A-Level

Topic : Discrete , Continuous Hypothesis and Types of Error

May 2013- May 2025

Questions

Question 1

A hockey player found that she scored a goal on 82% of her penalty shots. After attending a coaching course, she scored a goal on 19 out of 20 penalty shots. Making an assumption that should be stated, test at the 10% significance level whether she has improved. [5]

Question 2

In the past the weekly profit at a store had mean \$34 600 and standard deviation \$4500. Following a change of ownership, the mean weekly profit for 90 randomly chosen weeks was \$35 400.

(i) Stating a necessary assumption, test at the 5% significance level whether the mean weekly profit has increased. [6]

(ii) State, with a reason, whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

The mean weekly profit for another random sample of 90 weeks is found and the same test is carried out at the 5% significance level.

(iii) State the probability of a Type I error. [1]

(iv) Given that the population mean weekly profit is now \$36 500, calculate the probability of a Type II error. [5]

Question 3

The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of n plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.

(i) He finds that the value of the test statistic z is -1.563, correct to 3 decimal places. Calculate the value of n. State an assumption necessary for your calculation. [4]

(ii) Use this value of the test statistic to carry out the hypothesis test at the 6% significance level. [3]

Question 4

The number of cases of asthma per month at a clinic has a Poisson distribution. In the past the mean has been 5.3 cases per month. A new treatment is introduced. In order to test at the 5% significance level whether the mean has decreased, the number of cases in a randomly chosen month is noted.

(i) Find the critical region for the test and, given that the number of cases is 2, carry out the test. [5]

(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]

(iii) At another clinic the mean number of cases of asthma per month has the independent distribution Po(13.1). Assuming that the mean for the first clinic is still 5.3, use a suitable approximating distribution to estimate the probability that the total number of cases in the two clinics in a particular month is more than 20. [5]

Question 5

Leila suspects that a particular six-sided die is biased so that the probability, p, that it will show a six is greater than $\frac{1}{6}$. She tests the die by throwing it 5 times. If it shows a six on 3 or more throws she will conclude that it is biased.

(i) State what is meant by a Type I error in this situation and calculate the probability of a Type I error. [3]

(ii) Assuming that the value of p is actually $\frac{2}{3}$, calculate the probability of a Type II error. [3]

Leila now throws the die 80 times and it shows a six on 50 throws.

(iii) Calculate an approximate 96% confidence interval for p. [4]

Question 6

The times taken by students to complete a task are normally distributed with standard deviation 2.4 minutes. A lecturer claims that the mean time is 17.0 minutes. The times taken by a random sample of 5 students were 17.8, 22.4, 16.3, 23.1 and 11.4 minutes. Carry out a hypothesis test at the 5% significance level to determine whether the lecturer's claim should be accepted. [5]

Question 7

A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by X.

(i) Write down the value of $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = \frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

(ii) Find the probability of a Type I error. [5]

(iii) With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

Question 8

A traffic officer notes the speeds of vehicles as they pass a certain point. In the past the mean of these speeds has been $62.3\,\text{km}\,\text{h}^{-1}$ and the standard deviation has been $10.4\,\text{km}\,\text{h}^{-1}$. A speed limit is introduced, and following this, the mean of the speeds of 75 randomly chosen vehicles passing the point is found to be $59.9\,\text{km}\,\text{h}^{-1}$.

(i) Making an assumption that should be stated, test at the 2% significance level whether the mean speed has decreased since the introduction of the speed limit. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

Question 9

At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]

Question 10

A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.

(i) Carry out a test of Max's claim at the 2.5% significance level. [5]

(ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]

(iii) State what is meant by a Type II error in this context. [1]

Question 11

The weights, X kilograms, of rabbits in a certain area have population mean μ kg. A random sample of 100 rabbits from this area was taken and the weights are summarised by

$$\Sigma x = 165, \qquad \Sigma x^2 = 276.25.$$

Test at the 5% significance level the null hypothesis $H_0 : \mu = 1.6$ against the alternative hypothesis $H_1 : \mu \neq 1.6$. [6]

Question 12

A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean μ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part **(iii)**. [1]

Question 13

The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

Question 14

Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to find out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.

(i) Find the probability of a Type I error. [4]

(ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

Question 15

The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2. The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significance level whether the manager's claim is justified. [5]

Question 16

It is known that when seeds of a certain type are planted, on average 10% of the resulting plants reach a height of 1 metre. A gardener wishes to investigate whether a new fertiliser will increase this proportion. He plants a random sample of 18 seeds of this type, using the fertiliser, and notes how many of the resulting plants reach a height of 1 metre.

(i) In fact 4 of the 18 plants reach a height of 1 metre. Carry out a hypothesis test at the 8% significance level. [5]

(ii) Explain which of the errors, Type I or Type II, might have been made in part **(i)**. [2]

Later, the gardener plants another random sample of 18 seeds of this type, using the fertiliser, and again carries out a hypothesis test at the 8% significance level.

(iii) Find the probability of a Type I error. [3]

Question 17

A researcher wishes to investigate whether the mean height of a certain type of plant in one region is different from the mean height of this type of plant everywhere else. He takes a large random sample of plants from the region and finds the sample mean. He calculates the value of the test statistic, z, and finds that $z = 1.91$.

(i) Explain briefly why the researcher should use a two-tail test. [1]

(ii) Carry out the test at the 4% significance level. [3]

Question 18

The number of accidents on a certain road has a Poisson distribution with mean 3.1 per 12-week period.

(i) Find the probability that there will be exactly 4 accidents during an 18-week period. [3]

Following the building of a new junction on this road, an officer wishes to determine whether the number of accidents per week has decreased. He chooses 15 weeks at random and notes the number of accidents. If there are fewer than 3 accidents altogether he will conclude that the number of accidents per week has decreased. He assumes that a Poisson distribution still applies.

(ii) Find the probability of a Type I error. [3]

(iii) Given that the mean number of accidents per week is now 0.1, find the probability of a Type II error. [3]

(iv) Given that there were 2 accidents during the 15 weeks, explain why it is impossible for the officer to make a Type II error. [1]

Question 19

The number of hours that Mrs Hughes spends on her business in a week is normally distributed with mean μ and standard deviation 4.8. In the past the value of μ has been 49.5.

(i) Assuming that μ is still equal to 49.5, find the probability that in a random sample of 40 weeks the mean time spent on her business in a week is more than 50.3 hours. [4]

Following a change in her arrangements, Mrs Hughes wishes to test whether μ has decreased. She chooses a random sample of 40 weeks and notes that the total number of hours she spent on her business during these weeks is 1920.

(ii) **(a)** Explain why a one-tail test is appropriate. [1]

(b) Carry out the test at the 6% significance level. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in part **(ii)(b)**. [1]

Question 20

The mean breaking strength of cables made at a certain factory is supposed to be 5 tonnes. The quality control department wishes to test whether the mean breaking strength of cables made by a particular machine is actually less than it should be. They take a random sample of 60 cables. For each cable they find the breaking strength by gradually increasing the tension in the cable and noting the tension when the cable breaks.

(i) Give a reason why it is necessary to take a sample rather then testing all the cables produced by the machine. [1]

(ii) The mean breaking strength of the 60 cables in the sample is found to be 4.95 tonnes. Given that the population standard deviation of breaking strengths is 0.15 tonnes, test at the 1% significance level whether the population mean breaking strength is less than it should be. [4]

(iii) Explain whether it was necessary to use the Central Limit theorem in the solution to part **(ii)**. [2]

Question 21

Marie claims that she can predict the winning horse at the local races. There are 8 horses in each race. Nadine thinks that Marie is just guessing, so she proposes a test. She asks Marie to predict the winners of the next 10 races and, if she is correct in 3 or more races, Nadine will accept Marie's claim.

(i) State suitable null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) State the significance level of the test. [1]

Question 22

In the past, the flight time, in hours, for a particular flight has had mean 6.20 and standard deviation 0.80. Some new regulations are introduced. In order to test whether these new regulations have had any effect upon flight times, the mean flight time for a random sample of 40 of these flights is found.

(i) State what is meant by a Type I error in this context. [2]

(ii) The mean time for the sample of 40 flights is found to be 5.98 hours. Assuming that the standard deviation of flight times is still 0.80 hours, test at the 5% significance level whether the population mean flight time has changed. [4]

(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in your answer to part **(ii)**. [2]

Question 23

Cloth made at a certain factory has been found to have an average of 0.1 faults per square metre. Suki claims that the cloth made by her machine contains, on average, more than 0.1 faults per square metre. In a random sample of 5 m^2 of cloth from Suki's machine, it was found that there were 2 faults. Assuming that the number of faults per square metre has a Poisson distribution,

(i) state null and alternative hypotheses for a test of Suki's claim, [1]

(ii) test at the 10% significance level whether Suki's claim is justified. [4]

Question 24

In the past, the time taken by vehicles to drive along a particular stretch of road has had mean 12.4 minutes and standard deviation 2.1 minutes. Some new signs are installed and it is expected that the mean time will increase. In order to test whether this is the case, the mean time for a random sample of 50 vehicles is found. You may assume that the standard deviation is unchanged.

(i) The mean time for the sample of 50 vehicles is found to be 12.9 minutes. Test at the 2.5% significance level whether the population mean time has increased. [4]

(ii) State what is meant by a Type II error in this context. [2]

(iii) State what extra piece of information would be needed in order to find the probability of a Type II error. [1]

Question 25

Sami claims that he can read minds. He asks each of 50 people to choose one of the 5 letters A, B, C, D or E. He then tells each person which letter he believes they have chosen. He gets 13 correct. Sami says "This shows that I can read minds, because 13 is more than I would have got right if I were just guessing."

(i) State null and alternative hypotheses for a test of Sami's claim. [1]

(ii) Test at the 10% significance level whether Sami's claim is justified. [5]

Question 26

Parcels arriving at a certain office have weights W kg, where the random variable W has mean μ and standard deviation 0.2. The value of μ used to be 2.60, but there is a suspicion that this may no longer be true. In order to test at the 5% significance level whether the value of μ has increased, a random sample of 75 parcels is chosen. You may assume that the standard deviation of W is unchanged.

(i) The mean weight of the 75 parcels is found to be 2.64 kg. Carry out the test. [4]

(ii) Later another test of the same hypotheses at the 5% significance level, with another random sample of 75 parcels, is carried out. Given that the value of μ is now 2.68, calculate the probability of a Type II error. [5]

Question 27

(a) Narika has a die which is known to be biased so that the probability of throwing a 6 on any throw is $\frac{1}{100}$. She uses an approximating distribution to calculate the probability of obtaining no 6s in 450 throws. Find the percentage error in using the approximating distribution for this calculation. [4]

(b) Johan claims that a certain six-sided die is biased so that it shows a 6 less often than it would if the die were fair. In order to test this claim, the die is thrown 25 times and it shows a 6 on only 2 throws. Test at the 10% significance level whether Johan's claim is justified. [5]

Question 28

At a certain hospital it was found that the probability that a patient did not arrive for an appointment was 0.2. The hospital carries out some publicity in the hope that this probability will be reduced. They wish to test whether the publicity has worked.

(i) It is suggested that the first 30 appointments on a Monday should be used for the test. Give a reason why this is not an appropriate sample. [1]

A suitable sample of 30 appointments is selected and the number of patients that do not arrive is noted. This figure is used to carry out a test at the 5% significance level.

(ii) Explain why the test is one-tail and state suitable null and alternative hypotheses. [2]

(iii) State what is meant by a Type I error in this context. [1]

(iv) Use the binomial distribution to find the critical region, and find the probability of a Type I error. [5]

(v) In fact 3 patients out of the 30 do not arrive. State the conclusion of the test, explaining your answer. [2]

Question 29

In the past, Arvinder has found that the mean time for his journey to work is 35.2 minutes. He tries a different route to work, hoping that this will reduce his journey time. Arvinder decides to take a random sample of 25 journeys using the new route. If the sample mean is less than 34.7 minutes he will conclude that the new route is quicker. Assume that, for the new route, the journey time has a normal distribution with standard deviation 5.6 minutes.

(i) Find the probability that a Type I error occurs. [4]

(ii) Arvinder finds that the sample mean is 34.5 minutes. Explain briefly why it is impossible for him to make a Type II error. [1]

Question 30

Jill shoots arrows at a target. Last week, 65% of her shots hit the target. This week Jill claims that she has improved. Out of her first 20 shots this week, she hits the target with 18 shots. Assuming shots are independent, test Jill's claim at the 1% significance level. [5]

Question 31

The number of sightings of a golden eagle at a certain location has a Poisson distribution with mean 2.5 per week. Drilling for oil is started nearby. A naturalist wishes to test at the 5% significance level whether there are fewer sightings since the drilling began. He notes that during the following 3 weeks there are 2 sightings.

(i) Find the critical region for the test and carry out the test. [5]

(ii) State the probability of a Type I error. [1]

(iii) State why the naturalist could not have made a Type II error. [1]

Question 32

In the past, the mean annual crop yield from a particular field has been 8.2 tonnes. During the last 16 years, a new fertiliser has been used on the field. The mean yield for these 16 years is 8.7 tonnes. Assume that yields are normally distributed with standard deviation 1.2 tonnes. Carry out a test at the 5% significance level of whether the mean yield has increased. [5]

Question 33

At a certain company, computer faults occur randomly and at a constant mean rate. In the past this mean rate has been 2.1 per week. Following an update, the management wish to determine whether the mean rate has changed. During 20 randomly chosen weeks it is found that 54 computer faults occur. Use a suitable approximation to test at the 5% significance level whether the mean rate has changed. [6]

Question 34

Jacques is a chef. He claims that 90% of his customers are satisfied with his cooking. Marie suspects that the true percentage is lower than 90%. She asks a random sample of 15 of Jacques' customers whether they are satisfied. She then performs a hypothesis test of the null hypothesis $p = 0.9$ against the alternative hypothesis $p < 0.9$, where p is the population proportion of customers who are satisfied. She decides to reject the null hypothesis if fewer than 12 customers are satisfied.

(i) In the context of the question, explain what is meant by a Type I error. [1]

(ii) Find the probability of a Type I error in Marie's test. [3]

Question 35

In the past, the time spent by customers in a certain shop had mean 12.5 minutes and standard deviation 4.2 minutes. Following a change of layout in the shop, the mean time spent in the shop by a random sample of 50 customers is found to be 13.5 minutes.

(i) Assuming that the standard deviation remains at 4.2 minutes, test at the 5% significance level whether the mean time spent by customers in the shop has changed. [5]

(ii) Another random sample of 50 customers is chosen and a similar test at the 5% significance level is carried out. State the probability of a Type I error. [1]

Question 36

A six-sided die shows a six on 25 throws out of 200 throws. Test at the 10% significance level the null hypothesis: P(throwing a six) = $\frac{1}{6}$, against the alternative hypothesis: P(throwing a six) < $\frac{1}{6}$. [5]

Question 37

It is claimed that 30% of packets of Froogum contain a free gift. Andre thinks that the actual proportion is less than 30% and he decides to carry out a hypothesis test at the 5% significance level. He buys 20 packets of Froogum and notes the number of free gifts he obtains.

(i) State null and alternative hypotheses for the test. [1]

(ii) Use a binomial distribution to find the probability of a Type I error. [5]

Andre finds that 3 of the 20 packets contain free gifts.

(iii) Carry out the test. [2]

Question 38

In the past the time, in minutes, taken for a particular rail journey has been found to have mean 20.5 and standard deviation 1.2. Some new railway signals are installed. In order to test whether the mean time has decreased, a random sample of 100 times for this journey are noted. The sample mean is found to be 20.3 minutes. You should assume that the standard deviation is unchanged.

(i) Carry out a significance test, at the 4% level, of whether the population mean time has decreased. [5]

Later another significance test of the same hypotheses, using another random sample of size 100, is carried out at the 4% level.

(ii) Given that the population mean is now 20.1, find the probability of a Type II error. [5]

(iii) State what is meant by a Type II error in this context. [1]

Question 39

A die has six faces numbered 1, 2, 3, 4, 5, 6. Manjit suspects that the die is biased so that it shows a six on fewer throws than it would if it were fair. In order to test her suspicion, she throws the die a certain number of times and counts the number of sixes.

(i) State suitable null and alternative hypotheses for Manjit's test. [1]

(ii) There are no sixes in the first 15 throws. Show that this result is not significant at the 5% level. [2]

(iii) Find the smallest value of n such that, if there are no sixes in the first n throws, this result is significant at the 5% level. [2]

Question 40

In the past the time, in minutes, taken for a particular rail journey has been found to have mean 20.5 and standard deviation 1.2. Some new railway signals are installed. In order to test whether the mean time has decreased, a random sample of 100 times for this journey are noted. The sample mean is found to be 20.3 minutes. You should assume that the standard deviation is unchanged.

(i) Carry out a significance test, at the 4% level, of whether the population mean time has decreased. [5]

Later another significance test of the same hypotheses, using another random sample of size 100, is carried out at the 4% level.

(ii) Given that the population mean is now 20.1, find the probability of a Type II error. [5]

(iii) State what is meant by a Type II error in this context. [1]

Question 41

At a doctors' surgery, the number of missed appointments per day has a Poisson distribution. In the past the mean number of missed appointments per day has been 0.9. Following some publicity, the manager carries out a hypothesis test to determine whether this mean has decreased. If there are fewer than 3 missed appointments in a randomly chosen 5-day period, she will conclude that the mean has decreased.

(i) Find the probability of a Type I error. [3]

(ii) State what is meant by a Type I error in this context. [1]

(iii) Find the probability of a Type II error if the mean number of missed appointments per day is 0.2. [3]

Question 42

Karim has noted the lifespans, in weeks, of a large random sample of certain insects. He carries out a test, at the 1% significance level, for the population mean, μ. Karim's null hypothesis is $\mu = 6.4$.

(i) Given that Karim's test is two-tail, state the alternative hypothesis. [1]

Karim finds that the value of the test statistic is $z = 2.43$.

(ii) Explain what conclusion he should draw. [2]

(iii) Explain briefly when a one-tail test is appropriate, rather than a two-tail test. [1]

Question 43

In the past the number of accidents per month on a certain road was modelled by a random variable with distribution Po(0.47). After the introduction of speed restrictions, the government wished to test, at the 5% significance level, whether the mean number of accidents had decreased. They noted the number of accidents during the next 12 months. It is assumed that accidents occur randomly and that a Poisson model is still appropriate.

(i) Given that the total number of accidents during the 12 months was 2, carry out the test. [6]

(ii) Explain what is meant by a Type II error in this context. [1]

It is given that the mean number of accidents per month is now in fact 0.05.

(iii) Using another random sample of 12 months the same test is carried out again, with the same significance level. Find the probability of a Type II error. [4]

Question 44

It is claimed that 1 in every 4 packets of certain biscuits contains a free gift. Marisa and André both suspect that the true proportion is less than 1 in 4.

(i) Marisa chooses 20 packets at random. She decides that if fewer than 3 contain free gifts, she will conclude that the claim is not justified. Use a binomial distribution to find the probability of a Type I error. [2]

(ii) André chooses 25 packets at random. He decides to carry out a significance test at the 1% level, using a binomial distribution. Given that only 1 of the 25 packets contains a free gift, carry out the test. [5]

Question 45

The number of sports injuries per month at a certain college has a Poisson distribution. In the past the mean has been 1.1 injuries per month. The principal recently introduced new safety guidelines and she decides to test, at the 2% significance level, whether the mean number of sports injuries has been reduced. She notes the number of sports injuries during a 6-month period.

(i) Find the critical region for the test and state the probability of a Type I error. [6]

(ii) State what is meant by a Type I error in this context. [1]

(iii) During the 6-month period there are a total of 2 sports injuries. Carry out the test. [3]

(iv) Assuming that the mean remains 1.1, calculate the probability that there will be fewer than 30 sports injuries during a 36-month period. [4]

Question 46

Past experience has shown that the heights of a certain variety of plant have mean 64.0 cm and standard deviation 3.8 cm. During a particularly hot summer, it was expected that the heights of plants of this variety would be less than usual. In order to test whether this was the case, a botanist recorded the heights of a random sample of 100 plants and found that the value of the sample mean was 63.3 cm. Stating a necessary assumption, carry out the test at the 2.5% significance level. [6]

Question 47

In order to test the effect of a drug, a researcher monitors the concentration, X, of a certain protein in the blood stream of patients. For patients who are not taking the drug the mean value of X is 0.185. A random sample of 150 patients taking the drug was selected and the values of X were found. The results are summarised below.

$$n = 150 \qquad \Sigma x = 27.0 \qquad \Sigma x^2 = 5.01$$

The researcher wishes to test at the 1% significance level whether the mean concentration of the protein in the blood stream of patients taking the drug is less than 0.185.

(i) Carry out the test. [7]

(ii) Given that, in fact, the mean concentration for patients taking the drug is 0.175, find the probability of a Type II error occurring in the test. [5]

Question 48

In the past the number of cars sold per day at a showroom has been modelled by a random variable with distribution Po(0.7). Following an advertising campaign, it is hoped that the mean number of sales per day will increase. In order to test at the 10% significance level whether this is the case, the total number of sales during the first 5 days after the campaign is noted. You should assume that a Poisson model is still appropriate.

(i) Given that the total number of cars sold during the 5 days is 5, carry out the test. [6]

The number of cars sold per day at another showroom has the independent distribution Po(0.6). Assume that the distribution for the first showroom is still Po(0.7).

(ii) Find the probability that the total number of cars sold in the two showrooms during 3 days is exactly 2. [3]

Question 49

In a certain factory the number of items per day found to be defective has had the distribution Po(1.03). After the introduction of new quality controls, the management wished to test at the 10% significance level whether the mean number of defective items had decreased. They noted the total number of defective items produced in 5 randomly chosen days. It is assumed that defective items occur randomly and that a Poisson model is still appropriate.

(i) Given that the total number of defective items produced during the 5 days was 2, carry out the test. [6]

(ii) Using another random sample of 5 days the same test is carried out again, with the same significance level. Find the probability of a Type I error. [3]

(iii) Explain what is meant by a Type I error in this context. [1]

Question 50

The masses, m kg, of packets of flour are normally distributed. The mean mass is supposed to be 1.01 kg. A quality control officer measures the masses of a random sample of 100 packets. The results are summarised below.

$$n = 100 \qquad \Sigma m = 98.2 \qquad \Sigma m^2 = 104.52$$

(i) Test at the 5% significance level whether the population mean mass is less than 1.01 kg. [7]

(ii) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

Question 51

In order to test the effect of a drug, a researcher monitors the concentration, X, of a certain protein in the blood stream of patients. For patients who are not taking the drug the mean value of X is 0.185. A random sample of 150 patients taking the drug was selected and the values of X were found. The results are summarised below.

$$n = 150 \qquad \Sigma x = 27.0 \qquad \Sigma x^2 = 5.01$$

The researcher wishes to test at the 1% significance level whether the mean concentration of the protein in the blood stream of patients taking the drug is less than 0.185.

(i) Carry out the test. [7]

(ii) Given that, in fact, the mean concentration for patients taking the drug is 0.175, find the probability of a Type II error occurring in the test. [5]

Question 52

In the past the number of cars sold per day at a showroom has been modelled by a random variable with distribution Po(0.7). Following an advertising campaign, it is hoped that the mean number of sales per day will increase. In order to test at the 10% significance level whether this is the case, the total number of sales during the first 5 days after the campaign is noted. You should assume that a Poisson model is still appropriate.

(i) Given that the total number of cars sold during the 5 days is 5, carry out the test. [6]

The number of cars sold per day at another showroom has the independent distribution Po(0.6). Assume that the distribution for the first showroom is still Po(0.7).

(ii) Find the probability that the total number of cars sold in the two showrooms during 3 days is exactly 2. [3]

Question 53

A store sells two types of computer, laptops and tablets. The number of laptops sold per hour is modelled by a random variable with distribution Po(0.9). The number of tablets sold per hour is modelled by an independent random variable with distribution Po(1.5).

(i) Find the probability that, during a randomly chosen hour, the total number of laptops and tablets sold in the store is less than 4. [3]

(ii) The manager claims that on sunny Saturdays fewer laptops than usual are sold. In order to test this claim, an employee notes the number of laptops sold during a 4-hour period on a randomly chosen sunny Saturday. In fact only 1 laptop is sold during this period. Test the manager's claim at the 10% significance level. [5]

Question 54

Packets of Frugums contain 30 sweets. The manufacturer claims that, on average, 17% of the sweets are orange flavoured. Angela suspects that the average is actually less than 17%. In order to test the manufacturer's claim, she buys a packet of Frugums. If there are fewer than 3 orange flavoured sweets in the packet, she will conclude that the claim is false.

(i) State appropriate null and alternative hypotheses. [1]

(ii) Explain what is meant by a Type I error in this situation. [1]

(iii) Calculate the probability of a Type I error. [3]

(iv) Given that the true percentage of orange flavoured sweets is 5%, calculate the probability of a Type II error. [3]

Question 55

The time taken for a particular train journey is normally distributed. In the past, the time had mean 2.4 hours and standard deviation 0.3 hours. A new timetable is introduced and on 30 randomly chosen occasions the time for this journey is measured. The mean time for these 30 occasions is found to be 2.3 hours.

(i) Stating any assumption(s), test, at the 5% significance level, whether the mean time for this journey has changed. [6]

(ii) A similar test at the 5% significance level was carried out using the times from another randomly chosen 30 occasions.

(a) State the probability of a Type I error. [1]

(b) State what is meant by a Type II error in this context. [1]

Question 56

A ten-sided spinner has edges numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Sanjeev claims that the spinner is biased so that it lands on the 10 more often than it would if it were unbiased. In an experiment, the spinner landed on the 10 in 3 out of 9 spins.

(i) Test at the 1% significance level whether Sanjeev's claim is justified. [5]

(ii) Explain why a Type I error cannot have been made. [1]

In fact the spinner is biased so that the probability that it will land on the 10 on any spin is 0.5.

(iii) Another test at the 1% significance level, also based on 9 spins, is carried out. Calculate the probability of a Type II error. [6]

Question 57

The number of absences by girls from a certain class on any day is modelled by a random variable with distribution Po(0.2). The number of absences by boys from the same class on any day is modelled by an independent random variable with distribution Po(0.3).

(i) Find the probability that, during a randomly chosen 2-day period, the total number of absences is less than 3. [3]

(ii) Find the probability that, during a randomly chosen 5-day period, the number of absences by boys is more than 3. [2]

(iii) The teacher claims that, during the football season, there are more absences by boys than usual. In order to test this claim at the 5% significance level, he notes the number of absences by boys during a randomly chosen 5-day period during the football season.

(a) State what is meant by a Type I error in this context. [1]

(b) State appropriate null and alternative hypotheses and find the probability of a Type I error. [3]

(c) In fact there were 4 absences by boys during this period. Test the teacher's claim at the 5% significance level. [3]

Question 58

The mass, in kilograms, of rocks in a certain area has mean 14.2 and standard deviation 3.1.

(i) Find the probability that the mean mass of a random sample of 50 of these rocks is less than 14.0 kg. [3]

(ii) Explain whether it was necessary to assume that the population of the masses of these rocks is normally distributed. [1]

(iii) A geologist suspects that rocks in another area have a mean mass which is less than 14.2 kg. A random sample of 100 rocks in this area has sample mean 13.5 kg. Assuming that the standard deviation for rocks in this area is also 3.1 kg, test at the 2% significance level whether the geologist is correct. [5]

Question 59

A mill owner claims that the mean mass of sacks of flour produced at his mill is 51 kg. A quality control officer suspects that the mean mass is actually less than 51 kg. In order to test the owner's claim she finds the mass, x kg, of each of a random sample of 150 sacks and her results are summarised as follows.

$$n = 150 \qquad \Sigma x = 7480 \qquad \Sigma x^2 = 380\,000$$

(i) Carry out the test at the 2.5% significance level. [7]

You may now assume that the population standard deviation of the masses of sacks of flour is 6.856 kg. The quality control officer weighs another random sample of 150 sacks and carries out another test at the 2.5% significance level.

(ii) Given that the population mean mass is 49 kg, find the probability of a Type II error. [5]

Question 60

A headteacher models the number of children who bring a 'healthy' packed lunch to school on any day by the distribution $B(150, p)$. In the past, she has found that $p = \frac{1}{3}$. Following the opening of a fast food outlet near the school, she wishes to test, at the 1% significance level, whether the value of p has decreased.

(i) State the null and alternative hypotheses for this test. [1]

On a randomly chosen day she notes the number, N, of children who bring a 'healthy' packed lunch to school. She finds that $N = 36$ and then, assuming that the null hypothesis is true, she calculates that $P(N \leqslant 36) = 0.0084$.

(ii) State, with a reason, the conclusion that the headteacher should draw from the test. [2]

(iii) According to the model, what is the largest number of children who might bring a packed lunch to school? [1]

Question 61

In the past, Angus found that his train was late on 15% of his daily journeys to work. Following a timetable change, Angus found that out of 60 randomly chosen days, his train was late on 6 days.

(i) Test at the 10% significance level whether Angus' train is late less often than it was before the timetable change. [5]

Angus used his random sample to find an $\alpha\%$ confidence interval for the proportion of days on which his train is late. The upper limit of his interval was 0.150, correct to 3 significant figures.

(ii) Calculate the value of α correct to the nearest integer. [5]

Question 62

The numbers of basketball courts in a random sample of 70 schools in South Mowland are summarised in the table.

Number of basketball courts	0	1	2	3	4	>4
Number of schools	2	28	26	10	4	0

(i) Calculate unbiased estimates for the population mean and variance of the number of basketball courts per school in South Mowland. [4]

The mean number of basketball courts per school in North Mowland is 1.9.

(ii) Test at the 5% significance level whether the mean number of basketball courts per school in South Mowland is less than the mean for North Mowland. [5]

(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(ii)**. [1]

Question 63

The time taken by volunteers to complete a certain task is normally distributed. In the past the time, in minutes, has had mean 91.4 and standard deviation 6.4. A new, similar task is introduced and the times, t minutes, taken by a random sample of 6 volunteers to complete the new task are summarised by $\Sigma t = 568.5$. Andrea plans to carry out a test, at the 5% significance level, of whether the mean time for the new task is different from the mean time for the old task.

(i) Give a reason why Andrea should use a two-tail test. [1]

(ii) State the probability that a Type I error is made, and explain the meaning of a Type I error in this context. [2]

You may assume that the times taken for the new task are normally distributed.

(iii) Stating another necessary assumption, carry out the test. [7]

Question 64

At factory A the mean number of accidents per year is 32. At factory B the records of numbers of accidents before 2018 have been lost, but the number of accidents during 2018 was 21. It is known that the number of accidents per year can be well modelled by a Poisson distribution. Use an approximating distribution to test at the 2% significance level whether the mean number of accidents at factory B is less than at factory A. [6]

Question 65

The four sides of a spinner are A, B, C, D. The spinner is supposed to be fair, but Sonam suspects that the spinner is biased so that the probability, p, that it will land on side A is greater than $\frac{1}{4}$. He spins the spinner 10 times and finds that it lands on side A 6 times.

(i) Test Sonam's suspicion using a 1% significance level. [5]

Later Sonam carries out a similar test at the 1% significance level, using another 10 spins of the spinner.

(ii) Calculate the probability of a Type I error. [2]

(iii) Assuming that the value of p is actually $\frac{3}{5}$, calculate the probability of a Type II error. [3]

Question 66

The amount of money, in dollars, spent by a customer on one visit to a certain shop is modelled by the distribution $N(\mu, 1.94)$. In the past, the value of μ has been found to be 20.00, but following a rearrangement in the shop, the manager suspects that the value of μ has changed. He takes a random sample of 6 customers and notes how much they each spend, in dollars. The results are as follows.

17.60 23.50 17.30 22.00 31.00 15.50

The manager carries out a hypothesis test using a significance level of $\alpha\%$. The test does not support his suspicion. Find the largest possible value of α. [6]

Question 67

All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day is modelled by the distribution $B(320, 0.005)$.

(i) Explain what the number 320 represents in this context. [1]

(ii) The total number of passengers who have bought seats but fail to arrive for this flight on 2 randomly chosen days is denoted by X. Use a suitable approximating distribution to find $P(2 < X < 6)$. [3]

(iii) Justify the use of your approximating distribution. [2]

After some changes, the airline wishes to test whether the mean number of passengers per day who fail to arrive for this flight has decreased.

(iv) During 5 randomly chosen days, a total of 2 passengers failed to arrive. Carry out the test at the 2.5% significance level. [5]

Question 68

The manufacturer of a certain type of biscuit claims that 10% of packets include a free offer printed on the packet. Jyothi suspects that the true proportion is less than 10%. He plans to test the claim by looking at 40 randomly selected packets and, if the number which include the offer is less than 2, he will reject the manufacturer's claim.

(i) State suitable hypotheses for the test. [1]

(ii) Find the probability of a Type I error. [3]

On another occasion Jyothi looks at 80 randomly selected packets and finds that exactly 6 include the free offer.

(iii) Calculate an approximate 90% confidence interval for the proportion of packets that include the offer. [3]

(iv) Use your confidence interval to comment on the manufacturer's claim. [1]

Question 69

It is claimed that, on average, a particular train journey takes less than 1.9 hours. The times, t hours, taken for this journey on a random sample of 50 days were recorded. The results are summarised below.

$$n = 50 \qquad \Sigma t = 92.5 \qquad \Sigma t^2 = 175.25$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test the claim at the 5% significance level. [5]

Question 70

Sumitra has a six-sided die. She suspects that it is biased so that it shows a six less often than it would if it were fair. She decides to test the die by throwing it 30 times and noting the number of throws on which it shows a six.

(i) It shows a six on exactly 2 throws. Use a binomial distribution to carry out the test at the 5% significance level. [5]

(ii) Later, Sumitra repeats the test at the 5% significance level by throwing the die 30 times again. Find the probability of a Type I error in this second test. [2]

Question 71

The time, in minutes, that John takes to travel to work has a normal distribution. Last year the mean and standard deviation were 26.5 and 4.8 respectively. This year John uses a different route and he finds that the mean time for his first 150 journeys is 27.5 minutes.

(i) Stating a necessary assumption, test at the 1% significance level whether the mean time for his journey to work has increased. [6]

(ii) State, with a reason, whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

Question 72

Bob is a self-employed builder. In the past his weekly income had mean \$546 and standard deviation \$120. Following a change in Bob's working pattern, his mean weekly income for 40 randomly chosen weeks was \$581. You should assume that the standard deviation remains unchanged at \$120.

(i) Test at the 2.5% significance level whether Bob's mean weekly income has increased. [5]

Bob finds his mean weekly income for another random sample of 40 weeks and carries out a similar test at the 2.5% significance level.

(ii) Given that Bob's mean weekly income is now in fact \$595, find the probability of a Type II error. [5]

Question 73

A train company claims that 92% of trains on a particular line arrive on time. Sanjeep suspects that the true percentage is less than 92%. He chooses a random sample of 20 trains on this line and finds that exactly 16 of them arrive on time. Making an assumption that should be stated, test at the 5% significance level whether Sanjeep's suspicion is justified. [6]

Question 74

The number of accidents per month, X, at a factory has a Poisson distribution. In the past the mean has been 1.1 accidents per month. Some new machinery is introduced and the management wish to test whether the mean has increased. They note the number of accidents in a randomly chosen month and carry out a hypothesis test at the 1% significance level.

(i) Show that the critical region for the test is $X \geqslant 5$. Given that the number of accidents is 6, carry out the test. [6]

Later they carry out a similar test, also at the 1% significance level.

(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]

(iii) Given that the mean is now 7.0, find the probability of a Type II error. [2]

Question 75

The heights of a certain species of animal have been found to have mean 65.2 cm and standard deviation 7.1 cm. A researcher suspects that animals of this species in a certain region are shorter on average than elsewhere. She takes a large random sample of n animals of this species from this region and finds that their mean height is 63.2 cm. She then carries out an appropriate hypothesis test.

(i) She finds that the value of the test statistic z is -2.182, correct to 3 decimal places.

(a) Stating a necessary assumption, calculate the value of n. [4]

(b) Carry out the hypothesis test at the 4% significance level. [3]

(ii) Explain why it was necessary to use the Central Limit theorem in carrying out the test. [1]

Question 76

A national survey shows that 95% of year 12 students use social media. Arvin suspects that the percentage of year 12 students at his college who use social media is less than the national percentage. He chooses a random sample of 20 students at his college and notes the number who use social media. He then carries out a test at the 2% significance level.

(a) Find the rejection region for the test. [4]

(b) Find the probability of a Type I error. [1]

(c) Jimmy believes that the true percentage at Arvin's college is 70%. Assuming that Jimmy is correct, find the probability of a Type II error. [3]

Question 77

In the past, the mean time taken by Freda for a particular daily journey was 39.2 minutes. Following the introduction of a one-way system, Freda wishes to test whether the mean time for the journey has decreased. She notes the times, t minutes, for 40 randomly chosen journeys and summarises the results as follows.

$$n = 40 \qquad \Sigma t = 1504 \qquad \Sigma t^2 = 57\,760$$

(a) Calculate unbiased estimates of the population mean and variance of the new journey time. [3]

(b) Test, at the 5% significance level, whether the population mean time has decreased. [5]

Question 78

A market researcher is investigating the length of time that customers spend at an information desk. He plans to choose a sample of 50 customers on a particular day.

(a) He considers choosing the first 50 customers who visit the information desk.

Explain why this method is unsuitable. [1]

The actual lengths of time, in minutes, that customers spend at the information desk may be assumed to have mean μ and variance 4.8. The researcher knows that in the past the value of μ was 6.0. He wishes to test, at the 2% significance level, whether this is still true. He chooses a random sample of 50 customers and notes how long they each spend at the information desk.

(b) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(c) Given that the mean time spent at the information desk by the 50 customers is 6.8 minutes, carry out the test. [5]

(d) Give a reason why it was necessary to use the Central Limit theorem in your answer to part **(c)**. [1]

Question 79

The number of customers who visit a particular shop between 9.00 am and 10.00 am has the distribution $Po(\lambda)$. In the past the value of λ was 5.2. Following some new advertising, the manager wishes to test whether the value of λ has increased. He chooses a random sample of 20 days and finds that the total number of customers who visited the shop between 9.00 am and 10.00 am on those days is 125.

Use an approximating distribution to test at the 2.5% significance level whether the value of λ has increased. [6]

Question 80

The score on one spin of a 5-sided spinner is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Show that $Var(X) = 1.2$. [2]

The spinner is spun 200 times. The score on each spin is noted and the mean, $\overline{X}$, of the 200 scores is found.

(b) Given that $P(\overline{X} > a) = 0.1$, find the value of a. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(b)**. [1]

(d) Johann has another, similar, spinner. He suspects that it is biased so that the mean score is less than 2. He spins his spinner 200 times and finds that the mean of the 200 scores is 1.86.

Given that the variance of the score on one spin of this spinner is also 1.2, test Johann's suspicion at the 5% significance level. [5]

Question 81

A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are Grade A. Following a change in growing conditions at the farm, the shop management plan to carry out a hypothesis test to find out whether the proportion of Grade A apples has increased. They select 25 apples at random. If the number of Grade A apples is more than 3 they will conclude that the proportion has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the probability of a Type I error. [3]

In fact 2 of the 25 apples were Grade A.

(c) Which of the errors, Type I or Type II, is possible? Justify your answer. [2]

Question 82

A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $\text{Var}(X) = 2$. [1]

Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. [4]

Question 83

In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5% significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. [2]

(b) Carry out the test. [5]

Question 84

The time, in minutes, for Anjan's journey to work on Mondays has mean 38.4 and standard deviation 6.9.

(a) Find the probability that Anjan's mean journey time for a random sample of 30 Mondays is between 38 and 40 minutes. [5]

Anjan wishes to test whether his mean journey time is different on Tuesdays. He chooses a random sample of 30 Tuesdays and finds that his mean journey time for these 30 Tuesdays is 40.2 minutes. Assume that the standard deviation for his journey time on Tuesdays is 6.9 minutes.

(b) **(i)** State, with a reason, whether Anjan should use a one-tail or a two-tail test. [1]

(ii) Carry out the test at the 10% significance level. [5]

(iii) Explain whether it was necessary to use the Central Limit theorem in part **(b)(ii)**. [1]

Question 85

The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. [2]

Following a change in working conditions, the management wished to test whether the mean number of absences has decreased. They found that, in a randomly chosen 3-week period, there were exactly 2 absences.

(d) Carry out the test at the 10% significance level. [5]

(e) State, with a reason, which of the errors, Type I or Type II, might have been made in carrying out the test in part **(d)**. [2]

Question 86

A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.

(a) Carry out the test at the 10% significance level. [5]

(b) Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer.

Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3]

(c) Explain what is meant by a Type II error in this context. [1]

Question 87

The areas, $X\ \text{cm}^2$, of petals of a certain kind of flower have mean $\mu\ \text{cm}^2$. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

Question 88

It is known that 8% of adults in a certain town own a Chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He chooses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds that 4 of the 25 adults own a Chantor car.

Carry out a hypothesis test at the 5% significance level. [5]

(b) Explain which of the errors, Type I or Type II, might have been made in carrying out the test in part **(a)**. [2]

Later, the car dealer takes another random sample of 25 adults from the town and carries out a similar hypothesis test at the 5% significance level.

(c) Find the probability of a Type I error. [3]

Question 89

An architect wishes to investigate whether the buildings in a certain city are higher, on average, than buildings in other cities. He takes a large random sample of buildings from the city and finds the mean height of the buildings in the sample. He calculates the value of the test statistic, z, and finds that $z = 2.41$.

(a) Explain briefly whether he should use a one-tail test or a two-tail test. [1]

(b) Carry out the test at the 1% significance level. [3]

Question 90

The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for a test of Jane's claim. [1]

(ii) Test at the 5% significance level whether Jane's claim is justified. [4]

(b) Jane finds that the number of accidents per year has been gradually increasing over recent years.

State how this might affect the validity of the test carried out in part **(a)(ii)**. [1]

Question 91

In the past, the time, in hours, for a particular train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased.

(a) State what is meant by a Type II error in this context. [1]

(b) The mean time for a random sample of 50 journeys is found to be 1.36 hours.

Assuming that the standard deviation of journey times is still 0.12 hours, test at the 2.5% significance level whether the population mean journey time has decreased. [5]

(c) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(b)**. [2]

Question 92

The time, in minutes, spent by customers at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

(a) State what is meant by a Type I error in this context. [1]

(b) The mean time for a sample of 20 customers is found to be 45.6 minutes.

Test at the 2.5% significance level whether the value of μ has changed. [5]

Question 93

In a game, a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the ball will land in slot A is $\frac{1}{4}$.

(a) State suitable null and alternative hypotheses for Raju's test. [1]

The ball is thrown 100 times and it lands in slot A 15 times.

(b) Use a suitable approximating distribution to carry out the test at the 2% significance level. [5]

Question 94

At a certain large school it was found that the proportion of students not wearing correct uniform was 0.15. The school sent a letter to parents asking them to ensure that their children wear the correct uniform. The school now wishes to test whether the proportion not wearing correct uniform has been reduced.

(a) It is suggested that a random sample of the students in Grade 12 should be used for the test.

Give a reason why this would not be an appropriate sample. [1]

A suitable sample of 50 students is selected and the number not wearing correct uniform is noted. This figure is used to carry out a test at the 5% significance level.

(b) State suitable null and alternative hypotheses. [1]

(c) Use a binomial distribution to find the probability of a Type I error. You must justify your answer fully. [5]

(d) In fact 4 students out of the 50 are not wearing correct uniform.

State the conclusion of the test, explaining your answer. [2]

(e) State, with a reason, which of the errors, Type I or Type II, may have been made. [2]

Question 95

The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of the 100 apples is found to be 63.5 g.

Carry out the test at the 2.5% significance level. [5]

(b) Later another test of the same hypotheses at the 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out.

Given that the value of μ is in fact 62.7, calculate the probability of a Type II error. [5]

Question 96

(a) The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

Question 97

A machine is supposed to produce random digits. Bob thinks that the machine is not fair and that the probability of it producing the digit 0 is less than $\frac{1}{10}$. In order to test his suspicion he notes the number of times the digit 0 occurs in 30 digits produced by the machine. He carries out a test at the 10% significance level.

(a) State suitable null and alternative hypotheses. [1]

(b) Find the rejection region for the test. [4]

(c) State the probability of a Type I error. [1]

It is now given that the machine actually produces a 0 once in every 40 digits, on average.

(d) Find the probability of a Type II error. [3]

Question 98

A certain kind of firework is supposed to last for 30 seconds, on average, after it is lit. An inspector suspects that the fireworks actually last a shorter time than this, on average. He takes a random sample of 100 fireworks of this kind. Each firework in the sample is lit and the time it lasts is noted.

(a) Give a reason why it is necessary to take a sample rather than testing all the fireworks of this kind. [1]

It is given that the population standard deviation of the times that fireworks of this kind last is 5 seconds.

(b) The mean time lasted by the 100 fireworks in the sample is found to be 29 seconds.

Test the inspector's suspicion at the 1% significance level. [5]

(c) State with a reason whether the Central Limit theorem was needed in the solution to part **(b)**. [1]

Question 99

The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of the 100 apples is found to be 63.5 g.

Carry out the test at the 2.5% significance level. [5]

(b) Later another test of the same hypotheses at the 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out.

Given that the value of μ is in fact 62.7, calculate the probability of a Type II error. [5]

Question 100

(a) The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

Question 101

In the past the time, in minutes, taken by students to complete a certain challenge had mean 25.5 and standard deviation 5.2. A new challenge is devised and it is expected that students will take, on average, less than 25.5 minutes to complete this challenge. A random sample of 40 students is chosen and their mean time for the new challenge is found to be 23.7 minutes.

(a) Assuming that the standard deviation of the time for the new challenge is 5.2 minutes, test at the 1% significance level whether the population mean time for the new challenge is less than 25.5 minutes. [5]

(b) State, with a reason, whether it is possible that a Type I error was made in the test in part **(a)**. [1]

Question 102

Harry has a five-sided spinner with sectors coloured blue, green, red, yellow and black. Harry thinks the spinner may be biased. He plans to carry out a hypothesis test with the following hypotheses.

$$H_0\text{: P(the spinner lands on blue)} = \tfrac{1}{5}$$

$$H_1\text{: P(the spinner lands on blue)} \neq \tfrac{1}{5}$$

Harry spins the spinner 300 times. It lands on blue on 45 spins.

Use a suitable approximation to carry out Harry's test at the 5% significance level. [5]

Question 103

Batteries of type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carrying out this test. [1]

A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbiased estimate of the population variance is 225 hours^2.

(b) Test, at the 2% significance level, whether type B batteries have a shorter mean life than type A batteries. [5]

(c) Calculate a 94% confidence interval for the population mean life of type B batteries. [3]

Question 104

Anton believes that 10% of students at his college are left-handed. Aliya believes that this is an under-estimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against the alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if at least 5 of these students are left-handed.

(a) Explain what is meant by a Type I error in this context. [1]

(b) Find the probability of a Type I error in the test. [3]

(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test. [2]

Question 105

The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the critical region for the test. [4]

The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

(c) Carry out the test. [2]

(d) State, with a reason, whether it is possible that a Type I error has been made in carrying out the test in part **(c)**. [1]

The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

(e) Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

Question 106

In the past, the mean height of plants of a particular species has been 2.3 m. A random sample of 60 plants of this species was treated with fertiliser and the mean height of these 60 plants was found to be 2.4 m. Assume that the standard deviation of the heights of plants treated with fertiliser is 0.4 m.

Carry out a test at the 2.5% significance level of whether the mean height of plants treated with fertiliser is greater than 2.3 m. [5]

Question 107

In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypotheses for Jenny's test. [1]

(b) Find the probability that Jenny will make a Type I error. [3]

(c) Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both. [2]

Question 108

Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of the die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the correct score on exactly 5 out of 15 throws.

Test Arvind's claim at the 10% significance level. [5]

Question 109

A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

Question 110

In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

Question 111

In the past, the mean length of a particular variety of worm has been 10.3 cm, with standard deviation 2.6 cm. Following a change in the climate, it is thought that the mean length of this variety of worm has decreased. The lengths of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9.8 cm.

Assuming that the standard deviation remains at 2.6 cm, carry out a test at the 2% significance level of whether the mean length has decreased. [5]

Question 112

The number of faults in cloth made on a certain machine has a Poisson distribution with mean 2.4 per 10 m^2. An adjustment is made to the machine. It is required to test at the 5% significance level whether the mean number of faults has decreased. A randomly selected 30 m^2 of cloth is checked and the number of faults is found.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the probability of a Type I error. [3]

Exactly 3 faults are found in the randomly selected 30 m^2 of cloth.

(c) Carry out the test at the 5% significance level. [2]

Later a similar test was carried out at the 5% significance level, using another randomly selected 30 m^2 of cloth.

(d) Given that the number of faults actually has a Poisson distribution with mean 0.5 per 10 m^2, find the probability of a Type II error. [2]

Question 113

In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

Question 114

A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

Question 115

Last year, the mean time taken by students at a school to complete a certain test was 25 minutes. Akash believes that the mean time taken by this year's students was less than 25 minutes. In order to test this belief, he takes a large random sample of this year's students and he notes the time taken by each student. He carries out a test, at the 2.5% significance level, for the population mean time, μ minutes. Akash uses the null hypothesis H_0: $\mu = 25$.

(a) Give a reason why Akash should use a one-tailed test. [1]

Akash finds that the value of the test statistic is $z = -2.02$.

(b) Explain what conclusion he should draw. [2]

In a different one-tailed hypothesis test the z-value was found to be 2.14.

(c) Given that this value would lead to a rejection of the null hypothesis at the $\alpha\%$ significance level,

The population mean time taken by students at another school to complete a test last year was m minutes. Sorin carries out a one-tailed test to determine whether the population mean this year is less than m, using a random sample of 100 students. He assumes that the population standard deviation of the times is 3.9 minutes. The sample mean is 24.8 minutes, and this result just leads to the rejection of the null hypothesis at the 5% significance level.

(d) Find the value of m. [3]

Question 116

The number of accidents per 3-month period on a certain road has the distribution $\mathrm{Po}(\lambda)$. In the past the value of λ has been 5.7. Following some changes to the road, the council carries out a hypothesis test to determine whether the value of λ has decreased. If there are fewer than 3 accidents in a randomly chosen 3-month period, the council will conclude that the value of λ has decreased.

(a) Find the probability of a Type I error. [2]

(b) Find the probability of a Type II error if the mean number of accidents per 3-month period is now actually 0.9. [3]

Question 117

A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n, of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

(b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error. [2]

(c) State what is meant by a Type I error in this context. [1]

(d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation. [2]

Question 118

Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

(c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering **(b)**? [1]

Question 119

When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

(a) Use a binomial distribution to find the probability of a Type I error. [2]

The true probability of being awarded a medal is denoted by p.

(b) Given that the probability of a Type II error is 0.8926, find the value of p. [3]

Question 120

The masses, in kilograms, of newborn babies in country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, B, is different from that in country A. He chooses a random sample of 60 newborn babies in country B and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part **(a)** are the correct values for μ and σ^2. Assume also that the variance of the masses of newborn babies in country B is the same as in country A.

(b) Carry out the test at the 1% significance level. [5]

Question 121

The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the critical region for the test and state the probability of a Type I error. [6]

(c) State what is meant by a Type I error in this context. [1]

(d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

(e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

Question 122

In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level. [6]

Question 123

A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Question 124

In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

Question 125

The height H, in metres, of mature trees of a certain variety is normally distributed with standard deviation 0.67. In order to test whether the population mean of H is greater than 4.23, the heights of a random sample of 200 trees are measured.

(a) Write down suitable null and alternative hypotheses for the test. [1]

The sample mean height, $\bar{h}$ metres, of the 200 trees is found and the test is carried out. The result of the test is to reject the null hypothesis at the 5% significance level.

(b) Find the set of possible values of $\bar{h}$. [3]

(c) Ajit said, 'In **(b)** we had to assume that $\bar{H}$ is normally distributed, so it was necessary to use the Central Limit Theorem.'

Explain whether you agree with Ajit. [1]

Question 126

A researcher read a magazine article which stated that boys aged 1 to 3 prefer green to orange. It claimed that, when offered a green cube and an orange cube to play with, a boy is more likely to choose the green one.

The researcher disagrees with this claim. She believes that boys of this age are equally likely to choose either colour. In order to test her belief, the researcher carried out a hypothesis test at the 5% significance level. She offered a green cube and an orange cube to each of 10 randomly chosen boys aged 1 to 3, and recorded the number, X, of boys who chose the green cube.

Out of the 10 boys, 8 boys chose the green cube.

(a) **(i)** Assuming that the researcher's belief that either colour cube is equally likely to be chosen is valid, a student correctly calculates that $P(X = 8) = 0.0439$, correct to 3 significant figures. He says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

(ii) Carry out the test on the researcher's claim that either colour cube is equally likely to be chosen. [5]

(b) Another researcher claims that a Type I error was made in carrying out the test.

Explain why this cannot be true. [1]

A similar test, at the 5% significance level, was carried out later using 10 other randomly chosen boys aged 1 to 3.

(c) Find the probability of a Type I error. [2]

Question 127

The heights, in centimetres, of adult females in Litania have mean μ and standard deviation σ. It is known that in 2004 the values of μ and σ were 163.21 and 6.95 respectively. The government claims that the value of μ this year is greater than it was in 2004. In order to test this claim a researcher plans to carry out a hypothesis test at the 1% significance level. He records the heights of a random sample of 300 adult females in Litania this year and finds the value of the sample mean.

(a) State the probability of a Type I error. [1]

You should assume that the value of σ after 2004 remains at 6.95 .

(b) Given that the value of μ this year is actually 164.91, find the probability of a Type II error. [5]

Question 128

A teacher models the numbers of girls and boys who arrive late for her class on any day by the independent random variables $G \sim \text{Po}(0.10)$ and $B \sim \text{Po}(0.15)$ respectively.

(a) Find the probability that during a randomly chosen 2-day period no girls arrive late. [1]

(b) Find the probability that during a randomly chosen 5-day period the total number of students who arrive late is less than 3. [3]

(c) It is given that the values of $P(G = r)$ and $P(B = r)$ for $r \geqslant 3$ are very small and can be ignored.

Find the probability that on a randomly chosen day more girls arrive late than boys. [3]

Following a timetable change the teacher claims that on average more students arrive late than before the change. During a randomly chosen 5-day period a total of 4 students are late.

(d) Test the teacher's claim at the 5% significance level. [5]

Question 128

In this question you should **not** use an approximating distribution.

At an election in Menham last year, 24% of voters supported the Today Party. A student wishes to test whether support for the Today Party has decreased since last year. He chooses a random sample of 25 voters in Menham and finds that exactly 2 of them say that they support the Today Party.

Test at the 5% significance level whether support for the Today Party has decreased. [5]

Question 129

The masses of cereal boxes filled by a certain machine have mean 510 grams. An adjustment is made to the machine and an inspector wishes to test whether the mean mass of cereal boxes filled by the machine has decreased.

After the adjustment is made, he chooses a random sample of 120 cereal boxes. The mean mass of these boxes is found to be 508 grams.

Assume that the standard deviation of the masses is 10 grams.

(a) Test at the 2.5% significance level whether the mean mass of cereal boxes filled by the machine has decreased. [5]

Later the inspector carries out a similar test at the 2.5% significance level, using the same hypotheses and another 120 randomly chosen cereal boxes.

(b) Given that the mean mass is now actually 506 grams, find the probability of a Type II error. [5]

Question 130

Every July, as part of a research project, Rita collects data about sightings of a particular kind of bird. Each day in July she notes whether she sees this kind of bird or not, and she records the number X of days on which she sees it. She models the distribution of X by $B(31, p)$, where p is the probability of seeing this kind of bird on a randomly chosen day in July.

Data from previous years suggests that $p = 0.3$, but in 2022 Rita suspected that the value of p had been reduced. She decided to carry out a hypothesis test.

In July 2022, she saw this kind of bird on 4 days.

(a) Use the binomial distribution to test at the 5% significance level whether Rita's suspicion is justified. [5]

In July 2023, she noted the value of X and carried out another test at the 5% significance level using the same hypotheses.

(b) Calculate the probability of a Type I error. [2]

Rita models the number of sightings, Y, per year of a different, very rare, kind of bird by the distribution $B(365, 0.01)$.

(c) **(i)** Use a suitable approximating distribution to find $P(Y = 4)$. [3]

(ii) Justify your approximating distribution in this context. [1]

Question 131

The number of accidents per year on a certain road has the distribution $\mathrm{Po}(\lambda)$. In the past the value of λ was 3.3 . Recently, a new speed limit was imposed and the council wishes to test whether the value of λ has decreased. The council notes the total number, X, of accidents during **two** randomly chosen years after the speed limit was introduced and it carries out a test at the 5% significance level.

(a) Calculate the probability of a Type I error. [4]

(b) Given that $X = 2$, carry out the test. [3]

(c) The council decides to carry out another similar test at the 5% significance level using the same hypotheses and two different randomly chosen years.

Given that the true value of λ is 0.6, calculate the probability of a Type II error. [3]

(d) Using $\lambda = 0.6$ and a suitable approximating distribution, find the probability that there will be more than 10 accidents in 30 years. [4]

Question 132

The lengths, in centimetres, of worms of a certain kind are normally distributed with mean μ and standard deviation 2.3 . An article in a magazine states that the value of μ is 12.7 . A scientist wishes to test whether this value is correct. He measures the lengths, x cm, of a random sample of 50 worms of this kind and finds that $\Sigma x = 597.1$. He plans to carry out a test, at the 1% significance level, of whether the true value of μ is different from 12.7 .

(a) State, with a reason, whether he should use a one-tailed or a two-tailed test. [1]

(b) Carry out the test. [5]

Question 133

The heights of one-year-old trees of a certain variety are known to have mean 2.3 m. A scientist believes that, on average, trees of this age and variety in her region are slightly taller than in other places. She plans to carry out a hypothesis test, at the 2% significance level, in order to test her belief.

(a) State the probability that she will make a Type I error. [1]

She takes a random sample of 100 such trees in her region and measures their heights, h m. Her results are summarised below.

$$n = 100 \qquad \Sigma h = 238 \qquad \Sigma h^2 = 580$$

(b) Carry out the test at the 2% significance level. [7]

(c) The scientist carries out the test correctly, but another scientist claims that she has made a Type II error.

Comment on this claim. [1]

Question 134

A factory owner models the number of employees who use the factory canteen on any day by the distribution $\mathrm{B}(25, p)$. In the past the value of p was 0.8 . A new menu is introduced in the canteen and the owner wants to test whether the value of p has increased.

On a randomly chosen day he notes that the number of employees who use the canteen is 23.

(a) Use the binomial distribution to carry out the test at the 10% significance level. [5]

(b) Given that there are 30 employees at the factory comment on the suitability of the owner's model. [1]

Question 135

The lengths, in centimetres, of worms of a certain kind are normally distributed with mean μ and standard deviation 2.3 . An article in a magazine states that the value of μ is 12.7 . A scientist wishes to test whether this value is correct. He measures the lengths, x cm, of a random sample of 50 worms of this kind and finds that $\sum x = 597.1$. He plans to carry out a test, at the 1% significance level, of whether the true value of μ is different from 12.7 .

(a) State, with a reason, whether he should use a one-tailed or a two-tailed test. [1]

(b) Carry out the test. [5]

Question 136

Amir believes that 20% of the students at his college are left-handed. His friend believes that the true proportion, p, is less than 20%. Amir plans to use the binomial distribution to test the null hypothesis, H_0: $p = 0.2$, against the alternative hypothesis, H_1: $p < 0.2$.

He decides to choose 35 students at random. If 3 or fewer of these students are left-handed, Amir will reject his belief.

(a) Find the significance level of the test. [3]

(b) State the probability of a Type I error. [1]

It is now given that the true value of p is 0.05.

(c) Find the probability of a Type II error. [3]

Question 136

A researcher records the time, T seconds, taken by adults to complete a questionnaire.

The results for a random sample of 60 adults who completed the questionnaire this year are summarised as follows.

$$n = 60 \qquad \sum t = 3678 \qquad \sum t^2 = 226\,313.36$$

(a) Find an unbiased estimate of $E(T)$, and show that an unbiased estimate of $\mathrm{Var}(T)$ is 14.44. [3]

In the past, the population mean time was 62.4 seconds.

(b) Test at the 2% significance level whether the population mean time for this year is less than 62.4 seconds. [5]

(c) State, with a reason, whether it was necessary to use the Central Limit Theorem in your answer to part **(b)**. [1]

Question 137

In the past, one quarter of job applicants at a certain firm had first-class degrees. A change is made in the job description and a director of the firm believes that, on average, the proportion of job applicants with first class degrees has decreased.

In the month following the change, there were 35 job applicants, and r of these had first-class degrees. The firm carried out a hypothesis test at the 4% significance level to test the director's belief.

(a) Use a binomial distribution to find the largest value of r that would provide sufficient evidence that the director's belief is correct. [6]

In another month, the director carries out a similar test at the 4% significance level using the 35 job applicants from that month.

(b) Explain the meaning of a Type I error in this context, and state the probability of a Type I error. [2]

(c) Given that the proportion of job applicants with first class degrees this year is actually 0.05, find the probability of a Type II error. [2]

Question 138

The lengths of pencils made at a factory are normally distributed. The standard deviation of the lengths is σ cm, and the mean is supposed to be 10 cm. An inspector thinks that the mean is actually greater than 10 cm. He takes a random sample of 50 pencils produced at the factory and finds that the mean of these 50 lengths is 10.03 cm. He then carries out a hypothesis test.

(a) He finds that the value of the test statistic z is 1.995 correct to 3 decimal places.

(i) Calculate the value of σ. [3]

(ii) Carry out the hypothesis test at the 2.5% significance level. [3]

(b) Explain whether it was necessary to use the Central Limit Theorem in carrying out the test. [1]

Question 139

Birgitte has a six-sided dice. She suspects that the dice is biased so that the probability, p, that it will show a six on one throw is less than $\frac{1}{6}$. She throws the dice 30 times and finds that it shows a six on exactly 2 throws.

(a) Use a binomial distribution with a 5% significance level to test Birgitte's suspicion. [5]

Later, Birgitte carries out a similar test at the 5% significance level, using another 30 throws of the dice.

(b) Calculate the probability of a Type I error. [2]

(c) Given that the value of p is actually 0.02, calculate the probability of a Type II error. [3]

Question 140

The amount of time, in minutes, spent by a customer on one visit to a certain shop is modelled by the random variable $X \sim N(\mu, \sigma^2)$. In the past, the values of μ and σ were 10.5 and 3.8 respectively. The shop has recently moved to a new location, and the manager hopes that the new value of μ will be greater than 10.5. He takes a random sample of 10 customers and notes the time they each spend in the shop. He then calculates the sample mean $\overline{x}$ for these 10 times.

Using a hypothesis test at the 5% significance level, the manager finds that there is sufficient evidence to conclude that the new value of μ is greater than 10.5.

Stating a necessary assumption, find the smallest possible value of $\overline{x}$. [4]

Question 141

A manufacturer of cell phones claims that 25% of students own a Pumpkin phone. Jeyeraj thinks that the proportion of students at his large college who own a Pumpkin phone is less than 25%. He plans to test the manufacturer's claim. He chooses a random sample of 30 students at his college. If the number of students who own a Pumpkin phone is less than 5, Jeyeraj will reject the manufacturer's claim.

(a) State suitable hypotheses for the test. [1]

(b) Given that the true proportion of students at the college who own a Pumpkin phone is 10%, use a binomial distribution to find the probability of a Type II error. [3]

At Florence's college, in a random sample of 40 students, it was found that 5 own a Pumpkin phone.

(c) Calculate an approximate 95% confidence interval for the proportion of students at Florence's college who own a Pumpkin phone. [3]

Question 142

The time, T minutes, for a certain daily bus journey is normally distributed. The bus company claims that the mean of T is 45. A passenger believes that the mean of T is actually greater than 45. She notes the times taken for this journey on a random sample of 60 days. The results are summarised below.

$$n = 60 \qquad \sum t = 2750 \qquad \sum t^2 = 127000$$

(a) Calculate unbiased estimates of the population mean and variance. [3]

(b) Test the passenger's belief at the 5% significance level. [5]