A-Level

Topic : Continuous RandomVariable

May 2013-May 2025

Questions

Question 1

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^3} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find $P(1 \leqslant X \leqslant 2)$. [2]

(iii) Find $E(X)$. [3]

Question 2

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find $P(X < E(X))$. [2]

(iii) Hence explain whether the mean of X is less than, equal to or greater than the median of X. [2]

Question 3

The time in minutes taken by people to read a certain booklet is modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \frac{1}{2\sqrt{t}} & 4 \leqslant t \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the time within which 90% of people finish reading the booklet. [3]

(ii) Find $E(T)$ and $\mathrm{Var}(T)$. [6]

Question 4

The waiting time, T weeks, for a particular operation at a hospital has probability density function given by

$$f(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Given that $E(T) = \frac{16}{3}$, find $\text{Var}(T)$. [3]

(ii) 10% of patients have to wait more than n weeks for their operation. Find the value of n, giving your answer correct to the nearest integer. [5]

Question 5

The volume, in cm^3, of liquid left in a glass by people when they have finished drinking all they want is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{8}$. [2]

(ii) 20% of people leave at least $d\ \text{cm}^3$ of liquid in a glass. Find d. [3]

(iii) Find $E(X)$. [3]

Question 6

The lifetime, X years, of a certain type of battery has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

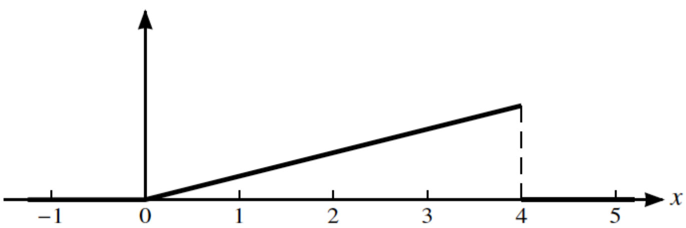
where k and a are positive constants.

(i) State what the value of a represents in this context. [1]

(ii) Show that $k = \dfrac{a}{a-1}$. [3]

(iii) Experience has shown that the longest that any battery of this type lasts is 2.5 years. Find the mean lifetime of batteries of this type. [3]

Question 7



A random variable X takes values between 0 and 4 only and has probability density function as shown in the diagram. Calculate the median of X. [3]

Question 8

The time, T hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} kt(16 - t^2) & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{64}$. [3]

(ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

(iii) Find the mean time spent on a visit to the museum. [3]

Question 9

A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

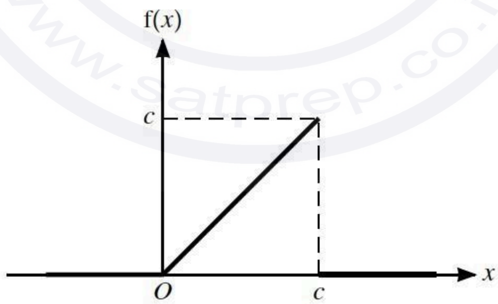
where k and a are positive constants.

(i) Show that $k = \dfrac{1}{\ln a}$. [3]

(ii) Find $E(X)$ in terms of a. [3]

(iii) Find the median of X in terms of a. [4]

Question 10



The diagram shows the graph of the probability density function, f, of a random variable X.

(i) Find the value of the constant c. [2]

(ii) Find the value of a such that $P(a < X < 1) = 0.1$. [4]

(iii) Find $E(X)$. [2]

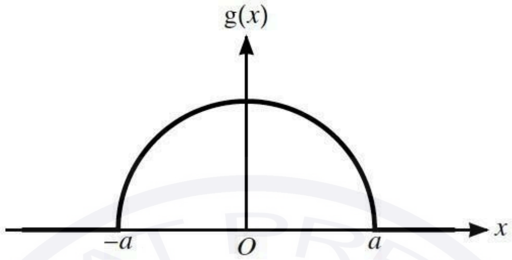
Question 11

(a) The time for which Lucy has to wait at a certain traffic light each day is T minutes, where T has probability density function given by

$$f(t) = \begin{cases} \frac{3}{2}t - \frac{3}{4}t^2 & 0 \leqslant t \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that, on a randomly chosen day, Lucy has to wait for less than half a minute at the traffic light. [3]

(b)



The diagram shows the graph of the probability density function, g, of a random variable X. The graph of g is a semicircle with centre (0, 0) and radius a. Elsewhere $g(x) = 0$.

(i) Find the value of a. [2]

(ii) State the value of $E(X)$. [1]

(iii) Given that $P(X < -c) = 0.2$, find $P(X < c)$. [2]

Question 12

The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{3}{4}x(c - x) & 0 \leqslant x \leqslant c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that $c = 2$. [3]

(ii) Sketch the graph of $y = f(x)$ and state the median of X. [3]

(iii) Find $P(X < 1.5)$. [4]

(iv) Hence write down the value of $P(0.5 < X < 1)$. [1]

Question 13

The waiting time, T minutes, for patients at a doctor's surgery has probability density function given by

$$f(t) = \begin{cases} k(225 - t^2) & 0 \leqslant t \leqslant 15, \\ 0 & \text{otherwise,} \end{cases}$$

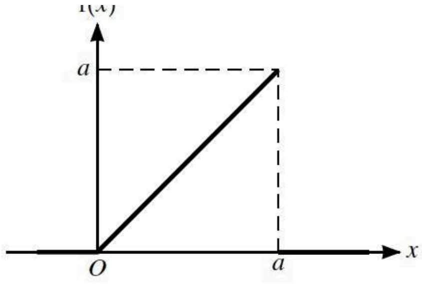
where k is a constant.

(i) Show that $k = \frac{1}{2250}$. [3]

(ii) Find the probability that a patient has to wait for more than 10 minutes. [3]

(iii) Find the mean waiting time. [4]

Question 14



The random variable X has probability density function, f, as shown in the diagram, where a is a constant. Find the value of a and hence show that $E(X) = 0.943$ correct to 3 significant figures. [5]

Question 15

A random variable X has probability density function given by

$$f(x) = \begin{cases} k(4 - x^2) & -2 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{32}$. [3]

(ii) Sketch the graph of $y = f(x)$ and hence write down the value of $E(X)$. [2]

(iii) Find $P(X < 1)$. [3]

Question 16

A random variable X has probability density function given by

$$f(x) = \begin{cases} k(3 - x) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

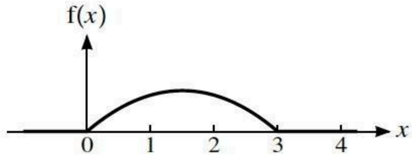
where k is a constant.

(i) Show that $k = \frac{2}{3}$. [3]

(ii) Find the median of X. [4]

Question 17

(a)



The diagram shows the graph of the probability density function, f, of a random variable X, where

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) State the value of $E(X)$ and find $Var(X)$. [4]

(ii) State the value of $P(1.5 \leqslant X \leqslant 4)$. [1]

(iii) Given that $P(1 \leqslant X \leqslant 2) = \frac{13}{27}$, find $P(X > 2)$. [2]

(b) A random variable, W, has probability density function given by

$$g(w) = \begin{cases} aw & 0 \leqslant w \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants. Given that the median of W is 2, find a and b. [4]

Question 18

The time, T minutes, taken by people to complete a test has probability density function given by

$$f(t) = \begin{cases} k(10t - t^2) & 5 \leqslant t \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{250}$. [3]

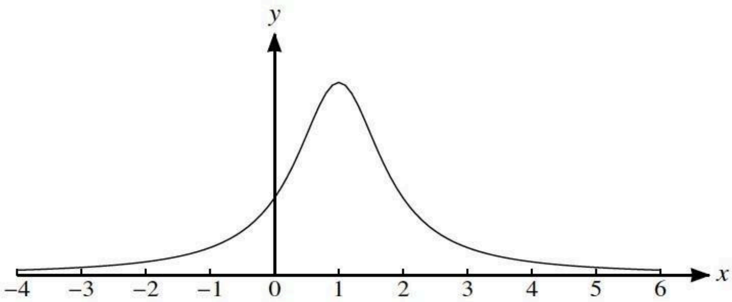
(ii) Find $E(T)$. [3]

(iii) Find the probability that a randomly chosen value of T lies between $E(T)$ and the median of T. [3]

(iv) State the greatest possible length of time taken to complete the test. [1]

Question 19

(a)



The diagram shows the graph of the probability density function of a variable X. Given that the graph is symmetrical about the line $x = 1$ and that $P(0 < X < 2) = 0.6$, find $P(X > 0)$. [2]

(b) A flower seller wishes to model the length of time that tulips last when placed in a jug of water. She proposes a model using the random variable X (in hundreds of hours) with probability density function given by

$$f(x) = \begin{cases} k(2.25 - x^2) & 0 \leqslant x \leqslant 1.5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{4}{9}$. [3]

(ii) Use this model to find the mean number of hours that a tulip lasts in a jug of water. [4]

The flower seller wishes to create a similar model for daffodils. She places a large number of daffodils in jugs of water and the longest time that any daffodil lasts is found to be 290 hours.

(iii) Give a reason why $f(x)$ would not be a suitable model for daffodils. [1]

(iv) The flower seller considers a model for daffodils of the form

$$g(x) = \begin{cases} c(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a and c are constants. State a suitable value for a. (There is no need to evaluate c.) [1]

Question 20

In each turn of a game, a coin is pushed and slides across a table. The distance, X metres, travelled by the coin has probability density function given by

$$f(x) = \begin{cases} kx^2(2 - x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

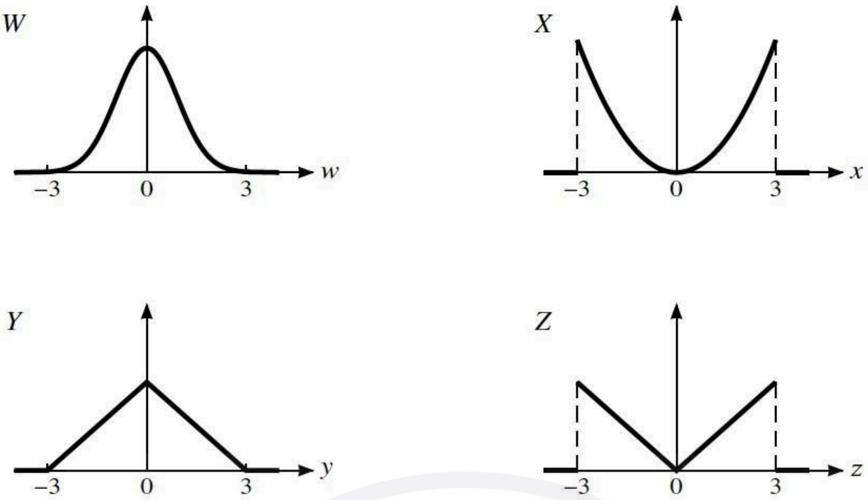
(i) State the greatest possible distance travelled by the coin in one turn. [1]

(ii) Show that $k = \frac{3}{4}$. [3]

(iii) Find the mean distance travelled by the coin in one turn. [3]

(iv) Out of 400 turns, find the expected number of turns in which the distance travelled by the coin is less than 1 metre. [3]

Question 21



The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between −3 and 3 only, and their standard deviations are σ_W, σ_X, σ_Y and σ_Z respectively.

(i) List σ_W, σ_X, σ_Y and σ_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

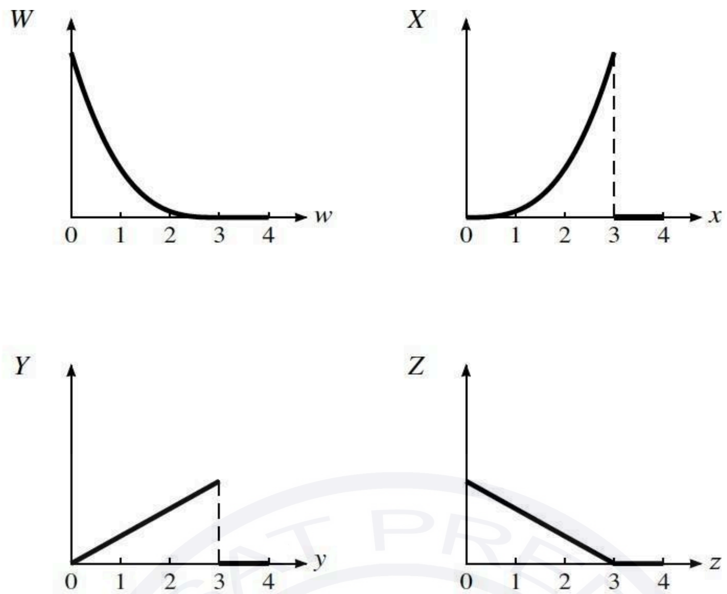
$$f(x) = \begin{cases} \frac{1}{18}x^2 & -3 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\sigma_X = 2.32$ correct to 3 significant figures. [3]

(b) Calculate $P(X > \sigma_X)$. [3]

(c) Write down the value of $P(X > 2\sigma_X)$. [1]

Question 22



The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between 0 and 3 only, and their medians are m_W, m_X, m_Y and m_Z respectively.

(i) List m_W, m_X, m_Y and m_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$f(x) = \begin{cases} \frac{4}{81}x^3 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

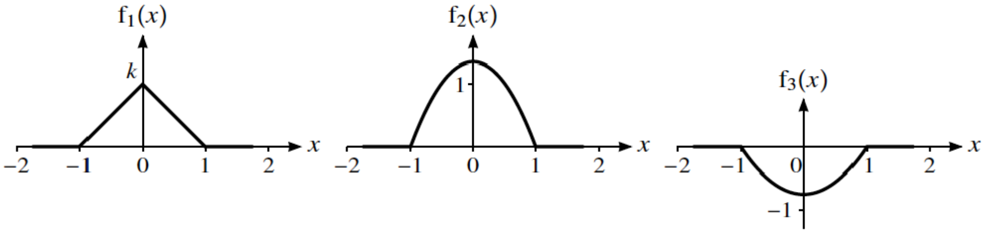
(a) Show that $E(X) = \frac{12}{5}$. [3]

(b) Calculate $P(X > E(X))$. [3]

(c) Write down the value of $P(X < 2E(X))$. [1]

Question 23

(a)



The diagram shows the graphs of three functions, f_1, f_2 and f_3. The function f_1 is a probability density function.

(i) State the value of k. [1]

(ii) For each of the functions f_2 and f_3, state why it cannot be a probability density function. [2]

(b) The probability density function g is defined by

$$g(x) = \begin{cases} 6(a^2 - x^2) & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

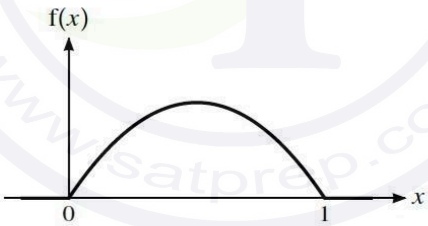
where a is a constant.

(i) Show that $a = \frac{1}{2}$. [3]

(ii) State the value of $E(X)$. [1]

(iii) Find $Var(X)$. [2]

Question 24



The diagram shows the graph of the probability density function, f, of a continuous random variable X, where f is defined by

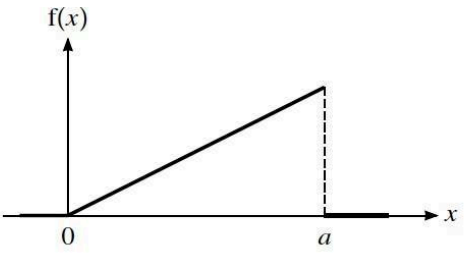
$$f(x) = \begin{cases} k(x - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the value of the constant k is 6. [3]

(ii) State the value of $E(X)$ and find $Var(X)$. [4]

(iii) Find $P(0.4 < X < 2)$. [3]

Question 25



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and a only. It is given that $P(X < 1) = 0.25$.

(i) Find, in any order,

(a) $P(X < 2)$,

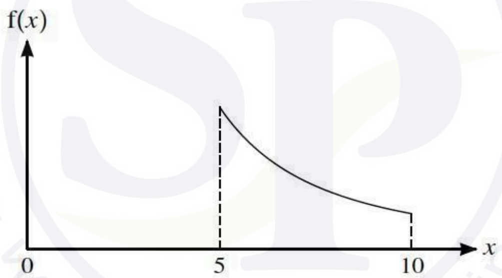
(b) the value of a,

(c) $f(x)$.

[5]

(ii) Find the median of X. [3]

Question 26



The time, X minutes, taken by a large number of runners to complete a certain race has probability density function f given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

(i) Without calculation, explain how you can tell that there were more runners whose times were below 7.5 minutes than above 7.5 minutes. [1]

(ii) Show that $k = 10$. [3]

(iii) Find $E(X)$. [3]

(iv) Find $Var(X)$. [2]

Question 27

A continuous random variable, X, has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find the median of X. [3]

Question 28

The random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{x}} & 0 < x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants. It is given that $E(X) = 3$.

(i) Find the value of a and show that $k = \frac{1}{6}$. [7]

(ii) Find the median of X. [3]

Question 29

A random variable X has probability density function given by

$$f(x) = \begin{cases} 6x(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the probability that X does not lie between 0.3 and 0.7. [4]

(ii) Sketch the graph of the probability density function and hence state the value of $E(X)$. [2]

(iii) Find $\text{Var}(X)$. [3]

Question 30

A random variable X has probability density function defined by

$$f(x) = \begin{cases} k\left(\dfrac{1}{x^2} + \dfrac{1}{x^3}\right) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

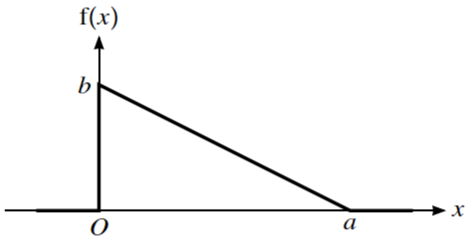
where k is a constant.

(i) Show that $k = \frac{8}{7}$. [3]

(ii) Find $E(X)$. [3]

(iii) Three values of X are chosen at random. Find the probability that one of these values is less than 1.5 and the other two are greater than 1.5. [5]

Question 31



The diagram shows the probability density function, f, of a random variable X, in terms of the constants a and b.

(i) Find b in terms of a. [2]

(ii) Show that $f(x) = \frac{2}{a} - \frac{2}{a^2}x$. [3]

(iii) Given that $E(X) = 0.5$, find a. [4]

Question 32

The time, in minutes, taken by people to complete a test is modelled by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 10$. [3]

(ii) Show that $E(X) = 10 \ln 2$. [2]

(iii) Find $P(X > 9)$. [3]

(iv) Given that $P(X < a) = 0.6$, find a. [3]

Question 33

The random variable X has probability density function given by

$$f(x) = \begin{cases} kx^{-1} & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{\ln 3}$. [2]

(ii) Show that $E(X) = 3.64$, correct to 3 significant figures. [3]

(iii) Given that the median of X is m, find $P(m < X < E(X))$. [4]

Question 34

The time, X hours, taken by a large number of runners to complete a race is modelled by the probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{(x+1)^2} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants.

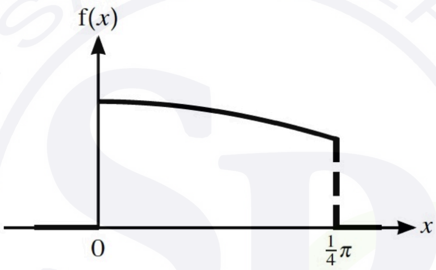
(i) Show that $k = \dfrac{a+1}{a}$. [3]

(ii) State what the constant a represents in this context. [1]

Three quarters of the runners take half an hour or less to complete the race.

(iii) Find the value of a. [3]

Question 35



A random variable X has probability density function given by

$$f(x) = \begin{cases} (\sqrt{2})\cos x & 0 \leqslant x \leqslant \frac{1}{4}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

as shown in the diagram.

(i) Find $P(X > \frac{1}{6}\pi)$. [2]

(ii) Find the median of X. [4]

(iii) Find $E(X)$. [4]

Question 36

A function f is defined by

$$f(x) = \begin{cases} \dfrac{3x^2}{a^3} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that f is a probability density function for all positive values of a. [3]

The random variable X has probability density function f and the median of X is 2.

(ii) Show that $a = 2.52$, correct to 3 significant figures. [3]

(iii) Find $E(X)$. [3]

Question 37

X is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{a}{x^2} & 1 \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

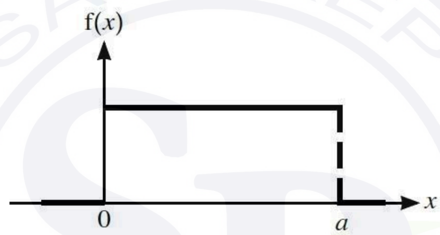
(i) Show that $b = \frac{a}{a-1}$. [3]

(ii) Given that the median of X is $\frac{3}{2}$, find the values of a and b. [3]

(iii) Use your values of a and b from part **(ii)** to find $E(X)$. [3]

Question 38

(a)



The diagram shows the graph of the probability density function, f, of a random variable X, where a is a constant greater than 0.5. The graph between $x = 0$ and $x = a$ is a straight line parallel to the x-axis.

(i) Find $P(X < 0.5)$ in terms of a. [2]

(ii) Find $E(X)$ in terms of a. [1]

(iii) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [2]

(b) A random variable T has probability density function given by

$$g(t) = \begin{cases} \frac{3}{2(t-1)^2} & 2 \leqslant t \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of b such that $P(T \leqslant b) = \frac{3}{4}$. [4]

Question 39

A random variable X has probability density function given by

$$f(x) = \begin{cases} k(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{2}{9}$. [3]

(ii) Find $P(1 \leqslant X \leqslant 2)$. [2]

(iii) Find $\text{Var}(X)$. [5]

Question 40

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Find a. [2]

(ii) Show that $E(X) = \frac{4}{3}$. [3]

The median of X is denoted by m.

(iii) Find $P(E(X) < X < m)$. [4]

Question 41

Bottles of Lanta contain approximately 300 ml of juice. The volume of juice, in millilitres, in a bottle is $300 + X$, where X is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{4000}(100 - x^2) & -10 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that a randomly chosen bottle of Lanta contains more than 305 ml of juice. [3]

(b) Given that 25% of bottles of Lanta contain more than $(300 + p)$ ml of juice, show that

$$p^3 - 300p + 1000 = 0.$$ [4]

Question 42

The length, X centimetres, of worms of a certain type is modelled by the probability density function

$$f(x) = \begin{cases} \frac{6}{125}(10 - x)(x - 5) & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(a) State the value of $E(X)$. [1]

(b) Find $\text{Var}(X)$. [3]

(c) Two worms of this type are chosen at random.

Find the probability that exactly one of them has length less than 6 cm. [5]

Question 43

A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(a) Show that $k = \dfrac{a}{a-1}$. [3]

(b) Find $E(X)$ in terms of a. [3]

(c) Find the 60th percentile of X in terms of a. [4]

Question 44

The length of time, T minutes, that a passenger has to wait for a bus at a certain bus stop is modelled by the probability density function given by

$$f(t) = \begin{cases} \frac{3}{4000}(20t - t^2) & 0 \leqslant t \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [1]

(b) Hence explain, without calculation, why $E(T) = 10$. [1]

(c) Find $Var(T)$. [3]

(d) It is given that $P(T < 10 + a) = p$, where $0 < a < 10$.

Find $P(10 - a < T < 10 + a)$ in terms of p. [2]

(e) Find $P(8 < T < 12)$. [3]

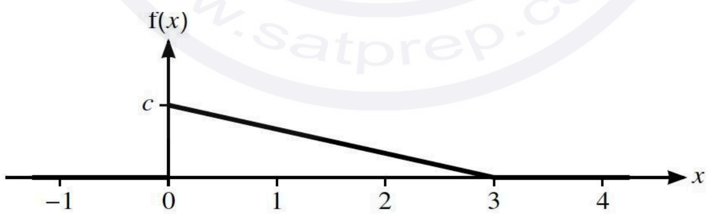
Question 45

The diagram shows the probability density function, $f(x)$, of a random variable X. For $0 \leqslant x \leqslant a$, $f(x) = k$; elsewhere $f(x) = 0$.

(a) Express k in terms of a. [1]

(b) Given that $Var(X) = 3$, find a. [4]

Question 46



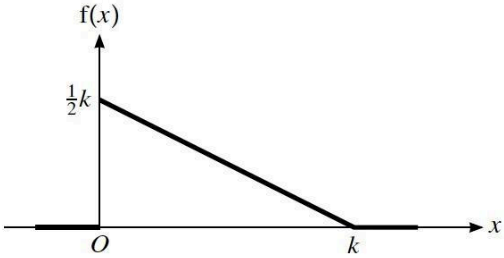
A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

(b) Find $P(X > 2)$. [2]

(c) Calculate $E(X)$. [4]

Question 47



The diagram shows the graph of the probability density function, f, of a random variable X.

(a) Find the value of the constant k. [2]

(b) Using this value of k, find $f(x)$ for $0 \leqslant x \leqslant k$ and hence find $E(X)$. [3]

(c) Find the value of p such that $P(p < X < 1) = 0.25$. [4]

Question 48

Alethia models the length of time, in minutes, by which her train is late on any day by the random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{3}{8000}(x-20)^2 & 0 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that the train is more than 10 minutes late on each of two randomly chosen days. [4]

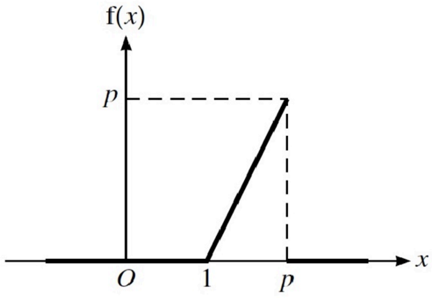
(b) Find $E(X)$. [4]

(c) The median of X is denoted by m.

Show that m satisfies the equation $(m-20)^3 = -4000$, and hence find m correct to 3 significant figures. [4]

(d) State one way in which Alethia's model may be unrealistic. [1]

Question 49



The random variable X takes values in the range $1 \leqslant x \leqslant p$, where p is a constant. The graph of the probability density function of X is shown in the diagram.

(a) Show that $p = 2$. [2]

(b) Find $E(X)$. [5]

Question 50

The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

State the value of $E(X)$ and show that $\text{Var}(X) = \frac{9}{5}$. [6]

Question 51

The graph of the probability density function of a random variable X is symmetrical about the line $x = 4$.

Given that $P(X < 5) = \frac{20}{27}$, find $P(3 < X < 5)$. [2]

Question 52

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{18}(9 - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 1.2)$. [3]

(b) Find $E(X)$. [3]

The median of X is m.

(c) Show that $m^3 - 27m + 27 = 0$. [3]

Question 53

(a) The probability density function of the random variable X is given by

$$f(x) = \begin{cases} kx(4-x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{16}$. [3]

(ii) Find $E(X)$. [3]

(b) The random variable Y has the following properties.

- Y takes values between 0 and 5 only.
- The probability density function of Y is symmetrical.

Given that $P(Y < a) = 0.2$, find $P(2.5 < Y < 5 - a)$ illustrating your method with a sketch on the axes provided. [3]

Question 54

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{18}(9 - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 1.2)$. [3]

(b) Find $E(X)$. [3]

The median of X is m.

(c) Show that $m^3 - 27m + 27 = 0$. [3]

Question 55

In a game a ball is rolled down a slope and along a track until it stops. The distance, in metres, travelled by the ball is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -k(x-1)(x-3) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Without calculation, explain why $E(X) = 2$. [1]

(b) Show that $k = \frac{3}{4}$. [3]

(c) Find $Var(X)$. [3]

One turn consists of rolling the ball 3 times and noting the largest value of X obtained. If this largest value is greater than 2.5, the player scores a point.

(d) Find the probability that on a particular turn the player scores a point. [4]

Question 56

The random variables X and W have probability density functions f and g defined as follows:

$$f(x) = \begin{cases} p(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

$$g(w) = \begin{cases} q(a^2 - w^2) & -a \leqslant w \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a, p and q are constants.

(a) **(i)** Write down the value of $P(X \geqslant 0)$. [1]

(ii) Write down the value of $P(W \geqslant 0)$. [1]

(iii) Write down an expression for q in terms of p only. [1]

(b) Given that $E(X) = 3$, find the value of a. [6]

Question 57

A random variable X has probability density function f. The graph of $f(x)$ is a straight line segment parallel to the x-axis from $x = 0$ to $x = a$, where a is a positive constant.

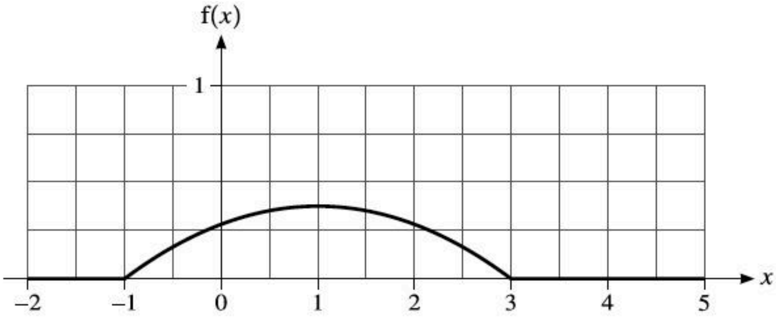
(a) State, in terms of a, the median of X. [1]

(b) Find $P(X > \frac{3}{4}a)$. [1]

(c) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [5]

(d) Given that $P(X < b) = p$, where $0 < b < \frac{1}{2}a$, find $P\left(\frac{1}{3}b < X < a - \frac{1}{3}b\right)$ in terms of p. [2]

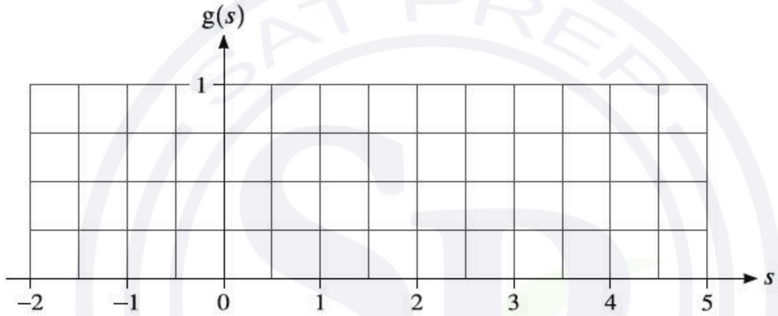
Question 58



The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

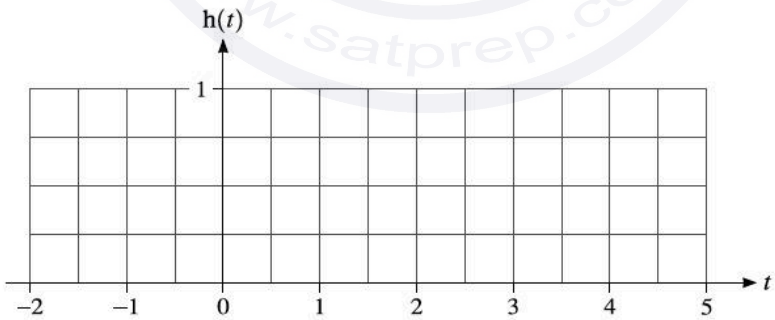
The random variable S is such that $E(S) = 2 \times E(X)$ and $Var(S) = Var(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $E(T) = E(X)$ and $Var(T) = \frac{1}{4}Var(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



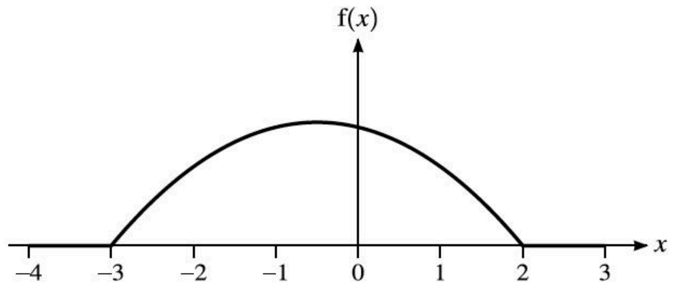
It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

(d) Hence verify that $0.69 < a < 0.70$. [1]

Question 59



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between -3 and 2 only.

(a) Given that the graph is symmetrical about the line $x = -0.5$ and that $P(X < 0) = p$, find $P(-1 < X < 0)$ in terms of p. [2]

(b) It is now given that the probability density function shown in the diagram is given by

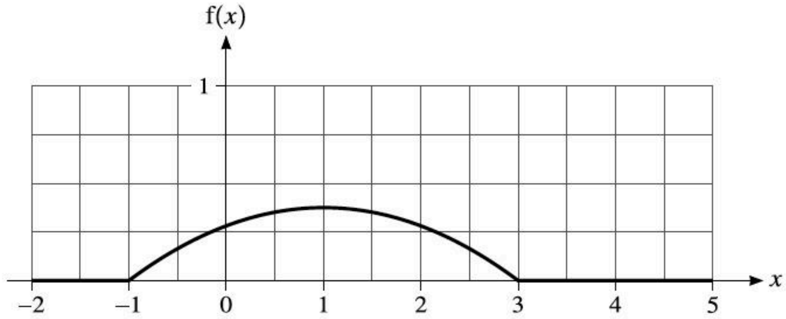
$$f(x) = \begin{cases} a - b(x^2 + x) & -3 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(i) Show that $30a - 55b = 6$. [3]

(ii) By substituting a suitable value of x into $f(x)$, find another equation relating a and b and hence determine the values of a and b. [3]

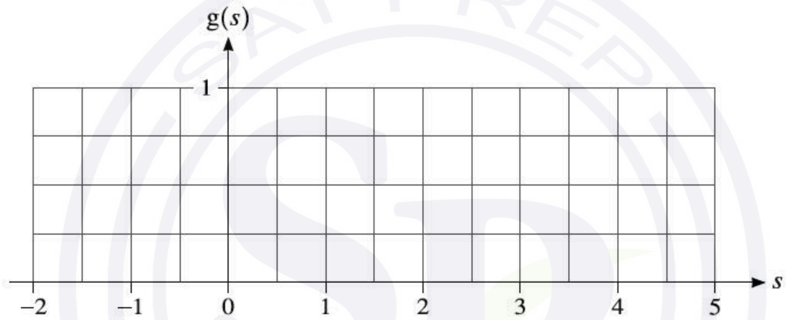
Question 60



The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

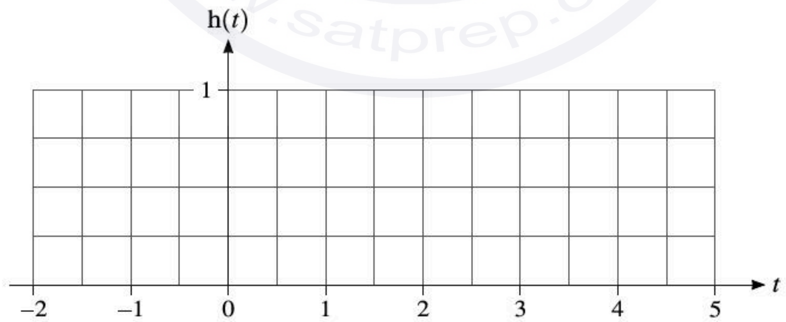
The random variable S is such that $\mathrm{E}(S) = 2 \times \mathrm{E}(X)$ and $\mathrm{Var}(S) = \mathrm{Var}(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $\mathrm{E}(T) = \mathrm{E}(X)$ and $\mathrm{Var}(T) = \frac{1}{4}\mathrm{Var}(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



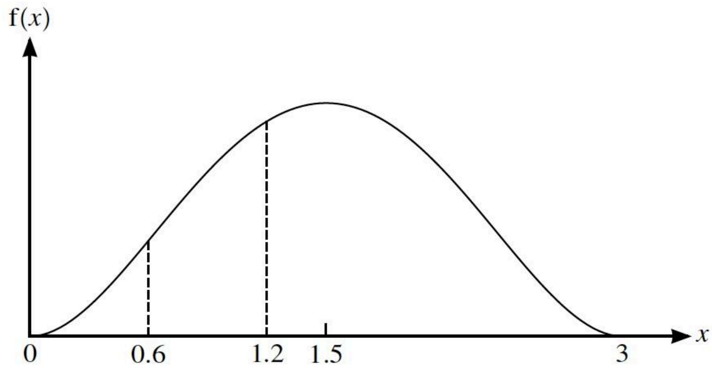
It is now given that

$$\mathrm{f}(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $\mathrm{P}(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

(d) Hence verify that $0.69 < a < 0.70$. [1]

Question 61



The diagram shows the graph of the probability density function, f, of a random variable X that takes values between $x = 0$ and $x = 3$ only. The graph is symmetrical about the line $x = 1.5$.

(a) It is given that $P(X < 0.6) = a$ and $P(0.6 < X < 1.2) = b$.

Find $P(0.6 < X < 1.8)$ in terms of a and b. [2]

(b) It is now given that the equation of the probability density function of X is

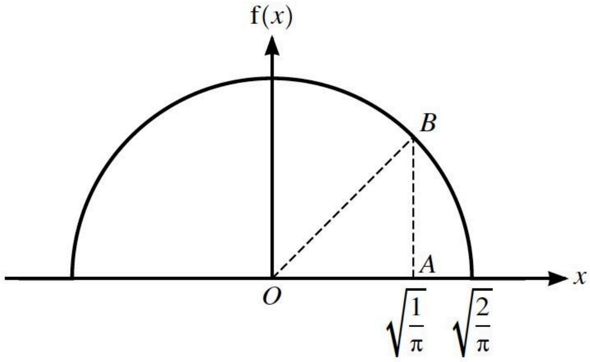
$$f(x) = \begin{cases} kx^2(3-x)^2 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{10}{81}$. [3]

(ii) Find $\text{Var}(X)$. [3]

Question 62



A random variable X has probability density function f, where the graph of $y = \mathrm{f}(x)$ is a semicircle with centre (0, 0) and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x-axis. Elsewhere $\mathrm{f}(x) = 0$ (see diagram).

(a) Verify that f can be a probability density function. [2]

A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x-axis and the semicircle respectively.

(b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $\mathrm{P}\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

Question 63

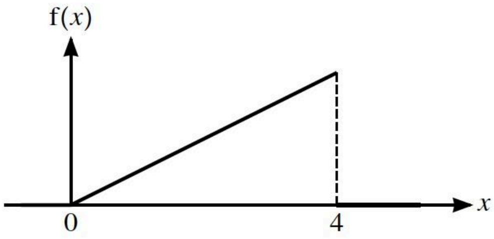
A random variable X has probability density function f, where

$$\mathrm{f}(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $\mathrm{E}(X)$. [3]

Question 64

(a)

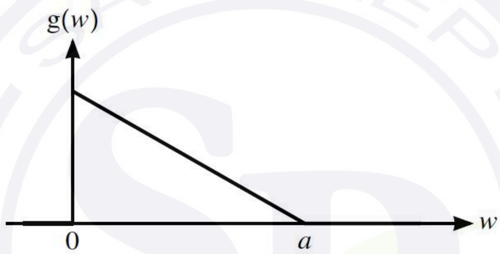


The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 4 only. Between these two values the graph is a straight line.

(i) Show that $f(x) = kx$ for $0 \leqslant x \leqslant 4$, where k is a constant to be determined. [2]

(ii) Hence, or otherwise, find $E(X)$. [3]

(b)

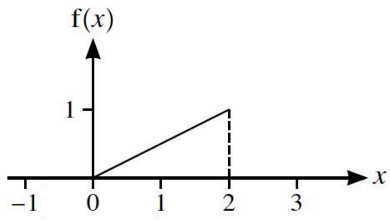


The diagram shows the graph of the probability density function, g, of a random variable W which takes values between 0 and a only, where $a > 0$. Between these two values the graph is a straight line.

Given that the median of W is 1, find the value of a. [3]

Question 65

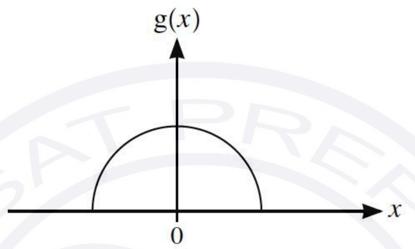
(a)



The graph of the function f is a straight line segment from (0, 0) to (2, 1).

Show that f could be a probability density function. [2]

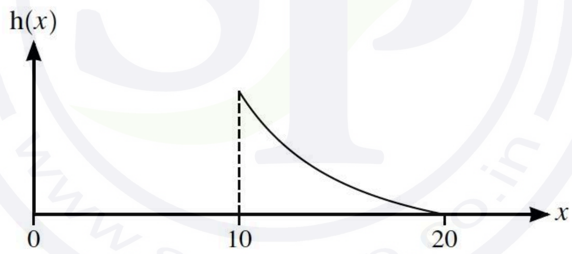
(b)



The graph of the function g is a semicircle, centre (0, 0), entirely above the x-axis.

Given that g is a probability density function, find the radius of the semicircle. [2]

(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h, as shown in the diagram.

(i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

It is now given that

$$h(x) = \begin{cases} \dfrac{40}{x^2} - \dfrac{1}{10} & 10 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) Find the mean time. [3]

Question 66

A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

Question 67

The random variable X has probability density function, f, given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a < x < b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(a) It is given that $E(X) = \ln 2$.

Show that $b = 2a$. [3]

(b) Show that $a = \frac{1}{2}$. [3]

(c) Find the median of X. [3]

Question 68

The graph of the probability density function f of a random variable X is symmetrical about the line $x = 2$. It is given that $P(2 < X < 5) = \frac{117}{256}$.

(a) Using only this information show that $P(X > -1) = \frac{245}{256}$. [2]

It is now given that, for x in a suitable domain,

$$f(x) = k(12 + 4x - x^2), \text{ where } k \text{ is a constant.}$$

(b) Find the value of k. [3]

(c) A different random variable X has probability density function $g(x) = \frac{2}{9}(2 + x - x^2)$. The domain of X is all values of x for which $g(x) \geqslant 0$.

Find $\text{Var}(X)$. [5]

Question 69

A random variable X has probability density function f given by

$$f(x) = \begin{cases} ax - x^3 & 0 \leqslant x \leqslant \sqrt{2}, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $a = 2$. [3]

(b) Find the median of X. [4]

(c) Find the exact value of $\mathrm{E}(X)$. [3]

Question 70

The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(1 + \cos x) & 0 \leqslant x \leqslant \pi, \\ 0 & \text{otherwise,} \end{cases}$$

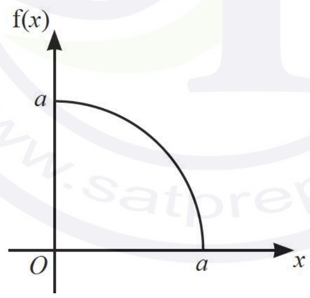
where k is a constant.

(a) Show that $k = \frac{1}{\pi}$. [3]

(b) Verify that the median of X lies between 0.83 and 0.84 . [3]

(c) Find the exact value of $\mathrm{E}(X)$. [4]

Question 71



The diagram shows the graph of the probability density function, f , of a random variable X . The graph is a quarter circle entirely in the first quadrant with centre $(0,0)$ and radius a, where a is a positive constant. Elsewhere $f(x) = 0$.

(a) Show that $a = \frac{2}{\sqrt{\pi}}$. [2]

(b) Show that $f(x) = \sqrt{\frac{4}{\pi} - x^2}$. [2]

(c) Show that $\mathrm{E}(X) = \frac{8}{3\sqrt{\pi^3}}$. [4]

Question 72

A random variable X has probability density function f defined by

$$f(x) = \begin{cases} \dfrac{a}{x^2} - \dfrac{18}{x^3} & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $a = \frac{27}{2}$. [3]

(b) Show that $E(X) = \frac{27}{2}\ln\frac{3}{2} - 3$. [3]

Question 73

The time, X hours, taken by a large number of people to complete a challenge is modelled by the probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(a) State what the constants a and b represent in this context. [1]

(b) Show that $a = \dfrac{b}{b+1}$. [3]

It is given that $E(X) = \ln 3$.

(c) Show that $b = 2$ and find the value of a. [4]

(d) Find the median of X. [3]

Question 74

A random variable X has probability density function f defined by

$$f(x) = \begin{cases} \dfrac{a}{x^2} - \dfrac{18}{x^3} & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

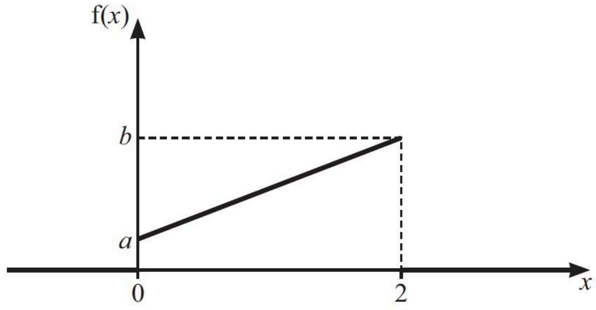
where a is a constant.

(a) Show that $a = \frac{27}{2}$. [3]

(b) Show that $E(X) = \frac{27}{2}\ln\frac{3}{2} - 3$. [3]

Question 75

(a)



The diagram shows the graph of the probability density function, f, of a random variable X. The graph is a straight line from $(0, a)$ to $(2, b)$, where a and b are positive constants. Elsewhere, $f(x) = 0$.

(i) Show that $b = 1 - a$. [2]

(ii) Given that $E(X) = 1.2$, find the value of a. [5]

(b) A random variable T has probability density function given by

$$g(t) = \begin{cases} \frac{1}{2}\cos t & -\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of c such that $P(-c < t < c) = \frac{1}{2}$. [4]

Question 76

A random variable X has probability density function given by

$$f(x) = \begin{cases} ax & 0 \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(a) Show that $a = \dfrac{2}{b^2}$. [3]

(b) Show that $P(X < E(X)) = \frac{4}{9}$. [6]

Question 77

The random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{kx^2}{a^2} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(a) Show that $k = \dfrac{3}{a}$. [3]

It is given that $E(X) = 1$.

(b) Find the value of a. [3]

(c) Find the median of X. [3]

Question 78

X is a random variable with probability density function given by

$$f(x) = \begin{cases} (1 + \cos \pi x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $P\left(X < \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\pi}$. [3]

(b) Show that $E(X) = \frac{1}{2} - \frac{2}{\pi^2}$. [5]