# A-Level

# Topic : Continuous Random Variable

# May 2013-May 2023

# Answers

Question 1

| | | | |
|---|---|---|---|
| (i) | $\int_1^\infty \frac{k}{x^3}dx = 1$ | M1 | All correct, including limits and an attempt to integrate |
| | $\left[-\frac{k}{2x^2}\right]_1^\infty = 1$ | | |
| | $0-(-\frac{k}{2}) = 1$ | A1 2 | or $0+\frac{k}{2}=1$ or $\frac{k}{2}=1$ AG must be convincing |
| (ii) | $\int_1^2 \frac{2}{x^3}dx$ | M1 | Attempt integ f($x$); ignore limits |
| | $=\left[-\frac{1}{x^2}\right]_1^2$ | | |
| | $=\frac{3}{4}$ | A1 2 | |
| (iii) | $\int_1^\infty \frac{2}{x^2}dx$ | M1 | Attempt integ $x$f($x$); ignore limits |
| | $=\left[-\frac{2}{x}\right]_1^\infty$ | A1 | Correct & correct limits |
| | $=2$ | A1 3 | |
| | [Total: 7] | | |

Question 2

| | | | |
|---|---|---|---|
| (i) | $\frac{2}{3}\int_1^2 x^2 dx$ | M1 | Attempt integ. $x$f($x$); ignore limits |
| | $=\frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$ | A1 | Correct integration and limits |
| | $=\frac{14}{9}$ or 1.56 o.e. | A1 [3] | |
| (ii) | $\frac{2}{3}\int_1^{\frac{14}{9}} x\,dx$ | M1 | Attempt integ. f($x$); with limits |
| | $(=\frac{2}{3}\left[\frac{x^3}{3}\right]_1^2)$ | | |
| | $=\frac{115}{243}$ or 0.473 (3 s.f.) | A1 [2] | |
| (iii) | $\frac{115}{243} < \frac{1}{2}$ o.e. | M1 | Comparison of prob. or values |
| | Hence mean < median | A1ft[2] | ft (i) or (ii) |
| | [Total: 7] | | |

## Question 3

| | | | |
|---|---|---|---|
| **(i)** | $\frac{1}{2}\int_4^t \frac{1}{\sqrt{t}}\mathrm{d}t = 0.9$ or $\frac{1}{2}\int_t^9 \frac{1}{\sqrt{t}}\mathrm{d}t = 0.1$ | M1 | Attempt integ f($t$) with unknown limit and 0.9/0.1. |
| | $\left[\sqrt{t}\right]_4^t = 0.9$ or $\left[\sqrt{t}\right]_t^9 = 0.1$ | A1 | Correct integration & limits = 0.9 or 0.1. |
| | $((\sqrt{t} - 2) = 0.9$ or $(3 - \sqrt{t}) = 0.1)$<br>$t = 8.41$ (mins) (3 sf) | A1<br>[3] | |
| **(ii)** | $\frac{1}{2}\int_4^9 \frac{t}{\sqrt{t}}\mathrm{d}t$ oe | M1 | Attempt integ $t$f($t$). Ignore limits |
| | $\frac{1}{2}\left[\frac{t1.5}{1.5}\right]_4^9$ oe | A1 | Correct integration & limits |
| | $= \frac{19}{3}$ | A1 | |
| | $\frac{1}{2}\int_4^9 \frac{t2}{\sqrt{t}}\mathrm{d}t$ oe | M1 | Attempt integ $t^2$f($t$). Ignore limits |
| | $(= \frac{1}{2}\left[\frac{t2.5}{2.5}\right]_4^9 = \frac{211}{5})$ | | |
| | $= \frac{'211'}{5} - (\frac{'19'}{3})^2$ | M1 | integ $t^2$f($t$) – (integ $t$f($t$))$^2$ attempted |
| | $= \frac{94}{45}$ or 2.09 (3 sf) | A1<br>[6] | |

## Question 4

| | | | | |
|---|---|---|---|---|
| **(i)** | $\int_0^{10} \frac{1}{2500}\left(100t^3 - t^5\right)\mathrm{d}t$ | **M1** | | Attempt integ $t^2$f($t$) |
| | $(= \frac{1}{2500}\left[25t^4 - \frac{t^6}{6}\right]_0^{10} = \frac{100}{3})$ | | | |
| | "$\frac{100}{3}$" – $(\frac{16}{3})^2$ | **M1** | | For E ( $T^2$ ) – ( E ( T ) )$^2$ |
| | $= \frac{44}{9}$ or 4.89 (3 sf) | **A1** | **3** | |

| | | | | |
|---|---|---|---|---|
| **(ii)** | $\int_n^{10} \frac{1}{2500}\left(100t - t^3\right)\mathrm{d}t$ | **M1** | | Attempt integ f($t$), ignore limits |
| | $\frac{1}{2500}\left[50t^2 - \frac{t_4}{4}\right] = 0.1$ | **M1** | | Attempt integ f($t$), limits $n$ to 10 or 0 to $n$<br>Equated to 0.1 or 0.9. Not need to be matched |
| | $\frac{1}{2500}\left[2500 - \left(50n^2 - \frac{n^4}{4}\right)\right] = 0.1$ | **M1** | | 0.1/0.9 matched to correct limits and used |
| | $(n^4 - 200n^2 + 9000 = 0)$ | **M1** | | Correct method of solution of a QE in $n^2$ |
| | $(n^2 = 68.3772, n = 8.27)$<br>$n = 8$ | **A1** | **5** | Must be single ans only |

Question 5

| | | | |
|---|---|---|---|
| **(i)** | $\int_0^2 k(x-2)^2\,dx = 1$<br>$\left(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1\right)$<br>$k\,[0-(-\frac{8}{3})] = 1$<br>$k = \frac{3}{8}$ **AG** | M1<br><br>A1<br><br>[2] | Attempt to integrate f($x$) with correct limits and = 1<br><br>Must see this line or better, e.g. $k \times \frac{8}{3} = 1$ |
| **(ii)** | $\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$<br>$\left(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2\right)$<br>$\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe<br>$((d-2)^3 = -1.6)$<br>$d = 0.83(0)$ (3 s.f.) | M1<br><br>M1<br><br>A1 [3] | ∫f($x$)d$x$ with limits d and 2 or 0 and d, and = 0.2 or =0.8<br>Condone missing 'k'<br><br>Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in $d^3$.<br>Condone missing 'k' |
| **(iii)** | $\frac{3}{8}\int_0^2 x(x-2)^2\,dx$<br>$(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx)$<br>$= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$<br>$= \frac{1}{2}$ | M1<br><br>A1<br><br>A1 [3] | Attempt integ $x$f($x$); ignore limits, condone missing k<br>$\left(\frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \int\frac{(x-2)^3}{3}\,dx\right]_0^2\right)$<br>$= \frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$<br>Correct integration & limits, condone missing k |

Question 6

| | | | |
|---|---|---|---|
| **(i)** | Longest lifetime | B1 [1] | Must be in context |
| **(ii)** | $\int_1^a \frac{k}{x^2}\,dx = 1$ | M1 | Int f($x$) and equate to 1. Ignore limits |
| | $k\left[-\frac{1}{x}\right]_1^a = 1$ | A1 | Correct integral and limits |
| | $\left(k\left[-\frac{1}{a}+1\right]=1\right)$ | | |
| | $k\left[\frac{-1+a}{a}\right] = 1$ or $k(-1 + a) = a$ | | |
| | $k = \frac{a}{a-1}$ **AG** | A1 [3] | Must be convinced (**AG**) |
| **(iii)** | $\frac{5}{3}\int_1^{2.5} \frac{1}{x}\,dx$ or $k\int_1^{2.5} \frac{1}{x}\,dx$ | M1 | Int $x$f($x$). Ignore limits |
| | $= \frac{5}{3}[\ln x]_1^{2.5}$ or $k[\ln x]_1^{2.5}$ | A1 | Correct integral and limits (Accept "$k$" or "their $k$") |
| | $= \frac{5}{3}\ln 2.5$ or 1.53 (3 s.f.) | A1 [3] | |

Question 7

| | | |
|---|---|---|
| ht $= \frac{1}{2}$ seen | B1 | or $y = \frac{1}{8}x$ |
| $\frac{1}{2} \times m \times \left(\frac{m}{4} \times "\frac{1}{2}"\right) = \frac{1}{2}$ | M1 | $\frac{1}{2} \times m \times ("\frac{1}{8}"m) = \frac{1}{2}$ or $\frac{m^2}{16} = \frac{1}{2}$ o.e. |
| **N.B. B1 M1 must be consistent** | | Or Integrating linear function of form $y = kx$ with limits 0 and $m$ or $m$ and 4 and equated to 0.5 |
| $m = \sqrt{8}$ or $2\sqrt{2}$ or 2.83 (3 s.f.) | A1 [3] | |

Question 8

| | | | | |
|---|---|---|---|---|
| (i) | $k\int_0^4 (16t - t^3)\,dt = 1$ | M1 | | Int f($t$) = 1 ignore limits |
| | $k\left[8t^2 - \frac{t^4}{4}\right]_0^4 = 1$ | A1 | | correct integration with correct limits |
| | $k(128 - 64) = 1$ o.e. | | | |
| | $k \times 64 = 1$ | | | |
| | $\left(k = \frac{1}{64}\right)$ **AG** | A1 | [3] | must be convinced (AG) |
| (ii) | $\frac{1}{64}\int_0^1 (16t - t^3)\,dt$ | M1 | | Int f($t$) between 0 and 1 (accept 0 and a value < 1 , 1 and 4) |
| | $= \frac{1}{64}\left[8t^2 - \frac{t^4}{4}\right]_0^1$ | A1 | | correct integration and correct limits (ignore "k") |
| | $= \frac{1}{64}\left[8 - \frac{1}{4}\right]$ | | | |
| | $= \frac{31}{256}$ or 0.121094 | A1 | | |
| | $\left(\frac{31}{256}\right)^2 = 0.0147$ (3 s.f.) o.e. | B1√ | [4] | ft their "$\frac{31}{256}$" |
| iii | $\frac{1}{64}\int_0^4 (16t^2 - t^4)\,dt$ | M1 | | Int $t$f($t$) ignore limits |
| | $= \frac{1}{64}\left[\frac{16t^3}{3} - \frac{t^5}{5}\right]_0^4$ | A1 | | correct integration and correct limits (ignore "k") |
| | $= \frac{1}{64}\left(\frac{1024}{3} - \frac{1024}{5}\right)$ | | | |
| | $= \frac{32}{15}$ or 2.13 (3 s.f.) o.e. | A1 | [3] | |

Question 9

| | | | | |
|---|---|---|---|---|
| **(i)** | $\int_1^a \frac{k}{x}\mathrm{d}x = 1$ | M1 | | Int f($x$) & equate to 1. Ignore limits |
| | $k[\ln x]_1^a = 1$ | A1 | | Correct integration and limits and = 1 |
| | $k\ln a = 1 \quad k = 1/\ln a$ | A1 | [3] | AG |
| **(ii)** | $\frac{1}{\ln a}\int_1^a 1\,\mathrm{d}x$ or $k\int_1^a 1\,\mathrm{d}x$ | M1 | | Int $x$f($x$). Ignore limits |
| | $= \frac{1}{\ln a}[x]_1^a$ or $k[x]_1^a$ | A1 | | Correct integration and limits (condone missing $k$) |
| | $= \frac{1}{\ln a}(a-1)$ | A1 | [3] | |
| **(iii)** | $\frac{1}{\ln a}\int_1^m \frac{1}{x}\mathrm{d}x = 0.5$ | M1 | | Int f($x$) and equate to 0.5. Ignore limits |
| | $\frac{1}{\ln a}[\ln x]_1^m = 0.5$ | A1 | | Correct integration and limits (1 to $m$ or $m$ to $a$) (condone missing $k$) |
| | $\frac{1}{\ln a}\ln m = 0.5$ | | | |
| | $\ln m = 0.5\ln a$ | A1 | | or $\ln m = \ln a^{0.5}$ |
| | $m = \sqrt{a}$ | A1 | [4] | |
| | | **[Total: 10]** | | |

Question 10

| | | | | |
|---|---|---|---|---|
| **(i)** | $\frac{1}{2}c^2 = 1$ | **M1** | | Area of triangle = 1 or integral of $kx$ with limits 0 and $c$ and equated to 1 |
| | $c = \sqrt{2}$ or 1.41 (3 sf) | **A1** | **[2]** | |
| **(ii)** | $f(x) = x$ or $y = x$ | **B1** | | Seen or implied, e.g. by next line. Can be awarded anywhere in the question. Implied by $(a+1)$ in area of trapezium. |
| | $\int_a^1 x\,\mathrm{d}x = 0.1$ | **M1** | | Ignore limits. Must be integral of $kx$ and equated to 0.1. Or trapezium area. |
| | $\left[\frac{x^2}{2}\right]_a^1 = 0.1$ | **A1√** | | Correct limits, ft incorrect $kx$. |
| | $1 - a^2 = 0.2$<br>$a = 0.894$ (3 sf) | **A1** | **[4]** | $\sqrt{\left(\frac{4}{5}\right)}$ oe |
| **(iii)** | $\int_0^{\sqrt{2}} x^2\,\mathrm{d}x$ | **M1** | | Ignore limits; ft their f($x$) but not $\int x\mathrm{d}x$ |
| | $\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$ | | | |
| | $= \frac{2}{3}\sqrt{2}$ **or 0.943 or** $\sqrt{\left(\frac{8}{3}\right)}$ | **A1√** | **[2]** | ft their $c$, dep 0 < ans < their $c$. Not ft their f($x$) |

Question 11

| | | | |
|---|---|---|---|
| **(a)** | $\int_0^{0.5}\left(1.5t-0.75t^2\right)\mathrm{d}t$ o.e. | M1 | Attempt int f($t$) |
| | $=\left[0.75t^2-0.25t^3\right]_0^{0.5}$ o.e. | A1 | Correct integration and limits |
| | $=\frac{5}{32}$ or 0.156 (3 sf) | A1 3 | |
| **(b) (i)** | $\frac{1}{2}\pi a^2=1$ or $\pi a^2=2$ oe | M1 | Attempt to find the area and equate to 1 |
| | $a=\sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf) | A1 2 | |
| **(ii)** | 0 | B1 1 | |
| **(iii)** | Symmetry stated, seen or implied<br>0.8 | M1<br>A1 2 | Could be a diagram<br>As final answer |
| | | **Total: 8** | |

Question 12

| | | | |
|---|---|---|---|
| **(i)** | $\frac{3}{4}\int_0^c\left(cx-x^2\right)dx=1$ | M1 | Attempt integ f($x$) and = 1. Ignore limits |
| | $\frac{3}{4}\left[\frac{cx^2}{2}-\frac{x^3}{3}\right]_0^c=1$ | A1 | Correct integration and limits (condone c = 2 |
| | $\frac{3}{4}\left(\frac{c^3}{2}-\frac{c^3}{3}\right)=1$ or $\frac{3}{4}\times\frac{c^3}{6}=1$ or $\frac{c^3}{8}=1$ | A1 [3] | No errors seen |
| | ($c=2$ **AG**) | | |
| **(ii)** | Inverted parabola<br>Through (0, 0) and (2, 0) and zero elsewhere<br>Median = 1 | B1<br>B1<br>B1 [3] | <br>Must not extend beyond [0,2] |
| **(iii)** | $\frac{3}{4}\int_0^{1.5}\left(2x-x^2\right)\mathrm{d}x$ | M1 | Attempt integ f($x$) ignore limits |
| | $=\frac{3}{4}\left[x^2-\frac{x^3}{3}\right]_0^{1.5}$ | A1 | Correct integration ignore limits |
| | $\frac{3}{4}\left(1.5^2-\frac{1.5^3}{3}\right)$ | B1 | Use of correct limits [0,1.5] or 1–[1.5,2] |
| | $=\frac{27}{32}$ or 0.844 (3 sf) | A1 [4] | |
| **(iv)** | $\left(\frac{27}{32}-\frac{1}{2}\text{ or } 0.844-0.5\right)$ | | |
| | $=\frac{11}{32}$ or 0.344 (3 sf) | B1f [1] | ft their (iii) For use of symmetry Note If do not use "hence" and start again B1 for cwo |
| | | **Total 11** | |

Question 13

| | | | | |
|---|---|---|---|---|
| (i) | $k\int_0^{15}(225-t^2)\,dt = 1$ | M1 | | Attempt integ f($x$) and = 1. Ignore limits |
| | $k\left[225t-\frac{t^3}{3}\right]_0^{15}=1$ | A1 | | Correct integration and limits |
| | $k\times[3375-1125\}=1$ or $k\times2250=1$ | A1 | 3 | No errors seen |
| | $(k=\frac{1}{2250}$ **AG**) | | | |
| (ii) | $\frac{1}{2250}\int_{10}^{15}(225-t^2)\,dt$ | M1 | | Attempt integ, ignore limits |
| | $(=\frac{1}{2250}\left[225t-\frac{t^3}{3}\right]_{10}^{15})$ | A1 | | Or $1-\int_0^{10}$ Correct integration and limits. Condone missing $k$ |
| | $=\frac{1}{2250}[2250-(2250-\frac{1000}{3})]$ | A1 | 3 | |
| | $=\frac{4}{27}$ or 0.148 (3 sf) | | | |
| (iii) | $\frac{1}{2250}\int_0^{15}(225t-t^3)\,dt$ | M1* | | Attempt integ $x$f($x$), ignore limits |
| | $=\frac{1}{2250}\left[\frac{225t^2}{2}-\frac{t^4}{4}\right]_0^{15}$ | A1 | | Correct integration and limits. Condone missing $k$ |
| | $=\frac{1}{2250}[\frac{50625}{2}-\frac{50625}{4}]$ | M1*dep | | Sub correct limits into their integral |
| | $=\frac{45}{8}$ or 5.625 or 5.63 (3 sf) | A1 | 4 | Accept 5 mins 37 or 38 secs |
| | **Total** | **10** | | |

Question 14

| | | | |
|---|---|---|---|
| $\frac{1}{2}a^2=1$ | M1 | | or $\int_0^a x\,dx = 1$ |
| $a=\sqrt{2}$ | A1 | | Allow 1.41 or better |
| $\int_0^{\sqrt{2}} x^2\,dx$ | M1 | | ignore limits |
| $=\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$ | A1f | | correct integral and limits, but ft their $a$ |
| $=\frac{(\sqrt{2})^3}{3}=$ or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428 (= 0.943 **AG**) | A1 | [5] | must see this numerical expression, or equiv SR Equating $\int x\,f(x)$ to 0.943 scores M1 Solving to find $a$ = 1.41 scores A1 |
| | **[Total 5]** | | |

Question 15

| | | | | |
|---|---|---|---|---|
| **(i)** | $k\int_{-2}^{2}(4-x^2)\mathrm{d}x = 1$ | **M1** | | attempt Integral f($x$) = 1, ignore limits |
| | $k[4x - \frac{x^3}{3}]_{-2}^{2} = 1$ | **A1** | | correct integration & limits |
| | $\left(k\left(8-\frac{8}{3}-\left(-8-\left(-\frac{8}{3}\right)=1\right)\right)\right)$ | | | |
| | $k \times \frac{32}{3} = 1$ oe Not e.g. $k\times 10.7 = k$ | | | |
| | $k = \frac{3}{32}$ **AG** | **A1** | [3] | exact answer correctly found |
| **(ii)** | Inverted parabola, vertex on $y$ axis | **B1** | | parabola must finish on x axis at ± 2, labelled (ignore markings on y axis ) |
| | E($X$) = 0 | **B1** | [2] | |
| **(iii)** | $\frac{3}{32}\int_{-2}^{1}(4-x^2)\mathrm{d}x$ | **M1** | | or $1-\frac{3}{32}\int_{1}^{2}(4-x^2)\mathrm{d}x$ ignore limits |
| | $\frac{3}{32}[4x - \frac{x^3}{3}]_{-2}^{1}$ | **A1** | | or $1-\frac{3}{32}[4x-\frac{x^3}{3}]_{1}^{2}$ |
| | $\frac{3}{32}(4-\frac{1}{3}-(-8-(-\frac{8}{3})))$ | | | correct integration and correct limits $= 1-\frac{3}{32}(8-\frac{8}{3}-(4-\frac{1}{3})$ |
| | $= \frac{27}{32}$ or 0.844 (3 sf) | **A1** | [3] | |

Question 16

| | | | |
|---|---|---|---|
| **(i)** | $k\int_1^2 (3-x)dx = 1$ | **M1** | Attempt $\int f(x) = 1$, ignore limits or $\frac{k}{2}(h_1 + h_2) = 1$ |
| | $k\left[3x - \frac{x^2}{2}\right]_1^2 = 1$ | **A1** | Correct integration & limits or $\frac{k}{2}(2+1) = 1$ |
| | $(k(6 - 2 - (3 - 0.5)) = 1)$ | | |
| | $k \times 1.5 = 1$ or $k \times \frac{3}{2} = 1$ or $k = \frac{1}{1.5}$ oe | | |
| | $k = \frac{2}{3}$ **AG** | **A1** [3] | No errors seen |
| **(ii)** | $\frac{2}{3}\int_1^m (3-x)dx = 0.5$ oe $\int$ from m to 2 | **M1*** | Attempt Int $f(x) = 0.5$, ignore limits oe<br>Or use of area of trapezium |
| | $\left(\frac{2}{3}\left[3x - \frac{x^2}{2}\right]_1^m = 0.5\right)$ | | |
| | $\frac{2}{3}\left[3m - \frac{m^2}{2} - 2.5\right] = 0.5$ | **dep M1*** | Sub of correct limits into their integral.<br>Or trapezium using 1 and m/m and 2<br>Any correct 3-term QE = 0 or $(m-3)^2 = 2.5$ |
| | $m^2 - 6m + 6.5 = 0$ oe | **A1** | |
| | $\left(m = \frac{6 \pm \sqrt{36 - 4 \times 6.5}}{2} = 1.42 \text{ or } 4.58\right)$ | | |
| | $m = 1.42$ (3 sf) | **A1** [4] | or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans |

Question 17

| | | | |
|---|---|---|---|
| **(a) (i)** | $E(X) = 1.5$ | **B1** | |
| | $\frac{2}{9}\int_0^3 (3x^3 - x^4)dx$ | **M1** | Attempt integ $x^2 f(x)$ ignore limits |
| | $= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ | | |
| | $= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ $(= 2.7)$ | **M1** | Sub correct limits into correct integral |
| | $Var(X)$ $(= 2.7 - 1.5^2) = 0.45$ oe | **A1√** [4] | Ft their $E(X)$, but no ft for –ve Var. |
| **(ii)** | 0.5 | **B1** [1] | |
| **(iii)** | $(1 - \frac{13}{27}) \div 2$ | **M1** | or $\frac{2}{9}\int_2^3 (3x - x^2)\,dx$ oe |
| | $= \frac{7}{27}$ or 0.259 | **A1** [2] | As final answer |
| **(b)** | $\frac{1}{2} \times 2 \times 2a = \frac{1}{2}$ or $\int_0^2 ax\,dx = \frac{1}{2}$ | **M1** | Attempt correct equation in ‘a’ |
| | $a = \frac{1}{4}$ | **A1** | |
| | $\frac{1}{2} \times b \times \frac{1}{4}b = 1$ or $\int_0^b \frac{1}{4}x\,dx = 1$ or $b = 2 \times \sqrt{2}$ | **M1** | or $\frac{1}{2} \times b \times ab = 1$ or $\int_0^b ax\,dx = 1$ attempt correct equation in (a and) b |
| | $b = 2\sqrt{2}$ | **A1√** [4] | Allow $b = \sqrt{8}$ or 2.83 (3 sf)<br>Ft incorrect $a$, both **M**s needed |

Question 18

| | | | |
|---|---|---|---|
| **(i)** | $k\int_5^{10}(10t-t^2)\mathrm{d}t=1$ | **M1** | Attempt to integrate, ignore limits |
| | $k\left[5t^2-\frac{t^3}{3}\right]_5^{10}=1$ | **A1** | Correct integral and limits |
| | $k(500-\frac{1000}{3}-(125-\frac{125}{3}))=1$ | | |
| | $k\times\frac{250}{3}=1$ | | |
| | $(k=\frac{3}{250}$ **AG**$)$ | **A1** [3] | No errors seen; No inexact decimals seen |
| **(ii)** | $\frac{3}{250}\int_5^{10}(10t^2-t^3)\mathrm{d}t$ | **M1** | Attempt to integrate, ignore limits |
| | $=\frac{3}{250}\left[\frac{10t^3}{3}-\frac{t^4}{4}\right]_5^{10}$ | **A1** | Correct integral and limit. Condone missing k |
| | $=\frac{3}{250}(\frac{10000}{3}-\frac{10000}{4}-(\frac{1250}{3}-\frac{625}{4}))$<br>$=6.875$ or $55/8$ | **A1** [3] | Allow 6.88 |
| **(iii)** | $\mathrm{P}(T<\mathrm{E}(T))=\frac{3}{250}\left[5t^2-\frac{t^3}{3}\right]_5^{"6.875"}$ | **M1*** | ft their E($T$) |
| | $=0.5361$<br>"0.5361" – 0.5<br>P($T$ between E($T$) & median = 0.0361 | **DM1***<br>**A1** [3] | allow 0.036<br>**Alternative Method**<br>Integrate f(t)limits 5 and m equated to 0.5 M1*<br>Integrate f(t)limits their 6.736 (provided between 5 and 10) and their 6.875DM1<br>Allow without "minutes" |
| **(iv)** | 10 (minutes) | **B1** [1] | |

Question 19

| | | | |
|---|---|---|---|
| (a) | 0.3 or 1 – 0.6 or 0.4 or 0.2 seen<br>0.8 | M1<br>A1 [2] | |
| (b) (i) | $k\int_0^{1.5}(2.25-x^2)\mathrm{d}x = 1$ | M1 | attempt integ f($x$) and '= 1'. Ignore limits |
| | $k\left[2.25x-\frac{x^3}{3}\right]_0^{1.5} = 1$ | A1 | correct integration and limits |
| | $k\times[3.375-1.125] = 1$ or $k\times\frac{9}{4} = 1$ oe | | |
| | $k = \frac{4}{9}$ **AG** | A1 [3] | No errors seen |
| (ii) | $\frac{4}{9}\int_0^{1.5}(2.25x-x^3)\mathrm{d}x$ | M1 | attempt integ $x$f($x$), ignore limits, condone missing $k$ |
| | $= \frac{4}{9}\left[2.25\frac{x^2}{2}-\frac{x^4}{4}\right]_0^{1.5}$ | A1 | correct integration and limits, condone missing $k$ |
| | = 0.5625 or 0.563 | A1 | |
| | Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins | A1√ [4] | ft their 0.5625 |
| (iii) | Max $x$ is 1.5, less than 2.9 or 150 < 290 | B1 [1] | Needs numerical justification |
| (iv) | any $a$ such that $2.9 \leqslant a \leqslant 5$ | B1 [1] | |

Question 20

| | | | |
|---|---|---|---|
| (i) | 2 m | B1 [1] | allow without units |
| (ii) | $k\int_0^2 x^2(2-x)\mathrm{d}x = 1$ | M1 | attempt integ f($x$) and '= 1'. Ignore limits |
| | $k\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_0^2$ | A1 | correct integration and limits |
| | $k\times\left[\frac{16}{3}-4\right] = 1$ or $k\times\frac{4}{3} = 1$ oe | | |
| | $k = \frac{3}{4}$ **AG** | A1 [3] | No errors seen |
| (iii) | $\frac{3}{4}\int_0^2 x^3(2-x)\mathrm{d}x$ | M1 | attempt integ $x$f($x$), condone missing $k$ |
| | $= \frac{3}{4}\times\left[\frac{2x^4}{4}-\frac{x^5}{5}\right]_0^2$ | A1 | correct integration and limits, condone missing $k$ |
| | 1.2 m oe | A1 [3] | allow without units |
| (iv) | $\frac{3}{4}\int_0^1 x^2(2-x)\mathrm{d}x$ | M1 | attempt integ f($x$), 0 to 1, condone missing $k$ |
| | $(= \frac{3}{4}\times(\frac{2}{3}-\frac{1}{4}))$<br>$= \frac{5}{16}$ or 0.3125 oe | A1 | |
| | $400\times\frac{5}{16} = 125$ | A1 ft [3] | ft their $\frac{5}{16}$ |

Question 20

| | | | | |
|---|---|---|---|---|
| **(a)** | | 0.3 or 1 – 0.6 or 0.4 or 0.2 seen<br>0.8 | **M1**<br>**A1** [2] | |
| **(b)** | **(i)** | $k\int_0^{1.5}(2.25-x^2)\mathrm{d}x = 1$ | **M1** | attempt integ f($x$) and '= 1'. Ignore limits |
| | | $k\left[2.25x-\frac{x^3}{3}\right]_0^{1.5} = 1$ | **A1** | correct integration and limits |
| | | $k\times[3.375-1.125] = 1$ or $k\times\frac{9}{4} = 1$ oe | | |
| | | $k = \frac{4}{9}$ **AG** | **A1** [3] | No errors seen |
| | **(ii)** | $\frac{4}{9}\int_0^{1.5}(2.25x-x^3)\mathrm{d}x$ | **M1** | attempt integ $x$f($x$), ignore limits, condone missing $k$ |
| | | $=\frac{4}{9}\left[2.25\frac{x^2}{2}-\frac{x^4}{4}\right]_0^{1.5}$ | **A1** | correct integration and limits, condone missing $k$ |
| | | = 0.5625 or 0.563 | **A1** | |
| | | Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins | **A1**√ [4] | ft their 0.5625 |
| | **(iii)** | Max $x$ is 1.5, less than 2.9 or 150 < 290 | **B1** [1] | Needs numerical justification |
| | **(iv)** | any $a$ such that $2.9 \leqslant a \leqslant 5$ | **B1** [1] | |

| | | | | |
|---|---|---|---|---|
| **(i)** | $\sigma_X, \sigma_Z, \sigma_Y, \sigma_W$ **or** $X, Z, Y, W$ | **B2** | [2] | B1 if two adjacent sds interchanged, ie $\sigma_Z, \sigma_X, \sigma_Y, \sigma_W$ **or** $\sigma_X, \sigma_Y, \sigma_Z, \sigma_W$ **or** $\sigma_X, \sigma_Z, \sigma_W, \sigma_Y$<br>B1 for correct order reversed |
| **(ii) (a)** | Mean = 0 stated or found or "– 0" seen | **B1** | | |
| | $\frac{1}{18}\int_{-3}^{3} x^4 \mathrm{d}x - 0$<br>$= \frac{1}{18}\left[\frac{x^5}{5}\right]_{-3}^{3}$<br>$= \frac{1}{18}\left[\frac{3^5}{5} + \frac{3^5}{5}\right]$ oe<br>$= 5.4$ | **M1** | | Attempt integral $^2$ f($x$). Ignore limits<br>Allow without "– 0" |
| | sd = √5.4 or $\sqrt{\frac{1}{18}\left[\frac{3^5}{5} + \frac{3^5}{5}\right]}$ or 2.324<br>sd = 2.32 (3 sf) AG | **A1** | [3] | Must see $\sqrt{\text{correct expression or 5.4}}$ or 2.324 or better |
| **(b)** | $\frac{1}{18}\int_{'2.324'}^{3} x^2 \mathrm{d}x$ | **M1** | | Attempt to integrate f($x$), ignore limits |
| | $\frac{1}{18}\left[\frac{x^3}{3}\right]_{'2.324'}^{3} = \frac{1}{18}\left[\frac{3^3}{3} - \frac{'2.324'^3}{3}\right]$ | **A1** | | Sub correct limits into correct integral |
| | = 0.268 (3 sf) | **A1** | [3] | Allow 0.269 |
| **(c)** | 0 | **B1** | [1] | |

Question 22

| | | | | |
|---|---|---|---|---|
| (i) | $m_X, m_Y, m_Z, m_W$ or $X, Y, Z, W$ | B2 | [2] | B1 if two adjacent means interchanged, i.e. $m_Y, m_X, m_Z, m_W$ or $m_X, m_Z, m_Y, m_W$ or $m_X, m_Y, m_W, m_Z$<br>B1 for correct order reversed. |
| (ii) (a) | $\int_0^3 \frac{4}{81}x^4 dx$ | M1 | | Attempt int $x$f($x$). Ignore limits |
| | $= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$ | A1 | | Correct integration and limits (condone missing 4/81) |
| | $= \frac{4}{81}\times\frac{3^5}{5}$ or $\frac{4}{81}\times\frac{243}{5}$ or $\frac{972}{405}$ oe | | | Must see correct expression as well as $\frac{12}{5}$ or 2.4 |
| | $= \frac{12}{5}$ or 2.4 AG | A1 | [3] | No errors seen |
| (b) | $\int_{2.4}^3 \frac{4}{81}x^3 dx$ or $1 - \int_0^{2.4} \frac{4}{81}x^3 dx$ | M1 | | Attempt int f($x$) ignore limits |
| | $= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$ | A1 | | Correct integration and limits (condone missing 4/81) |
| | $= 1 - \frac{4}{81}\times\frac{2.4^4}{4}$ oe | | | |
| | $= \frac{369}{625}$ or 0.59(0) (3 sf) | A1 | [3] | As final answer |
| (c) | 1 | B1 | [1] | |

Question 23

| | | | |
|---|---|---|---|
| (a)(i) | $k = 1$ | B1 | |
| | **Total:** | 1 | |
| (a)(ii) | $f_2$: area > 1 (area ≠ 1) | B1 | oe |
| | $f_3$: includes negative values of $f_3$ | B1 | oe |
| | **Total:** | 2 | |
| (b)(i) | $6\int_{-a}^{a}(a^2 - x^2)dx = 1$ | M1 | Integ f($x$) = 1, ignore limits |
| | $6\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = 1$ | A1 | Correct integral and limits |
| | $6(2a^3 - \frac{2a^3}{3}) = 1$<br>$\frac{24a^3}{3} = 1$ or $8a^3 = 1$<br>$a = 1/2$ **AG** | A1 | Correctly obtained. No errors seen.<br>(SR Verification scores M1A1 only max 2/3) |

| | | | |
|---|---|---|---|
| (b)(ii) | 0 | **B1** | |
| | **Total:** | **1** | |
| (b)(iii) | $6\int_{-0.5}^{0.5} (\frac{x^2}{4} - x^4)\mathrm{d}x$<br>$(= 6\left[\frac{x^3}{12} - \frac{x^5}{5}\right]_{-0.5}^{0.5} = 0.05)$<br>Var = $0.05 - 0^2$ | **M1** | attempt int $x^2\mathrm{f}(x)$ & correct limits |
| | = 0.05 oe | **A1** | cao; allow omission of $- 0^2$ |

Question 24

| | | | |
|---|---|---|---|
| 5(i) | $k\int_0^1 (x - x^2)\,\mathrm{d}x = 1$ | **M1** | Attempt integ $\mathrm{f}(x)$ and "= 1", ignore limits |
| | $= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$ | **A1** | correct integration, limits 0 and 1 |
| | $= k\left[\frac{1}{2} - \frac{1}{3}\right] = 1$ or $\frac{k}{6} = 1$ | **A1** | correctly obtained, no errors seen |
| | **Total:** | **3** | |
| (ii) | $\mathrm{E}(X) = 0.5$ | **B1** | |
| | $6\int_0^1 (x^3 - x^4)\,\mathrm{d}x$ | **M1** | Attempt integ $x^2\mathrm{f}(x)$, limits 0 to 1 |
| | $(= 6\left[\frac{1}{4} - \frac{1}{5}\right] = 0.3)$<br>"0.3" – "0.5"$^2$ | **M1** | their int $x^2\mathrm{f}(x)$ – their $(\mathrm{E}(X))^2$ dep +ve result |
| | = 0.05(= 1/20) | **A1** | |
| | **Total:** | **4** | |
| (iii) | $6\int_{0.4}^{1} (x - x^2)\,\mathrm{d}x$ | **M1** | ignore limits, eg **M1** for $6\int_{0.4}^{2} (x - x^2)\,\mathrm{d}x$ |
| | $= 6\{\frac{1}{2} - \frac{1}{3} - (\frac{0.4^2}{2} - \frac{0.4^3}{3})\}$ | **A1FT** | subst correct limits into correct integration |
| | = 0.648(= 81/125) | **A1** | condone incorrect "k" for **A1** |

## Question 25

| | | | |
|---|---|---|---|
| (i) | $0.5 \times 1 \times h = 0.25$<br>$h = 0.5$<br>grad = 0.5 | M1 | $P(X < 2) = 4 \times P(X < 1)$ **M1** |
| | $f(x) = 0.5x$ | A1 | $P(X < 2) = 1$ **A1**<br>$a = 2$ **A1** |
| | $0.5 \times a \times 0.5a = 1$ | M1 | $0.5 \times 2 \times h' = 1$ **M1**<br>$h' = 1$ |
| | $a = 2$ | A1 | grad = 0.5 |
| | $P(X < 2) = 1$ | A1 | $f(x) = 0.5x$ **A1** |
| | **Total:** | 5 | |
| (ii) | $\int_0^m 0.5x\,dx = 0.5$ | M1 | Attempt $\int f(x)\,dx = 0.5$ Ignore limits |
| | $= \left[\frac{x^2}{4}\right]_0^m = 0.5$ | A1FT | Correct integration (ft f(x) ) & limits = 0.5 |
| | $m = \sqrt{2}$ or 1.41 (3 sf) | A1 | or by similarity $m = \frac{1}{\sqrt{2}} \times 2$ **M2**<br>$= \sqrt{2}$ **A1** |

## Question 26

| | | | |
|---|---|---|---|
| (i) | Greater area where $x < 7.5$ than $x > 7.5$ | B1 | Allow Graph higher for $x < 7.5$ than for $x > 7.5$ or Graph decreasing or equiv expl'n |
| | **Total:** | 1 | |
| (ii) | $\int_5^{10} \frac{k}{x^2}\,dx = 1$ | M1 | Attempt Integ f(x) = 1 ignore limits |
| | $k\left[-\frac{1}{x}\right]_5^{10} = 1$<br>$k \times \frac{1}{10} = 1$ | A1 | Correct integration and limits |
| | $k = 10$ AG | A1 | No errors seen |
| (iii) | $10\int_5^{10} \frac{1}{x}\,dx$ | M1 | Attempt Integ $xf(x)$ ignore limits |
| | $= 10[\ln x]_5^{10}$<br>$= 10(\ln 10 - \ln 5)$ | M1 | Correct integration and limits |
| | $= 10\ln 2$ or 6.93 (3 sf) | A1 | OE |
| | **Total:** | 3 | |
| (iv) | $10\int_5^{10} 1\,dx - \text{"}6.93\text{"}^2$ | M1 | Attempt (Integ $x^2f(x)$) – $(E(x))^2$. No limits **M0** |
| | = 1.95 (accept 1.96) | A1 | *Use of 6.93 gives 1.97* **A0** |

Question 27

| | | | |
|---|---|---|---|
| (i) | $\frac{1}{4}\int_0^2 (x^2+x)\,dx \qquad (= \frac{1}{4}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^2)$ | M1 | Attempt integ $x$f($x$), ignore limits |
| | $= \frac{1}{4}(\frac{8}{3}+2) \quad (-\ 0)$ | A1 | Subst correct limits in correct integration |
| | $= \frac{7}{6}$ OE or 1.17 (3 sf) | A1 | |
| | | **3** | |
| (ii) | $\frac{1}{4}\int_0^m (x+1)\,dx = 0.5 \quad (=\frac{1}{4}\left[\frac{x^2}{2}+x\right]_0^m = 0.5)$ | M1 | attempt integ f($x$), limits 0 to unknown (or unknown to 2) and = 0.5 |
| | $\frac{1}{4}(\frac{m^2}{2}+m)=0.5$<br>$m^2+2m-4=0$<br>$m = \frac{-2\pm\sqrt{4+16}}{2}$ OE | A1 | a correct equation in $m$ (any form)<br>or $\sqrt{5}-1$ |
| | $m = 1.24$ | A1 | must reject the negative value if there |

Question 28

| | | | |
|---|---|---|---|
| (i) | $k\int_0^a \frac{1}{\sqrt{x}}dx = 1$ | M1 | Attempt int f($x$) and = 1 ignore limits |
| | $(2k[x^{0.5}]_0^a = 1)$<br>$2ka^{0.5}=1 \qquad$ or $a = \frac{1}{4k^2}$ | A1 | OE; a correct eqn in $k$ & $a$ after sub limits |
| | $k\int_0^a \frac{x}{\sqrt{x}}dx = 3$ | M1 | Attempt int $x$f($x$) and = 3 |
| | e.g. $\frac{2}{3}ka^{1.5}=3 \qquad$ or $a^3 = \frac{81}{4k^2}$ | A1 | OE; a correct eqn in $k$ and $a$ after sub limits |
| | e.g. $a^2 = 81 \qquad$ or e.g. $k^2 = \frac{81}{4\times 9^3}$ | M1 | Attempt eliminate one letter |
| | $a = 9$ | A1 | Convincingly obtained |
| | e.g. $k = \frac{9}{54}$<br>$k = \frac{1}{6}$ AG | A1 | |
| (ii) | $\frac{1}{6}\int_0^m \frac{1}{\sqrt{x}}dx = 0.5$ OE | M1 | Attempt int f($x$), unknown limit and = 0.5 |
| | $\frac{1}{3}m^{0.5} = 0.5$ | A1 | a correct equn in $m$ after sub limits |
| | $m = 2.25$ | A1 | |

Question 29

| | | | |
|---|---|---|---|
| (i) | $1-6\int_{0.3}^{0.7}\left(x-x^2\right)\mathrm{d}x$ | **M1** | or $2\times 6\int_{0}^{0.3}\left(x-x^2\right)\mathrm{d}x$ or similar<br>correct expression before integration |
| | $1-\left[6\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\right]_{0.3}^{0.7}$ | **A1** | or similar correct expression after integration |
| | $1-6\left[\frac{0.7^2}{2}-\frac{0.7^3}{3}-\frac{0.3^2}{2}+\frac{0.3^3}{3}\right]$ | **M1** | Attempt subst correct limits in this or other correct expression |
| | = 0.432 (or 54/125) | **A1** | (SR1 Omission of '1–' scores **B2** for 0.568 or 71/125)<br>(SR2 Omission of '2x' scores **B2** for 0.216 or 27/125) |
| | | **4** | |
| (ii) | Correct shape between $x = 0$ and 1 | **B1** | No curve outside this range. |
| | $E(X) = 0.5$ | **B1** | |
| | | **2** | |
| (iii) | $6\int_0^1\left(x^3-x^4\right)\mathrm{d}x$<br>$=\left[6\left(\frac{x^4}{4}-\frac{x^5}{5}\right)\right]_0^1$ | **M1** | attempt int $x^2f(x)$, ignore limits |
| | $6\left[\frac{1^4}{4}-\frac{1^5}{5}\right]$ (= 0.3) | **M1** | attempt subst correct limits in correct integ |
| | $Var(X) = '0.3' - '0.5'^2$<br>= 0.05 | **A1FT** | FT their mean, dep their $Var(X) > 0$ |

## Question 30

| | | | |
|---|---|---|---|
| (i) | $k\int_1^2 (\frac{1}{x^2}+\frac{1}{x^3})\,dx = 1$ | M1 | Attempt integ f($x$) & '= 1'; ignore limits |
| | $k\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^2 = 1$ | A1 | Correct integral & limits & '= 1' |
| | $k\left[-\frac{1}{2}-\frac{1}{8}+1+\frac{1}{2}\right] = 1$ <br> $k = \frac{8}{7}$ **AG** | A1 | Sufficient working must be shown, no errors seen |
| | | 3 | |
| (ii) | $\frac{8}{7}\int_1^2 (\frac{1}{x}+\frac{1}{x^2})\,dx$ | M1 | Attempt integ $x$f($x$), ignore limits |
| | $= \frac{8}{7}\left[\ln x - \frac{1}{x}\right]_1^2$ | A1 | Correct integral & limits, condone missing k |
| | $= \frac{8}{7}(\ln 2 + \frac{1}{2})$ or 1.36 (3 sf) | A1 | |
| | | 3 | |
| (iii) | $\frac{8}{7}\int_1^{1.5} (\frac{1}{x^2}+\frac{1}{x^3})\,dx$ <br> $= \frac{8}{7}\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^{1.5}$ | M1 | Attempt integration f($x$) between 1 and 1.5 or between 1.5 and 2 |
| | $= \frac{44}{63}$ or 0.698....... | A1 | Or $\frac{19}{63}$ or 0.302 |
| | '$\frac{44}{63}$' (1 – '$\frac{44}{63}$')$^2$ | M1 | FT their $\frac{44}{63}$ |
| | × 3 | M1 | Independent provided answer is <1 |
| | = 0.191 | A1 | |
| | | 5 | |

## Question 31

| | | | |
|---|---|---|---|
| (i) | $\frac{1}{2}\times a\times b = 1$ | M1 | Attempt Δ area = 1 or $\int(b - bx/a)\,dx = 1$ with correct limits |
| | $b = \frac{2}{a}$ | A1 | |
| | | 2 | |
| (ii) | grad $= -\frac{2}{a^2}$ or $-\frac{b}{a}$ | B1 | allow without '–' sign (could be implied or seen in **(i)**) |
| | $y - (\frac{2}{a}) =$ grad $\times x$ or $y =$ grad $\times (x - a)$ | M1 | correct use of $y = mx + c$ or $y - y_1 = m(x - x_1)$ with (0,b) or (a,0) including attempt at substitution of their b |
| | $y - (\frac{2}{a}) = -\frac{2}{a^2}x$ or $y = -\frac{2}{a^2}(x-a)$ <br> and $y = \frac{2}{a} - \frac{2}{a^2}x$ **AG** | A1 | No errors seen |
| (iii) | $\int_0^a (\frac{2}{a}x - \frac{2}{a^2}x^2)\,dx$ | M1 | Attempt int $x$f($x$) ignore limits |
| | $= [\frac{1}{a}x^2 - \frac{2}{3a^2}x^3]_0^a$ | A1 | Correct integration ignore limits |
| | $a - \frac{2}{3}a = 0.5$ | M1 | Sub correct limits into their integral and = 0.5 |
| | $a = 1.5$ | A1 | |

Question 32

| | | | |
|---|---|---|---|
| (i) | $\int_5^{10} \frac{k}{x^2} dx = 1$ | **M1** | Attempt integration f($x$) and '= 1'; ignore limits |
| | $\left[-\frac{k}{x}\right]_5^{10} = 1$ oe<br>$(\frac{k}{5} - \frac{k}{10} = 1)$ | **A1** | Correct integration and limits and '= 1' |
| | $k = 10$ **AG** | **A1** | No errors seen |
| | | **3** | |
| (ii) | $10\int_5^{10} \frac{1}{x} dx$<br>$10\left[\ln x\right]_5^{10}$ | **M1** | Attempt integ $x$f($x$); ignore limits.<br>or 10(ln 10 – ln 5) |
| | = 10ln 2 **AG** | **A1** | No errors seen |
| | | **2** | |
| (iii) | $10\int_9^{10} \frac{1}{x^2} dx$<br>$(10\left[-\frac{1}{x}\right]_9^{10})$ | **M1** | Attempt integ f(x) with correct limits |
| | $10\left[-\frac{1}{10} + \frac{1}{9}\right]$ | **A1** | Substitute correct limits in correct integration |
| | $= \frac{1}{9}$ or 0.111 (3 sf) | **A1** | |
| | | **3** | |
| (iv) | $\int_5^{a} \frac{k}{x^2} dx = 0.6$<br>$10\left[-\frac{1}{x}\right]_5^{a} = 0.6$ | **M1** | Attempt integration of f($x$) with correct limits and = 0.6 |
| | $10[\frac{1}{5} - \frac{1}{a}] = 0.6$ | **A1** | Substitute correct limits in correct integration |
| | $a = \frac{50}{7}$ or 7.14 (3 sf) | **A1** | |
| | | **3** | |

Question 33

| | | | |
|---|---|---|---|
| (i) | $k\int_2^6 x^{-1}\mathrm{d}x = 1$ | **M1** | Attempt integrate f($x$) & = 1. Ignore limits |
| | $k[\ln x]_2^6 = 1$<br>$k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$<br>$k = \frac{1}{\ln 3}$ **AG** | **A1** | correct sub of correct limits in correct integral leading to correct ans. No errors seen. |
| | | **2** | |
| (ii) | $\frac{1}{\ln 3}\int_2^6 1\,\mathrm{d}x$ | **M1** | Attempt integ $x$f($x$). Ignore limits |
| | $= \frac{1}{\ln 3}[x]_2^6 \quad (= \frac{1}{\ln 3}(6-2))$ | **A1** | Correct integral and limits |
| | $= \frac{4}{\ln 3} = 3.64$ **AG** | **A1** | No errors seen |
| | | **3** | |
| (iii) | $P(X < E(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$ | **M1** | Attempt integ f($x$) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe |
| | $= \frac{1}{\ln 3}[\ln x]_2^{3.64}$<br>$= \frac{1}{\ln 3}(\ln 3.64 - \ln 2) \quad (= 0.545)$ | **A1** | Correct sub correct limits into correct integral |
| | $P(m < X < E(X)) =$ "0.545" – 0.5 | **M1** | Subt 0.5 from their $P(X < E(X))$<br>art 0.045 . ft their $P(X < E(X)$ (> 0.5) |
| | = 0.045 (2 sfs) | **A1** | equivalent method<br>M1 method for median-need 0.5 and limits 2 to m or m to 6<br>A1 sqrt 12 or 3.464<br>M1 calc area from "3.464" to 3.64<br>A1 0.045 or better, not 0.046 |
| | | **4** | |

Question 34

| | | | |
|---|---|---|---|
| (i) | $\int_0^a \frac{k}{(x+1)^2}dx = 1$ | **M1** | Any attempt integ f($x$) and = 1. Ignore limits |
| | $-\left[\frac{k}{(x+1)}\right]_0^a = 1$ <br> $-k(\frac{1}{a+1}-1) = 1$ | **M1** | Attempt subst correct limits into correct integral |
| | $k\times\frac{a}{a+1} = 1$ and $k=\frac{a+1}{a}$ **AG** | **A1** | No errors seen |
| | | **3** | |
| (ii) | Max time allowed by model (for runners to finish) | **B1** | Allow: All runners finish in time $a$ or less or Longest time (taken by any runner) oe |
| | | **1** | |
| (iii) | $\frac{a+1}{a}\int_0^{0.5}\frac{1}{(x+1)^2}dx = \frac{3}{4}$ | **M1** | Attempt integ f($x$) and $=\frac{3}{4}$; ignore limits oe. Condone missing / incorrect k |
| | $-\frac{a+1}{a}\left[\frac{1}{(x+1)}\right]_0^{0.5} = \frac{3}{4}$ <br> $-\frac{a+1}{a}(\frac{2}{3}-1) = \frac{3}{4}$ | **M1** | Attempt subst correct limits into correct integral. Condone missing / incorrect k |
| | $a = 0.8$ oe | **A1** | |
| | | **3** | |

Question 35

| | | | |
|---|---|---|---|
| (i) | $\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\cos x\,dx$ <br> $=\sqrt{2}\,[\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ | **M1** | Attempt integ f($x$) with correct limits |
| | $=\frac{2-\sqrt{2}}{2}$ oe or 0.293 (3 sf) | **A1** | SC Final answer of 0.707 scores B1sc |
| | | **2** | |
| (ii) | $\sqrt{2}\int_0^m \cos x\,dx = 0.5$ | **M1** | Attempt to integ f(x) & = 0.5. Ignore limits. Condone missing √2 |
| | $\sqrt{2}\,[\sin x]_0^m = 0.5$ <br> $\sqrt{2}\,\sin m = 0.5$ | **A1** | Correct integral and limits 0 to unknown & = 0.5 Condone missing √2 |
| | $\sin m = \frac{1}{2\sqrt{2}}$ oe | **M1** | For rearranging their expression to the form sin$m$=… (sin$m$ = 0.35355.... or 0.354) seen or implied |
| | $m = 0.361$ (3 sfs) | **A1** | No errors seen <br> (Note 20.705 can score M1 A1 M1 A0) |
| | | **4** | |

| | | | |
|---|---|---|---|
| '(iii) | $\sqrt{2}\int_0^{\frac{\pi}{4}} x\cos x\,dx$ | M1 | Attempt to integ $x$f($x$). Ignore limits. Condone missing √2 |
| | $= \sqrt{2}\ \{[x(\sin x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x\,dx\ \}$ | M1 | Attempt to integ by parts leading to expression of form $\pm x\sin x \pm \cos x$ with correct limits |
| | $= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} - 0 - [-\cos x]_0^{\frac{\pi}{4}}\}$ | A1 | For $\sqrt{2}(x\sin x - (-\cos x))$ with correct limits |
| | $= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} + \cos\frac{\pi}{4} - 1\}$<br>$= \frac{\pi}{4} + 1 - \sqrt{2}$ oe or 0.371 (3 sf) | A1 | |
| | | 4 | |

Question 36

| | | | |
|---|---|---|---|
| ;(i) | $\frac{3}{a^3}\int_0^a x^2 dx$<br>$(= \frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^a)$ | M1 | Attempt to integrate f(x) with limits 0 and a (condone missing $\frac{3}{a^3}$) |
| | $= \frac{3a^3}{3a^3}$ | A1 | $\frac{3a^3}{3a^3} - 0$ or better seen |
| | = 1 Hence f is pdf for all $a$ | A1 | Answer = 1 and comment |
| | | 3 | |
| (ii) | $\frac{3}{a^3}\int_0^2 x^2 dx = 0.5$<br>$\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^2 = 0.5$ | M1 | Attempt to integrate f(x)=0.5, limits 0 and 2 oe, condone missing $\frac{3}{a^3}$ |
| | $\frac{3}{a^3} \times \frac{8}{3} = 0.5$ oe | A1 | $\frac{2^3}{3} - 0$ or better, condone missing $\frac{3}{a^3}$ |
| | $a^3 = 16$ or $a = \sqrt[3]{16}$<br>( = 2.52 **AG**) | A1 | Convincingly obtained<br>Note: Attempt to verify 2.52, M1 as stated except not equated to 0.5.A1 as stated, A1 for evaluation to 0.499..apprx 0.5 |
| | | 3 | |
| ;(iii) | $\frac{3}{16}\int_0^{2.52} x^3 dx$ or $\frac{3}{16}\int_0^a x^3 dx$<br>$= \frac{3}{16}\left[\frac{x^4}{4}\right]_0^{2.52}$ or $\frac{3}{16}\left[\frac{x^4}{4}\right]_0^a$ | M1 | Attempt integ $x$f($x$), correct limits, condone missing $\frac{3}{a^3}$ |
| | $= \frac{3}{16} \times \frac{40.317}{4}$ | A1 | $\frac{2.52^4}{4} - 0$ or better, condone missing $\frac{3}{a^3}$ |
| | = 1.89 (3 sf) | A1 | |
| | | 3 | |

Question 37

| | | | |
|---|---|---|---|
| (i) | $a\int_1^b \frac{1}{x^2}dx = 1$ | **M1** | Attempt int f($x$) and = 1, ignore limits |
| | $a[-\frac{1}{x}]_1^b = 1$ | **A1** | correct integ and limits = 1 |
| | $a[1-\frac{1}{b}] = 1$ or $a\times\frac{b-1}{b} = 1$<br>$b = \frac{a}{a-1}$ **AG** | **A1** | No errors seen |
| | | **3** | |
| (ii) | $a\int_1^{\frac{3}{2}} \frac{1}{x^2}dx = \frac{1}{2}$<br>$a[-\frac{1}{x}]_1^{\frac{3}{2}} = \frac{1}{2}$ | **M1** | Attempt int f($x$) with limits 1 to $\frac{3}{2}$ and = $\frac{1}{2}$ |
| | $a[1-\frac{2}{3}] = \frac{1}{2}$ | **A1** | oe correct equn in $a$ |
| | $a = \frac{3}{2}$, $b = 3$ | **A1** | Both |
| | | **3** | |
| (iii) | $\frac{3}{2}\int_1^3 \frac{1}{x}dx$ | **M1** | Attempt int $x$f($x$), ignore limits – condone missing a |
| | $= \frac{3}{2}[\ln x]_1^3$ | **A1** | **FT** Correct integ and *their* limits 1 to b – condone missing a |
| | $= \frac{3}{2}\ln 3$ or 1.65 (3 sf) | **A1** | **FT** *their a* and *b* (valid *b* i.e. >1) |
| | | **3** | |

Question 38

| | | | |
|---|---|---|---|
| (a)(i) | $0.5 \times 1/a = (\frac{0.5}{a})$ | **M1** | Or attempt to integrate f($x$) (=1/a) between 0 and 0.5 |
| | $= \frac{1}{2a}$ oe | **A1** | Accept 0.5/a for A1 |
| | | **2** | |
| (a)(ii) | $\frac{a}{2}$ | **B1** | |
| | | **1** | |
| (a)(iii) | $\int_0^a \frac{x^2}{a}\mathrm{d}x - (\text{'}\frac{a}{2}\text{'})^2$ | **M1** | Integ their $x^2$f($x$) from 0 to $a$ and sub their mean$^2$ |
| | $\text{Var}(X) = \frac{a^2}{3} - \frac{a^2}{4}$<br>$(\text{Var}(X) = \frac{a^2}{12}$ **AG**) | **A1** | Must see this line oe |
| | | **2** | |

| | | | |
|---|---|---|---|
| (b) | $\int_2^b \frac{3}{2(t-1)^2}\mathrm{d}t$ | **M1** | Attempt integ g($t$) ignore limits |
| | $\left[-\frac{3}{2(t-1)}\right]_2^b$ | **A1** | Correct integral |
| | $-\frac{3}{2}(\frac{1}{(b-1)} - 1) = \frac{3}{4}$<br>$(1 - \frac{1}{(b-1)} = \frac{1}{2})$ | **M1** | Attempt subst correct limits in their integ and = $\frac{3}{4}$ |
| | $b = 3$ | **A1** | |
| | | **4** | |

## Question 39

| | | | |
|---|---|---|---|
| (i) | $k\int_0^3 \left(3x - x^2\right) dx = 1$ | **M1** | Attempt to integrate f($x$) and = 1 |
| | $k\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3$ <br> $k\left(\frac{27}{2} - \frac{27}{3}\right) = 1$ | **A1** | Correct integral and limits |
| | $k = \frac{2}{9}$ | **A1** | AG <br> No errors seen |
| | | **3** | |
| (ii) | $\frac{2}{9}\int_1^2 \left(3x - x^2\right) dx = \frac{2}{9}\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_1^2 = \frac{2}{9} \times \left(6 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3}\right)$ | **M1** | Attempt to integrate f($x$) d$x$ with limits 1 and 2 <br> OE |
| | $\frac{13}{27}$ or 0.481 (3 sf) | **A1** | |
| | | **2** | |
| (iii) | $y = 3x - x^2$ symmetrical about $x = \frac{3}{2}$ | **M1** | Attempt $\frac{2}{9}\int_0^3 (3x^2 - x^3)\, dx$ |
| | $E(X) = \frac{3}{2}$ | **A1** | |
| | $\frac{2}{9}\int_0^3 (3x^3 - x^4)\, dx$ | **M1** | Attempt to integrate $x^2$f($x$) |
| | $= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3 (= \frac{2}{9} \times \frac{243}{20} = \frac{27}{10})$ <br> $\frac{'27'}{10} - \left(\frac{'3'}{2}\right)^2$ | **M1** | Subtract their $(E(X))^2$ from their integral $x^2$f($x$) with correct limits substituted |
| | $\frac{9}{20}$ or 0.45 | **A1** | |
| | | **5** | |

Question 40

| | | | |
|---|---|---|---|
| (i) | $\frac{1}{2}\times a\times\frac{a}{2}=1$ **or** $\frac{1}{2}\int_0^a x\mathrm{d}x=1$<br>$\frac{a^2}{4}=1$ OE | **M1** | Attempt at triangle area or integral f($x$) **and** = 1, |
| | $a=2$ | **A1** | |
| | | **2** | |
| (ii) | $\frac{1}{2}\int_0^2 x^2\mathrm{d}x$ | **M1** | Attempt integral $x$f($x$) |
| | $=\left[\frac{x^3}{6}\right]_0^2$ | **M1** | Correct integral and limits 0 to their 'a' |
| | $\left(=\frac{8}{6}\right)=\frac{4}{3}$ | **A1** | AG<br>CWO |
| | | **3** | |

| | | | |
|---|---|---|---|
| (iii) | $P\left(X<\frac{4}{3}\right)=\frac{1}{2}\int_0^{\frac{4}{3}} x\mathrm{d}x$ | **M1** | Attempt integral f($x$) between correct limits |
| | $=\frac{4}{9}$ | **A1** | or $\frac{5}{9}$ |
| | $P(E(X)<X<m)=\frac{1}{2}-\text{'}\frac{4}{9}\text{'}$ | **M1** | or $\frac{5}{9}-\frac{1}{2}$ |
| | $\frac{1}{18}$ | **A1** | |
| | **Alternative method for question 4(iii)** | | |
| | Attempt to find $m$ | **M1** | |
| | $m=\sqrt{2}$ | **A1** | |
| | Integrate f($x$) between $\frac{4}{3}$ and '$\sqrt{2}$' | **M1** | |
| | $\frac{1}{18}$ | **A1** | |
| | | **4** | |

Question 41

| | | | |
|---|---|---|---|
| (a) | $\frac{3}{4000}\int_5^{10}(100-x^2)\mathrm{d}x$ <br> $=\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_5^{10}$ | M1 | Attempt integration of f($x$), ignore limits. Condone omission of $\frac{3}{4000}$ |
| | $=\frac{3}{4000}\left(1000-\frac{1000}{3}-500+\frac{125}{3}\right)$ | M1 | Correct limits 5 and 10. OE SOI |
| | $= 0.156$ (3 sf) or $\frac{5}{32}$ | A1 | For fully correct working seen including substitution of limits |
| | | 3 | |

| | | | |
|---|---|---|---|
| (b) | $\frac{3}{4000}\int_p^{10}(100-x^2)\mathrm{d}x = \frac{1}{4}$ | M1 | Attempt integration of f($x$) with any limits and $=\frac{1}{4}$ or $=\frac{3}{4}$ seen. Condone omission of $\frac{3}{4000}$ |
| | $\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_p^{10} = \frac{1}{4}$ | A1 | Correct integration with correct limits seen (or implied for limits p and 10) and $=\frac{1}{4}$ OE <br> Condone omission of $\frac{3}{4000}$ |
| | $\frac{3}{4000}\left(1000-\frac{1000}{3}-100p+\frac{p^3}{3}\right)=\frac{1}{4}$ | M1 | Attempt substitution correct limits in their integration of f($x$). Accept limits 0 to $p$ if clearly seen, accept limits –10 and $p$. Substitution must be seen. |
| | e.g. $\frac{2000}{3}-100p+\frac{p^3}{3}=\frac{1000}{3}$ <br> $p^3-300p+1000=0$ | A1 | AG <br> No errors seen |
| | | 4 | |

| | | | |
|---|---|---|---|
| (c) | Curve is symmetrical about $x = 0$ | B1 | May be implied by sketch. No contradictions or integrate f($x$) between $-q$ and $+q$ and equate to 0.5 leading to $q^3$ -300$q$ + 1000 = 0 oe |
| | $q = 3.47$ | B1 | |
| | | 2 | |

Question 42

| | | |
|---|---|---|
| (a) | 7.5 | **B1** |
| | | **1** |
| (b) | $\frac{6}{125}\int_5^{10}(-x^4+15x^3-50x^2)\mathrm{d}x$ | **M1** |
| | $\frac{6}{125}\left[-\frac{x^5}{5}+15\frac{x^4}{4}-50\frac{x^3}{3}\right]_5^{10} - '7.5'^2$ | **M1** |
| | 1.25 (3 sf) | **A1** |
| | | **3** |

| | | |
|---|---|---|
| (c) | $\frac{6}{125}\int_5^{6}(-x^2+15x-50)\mathrm{d}x$<br>$\frac{6}{125}\left[-\frac{x^3}{3}+15\frac{x^2}{2}-50x\right]_5^{6}$ | **M1** |
| | $\frac{6}{125}(-102+\frac{625}{6})$ oe | **M1** |
| | 0.104 | **A1** |
| | $2\times('0.104'\times(1-'0.104'))$ | **M1** |
| | 0.186 (3 sf) | **A1ft** |
| | | **5** |

Question 43

| | | |
|---|---|---|
| (a) | $\int_1^a \frac{k}{x^2}\,dx = 1$ | **M1** |
| | $k\left[-\frac{1}{x}\right]_1^a = 1$<br>$k\left[1-\frac{1}{a}\right] = 1$ | **A1** |
| | $k\left[\frac{a-1}{a}\right] = 1$<br>$\left(k = \frac{1}{a-1}\right)$ **AG** | **A1** |
| | | **3** |
| (b) | $\frac{a}{a-1}\int_1^a \frac{1}{x}\,dx$ | **M1** |
| | $\frac{a}{a-1}[\ln x]_1^a$ | **A1** |
| | $\frac{a\ln a}{a-1}$ | **A1** |
| | | **3** |

| | | |
|---|---|---|
| (c) | $\frac{a}{a-1}\int_1^m \frac{1}{x^2}\,dx = \frac{3}{5}$ | **M1** |
| | $\frac{a}{a-1}\left[-\frac{1}{x}\right]_1^m = \frac{3}{5}$<br>$\frac{a}{a-1}\left[1-\frac{1}{m}\right] = \frac{3}{5}$ | **A1** |
| | $\frac{1}{m} = 1 - \frac{3(a-1)}{5a}$ or $\frac{1}{m} = \frac{2a+3}{5a}$ | **A1** |
| | $m = \frac{5a}{2a+3}$ | **A1** |
| | | **4** |

Question 44

| | | |
|---|---|---|
| (a) | 'Tails down' parabola only from $x = 0$ to 20 shown | **B1** |
| | | **1** |
| (b) | Symmetrical | **B1** |
| | | **1** |
| (c) | $\frac{3}{4000}\int_0^{20}\left(20t^3 - t^4\right)dx = \frac{3}{4000}\left[20\frac{t^4}{4} - \frac{t^5}{5}\right]_0^{20}$ | **M1** |
| | $\text{Var}(T) = \frac{3}{4000}\times 160000 - 10^2$ | **M1** |
| | 20 | **A1** |
| | | **3** |
| (d) | $(p - 0.5)\times 2$ or $1 - 2(1 - p)$ | **M1** |
| | $2p - 1$ | **A1** |
| | | **2** |
| (e) | $\frac{3}{4000}\int_8^{12}\left(20t - t^2\right)dx$ | **M1** |
| | $\frac{3}{4000}\left[20\frac{t^2}{2} - \frac{t^3}{3}\right]_8^{12} = \frac{3}{4000}\left(1440 - 576 - 640 + \frac{512}{3}\right)$ | **A1** |
| | $\frac{37}{125}$ or 0.296 | **A1** |
| | | **3** |
| (f) | Does not allow times greater than 20 minutes | **B1** |
| | | **1** |

Question 45

| | | |
|---|---|---|
| (a) | $(k =)\ \frac{1}{a}$ | **B1** |
| | | **1** |
| (b) | (Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$ | **B1 FT** |
| | $\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \text{'}\frac{a^2}{3}\text{'}\right)$ | **M1** |
| | $-\left(\text{'}\frac{a}{2}\text{'}\right)^2 \left(= \frac{a^2}{12}\right)$ | **M1** |
| | $\left(\frac{a^2}{12} = 3\right)\ a = 6$ | **A1** |
| | | **4** |

Question 46

| | | |
|---|---|---|
| (a) | $\frac{1}{2} \times 3 \times c = 1$<br>($c = \frac{2}{3}$ **AG**) | **B1** |
| | | **1** |
| (b) | $\left(\frac{1}{3}\right)^2$ | **M1** |
| | $= \frac{1}{9}$ or 0.111(3sf) | **A1** |
| | | **2** |

| | | |
|---|---|---|
| (c) | Equation of line is $y = \frac{2}{3} - \left(\frac{2}{3} \div 3\right)x$ | ***M1** |
| | $E(X) = \int_0^3 \left(\frac{2}{3}x - \frac{2}{9}x^2\right) dx$ | **DM1** |
| | $= \left[\frac{x^2}{3} - \frac{2x^3}{27}\right]_0^3$ | **A1 FT** |
| | $= 1$ | **A1** |
| | | **4** |

Question 47

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{2}\times\frac{1}{2}k\times k = 1$ | M1 | Or use of $\int_0^k\left(-\frac{1}{2}x+\frac{1}{2}k\right)\mathrm{d}x=1$ and attempt at integral. |
| | $k=2$ | A1 | Unsupported answers M0 A0. Do not accept $\pm 2$. |
| | | 2 | |
| (b) | $\mathrm{f}(x) = -\frac{1}{2}x + 1$ | B1 FT | FT *their k* from $y=-\frac{1}{2}x+\frac{1}{2}k$ . |
| | $\int_0^2(-\frac{1}{2}x^2 + x)\mathrm{d}x = \left[-\frac{x^3}{6}+\frac{x^2}{2}\right]_0^2$ | M1 | Attempt integration of $x\mathrm{f}(x)$ limits 0 to $k$.<br>FT *their* f($x$).<br>Could be in terms of $k$. |
| | $\frac{2}{3}$ or 0.667 (3 sf) | A1 | |
| | | 3 | |

| | | | |
|---|---|---|---|
| (c) | $\int_p^1(-\frac{1}{2}x+1)\mathrm{d}x$ [= 0.25] | M1 | FT their equation of **line**; correct integral and limits (could be reversed) stated or $\frac{1}{2}(1-p)(1-\frac{1}{2}p+\frac{1}{2})$ [= 0.25]. |
| | $\left[-\frac{x^2}{4}+x\right]_p^1 = 0.25$<br>$-\frac{1}{4}+1+\frac{p^2}{4}-p=0.25$ | M1 | Attempt substitution of correct limits (not reversed) into their integral or attempt expand must equal 0.25.<br>OE |
| | $p^2 - 4p + 2 = 0$ | M1 | Obtain 3-term quadratic set equal to 0, obtain at least 1 solution. |
| | $p = 2 - \sqrt{2}$ or 0.586 | A1 | CAO |
| | | 4 | |

Question 48

| | | | |
|---|---|---|---|
| (a) | $P(X > 10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2\,dx$ | **M1** | Attempt integration of f(x), ignore limits. |
| | $= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_{10}^{20}$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_{10}^{20}$ <br> $= \frac{1}{8000}\left[0 - (-10)^3\right]$ | **M1** | Substitute correct limits 10 to 20 or 1 – … limits 0 to 10 in *their* integral |
| | $\frac{1}{8}$ or 0.125 | **A1** | **SC** Unsupported answer of $\frac{1}{8}$ scores B1 only |
| | $(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf) | **B1 FT** | FT *their* P(X > 10) dependent on first M1 gained |
| | | **4** | |
| (b) | $\int_0^{20} \frac{3}{8000}(x^3 - 40x^2 + 400x)\,dx$ | **M1** | Attempt integration of xf(x). Ignore limits. |
| | $\frac{3}{8000}\left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2}\right]_0^{20}$ <br> or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3}\right) - \frac{1}{8000}\left(\frac{(x-20)^4}{4}\right)$ | **A1** | Correct integral (by expanding or by parts) |
| | $\frac{3}{8000}\left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400\right]$ | **M1** | Subst correct limits in their (4th degree) integral |
| | 5 | **A1** | |
| | | **4** | |
| (c) | $\int_0^m \frac{3}{8000}(x-20)^2\,dx = 0.5$ | **M1** | Attempt to integrate f(x) and equate to 0.5. Ignore limits. |
| | $\left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_0^m = 0.5$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_0^m = 0.5$ <br> $\frac{1}{8000}\left[(m-20)^3 - (-20)^3\right] = 0.5$ | **M1** | Attempt integral and substitute limits 0 and *m* or *m* and 20 and = 0.5 |
| | $(m-20)^3 = -4000$ | **A1** | AG. Found convincingly. |
| | $(m = 20 + \sqrt[3]{-4000})$ <br> $m = 4.13$ (3 sf) | **B1** | |
| | | **4** | |
| (d) | Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early | **B1** | or any relevant comment e.g. trains on Sun may be different to trains on Mon |
| | | **1** | |

Question 49

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{2}p(p-1)=1$ | M1 | For area =1<br>For verification methods accept $\frac{1}{2}\times 2\times 1=1$ or $\frac{1}{2}\times 2\times(p-1)=1$ or $\frac{1}{2}\times 1\times p=1$ as indication that area=1 |
| | $p=2$ | A1 | AG - Convincing method and answer.<br>Must see quadratic rearranged to =0 and no errors seen.<br>N.B. Accept convincing verification methods (e.g. statement such as 'assume $p = 2$' or 'if $p = 2$' or 'using $p = 2$' or showing by clear substitution that $p = 2$ fits $\frac{1}{2}p(p-1)=1$ with clear conclusion) |
| | | **2** | |
| (b) | Gradient = 2<br>equation of line is $y = 2x + c$<br>line passes through (1, 0), hence $c = -2$ | M1 | Award for attempting equation of line $y=mx+c$ with $m$ = 2, -2, $\frac{1}{2}$ or $-\frac{1}{2}$ and numerical $c$ ($c\neq 0$) |
| | $y = 2x - 2$ | A1 | May be seen in **(a)**<br>M1 can be implied by correct answer |
| | $2\int_1^2 (x^2 - x)\,dx$ | M1 | For attempting $\int x\text{f}(x)\,dx$.<br>Ignore limits, FT *their* equation. |
| | $2\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2$ | A1 FT | Correct integration FT *their* f($x$)and correct limits |
| | $\frac{5}{3}$ or 1.67 (3 sf) | A1 | |
| | | **5** | |

## Question 50

| | | |
|---|---|---|
| $E(X) = 3$ | B1 | N.B. $E(X)=108k$ is B0 until correct $k$ substituted in. |
| $k\int_0^6 (6x - x^2)\mathrm{d}x = 1$<br>$k\left[3x^2 - \frac{x^3}{3}\right]_0^6$ [= 1] | M1 | Attempt integration of f($x$) and =1.<br>Ignore limits at this stage. |
| $k\left(108 - \frac{216}{3}\right) = 1$<br>$k = \frac{3}{108}$ or $\frac{1}{36}$ | A1 | |
| $'\frac{3}{108}'\int_0^6 (6x^3 - x^4)\mathrm{d}x$<br>$= \frac{3}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = 10.8$ | *M1 | Attempt integration of *their* $k \times x^2 f(x)$.<br>Ignore limits at this stage. Accept in terms of $k$. |
| $'10.8' - '3'^2$ | DM1 | *Their* 10.8 (from use of limits 0 and 6) minus *their* $(E(X))^2$.<br>Accept in terms of $k$: $388.8k-(108k)^2$ |
| $\frac{9}{5}$ or 1.8 | A1 | CWO. Must be convincingly obtained as AG. |
| | **6** | |

## Question 51

| | | |
|---|---|---|
| $1 - \frac{20}{27}$ or $\frac{20}{27} - \frac{1}{2}$<br>$\frac{20}{27} - \left(1 - \frac{20}{27}\right)$ or $\left(\frac{20}{27} - \frac{1}{2}\right)$ | M1 | For either expression seen. |
| $\frac{13}{27}$ | A1 | OE. Accept 0.481 or 0.482. |
| | **2** | |

## Question 52

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{18}\int_0^{1.2}(9-x^2)\mathrm{d}x$ | M1 | Attempt to integrate f($x$), ignore limits. Must see an increase of power. |
| | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$ | A1 | Correct integration and correct limits. |
| | $\frac{71}{125}$ or 0.568 | A1 | SC unsupported answer scores **B2** only. |
| | | 3 | |
| (b) | $\frac{1}{18}\int_0^3(9x-x^3)\mathrm{d}x$ | M1 | Attempt to integrate $x$f($x$), ignore limits. Must see an increase of power. |
| | $\frac{1}{18}\left[\frac{9x^2}{2}-\frac{x^4}{4}\right]_0^3$ | A1 | Correct integration and correct limits. |
| | $\frac{9}{8}$ or 1.125 | A1 | SC unsupported answer scores **B2** only. |
| | | 3 | |
| (c) | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^m=0.5$ | M1 | Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits $m$ to 3. Allow $x$ instead of $m$. |
| | $\frac{1}{18}\left[9m-\frac{m^3}{3}\right]-0.5=0$ | A1 | Any correct cubic equation in $m$ or $x$. |
| | $m^3-27m+27=0$ | A1 | AG. Correctly obtain this equation. No errors seen. |
| | | 3 | |

## Question 53

| | | | |
|---|---|---|---|
| '(a)(i) | $k\int_0^2(4x-x^2)\mathrm{d}x=1$ | M1 | Attempt integral f($x$) and = 1. Ignore limits (must see a power increase for attempted integration). |
| | $k\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^2=1$ | A1 | Correct integration and correct limits. |
| | $k\times\frac{16}{3}=1\ \left[k=\frac{3}{16}\right]$ | A1 | OE AG<br>Convincingly obtained. At least one interim step. No errors seen. |
| | | 3 | |
| (a)(ii) | $\frac{3}{16}\int_0^2(4x^2-x^3)\mathrm{d}x$ | M1 | Attempt integral $x$f($x$). Ignore limits. (must see a power increase for attempted integration). Condone missing $k$. |
| | $\frac{3}{16}\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^2$ | A1 | Correct integration and correct limits. Condone missing $k$. |
| | $\frac{5}{4}$ | A1 | Unsupported correct answer scores **SC B2** only. |
| | | 3 | |

Question 54

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{18}\int_0^{1.2}(9-x^2)\mathrm{d}x$ | **M1** | Attempt to integrate f($x$), ignore limits. Must see an increase of power. |
| | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$ | **A1** | Correct integration and correct limits. |
| | $\frac{71}{125}$ or 0.568 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |
| (b) | $\frac{1}{18}\int_0^{3}(9x-x^3)\mathrm{d}x$ | **M1** | Attempt to integrate $x$f($x$), ignore limits. Must see an increase of power. |
| | $\frac{1}{18}\left[\frac{9x^2}{2}-\frac{x^4}{4}\right]_0^{3}$ | **A1** | Correct integration and correct limits. |
| | $\frac{9}{8}$ or 1.125 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |
| (c) | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{m}=0.5$ | **M1** | Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits $m$ to 3. Allow $x$ instead of $m$. |
| | $\frac{1}{18}\left[9m-\frac{m^3}{3}\right]-0.5=0$ | **A1** | Any correct cubic equation in $m$ or $x$. |
| | $m^3-27m+27=0$ | **A1** | AG. Correctly obtain this equation. No errors seen. |
| | | **3** | |

Question 55

| | | | |
|---|---|---|---|
| (a) | Quadratic curve, hence symmetrical | B1 | OE. Allow sketch and 'symmetrical' or just 'curve symmetrical' |
| | | 1 | |
| (b) | $-k\int_1^3 (x^2-4x+3)\mathrm{d}x = 1$ | M1 | Attempt to integrate f($x$) and '= 1'. Ignore limits at this stage |
| | $-k\left[\frac{x^3}{3}-2x^2+3x\right]_1^3$ | A1 | Fully correct expression (correct integration and limits) |
| | $-k\times\left[0-\frac{4}{3}\right]=1$ or $k\times\frac{4}{3}=1$ <br> $\left[k=\frac{3}{4}\right]$ | A1 | AG, OE. Correctly substitute limits and '= 1' and correctly obtain result with no errors seen. |
| | | 3 | |
| (c) | $-\frac{3}{4}\int_1^3\left(x^4-4x^3+3x^2\right)\mathrm{d}x$ | M1 | Attempt to integrate $x^2$ f($x$) from 1 to 3 |
| | $-\frac{3}{4}\times\left[\frac{x^5}{5}-x^4+x^3\right]_1^3$ <br> $\left[=\frac{3}{4}\times\frac{28}{5}=\frac{21}{5}\right]$ | A1 | Correct integration and limits |
| | $\left[\frac{21}{5}-2^2\right]=0.2$ | A1 | |
| | | 3 | |
| (d) | $-\frac{3}{4}\int_{2.5}^{3}\left(x^2-4x+3\right)\mathrm{d}x$ | M1 | OE. Attempt to integrate f($x$), from 2.5 to 3 (or 1 to 2.5) |
| | $=-\frac{3}{4}\times\left[\frac{x^3}{3}-2x^2+3x\right]_{2.5}^{3}=\frac{5}{32}$ or 0.15625 | A1 | |
| | $1-\left(1-\text{'}\frac{5}{32}\text{'}\right)^3$ | M1 | OE. FT *their* $\frac{5}{32}$ . |
| | = 0.399 (3 sf) | A1 | |
| | | 4 | |

## Question 56

| | | | |
|---|---|---|---|
| (a)(i) | 1 | **B1** | no ambiguity |
| | | **1** | |
| (a)(ii) | $\frac{1}{2}$ | **B1** | No ambiguity |
| | | **1** | |
| (a)(iii) | $[q=]\ \frac{1}{2}p$ | **B1** | Accept $2q = p$ |
| | | **1** | |
| (b) | $p\int_0^a (a^2 - x^2)\mathrm{d}x = 1$ | **M1** | Attempt to integrate $\mathrm{f}(x)$ and equated to 1 |
| | $\frac{2}{3}a^3 p = 1$ | **A1** | OE, simplified |
| | $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ or $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ | **M1** | Attempt to integrate $x\mathrm{f}(x)$, with multiplier $p$ or $\frac{3}{2a^3}$ or *their* $p$, and equate to 3 |
| | $p \times \frac{a^4}{4} = 3$ | **A1** | May be implied by next line |
| | $"\frac{3}{2a^3}" \times \frac{a^4}{4} = 3$ | **M1** | OE. Substitute from one equation into the other. FT *their* equations |
| | $a = 8$ | **A1** | |
| | | **6** | |

## Question 57

| | | | |
|---|---|---|---|
| (a) | $\frac{a}{2}$ | **B1** | |
| | | **1** | |
| (b) | $\frac{1}{4}$ | **B1** | |
| | | **1** | |
| (c) | $\mathrm{f}(x) = \frac{1}{a}$ | **B1** | SOI (may be seen in part **(a)** or part **(b)**) |
| | $\mathrm{E}(X) = \frac{a}{2}$ | **B1** | SOI |
| | $\int_0^a \frac{1}{a}x^2\mathrm{d}x$ | **M1** | Attempt integrate *their* $\mathrm{f}(x) \times x^2$ with correct limits |
| | $= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$ | **A1** | |
| | $\frac{a^2}{3} - (\frac{a}{2})^2$ or $\frac{a^2}{3} - \frac{a^2}{4}$ $[= \frac{a^2}{12}$ **AG** $]$ | **A1** | Must see previous line and answer<br>No errors seen |
| | | **5** | |
| (d) | $\mathrm{P}(X < \frac{b}{3}) = \frac{p}{3}$ | **M1** | SOI (could be on a diagram)<br>OR by integration: prob = 1-(2/3)(b/a) |
| | $\mathrm{P}(\frac{b}{3} < X < a - .\frac{b}{3}) = 1 - \frac{2p}{3}$ | **A1** | |
| | | **2** | |

## Question 58

| | | | |
|---|---|---|---|
| (a) | Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375) | B1 | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square.<br>Must not go below the x-axis. |
| | | 1 | |
| (b) | Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$ | B1 | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 2$. Highest point must be at $x = 1$. |
| | Highest point (1, 0.75) | B1 | |
| | | 2 | |
| (c) | $\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\mathrm{d}x = \frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\mathrm{d}x = \frac{1}{2}$ | M1 | OE<br>Attempt to integrate f(x) and correct limits with correct RHS. |
| | $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3} = \frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a} = \frac{1}{2}$ | A1 | Correct integration. |
| | $a^3 - 12a + 8 = 0$ | A1 | AG Substitute limits and correctly obtain equation.<br>May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$<br>No errors seen.. |
| | | 3 | |
| (d) | $0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) $> 0$<br>$0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) $< 0$<br>Hence $0.69 < a < 0.70$ | B1 | AG<br>Must state either the correct expression and > 0 and < 0 or both answers to 2 sf.<br>Both answers correct and conclusion.<br>Accept equivalent expressions.<br><br>OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70. |
| | | 1 | |

## Question 59

| | | | |
|---|---|---|---|
| (a) | $1-p$ or $p-0.5$ | M1 | SOI, e.g. on diagram. |
| | $[\mathrm{P}(-1 < X < 0) =]\ 2p-1$ | A1 | Clearly as final answer. |
| | | 2 | |
| (b)(i) | $\int_{-3}^{2}(a-b(x^2+x))\mathrm{d}x = 1$ or $\int_{-3}^{2}(ax-b(x^3+x^2))\mathrm{d}x = -0.5$ | M1 | OE Attempt integral, with correct limits and RHS. |
| | $\left[ax-b\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\right]_{-3}^{2}$ (= 1) or $\left[a\frac{x^2}{2}-b\left(\frac{x^4}{4}+\frac{x^3}{3}\right)\right]_{-3}^{2}$ (= –0.5) | A1 | OE Correct integration. |
| | 2a – 8b/3 – 2b +3a – 9b +9b / 2 = 1 or 2a – 4b – 8b / 3 – 9a / 2 + 81b / 4 – 9b = –0.5 leading to $30a - 55b = 6$ **AG** | A1 | Correctly obtained. No errors seen. |
| | | 3 | |
| (b)(ii) | $a - b(9-3) = 0$ or $a - b(4+2) = 0$ [hence $a - 6b = 0$] | *M1 | Use f(–3) = 0 or f(2) = 0.<br>Further attempts at integration M0. |
| | Attempt to solve $30a - 55b = 6$ and their a – 6b = 0 | DM1 | |
| | $a = \frac{36}{125}$ or 0.288 $b = \frac{6}{125}$ or 0.048 | A1 | |
| | | 3 | |

## Question 60

| | | | |
|---|---|---|---|
| (a) | Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375) | B1 | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square.<br>Must not go below the x-axis. |
| | | 1 | |
| (b) | Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$ | B1 | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 2$. Highest point must be at $x = 1$. |
| | Highest point (1, 0.75) | B1 | |
| | | 2 | |
| (c) | $\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\mathrm{d}x = \frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\mathrm{d}x = \frac{1}{2}$ | M1 | OE<br>Attempt to integrate f(x) and correct limits with correct RHS. |
| | $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3} = \frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a} = \frac{1}{2}$ | A1 | Correct integration. |
| | $a^3 - 12a + 8 = 0$ | A1 | AG Substitute limits and correctly obtain equation.<br>May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$<br>No errors seen.. |
| | | 3 | |
| (d) | $0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) $> 0$<br>$0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) $< 0$<br>Hence $0.69 < a < 0.70$ | B1 | AG<br>Must state either the correct expression and > 0 and < 0 or both answers to 2 sf.<br>Both answers correct and conclusion.<br>Accept equivalent expressions.<br><br>OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70. |
| | | 1 | |

## Question 61

| | | | |
|---|---|---|---|
| (a) | $1 - 2(a + b)$ **or** $1 - 2a$ **or** $0.5 - a - b$ **or** 1-(a+b) **or** a+a+b | M1 | OE. Seen or implied – may be on the diagram (or for correct un-simplified final expression). |
| | $P(0.6 \le X \le 1.8) = 1 - 2a - b$ | A1 | Accept $1 - (2a + b)$. |
| | | 2 | |
| (b)(i) | $k\int_0^3(9x^2-6x^3+x^4)\mathrm{d}x = 1$ | M1 | Attempt integrate $f(x)$ ignore limits and '= 1'. |
| | $k\left[\frac{9x^3}{3}-\frac{6x^4}{4}+\frac{x^5}{5}\right]_0^3 = 1$ | A1 | Correct integration seen, correct limits. |
| | $k \times \frac{81}{10} = 1,\ k = \frac{10}{81}$ | A1 | AG. Convincingly obtained. No errors seen. (Must see integration). |
| | | 3 | |
| (b)(ii) | $\frac{10}{81}\int_0^3(9x^4-6x^5+x^6)\mathrm{d}x$<br>$\left[\frac{10}{81}\left[\frac{9x^5}{5}-x^6+\frac{x^7}{7}\right]_0^3\right]$ $[= \frac{18}{7}$ or 2.57...] | M1 | Attempt integrate $x^2f(x)$ between 0 and 3 condone missing k.<br>Must see integration or correct answer of 18/7 seen or implied. |
| | $\frac{18}{7} - $ '1.5'$^2$ | M1 | Their integral of $x^2f(x) - 1.5^2$ (or their mean$^2$). |
| | $= \frac{9}{28}$ or 0.321 | A1 | |
| | | 3 | |

## Question 62

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$ | M1 | |
| | = 1, which is the area under a PDF [and $f(x) \geq 0$] | A1 | Result and statement are both needed. |
| | | 2 | |
| (b) | $\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right) = \frac{\pi}{4}$ | B1 | AG.<br>Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles.<br>Answer should be convincingly obtained and all correct. |
| | Area of sector = $\frac{1}{4}$ | B1 | |
| | Area of triangle $AOB = \frac{1}{2}OA \times OB = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi} - \frac{1}{\pi}}$<br>or<br>Area of triangle $AOB = \frac{1}{2}OA \times OB \times \sin(AOB) = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi}} \sin\frac{\pi}{4}$ | M1 | Accept alternative approaches.<br>Note: AB = $\sqrt{0.7979^2 - 0.5642^2}$ [= 0.5642]<br>Allow values to 3sf. |
| | $\frac{1}{2\pi}$ or 0.1592 | A1 | |
| | '$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592' | M1 | Attempt area of sector – area of triangle *AOB*. |
| | $= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf) | A1 | |
| (b) | **Alternative Method for Question Q7(b): Using integration** | | |
| | Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$ | M1 | |
| | $y = \sqrt{\frac{2}{\pi} - x^2}$ | A1 | |
| | Attempt to integrate (any limits) | M1 | |
| | Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$ | B1 | |
| | Correct integration with correct limits | A1 | |
| | $= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf) | A1 | Correct final answer. |
| | | 6 | |

## Question 63

| | | |
|---|---|---|
| $\frac{3}{2}\int_0^1 (x - x^3)\,dx$ | M1 | Attempt to integrate $xf(x)$; ignore limits. |
| $= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$ | A1 | Correct integration and limits. |
| $= \frac{3}{8}$ | A1 | |
| | 3 | |

Question 64

| | | | |
|---|---|---|---|
| (a)(i) | $\frac{1}{2} \times 4 \times a = 1$ | **M1** | For use of area = 1<br>or let $f(x) = kx$ and attempt $\int_0^4 kx \, dx = 1$. |
| | $[a = \frac{1}{2}]$ $f(x) = \frac{1}{8}x$ | **A1** | $k\left[\frac{x^2}{2}\right]_0^4 = 1$; $8k = 1$; $k = \frac{1}{8}$.<br>$f(x) = \frac{1}{8}x$ or $k = \frac{1}{8}$. |
| | | **2** | |
| (a)(ii) | $\int_0^4 x \times \frac{1}{8}x \, dx$ | **M1** | Attempt to integrate $x \times$ *their* $f(x)$. Ignore limits accept in terms of $k$. |
| | $\left[\frac{x^3}{24}\right]_0^4$ | **A1ft** | Their integral and correct limits accept in terms of $k$. |
| | $= \frac{8}{3}$ or 2.67 (3 sf) | **A1** | Note: Final answer of $64k/3$ scores 2/3. |
| | | **3** | |
| (b) | $\frac{a-1}{a} = \frac{1}{\sqrt{2}}$ | **M1** | Or attempt $\int_0^1 g(w)dw = \frac{1}{2}$ i.e. $\int_0^1 (\frac{2}{a} - \frac{2}{a^2}w)dw = \frac{1}{2}$,<br>or integral from 1 to $a$.<br>$g(w)$ must be linear of form $g(w) = mw$ $(+c)$.<br>Or area attempt: attempt to calculate heights using their linear equation ($h_1 = 2/a$ and $h_2 = -2/a^2 + 2/a$) and use in either area trapezium = 0.5, or area trapezium =area small triangle or area small triangle = 0.5 .<br>Area trapezium $= 1/2 \times 1$ $(2/a + -2/a^2 + 2/a)$<br>Area triangle $= 1/2(a-1)(-2/a^2 + 2/a))$<br>Note: alternative expression for $h_1 = (a-2)/(a-1)$. |
| | $a\sqrt{2} - \sqrt{2} = a$ | **A1** | Or $a^2 - 4a + 2 = 0$.<br>Any correct equation in $a$, $a$ not in denominator. |
| | $a = 2 + \sqrt{2} = 3.41$ | **A1** | |
| | | **3** | |

Question 65

| | | | |
|---|---|---|---|
| (a) | $\frac{1}{2}\times 2\times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf. | **B1** | Calculation and result. |
| | $f(x) \geqslant 0$ | **B1** | Condone $f(x) > 0$ or 'Line is above $x$-axis' OE. |
| | | **2** | |
| (b) | $\frac{1}{2}\pi r^2 = 1$ | **M1** | Area of semi-circle equated to 1 OE.<br>Missing factor of ½ gets M1A0. |
| | $r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf) | **A1** | |
| | | **2** | |
| (c)(i) | Area to the left of 15 is greater than 0.5 | **B1** | OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'. |
| | | **1** | |
| (c)(ii) | $\int_{10}^{20} (\frac{40}{x} - \frac{x}{10})dx$ | **M1** | Integration of $xh(x)$ attempted. Ignore limits. |
| | $\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$ | **A1** | Correct integration and limits (can be implied by final answer). |
| | $= 40\ln 2 - 15$ or 12.7 (3sf) | **A1** | |
| | | **3** | |