A-Level

Topic : Linear Combination of Random Variable

May 2013-May 2023

Questions

Question 1

The mean and variance of the random variable X are 5.8 and 3.1 respectively. The random variable S is the sum of three independent values of X. The independent random variable T is defined by $T = 3X + 2$.

(i) Find the variance of S. [1]

(ii) Find the variance of T. [1]

(iii) Find the mean and variance of $S - T$. [3]

Question 2

Packets of cereal are packed in boxes, each containing 6 packets. The masses of the packets are normally distributed with mean 510 g and standard deviation 12 g. The masses of the empty boxes are normally distributed with mean 70 g and standard deviation 4 g.

(i) Find the probability that the total mass of a full box containing 6 packets is between 3050 g and 3150 g. [5]

(ii) A packet and an empty box are chosen at random. Find the probability that the mass of the packet is at least 8 times the mass of the empty box. [5]

Question 3

Weights of cups have a normal distribution with mean 91 g and standard deviation 3.2 g. Weights of saucers have an independent normal distribution with mean 72 g and standard deviation 2.6 g. Cups and saucers are chosen at random to be packed in boxes, with 6 cups and 6 saucers in each box. Given that each empty box weighs 550 g, find the probability that the total weight of a box containing 6 cups and 6 saucers exceeds 1550 g. [5]

Question 4

The lifetimes, in hours, of Longlive light bulbs and Enerlow light bulbs have the independent distributions $N(1020, 45^2)$ and $N(2800, 52^2)$ respectively.

(i) Find the probability that the total of the lifetimes of 5 randomly chosen Longlive bulbs is less than 5200 hours. [4]

(ii) Find the probability that the lifetime of a randomly chosen Enerlow bulb is at least 3 times that of a randomly chosen Longlive bulb. [6]

Question 5

Kieran and Andreas are long-jumpers. They model the lengths, in metres, that they jump by the independent random variables $K \sim N(5.64, 0.0576)$ and $A \sim N(4.97, 0.0441)$ respectively. They each make a jump and measure the length. Find the probability that

(i) the sum of the lengths of their jumps is less than 11 m, [4]

(ii) Kieran jumps more than 1.2 times as far as Andreas. [6]

Question 6

In an examination, the marks in the theory paper and the marks in the practical paper are denoted by the random variables X and Y respectively, where $X \sim N(57, 13)$ and $Y \sim N(28, 5)$. You may assume that each candidate's marks in the two papers are independent. The final score of each candidate is found by calculating $X + 2.5Y$. A candidate is chosen at random. Without using a continuity correction, find the probability that this candidate

(i) has a final score that is greater than 140, [5]

(ii) obtains at least 20 more marks in the theory paper than in the practical paper. [5]

Question 7

Each day Samuel travels from A to B and from B to C. He then returns directly from C to A. The times, in minutes, for these three journeys have the independent distributions $N(20, 2^2)$, $N(18, 1.5^2)$ and $N(30, 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A. [5]

Question 8

The masses, in grams, of tomatoes of type A and type B have the distributions $N(125, 30^2)$ and $N(130, 32^2)$ respectively.

(i) Find the probability that the total mass of 4 randomly chosen tomatoes of type A and 6 randomly chosen tomatoes of type B is less than 1.5 kg. [5]

(ii) Find the probability that a randomly chosen tomato of type A has a mass that is at least 90% of the mass of a randomly chosen tomato of type B. [5]

Question 9

The masses, in grams, of potatoes of types A and B have the distributions $N(175, 60^2)$ and $N(105, 28^2)$ respectively. Find the probability that a randomly chosen potato of type A has a mass that is at least twice the mass of a randomly chosen potato of type B. [5]

Question 10

The independent random variables X and Y have standard deviations 3 and 6 respectively. Calculate the standard deviation of $4X - 5Y$. [3]

Question 11

The independent variables X and Y are such that $X \sim B(10, 0.8)$ and $Y \sim Po(3)$. Find

(i) $E(7X + 5Y - 2)$, [2]

(ii) $Var(4X - 3Y + 3)$, [4]

(iii) $P(2X - Y = 18)$. [4]

Question 12

The weights, in kilograms, of men and women have the distributions $N(78, 7^2)$ and $N(66, 5^2)$ respectively.

(i) The maximum load that a certain cable car can carry safely is 1200 kg. If 9 randomly chosen men and 7 randomly chosen women enter the cable car, find the probability that the cable car can operate safely. [5]

(ii) Find the probability that a randomly chosen woman weighs more than a randomly chosen man. [4]

Question 13

The masses, in grams, of large bags of sugar and small bags of sugar are denoted by X and Y respectively, where $X \sim N(5.1, 0.2^2)$ and $Y \sim N(2.5, 0.1^2)$. Find the probability that the mass of a randomly chosen large bag is less than twice the mass of a randomly chosen small bag. [5]

Question 14

A fair six-sided die is thrown 20 times and the number of sixes, X, is recorded. Another fair six-sided die is thrown 20 times and the number of odd-numbered scores, Y, is recorded. Find the mean and standard deviation of $X + Y$. [5]

Question 15

Bags of sugar are packed in boxes, each box containing 20 bags. The masses of the boxes, when empty, are normally distributed with mean 0.4 kg and standard deviation 0.01 kg. The masses of the bags are normally distributed with mean 1.02 kg and standard deviation 0.03 kg.

(i) Find the probability that the total mass of a full box of 20 bags is less than 20.6 kg. [5]

(ii) Two full boxes are chosen at random. Find the probability that they differ in mass by less than 0.02 kg. [5]

Question 16

Each box of Fruity Flakes contains X grams of flakes and Y grams of fruit, where X and Y are independent random variables, having distributions $N(400, 50)$ and $N(100, 20)$ respectively. The weight of each box, when empty, is exactly 20 grams. A full box of Fruity Flakes is chosen at random.

(i) Find the probability that the total weight of the box and its contents is less than 530 grams. [5]

(ii) Find the probability that the weight of flakes in the box is more than 4.1 times the weight of fruit in the box. [5]

Question 17

The thickness of books in a large library is normally distributed with mean 2.4 cm and standard deviation 0.3 cm.

(i) Find the probability that the total thickness of 6 randomly chosen books is more than 16 cm. [4]

(ii) Find the probability that the thickness of a book chosen at random is less than 1.1 times the thickness of a second book chosen at random. [5]

Question 18

A men's triathlon consists of three parts: swimming, cycling and running. Competitors' times, in minutes, for the three parts can be modelled by three independent normal variables with means 34.0, 87.1 and 56.9, and standard deviations 3.2, 4.1 and 3.8, respectively. For each competitor, the total of his three times is called the race time. Find the probability that the mean race time of a random sample of 15 competitors is less than 175 minutes. [5]

Question 19

Each week a farmer sells X litres of milk and Y kg of cheese, where X and Y have the independent distributions $N(1520, 53^2)$ and $N(175, 12^2)$ respectively.

(i) Find the mean and standard deviation of the total amount of milk that the farmer sells in 4 randomly chosen weeks. [2]

During a year when milk prices are low, the farmer makes a loss of 2 cents per litre on milk and makes a profit of 21 cents per kg on cheese, so the farmer's overall weekly profit is $(21Y - 2X)$ cents.

(ii) Find the probability that, in a randomly chosen week, the farmer's overall profit is positive. [5]

Question 20

The masses, in kilograms, of cartons of sugar and cartons of flour have the distributions $N(78.8, 12.6^2)$ and $N(62.0, 10.0^2)$ respectively.

(i) The standard load for a certain crane is 8 cartons of sugar and 3 cartons of flour. The maximum load that can be carried safely by the crane is 900 kg. Stating a necessary assumption, find the percentage of standard loads that will exceed the maximum safe load. [5]

(ii) Find the probability that a randomly chosen carton of sugar has a smaller mass than a randomly chosen carton of flour. [5]

Question 21

(a) A random variable X is normally distributed with mean 4.2 and standard deviation 1.1. Find the probability that the sum of two randomly chosen values of X is greater than 10. [4]

(b) Each candidate's overall score for an essay is calculated as follows. The mark for creativity is denoted by C, the penalty mark for spelling errors is denoted by S and the overall score is defined by $C - \frac{1}{2}S$. The variables C and S are independent and have distributions $N(29, 105)$ and $N(17, 15)$ respectively. Find the proportion of candidates receiving a negative overall score. [5]

Question 22

Large packets of sugar are packed in cartons, each containing 12 packets. The weights of these packets are normally distributed with mean 505 g and standard deviation 3.2 g. The weights of the cartons, when empty, are independently normally distributed with mean 150 g and standard deviation 7 g.

(i) Find the probability that the total weight of a full carton is less than 6200 g. [5]

Small packets of sugar are packed in boxes. The total weight of a full box has a normal distribution with mean 3130 g and standard deviation 12.1 g.

(ii) Find the probability that the weight of a randomly chosen full carton is less than double the weight of a randomly chosen full box. [5]

Question 23

The numbers of barrels of oil, in millions, extracted per day in two oil fields A and B are modelled by the independent random variables X and Y respectively, where $X \sim N(3.2, 0.4^2)$ and $Y \sim N(4.3, 0.6^2)$. The income generated by the oil from the two fields is \$90 per barrel for A and \$95 per barrel for B.

(i) Find the mean and variance of the daily income, in millions of dollars, generated by field A. [3]

(ii) Find the probability that the total income produced by the two fields in a day is at least \$670 million. [5]

Question 24

The marks in paper 1 and paper 2 of an examination are denoted by X and Y respectively, where X and Y have the independent continuous distributions $N(56, 6^2)$ and $N(43, 5^2)$ respectively.

(i) Find the probability that a randomly chosen paper 1 mark is more than a randomly chosen paper 2 mark. [5]

(ii) Each candidate's overall mark is M where $M = X + 1.5Y$. The minimum overall mark for grade A is 135. Find the proportion of students who gain a grade A. [5]

Question 25

The heights of plants of type A have mean 1.2 m and standard deviation 0.03 m. A random sample of 5 plants of type A is selected. The sum of the heights of these 5 plants is denoted by H_A m.

(i) Find the mean and variance of H_A. [3]

The heights of plants of type B have mean 0.6 m and standard deviation 0.02 m. A random sample of 5 plants of type B is selected. The sum of the heights of these 5 plants is denoted by H_B m.

(ii) Find the mean and variance of $H_A - 2H_B$. [4]

Question 26

The random variable X has the distribution $N(3, 1.2)$. The random variable A is defined by $A = 2X$. The random variable B is defined by $B = X_1 + X_2$, where X_1 and X_2 are independent random values of X. Describe fully the distribution of A and the distribution of B. [3]

Question 27

The volume, in millilitres, of a small cup of coffee has the distribution $N(103.4, 10.2)$. The volume of a large cup of coffee is 1.5 times the volume of a small cup of coffee.

(i) Find the mean and standard deviation of the volume of a large cup of coffee. [3]

(ii) Find the probability that the total volume of a randomly chosen small cup of coffee and a randomly chosen large cup of coffee is greater than 250 ml. [4]

Question 28

The times, in months, taken by a builder to build two types of house, P and Q, are represented by the independent variables $T_1 \sim N(2.2, 0.4^2)$ and $T_2 \sim N(2.8, 0.5^2)$ respectively.

(i) Find the probability that the total time taken to build one house of each type is less than 6 months. [4]

(ii) Find the probability that the time taken to build a type Q house is more than 1.2 times the time taken to build a type P house. [5]

Question 29

Sugar and flour for making cakes are measured in cups. The mass, in grams, of one cup of sugar has the distribution N(250, 10). The mass, in grams, of one cup of flour has the independent distribution N(160, 9). Each cake contains 2 cups of sugar and 5 cups of flour. Find the probability that the total mass of sugar and flour in one cake exceeds 1310 grams. [5]

Question 30

The independent random variables X and Y have the distributions N(9.2, 12.1) and N(3.0, 8.6) respectively. Find $P(X > 3Y)$. [5]

Question 31

A factory supplies boxes of children's bricks. Each box contains 10 randomly chosen large bricks and 20 randomly chosen small bricks. The masses, in grams, of large and small bricks have the distributions N(60, 1.2) and N(30, 0.7) respectively. The mass of an empty box is 8 g. Find the probability that the total weight of a box and its contents is less than 1200 g. [5]

Question 32

The heights of a certain variety of plant are normally distributed with mean 110 cm and variance 1050 cm^2. Two plants of this variety are chosen at random. Find the probability that the height of one of these plants is at least 1.5 times the height of the other. [7]

Question 33

The random variable X has mean 2.4 and variance 3.1.

(i) The random variable Y is the sum of four independent values of X. Find the mean and variance of Y. [2]

(ii) The random variable Z is defined by $Z = 4X - 3$. Find the mean and variance of Z. [2]

Question 34

The masses, in grams, of large boxes of chocolates and small boxes of chocolates have the distributions N(325, 6.1) and N(167, 5.6) respectively.

(i) Find the probability that the total mass of 10 randomly chosen large boxes of chocolates is less than 3240 g. [4]

(ii) Find the probability that the mass of a randomly chosen large box of chocolates is more than twice the mass of a randomly chosen small box of chocolates. [5]

Question 35

The volumes, in millilitres, of large and small cups of tea are modelled by the distributions $N(200, 30)$ and $N(110, 20)$ respectively.

(a) Find the probability that the total volume of a randomly chosen large cup of tea and a randomly chosen small cup of tea is less than 300 ml. [4]

(b) Find the probability that the volume of a randomly chosen large cup of tea is more than twice the volume of a randomly chosen small cup of tea. [6]

Question 36

Each day at the gym, Sarah completes three runs. The distances, in metres, that she completes in the three runs have the independent distributions $W \sim N(1520, 450)$, $X \sim N(2250, 720)$ and $Y \sim N(3860, 1050)$.

Find the probability that, on a particular day, Y is less than the total of W and X. [5]

Question 37

The masses, in grams, of plums of a certain type have the distribution $N(40.4, 5.2^2)$. The plums are packed in bags, with each bag containing 6 randomly chosen plums. If the total weight of the plums in a bag is less than 220 g the bag is rejected.

Find the percentage of bags that are rejected. [4]

Question 38

The masses, in kilograms, of large sacks of flour and small sacks of flour have the independent distributions $N(40, 1.5^2)$ and $N(12, 0.7^2)$ respectively.

(a) Find the probability that the total mass of 6 randomly chosen large sacks of flour is more than 245 kg. [4]

(b) Find the probability that the mass of a randomly chosen large sack of flour is less than 4 times the mass of a randomly chosen small sack of flour. [6]

Question 39

The masses, in kilograms, of female and male animals of a certain species have the distributions $N(102, 27^2)$ and $N(170, 55^2)$ respectively.

Find the probability that a randomly chosen female has a mass that is less than half the mass of a randomly chosen male. [6]

Question 40

Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions $N(380, 140)$ and $N(210, 80)$ respectively. You should assume that no continuity corrections are needed when using these models.

A book of this type is chosen at random.

(a) Find the probability that the number of minor errors is at least 200 more than the number of major errors. [5]

The costs of correcting a minor error and a major error are 20 cents and 50 cents respectively.

(b) Find the probability that the total cost of correcting the errors in the book is less than \$190. [5]

Question 41

The volumes, in litres, of juice in large and small bottles have the distributions N(5.10, 0.0102) and N(2.51, 0.0036) respectively.

(a) Find the probability that the total volume of juice in 3 randomly chosen large bottles and 4 randomly chosen small bottles is less than 25.5 litres. [5]

(b) Find the probability that the volume of juice in a randomly chosen large bottle is at least twice the volume of juice in a randomly chosen small bottle. [5]

Question 42

The number of goals scored by a team in a match is independent of other matches, and is denoted by the random variable X, which has a Poisson distribution with mean 1.36. A supporter offers to make a donation of \$5 to the team for each goal that they score in the next 10 matches.

Find the expectation and standard deviation of the amount that the supporter will pay. [5]

Question 43

Wendy's journey to work consists of three parts: walking to the train station, riding on the train and then walking to the office. The times, in minutes, for the three parts of her journey are independent and have the distributions $N(15.0, 1.1^2)$, $N(32.0, 3.5^2)$ and $N(8.6, 1.2^2)$ respectively.

(a) Find the mean and variance of the total time for Wendy's journey. [2]

If Wendy's journey takes more than 60 minutes, she is late for work.

(b) Find the probability that, on a randomly chosen day, Wendy will be late for work. [3]

(c) Find the probability that the mean of Wendy's journey times over 15 randomly chosen days will be less than 54.5 minutes. [3]

Question 44

The random variable X has the distribution B(400, 0.01).

(a) Find $\text{Var}(4X + 2)$. [3]

(b) **(i)** State an appropriate approximating distribution for X, giving the values of any parameters. Justify your choice of approximating distribution. [2]

(ii) Use your approximating distribution to find $P(2 \leqslant X \leqslant 5)$. [2]

Question 45

The masses, in kilograms, of large and small sacks of flour have the distributions $N(55, 3^2)$ and $N(27, 2.5^2)$ respectively.

(a) Some sacks are loaded onto a boat. The maximum load of flour that the boat can carry safely is 340 kg.

Find the probability that the boat can carry safely 3 randomly chosen large sacks of flour and 6 randomly chosen small sacks of flour. [5]

(b) Find the probability that the mass of a randomly chosen large sack of flour is greater than the total mass of two randomly chosen small sacks of flour. [5]

Question 46

The random variable T denotes the time, in seconds, for 100 m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the following results.

$$n = 40 \qquad \Sigma t = 560 \qquad \Sigma t^2 = 7850$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

The random variable S denotes the time, in seconds, for 100 m races run by Suki. S has the independent distribution $\mathrm{N}(14.2, 0.3)$.

(b) Using your answers to part **(a)**, find the probability that, in a randomly chosen 100 m race, Suki's time will be at least 0.1 s more than Tania's time. [5]

Question 47

It is known that the height H, in metres, of trees of a certain kind has the distribution $\mathrm{N}(12.5, 10.24)$. A scientist takes a random sample of 25 trees of this kind and finds the sample mean, $\overline{H}$, of the heights.

(a) State the distribution of $\overline{H}$, giving the values of any parameters. [2]

(b) Find $\mathrm{P}(12 < \overline{H} < 13)$. [3]

Question 48

The heights of buildings in a large city are normally distributed with mean 18.3 m and standard deviation 2.5 m.

(a) Find the probability that the total height of 5 randomly chosen buildings in the city is more than 95 m. [4]

(b) Find the probability that the difference between the heights of two randomly chosen buildings in the city is less than 1 m. [5]

Question 49

Each box of Seeds & Raisins contains S grams of seeds and R grams of raisins. The weight of a box, when empty, is B grams. S, R and B are independent random variables, where $S \sim \mathrm{N}(300, 45)$, $R \sim \mathrm{N}(200, 25)$ and $\mathrm{B} \sim \mathrm{N}(15, 4)$. A full box of Seeds & Raisins is chosen at random.

(a) Find the probability that the total weight of the box and its contents is more than 500 grams. [5]

(b) Find the probability that the weight of seeds in the box is less than 1.4 times the weight of raisins in the box. [5]

Question 50

The masses, in kilograms, of large and small sacks of grain have the distributions $\mathrm{N}(53, 11)$ and $\mathrm{N}(14, 3)$ respectively.

(a) Find the probability that the mass of a randomly chosen large sack is greater than four times the mass of a randomly chosen small sack. [5]

(b) A lift can safely carry a maximum mass of 1000 kg.

Find the probability that the lift can safely carry 12 randomly chosen large sacks and 25 randomly chosen small sacks. [5]

Question 51

The independent random variables X and Y have distributions Po(2) and $\mathrm{B}\left(20, \frac{1}{4}\right)$ respectively.

(a) Find the mean and standard deviation of $X - 3Y$. [5]

(b) Find $\mathrm{P}(Y = 15X)$. [3]

Question 52

The lengths, in centimetres, of two types of insect, A and B, are modelled by the random variables $X \sim \mathrm{N}(6.2, 0.36)$ and $Y \sim \mathrm{N}(2.4, 0.25)$ respectively.

Find the probability that the length of a randomly chosen type A insect is greater than the sum of the lengths of 3 randomly chosen type B insects. [5]

Question 53

Each month a company sells X kg of brown sugar and Y kg of white sugar, where X and Y have the independent distributions $\mathrm{N}(2500, 120^2)$ and $\mathrm{N}(3700, 130^2)$ respectively.

(a) Find the mean and standard deviation of the total amount of sugar that the company sells in 3 randomly chosen months. [3]

The company makes a profit of \$1.50 per kilogram of brown sugar sold and makes a loss of \$0.20 per kilogram of white sugar sold.

(b) Find the probability that, in a randomly chosen month, the total profit is less than \$3000. [5]

Question 54

The masses, in grams, of small and large bags of flour have the distributions $\mathrm{N}(510, 100)$ and $\mathrm{N}(1015, 324)$ respectively. André selects 4 small bags of flour and 2 large bags of flour at random.

(a) Find the probability that the total mass of these 6 bags of flour is less than 4130 g. [5]

(b) Find the probability that the total mass of the 4 small bags is more than the total mass of the 2 large bags. [5]

Question 55

Each month a company sells X kg of brown sugar and Y kg of white sugar, where X and Y have the independent distributions $\mathrm{N}(2500, 120^2)$ and $\mathrm{N}(3700, 130^2)$ respectively.

(a) Find the mean and standard deviation of the total amount of sugar that the company sells in 3 randomly chosen months. [3]

The company makes a profit of \$1.50 per kilogram of brown sugar sold and makes a loss of \$0.20 per kilogram of white sugar sold.

(b) Find the probability that, in a randomly chosen month, the total profit is less than \$3000. [5]

Question 56

The masses, in grams, of large and small packets of Maxwheat cereal have the independent distributions $N(410.0, 3.6^2)$ and $N(206.0, 3.7^2)$ respectively.

(a) Find the probability that a randomly chosen large packet has a mass that is more than double the mass of a randomly chosen small packet. [5]

The packets are placed in boxes. The boxes are identical in appearance. 60% of the boxes contain exactly 10 randomly chosen large packets. 40% of the boxes contain exactly 20 randomly chosen small packets.

(b) Find the probability that a randomly chosen box contains packets with a total mass of more than 4080 grams. [6]

Question 57

The mass, in tonnes, of steel produced per day at a factory is normally distributed with mean 65.2 and standard deviation 3.6. It can be assumed that the mass of steel produced each day is independent of other days. The factory makes \$50 profit on each tonne of steel produced.

Find the probability that the total profit made in a randomly chosen 7-day week is less than \$22 000. [6]

Question 58

(a) Two random variables X and Y have the independent distributions $N(7, 3)$ and $N(6, 2)$ respectively. A random value of each variable is taken.

Find the probability that the two values differ by more than 2. [5]

(b) Each candidate's overall score in a science test is calculated as follows. The mark for theory is denoted by T, the mark for practical is denoted by P, and the overall score is given by $T + 1.5P$. The variables T and P are assumed to be independent with distributions $N(62, 158)$ and $N(42, 108)$ respectively. You should assume that no continuity corrections are needed when using these distributions.

(i) A pass is awarded to candidates whose overall score is at least 90.

Find the proportion of candidates who pass. [5]

(ii) Comment on the assumption that the variables T and P are independent. [1]

Question 59

Large packets of rice are packed in cartons, each containing 20 randomly chosen packets. The masses of these packets are normally distributed with mean 1010 g and standard deviation 3.4 g. The masses of the cartons, when empty, are independently normally distributed with mean 50 g and standard deviation 2.0 g.

(a) Find the variance of the masses of full cartons. [2]

Small packets of rice are packed in boxes. The total masses of full boxes are normally distributed with mean 6730 g and standard deviation 15.0 g. The masses of the boxes and cartons are distributed independently of each other.

(b) Find the probability that the mass of a randomly chosen full carton is more than three times the mass of a randomly chosen full box. [5]