A-Level

Topic : Poisson Distribution

May 2013-May 2023

Questions

Question 1

Calls arrive at a helpdesk randomly and at a constant average rate of 1.4 calls per hour. Calculate the probability that there will be

(i) more than 3 calls in $2\frac{1}{2}$ hours, [3]

(ii) fewer than 1000 calls in four weeks (672 hours). [4]

Question 2

The independent random variables X and Y have the distributions Po(2) and Po(3) respectively.

(i) Given that $X + Y = 5$, find the probability that $X = 1$ and $Y = 4$. [4]

(ii) Given that $P(X = r) = \frac{2}{3}P(X = 0)$, show that $3 \times 2^{r-1} = r!$ and verify that $r = 4$ satisfies this equation. [2]

Question 3

It is known that 1.2% of rods made by a certain machine are bent. The random variable X denotes the number of bent rods in a random sample of 400 rods.

(i) State the distribution of X. [2]

(ii) State, with a reason, a suitable approximate distribution for X. [2]

(iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods. [2]

Question 4

The probability that a new car of a certain type has faulty brakes is 0.008. A random sample of 520 new cars of this type is chosen, and the number, X, having faulty brakes is noted.

(i) Describe fully the distribution of X and describe also a suitable approximating distribution. Justify this approximating distribution. [4]

(ii) Use your approximating distribution to find

(a) $P(X > 3)$, [2]

(b) the smallest value of n such that $P(X = n) > P(X = n + 1)$. [3]

Question 5

Goals scored by Femchester United occur at random with a constant average of 1.2 goals per match. Goals scored against Femchester United occur independently and at random with a constant average of 0.9 goals per match.

(i) Find the probability that in a randomly chosen match involving Femchester,

(a) a total of 3 goals are scored, [2]

(b) a total of 3 goals are scored and Femchester wins. [3]

The manager promises the Femchester players a bonus if they score at least 35 goals in the next 25 matches.

(ii) Find the probability that the players receive the bonus. [4]

Question 6

The number of radioactive particles emitted per 150-minute period by some material has a Poisson distribution with mean 0.7.

(i) Find the probability that at most 2 particles will be emitted during a randomly chosen 10-hour period. [3]

(ii) Find, in minutes, the longest time period for which the probability that no particles are emitted is at least 0.99. [5]

Question 7

Each computer made in a factory contains 1000 components. On average, 1 in 30 000 of these components is defective. Use a suitable approximate distribution to find the probability that a randomly chosen computer contains at least 1 faulty component. [4]

Question 8

A Lost Property office is open 7 days a week. It may be assumed that items are handed in to the office randomly, singly and independently.

(i) State another condition for the number of items handed in to have a Poisson distribution. [1]

It is now given that the number of items handed in per week has the distribution Po(4.0).

(ii) Find the probability that exactly 2 items are handed in on a particular day. [2]

(iii) Find the probability that at least 4 items are handed in during a 10-day period. [3]

(iv) Find the probability that, during a certain week, 5 items are handed in altogether, but no items are handed in on the first day of the week. [3]

Question 9

On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]

Question 10

(i) The random variable W has the distribution Po(1.5). Find the probability that the sum of 3 independent values of W is greater than 2. [3]

(ii) The random variable X has the distribution Po(λ). Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

(iii) The random variable Y has the distribution Po(μ), where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ. [3]

Question 11

(i) The following tables show the probability distributions for the random variables V and W.

v	-1	0	1	>1
$P(V = v)$	0.368	0.368	0.184	0.080

w	0	0.5	1	>1
$P(W = w)$	0.368	0.368	0.184	0.080

For each of the variables V and W state how you can tell from its probability distribution that it does NOT have a Poisson distribution. [2]

(ii) The random variable X has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 0) = p \quad \text{and} \quad P(X = 1) = 2.5p,$$

where p is a constant.

(a) Show that $\lambda = 2.5$. [1]

(b) Find $P(X \geqslant 3)$. [2]

(iii) The random variable Y has the distribution $\text{Po}(\mu)$, where $\mu > 30$. Using a suitable approximating distribution, it is found that $P(Y > 40) = 0.5793$ correct to 4 decimal places. Find μ. [5]

Question 12

The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X, having the gene is found.

(i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $P(X \leqslant 3)$. [2]

Question 13

The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period. Find the probability of

(i) exactly 4 calls in an 8-minute period, [2]

(ii) at least 3 calls in a 3-minute period. [3]

The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(iii) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive. [5]

Question 14

The probability that a randomly chosen plant of a certain kind has a particular defect is 0.01. A random sample of 150 plants is taken.

(i) Use an appropriate approximating distribution to find the probability that at least 1 plant has the defect. Justify your approximating distribution. [4]

The probability that a randomly chosen plant of another kind has the defect is 0.02. A random sample of 100 of these plants is taken.

(ii) Use an appropriate approximating distribution to find the probability that the total number of plants with the defect in the two samples together is more than 3 and less than 7. [3]

Question 15

People arrive at a checkout in a store at random, and at a constant mean rate of 0.7 per minute. Find the probability that

(i) exactly 3 people arrive at the checkout during a 5-minute period, [2]

(ii) at least 30 people arrive at the checkout during a 1-hour period. [4]

People arrive independently at another checkout in the store at random, and at a constant mean rate of 0.5 per minute.

(iii) Find the probability that a total of more than 3 people arrive at this pair of checkouts during a 2-minute period. [4]

Question 16

In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for X. [3]

(ii) Use your approximating distribution to find $P(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geqslant 1$. [4]

Question 17

In a golf tournament, the number of times in a day that a 'hole-in-one' is scored is denoted by the variable X, which has a Poisson distribution with mean 0.15. Mr Crump offers to pay \$200 each time that a hole-in-one is scored during 5 days of play. Find the expectation and variance of the amount that Mr Crump pays. [5]

Question 18

A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are

(i) exactly 2 errors in 10 pages, [2]

(ii) at least 3 errors in 6 pages, [3]

(iii) fewer than 50 errors in 200 pages. [4]

Question 19

The number of calls received per 5-minute period at a large call centre has a Poisson distribution with mean λ, where $\lambda > 30$. If more than 55 calls are received in a 5-minute period, the call centre is overloaded. It has been found that the probability of being overloaded during a randomly chosen 5-minute period is 0.01. Use the normal approximation to the Poisson distribution to obtain a quadratic equation in $\sqrt{\lambda}$ and hence find the value of λ. [5]

Question 20

On average, 1 in 2500 adults has a certain medical condition.

(i) Use a suitable approximation to find the probability that, in a random sample of 4000 people, more than 3 have this condition. [3]

(ii) In a random sample of n people, where n is large, the probability that none has the condition is less than 0.05. Find the smallest possible value of n. [4]

Question 21

Failures of two computers occur at random and independently. On average the first computer fails 1.2 times per year and the second computer fails 2.3 times per year. Find the probability that the total number of failures by the two computers in a 6-month period is more than 1 and less than 4. [4]

Question 22

The battery in Sue's phone runs out at random moments. Over a long period, she has found that the battery runs out, on average, 3.3 times in a 30-day period.

(i) Find the probability that the battery runs out fewer than 3 times in a 25-day period. [3]

(ii) **(a)** Use an approximating distribution to find the probability that the battery runs out more than 50 times in a year (365 days). [4]

(b) Justify the approximating distribution used in part **(ii)(a)**. [1]

(iii) Independently of her phone battery, Sue's computer battery also runs out at random moments. On average, it runs out twice in a 15-day period. Find the probability that the total number of times that her phone battery and her computer battery run out in a 10-day period is at least 4. [3]

Question 23

1% of adults in a certain country own a yellow car.

(i) Use a suitable approximating distribution to find the probability that a random sample of 240 adults includes more than 2 who own a yellow car. [4]

(ii) Justify your approximation. [2]

Question 24

At a certain shop the demand for hair dryers has a Poisson distribution with mean 3.4 per week.

(i) Find the probability that, in a randomly chosen two-week period, the demand is for exactly 5 hair dryers. [3]

(ii) At the beginning of a week the shop has a certain number of hair dryers for sale. Find the probability that the shop has enough hair dryers to satisfy the demand for the week if

(a) they have 4 hair dryers in the shop, [2]

(b) they have 5 hair dryers in the shop. [2]

(iii) Find the smallest number of hair dryers that the shop needs to have at the beginning of a week so that the probability of being able to satisfy the demand that week is at least 0.9. [3]

Question 25

(a) A large number of spoons and forks made in a factory are inspected. It is found that 1% of the spoons and 1.5% of the forks are defective. A random sample of 140 items, consisting of 80 spoons and 60 forks, is chosen. Use the Poisson approximation to the binomial distribution to find the probability that the sample contains

(i) at least 1 defective spoon and at least 1 defective fork, [3]

(ii) fewer than 3 defective items. [3]

(b) The random variable X has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 1) = p \quad \text{and} \quad P(X = 2) = 1.5p,$$

where p is a non-zero constant. Find the value of λ and hence find the value of p. [4]

Question 26

Men arrive at a clinic independently and at random, at a constant mean rate of 0.2 per minute. Women arrive at the same clinic independently and at random, at a constant mean rate of 0.3 per minute.

(i) Find the probability that at least 2 men and at least 3 women arrive at the clinic during a 5-minute period. [4]

(ii) Find the probability that fewer than 36 people arrive at the clinic during a 1-hour period. [5]

Question 27

The random variable X has the distribution $\text{Po}(3.5)$. Find $P(X < 3)$. [3]

Question 28

Particles are emitted randomly from a radioactive substance at a constant average rate of 3.6 per minute. Find the probability that

(i) more than 3 particles are emitted during a 20-second period, [3]

(ii) more than 240 particles are emitted during a 1-hour period. [4]

Question 29

The number of planes arriving at an airport every hour during daytime is modelled by the random variable X with distribution $\text{Po}(5.2)$.

(i) State two assumptions required for the Poisson model to be valid in this context. [2]

(ii) **(a)** Find the probability that the number of planes arriving in a 15-minute period is greater than 1 and less than 4, [3]

(b) Find the probability that more than 3 planes will arrive in a 40-minute period. [2]

(iii) The airport has enough staff to deal with a maximum of 60 planes landing during a 10-hour day. Use a suitable approximation to find the probability that, on a randomly chosen 10-hour day, staff will be able to deal with all the planes that land. [4]

Question 30

(i) A random variable X has the distribution Po(42).

(a) Use an appropriate approximating distribution to find $P(X \geqslant 40)$. [4]

(b) Justify your use of the approximating distribution. [1]

(ii) A random variable Y has the distribution B(60, 0.02).

(a) Use an appropriate approximating distribution to find $P(Y > 2)$. [3]

(b) Justify your use of the approximating distribution. [1]

Question 31

Old televisions arrive randomly and independently at a recycling centre at an average rate of 1.2 per day.

(i) Find the probability that exactly 2 televisions arrive in a 2-day period. [2]

(ii) Use an appropriate approximating distribution to find the probability that at least 55 televisions arrive in a 50-day period. [4]

Independently of televisions, old computers arrive randomly and independently at the same recycling centre at an average rate of 4 per 7-day week.

(iii) Find the probability that the total number of televisions and computers that arrive at the recycling centre in a 3-day period is less than 4. [3]

Question 32

Javier writes an article containing 52 460 words. He plans to upload the article to his website, but he knows that this process sometimes introduces errors. He assumes that for each word in the uploaded version of his article, the probability that it contains an error is 0.000 08. The number of words containing an error is denoted by X.

(i) Find $E(X)$ and $\mathrm{Var}(X)$, giving your answers correct to three decimal places. [2]

Javier wants to use the Poisson distribution as an approximating distribution to calculate the probability that there will be fewer than 5 words containing an error in his uploaded article.

(ii) Explain how your answers to part **(i)** are consistent with the use of the Poisson distribution as an approximating distribution. [1]

(iii) Use the Poisson distribution to calculate $P(X < 5)$. [2]

Question 33

On average, 1 clover plant in 10 000 has four leaves instead of three.

(i) Use an approximating distribution to calculate the probability that, in a random sample of 2000 clover plants, more than 2 will have four leaves. [3]

(ii) Justify your approximating distribution. [2]

Question 34

An airline has found that, on average, 1 in 100 passengers do not arrive for each flight, and that this occurs randomly. For one particular flight the airline always sells 403 seats. The plane only has room for 400 passengers, so the flight is overbooked if the number of passengers who do not arrive is less than 3. Use a suitable approximation to find the probability that the flight is overbooked. [4]

Question 35

A random variable, X, has the distribution $\text{Po}(31)$. Use the normal approximation to the Poisson distribution to find $\text{P}(X > 40)$. [3]

Question 36

(a) **(i)** A random variable X has the distribution $\text{B}(2540, 0.001)$. Use the Poisson approximation to the binomial distribution to find $\text{P}(X > 1)$. [3]

(ii) Explain why the Poisson approximation is appropriate in this case. [1]

(b) Two independent random variables, S and T, have distributions $\text{Po}(2.1)$ and $\text{Po}(3.5)$ respectively. Find the mean and standard deviation of $S + T$. [2]

Question 30

The number of phone calls arriving in a 10-minute period at a switchboard is modelled by the random variable X which has the distribution $\text{Po}(4.1)$. Use an approximating distribution to find the probability that more than 90 calls arrive in a 4-hour period. [5]

Question 31

The times, in minutes, taken to complete the two parts of a task are normally distributed with means 4.5 and 2.3 respectively and standard deviations 1.1 and 0.7 respectively.

(i) Find the probability that the total time taken for the task is less than 8.5 minutes. [4]

(ii) Find the probability that the time taken for the first part of the task is more than twice the time taken for the second part. [5]

Question 32

The numbers, M and F, of male and female students who leave a particular school each year to study engineering have means 3.1 and 0.8 respectively.

(i) State, in context, one condition required for M to have a Poisson distribution. [1]

Assume that M and F can be modelled by independent Poisson distributions.

(ii) Find the probability that the total number of students who leave to study engineering in a particular year is more than 3. [3]

(iii) Given that the total number of students who leave to study engineering in a particular year is more than 3, find the probability that no female students leave to study engineering in that year. [3]

Question 33

A random variable X has the distribution $\text{B}(75, 0.03)$.

(i) Use the Poisson approximation to the binomial distribution to calculate $\text{P}(X < 3)$. [3]

(ii) Justify the use of the Poisson approximation. [1]

Question 34

Accidents on a particular road occur at a constant average rate of 1 every 4.8 weeks.

(i) State, in context, one condition for the number of accidents in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 4 accidents will occur during a randomly chosen 12-week period. [2]

(iii) Find the probability that more than 3 accidents will occur during a randomly chosen 10-week period. [3]

(iv) Use a suitable approximating distribution to find the probability that fewer than 30 accidents will occur during a randomly chosen 2-year period ($104\frac{2}{7}$ weeks). [4]

Question 35

The numbers of alpha, beta and gamma particles emitted per minute by a certain piece of rock have independent distributions Po(0.2), Po(0.3) and Po(0.6) respectively. Find the probability that the total number of particles emitted during a 4-minute period is less than 4. [3]

Question 38

The number of e-readers sold in a 10-day period in a shop is modelled by the distribution Po(5.1). Use an approximating distribution to find the probability that fewer than 140 e-readers are sold in a 300-day period. [4]

Question 37

Small drops of two liquids, *A* and *B*, are randomly and independently distributed in the air. The average numbers of drops of *A* and *B* per cubic centimetre of air are 0.25 and 0.36 respectively.

(i) A sample of $10\,\text{cm}^3$ of air is taken at random. Find the probability that the total number of drops of *A* and *B* in this sample is at least 4. [3]

(ii) A sample of $100\,\text{cm}^3$ of air is taken at random. Use an approximating distribution to find the probability that the total number of drops of *A* and *B* in this sample is less than 60. [5]

Question 38

The independent random variables X and Y have the distributions Po(2.1) and Po(3.5) respectively.

(i) Find $P(X + Y = 3)$. [2]

(ii) Given that $X + Y = 3$, find $P(X = 2)$. [3]

(iii) A random sample of 100 values of X is taken. Find the probability that the sample mean is more than 2.2. [6]

Question 39

The random variable X has the distribution Po(2.3). Find $P(2 \leqslant X < 5)$. [3]

Question 40

The number of eagles seen per hour in a certain location has the distribution Po(1.8). The number of vultures seen per hour in the same location has the independent distribution Po(2.6).

(i) Find the probability that, in a randomly chosen hour, at least 2 eagles are seen. [2]

(ii) Find the probability that, in a randomly chosen half-hour period, the total number of eagles and vultures seen is less than 5. [3]

Alex wants to be at least 99% certain of seeing at least 1 eagle.

(iii) Find the minimum time for which she should watch for eagles. [3]

Question 41

Each day at a certain doctor's surgery there are 70 appointments available in the morning and 60 in the afternoon. All the appointments are filled every day. The probability that any patient misses a particular morning appointment is 0.04, and the probability that any patient misses a particular afternoon appointment is 0.05. All missed appointments are independent of each other.

Use suitable approximating distributions to answer the following.

(i) Find the probability that on a randomly chosen morning there are at least 3 missed appointments. [3]

(ii) Find the probability that on a randomly chosen day there are a total of exactly 6 missed appointments. [3]

(iii) Find the probability that in a randomly chosen 10-day period there are more than 50 missed appointments. [4]

Question 42

All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day is modelled by the distribution B(320, 0.005).

(i) Explain what the number 320 represents in this context. [1]

(ii) The total number of passengers who have bought seats but fail to arrive for this flight on 2 randomly chosen days is denoted by X. Use a suitable approximating distribution to find $P(2 < X < 6)$. [3]

Question 43

The random variable X has the distribution Po(5).

(i) Find $P(X = 2)$. [1]

It is given that $P(X = n) = P(X = n + 1)$.

(ii) Write down an equation in n. [1]

(iii) Hence or otherwise find the value of n. [1]

Question 44

(a) The random variable X has the distribution Po(2.3).

(i) Find $P(2 \leqslant X \leqslant 4)$. [2]

(ii) Find the probability that the sum of two independent values of X is greater than 2. [3]

(iii) The random variable S is the sum of 50 independent values of X. Use a suitable approximating distribution to find $P(S \leqslant 110)$. [4]

(b) The random variable Y has the distribution Po(λ). Given that $P(Y = 3) = P(Y = 5)$, find λ. [3]

Question 45

At an internet café, the charge for using a computer is 5 cents per minute. The number of minutes for which people use a computer has mean 23 and standard deviation 8.

(i) Find, in cents, the mean and standard deviation of the amount people pay when using a computer. [2]

(ii) Each day, 15 people use computers independently. Find, in cents, the mean and standard deviation of the total amount paid by 15 people. [3]

Question 46

(i) The random variable X has the distribution $B(300, 0.01)$. Use a Poisson approximation to find $P(2 < X < 6)$. [3]

(ii) The random variable Y has the distribution $Po(\lambda)$, and $P(Y = 0) = P(Y = 2)$. Find λ. [2]

(iii) The random variable Z has the distribution $Po(5.2)$ and it is given that $P(Z = n) < P(Z = n + 1)$.

(a) Write down an inequality in n. [1]

(b) Hence or otherwise find the largest possible value of n. [2]

Question 47

Cars arrive at a filling station randomly and at a constant average rate of 2.4 cars per minute.

(i) Calculate the probability that fewer than 4 cars arrive in a 2-minute period. [2]

(ii) Use a suitable approximating distribution to calculate the probability that at least 140 cars arrive in a 1-hour period. [4]

Question 48

On average, 1 in 150 components made by a certain machine are faulty. The random variable X denotes the number of faulty components in a random sample of 500 components.

(i) Describe fully the distribution of X. [2]

(ii) State a suitable approximating distribution for X, giving a justification for your choice. [2]

(iii) Use your approximating distribution to find the probability that the sample will include at least 3 faulty components. [3]

Question 49

The number of accidents on a certain road has a Poisson distribution with mean 0.4 per 50-day period.

(a) Find the probability that there will be fewer than 3 accidents during a year (365 days). [3]

(b) The probability that there will be no accidents during a period of n days is greater than 0.95.

Find the largest possible value of n. [4]

Question 50

The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30 000 letters, and these errors occur randomly. A randomly chosen booklet from this publisher contains 12 500 letters.

Use a suitable approximating distribution to find the probability that this booklet contains at least 2 errors. [3]

Question 51

The random variable A has the distribution $Po(1.5)$. A_1 and A_2 are independent values of A.

(a) Find $P(A_1 + A_2 < 2)$. [3]

(b) Given that $A_1 + A_2 < 2$, find $P(A_1 = 1)$. [4]

(c) Give a reason why $A_1 - A_2$ cannot have a Poisson distribution. [1]

Question 52

(a) The random variable X has the distribution $\text{Po}(\lambda)$.

(i) State the values that X can take. [1]

It is given that $P(X = 1) = 3 \times P(X = 0)$.

(ii) Find λ. [1]

(iii) Find $P(4 \leqslant X \leqslant 6)$. [2]

(b) The random variable Y has the distribution $\text{Po}(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $P(Y < 46) = 0.0668$, correct to 4 decimal places.

Find μ. [5]

Question 53

In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution of X, including the values of any parameters. [1]

(ii) State an appropriate approximating distribution for X, including the values of any parameters.

Justify your choice of approximating distribution. [3]

(iii) Use your approximating distribution to find $P(X > 2)$. [2]

(b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n. [3]

Question 54

Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

(i) Find the probability that the team scores a total of 4 goals in their two matches. [2]

(ii) Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

(b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

(c) Justify the use of the approximating distribution used in part **(b)**. [1]

Question 55

It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an appropriate approximating distribution to find the probability that more than 1 flower in the sample is white. [3]

(b) Justify the approximating distribution used in part **(a)**. [1]

The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in the two samples is less than 4. [3]

Question 56

Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. [1]

It is now given that the number of customers arriving per minute has the distribution Po(2.3).

(b) Find the probability that exactly 3 customers arrive during a 1-minute period. [2]

(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3]

(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3]

Question 57

On average, 1 in 50 000 people have a certain gene.

Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3]

Question 58

On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters. [1]

(b) State an approximating distribution for X, giving the values of any parameters. [2]

(c) Use this approximating distribution to find each of the following.

(i) $P(X = 4)$. [2]

(ii) $P(2 \leqslant X \leqslant 4)$. [2]

(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one. [3]

Question 59

Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10 000 of these plants have four leaves, and plants with four leaves are called 'lucky'. The number of lucky plants in a random sample of 25 000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. [2]

Use your approximating distribution to answer parts **(b)** and **(c)**.

(b) Find $P(X \leqslant 3)$. [2]

(c) Given that $P(X = k) = 2P(X = k + 1)$, find k. [2]

The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geqslant 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. [3]

Question 60

Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. [2]

(b) Find the probability that at least 4 customers arrive during a 20-minute interval. [2]

(c) Use a suitable approximating distribution to find the probability that fewer than 40 customers arrive during a 2-hour interval. [4]

Question 61

On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10 000 people, at least 1 has the genetic disorder. [3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0.9.

Find the largest possible value of n. [4]

Question 62

Accidents at two factories occur randomly and independently. On average, the numbers of accidents per month are 3.1 at factory A and 1.7 at factory B.

Find the probability that the total number of accidents in the two factories during a 2-month period is more than 3. [4]

Question 63

The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

Question 64

The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

Question 65

In a certain large document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. [2]

(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. [4]

In the same document, formatting errors occur at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. [3]

Question 66

(a) The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

Question 67

The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

Question 68

(a) Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.

Use appropriate Poisson approximations to find the following probabilities.

(i) The samples contain at least 2 carp from pond A and at least 2 carp from pond B. [3]

(ii) The samples contain at least 4 carp altogether. [3]

(b) The random variables X and Y have the distributions $\text{Po}(\lambda)$ and $\text{Po}(\mu)$ respectively. It is given that

- $P(X = 0) = [P(Y = 0)]^2$,
- $P(X = 2) = k[P(Y = 1)]^2$, where k is a non-zero constant.

Find the value of k. [4]

Question 69

The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]

Question 70

X is a random variable with distribution $\text{Po}(2.90)$. A random sample of 100 values of X is taken.

Find the probability that the sample mean is less than 2.88. [5]

Question 71

It is known that 1.8% of children in a certain country have not been vaccinated against measles. A random sample of 200 children in this country is chosen.

(a) Use a suitable approximating distribution to find the probability that there are fewer than 3 children in the sample who have not been vaccinated against measles. [4]

(b) Justify your approximating distribution. [2]

Question 72

Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

Question 73

Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

Question 74

1.6% of adults in a certain town ride a bicycle. A random sample of 200 adults from this town is selected.

(a) Use a suitable approximating distribution to find the probability that more than 3 of these adults ride a bicycle. [4]

(b) Justify your approximating distribution. [2]

Question 75

Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

Question 76

The number of orders arriving at a shop during an 8-hour working day is modelled by the random variable X with distribution $\text{Po}(25.2)$.

(a) State **two** assumptions that are required for the Poisson model to be valid in this context. [2]

(b) **(i)** Find the probability that the number of orders that arrive in a randomly chosen 3-hour period is between 3 and 5 inclusive. [3]

(ii) Find the probability that, in two randomly chosen 1-hour periods, exactly 1 order will arrive in one of the 1-hour periods, and at least 2 orders will arrive in the other 1-hour period. [4]

(c) The shop can only deal with a maximum of 120 orders during any 36-hour period.

Use a suitable approximating distribution to find the probability that, in a randomly chosen 36-hour period, there will be too many orders for the shop to deal with. [4]

Question 77

It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an appropriate approximating distribution to find $P(X \leqslant 3)$. [3]

(b) Find the values of $E(X)$ and $\text{Var}(X)$, and explain how your answers suggest that the approximating distribution used in **(a)** is likely to be appropriate. [2]

Question 78

The number, X, of books received at a charity shop has a constant mean of 5.1 per day.

(a) State, in context, one condition for X to be modelled by a Poisson distribution. [1]

Assume now that X can be modelled by a Poisson distribution.

(b) Find the probability that exactly 10 books are received in a 3-day period. [2]

(c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

(d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

Question 79

(a) The random variable W has a Poisson distribution.

State the relationship between $E(W)$ and $Var(W)$. [1]

(b) The random variable X has the distribution $B(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for X.

Use the formulae for $E(X)$ and $Var(X)$ to explain why it is necessary for p to be close to 0 for this to be a reasonable approximation. [1]

(c) Given that Y has the distribution $B(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $P(Y > 2)$. [3]

Question 80

In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

(b) Justify the approximating distribution used in part **(a)**. [2]