A-Level

Topic : Continuous Random Variable

May 2013-May 2025

Answers

Question 1

(i)	$\int^{\infty} \frac{k}{x^3} \mathrm{d}x = 1$	M1		All correct, including limits and an attempt to integrate
	$\left[-\frac{k}{2x^2}\right]_1^{\infty} = 1$			
	$0 - (-\frac{k}{2}) = 1$	A1	2	or $0 + \frac{k}{2} = 1$ or $\frac{k}{2} = 1$ AG must be convincing
(ii)	$\int_1^2 \frac{2}{x^3} \mathrm{d}x$	M1		Attempt integ f(x); ignore limits
	$= \left[-\frac{1}{x^2}\right]_1^2$			
	$= \frac{3}{4}$	A1	2	
(iii)	$\int_1^{\infty} \frac{2}{x^2} \mathrm{d}x$	M1		Attempt integ xf(x); ignore limits
	$= \left[-\frac{2}{x}\right]_1^{\infty}$	A1		Correct & correct limits
	$= 2$	A1	3	
		[Total: 7]		

Question 2

(i)	$\frac{2}{3} \int_1^2 x^2 \mathrm{d}x$	M1	Attempt integ. xf(x); ignore limits
	$= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$	A1	Correct integration and limits
	$= \frac{14}{9}$ or 1.56 o.e.	A1 [3]	
(ii)	$\frac{2}{3} \int_1^{\frac{14}{9}} x\, dx$	M1	Attempt integ. f(x); with limits
	$(= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2)$		
	$= \frac{115}{243}$ or 0.473 (3 s.f.)	A1 [2]	
(iii)	$\frac{115}{243} < \frac{1}{2}$ o.e.	M1	Comparison of prob. or values
	Hence mean < median	A1ft[2]	ft (i) or (ii)
		[Total: 7]	

Question 3

(i)	$\frac{1}{2}\int_4^t \frac{1}{\sqrt{t}}dt = 0.9$ or $\frac{1}{2}\int_t^9 \frac{1}{\sqrt{t}}dt = 0.1$	M1	Attempt integ f(t) with unknown limit and 0.9/0.1.
	$\left[\sqrt{t}\right]_4^t = 0.9$ or $\left[\sqrt{t}\right]_t^9 = 0.1$	A1	Correct integration & limits = 0.9 or 0.1.
	$((\sqrt{t} - 2) = 0.9$ or $(3 - \sqrt{t}) = 0.1)$ $t = 8.41$ (mins) (3 sf)	A1 [3]	
(ii)	$\frac{1}{2}\int_4^9 \frac{t}{\sqrt{t}}dt$ oe	M1	Attempt integ tf(t). Ignore limits
	$\frac{1}{2}\left[\frac{t1.5}{1.5}\right]_4^9$ oe	A1	Correct integration & limits
	$= \frac{19}{3}$	A1	
	$\frac{1}{2}\int_4^9 \frac{t2}{\sqrt{t}}dt$ oe	M1	Attempt integ t^2f(t). Ignore limits
	$(= \frac{1}{2}\left[\frac{t2.5}{2.5}\right]_4^9 = \frac{211}{5})$ $= \frac{'211'}{5} - (\frac{'19'}{3})^2$	M1	integ t^2f(t) – (integ tf(t))^2 attempted
	$= \frac{94}{45}$ or 2.09 (3 sf)	A1 [6]	

Question 4

(i)	$\int_0^{10} \frac{1}{2500}\left(100t^3 - t^5\right)dt$	M1		Attempt integ t^2f(t)
	$(= \frac{1}{2500}\left[25t^4 - \frac{t^6}{6}\right]_0^{10} = \frac{100}{3})$			
	"$\frac{100}{3}$" $- (\frac{16}{3})^2$	M1		For E (T^2) – (E (T))^2
	$= \frac{44}{9}$ or 4.89 (3 sf)	A1	3	
(ii)	$\int_n^{10} \frac{1}{2500}\left(100t - t^3\right)dt$	M1		Attempt integ f(t), ignore limits
	$\frac{1}{2500}\left[50t^2 - \frac{t_4}{4}\right] = 0.1$	M1		Attempt integ f(t), limits n to 10 or 0 to n Equated to 0.1 or 0.9. Not need to be matched
	$\frac{1}{2500}\left[2500 - \left(50n^2 - \frac{n^4}{4}\right)\right] = 0.1$	M1		0.1/0.9 matched to correct limits and used
	$(n^4 - 200n^2 + 9000 = 0)$	M1		Correct method of solution of a QE in n^2
	$(n^2 = 68.3772, n = 8.27)$ $n = 8$	A1	5	Must be single ans only

Question 5

(i)	$\int_0^2 k(x-2)^2\,dx = 1$ $\left(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1\right)$ $k\,[0-(-\frac{8}{3})] = 1$ $k = \frac{3}{8}$ **AG**	M1 A1 [2]	Attempt to integrate f(x) with correct limits and = 1 Must see this line or better, e.g. $k \times \frac{8}{3} = 1$
(ii)	$\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$ $\left(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2\right)$ $\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe $((d-2)^3 = -1.6)$ $d = 0.83(0)$ (3 s.f.)	M1 M1 A1 [3]	$\int$f(x)dx with limits d and 2 or 0 and d, and = 0.2 or =0.8 Condone missing 'k' Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in d^3. Condone missing 'k'
(iii)	$\frac{3}{8}\int_0^2 x(x-2)^2\,dx$ $(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx)$ $= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$ $= \frac{1}{2}$	M1 A1 A1 [3]	Attempt integ xf(x); ignore limits, condone missing k $\left(\frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \int \frac{(x-2)^3}{3}\,dx\right]_0^2\right)$ $= \frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$ Correct integration & limits, condone missing k

Question 6

(i)	Longest lifetime	B1 [1]	Must be in context
(ii)	$\int_1^a \frac{k}{x^2}\,dx = 1$	M1	Int f(x) and equate to 1. Ignore limits
	$k\left[-\frac{1}{x}\right]_1^a = 1$	A1	Correct integral and limits
	$\left(k\left[-\frac{1}{a}+1\right]=1\right)$		
	$k\left[\frac{-1+a}{a}\right] = 1$ or $k(-1+a) = a$		
	$k = \frac{a}{a-1}$ **AG**	A1 [3]	Must be convinced (**AG**)
(iii)	$\frac{5}{3}\int_1^{2.5} \frac{1}{x}\,dx$ or $k\int_1^{2.5} \frac{1}{x}\,dx$	M1	Int xf(x). Ignore limits
	$= \frac{5}{3}[\ln x]_1^{2.5}$ or $k[\ln x]_1^{2.5}$	A1	Correct integral and limits (Accept "k" or "their k")
	$= \frac{5}{3}\ln 2.5$ or 1.53 (3 s.f.)	A1 [3]	

Question 7

ht = $\frac{1}{2}$ seen	B1	or $y = \frac{1}{8}x$
$\frac{1}{2} \times m \times \left(\frac{m}{4} \times "\frac{1}{2}"\right) = \frac{1}{2}$	M1	$\frac{1}{2} \times m \times ("\frac{1}{8}"m) = \frac{1}{2}$ or $\frac{m^2}{16} = \frac{1}{2}$ o.e.
N.B. B1 M1 must be consistent		Or Integrating linear function of form $y = kx$ with limits 0 and m or m and 4 and equated to 0.5
$m = \sqrt{8}$ or $2\sqrt{2}$ or 2.83 (3 s.f.)	A1 [3]	

Question 8

(i)	$k\int_0^4 (16t - t^3)\,dt = 1$	M1		Int f(t) = 1 ignore limits
	$k\left[8t^2 - \frac{t^4}{4}\right]_0^4 = 1$	A1		correct integration with correct limits
	$k(128 - 64) = 1$ o.e.			
	$k \times 64 = 1$			
	$\left(k = \frac{1}{64}\right)$ **AG**	A1	[3]	must be convinced (AG)
(ii)	$\frac{1}{64}\int_0^1 (16t - t^3)\,dt$	M1		Int f(t) between 0 and 1 (accept 0 and a value < 1 , 1 and 4)
	$= \frac{1}{64}\left[8t^2 - \frac{t^4}{4}\right]_0^1$	A1		correct integration and correct limits (ignore "k")
	$= \frac{1}{64}\left[8 - \frac{1}{4}\right]$			
	$= \frac{31}{256}$ or 0.121094	A1		
	$\left(\frac{31}{256}\right)^2 = 0.0147$ (3 s.f.) o.e.	B1√	[4]	ft their "$\frac{31}{256}$"
iii	$\frac{1}{64}\int_0^4 (16t^2 - t^4)\,dt$	M1		Int tf(t) ignore limits
	$= \frac{1}{64}\left[\frac{16t^3}{3} - \frac{t^5}{5}\right]_0^4$	A1		correct integration and correct limits (ignore "k")
	$= \frac{1}{64}\left(\frac{1024}{3} - \frac{1024}{5}\right)$			
	$= \frac{32}{15}$ or 2.13 (3 s.f.) o.e.	A1	[3]	

Question 9

(i)	$\int_1^a \frac{k}{x}dx = 1$	M1		Int f(x) & equate to 1. Ignore limits
	$k[\ln x]_1^a = 1$	A1		Correct integration and limits and = 1
	$k\ln a = 1 \quad k = 1/\ln a$	A1	[3]	AG
(ii)	$\frac{1}{\ln a}\int_1^a 1\,dx \quad$ or $k\int_1^a 1\,dx$	M1		Int xf(x). Ignore limits
	$= \frac{1}{\ln a}[x]_1^a \quad$ or $k[x]_1^a$	A1		Correct integration and limits (condone missing k)
	$= \frac{1}{\ln a}(a-1)$	A1	[3]	
(iii)	$\frac{1}{\ln a}\int_1^m \frac{1}{x}dx = 0.5$	M1		Int f(x) and equate to 0.5. Ignore limits
	$\frac{1}{\ln a}[\ln x]_1^m = 0.5$	A1		Correct integration and limits (1 to m or m to a) (condone missing k)
	$\frac{1}{\ln a}\ln m = 0.5$			
	$\ln m = 0.5\ln a$	A1		or $\ln m = \ln a^{0.5}$
	$m = \sqrt{a}$	A1	[4]	
		[Total: 10]		

Question 10

(i)	$\frac{1}{2}c^2 = 1$	M1		Area of triangle = 1 or integral of kx with limits 0 and c and equated to 1
	$c = \sqrt{2}$ or 1.41 (3 sf)	A1	[2]	
(ii)	$f(x) = x$ or $y = x$	B1		Seen or implied, e.g. by next line. Can be awarded anywhere in the question. Implied by $(a+1)$ in area of trapezium.
	$\int_a^1 x\,dx = 0.1$	M1		Ignore limits. Must be integral of kx and equated to 0.1. Or trapezium area.
	$\left[\frac{x^2}{2}\right]_a^1 = 0.1$	A1√		Correct limits, ft incorrect kx.
	$1 - a^2 = 0.2$ $a = 0.894$ (3 sf)	A1	[4]	$\sqrt{\left(\frac{4}{5}\right)}$ oe
(iii)	$\int_0^{\sqrt{2}} x^2\,dx$	M1		Ignore limits; ft their f(x) but not $\int x\,dx$
	$\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$			
	$= \frac{2}{3}\sqrt{2}$ **or 0.943 or** $\sqrt{\left(\frac{8}{3}\right)}$	A1√	[2]	ft their c, dep 0 < ans < their c. Not ft their f(x)

Question 11

(a)	$\int_0^{0.5}(1.5t-0.75t^2)\mathrm{d}t$ o.e.	M1	Attempt int f(t)
	$=\left[0.75t^2-0.25t^3\right]_0^{0.5}$ o.e.	A1	Correct integration and limits
	$=\frac{5}{32}$ or 0.156 (3 sf)	A1 3	
(b) (i)	$\frac{1}{2}\pi a^2=1$ or $\pi a^2=2$ oe	M1	Attempt to find the area and equate to 1
	$a=\sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf)	A1 2	
(ii)	0	B1 1	
(iii)	Symmetry stated, seen or implied 0.8	M1 A1 2	Could be a diagram As final answer
		Total: 8	

Question 12

(i)	$\frac{3}{4}\int_0^c(cx-x^2)dx=1$	M1	Attempt integ f(x) and = 1. Ignore limits
	$\frac{3}{4}\left[\frac{cx^2}{2}-\frac{x^3}{3}\right]_0^c=1$	A1	Correct integration and limits (condone c = 2
	$\frac{3}{4}\left(\frac{c^3}{2}-\frac{c^3}{3}\right)=1$ or $\frac{3}{4}\times\frac{c^3}{6}=1$ or $\frac{c^3}{8}=1$	A1 [3]	No errors seen
	($c=2$ **AG**)		
(ii)	Inverted parabola Through (0, 0) and (2, 0) and zero elsewhere Median = 1	B1 B1 B1 [3]	 Must not extend beyond [0,2]
(iii)	$\frac{3}{4}\int_0^{1.5}(2x-x^2)\mathrm{d}x$	M1	Attempt integ f(x) ignore limits
	$=\frac{3}{4}\left[x^2-\frac{x^3}{3}\right]_0^{1.5}$	A1	Correct integration ignore limits
	$\frac{3}{4}\left(1.5^2-\frac{1.5^3}{3}\right)$	B1	Use of correct limits [0,1.5] or 1–[1.5,2]
	$=\frac{27}{32}$ or 0.844 (3 sf)	A1 [4]	
(iv)	$\left(\frac{27}{32}-\frac{1}{2}\text{ or } 0.844-0.5\right)$		
	$=\frac{11}{32}$ or 0.344 (3 sf)	B1f [1]	ft their (iii) For use of symmetry Note If do not use "hence" and start again B1 for cwo
		Total 11	

Question 13

(i)	$k\int_0^{15}(225-t^2)\mathrm{d}t = 1$	M1		Attempt integ f(x) and = 1. Ignore limits
	$k\left[225t-\frac{t^3}{3}\right]_0^{15} = 1$	A1		Correct integration and limits
	$k\times[3375 - 1125\} = 1$ or $k\times2250 = 1$ ($k = \frac{1}{2250}$ **AG**)	A1	3	No errors seen
(ii)	$\frac{1}{2250}\int_{10}^{15}(225-t^2)\mathrm{d}t$	M1		Attempt integ, ignore limits
	$(= \frac{1}{2250}\left[225t-\frac{t^3}{3}\right]_{10}^{15})$	A1		Or $1-\int_0^{10}$ Correct integration and limits. Condone missing k
	$= \frac{1}{2250}[2250-(2250-\frac{1000}{3})]$ $= \frac{4}{27}$ or 0.148 (3 sf)	A1	3	
(iii)	$\frac{1}{2250}\int_0^{15}(225t-t^3)\mathrm{d}t$	M1*		Attempt integ xf(x) , ignore limits
	$= \frac{1}{2250}\left[\frac{225t^2}{2}-\frac{t^4}{4}\right]_0^{15}$	A1		Correct integration and limits. Condone missing k
	$= \frac{1}{2250}[\frac{50625}{2}-\frac{50625}{4}]$	M1*dep		Sub correct limits into their integral
	$= \frac{45}{8}$ or 5.625 or 5.63 (3 sf)	A1	4	Accept 5 mins 37 or 38 secs
	Total	**10**		

Question 14

$\frac{1}{2}a^2 = 1$	M1		or $\int_0^a x\mathrm{d}x = 1$
$a = \sqrt{2}$	A1		Allow 1.41 or better
$\int_0^{\sqrt{2}} x^2\mathrm{d}x$	M1		ignore limits
$= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$	A1f		correct integral and limits, but ft their a
$= \frac{(\sqrt{2})^3}{3}$ = or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428 (= 0.943 **AG**)	A1	[5]	must see this numerical expression, or equiv SR Equating $\int x$ f(x) to 0.943 scores M1 Solving to find a = 1.41 scores A1
	[Total 5]		

Question 15

(i)	$k\int_{-2}^{2}(4-x^2)\mathrm{d}x = 1$	**M1**		attempt Integral f(x) = 1, ignore limits
	$k[4x - \frac{x^3}{3}]_{-2}^{2} = 1$	**A1**		correct integration & limits
	$\left(k\left(8-\frac{8}{3}-\left(-8-\left(-\frac{8}{3}\right)=1\right)\right)\right)$			
	$k \times \frac{32}{3} = 1$ oe Not e.g. $k\times 10.7 = k$			
	$k = \frac{3}{32}$ **AG**	**A1**	[3]	exact answer correctly found
(ii)	Inverted parabola, vertex on y axis	**B1**		parabola must finish on x axis at ± 2, labelled (ignore markings on y axis)
	E(X) = 0	**B1**	[2]	
(iii)	$\frac{3}{32}\int_{-2}^{1}(4-x^2)\mathrm{d}x$	**M1**		or $1 - \frac{3}{32}\int_{1}^{2}(4-x^2)\mathrm{d}x$ ignore limits
	$\frac{3}{32}[4x - \frac{x^3}{3}]_{-2}^{1}$	**A1**		or $1 - \frac{3}{32}[4x - \frac{x^3}{3}]_{1}^{2}$ correct integration and correct limits
	$\frac{3}{32}(4 - \frac{1}{3} - (-8 - (-\frac{8}{3})))$			$= 1 - \frac{3}{32}(8 - \frac{8}{3} - (4 - \frac{1}{3})$
	$= \frac{27}{32}$ or 0.844 (3 sf)	**A1**	[3]	

Question 16

(i)	$k\int_1^2(3-x)dx=1$	**M1**	Attempt $\int f(x)=1$, ignore limits or $\frac{k}{2}(h_1+h_2)=1$
	$k\left[3x-\frac{x^2}{2}\right]_1^2=1$	**A1**	Correct integration & limits or $\frac{k}{2}(2+1)=1$
	$(k(6-2-(3-0.5))=1)$		
	$k\times 1.5=1$ or $k\times\frac{3}{2}=1$ or $k=\frac{1}{1.5}$ oe		
	$k=\frac{2}{3}$ **AG**	**A1** [3]	No errors seen
(ii)	$\frac{2}{3}\int_1^m(3-x)dx=0.5$ oe $\int$ from m to 2	**M1***	Attempt Int $f(x)=0.5$, ignore limits oe Or use of area of trapezium
	$\left(\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_1^m=0.5\right)$		
	$\frac{2}{3}\left[3m-\frac{m^2}{2}-2.5\right]=0.5$	**dep M1***	Sub of correct limits into their integral. Or trapezium using 1 and m/m and 2 Any correct 3-term QE = 0 or $(m-3)^2=2.5$
	$m^2-6m+6.5=0$ oe	**A1**	
	$\left(m=\frac{6\pm\sqrt{36-4\times 6.5}}{2}=1.42 \text{ or } 4.58\right)$		
	$m=1.42$ (3 sf)	**A1** [4]	or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans

Question 17

(a) (i)	$E(X)=1.5$	**B1**	
	$\frac{2}{9}\int_0^3(3x^3-x^4)dx$	**M1**	Attempt integ $x^2f(x)$ ignore limits
	$=\frac{2}{9}\left[\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^3$		
	$=\frac{2}{9}\left[\frac{243}{4}-\frac{243}{5}\right]$ $(=2.7)$	**M1**	Sub correct limits into correct integral
	$\text{Var}(X)\ (=2.7-1.5^2)=0.45$ oe	**A1**√ [4]	Ft their $E(X)$, but no ft for –ve Var.
(ii)	0.5	**B1** [1]	
(iii)	$(1-\frac{13}{27})\div 2$	**M1**	or $\frac{2}{9}\int_2^3(3x-x^2)\,dx$ oe
	$=\frac{7}{27}$ or 0.259	**A1** [2]	As final answer
(b)	$\frac{1}{2}\times 2\times 2a=\frac{1}{2}$ or $\int_0^2 ax\,dx=\frac{1}{2}$	**M1**	Attempt correct equation in 'a'
	$a=\frac{1}{4}$	**A1**	
	$\frac{1}{2}\times b\times\frac{1}{4}b=1$ or $\int_0^b\frac{1}{4}x\,dx=1$ or $b=2\times\sqrt{2}$	**M1**	or $\frac{1}{2}\times b\times ab=1$ or $\int_0^b ax\,dx=1$ attempt correct equation in (a and) b
	$b=2\sqrt{2}$	**A1**√ [4]	Allow $b=\sqrt{8}$ or 2.83 (3 sf) Ft incorrect a, both **M**s needed

Question 18

(i)	$k\int_{5}^{10}(10t-t^2)\mathrm{d}t = 1$	**M1**	Attempt to integrate, ignore limits
	$k\left[5t^2-\frac{t^3}{3}\right]_5^{10}=1$	**A1**	Correct integral and limits
	$k(500-\frac{1000}{3}-(125-\frac{125}{3}))=1$		
	$k\times\frac{250}{3}=1$		
	$(k=\frac{3}{250}$ **AG**)	**A1** [3]	No errors seen; No inexact decimals seen
(ii)	$\frac{3}{250}\int_{5}^{10}(10t^2-t^3)\mathrm{d}t$	**M1**	Attempt to integrate, ignore limits
	$=\frac{3}{250}\left[\frac{10t^3}{3}-\frac{t^4}{4}\right]_5^{10}$	**A1**	Correct integral and limit. Condone missing k
	$=\frac{3}{250}(\frac{10000}{3}-\frac{10000}{4}-(\frac{1250}{3}-\frac{625}{4})$		
	= 6.875 or 55/8	**A1** [3]	Allow 6.88
(iii)	$\mathrm{P}(T<\mathrm{E}(T))=\frac{3}{250}\left[5t^2-\frac{t^3}{3}\right]_5^{\text{"6.875"}}$	**M1***	ft their E(*T*)
	= 0.5361		
	"0.5361" – 0.5	**DM1***	allow 0.036
	P(*T* between E(*T*) & median = 0.0361	**A1** [3]	**Alternative Method** Integrate f(t)limits 5 and m equated to 0.5 M1* Integrate f(t)limits their 6.736 (provided between 5 and 10) and their 6.875DM1 Allow without "minutes"
(iv)	10 (minutes)	**B1** [1]	

Question 19

(a)	0.3 or 1 – 0.6 or 0.4 or 0.2 seen 0.8	**M1** **A1** [2]	
(b) (i)	$k\int_0^{1.5}(2.25-x^2)\mathrm{d}x = 1$	**M1**	attempt integ f(x) and '= 1'. Ignore limits
	$k\left[2.25x-\frac{x^3}{3}\right]_0^{1.5} = 1$	**A1**	correct integration and limits
	$k\times[3.375-1.125] = 1$ or $k\times\frac{9}{4} = 1$ oe		
	$k = \frac{4}{9}$ **AG**	**A1** [3]	No errors seen
(ii)	$\frac{4}{9}\int_0^{1.5}(2.25x-x^3)\mathrm{d}x$	**M1**	attempt integ xf(x), ignore limits, condone missing k
	$=\frac{4}{9}\left[2.25\frac{x^2}{2}-\frac{x^4}{4}\right]_0^{1.5}$	**A1**	correct integration and limits, condone missing k
	= 0.5625 or 0.563	**A1**	
	Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins	**A1**√ [4]	ft their 0.5625
(iii)	Max x is 1.5, less than 2.9 or 150 < 290	**B1** [1]	Needs numerical justification
(iv)	any a such that $2.9 \leqslant a \leqslant 5$	**B1** [1]	

Question 20

(i)	2 m	**B1** [1]	allow without units
(ii)	$k\int_0^2 x^2(2-x)\mathrm{d}x = 1$	**M1**	attempt integ f(x) and '= 1'. Ignore limits
	$k\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_0^2$	**A1**	correct integration and limits
	$k\times\left[\frac{16}{3}-4\right] = 1$ or $k\times\frac{4}{3} = 1$ oe		
	$k = \frac{3}{4}$ **AG**	**A1** [3]	No errors seen
(iii)	$\frac{3}{4}\int_0^2 x^3(2-x)\mathrm{d}x$	**M1**	attempt integ xf(x), condone missing k
	$=\frac{3}{4}\times\left[\frac{2x^4}{4}-\frac{x^5}{5}\right]_0^2$	**A1**	correct integration and limits, condone missing k
	1.2 m oe	**A1** [3]	allow without units
(iv)	$\frac{3}{4}\int_0^1 x^2(2-x)\mathrm{d}x$	**M1**	attempt integ f(x), 0 to 1, condone missing k
	$(=\frac{3}{4}\times(\frac{2}{3}-\frac{1}{4}))$ $=\frac{5}{16}$ or 0.3125 oe	**A1**	
	$400\times\frac{5}{16} = 125$	**A1 ft** [3]	ft their $\frac{5}{16}$

Question 20

(a)		0.3 or 1 – 0.6 or 0.4 or 0.2 seen	**M1**	
		0.8	**A1** [2]	
(b)	**(i)**	$k\int_0^{1.5}(2.25-x^2)\mathrm{d}x = 1$	**M1**	attempt integ f(x) and '= 1'. Ignore limits
		$k\left[2.25x-\frac{x^3}{3}\right]_0^{1.5} = 1$	**A1**	correct integration and limits
		$k\times[3.375-1.125]=1$ or $k\times\frac{9}{4}=1$ oe		
		$k=\frac{4}{9}$ **AG**	**A1** [3]	No errors seen
	(ii)	$\frac{4}{9}\int_0^{1.5}(2.25x-x^3)\mathrm{d}x$	**M1**	attempt integ xf(x), ignore limits, condone missing k
		$=\frac{4}{9}\left[2.25\frac{x^2}{2}-\frac{x^4}{4}\right]_0^{1.5}$	**A1**	correct integration and limits, condone missing k
		= 0.5625 or 0.563	**A1**	
		Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins	**A1**√ [4]	ft their 0.5625
	(iii)	Max x is 1.5, less than 2.9 or 150 < 290	**B1** [1]	Needs numerical justification
	(iv)	any a such that $2.9 \leqslant a \leqslant 5$	**B1** [1]	

(i)	$\sigma_X, \sigma_Z, \sigma_Y, \sigma_W$ **or** X, Z, Y, W	**B2**	[2]	B1 if two adjacent sds interchanged, ie $\sigma_Z, \sigma_X, \sigma_Y, \sigma_W$ **or** $\sigma_X, \sigma_Y, \sigma_Z, \sigma_W$ **or** $\sigma_X, \sigma_Z, \sigma_W, \sigma_Y$ B1 for correct order reversed
(ii) (a)	Mean = 0 stated or found or "– 0" seen	**B1**		
	$\frac{1}{18}\int_{-3}^{3} x^4 \mathrm{d}x - 0$ $= \frac{1}{18}\left[\frac{x^5}{5}\right]_{-3}^{3}$ $= \frac{1}{18}\left[\frac{3^5}{5} + \frac{3^5}{5}\right]$ oe $= 5.4$	**M1**		Attempt integral x^2 f(x). Ignore limits Allow without "– 0"
	sd = √5.4 or $\sqrt{\frac{1}{18}\left[\frac{3^5}{5} + \frac{3^5}{5}\right]}$ or 2.324 sd = 2.32 (3 sf) AG	**A1**	[3]	Must see $\sqrt{\text{correct expression or } 5.4}$ or 2.324 or better
(b)	$\frac{1}{18}\int_{'2.324'}^{3} x^2 \mathrm{d}x$	**M1**		Attempt to integrate f(x), ignore limits
	$\frac{1}{18}\left[\frac{x^3}{3}\right]_{'2.324'}^{3} = \frac{1}{18}\left[\frac{3^3}{3} - \frac{'2.324'^3}{3}\right]$	**A1**		Sub correct limits into correct integral
	= 0.268 (3 sf)	**A1**	[3]	Allow 0.269
(c)	0	**B1**	[1]	

Question 22

(i)	m_X, m_Y, m_Z, m_W or X, Y, Z, W	B2	[2]	B1 if two adjacent means interchanged, i.e. m_Y, m_X, m_Z, m_W or m_X, m_Z, m_Y, m_W or m_X, m_Y, m_W, m_Z B1 for correct order reversed.
(ii) (a)	$\int_0^3 \frac{4}{81}x^4 \mathrm{d}x$	M1		Attempt int $x\mathrm{f}(x)$. Ignore limits
	$= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$	A1		Correct integration and limits (condone missing 4/81)
	$= \frac{4}{81} \times \frac{3^5}{5}$ or $\frac{4}{81} \times \frac{243}{5}$ or $\frac{972}{405}$ oe			Must see correct expression as well as $\frac{12}{5}$ or 2.4
	$= \frac{12}{5}$ or 2.4 AG	A1	[3]	No errors seen
(b)	$\int_{2.4}^3 \frac{4}{81}x^3 \mathrm{d}x$ or $1 - \int_0^{2.4} \frac{4}{81}x^3 \mathrm{d}x$	M1		Attempt int $\mathrm{f}(x)$ ignore limits
	$= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$	A1		Correct integration and limits (condone missing 4/81)
	$= 1 - \frac{4}{81} \times \frac{2.4^4}{4}$ oe			
	$= \frac{369}{625}$ or 0.59(0) (3 sf)	A1	[3]	As final answer
(c)	1	B1	[1]	

Question 23

(a)(i)	$k = 1$	B1	
	Total:	1	
(a)(ii)	f_2: area > 1 (area ≠ 1)	B1	oe
	f_3: includes negative values of f_3	B1	oe
	Total:	2	
(b)(i)	$6\int_{-a}^{a} (a^2 - x^2)\mathrm{d}x = 1$	M1	Integ $\mathrm{f}(x) = 1$, ignore limits
	$6\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = 1$	A1	Correct integral and limits
	$6(2a^3 - \frac{2a^3}{3}) = 1$ $\frac{24a^3}{3} = 1$ or $8a^3 = 1$ $a = 1/2$ **AG**	A1	Correctly obtained. No errors seen. (SR Verification scores M1A1 only max 2/3)

(b)(ii)	0	**B1**	
	Total:	**1**	
(b)(iii)	$6\int_{-0.5}^{0.5}(\frac{x^2}{4}-x^4)dx$ $(=6\left[\frac{x^3}{12}-\frac{x^5}{5}\right]_{-0.5}^{0.5}=0.05)$ Var $= 0.05 - 0^2$	**M1**	attempt int x^2f(x) & correct limits
	$= 0.05$ oe	**A1**	cao; allow omission of $- 0^2$

Question 24

5(i)	$k\int_0^1(x-x^2)\,dx=1$	**M1**	Attempt integ f(x) and "= 1", ignore limits
	$=k\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=1$	**A1**	correct integration, limits 0 and 1
	$=k\left[\frac{1}{2}-\frac{1}{3}\right]=1$ or $\frac{k}{6}=1$	**A1**	correctly obtained, no errors seen
	Total:	**3**	
(ii)	E(X) = 0.5	**B1**	
	$6\int_0^1(x^3-x^4)\,dx$	**M1**	Attempt integ x^2f(x), limits 0 to 1
	$(=6\left[\frac{1}{4}-\frac{1}{5}\right]=0.3)$ "0.3" – "0.5"^{2}	**M1**	their int x^2f(x) – their (E(X))^2 dep +ve result
	$= 0.05 (= 1/20)$	**A1**	
	Total:	**4**	
(iii)	$6\int_{0.4}^{1}(x-x^2)\,dx$	**M1**	ignore limits, eg **M1** for $6\int_{0.4}^{2}(x-x^2)\,dx$
	$=6\{\frac{1}{2}-\frac{1}{3}-(\frac{0.4^2}{2}-\frac{0.4^3}{3})\}$	**A1FT**	subst correct limits into correct integration
	$= 0.648 (= 81/125)$	**A1**	condone incorrect "k" for **A1**

Question 25

(i)	$0.5\times1\times h=0.25$ $h = 0.5$ grad = 0.5	M1	$P(X<2) = 4 \times P(X<1)$ **M1**
	$f(x) = 0.5x$	A1	$P(X<2) = 1$ **A1** $a = 2$ **A1**
	$0.5 \times a \times 0.5a = 1$	M1	$0.5 \times 2 \times h' = 1$ **M1** $h' = 1$
	$a = 2$	A1	grad = 0.5
	$P(X<2) = 1$	A1	$f(x) = 0.5x$ **A1**
	Total:	**5**	
(ii)	$\int_0^m 0.5x\,dx = 0.5$	M1	Attempt $\int f(x)dx = 0.5$ Ignore limits
	$=\left[\frac{x^2}{4}\right]_0^m = 0.5$	A1FT	Correct integration (ft f(x)) & limits = 0.5
	$m=\sqrt{2}$ or 1.41 (3 sf)	A1	or by similarity $m=\frac{1}{\sqrt{2}}\times 2$ **M2** $=\sqrt{2}$ **A1**

Question 26

(i)	Greater area where $x<7.5$ than $x>7.5$	B1	Allow Graph higher for $x<7.5$ than for $x>7.5$ or Graph decreasing or equiv expl'n
	Total:	**1**	
(ii)	$\int_5^{10} \frac{k}{x^2}\,dx = 1$	M1	Attempt Integ $f(x) = 1$ ignore limits
	$k\left[-\frac{1}{x}\right]_5^{10} = 1$ $k \times \frac{1}{10} = 1$	A1	Correct integration and limits
	$k = 10$ AG	A1	No errors seen
(iii)	$10\int_5^{10} \frac{1}{x}\,dx$	M1	Attempt Integ $xf(x)$ ignore limits
	$= 10[\ln x]_5^{10}$ $= 10(\ln10 - \ln5)$	M1	Correct integration and limits
	$= 10\ln2$ or 6.93 (3 sf)	A1	OE
	Total:	**3**	
(iv)	$10\int_5^{10} 1\,dx - \text{"}6.93\text{"}^2$	M1	Attempt (Integ $x^2f(x)$) – $(E(x))^2$. No limits **M0**
	= 1.95 (accept 1.96)	A1	*Use of 6.93 gives 1.97* **A0**

Question 27

(i)	$\frac{1}{4}\int_0^2 (x^2+x)\,dx$ $\quad(= \frac{1}{4}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^2)$	M1	Attempt integ xf(x), ignore limits
	$= \frac{1}{4}(\frac{8}{3}+2)\quad(-\ 0)$	A1	Subst correct limits in correct integration
	$= \frac{7}{6}$ OE or 1.17 (3 sf)	A1	
		3	
(ii)	$\frac{1}{4}\int_0^m (x+1)\,dx = 0.5\quad(=\frac{1}{4}\left[\frac{x^2}{2}+x\right]_0^m = 0.5)$	M1	attempt integ f(x), limits 0 to unknown (or unknown to 2) and = 0.5
	$\frac{1}{4}(\frac{m^2}{2}+m)=0.5$ $m^2+2m-4=0$ $m = \frac{-2\pm\sqrt{4+16}}{2}$ OE	A1	a correct equation in m (any form) or $\sqrt{5}-1$
	$m = 1.24$	A1	must reject the negative value if there

Question 28

(i)	$k\int_0^a \frac{1}{\sqrt{x}}dx = 1$	M1	Attempt int f(x) and = 1 ignore limits
	$(2k[x^{0.5}]_0^a = 1)$ $2ka^{0.5}=1$ $\quad$ or $a = \frac{1}{4k^2}$	A1	OE; a correct eqn in k & a after sub limits
	$k\int_0^a \frac{x}{\sqrt{x}}dx = 3$	M1	Attempt int xf(x) and = 3
	e.g. $\frac{2}{3}ka^{1.5}=3$ $\quad$ or $a^3 = \frac{81}{4k^2}$	A1	OE; a correct eqn in k and a after sub limits
	e.g. $a^2 = 81$ $\quad$ or e.g. $k^2 = \frac{81}{4\times 9^3}$	M1	Attempt eliminate one letter
	$a = 9$	A1	Convincingly obtained
	e.g. $k = \frac{9}{54}$ $k = \frac{1}{6}$ $\quad$ AG	A1	
(ii)	$\frac{1}{6}\int_0^m \frac{1}{\sqrt{x}}dx = 0.5$ OE	M1	Attempt int f(x), unknown limit and = 0.5
	$\frac{1}{3}m^{0.5}=0.5$	A1	a correct equn in m after sub limits
	$m = 2.25$	A1	

Question 29

(i)	$1 - 6\int_{0.3}^{0.7}\left(x - x^2\right)dx$	M1	or $2 \times 6\int_{0}^{0.3}\left(x - x^2\right)dx$ or similar correct expression before integration
	$1 - \left[6(\frac{x^2}{2} - \frac{x^3}{3})\right]_{0.3}^{0.7}$	A1	or similar correct expression after integration
	$1 - 6\left[\frac{0.7^2}{2} - \frac{0.7^3}{3} - \frac{0.3^2}{2} + \frac{0.3^3}{3}\right]$	M1	Attempt subst correct limits in this or other correct expression
	= 0.432 (or 54/125)	A1	(SR1 Omission of '1–' scores **B2** for 0.568 or 71/125) (SR2 Omission of '2x' scores **B2** for 0.216 or 27/125)
		4	
(ii)	Correct shape between $x = 0$ and 1	B1	No curve outside this range.
	$E(X) = 0.5$	B1	
		2	
(iii)	$6\int_0^1\left(x^3 - x^4\right)dx$ $= \left[6\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\right]_0^1$	M1	attempt int $x^2f(x)$, ignore limits
	$6[\frac{1^4}{4} - \frac{1^5}{5}]$ (= 0.3)	M1	attempt subst correct limits in correct integ
	$Var(X) = '0.3' - '0.5'^2$ = 0.05	A1FT	FT their mean, dep their $Var(X) > 0$

Question 30

(i)	$k\int_1^2 (\frac{1}{x^2}+\frac{1}{x^3})\,dx = 1$	M1	Attempt integ f(x) & '= 1'; ignore limits
	$k\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^2 = 1$	A1	Correct integral & limits & '= 1'
	$k\left[-\frac{1}{2}-\frac{1}{8}+1+\frac{1}{2}\right] = 1$ $k = \frac{8}{7}$ **AG**	A1	Sufficient working must be shown, no errors seen
		3	
(ii)	$\frac{8}{7}\int_1^2 (\frac{1}{x}+\frac{1}{x^2})\,dx$	M1	Attempt integ xf(x), ignore limits
	$= \frac{8}{7}\left[\ln x - \frac{1}{x}\right]_1^2$	A1	Correct integral & limits, condone missing k
	$= \frac{8}{7}(\ln 2 + \frac{1}{2})$ or 1.36 (3 sf)	A1	
		3	
(iii)	$\frac{8}{7}\int_1^{1.5} (\frac{1}{x^2}+\frac{1}{x^3})\,dx$ $= \frac{8}{7}\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^{1.5}$	M1	Attempt integration f(x) between 1 and 1.5 or between 1.5 and 2
	$= \frac{44}{63}$ or 0.698.......	A1	Or $\frac{19}{63}$ or 0.302
	'$\frac{44}{63}$' $(1 -$ '$\frac{44}{63}$'$)^2$	M1	FT their $\frac{44}{63}$
	× 3	M1	Independent provided answer is <1
	= 0.191	A1	
		5	

Question 31

(i)	$\frac{1}{2} \times a \times b = 1$	M1	Attempt Δ area = 1 or $\int(b - bx/a)\,dx = 1$ with correct limits
	$b = \frac{2}{a}$	A1	
		2	
(ii)	grad $= -\frac{2}{a^2}$ or $-\frac{b}{a}$	B1	allow without '–' sign (could be implied or seen in **(i)**)
	$y - (\frac{2}{a}) =$ grad × x or $y =$ grad × $(x - a)$	M1	correct use of $y = mx + c$ or $y - y_1 = m(x - x_1)$ with (0,b) or (a,0) including attempt at substitution of their b
	$y - (\frac{2}{a}) = -\frac{2}{a^2}x$ or $y = -\frac{2}{a^2}(x - a)$ and $y = \frac{2}{a} - \frac{2}{a^2}x$ **AG**	A1	No errors seen
(iii)	$\int_0^a (\frac{2}{a}x - \frac{2}{a^2}x^2)\,dx$	M1	Attempt int xf(x) ignore limits
	$= [\frac{1}{a}x^2 - \frac{2}{3a^2}x^3]_0^a$	A1	Correct integration ignore limits
	$a - \frac{2}{3}a = 0.5$	M1	Sub correct limits into their integral and = 0.5
	$a = 1.5$	A1	

Question 32

(i)	$\int_5^{10} \frac{k}{x^2}\,dx = 1$	**M1**	Attempt integration f(x) and '= 1'; ignore limits
	$\left[-\frac{k}{x}\right]_5^{10} = 1$ oe $(\frac{k}{5} - \frac{k}{10} = 1)$	**A1**	Correct integration and limits and '= 1'
	$k = 10$ **AG**	**A1**	No errors seen
		3	
(ii)	$10\int_5^{10} \frac{1}{x}\,dx$ $10\left[\ln x\right]_5^{10}$	**M1**	Attempt integ xf(x); ignore limits. or 10(ln 10 – ln 5)
	= 10ln 2 **AG**	**A1**	No errors seen
		2	
(iii)	$10\int_9^{10} \frac{1}{x^2}\,dx$ $(10\left[-\frac{1}{x}\right]_9^{10})$	**M1**	Attempt integ f(x) with correct limits
	$10\left[-\frac{1}{10} + \frac{1}{9}\right]$	**A1**	Substitute correct limits in correct integration
	$= \frac{1}{9}$ or 0.111 (3 sf)	**A1**	
		3	
(iv)	$\int_5^{a} \frac{k}{x^2}\,dx = 0.6$ $10\left[-\frac{1}{x}\right]_5^{a} = 0.6$	**M1**	Attempt integration of f(x) with correct limits and = 0.6
	$10[\frac{1}{5} - \frac{1}{a}] = 0.6$	**A1**	Substitute correct limits in correct integration
	$a = \frac{50}{7}$ or 7.14 (3 sf)	**A1**	
		3	

Question 33

(i)	$k\int_2^6 x^{-1}\mathrm{d}x = 1$	**M1**	Attempt integrate f(x) & = 1. Ignore limits
	$k[\ln x]_2^6 = 1$ $k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$ $k = \frac{1}{\ln 3}$ **AG**	**A1**	correct sub of correct limits in correct integral leading to correct ans. No errors seen.
		2	
(ii)	$\frac{1}{\ln 3}\int_2^6 1\,\mathrm{d}x$	**M1**	Attempt integ xf(x). Ignore limits
	$= \frac{1}{\ln 3}[x]_2^6 \quad (= \frac{1}{\ln 3}(6-2))$	**A1**	Correct integral and limits
	$= \frac{4}{\ln 3} = 3.64$ **AG**	**A1**	No errors seen
		3	
(iii)	$P(X < E(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$	**M1**	Attempt integ f(x) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe
	$= \frac{1}{\ln 3}[\ln x]_2^{3.64}$ $= \frac{1}{\ln 3}(\ln 3.64 - \ln 2) \quad (= 0.545)$	**A1**	Correct sub correct limits into correct integral
	$P(m < X < E(X)) =$ "0.545" – 0.5	**M1**	Subt 0.5 from their $P(X < E(X))$ art 0.045 . ft their $P(X < E(X)$ (> 0.5)
	= 0.045 (2 sfs)	**A1**	equivalent method M1 method for median-need 0.5 and limits 2 to m or m to 6 A1 sqrt 12 or 3.464 M1 calc area from "3.464" to 3.64 A1 0.045 or better, not 0.046
		4	

Question 34

(i)	$\int_0^a \frac{k}{(x+1)^2}dx = 1$	M1	Any attempt integ f(x) and = 1. Ignore limits
	$-\left[\frac{k}{(x+1)}\right]_0^a = 1$ $-k(\frac{1}{a+1} - 1) = 1$	M1	Attempt subst correct limits into correct integral
	$k \times \frac{a}{a+1} = 1$ and $k = \frac{a+1}{a}$ **AG**	A1	No errors seen
		3	
(ii)	Max time allowed by model (for runners to finish)	B1	Allow: All runners finish in time a or less or Longest time (taken by any runner) oe
		1	
(iii)	$\frac{a+1}{a}\int_0^{0.5} \frac{1}{(x+1)^2}dx = \frac{3}{4}$	M1	Attempt integ f(x) and $= \frac{3}{4}$; ignore limits oe. Condone missing / incorrect k
	$-\frac{a+1}{a}\left[\frac{1}{(x+1)}\right]_0^{0.5} = \frac{3}{4}$ $-\frac{a+1}{a}(\frac{2}{3} - 1) = \frac{3}{4}$	M1	Attempt subst correct limits into correct integral. Condone missing / incorrect k
	$a = 0.8$ oe	A1	
		3	

Question 35

(i)	$\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx$ $= \sqrt{2}\,[\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1	Attempt integ f(x) with correct limits
	$= \frac{2-\sqrt{2}}{2}$ oe or 0.293 (3 sf)	A1	SC Final answer of 0.707 scores B1sc
		2	
(ii)	$\sqrt{2}\int_0^m \cos x dx = 0.5$	M1	Attempt to integ f(x) & = 0.5. Ignore limits. Condone missing √2
	$\sqrt{2}\,[\sin x]_0^m = 0.5$ $\sqrt{2}\,\sin m = 0.5$	A1	Correct integral and limits 0 to unknown & = 0.5 Condone missing √2
	$\sin m = \frac{1}{2\sqrt{2}}$ oe	M1	For rearranging their expression to the form sin$m$=… (sin$m$ = 0.35355.... or 0.354) seen or implied
	$m = 0.361$ (3 sfs)	A1	No errors seen (Note 20.705 can score M1 A1 M1 A0)
		4	

'(iii)	$\sqrt{2}\int_0^{\frac{\pi}{4}} x\cos x\,dx$	M1	Attempt to integ xf(x). Ignore limits. Condone missing √2
	$= \sqrt{2}\ \{[x(\sin x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x\,dx\ \}$	M1	Attempt to integ by parts leading to expression of form $\pm x\sin x \pm \cos x$ with correct limits
	$= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} - 0 - [-\cos x]_0^{\frac{\pi}{4}}\}$	A1	For $\sqrt{2}(x\sin x - (-\cos x))$ with correct limits
	$= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} + \cos\frac{\pi}{4} - 1\}$ $= \frac{\pi}{4} + 1 - \sqrt{2}$ oe or 0.371 (3 sf)	A1	
		4	

Question 36

·(i)	$\frac{3}{a^3}\int_0^a x^2dx$ $(= \frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^a)$	M1	Attempt to integrate f(x) with limits 0 and a (condone missing $\frac{3}{a^3}$)
	$= \frac{3a^3}{3a^3}$	A1	$\frac{3a^3}{3a^3} - 0$ or better seen
	= 1 Hence f is pdf for all a	A1	Answer = 1 and comment
		3	
(ii)	$\frac{3}{a^3}\int_0^2 x^2dx = 0.5$ $\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^2 = 0.5$	M1	Attempt to integrate f(x)=0.5, limits 0 and 2 oe, condone missing $\frac{3}{a^3}$
	$\frac{3}{a^3} \times \frac{8}{3} = 0.5$ oe	A1	$\frac{2^3}{3} - 0$ or better, condone missing $\frac{3}{a^3}$
	$a^3 = 16$ or $a = \sqrt[3]{16}$ (= 2.52 **AG**)	A1	Convincingly obtained Note: Attempt to verify 2.52, M1 as stated except not equated to 0.5.A1 as stated, A1 for evaluation to 0.499..apprx 0.5
		3	
·(iii)	$\frac{3}{16}\int_0^{2.52} x^3dx$ or $\frac{3}{16}\int_0^a x^3dx$ $= \frac{3}{16}\left[\frac{x^4}{4}\right]_0^{2.52}$ or $\frac{3}{16}\left[\frac{x^4}{4}\right]_0^a$	M1	Attempt integ xf(x), correct limits, condone missing $\frac{3}{a^3}$
	$= \frac{3}{16} \times \frac{40.317}{4}$	A1	$\frac{2.52^4}{4} - 0$ or better, condone missing $\frac{3}{a^3}$
	= 1.89 (3 sf)	A1	
		3	

Question 37

(i)	$a\int_1^b \frac{1}{x^2}dx = 1$	**M1**	Attempt int f(x) and = 1, ignore limits
	$a[-\frac{1}{x}]_1^b = 1$	**A1**	correct integ and limits = 1
	$a[1-\frac{1}{b}] = 1$ or $a\times\frac{b-1}{b} = 1$ $b = \frac{a}{a-1}$ **AG**	**A1**	No errors seen
		3	
(ii)	$a\int_1^{\frac{3}{2}} \frac{1}{x^2}dx = \frac{1}{2}$ $a[-\frac{1}{x}]_1^{\frac{3}{2}} = \frac{1}{2}$	**M1**	Attempt int f(x) with limits 1 to $\frac{3}{2}$ and = $\frac{1}{2}$
	$a[1-\frac{2}{3}] = \frac{1}{2}$	**A1**	oe correct equn in a
	$a = \frac{3}{2}$, $b = 3$	**A1**	Both
		3	
(iii)	$\frac{3}{2}\int_1^3 \frac{1}{x}dx$	**M1**	Attempt int xf(x), ignore limits – condone missing a
	$= \frac{3}{2}\ [\ln x]_1^3$	**A1**	**FT** Correct integ and *their* limits 1 to b – condone missing a
	$= \frac{3}{2}\ln 3$ or 1.65 (3 sf)	**A1**	**FT** *their* a and b (valid b i.e. >1)
		3	

Question 38

(a)(i)	$0.5 \times 1/a = (\frac{0.5}{a})$	**M1**	Or attempt to integrate $f(x)$ $(=1/a)$ between 0 and 0.5
	$= \frac{1}{2a}$ oe	**A1**	Accept 0.5/a for A1
		2	
(a)(ii)	$\frac{a}{2}$	**B1**	
		1	
(a)(iii)	$\int_0^a \frac{x^2}{a}\,dx - (\text{'}\frac{a}{2}\text{'})^2$	**M1**	Integ their $x^2f(x)$ from 0 to a and sub their mean^2
	$\text{Var}(X) = \frac{a^2}{3} - \frac{a^2}{4}$ $(\text{Var}(X) = \frac{a^2}{12}$ **AG**)	**A1**	Must see this line oe
		2	

(b)	$\int_2^b \frac{3}{2(t-1)^2}\,dt$	**M1**	Attempt integ $g(t)$ ignore limits
	$\left[-\frac{3}{2(t-1)}\right]_2^b$	**A1**	Correct integral
	$-\frac{3}{2}(\frac{1}{(b-1)} - 1) = \frac{3}{4}$ $(1 - \frac{1}{(b-1)} = \frac{1}{2})$	**M1**	Attempt subst correct limits in their integ and $= \frac{3}{4}$
	$b = 3$	**A1**	
		4	

Question 39

(i)	$k\int_0^3 (3x - x^2)\,dx = 1$	M1	Attempt to integrate f(x) and = 1
	$k\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3$ $k\left(\frac{27}{2} - \frac{27}{3}\right) = 1$	A1	Correct integral and limits
	$k = \frac{2}{9}$	A1	AG No errors seen
		3	
(ii)	$\frac{2}{9}\int_1^2 (3x - x^2)\,dx = \frac{2}{9}\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_1^2 = \frac{2}{9} \times \left(6 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3}\right)$	M1	Attempt to integrate f(x) dx with limits 1 and 2 OE
	$\frac{13}{27}$ or 0.481 (3 sf)	A1	
		2	
(iii)	$y = 3x - x^2$ symmetrical about $x = \frac{3}{2}$	M1	Attempt $\frac{2}{9}\int_0^3 (3x^2 - x^3)\,dx$
	$E(X) = \frac{3}{2}$	A1	
	$\frac{2}{9}\int_0^3 (3x^3 - x^4)\,dx$	M1	Attempt to integrate x^2f(x)
	$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3 \left(= \frac{2}{9} \times \frac{243}{20} = \frac{27}{10}\right)$ $\frac{'27'}{10} - \left(\frac{'3'}{2}\right)^2$	M1	Subtract their $(E(X))^2$ from their integral x^2f(x) with correct limits substituted
	$\frac{9}{20}$ or 0.45	A1	
		5	

Question 40

(i)	$\frac{1}{2}\times a\times\frac{a}{2}=1$ **or** $\frac{1}{2}\int_0^a x\,dx=1$ $\frac{a^2}{4}=1$ OE	**M1**	Attempt at triangle area or integral f(x) **and** = 1,
	$a=2$	**A1**	
		2	
(ii)	$\frac{1}{2}\int_0^2 x^2\,dx$	**M1**	Attempt integral xf(x)
	$=\left[\frac{x^3}{6}\right]_0^2$	**M1**	Correct integral and limits 0 to their 'a'
	$\left(=\frac{8}{6}\right)=\frac{4}{3}$	**A1**	AG CWO
		3	

(iii)	$P\left(X<\frac{4}{3}\right)=\frac{1}{2}\int_0^{\frac{4}{3}} x\,dx$	**M1**	Attempt integral f(x) between correct limits
	$=\frac{4}{9}$	**A1**	or $\frac{5}{9}$
	$P(E(X)<X<m)=\frac{1}{2}-\text{'}\frac{4}{9}\text{'}$	**M1**	or $\frac{5}{9}-\frac{1}{2}$
	$\frac{1}{18}$	**A1**	
	Alternative method for question 4(iii)		
	Attempt to find m	**M1**	
	$m=\sqrt{2}$	**A1**	
	Integrate f(x) between $\frac{4}{3}$ and '$\sqrt{2}$'	**M1**	
	$\frac{1}{18}$	**A1**	
		4	

Question 41

(a)	$\frac{3}{4000}\int_5^{10}(100-x^2)\mathrm{d}x$ $=\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_5^{10}$	**M1**	Attempt integration of f(x), ignore limits. Condone omission of $\frac{3}{4000}$
	$=\frac{3}{4000}\left(1000-\frac{1000}{3}-500+\frac{125}{3}\right)$	**M1**	Correct limits 5 and 10. OE SOI
	$= 0.156$ (3 sf) or $\frac{5}{32}$	**A1**	For fully correct working seen including substitution of limits
		3	

(b)	$\frac{3}{4000}\int_p^{10}(100-x^2)\mathrm{d}x = \frac{1}{4}$	**M1**	Attempt integration of f(x) with any limits and $=\frac{1}{4}$ or $=\frac{3}{4}$ seen. Condone omission of $\frac{3}{4000}$
	$\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_p^{10} = \frac{1}{4}$	**A1**	Correct integration with correct limits seen (or implied for limits p and 10) and $=\frac{1}{4}$ OE Condone omission of $\frac{3}{4000}$
	$\frac{3}{4000}\left(1000-\frac{1000}{3}-100p+\frac{p^3}{3}\right)=\frac{1}{4}$	**M1**	Attempt substitution correct limits in their integration of f(x). Accept limits 0 to p if clearly seen, accept limits -10 and p. Substitution must be seen.
	e.g. $\frac{2000}{3}-100p+\frac{p^3}{3}=\frac{1000}{3}$ $p^3-300p+1000=0$	**A1**	AG No errors seen
		4	

(c)	Curve is symmetrical about $x = 0$	**B1**	May be implied by sketch. No contradictions or integrate f(x) between $-q$ and $+q$ and equate to 0.5 leading to $q^3 - 300q + 1000 = 0$ oe
	$q = 3.47$	**B1**	
		2	

Question 42

(a)	7.5	B1
		1
(b)	$\frac{6}{125}\int_{5}^{10}(-x^4+15x^3-50x^2)\mathrm{d}x$	M1
	$\frac{6}{125}\left[-\frac{x^5}{5}+15\frac{x^4}{4}-50\frac{x^3}{3}\right]_5^{10}-'7.5'^2$	M1
	1.25 (3 sf)	A1
		3

(c)	$\frac{6}{125}\int_{5}^{6}(-x^2+15x-50)\mathrm{d}x$ $\frac{6}{125}\left[-\frac{x^3}{3}+15\frac{x^2}{2}-50x\right]_5^6$	M1
	$\frac{6}{125}(-102+\frac{625}{6})$ oe	M1
	0.104	A1
	$2\times('0.104'\times(1-'0.104')$	M1
	0.186 (3 sf)	A1ft
		5

Question 43

(a)	$\int_1^a \frac{k}{x^2}\,dx = 1$	M1
	$k\left[-\frac{1}{x}\right]_1^a = 1$ $k\left[1-\frac{1}{a}\right] = 1$	A1
	$k\left[\frac{a-1}{a}\right] = 1$ $\left(k = \frac{1}{a-1}\right)$ **AG**	A1
		3
(b)	$\frac{a}{a-1}\int_1^a \frac{1}{x}\,dx$	M1
	$\frac{a}{a-1}[\ln x]_1^a$	A1
	$\frac{a\ln a}{a-1}$	A1
		3

(c)	$\frac{a}{a-1}\int_1^m \frac{1}{x^2}\,dx = \frac{3}{5}$	M1
	$\frac{a}{a-1}\left[-\frac{1}{x}\right]_1^m = \frac{3}{5}$ $\frac{a}{a-1}\left[1-\frac{1}{m}\right] = \frac{3}{5}$	A1
	$\frac{1}{m} = 1 - \frac{3(a-1)}{5a}$ or $\frac{1}{m} = \frac{2a+3}{5a}$	A1
	$m = \frac{5a}{2a+3}$	A1
		4

Question 44

(a)	'Tails down' parabola only from $x = 0$ to 20 shown	**B1**
		1
(b)	Symmetrical	**B1**
		1
(c)	$\frac{3}{4000}\int_0^{20}\left(20t^3 - t^4\right)\mathrm{d}x = \frac{3}{4000}\left[20\frac{t^4}{4} - \frac{t^5}{5}\right]_0^{20}$	**M1**
	$\mathrm{Var}(T) = \frac{3}{4000}\times 160000 - 10^2$	**M1**
	20	**A1**
		3
(d)	$(p-0.5)\times 2$ or $1-2(1-p)$	**M1**
	$2p-1$	**A1**
		2
(e)	$\frac{3}{4000}\int_8^{12}\left(20t - t^2\right)\mathrm{d}x$	**M1**
	$\frac{3}{4000}\left[20\frac{t^2}{2} - \frac{t^3}{3}\right]_8^{12} = \frac{3}{4000}\left(1440 - 576 - 640 + \frac{512}{3}\right)$	**A1**
	$\frac{37}{125}$ or 0.296	**A1**
		3
(f)	Does not allow times greater than 20 minutes	**B1**
		1

Question 45

(a)	$(k =)\ \frac{1}{a}$	**B1**
		1
(b)	(Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$	**B1 FT**
	$\frac{1}{a}\int_0^a x^2\,dx\left(= \frac{a^2}{3}\right)$	**M1**
	$-\left(\frac{a}{2}\right)^2\left(= \frac{a^2}{12}\right)$	**M1**
	$\left(\frac{a^2}{12} = 3\right)\ a = 6$	**A1**
		4

Question 46

(a)	$\frac{1}{2} \times 3 \times c = 1$ ($c = \frac{2}{3}$ **AG**)	**B1**
		1
(b)	$\left(\frac{1}{3}\right)^2$	**M1**
	$= \frac{1}{9}$ or 0.111(3sf)	**A1**
		2

(c)	Equation of line is $y = \frac{2}{3} - \left(\frac{2}{3} \div 3\right)x$	***M1**
	$\mathrm{E}(X) = \int_0^3 \left(\frac{2}{3}x - \frac{2}{9}x^2\right)\mathrm{d}x$	**DM1**
	$= \left[\frac{x^2}{3} - \frac{2x^3}{27}\right]_0^3$	**A1 FT**
	$= 1$	**A1**
		4

Question 47

(a)	$\frac{1}{2}\times\frac{1}{2}k\times k = 1$	M1	Or use of $\int_0^k\left(-\frac{1}{2}x+\frac{1}{2}k\right)dx=1$ and attempt at integral.
	$k=2$	A1	Unsupported answers M0 A0. Do not accept ± 2.
		2	
(b)	$f(x) = -\frac{1}{2}x+1$	B1 FT	FT *their* k from $y=-\frac{1}{2}x+\frac{1}{2}k$.
	$\int_0^2(-\frac{1}{2}x^2+x)dx = \left[-\frac{x^3}{6}+\frac{x^2}{2}\right]_0^2$	M1	Attempt integration of $xf(x)$ limits 0 to k. FT *their* $f(x)$. Could be in terms of k.
	$\frac{2}{3}$ or 0.667 (3 sf)	A1	
		3	

(c)	$\int_p^1(-\frac{1}{2}x+1)dx$ [= 0.25]	M1	FT their equation of **line**; correct integral and limits (could be reversed) stated or $\frac{1}{2}(1-p)(1-\frac{1}{2}p+\frac{1}{2})$ [= 0.25].
	$\left[-\frac{x^2}{4}+x\right]_p^1 = 0.25$ $-\frac{1}{4}+1+\frac{p^2}{4}-p = 0.25$	M1	Attempt substitution of correct limits (not reversed) into their integral or attempt expand must equal 0.25. OE
	$p^2-4p+2=0$	M1	Obtain 3-term quadratic set equal to 0, obtain at least 1 solution.
	$p = 2-\sqrt{2}$ or 0.586	A1	CAO
		4	

Question 48

(a)	$P(X > 10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2\,dx$	M1	Attempt integration of f(x), ignore limits.
	$= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_{10}^{20}$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_{10}^{20}$ $= \frac{1}{8000}\left[0 - (-10)^3\right]$	M1	Substitute correct limits 10 to 20 or 1 – … limits 0 to 10 in *their* integral
	$\frac{1}{8}$ or 0.125	A1	**SC** Unsupported answer of $\frac{1}{8}$ scores B1 only
	$(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf)	B1 FT	FT *their* $P(X > 10)$ dependent on first M1 gained
		4	
(b)	$\int_0^{20} \frac{3}{8000}(x^3 - 40x^2 + 400x)\,dx$	M1	Attempt integration of xf(x). Ignore limits.
	$\frac{3}{8000}\left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2}\right]_0^{20}$ or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3}\right) - \frac{1}{8000}\left(\frac{(x-20)^4}{4}\right)$	A1	Correct integral (by expanding or by parts)
	$\frac{3}{8000}\left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400\right]$	M1	Subst correct limits in their (4th degree) integral
	5	A1	
		4	
(c)	$\int_0^{m} \frac{3}{8000}(x-20)^2\,dx = 0.5$	M1	Attempt to integrate f(x) and equate to 0.5. Ignore limits.
	$\left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_0^{m} = 0.5$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_0^{m} = 0.5$ $\frac{1}{8000}\left[(m-20)^3 - (-20)^3\right] = 0.5$	M1	Attempt integral and substitute limits 0 and m or m and 20 and = 0.5
	$(m-20)^3 = -4000$	A1	AG. Found convincingly.
	$(m = 20 + \sqrt[3]{-4000})$ $m = 4.13$ (3 sf)	B1	
		4	
(d)	Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early	B1	or any relevant comment e.g. trains on Sun may be different to trains on Mon
		1	

Question 49

(a)	$\frac{1}{2}p(p-1)=1$	M1	For area =1 For verification methods accept $\frac{1}{2}\times 2\times 1=1$ or $\frac{1}{2}\times 2\times(p-1)=1$ or $\frac{1}{2}\times 1\times p=1$ as indication that area=1
	$p=2$	A1	AG - Convincing method and answer. Must see quadratic rearranged to =0 and no errors seen. N.B. Accept convincing verification methods (e.g. statement such as 'assume $p=2$' or 'if $p=2$' or 'using $p=2$' or showing by clear substitution that $p=2$ fits $\frac{1}{2}p(p-1)=1$ with clear conclusion)
		2	
(b)	Gradient = 2 equation of line is $y=2x+c$ line passes through (1, 0), hence $c=-2$	M1	Award for attempting equation of line $y=mx+c$ with $m=2$, -2, $\frac{1}{2}$ or $-\frac{1}{2}$ and numerical c ($c\neq 0$)
	$y=2x-2$	A1	May be seen in **(a)** M1 can be implied by correct answer
	$2\int_1^2 (x^2-x)\,dx$	M1	For attempting $\int x f(x)\,dx$. Ignore limits, FT *their* equation.
	$2\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2$	A1 FT	Correct integration FT *their* f(x)and correct limits
	$\frac{5}{3}$ or 1.67 (3 sf)	A1	
		5	

Question 50

$E(X) = 3$	B1	N.B. $E(X)=108k$ is B0 until correct k substituted in.
$k\int_0^6 (6x - x^2)dx = 1$ $k\left[3x^2 - \frac{x^3}{3}\right]_0^6$ [= 1]	M1	Attempt integration of f(x) and =1. Ignore limits at this stage.
$k\left(108 - \frac{216}{3}\right) = 1$ $k = \frac{3}{108}$ or $\frac{1}{36}$	A1	
$'\frac{3}{108}'\int_0^6 (6x^3 - x^4)dx$ $= \frac{3}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = 10.8$	*M1	Attempt integration of *their* $k \times x^2 f(x)$. Ignore limits at this stage. Accept in terms of k.
$'10.8' - '3'^2$	DM1	*Their* 10.8 (from use of limits 0 and 6) minus *their* $(E(X))^2$. Accept in terms of k: $388.8k-(108k)^2$
$\frac{9}{5}$ or 1.8	A1	CWO. Must be convincingly obtained as AG.
	6	

Question 51

$1 - \frac{20}{27}$ or $\frac{20}{27} - \frac{1}{2}$ $\frac{20}{27} - \left(1 - \frac{20}{27}\right)$ or $\left(\frac{20}{27} - \frac{1}{2}\right)$	M1	For either expression seen.
$\frac{13}{27}$	A1	OE. Accept 0.481 or 0.482.
	2	

Question 52

(a)	$\frac{1}{18}\int_0^{1.2}(9-x^2)\,dx$	M1	Attempt to integrate f(x), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$	A1	Correct integration and correct limits.
	$\frac{71}{125}$ or 0.568	A1	SC unsupported answer scores **B2** only.
		3	
(b)	$\frac{1}{18}\int_0^3(9x-x^3)\,dx$	M1	Attempt to integrate xf(x), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[\frac{9x^2}{2}-\frac{x^4}{4}\right]_0^3$	A1	Correct integration and correct limits.
	$\frac{9}{8}$ or 1.125	A1	SC unsupported answer scores **B2** only.
		3	
(c)	$\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^m=0.5$	M1	Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits m to 3. Allow x instead of m.
	$\frac{1}{18}\left[9m-\frac{m^3}{3}\right]-0.5=0$	A1	Any correct cubic equation in m or x.
	$m^3-27m+27=0$	A1	AG. Correctly obtain this equation. No errors seen.
		3	

Question 53

'(a)(i)	$k\int_0^2(4x-x^2)\,dx=1$	M1	Attempt integral f(x) and = 1. Ignore limits (must see a power increase for attempted integration).
	$k\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^2=1$	A1	Correct integration and correct limits.
	$k\times\frac{16}{3}=1$ $\left[k=\frac{3}{16}\right]$	A1	OE AG Convincingly obtained. At least one interim step. No errors seen.
		3	
(a)(ii)	$\frac{3}{16}\int_0^2(4x^2-x^3)\,dx$	M1	Attempt integral xf(x). Ignore limits. (must see a power increase for attempted integration). Condone missing k.
	$\frac{3}{16}\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^2$	A1	Correct integration and correct limits. Condone missing k.
	$\frac{5}{4}$	A1	Unsupported correct answer scores **SC B2** only.
		3	

Question 54

(a)	$\frac{1}{18}\int_0^{1.2}(9-x^2)\mathrm{d}x$	**M1**	Attempt to integrate f(x), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$	**A1**	Correct integration and correct limits.
	$\frac{71}{125}$ or 0.568	**A1**	SC unsupported answer scores **B2** only.
		3	
(b)	$\frac{1}{18}\int_0^{3}(9x-x^3)\mathrm{d}x$	**M1**	Attempt to integrate xf(x), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[\frac{9x^2}{2}-\frac{x^4}{4}\right]_0^{3}$	**A1**	Correct integration and correct limits.
	$\frac{9}{8}$ or 1.125	**A1**	SC unsupported answer scores **B2** only.
		3	
(c)	$\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{m}=0.5$	**M1**	Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits m to 3. Allow x instead of m.
	$\frac{1}{18}\left[9m-\frac{m^3}{3}\right]-0.5=0$	**A1**	Any correct cubic equation in m or x.
	$m^3-27m+27=0$	**A1**	AG. Correctly obtain this equation. No errors seen.
		3	

Question 55

(a)	Quadratic curve, hence symmetrical	B1	OE. Allow sketch and 'symmetrical' or just 'curve symmetrical'
		1	
(b)	$-k\int_1^3 (x^2-4x+3)\mathrm{d}x = 1$	M1	Attempt to integrate f(x) and '= 1'. Ignore limits at this stage
	$-k\left[\frac{x^3}{3}-2x^2+3x\right]_1^3$	A1	Fully correct expression (correct integration and limits)
	$-k\times\left[0-\frac{4}{3}\right]=1$ or $k\times\frac{4}{3}=1$ $\left[k=\frac{3}{4}\right]$	A1	AG, OE. Correctly substitute limits and '= 1' and correctly obtain result with no errors seen.
		3	
(c)	$-\frac{3}{4}\int_1^3\left(x^4-4x^3+3x^2\right)\mathrm{d}x$	M1	Attempt to integrate x^2 f(x) from 1 to 3
	$-\frac{3}{4}\times\left[\frac{x^5}{5}-x^4+x^3\right]_1^3$ $\left[=\frac{3}{4}\times\frac{28}{5}=\frac{21}{5}\right]$	A1	Correct integration and limits
	$\left[\frac{21}{5}-2^2\right]=0.2$	A1	
		3	
(d)	$-\frac{3}{4}\int_{2.5}^{3}\left(x^2-4x+3\right)\mathrm{d}x$	M1	OE. Attempt to integrate f(x), from 2.5 to 3 (or 1 to 2.5)
	$=-\frac{3}{4}\times\left[\frac{x^3}{3}-2x^2+3x\right]_{2.5}^{3}=\frac{5}{32}$ or 0.15625	A1	
	$1-\left(1-\text{'}\frac{5}{32}\text{'}\right)^3$	M1	OE. FT *their* $\frac{5}{32}$.
	= 0.399 (3 sf)	A1	
		4	

Question 56

(a)(i)	1	**B1**	no ambiguity
		1	
(a)(ii)	$\frac{1}{2}$	**B1**	No ambiguity
		1	
(a)(iii)	$[q =]\ \frac{1}{2}p$	**B1**	Accept $2q = p$
		1	
(b)	$p\int_0^a (a^2 - x^2)\mathrm{d}x\ = 1$	**M1**	Attempt to integrate f(x) and equated to 1
	$\frac{2}{3}a^3 p\ = 1$	**A1**	OE, simplified
	$"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ or $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x\ = 3$	**M1**	Attempt to integrate xf(x), with multiplier p or $\frac{3}{2a^3}$ or *their* p, and equate to 3
	$p \times \frac{a^4}{4}\ = 3$	**A1**	May be implied by next line
	$"\frac{3}{2a^3}" \times \frac{a^4}{4}\ = 3$	**M1**	OE. Substitute from one equation into the other. FT *their* equations
	$a = 8$	**A1**	
		6	

Question 57

(a)	$\frac{a}{2}$	**B1**	
		1	
(b)	$\frac{1}{4}$	**B1**	
		1	
(c)	$\mathrm{f}(x) = \frac{1}{a}$	**B1**	SOI (may be seen in part **(a)** or part **(b)**)
	$\mathrm{E}(X) = \frac{a}{2}$	**B1**	SOI
	$\int_0^a \frac{1}{a}x^2\mathrm{d}x$	**M1**	Attempt integrate *their* f(x) $\times x^2$ with correct limits
	$= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$	**A1**	
	$\frac{a^2}{3} - (\frac{a}{2})^2$ or $\frac{a^2}{3} - \frac{a^2}{4}$ $[= \frac{a^2}{12}$ **AG** $]$	**A1**	Must see previous line and answer No errors seen
		5	
(d)	$\mathrm{P}(X < \frac{b}{3}) = \frac{p}{3}$	**M1**	SOI (could be on a diagram) OR by integration: prob = 1-(2/3)(b/a)
	$\mathrm{P}(\frac{b}{3} < X < a - .\frac{b}{3}) = 1 - \frac{2p}{3}$	**A1**	
		2	

Question 58

(a)	Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375)	B1	Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square. Must not go below the x-axis.
		1	
(b)	Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$	B1	Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 2$. Highest point must be at $x = 1$.
	Highest point (1, 0.75)	B1	
		2	
(c)	$\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\mathrm{d}x = \frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\mathrm{d}x = \frac{1}{2}$	M1	OE Attempt to integrate f(x) and correct limits with correct RHS.
	$\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3} = \frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a} = \frac{1}{2}$	A1	Correct integration.
	$a^3 - 12a + 8 = 0$	A1	AG Substitute limits and correctly obtain equation. May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$ No errors seen..
		3	
(d)	$0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) > 0 $0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) < 0 Hence $0.69 < a < 0.70$	B1	AG Must state either the correct expression and > 0 and < 0 or both answers to 2 sf. Both answers correct and conclusion. Accept equivalent expressions. OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70.
		1	

Question 59

(a)	$1 - p$ or $p - 0.5$	M1	SOI, e.g. on diagram.
	$[\mathrm{P}(-1 < X < 0) =]\ 2p - 1$	A1	Clearly as final answer.
		2	
(b)(i)	$\int_{-3}^{2}(a - b(x^2+x))\mathrm{d}x = 1$ or $\int_{-3}^{2}(ax - b(x^3+x^2))\mathrm{d}x = -0.5$	M1	OE Attempt integral, with correct limits and RHS.
	$\left[ax - b\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\right]_{-3}^{2}$ (= 1) or $\left[a\frac{x^2}{2} - b\left(\frac{x^4}{4}+\frac{x^3}{3}\right)\right]_{-3}^{2}$ (= –0.5)	A1	OE Correct integration.
	2a – 8b/3 – 2b +3a – 9b +9b / 2 = 1 or 2a – 4b – 8b / 3 – 9a / 2 + 81b / 4 – 9b = –0.5 leading to $30a - 55b = 6$ **AG**	A1	Correctly obtained. No errors seen.
		3	
(b)(ii)	$a - b(9-3) = 0$ or $a - b(4+2) = 0$ [hence $a - 6b = 0$]	*M1	Use f(–3) = 0 or f(2) = 0. Further attempts at integration M0.
	Attempt to solve $30a - 55b = 6$ and their a – 6b = 0	DM1	
	$a = \frac{36}{125}$ or 0.288 $b = \frac{6}{125}$ or 0.048	A1	
		3	

Question 60

(a)	Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375)	B1	Not straight lines, not bell shaped. Must be correct at $x =$ 0 and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square. Must not go below the x-axis.
		1	
(b)	Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$	B1	Not straight lines, not bell shaped. Must be correct at $x =$ 0 and $x = 2$. Highest point must be at $x = 1$.
	Highest point (1, 0.75)	B1	
		2	
(c)	$\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\,dx=\frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\,dx=\frac{1}{2}$	M1	OE Attempt to integrate f(x) and correct limits with correct RHS.
	$\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3}=\frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a}=\frac{1}{2}$	A1	Correct integration.
	$a^3 - 12a + 8 = 0$	A1	AG Substitute limits and correctly obtain equation. May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$ No errors seen..
		3	
(d)	$0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) > 0 $0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) < 0 Hence $0.69 < a < 0.70$	B1	AG Must state either the correct expression and > 0 and < 0 or both answers to 2 sf. Both answers correct and conclusion. Accept equivalent expressions. OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70.
		1	

Question 61

(a)	$1 - 2(a + b)$ **or** $1 - 2a$ **or** $0.5 - a - b$ **or** 1-(a+b) **or** a+a+b	M1	OE. Seen or implied – may be on the diagram (or for correct un-simplified final expression).
	$P(0.6 \le X \le 1.8) = 1 - 2a - b$	A1	Accept $1 - (2a + b)$.
		2	
(b)(i)	$k\int_{0}^{3}(9x^2-6x^3+x^4)\,dx = 1$	M1	Attempt integrate $f(x)$ ignore limits and '= 1'.
	$k\left[\frac{9x^3}{3}-\frac{6x^4}{4}+\frac{x^5}{5}\right]_0^3 = 1$	A1	Correct integration seen, correct limits.
	$k\times\frac{81}{10}=1$, $k=\frac{10}{81}$	A1	AG. Convincingly obtained. No errors seen. (Must see integration).
		3	
(b)(ii)	$\frac{10}{81}\int_{0}^{3}(9x^4-6x^5+x^6)\,dx$ $\left[\frac{10}{81}\left[\frac{9x^5}{5}-x^6+\frac{x^7}{7}\right]_0^3\right]$ [$=\frac{18}{7}$ or 2.57...]	M1	Attempt integrate $x^2f(x)$ between 0 and 3 condone missing k. Must see integration or correct answer of 18/7 seen or implied.
	$\frac{18}{7} - \text{'}1.5\text{'}^2$	M1	Their integral of $x^2f(x) - 1.5^2$ (or their mean^2).
	$=\frac{9}{28}$ or 0.321	A1	
		3	

Question 62

(a)	$\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$	M1	
	= 1, which is the area under a PDF [and $f(x) \geq 0$]	A1	Result and statement are both needed.
		2	
(b)	$\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right) = \frac{\pi}{4}$	B1	AG. Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles. Answer should be convincingly obtained and all correct.
	Area of sector = $\frac{1}{4}$	B1	
	Area of triangle $AOB = \frac{1}{2}OA \times OB = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi} - \frac{1}{\pi}}$ or Area of triangle $AOB = \frac{1}{2}OA \times OB \times \sin(AOB) = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi}} \sin\frac{\pi}{4}$	M1	Accept alternative approaches. Note: AB = $\sqrt{0.7979^2 - 0.5642^2}$ [= 0.5642] Allow values to 3sf.
	$\frac{1}{2\pi}$ or 0.1592	A1	
	'$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592'	M1	Attempt area of sector – area of triangle *AOB*.
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	A1	
(b)	**Alternative Method for Question Q7(b): Using integration**		
	Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$	M1	
	$y = \sqrt{\frac{2}{\pi} - x^2}$	A1	
	Attempt to integrate (any limits)	M1	
	Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$	B1	
	Correct integration with correct limits	A1	
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	A1	Correct final answer.
		6	

Question 63

$\frac{3}{2}\int_0^1 (x - x^3)\,dx$	M1	Attempt to integrate $xf(x)$; ignore limits.
$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$	A1	Correct integration and limits.
$= \frac{3}{8}$	A1	
	3	

Question 64

(a)(i)	$\frac{1}{2} \times 4 \times a = 1$	M1	For use of area = 1 or let f(x) = kx and attempt $\int_0^4 kx \, dx = 1$.
	$[a = \frac{1}{2}]$ $f(x) = \frac{1}{8}x$	A1	$k\left[\frac{x^2}{2}\right]_0^4 = 1$; $8k = 1$; $k = \frac{1}{8}$. f(x) = $\frac{1}{8}x$ or $k = \frac{1}{8}$.
		2	
(a)(ii)	$\int_0^4 x \times \frac{1}{8}x \, dx$	M1	Attempt to integrate $x \times$ *their* f(x). Ignore limits accept in terms of k.
	$\left[\frac{x^3}{24}\right]_0^4$	A1ft	Their integral and correct limits accept in terms of k.
	$= \frac{8}{3}$ or 2.67 (3 sf)	A1	Note: Final answer of $64k/3$ scores 2/3.
		3	
(b)	$\frac{a-1}{a} = \frac{1}{\sqrt{2}}$	M1	Or attempt $\int_0^1 g(w)dw = \frac{1}{2}$ i.e. $\int_0^1 (\frac{2}{a} - \frac{2}{a^2}w)dw = \frac{1}{2}$, or integral from 1 to a. g(w) must be linear of form g(w) = mw (+c). Or area attempt: attempt to calculate heights using their linear equation (h_1=2/a and h_2= $-2/a^2 + 2/a$) and use in either area trapezium = 0.5, or area trapezium =area small triangle or area small triangle = 0.5 . Area trapezium =1/2 × 1 ($2/a + -2/a^2 + 2/a$) Area triangle = 1/2(a – 1)($-2/a^2 + 2/a$)) Note: alternative expression for $h_1 = (a-2)/(a-1)$.
	$a\sqrt{2} - \sqrt{2} = a$	A1	Or $a^2 - 4a + 2 = 0$. Any correct equation in a, a not in denominator.
	$a = 2 + \sqrt{2} = 3.41$	A1	
		3	

Question 65

(a)	$\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf.	**B1**	Calculation and result.
	$f(x) \geqslant 0$	**B1**	Condone $f(x) > 0$ or 'Line is above x-axis' OE.
		2	
(b)	$\frac{1}{2}\pi r^2 = 1$	**M1**	Area of semi-circle equated to 1 OE. Missing factor of ½ gets M1A0.
	$r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	**A1**	
		2	
(c)(i)	Area to the left of 15 is greater than 0.5	**B1**	OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'.
		1	
(c)(ii)	$\int_{10}^{20} \left(\frac{40}{x} - \frac{x}{10}\right)dx$	**M1**	Integration of $xh(x)$ attempted. Ignore limits.
	$\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$	**A1**	Correct integration and limits (can be implied by final answer).
	$= 40\ln 2 - 15$ or 12.7 (3sf)	**A1**	
		3	

Question 66

(a)	$p + \frac{13}{10}p \leqslant \frac{1}{2} \Rightarrow p \leqslant \frac{5}{23}$ AG	**B1**	Allow '=' in working but need an inequality in the answer. Allow $0 < p \leqslant \frac{5}{23}$.
		1	
(b)	e.g. $0.5 - 2.3p$, 0 to a; $p + 1.3p$, a to 3; $2 \times 1.3p$, $2 \times$ (b to 3); $2.3p + 1.3p$, a to 3 + b to 3; $2p + 2.6p$, $2 \times$ (a to 3); $0.5 - 1.3p$, 0 to b; $0.5 + 1.3p$, b to 6	**M1**	Any correct expression for the probability of a relevant region.
	$\frac{18}{5}p$ or $3.6p$	**A1**	
		2	

(c)	$\frac{1}{36}\int_a^2 (6x - x^2)\,dx = \frac{5}{27}$	M1	Attempt to integrate with correct limits and equate to $\frac{5}{27}$, oe. Integrate from 2 to 6 – a and equate $\frac{18}{5}p = \frac{2}{3}$. Integrate from a to 3 and equate to $\frac{23}{54}$. Integrate from 0 to a and equate to $\frac{2}{27}$.
	$\left[\Rightarrow \frac{1}{36}\left[3x^2 - \frac{x^3}{3}\right]_a^2 = \frac{5}{27} \Rightarrow \frac{1}{36}\left(12 - \frac{8}{3} - 3a^2 + \frac{a^3}{3}\right) = \frac{5}{27}\right]$	M1	For integrating and substitution of limits to form cubic in a.
	$a^3 - 9a^2 + 8 = 0$	A1	Any correct three term cubic equation in a.
	$(a - 1)(a^2 - 8a - 8) = 0$	M1	Attempt to factorise their cubic equation.
	$a = \frac{8 \pm \sqrt{96}}{2} = 4 \pm \sqrt{24}$ or –0.899 or 8.90, [not between 0 and 6]		
	$a = 1$ only [other two values rejected]	A1	**SC B1** for $a = 1$ only, if no method seen for solving the cubic.
		5	

Question 67

(a)	$\int_a^b \frac{x}{x^2}\,dx$	M1	Attempt to integrate $x\text{f}(x)$ from a to b.
	$= [\ln x]_a^b$ or $\ln b - \ln a$	A1	
	$\ln \frac{b}{a} = \ln 2$ or $\ln 2a = \ln b$ oe Hence $b = 2a$ (AG)	A1	Must see both statements. No errors seen.
		3	
(b)	$\int_a^b \frac{1}{x^2}\,dx = 1$	M1	Attempt to integrate f(x) and equate to 1. Ignore limits.
	$\left[\left[-\frac{1}{x}\right]_a^b = 1 \text{ or } \frac{1}{a} - \frac{1}{b} = 1\right] \frac{1}{a} - \frac{1}{2a} = 1$	A1	Integrate with correct limits and substitute $b = 2a$.
	e.g. $\frac{1}{2a} = 1$ or $2 + (-1) = 2a$ Hence $a = a = \frac{1}{2}$ (AG)	A1	Obtain convincingly (at least one step from previous answer), no errors seen (ignore $a = 0$).
		3	
(c)	$\int_{0.5}^m \frac{1}{x^2}\,dx = \frac{1}{2}$ or $\int_m^1 \frac{1}{x^2}\,dx = \frac{1}{2}$	M1	Attempt integrate f(x), with correct limits, and equate to $\frac{1}{2}$.
	$\left[-\frac{1}{x}\right]_{0.5}^m = \frac{1}{2}$ or $\left[-\frac{1}{x}\right]_m^1 = \frac{1}{2}$		
	$2 - \frac{1}{m} = \frac{1}{2}$ or $\frac{1}{m} - 1 = \frac{1}{2}$	A1	oe. Correct equation after substituting limits.
	$m = \frac{2}{3}$ or 0.667 (3 sf)	A1	
		3	

Question 68

(a)	$\frac{1}{2} - \frac{117}{256}$ $\left[= \frac{11}{256}\right]$	**M1**	For use of symmetry about $x = 2$, oe. E.g. $(1 - 2 \times \frac{117}{256}) \div 2$ or $\frac{1}{2} + \frac{117}{256}$.
	$1 - \frac{11}{256}$ or $2 \times \frac{117}{256} + \frac{11}{256}$ $= \frac{245}{256}$ **AG**	**A1**	Any correct numerical expression seen leading to AG.
		2	
(b)	$k\int_{2}^{5}(12+4x-x^2)\mathrm{d}x$ $\left[= k\left[12x + 2x^2 - \frac{x^3}{3}\right]_2^5\right]$	**M1**	Attempt to integrate $f(x)$ with any limits.
	$39k = \frac{117}{256}$	**M1**	Use of limits 2 and 5 and equating *their* integration attempt to $\frac{117}{256}$. Or limits –2 and 6 equated to 1. Or limits –1 and 6 equated to $\frac{245}{256}$. Or limits –1 to 5 equated to $\frac{234}{256}$. Oe. No mixed methods.
	$k = \frac{3}{256}$ or 0.0117	**A1**	
		3	
(c)	$[2 + x - x^2 = 0]$ $x = -1$ and $x = 2$ seen or implied [Domain is $-1 \leqslant x \leqslant 2$]	**B1**	
	Mean = 0.5	**B1**	
	$\frac{2}{9}\int_{-1}^{2}(2x^2+x^3-x^4)\mathrm{d}x$ $\left[= \frac{2}{9}\left[\frac{2}{3}x^3 + \frac{x^4}{4} - \frac{x^5}{5}\right]_{-1}^{2}\right]$ [= 0.7]	***M1**	Attempt to integrate $x^2 g(x)$ with any limits.
	their '0.7' – *their* '0.5'^{2}	**DM1**	Subtract *their* mean^2 from *their* $\int x^2 g(x)\mathrm{d}x$ (both must be numerical).
	$= 0.45$ or $\frac{9}{20}$	**A1**	
		5	

Question 69

(a)	$\int_0^{\sqrt{2}} (ax - x^3)\mathrm{d}x = 1$	**M1**	Attempted integration of $\mathrm{f}(x)$ and equated to 1.
	$\left[a\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{2}} = 1$ $a - \frac{4}{4} = 1$	**A1**	Correct integration and substitute correct limits.
	$a = 2$	**A1**	AG Convincingly obtained and no errors seen.
		3	
(b)	$\int_0^m \left(2x - x^3\right)dx = \quad = \frac{1}{2}$	**M1**	Attempt integrate $\mathrm{f}(x)$ with limits 0 to m (or m to $\sqrt{2}$) and equate to $\frac{1}{2}$.
	$m^2 - \frac{m^4}{4} = \frac{1}{2}$	**A1**	For correct quartic in any form.
	$m^4 - 4m^2 + 2 = 0 \Rightarrow m^2 = \frac{4 \pm \sqrt{16-8}}{2}$ $\quad [= 2 \pm \sqrt{2}\,]$	**M1**	For solving their three term quartic to find m^2.
	$m = \sqrt{2 - \sqrt{2}}$ or 0.765 (3sf)	**A1**	
		4	
(c)	$\int_0^{\sqrt{2}} (2x^2 - x^4)\mathrm{d}x$	**M1**	Attempt to integrate $x\mathrm{f}(x)$. Ignore limits.
	$\left[\frac{2x^3}{3} - \frac{x^5}{5}\right]_0^{\sqrt{2}}$	**A1**	Correct integration and correct limits.
	$[= \frac{4\sqrt{2}}{3} - \frac{4\sqrt{2}}{5}\,] = \frac{8}{15}\sqrt{2}$	**A1**	OE For single exact term.
		3	

Question 70

(a)	$k\int_0^{\pi} (1 + \cos x)\mathrm{d}x = 1$	**M1**	Attempt integrate $\mathrm{f}(x)$ with correct limits and equate to 1.
	$k\left[x + \sin x\right]_0^{\pi} = 1$	**A1**	Correct integration.
	[e.g. $k(\pi + \sin\pi - (0 + 0)) = 1$], $k\pi = 1$, $k = \frac{1}{\pi}$	**A1**	AG Some evidence of substitution of limits, i.e. at least one interim step (e.g. $k\pi = 1$) as minimum requirement. Convincingly obtained; no errors seen.
		3	

(b)	$\frac{1}{\pi}\left[x+\sin x\right]_0^{0.83}$ or $\frac{1}{\pi}(0.83 + \sin 0.83)$	$\frac{1}{\pi}\left[x+\sin x\right]_0^{0.84}$ or $\frac{1}{\pi}(0.84 + \sin 0.84)$	**M1**	Substitute correct limits into their integral. OR_1: integrate 0 to 0.83 and 0.84 to π. OR_2: use $g(m) = m + \sin m - (\pi/2)$ and find g(0.83) and g(0.84). OR_3: use $h(m) = m + \sin m$ and find h(0.83) and h(0.84). Both attempted.
	= 0.499 (3 sf)	= 0.504 (3 sf)	**A1**	OR_1: 0.499 and 0.496. OR_2: g(0.83) = –0.00286/7 and g(0.84) = 0.0138/9. OR_3: h(0.83) = 1.57 and h(0.84) = 1.58 or 1.59. Both correct.
	'0.499' < 0.5 < '0.504' hence 0.83 < median < 0.84 Equivalent to –0.000912 < 0 < 0.00441 hence 0.83 < median < 0.84	**A1FT**	FT their areas; dep 0.5 is between their areas OE. OR_1: 0.499 < 0.5 and 0.496 < 0.5, so $0.83 < m < 0.84$. OR_2: g(0.83) > 0 g(0.84) < 0 OE, so 0.83 < m < 0.84. OR_3: $h(0.83) < \frac{\pi}{2}$ $h(0.84) > \frac{\pi}{2}$, so $0.83 < m < 0.84$. Both statements needed. Note: A score of M1 A0 A1FT is possible.
			If 0 scored, **SC:** $\frac{1}{\pi}(m + \sin m) = 0.5$ **B1** and m = 0.831 to 0.832, so $0.83 < m < 0.84$ **B1**.
		3	
(c)	$\frac{1}{\pi}\int_0^{\pi}(x + x\cos x)\mathrm{d}x$	**M1***	Attempt integrate $x\mathrm{f}(x)$. Ignore limits.
	$= \frac{1}{\pi}\left(\left[\frac{x^2}{2}\right]_0^{\pi} + \left[x\sin x\right]_0^{\pi} - \int_0^{\pi}(\sin x\mathrm{d}x)\right)$	**DM1**	OE Attempt to integrate (using 'parts') with correct limits, reaching an expression of the form $ax^2 + uv - \int v\,\mathrm{d}u$. OR using parts to integrate $x(1+\cos x)$ reaching an expression of the form $uv - \int v\,\mathrm{d}u$ i.e. $\frac{1}{\pi}(x^2 + x\sin x - \int(x + \sin x)\,\mathrm{d}x))$
	$= \frac{1}{\pi}\left(\frac{x^2}{2} + x\sin x + \cos x\right)$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi}(0 - [-\cos x]_0^{\pi})$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi}((-1 - 1))$	**A1**	Integration fully correct.
	$= \frac{\pi}{2} - \frac{2}{\pi}$	**A1**	OE ISW after correct exact value seen. **SC1**: Unsupported answer of $\frac{\pi}{2} - \frac{2}{\pi}$ scores **B3**. **SC2**: Unsupported answer of 0.934 scores **B2**.
		4	

Question 71

(a)	$\frac{1}{4}\pi a^2 = 1$	**M1**	OE Attempt to set area = 1.
	$a = \frac{2}{\sqrt{\pi}}$	**A1**	AG Correct equation and correctly rearranged to $a = \ldots$ No errors seen.
		2	
(b)	$x^2 + y^2 = \left(\frac{2}{\sqrt{\pi}}\right)^2$	**M1**	Or $x^2 + y^2 = a^2$.
	$[y^2 = \frac{4}{\pi} - x^2\,]$		Must see at least one intermediate step
	$f(x) = \sqrt{\frac{4}{\pi} - x^2}$	**A1**	AG Convincingly rearranged to reach given answer. No errors seen.
		2	
(c)	$\int_0^{\frac{2}{\sqrt{\pi}}} x\sqrt{\frac{4}{\pi} - x^2}\,dx$	**B1**	Correct expression for $E(X) = \int x f(x)\,dx$ with correct limits (accept limits 0 and a).
	$-\frac{1}{3}\left[\left(\frac{4}{\pi} - x^2\right)^{\frac{3}{2}}\right]_0^{\frac{2}{\sqrt{\pi}}}$	**M1**	Integrate $x f(x)$ with any limits or none. Must reach expression of form: any constant × $\left(\frac{4}{\pi} - x^2\right)^{\frac{3}{2}}$.
		A1	Wholly correct integration and limits.
	$\frac{8}{3\sqrt{\pi^3}}$	**A1**	AG Correctly obtained with no errors seen.
		4	

Question 72

(a)	$\int_2^3 \left(\frac{a}{x^2} - \frac{18}{x^3}\right)dx = 1$	**M1**	Attempt integrate f(x), ignore limits and '= 1'.
	$\left[-\frac{a}{x} + \frac{9}{x^2}\right]_2^3 = 1$	**A1**	OE Correct integration and limits.
	$\left[-\frac{a}{3} + 1 + \frac{a}{2} - \frac{9}{4}\right] = 1$ $a = \frac{27}{2}$ **(AG)**	**A1**	Must see correct substitution of limits. Correct working no errors seen.
		3	
(b)	$\int_2^3 \left(\frac{27}{2x} - \frac{18}{x^2}\right)dx$	**M1**	Attempt to integrate xf(x), ignore limits.
	$\left[\frac{27}{2}\ln x + \frac{18}{x}\right]_2^3$ or $\left[\frac{27}{2}\ln 2x + \frac{18}{x}\right]_2^3$	**A1**	Correct integration and limits. OE e.g. using ln 2x.
	$= \frac{27}{2}\ln 3 + 6 - \frac{27}{2}\ln 2 - 9 = \frac{27}{2}\ln\frac{3}{2} - 3$ **AG**	**A1**	Must see correct substitution of limits. Correct working no errors seen.
		3	

Question 73

(a)	Min and max times [to complete challenge]	**B1**	In context (e.g. min and max x scores **B0**).
		1	
(b)	$\int_a^b \frac{1}{x^2}\,dx = 1$	**M1**	Attempt to integrate f(x) and =1, ignore limits.
	$\left[\left[-\frac{1}{x}\right]_a^b = 1\right] \; -\frac{1}{b}+\frac{1}{a} = 1$	**A1**	For correct equation using correct limits into correct integration and = 1.
	$-a + b = ab$ or $b = a(b + 1)$	**A1**	Convincingly obtained. No errors seen. OE $\Rightarrow a = \frac{b}{b+1}$ **AG.**
		3	
(c)	$E(X) = \int_a^b \frac{1}{x}\,dx$	**M1**	Attempt to integrate xf(x). Limits a and b or b/(b+1) and b (condone a and 2 for **M1**) See SC for use of limits 2/3 and 2
	$= \ln b - \ln a$ or $\ln b - \ln(b/(b+1))$	**A1**	Correct integration and limits substituted. Condone ln 2 – ln a.
	$[= \ln b - (\ln b - \ln(b + 1))] = \ln b - \ln(b/(b+1)) = \ln 3$ $= \ln(b + 1) = \ln 3$ or $b+1 = 3$ or $b^2 + b = 3b$ or $\frac{b}{\left(\frac{b}{b+1}\right)} = 3$	**A1**	For correct equation in b only (i.e. using part **(b)**).
	$b = 2$ **(AG)** $a = \frac{2}{3}$	**A1**	Both obtained correctly (Note: if b=2 not shown but used can score **M1 A1, A1/A0** depending on where b=2 is introduced, **A0**)
			SC verification: using b=2 and a=2/3 then integrating xf(x) from 2/3 to 2 scores **M1 A1** for integration and limits substituted, then **A1** for showing =ln 3 Final **A0** (as verified not shown) max ¾.
		4	
(d)	$\int_{\frac{2}{3}}^{m} \frac{1}{x^2}\,dx = 0.5$ or $\int_m^2 \frac{1}{x^2}\,dx = 0.5$	**M1**	Attempt to integrate f(x) equated to 0.5 and correct limits stated.
	$\left[-\frac{1}{x}\right]_{\frac{2}{3}}^{m} = 0.5$ or $\left[-\frac{1}{x}\right]_m^2 = 0.5$	**A1FT**	Correct integration FT *their a* .
	$\left[-\frac{1}{m}+\frac{3}{2} = 0.5\right]$ or $\left[-\frac{1}{2}+\frac{1}{m} = 0.5\right]$ $m = 1$	**A1**	
		3	

Question 74

(a)	$\int_2^3 (\frac{a}{x^2} - \frac{18}{x^3})\,dx = 1$	M1	Attempt integrate f(x), ignore limits and '= 1'.
	$\left[-\frac{a}{x} + \frac{9}{x^2}\right]_2^3 = 1$	A1	OE Correct integration and limits.
	$\left[-\frac{a}{3} + 1 + \frac{a}{2} - \frac{9}{4}\right] = 1$ $a = \frac{27}{2}$ **(AG)**	A1	Must see correct substitution of limits. Correct working no errors seen.
		3	
(b)	$\int_2^3 (\frac{27}{2x} - \frac{18}{x^2})\,dx$	M1	Attempt to integrate xf(x), ignore limits.
	$\left[\frac{27}{2}\ln x + \frac{18}{x}\right]_2^3$ or $\left[\frac{27}{2}\ln 2x + \frac{18}{x}\right]_2^3$	A1	Correct integration and limits. OE e.g. using ln 2x.
	$= \frac{27}{2}\ln 3 + 6 - \frac{27}{2}\ln 2 - 9 = \frac{27}{2}\ln\frac{3}{2} - 3$ **AG**	A1	Must see correct substitution of limits. Correct working no errors seen.
		3	

Question 75

(a)(i)	$\frac{a+b}{2} \times 2 = 1$ or ½(b - a) x 2 + 2a = 1 or 2b - 1/2 x 2(b - a) = 1 or $\int_0^2 (a + \frac{b-a}{2}x)\,dx = 1$ and attempt to integrate	M1	
	eg $a + b = 1$ or $2a + b - a = 1$ or $2b - b + a = 1 \Rightarrow b = 1 - a$	A1	Must see correct intermediate step and answer correctly obtained no errors seen.
		2	
(a)(ii)	$y = a + \frac{1-2a}{2}x$ or $y = a + \frac{b-a}{2}x$	B1	Could be seen in 4**(a)(i).**
	$\int_0^2 (ax + \frac{1-2a}{2}x^2)\,dx$ or $\int_0^2 (ax + \frac{b-a}{2}x^2)\,dx$	M1	Attempt to integrate $xf(x)$ ignore limits, ft their line equation (of form y = mx + c and in terms of a or a and b).
	$\left[\frac{ax^2}{2} + \frac{(1-2a)x^3}{6}\right]_0^2$ or $\left[\frac{ax^2}{2} + \frac{(b-a)x^3}{6}\right]_0^2$ [=1.2]	A1	Correct integration, ignore limits.
	$[2a + \frac{4}{3}(1-2a) = 1.2]$	M1	Substitute correct limits into their integral in terms of a, not a and b (their integral must come from xf(x)) and equated to 1.2.
	$[\frac{4-2a}{3} = 1.2]$ $a = 0.2$	A1	
		5	
(b)	$\frac{1}{2}\int_0^c \cos t\,dt = \frac{1}{4}$ or $\frac{1}{2}\int_{-c}^c \cos t\,dt = \frac{1}{2}$	M1	OE. Attempt integrate cos t with correct limits and RHS.
	$\frac{1}{2}[\sin t]_0^c = \frac{1}{4}$ or $\frac{1}{2}[\sin t]_{-c}^c = \frac{1}{2}$	A1	OE. Correct integral with correct limits and RHS.
	$\sin c = \frac{1}{2}$	A1	
	$c = \frac{\pi}{6}$	A1	Alone. Accept c = 0.5236.
		4	

Question 76

(a)	$\frac{1}{2}\times b\times ab=1$ or $a\int_0^b x\,dx=1$	**M1**	Attempt triangle area or integrate f(x) (ignore limits) and = 1.
	$a\left[\frac{x^2}{2}\right]_0^b=1$	**A1**	Correct integration. ½ × b × ab = 1.
	$\frac{ab^2}{2}=1\Rightarrow\ a=\frac{2}{b^2}$	**A1**	Convincingly obtained. No errors seen.
		3	

(b)	$E(X)=\frac{2}{b^2}\int_0^b x^2\,dx$	**M1**	Attempt to integrate $xf(x)$. Ignore limits.
	$=\frac{2}{b^2}\left[\frac{x^3}{3}\right]_0^b$	**A1**	Correct integration and limits.
	$=\frac{2}{3}b$	**A1**	OE. E.g. $ab^3/3$
	$P(X<E(X))=\frac{2}{b^2}\int_0^{\frac{2}{3}b} x\,dx$	**M1**	OE. Integrate $f(x)$ with limits 0 and their E(X) Condone a instead of $\frac{2}{b^2}$ Or using areas, prob $=\frac{1}{2}\times\frac{2b}{3}\times\frac{2}{3}ab$
	$=\frac{2}{b^2}\left[\frac{x^2}{2}\right]_0^{\frac{2}{3}b}$	**A1FT**	Correct integration and limits, **FT** their $E(X)$. Need $\frac{2}{b^2}$. Or prob $=\frac{1}{2}\times\frac{2b}{3}\times\frac{2}{3}\times\frac{2b}{b^2}$.
	$=\frac{2}{b^2}\times\frac{\frac{4}{9}b^2}{2}=\frac{4}{9}$	**A1**	Must see correct expression after substituting limits. Convincingly obtained. No errors seen. Or expression correctly simplified.
		6	

Question 77

(a)	$\frac{k}{a^2}\int_0^a x^2\,dx=1$	**M1**	Attempt to integrate f(x) and =1 ignore limits.
	$\left[\frac{kx^3}{3a^2}\right]_0^a=1$	**A1**	Correct integration and correct limits.
	$\Rightarrow\frac{ka}{3}=1\Rightarrow k=\frac{3}{a}$	**A1**	AG. Must see an intermediate step and answer. Convincingly obtained no errors seen.
		3	
(b)	$\frac{3}{a^3}\int_0^a x^3\,dx$	**M1**	Attempt to integrate xf(x) with limits 0 and a. Condone k instead of 3/a or missing 'k'.
	$\frac{3}{a^3}\left[\frac{x^4}{4}\right]_0^a=1$ $[\Rightarrow\frac{3}{4}a=1]$	**A1**	Correct expression after correct integration and = 1. Condone k instead of $\frac{3}{a}$.
	$a=\frac{4}{3}$ or 1.33 (3 sf)	**A1**	
		3	

(c)	$\frac{81}{64}\int_0^m x^2\,dx = 0.5$	**M1**	Attempt to integrate f(x), limits 0 and m or limits m and a. and =0.5 (accept in terms of a and/or k. Condone missing k. **FT** their a.
	$\frac{81}{64}\left[\frac{x^3}{3}\right]_0^m = 0.5 \Rightarrow \frac{27}{64}m^3 = 0.5$	**A1FT**	Correct expression after correct integration with a and k correctly substituted (at some point) not necessarily simplified. **FT** their a.
	$m = \sqrt[3]{\frac{32}{27}}$ or 1.06 (3 sf)	**A1**	OE. CWO.
		3	

Question 78

(a)	$\int_0^{\frac{1}{2}}(1+\cos\pi x)\,dx$	**M1**	Attempt to integrate f(x). Ignore limits.
	$= \left[x+\frac{\sin\pi x}{\pi}\right]_0^{\frac{1}{2}}$	**A1**	Correct integration and limits.
	$= \frac{1}{2}+\frac{\sin\frac{\pi}{2}}{\pi} = \frac{1}{2}+\frac{1}{\pi}$	**A1**	**AG**. Must see intermediate step. Convincingly obtained. No errors seen.
		3	
(b)	$\int_0^1(x+x\cos\pi x)\,dx$	**M1***	Attempt to integrate xf(x). Ignore limits.
	$= \left[\frac{x^2}{2}+x\times\frac{\sin\pi x}{\pi}-\int\frac{\sin\pi x}{\pi}\,dx\right]_0^1$	**M1***	Attempt to integrate by parts to reach expression with at least two terms correct (must be more than two terms). May be implied by next line.
	$= \left[\frac{x^2}{2}+x\times\frac{\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^1$	**A1**	
	$= \frac{1}{2}+\frac{\sin\pi}{\pi}+\frac{\cos\pi}{\pi^2}-\frac{\cos 0}{\pi^2}$	**DM1**	Substitute correct limits, dep **M1M1**.
	$= \frac{1}{2}-\frac{2}{\pi^2}$	**A1**	**AG**. Legitimately obtained. No errors seen
		5	