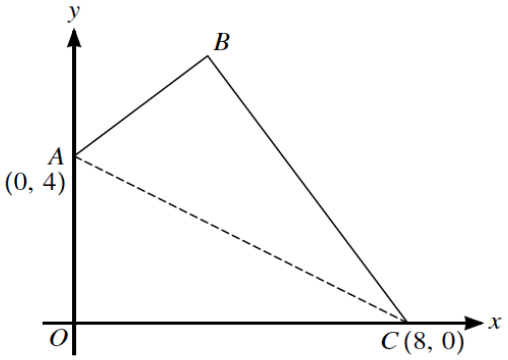
(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

Question 33

Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

Question 34



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are $(0, 4)$ and $(8, 0)$ respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

(ii) Find, by calculation, the coordinates of B. [3]

Question 35

Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

Question 36

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

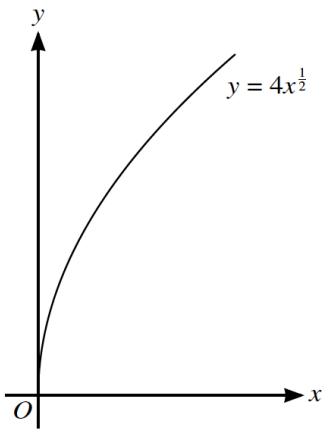
(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

Question 37



The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

Question 38

The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB. A point C has coordinates (x, y), where x and y are variables.

(i) State the coordinates of D. [1]

(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

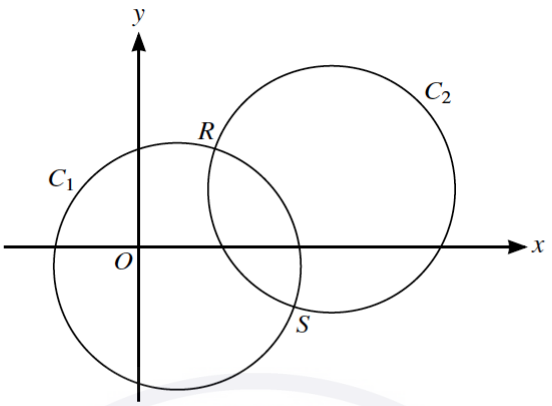
Question 39

The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

Question 40

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

Question 41

(a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

Question 42

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

Question 43

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

Question 44

A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

(d) Find the x-coordinates of A and B. [3]

Question 45

A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

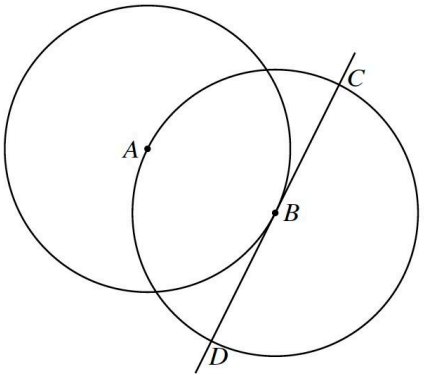
Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

Question 46



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

Question 47

The points $A\,(7, 1)$, $B\,(7, 9)$ and $C\,(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

Question 48

Points $A\,(-2, 3)$, $B\,(3, 0)$ and $C\,(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

(b) Hence state the coordinates of D. [1]

(c) Find an equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

Question 49

The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

Question 50

Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

Question 51

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

Question 52

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

Question 53

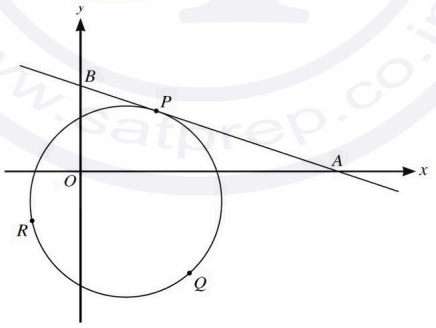
The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

Question 54



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P\,(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

Question 55

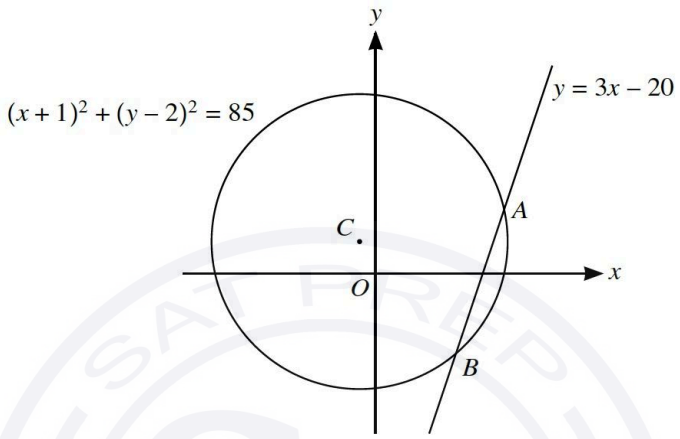
A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

Question 56

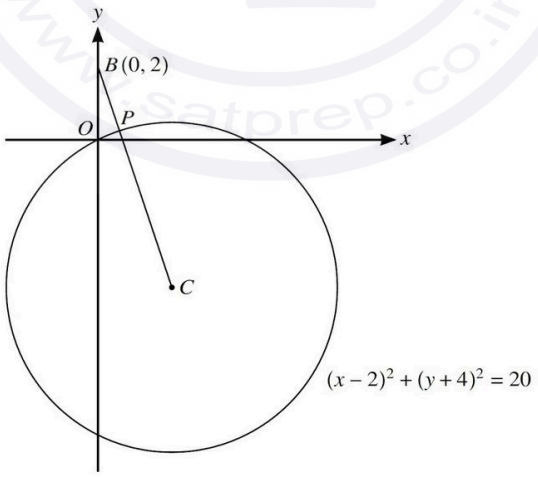


The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

Question 57



The diagram shows the circle with equation $(x - 2)^2 + (y + 4)^2 = 20$ and with centre C. The point B has coordinates (0, 2) and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

Question 58

The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

Question 59

The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

Question 60

The coordinates of points A, B and C are $A\,(5,\ -2)$, $B\,(10,\ 3)$ and $C\,(2p,\ p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]

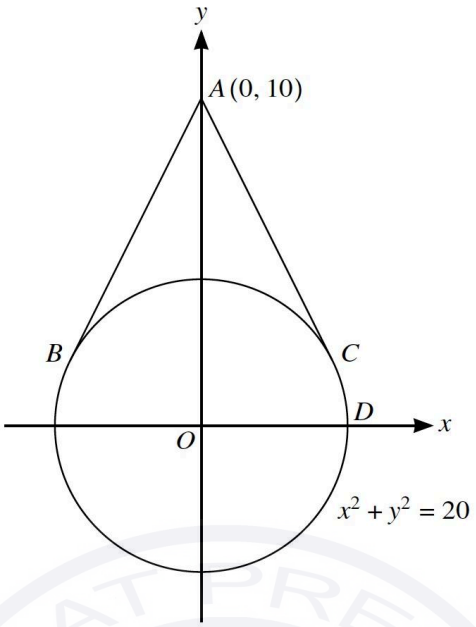
Question 61

Points A and B have coordinates $(5,\ 2)$ and $(10,\ -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle with centre A which passes through B. [3]

Question 62



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A\,(0,\ 10)$.

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

Question 63

Points $A\,(7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

Question 64

A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

(b) Find an equation of the circle with diameter AB. [3]

Question 65

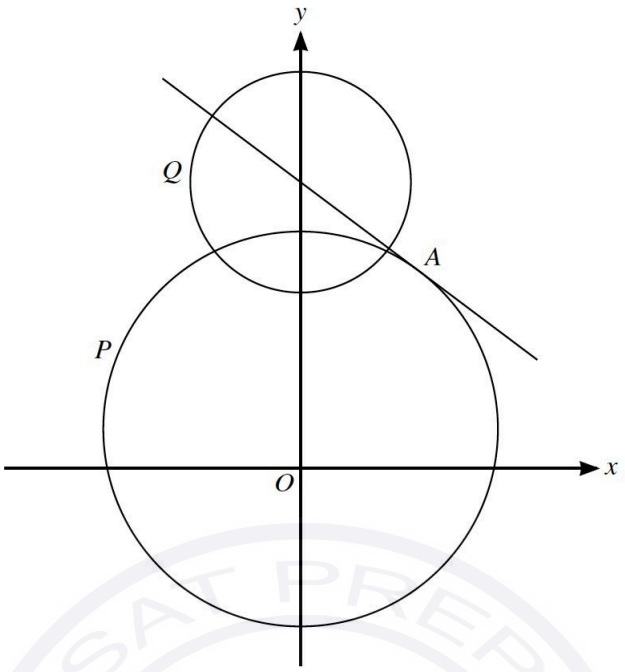
The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

Question 66



The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

(b) Find the equation of the tangent to the circle P at A. [2]

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

Question 67

The circle with equation $(x-3)^2 + (y-5)^2 = 40$ intersects the y-axis at points A and B.

(a) Find the y-coordinates of A and B, expressing your answers in terms of surds. [2]

(b) Find the equation of the circle which has AB as its diameter. [2]

Question 68

The coordinates of points A, B and C are (6, 4), (p, 7) and (14, 18) respectively, where p is a constant. The line AB is perpendicular to the line BC.

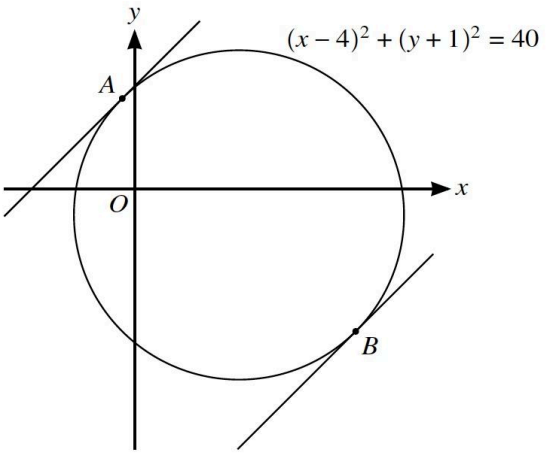
(a) Given that $p < 10$, find the value of p. [4]

A circle passes through the points A, B and C.

(b) Find the equation of the circle. [3]

(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d, e and f are integers. [3]

Question 69



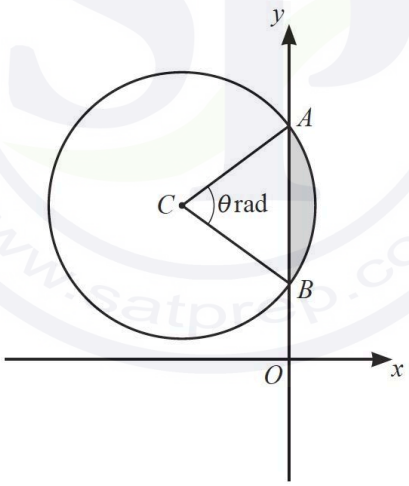
The diagram shows the circle with equation $(x-4)^2+(y+1)^2=40$. Parallel tangents, each with gradient 1, touch the circle at points A and B.

(a) Find the equation of the line AB, giving the answer in the form $y=mx+c$. [3]

(b) Find the coordinates of A, giving each coordinate in surd form. [4]

(c) Find the equation of the tangent at A, giving the answer in the form $y=mx+c$, where c is in surd form. [2]

Question 70



The diagram shows the circle with centre $C(-4,5)$ and radius $\sqrt{20}$ units. The circle intersects the y-axis at the points A and B. The size of angle ACB is θ radians.

(a) Find the equation of the tangent to the circle at the point $(-6,9)$. [3]

(b) Find the equation of the circle in the form $x^2+y^2+ax+by+c=0$. [2]

(c) Find the value of θ correct to 4 significant figures. [3]

(d) Find the perimeter and area of the segment shaded in the diagram. [4]

Question 71

A circle with equation $x^2+y^2-6x+2y-15=0$ meets the y-axis at the points A and B. The tangents to the circle at A and B meet at the point P.

Find the coordinates of P. [8]

Question 72

The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

Question 73

The equation of a circle is $(x-3)^2+y^2=18$. The line with equation $y=mx+c$ passes through the point $(0,-9)$ and is a tangent to the circle.

Find the two possible values of m and, for each value of m, find the coordinates of the point at which the tangent touches the circle. [8]

Question 74

The equation of a curve is $y=\mathrm{f}(x)$, where $\mathrm{f}(x)=(2x-1)\sqrt{3x-2}-2$. The following points lie on the curve. Non-exact values have been given correct to 5 decimal places.

$A(2,4)$, $B(2.0001,k)$, $C(2.001,4.00625)$, $D(2.01,4.06261)$, $E(2.1,4.63566)$, $F(3,11.22876)$

(a) Find the value of k. Give your answer correct to 5 decimal places. [1]

The table shows the gradients of the chords AB, AC, AD and AF.

Chord	AB	AC	AD	AE	AF
Gradient of chord	6.2501	6.2511	6.2608		7.2288

(b) Find the gradient of the chord AE. Give your answer correct to 4 decimal places. [1]

(c) Deduce the value of $\mathrm{f}'(2)$ using the values in the table. [1]

Question 75

Points A and B have coordinates (4, 3) and (8, −5) respectively. A circle with radius 10 passes through the points A and B.

(a) Show that the centre of the circle lies on the line $y=\frac{1}{2}x-4$. [4]

(b) Find the two possible equations of the circle. [5]

Question 76

The equation of a circle is $x^2+y^2+px+2y+q=0$, where p and q are constants.

(a) Express the equation in the form $(x-a)^2+(y-b)^2=r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q. [2]

The line with equation $x+2y=10$ is the tangent to the circle at the point $A(4, 3)$.

(b) **(i)** Find the equation of the normal to the circle at the point A. [3]

(ii) Find the values of p and q. [5]

Question 77

Circles C_1 and C_2 have equations

$$x^2+y^2+6x-10y+18=0 \text{ and } (x-9)^2+(y+4)^2-64=0$$

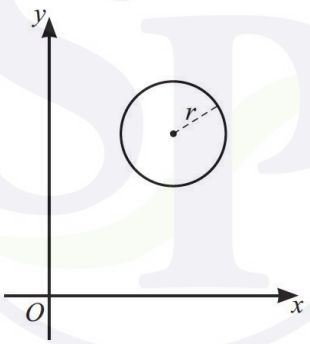
respectively.

(a) Find the distance between the centres of the circles. [4]

P and Q are points on C_1 and C_2 respectively. The distance between P and Q is denoted by d.

(b) Find the greatest and least possible values of d. [3]

Question 78



The diagram shows a circle C of radius r, where $x>0$ and $y>0$ for all points on C. The least distance between any point on C and the x-axis is 8 units, and the least distance between any point on C and the y-axis is 5 units.

(a) State the coordinates of the centre of the circle in terms of r. [1]

(b) Given that the distance between the origin and the centre of the circle is 15 units, find the value of r. [3]

(c) The point on the circle furthest from the origin is denoted by P.

Find the gradient of the tangent to the circle at P. [2]

Question 79

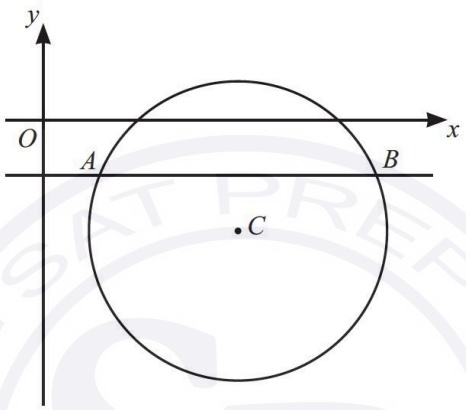
Three points P, Q and R have coordinates $P(-13, 5)$, $Q(5, 1)$ and $R(2, k)$, where k is a constant. It is given that the angle PRQ is a right angle.

(a) Show that one of the possible values of k is 10, and find the other possible value. [4]

(b) It is now given that $k = 10$. A circle passes through the points P, Q and R.

Find the equation of the tangent to the circle at R. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. [5]

Question 80



The diagram shows the circle with equation $x^2+y^2-14x+8y+36=0$ and the line $y=-2$. The line intersects the circle at the points A and B. The centre of the circle is C.

(a) Find the coordinates of A, B and C. [3]

(b) Find the angle ACB in radians. Give your answer correct to 3 significant figures. [2]

(c) The chord AB divides the circle into two segments.

Find the area of the larger segment. [4]

Question 81

Find the coordinates of the points of intersection of the curve and the line with equations

$$2xy+5y^2=24 \quad \text{and} \quad 2x+y+4=0.$$ [4]

Question 82

The circle with equation $x^2+y^2-6x+10y-27=0$ intersects the line $x=-2$ at the points P and Q.

Find the area of the triangle formed by the tangents to the circle at P and Q, and the line $x=-2$. [8]