A-Level

Topic : Poisson Distribution

May 2013-May 2025

Answers

Question 1

(i)	$\lambda (= 1.4 \times 2.5) = 3.5$	B1	
	$1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$	M1	Any λ allow one end error
	$= 0.463$ (3 sf)	A1	3
(ii)	$(\lambda = 672 \times 1.4 = 940.8)$		
	N(940.8, 940.8)	B1	Seen or implied
	$\frac{999.5 - 940.8}{\sqrt{940.8}} (= 1.914)$	M1	Allow with wrong or no cc . no sd/var
		M1	mixes
	$\Phi('1.914')$		
	$= 0.972$ (3 sf)	A1	4

[Total: 7]

Question 2

(i)	$e^{-2} \times 2 \ (\times) \ e^{-3} \times \frac{3^4}{4!}$	M1	Correct exp'n for P(1) with $\lambda=2$ OR P(4) with $\lambda=3$
	$e^{-5} \times \frac{5^4}{5!}$	B1	Correct exp'n
	$\div$	M1	dep M1B1
	$\frac{162}{625}$ or 0.259 (3 sf)	A1	4
(ii)	$(e^{-2} \times \frac{2^r}{r!} = \frac{2}{3}e^{-2} \Rightarrow)$		
	$3 \times 2^r = 2 \times r!$ OR $2^{r-1} = \frac{1}{3} \times r!$	B1	Legitimately shown
	$(\Rightarrow 3 \times 2^{r-1} = r!)$		
	$3 \times 2^3 = 24$ OR $3! = 24$ seen	B1	2	Legitimately shown on either equation

[Total: 6]

Question 3

(i)	Binomial		B1	
	$n = 400, \quad p = 0.012$		B1 [2]	Both. Not $p = 1.2\%$ Or B(400, 0.012): B1B1
(ii)	Poisson		B1	
	n large and mean = 4.8, which is < 5		B1 [2]	n large, p small
(iii)	$1 - e^{-4.8}(1 + 4.8 + \dfrac{4.8^2}{2})$		M1	$P(X = 0, 1, 2)$; allow any λ; allow one end error
	$= 0.857/0.858$		A1 [2]	(Normal/Binomial in (ii) can score M1 only)

Question 4

(i)	B(520, 0.008)		B1	
	Po(4.16)		B1B1	Po: B1, $\lambda = 4.16$: B1
	$n = 500$ which is large,			
	$np = 4.16$ which is < 5 or p small <0.1		B1 [4]	Both needed
(ii) (a)	$1 - e^{-4.16}(1 + 4.16 + \dfrac{4.16^2}{2} + \dfrac{4.16^3}{3!})$		M1	1–P(0,1,2,3) any λ allow one end error
	$= 0.597$ (3 sf)		A1 [2]	
(ii) (b)	$e^{-4.16} \times \dfrac{4.16^n}{n!} > e^{-4.16} \times \dfrac{4.16^{n+1}}{(n+1)!}$		M1	any λ
	$1 > \dfrac{4.16}{n+1}$		A1	or equiv equn without e and without factorials
	$n > 3.16$			
	Smallest n is 4		A1 [3]	(Calculation of P(0), P(1),....P(5) scores M1 for at least 3 attempted, A1 all correct, A1 for n = 4)

Question 5

(i) (a)	$e^{-2.1} \times \dfrac{2.1^3}{3!}$ alone	M1		Allow any λ. Allow sum of 3 or 4 rel products, e.g. P (3, 0)
	$= 0.189$	A1	2	
(b)	$e^{-1.2} \times \dfrac{1.2^3}{3!} \times e^{-0.9}$	M1		P (Fem = 3) × P (Opp = 0) or P (Fem = 2) × P (Opp = 1)
	$+ e^{-1.2} \times \dfrac{1.2^2}{2!} \times e^{-0.9} \times 0.9$	M1		P (3 , 0) + P (2 , 1)
	$= 0.115$	A1	3	As final answer
(ii)	N (30, 30)	B1		seen or implied
	$\dfrac{34.5 - 30}{\sqrt{30}} \quad (= 0.8216)$	M1		standardising with their N (λ , λ)
	$1 - \Phi(\text{“0.822”})$	M1		Allow with no or incorrect cc or no √
	$= 0.206$ (3sf)	A1	4	Area consistent with their working

Question 6

(i)	$\lambda = 2.8$	B1		seen
	$e^{-2.8}(1 + 2.8 + \dfrac{2.8^2}{2})$	M1		any λ allow one end error
	$= 0.469$ (3 s.f.) or 0.47(0)	A1	[3]	As final answer
(ii)	$e^{-0.7n} \geqslant 0.99$ $\quad$ or $e^{-\lambda} \geqslant 0.99$	M1		Allow '=' throughout
	$-0.7n \geqslant \ln 0.99$ $\quad$ or $-\lambda \geqslant \ln 0.99$	M1		Attempt ln both sides
	$n \leqslant 0.01436$ $\quad$ or $\lambda \leqslant 0.01005$	A1		Can be implied. Accept 3 s.f.
	'0.01436' $\times$ 150			
	or '0.01005' $\times$ 150 $\div$ 0.7	M1		Note $e^{-(0.7/150)n} \geqslant 0.99$ scores 1st and 3rd M1
	Max period is 2.15 mins (3 sf)	A1	[5]	T & I leading to ans 2.2 mins, SC: B2

Question 7

$\lambda = \dfrac{1}{30}$	B1		o.e.
$1 - e^{-\frac{1}{30}}$	M1		$1 - P(X = 0)$ by Poisson, any λ allow 1 end error
$= 0.0328$ (3 s.f.)	M1		$1 - P(X = 0)$ by Poisson, correct λ no end errors
	A1		
			S.R. Binomial with final answer 0.0328 B2
		[4]	Correct answer, no working scores B2

Question 8

(i)	Constant mean (or average) rate	B1	[1]	Constant mean per day (or week, etc.) o.e.
(ii)	$e^{-\frac{4}{7}} \times \dfrac{\frac{4^2}{7}}{2!}$ $\quad$ or $\quad e^{-0.571} \times \dfrac{0.571^2}{2!}$	M1		Expression for P(2) allow any λ
	$= 0.0922$ or 0.0921 (3 s.f.)	A1	[2]	
(iii)	$\lambda = \dfrac{40}{7}$ or 5.71...	B1		
	$1 - e^{-\frac{40}{7}}\left(1 + \dfrac{40}{7} + \dfrac{\frac{40^2}{7}}{2!} + \dfrac{\frac{40^3}{7}}{3!}\right)$	M1		Allow any λ allow one end error
	$= 0.821$ (3 s.f.)	A1	[3]	
(iv)	$\dfrac{24}{7}$ o.e. 3 s.f. or better seen	B1		
	$e^{-\frac{4}{7}} \times e^{-\frac{24}{7}} \times \dfrac{\frac{24^5}{7}}{5!}$	M1		M1 for P(0) $\times$ P(5) any consistent λ
	$= 0.0723$ (3 s.f.)	A1	[3]	

Question 9

$e^{-4}(1+4)$	M1 M1	M1 for P(0 or 1) using Poisson, any λ Expression of correct form correct λ (allow 1 end error)
$= 0.0916$ (3 s.f.)	A1　[3]	SR Use of Bin(100000, 1/25000) scores M1 for P(0,1) allow one end error. A1 0.0916

Question 10

(i)	$\lambda = 4.5$	B1	seen
	$1 - e^{-4.5}\left(1+4.5+\dfrac{4.5^2}{2}\right)$	M1	any λ. Allow one end error
	$= 0.826$ (3 s.f.)	A1　[3]	
(ii)	$e^{-\lambda} = 0.523$	B1	
	$(-\lambda = \ln 0.523)$		
	$\lambda = 0.648$ (3 s.f.)	B1　[2]	
(iii)	$e^{-\mu} \times \dfrac{\mu^3}{3!} = 24 \times e^{-\mu} \times \mu$	B1	
	$\dfrac{\mu^2}{6} = 24$	M1	For a simplified expression in μ^2 with $e^{-\mu}$ and μ cancelled and no factorials.
	$\mu = 12$	A1　[3]	

Question 11

(i)	V: cannot have neg value W: cannot have non-integer value	B1 B1　[2]	
(ii)	(a) $e^{-\lambda} = p$ and $\lambda e^{-\lambda} = 2.5p$ (Hence $\lambda = 2.5$ **AG**)	B1　[1]	or equiv explanation
(ii)	(b) $1 - e^{-2.5}(1 + 2.5 + \dfrac{2.5^2}{2})$	M1	Allow one end error
	$= 0.456$ (3 sf)	A1　[2]	
(iii)	$\Phi^{-1}(0.5793) = -0.2$	B1	
	$N(\mu, \mu)$ seen or implied	M1	
	$\dfrac{40.5 - \mu}{\sqrt{\mu}} = \text{``}{-}0.2\text{''}$	M1	Allow no cc or incorrect cc
	$\mu + \text{``}{-}0.2\text{''}\sqrt{\mu} - 40.5 = 0$		
	$\sqrt{\mu} = \dfrac{\text{"}0.2\text{"} \pm \sqrt{\text{"}0.2\text{"}^2 + 4\times40.5}}{2}$	M1	For solving quadratic in $\sqrt{\mu}$ (or μ)
	$(= 6.4647..)$		Ignore other answer for $\sqrt{\mu}$, but not for μ
	$\mu = 41.8$ (3 sf)	A1　[5]	
		[Total: 10]	

Question 12

(i)	B(3500, 0.001)	B1			
	Poisson with mean = 3.5	B1		or Po(3.5)	
	$n > 50$ and $np < 5$	B1	[3]	Both. Or $n > 50$ and $\lambda < 5$ or $3.5 < 5$	
(ii)	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$	M1		Allow any λ	
	$= 0.537$ (3 dp)	A1	[2]		
		[Total: 5)			

Question 13

(i)	$e^{-3.84} \times \frac{3.84^4}{4!}$	M1		Poisson P($X = 4$), any λ	
	$= 0.195$ (3 sf)	A1	[2]		
(ii)	1.44	B1		Seen	
	$1 - e^{-1.44}\left(1 + 1.44 + \frac{1.44^2}{2}\right)$	M1		Any λ, allow one end error, need "1 $-$…"	
	$= 0.176$	A1	[3]		
(iii)	$X \sim N(41, 41)$	B1		Seen or implied	
	$\frac{40.5 - 41}{\sqrt{41}}$ $(= -0.078)$ $\frac{59.5 - 41}{\sqrt{41}}$ $(= 2.889)$	M1M1		M1M0 if no cc or incorrect cc OR no $\sqrt{}$ in both	
	$\Phi('2.889') - \Phi('-0.078')$ $= \Phi('2.889') - (1 - \Phi('0.078'))$ $= 0.9981 - (1 - 0.5311)$	M1		Use of tables and correct area consistent with their working.	
	$= 0.529$ (3sf)	A1	[5]	cwo	

Question 14

(i)	(Bin) with $n > 50$ and mean (or np) < 5	B1		Accept n 'large', p 'small'
	Po(1.5)	B1		Poisson with correct mean stated or implied
	$1 - e^{-1.5}$	M1		Poisson $1 - P(X = 0)$; allow incorrect λ; allow 1 end error
	$= 0.777$ (3 sf)	A1	4	SR If zero scored use of Bin leading to 0.778 / 0.779 scores B1
(ii)	3.5	B1		Correct mean stated or implied
	$e^{-3.5}\left(\frac{3.5^4}{4!} + \frac{3.5^5}{5!} + \frac{3.5^6}{6!}\right)$	M1		Poisson P($X = 4, 5, 6$); allow incorrect λ; allow 1 end error
	$= 0.398$ (3 sf)	A1	3	
		Total: 7		

Question 15

(i)	$e^{-3.5} \times \dfrac{3.5^3}{3!}$	M1		P($X = 3$) any λ
	$= 0.216$ (3 sf)	A1	[2]	
(ii)	N(42, 42) stated or implied	B1		
	$\dfrac{29.5 - 42}{\sqrt{42}}$ $\quad (= -1.929)$	M1		Allow with wrong or no cc OR without $\sqrt{}$
	P($z > $ '-1.929') $= \Phi$('1.929')	M1		For correct area consistent with their working
	$= 0.973$ (3 sf)	A1	[4]	
(iii)	(λ) $= 2.4$	B1		
	$1 - e^{-2.4}\left(1 + 2.4 + \dfrac{2.4^2}{2} + \dfrac{2.4^3}{3!}\right)$	M1 M1		for $1 - $P($X \leq 3$), any λ allow one end error Correct expression any λ
	$= 0.221$ (3 sf)	A1	4	NB For combination method B1 attempting 10 combinations with $\lambda = 1$, $\lambda = 1.4$ M1 6 expressions M1 10 expressions 0.221 A1
		Total 10		

Question 16

(i)	Poisson	B1		
	(Actually binomial with)			Allow without "binomial"
	$n > 50$	B1		Accept n large
	and np (or λ) $(= 2.1)$ which is < 5	B1	3	Accept p small ($p < 0.1$)
(ii)	$\lambda = 2.1$	B1		
	$e^{-2.1}\left(1 + 2.1 + \dfrac{2.1^2}{2} + \dfrac{2.1^3}{3!}\right)$	M1		Attempt P(0,1,2,3) any λ allow 1 end error SR₁ Ft Normal N(2.1,2.1) B1 standardising M1
	$= 0.839$ (3 sf)	A1	3	0.833 A1 SR₂ Ft Binomial B(10500,0.0002) B1 calculating binomial prob P(0,1,2,3) M1 $= 0.8386$ A1
(iii)	P($X \geq 1$) $= 1 - e^{-2.1}$ $\quad (= 0.87754)$	M1		Any λ
	P($X = 1,2,3$) $= e^{-2.1}\left(2.1 + \dfrac{2.1^2}{2} + \dfrac{2.1^3}{3!}\right)$ $\quad (= 0.71619)$	M1		Or '0.839' $- e^{-2.1}$ Any λ
	$\dfrac{P(X=1,2,3)}{P(X>1)}$ $\left(= \dfrac{0.71619}{0.87754}\right)$	M1		Allow any attempted $\dfrac{P(X=1,2,3)}{P(X>1)}$ Any λ
	$= 0.816$ (3 sf)	A1	4	SR₁ Ft Normal P(>0.5)=0.86523 M1 P(1,2,3)=0.698 M1 0.698/0.86523 = 0.807 M1A1 SR₂ FT Binomial M1 M1 M1 A1
		Total	10	

Question 17

$\lambda = 5 \times 0.15$ (= 0.75)	M1		
E(amount) = 200×0.75 = 150	A1		
Var(weekly no of hole–in–ones) = 0.75	B1✓		
Var(amount) = $200^2 \times 0.75$	M1		Allow $200^2 \times$ their variance (with nothing added/subtracted at any stage)
= 30,000	A1	5	(SR probability table can score M1A0 srB1 if var rounds to 30,000 (2sf))
Total		**5**	

Question 18

(i)	$e^{-\frac{10}{3}} \times \dfrac{\left(\frac{10}{3}\right)^2}{2}$	M1		P(2), allow any λ
	= 0.198 (3 sf)	A1	[2]	
(ii)	$1 - e^{-2}\left(1 + 2 + \dfrac{2^2}{2}\right)$	M1 M1		M1 allow any λ and/or 1end error Correct expression , correct λ
	= 0.323 (3 sf)	A1	[3]	
(iii)	$N\left(\dfrac{200}{3}, \dfrac{200}{3}\right)$	M1		seen or implied
	$\dfrac{49.5 - \frac{200}{3}}{\sqrt{\frac{200}{3}}}$ (= –2.102)	M1		For standardising allow either wrong or no cc No sd/var mix
	$\Phi('{-}2.102') = 1 - \Phi('2.102')$ = 0.0178 (3 sf)	M1 A1	[4]	For finding area consistent with their working
		[Total: 9]		

Question 19

$(\Phi^{-1}(0.99) =) \; 2.326$ seen	B1		must be Φ^{-1}, not Φ
$N(\lambda, \lambda)$ seen or implied	M1		
$\dfrac{55.5 - \lambda}{\sqrt{\lambda}} = +\text{``2.326''}$	M1		allow with wrong or no cc & $\Phi(0.99)$ (= 0.8389) must = " z " or attempt at z (0.99 / 0.01 M0)
$\lambda + \text{``2.326''}\sqrt{\lambda} - 55.5 = 0$			
$\sqrt{\lambda} = \dfrac{-\text{``2.326''} \pm \sqrt{\text{``2.326''}^2 + 4 \times 55.5}}{2}$ (= 6.377.. or –8.703..))	M1		for correct method of solving their quad in $\sqrt{\lambda}$ and squaring to find λ
$\lambda = 40.7$ (3 sf)	A1	[5]	cao, one ans only Without cc, $\lambda = 40.2$: lose final A1

Question 20

(i)	Po(1.6) stated or implied	**M1**	
	$P(X > 3) = 1 - e^{-1.6}\left(1 + 1.6 + \dfrac{1.6^2}{2} + \dfrac{1.6^3}{3!}\right)$	**M1**	Allow M1 for $1 - P(X \leqslant 3)$, incorrect λ and allow one end error
	$= 0.0788$ (3 sf)	**A1** [3]	SR Use of Bin scores B1 only for 0.0788

(ii)	$\lambda = \dfrac{n}{2500}$	**B1**	or $e^{-\mu} < 0.05$ M1	or $\dfrac{2499}{2500}$ B1
	$e^{-\frac{n}{2500}} < 0.05$ Allow = Allow incorrect λ	**M1**		$\left(\dfrac{2499}{2500}\right)^n < 0.05$ M1
	$-\dfrac{n}{2500} < \ln 0.05$ Attempt ln bs	**M1**	$-\mu < \ln 0.05$ M1 $(\mu > 2.9957)$	$n \ln \dfrac{2499}{2500} < \ln 0.05$ M1
	$n > 7489.3$ (1 dp) Smallest $n = 7490$	**A1** [4]	$n = \mu \times 2500$ B1 Smallest $n = 7490$ A1	Smallest $n = 7488$ A1

Question 21

	$\lambda = (1.2 + 2.3) \div 2$	**M1**	Attempt combined mean, allow $1.2 + 2.3$
	$= 1.75$	**A1**	Correct mean
	$e^{-1.75}\left(\dfrac{1.75^2}{2} + \dfrac{1.75^3}{3!}\right)$	**M1**	Allow incorrect mean. Allow end errors (1 and/or 4)
	$= 0.421$ (3 sf)	**A1** [4]	

Question 22

(i)		$\lambda = 3.3 \times \frac{25}{30} = 2.75$	**B1**	
		$e^{-2.75}(1 + 2.75 + \frac{2.75^2}{2})$	**M1**	Allow any λ Allow one end error
		$= 0.481$ (3 sf)	**A1** [3]	As final answer. Accept 0.482
(ii)	**(a)**	$\lambda \left(= 3.3 \times \frac{365}{30}\right) = 40.15$	**B1**	Accept 40.1 or 40.2
		$(X \sim Po(40.15) \Rightarrow X \sim N(40.15, 40.15))$		
		$\frac{50.5 - "40.15"}{\sqrt{"40.15"}}$ $(= 1.633)$	**M1**	Allow with incorrect or no cc OR no $\sqrt{}$ sign
		$1 - \Phi("1.633")$	**M1**	For correct area consistent with their working
		$= 0.0513$ (3 sf)	**A1** [4]	Accept 0.0512
	(b)	$\lambda > 15$	**B1** [1]	or similar

Question 23

(i)	Use of Poisson	**B1**	
	Mean = 2.4	**B1**	
	$1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2})$	**M1**	Allow any λ (*Allow one end error*)
	$= 0.43(0)$ (3 sf)	**A1** [4]	Final answer
			SR Use of binomial: B1 for ans 0.431 (3 sf)
(ii)	240 > 50 or n>50	**B1**	
	$240 \times 0.01 = 2.4 < 5$ or np<5 or p<0.1	**B1** [2]	SR n large, p small: B1

Question 24

(i)		$\lambda = 6.8$	**B1**	
		$e^{-6.8} \times \frac{6.8^5}{5!}$	**M1**	any λ
		$= 0.135$ (3 sf)	**A1** [3]	
(ii)	(a)	$e^{-3.4}(1 + 3.4 + \frac{3.4^2}{2} + \frac{3.4^3}{3!} + \frac{3.4^4}{4!})$	**M1**	any λ, allow one end-error
		$= 0.744$ (3 sf)	**A1** [2]	
	(b)	'0.744' $+ e^{-3.4} \times \frac{3.4^5}{5!}$	**M1**	or complete method, any λ, allow one end-error
		$= 0.87(0)$ (3 sf) or 0.871	**A1** [2]	
(iii)		$P(X \leqslant 6) = $ '0.870' $+ e^{-3.4} \times \frac{3.4^6}{6!}$	**M1**	or complete method, any λ
		$= 0.94$	**A1**	fully correct un-simplified expression or better
		Need 6 hair driers	**A1** [3]	dep M1A1 with numerical justification (0.94 or better)

Question 25

(a)	(i)	0.01×80 and 0.015×60	**M1**	$(1 - e^{-\lambda}) \times (1 - e^{-\mu})$ any λ, μ $(\lambda \neq \mu)$
		$(1 - e^{-0.8}) \times (1 - e^{-0.9})$	**M1**	allow one end error
		$= 0.327$ (3 sf)	**A1** [3]	
	(ii)	$\lambda = 0.02 \times 40 + 0.015 \times 60$	**M1**	or their $0.8 + 0.9$
		$e^{-1.7} \times (1 + 1.7 + \frac{1.7^2}{2})$	**M1**	
		$= 0.757$ (3 sf)	**A1** [3]	
(b)		$e^{-\lambda} \times \lambda = p$ and $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5p$	**M1**	or $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5 \times e^{-\lambda} \times \lambda$ seen or implied
		$\lambda = 3$	**A1**	
		$p = e^{-3} \times 3$	**M1**	their λ
		$= 0.149$ (3 sf)	**A1** [4]	

Question 26

(i)	$1 - e^{-1}(1 + 1)$ $\quad\quad (= 0.26424)$	**B1**		B1 for either λ correct.
	$1 - e^{-1.5}(1 + 1.5 + \frac{1.5^2}{2!})$ $(= 0.19115)$	**B1**		B1 for either correct expression with correct λ
	'0.26424' $\times$ '0.19115'	**M1**		product of their values for $\leqslant 2$ and $\leqslant 3$ from Poisson, need correct form " 1 - .. " , but allow incorrect λ values and end errors
	= 0.0505 (3 sf)	**A1**	[4]	accept 0.0504
(ii)	$\lambda = 30$	**B1**		
	N(30, 30)	**B1**✓		seen or implied, need N(λ,λ)
	$\frac{35.5 - 30}{\sqrt{30}}$ $\quad (= 1.004)$	**M1**		allow with wrong or no cc or no $\sqrt{}$
	Φ ('1.004')	**M1**		consistent with their working
	= 0.842 (3 sf)	**A1**	[5]	

Question 27

	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!})$	**M2**		Allow M1 if extra term $e^{-3.5} \times \frac{3.5^3}{3!}$ or '
	= 0.321 (3 sf)	**A1**	[3]	1 - . " or omit P(0)

Question 28

(i)	$(\lambda) = 3.6 \div 3 = 1.2$	**B1**		1.2 seen
	$1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2} + \frac{1.2^3}{3!}\right)$	**M1**		Allow any λ
	= 0.0338 (3 sf)	**A1**	[3]	As final answer
(ii)	N(60 × 3.6, 60 × 3.6)	**M1**		Stated or implied
	$\frac{240.5 - '216'}{\sqrt{'216'}}$ $\quad (= 1.667)$	**M1**		Allow with no or wrong cc (no sd/var mixes)
	$1 - \Phi('1.667')$	**M1**		Area consistent with their working
	= 0.0478 (3 sf)	**A1**	[4]	SR use of Poisson 0.0497 scores 4/4

Question 29

7(i)	Planes arrive at constant mean rate	**B1**	
	Planes arrive at random	**B1**	or Planes arrive independently Must be in context
	Total:	**2**	
ii)(a)	$(\lambda =)\ 5.2 \div 4$	**M1**	
	$e^{-1.3}(\frac{1.3^2}{2} + \frac{1.3^3}{3!})$	**M1**	Allow any λ , allow one end error
	$= 0.330$ (3 sfs)	**A1**	Accept 0.33
	Total:	**3**	
ii)(b)	$1 - e^{-3.467} \times (1 + 3.467 + \frac{3.467^2}{2!} + \frac{3.467^3}{3!})$	**M1**	Allow any λ except 5.2 or 1.3, allow one end error
	$= 0.456$ (3 sfs)	**A1**	
	Total:	**2**	
(iii)	N(52, 52) stated or implied	**B1**	
	$\frac{60.5 - 52}{\sqrt{52}}\ (= 1.179)$	**M1**	ft their mean and var. Allow wrong or no cc or no $\sqrt{}$
	$\Phi("1.179")$	**M1**	
	$= 0.881$ (3 sf)	**A1**	
	Total:	**4**	

Question 30

5(i)(a)	$X \sim N(42, 42)$	**B1**	stated or implied
	$\frac{39.5 - "42"}{\sqrt{"42"}}\ (= -0.386)$	**M1**	allow with wrong or no cc
	$1 - \phi("-0.386") = \phi("0.386")$	**M1**	correct area consistent with their working
	$= 0.65(0)$ (3 sf)	**A1**	
	Total:	**4**	
5(i)(b)	$42 > (\text{e.g. } 15)$ or mean is large	**B1**	$\lambda > 15$ or higher, $\lambda = $ large ignore subsequent work if not undermining what alrea written
	Total:	**1**	
(ii)(a)	$Y \sim Po(1.2)$	**B1**	stated or implied
	$1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$	**M1**	allow any λ allow one end error
	$= 0.121$ (3 sf)	**A1**	Using binomial: 0.119 SR **B1**
	Total:	**3**	
(ii)(b)	$60 \times 0.02 = 1.2 < 5$ or mean is small	**B1FT**	or large n small p FT Poisson only

Question 31

(i)	$e^{-2.4} \times \frac{2.4^2}{2!}$		M1	Allow incorrect λ
	$= 0.261$ (3 sfs)		A1	
		Total:	2	
(ii)	N(60, 60)		B1	seen or implied
	$\frac{54.5-60}{\sqrt{60}}$	$(=-0.710)$	M1	allow with wrong or missing cc
	$1-\phi("-0.710") = \phi("0.710")$		M1	For area consistent with their working
	$= 0.761$ (3 sf)		A1	
		Total:	4	
(iii)	$\lambda = 3.6 + 12 \div 7 \ (= 186/35)$ $(= 5.314)$		M1	
	$e^{-5.314}\left(1 + 5.314 + \frac{5.314^2}{2} + \frac{5.314^3}{3!}\right)$		M1	Allow incorrect λ. Allow one end error.
	$= 0.224$ (3 sfs)		A1	
		Total:	3	

Question 32

(i)	E(X) = 4.197	B1	
	Var (X) = 4.196	B1	Both to 3dp or better
	Total:	4	
(ii)	$E(X) \approx Var(X)$	B1	Condone =
(iii)	$e^{-4.1968}\left(1 + 4.1968 + \frac{4.1968^2}{2} + \frac{4.1968^3}{3!} + \frac{4.1968^4}{4!}\right)$	M1	Any λ. Allow with one end error
	$= 0.59(0)$ (3 sfs)	A1	Allow 0.591

Question 33

(i)	Poisson with $\lambda = 0.2$	B1	
	$1 - e^{-0.2}\left(1 + 0.2 + \frac{0.2^2}{2}\right)$	M1	1 – Poisson P(0, 1, 2, 3) attempted, any λ, allow one end error
	$= 0.00115$ (3 sf)	A1	SR: using Bin, ans 0.00115: **B1**
		Total:	3	
(ii)	n large (n > 50)	B1	
	$np = 0.2 < 5$ or p small	B1	

Question 34

Poisson	B1	seen or implied
$\lambda = 4.03$	B1	seen or implied
$e^{-4.03}(1 + 4.03 + \frac{4.03^2}{2!})$	M1	any λ; e.g. allow $\lambda = 4$ no extra or missing terms
$= 0.234$ (3 sf)	A1	

Question 35

$\dfrac{40.5-31}{\sqrt{31}}$	(= 1.706)	**M1**	standn correct but allow with no or incorrect cc
$1 - \phi("1.706")$		**M1**	indep correct area consistent with working
= 0.0441 (3 sf) or 0.0440		**A1**	not 0.044

Question 36

(a)(i)	Po(2.54)	**M1**	seen or implied Po(2540 × 0.001)
	$1 - e^{-2.54}(1 + 2.54)$	**M1**	any λ Allow 1 end error
	= 0.721 (3 sf)	**A1**	
		3	
(a)(ii)	n large and p small (or np (= 2.54) < 5)	**B1**	$n > 50$, $p < 0.1$
		1	
1(b)	$\mu = 5.6$	**B1**	
	$\sigma = 2.37$ (3 sf)	**B1**	Accept $\sqrt{5.6}$

Question 30

$\lambda = 98.4$		**B1**	
N(98.4, 98.4) seen or implied		**B1**	
$\dfrac{90.5-"98.4"}{\sqrt{"98.4"}}$	(= −0.796)	**M1**	allow with wrong or no cc. No sd/var mix.
$\phi("0.796")$		**M1**	Correct area consistent with working
= 0.787 (3 sf)		**A1**	

Question 31

(i)	$E(T) = 4.5 + 2.3$ $Var(T) = 1.1^2 + 0.7^2$	$(= 6.8)$ $(= 1.7)$		M1	Both methods seen or implied
	$\dfrac{8.5 - "6.8"}{\sqrt{"1.7"}}$	$(= 1.304)$		M1	Correct stand'n using their μ and σ^2 must be a combination of the two variables
	$\phi("1.304")$			M1	Area consistent with their working
	$= 0.904$ (3 sf)			A1	
				4	
(ii)	$E(D) = 4.5 - 2 \times 2.3$	or -0.1		M1	
	$Var(D) = 1.1^2 + 2^2 \times 0.7^2$	or 3.17		M1	Both can seen or implied
	$\dfrac{0 - ('-0.1')}{\sqrt{'3.17'}}$	$(= 0.056)$		M1	Correct stand'n using their μ and σ^2 must be a Combination of the two variables
	$1 - \phi("0.056")$			M1	Area consistent with their working
	$= 0.478$ (3 sf)			A1	
				5	

Question 32

(i)	No of males leaving (to do eng) each yr has const mean or Males leave (to do eng) indep of other males leaving (to do eng) or Males leave (to do eng) at random		B1	One of these or any equiv statement in context.
			1	
(ii)	$\lambda = 3.9$		B1	
	$1 - e^{-3.9}(1 + 3.9 + \dfrac{3.9^2}{2!} + \dfrac{3.9^3}{3!})$		M1	Any λ. Allow one end error or extra term.
	0.546753 or 0.547 (3 sf)		A1	
			3	
(iii)	$P(F = 0 \text{ and } M > 3) =$ $e^{-0.8} \times \left[1 - e^{-3.1}\left(1 + 3.1 + \dfrac{3.1^2}{2!} + \dfrac{3.1^3}{3!}\right) \right]$ $(= 0.16857)$		M1	Attempt $P(F = 0) \times P(M > 3)$ allow one end error for $P(M > 3)$ provided $\lambda = 3.1$
	$\dfrac{P(F=0 \text{ and } M>3)}{P(M+F>3)}$ $\dfrac{"0.16857"}{"0.54675"}$		M1	Attempted, allow any probability/their (ii) provided the answer is <1
	$= 0.308$ (3 sf)		A1	
			3	

Question 33

(i)	Po(2.25)	**B1**
	$e^{-2.25}(1 + 2.25 + \frac{2.25^2}{2})$	**M1**
	$= 0.609$ (3 sf)	**A1**
		3
(ii)	$\mu = 2.25$, which is less than 5; n large	**B1**
		1

Question 34

(i)	Accidents occur independently or randomly	**B1**	In context. Allow 'singly'.
		1	
(ii)	$e^{-2.5} \times \frac{2.5^4}{4!}$	**M1**	Poisson P(4), allow any λ
	$= 0.134$ (3 sfs)	**A1**	
		2	
(iii)	$\lambda = \frac{25}{12}$ or 2.08(333)	**B1**	
	$1 - e^{-\frac{25}{12}}(1 + \frac{25}{12} + \frac{25^2}{2!} + \frac{25^3}{3!})$	**M1**	1 – Poisson P(0, 1, 2, 3), allow any λ allow one end error
	$= 0.158$ (3 sfs)	**A1**	As final answer
		3	
(iv)	$N(\frac{1825}{84}, \frac{1825}{84})$ or $N(21.7(26), 21.7(26))$	**B1**	Stated or implied
	$\frac{29.5 - \frac{1825}{84}}{\sqrt{\frac{1825}{84}}}$	**M1**	Allow with wrong or no cc with their mean/sd
	$\Phi("1.668")$	**M1**	Correct area consistent with their working
	$= 0.952$ (3 sfs)	**A1**	

Question 35

$\lambda = 4.4$		**B1**	
$P(X<4) = e^{-4.4}(1 + 4.4 + \frac{4.4^2}{2} + \frac{4.4^3}{3!})$		**M1**	Allow any λ allow one end error
$= 0.359$		**A1**	
		3	

Question 36

N(153, 153)		**B1**	Seen or implied
$\dfrac{139.5-153}{\sqrt{"153"}}$ $(=-1.091)$		**M1**	Allow with wrong or no cc
$\phi("-1.091") = 1 - \phi("1.091")$		**M1**	For area consistent with their working
$= 0.138$ (3 sf)		**A1**	

Question 37

(i)	$\lambda = 10 \times 0.25 + 10 \times 0.36$ $(= 6.1)$		**B1**	
	$1 - e^{-6.1}\left(1 + 6.1 + \dfrac{6.1^2}{2} + \dfrac{6.1^3}{3!}\right)$		**M1**	$1 - P(X \leqslant 3)$, any λ Allow one end error
	$= 0.857$		**A1**	Allow 0.858
			3	
(ii)	$\lambda = 61$		**B1 ft**	Ft from (i)
	N('61', '61')		**M1**	N with $\mu = \lambda$, any λ. May be implied
	$\dfrac{59.5-61}{\sqrt{'61'}}$ $(=-0.192)$		**M1**	Standardise with their mean and variance Allow no or wrong cc. not 61/100
	$\Phi('-0.192') = 1 - \Phi('0.192')$		**M1**	Correct area consistent with their working
	$= 0.424$		**A1**	
			5	

Question 38

(i)	$e^{-5.6} \times \frac{5.6^3}{3!}$	M1	Allow any λ
	$= 0.108$ (3 sf)	A1	
		2	
(ii)	P(X=2 & Y=1) = $e^{-2.1} \times \frac{2.1^2}{2} \times e^{-3.5} \times 3.5$ $(0.2700 \times 0.10569 = 0.028538)$	M1	
	$\frac{P(X = 2\ \&\ Y = 1)}{P(X+Y=3)}$ attempted $= \frac{'0.028538'}{'0.108234'}$	M1	For attempt at fraction with their **(i)** as denominator or $\frac{2.1^2}{2} \times 3.5 \div \frac{5.6^3}{3}$ M2
	$= 0.264$ (3 sf)	A1	
		3	
(iii)	Var(X) = 2.1	B1	soi
	$\bar{X} \sim N(2.1, \frac{2.1}{100})$ or N(210,210)	B1	soi B1 for N(2.1, ...)
		B1	B1 for $\frac{2.1}{100}$ oe Standardise with their values. Allow with or without cc or with incorrect cc
	$\frac{2.2-2.1}{\frac{\sqrt{2.1}}{\sqrt{100}}}$ oe $(220 - 210)/\sqrt{210}$ $(=0.690)$	M1	or $\frac{2.2+0.5\div100-2.1}{\frac{\sqrt{2.1}}{\sqrt{100}}}$ or $(220.5 - 210)/\sqrt{210})$ $(=0.725)$ no mixed methods
	$1 - \phi('0.690')$	M1	Correct area consistent with their working or $1 - \phi('0.725')$
	$= 0.245$ (3 sf)	A1	$= 0.234$ (3 sf)
		6	

Question 39

$e^{-2.3}(\frac{2.3^2}{2} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!})$	M2	M1 for one term wrong or one end error or $1 - P(2, 3, 4)$
$= 0.585$ (3 sf)	A1	
	3	

Question 40

(i)	$1 - e^{-1.8}(1 + 1.8)$	M1	Accept any λ. Accept $1-P(0,1,2)$
	$= 0.537$ (3 sf)	A1	
		2	
(ii)	$\lambda = 2.2$	B1	
	$e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2!} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$	M1	Attempt expr'n for $P(X \leqslant 4)$,allow one end error, allow any λ
	$= 0.928$ (3 sf) or 0.927	A1	
		3	
(iii)	$1 - e^{-1.8t} \geqslant 0.99$ $\quad$ or $1 - e^{-\lambda} \geqslant 0.99$	M1	Condone = signs/incorrect inequality signs
	$e^{-1.8t} \leqslant 0.01$ $\quad$ or $e^{-\lambda} \leqslant 0.01$ $-1.8t \leqslant \ln 0.01$	M1	Valid attempt take logs (must have single term on each side)
	$t \geqslant 2.56$ She must watch for at least 2.56 (hours)	A1	or 2 hours, 34 mins or better. No errors seen
		3	

Question 41

(i)	Use of Po(2.8)	M1	May be implied
	$1 - e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2}))$	M1	Any λ allowing one end error
	$= 0.531$ or $0.53(0)$ (3 sf)	A1	SC Binomial 0.534 B1
		3	
(ii)	Use of Po(5.8)	M1	May be implied
	$e^{-5.8} \times \frac{5.8^6}{6!}$	M1	Any λ
	$= 0.16(0)$ (3 sf)	A1	
		3	
(iii)	Use of N(58, 58)	M1	May be implied or N(58, 55.38)
	$\frac{50.5 - '58'}{\sqrt{'58'}}$ $(= -0.985)$	M1	Standardised with their values, allow wrong or incorrect cc
	$\Phi('0.985')$	M1	Correct area consistent with their working or $\Phi("1.008)$
	$= 0.838$ (3 sf)	A1	or 0.843
		4	

Question 42

(i)	Max no. of passengers plane can take oe	B1	oe e.g. No of passengers who bought tickets
		1	
(ii)	$\lambda = 3.2$	B1	
	$e^{-3.2}(\frac{3.2^3}{3!} + \frac{3.2^4}{4!} + \frac{3.2^5}{5!})$	M1	Any λ. Allow one end error
	$= 0.5146 = 0.515$ (3 sfs)	A1	SR Use of Bin(640,0.005) scores B1 (only) for 0.516
		3	

Question 43

(i)	0.0842 (3 sf)	B1	
		1	
(ii)	$e^{-5} \times \frac{5^n}{n!} = e^{-5} \times \frac{5^{n+1}}{(n+1)!}$	B1	or $\frac{5^n}{n!} = \frac{5^{n+1}}{(n+1)!}$ or better ISW
		1	
(iii)	$1 = \frac{5}{n+1}$ $n = 4$	B1	
		1	

Question 44

(a)(i)	$e^{-2.3}(\frac{2.3^2}{2} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!}$	M1	Allow one end error
	$= 0.585$	A1	
		2	
(a)(ii)	$(\lambda) = 4.6$	B1	
	$1 - e^{-4.6}(1 + 4.6 + \frac{4.6^2}{2})$	M1	any λ, Allow one end error
	$= 0.837$ (3 sf)	A1	
		3	
(a)(iii)	$S \sim N(115, 115)$	B1	May be implied
	$\frac{110.5 - 115}{\sqrt{115}}$ $(= -0.420)$	M1	Allow with wrong or no cc OR no $\sqrt{}$
	$1 - \Phi('0.420')$ $(= 1 - 0.663)$	M1	
	$= 0.337$	A1	Accept alternative method using N(2.3, 2.3) no mixed methods.
		4	
(b)	$e^{-\lambda} \times \frac{\lambda^3}{3!} = e^{-\lambda} \times \frac{\lambda^5}{5!}$	M1	
	$\lambda^3 = \frac{\lambda^5}{4 \times 5}$ or $\lambda^2 = 20$ oe	A1	any correct simplification without $e^{-\lambda}$ or !
	$\lambda = \sqrt{20}$ or $2\sqrt{5}$ or 4.47 (3 sf)	A1	
		3	

Question 45

(i)	Mean = 115	B1	
	SD = 40	B1	
		2	
(ii)	Mean = 15 × '115' = 1725	B1ft	
	15 × '40'² ² (= 24000)	M1	or SD = $\sqrt{15 \times '40'}$. ft their (i)
	SD = $\sqrt{24000}$ SD = 155 (cents) (3 sf)	A1	Accept $\sqrt{24000}$ SC: Allow correct answers in dollars
		3	

Question 46

(i)	Po(3)	B1	SOI
	$e^{-3}\left(\dfrac{3^3}{3!}+\dfrac{3^4}{4!}+\dfrac{3^5}{5!}\right)$	M1	Allow one or two extra terms (2 or 6 or both)
	0.493 (3 sf)	A1	
		3	
(ii)	A correct equation from $P(0)=P(2)$ $\left(\text{leading to } 1=\dfrac{\lambda^2}{2}\right)$	M1	
	$\lambda=\sqrt{2}$ or 1.41 (3 sf)	A1	CWO
		2	

Question 47

(i)	$\left(\lambda\left(=2\times2.4\right)=4.8\right)$ $e^{-4.8}\left(1+4+\dfrac{4.8^2}{2}+\dfrac{4.8^3}{3!}\right)$	M1	Any λ
	0.294 (3 sf)	A1	
		2	
(ii)	$\left(\lambda\left(=60\times2.4\right)=144\right)$ N('144', '144')	M1	N and $\sigma^2=\mu$ SOI
	$\dfrac{139.5-'144'}{\sqrt{'144'}}(=-0.375)$	M1	Allow with no continuity correction
	$\phi('0.375')$	M1	Correct area consistent with their working
	0.646 (3 sf)	A1	
		4	

Question 48

(i)	Binomial	B1	
	$n=500$ **and** $p=\dfrac{1}{150}$ or 0.00667	B1	Or $B\left(500,\dfrac{1}{150}\right)$ for B1B1
		2	
(ii)	Poisson	B1	
	n large and mean $=\dfrac{10}{3}$ or 3.3 or better, which is <5	B1	Accept $n>50$
		2	
(iii)	$1-e^{-\frac{10}{3}}\times\left(1+\dfrac{10}{3}+\dfrac{\left(\frac{10}{3}\right)^2}{2}\right)$	M1	1-P($X=0$, 1, 2)
	$=1-0.353$	A1	Correct expression with $\lambda=3.3$ or better
	$=0.647$ (3 sf)	A1	**SC** Use of Binomial scores B1 for 0.648. Use of Normal scores B1 for 0.67(0) to 0.677
		3	

Question 49

(a)	$\lambda\ (= 0.4 \times 365 \div 50) = 2.92$	**B1**	
	$e^{-2.92}(1 + 2.92 + \frac{2.92^2}{2})$	**M1**	Any λ. Allow one end error
	$= 0.441$ (3 sf)	**A1**	
		3	

(b)	$e^{-\lambda} > 0.95$	**M1**	Allow '=' throughout
	$-\lambda > \ln 0.95$ or $\lambda < 0.051293$ OE	**M1**	Attempt ln both sides
	'0.051293' $\times 50 \div 0.4\ (= 6.411)$	**M1**	
	Largest n is 6 (3 sf) Allow $n = 6$ or $n \leqslant 6$ (NOT $n < 6$ or $n \geqslant 6$ as final answer)	**A1**	**SC** Trial and Improvement M1 for $e^{-\lambda} > 0.95$ SOI; M1 for $\lambda = n \times \frac{0.4}{50}$; M1 for use of both $n = 6$ giving 0.9531 and $n = 7$ giving 0.9455; A1 $n = 6$
		4	

Question 50

$(\lambda =) \frac{5}{12} = 0.417$ or better	**B1**	
$1 - e^{-\frac{5}{12}}(1 + \frac{5}{12})$	**M1**	$1 - P(X = 0$ or 1), by Poisson, using any λ, allow $1 - P(X = 0$ or 1 or 2) for M1
$= 0.0661$ or 0.0662 (3 sf)	**A1**	Final answer **SC** use of Binomial (from 0.06607…) B1 only
	3	

Question 51

(a)	$\lambda = 3$	**B1**
	$e^{-3}(1 + 3)$	**M1**
	$= 0.199$ (3 sf)	**A1**
		3

(b)	$P(A_1 = 1 \text{ and } A_1 + A_2 < 2) = P(A_1 = 1) \times P(A_2 = 0)$	**M1**
	$e^{-1.5} \times 1.5 \times e^{-1.5} = 0.0747$	**A1**
	$P(A_1 = 1 \mid A_1 + A_2 < 2) = \dfrac{P(A1 = 1 \text{ and } A1 + A2 < 2)}{P(A1 + A2 < 2)}$	**M1**
	$= \dfrac{1.5 \times (e^{-1.5})^2}{4e^{-3}} = \dfrac{'0.0747'}{'0.199'}$	
	$\dfrac{3}{8}$ or 0.375 (3 sf)	**A1**
		4

(c)	Takes negative values	**B1**
		1

Question 52

(a)(i)	$0, 1, 2, 3, \ldots.$	**B1**
		1
(a)(ii)	3	**B1**
		1
(a)(iii)	$e^{-3}\left(\dfrac{3^4}{4!}+\dfrac{3^5}{5!}+\dfrac{3^6}{6!}\right)$	**M1**
	0.319 (3 sf)	**A1**
		2

(b)	$\Phi^{-1}(0.0668)\;(=-1.500)$	**M1**
	$\text{N}(\mu, \mu)$	**M1**
	$\dfrac{45.5-\mu}{\sqrt{\mu}} = \text{“}-1.500\text{”}$	**M1**
	$\mu - \text{“}1.500\text{”}\sqrt{\mu} - 45.5 = 0$ $\sqrt{\mu} = \dfrac{\text{"}1.5\text{"}\pm\sqrt{\left(\text{"}-1.5\text{"}\right)^2 + 4\times 45.5}}{2}\;(=7.5369)$	**M1**
	$\mu = 56.8$ (3 sf)	**A1**
		5

Question 53

(a)(i)	$\text{B}(3600, 0.0012)$	**B1**
		1
(a)(ii)	$\text{Po}(4.32)$ (**B1** for Po. **B1** for $\lambda = 4.32$)	**B2**
	$n = 3600$ which is large, $p = 0.12$ which is small and $np = 4.32$ which is < 5	**B1**
		3
(a)(iii)	$1 - e^{-4.32}\left(1 + 4.32 + \dfrac{4.32^2}{2}\right)$	**M1**
	0.805 (3 sf)	**A1**
		2
(b)	$e^{-\lambda} > 0.1$	**M1**
	$(-\lambda > \ln 0.1)$ $(\lambda < \ln 10)$ $0.0012n < \ln 10$	**A1**
	$(n < 1918.8)$ largest n is 1918	**A1**
		3

Question 54

(a)(i)	$e^{-2.9} \times \dfrac{2.9^4}{4!}$	**M1**
	0.162 (3 sf)	**A1**
		2
(a)(ii)	$e^{-2.1} \times \dfrac{2.1^4}{4!} \times e^{-0.8} + e^{-2.1} \times \dfrac{2.1^3}{3!} \times e^{-0.8} \times 0.8$	**B1**
	(**B1** for either expression correct, **M1** for P(4, 0) + P(3, 1))	**M1**
	0.113 (3 sf)	**A1**
		3
(b)	N(29, 29)	**M1**
	$\dfrac{24.5 - 29}{\sqrt{29}} (= -0.83563)$	**M1**
	$1 - \Phi("0.836")$	**M1**
	0.202 (3sf)	**A1**
		4
(c)	29 is large or 29 > 15	**B1**
		1
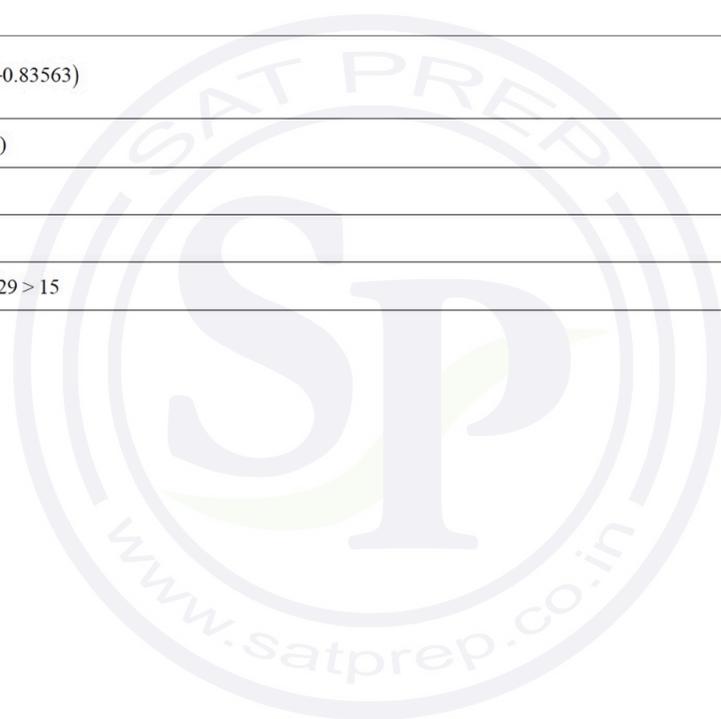
Question 55

(a)	$\text{Po}\left(\dfrac{2}{3}\right)$	**B1**
	$1 - e^{-\frac{2}{3}}\left(1 + \dfrac{2}{3}\right)$	**M1**
	$= 0.144 \ (3\ \text{sf})$	**A1**
		3
(b)	$n > 50$ and $np = \dfrac{2}{3} < 5$ **or** $n > 50$ and $p = \dfrac{1}{300} < 0.1$	**B1**
		3
(c)	$\text{Po}\left(\dfrac{11}{3}\right)$	**B1**
	$e^{-\frac{11}{3}}\left(1 + \dfrac{11}{3} + \dfrac{\left(\dfrac{11}{3}\right)^2}{2!} + \dfrac{\left(\dfrac{11}{3}\right)^3}{3!}\right)$	**M1**
	$= 0.501 \ \ (3\ \text{sf})$	**A1**
		3

Question 56

i(a)	Customers arrive independently or singly or at random	B1
		1
i(b)	$e^{-2.3} \times \dfrac{2.3^3}{3!}$	M1
	$= 0.203 \text{ (3sf)}$	A1
		2
i(c)	Po(4.6)	B1
	$1 - e^{-4.6}\left(1 + 4.6 + \dfrac{4.6^2}{2!} + \dfrac{4.6^3}{3!}\right)$	M1
	$= 0.674 \text{ (3sf)}$	A1
		3
i(d)	P(none arrive) $= e^{-2.3}$ $(= 0.10026)$	M1
	$^{5}C_2(e^{-2.3})^2(1 - e^{-2.3})^3$	M1
	$= 0.0732 \text{ or } 0.0733 \text{ (3sf)}$	A1
		3

Question 57

Poisson, any λ		M1
$1 - e^{-3}\left(1 + 3 + \dfrac{3^2}{2}\right)$		M1
$= 0.577 \text{ (3sf)}$		A1
		3

Question 58

(a)	$B(1000, \frac{1}{400})$	**B1**	Accept Bin and $n = 1000$, $p = \frac{1}{400}$.
		1	
(b)	Po(2.5)	**B2**	B1 for Po. B1 for $\lambda = 2.5$.
		2	
c)(i)	$e^{-2.5} \times \frac{2.5^4}{4!}$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow any λ
	0.134 (3 sf)	**A1**	CWO
		2	
c)(ii)	$e^{-2.5}(\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!})$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow with one term extra or omitted or wrong. Allow any λ.
	0.604 (3 sf)	**A1**	CWO
		2	
(d)	$\lambda = 2.5 \times 0.7$ or $\lambda = 700 \times \frac{1}{400}$ [= 1.75]	**M1**	Must see λ or use of Poisson.
	$1 - e^{-1.75}$	**M1**	Allow any λ. Allow $1 - P(0,1)$.
	0.826	**A1**	**SC B1** Use of B(700,0.0025) leading to 0.826.
		3	

Question 59

(a)	Po(2.5)	**B1**	Accept Poisson with mean = 2.5 not just np = 2.5
	$n = 25\,000 > 50$ and np (or λ) = 2.5 which is < 5 or $n = 25\,000 > 50$ and $p = 0.0001 < 0.1$	**B1**	Must see 2.5 (or 0.0001) and 25000 OE, not just $np < 5$ (or $p < 0.1$) and $n > 50$
		2	
(b)	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ, accept one end error. FT binomial from part **(a)** scores M1 only for equivalent binomial expressions FT normal from part **(a)** must use correct continuity correction and tables scores M1 only for complete method
	0.758 (3 sf)	**A1**	Unsupported answer of 0.758 scores B1 instead of M1A1
		2	
(c)	$e^{-2.5} \times \frac{2.5^k}{(k)!} = 2e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$	**M1**	Any λ FT binomial from **(a)** scores M1 only for equivalent binomial expression FT from **(a)** normal for equivalent expressions continuity correction must be included
	$k = 4$	**A1**	No errors seen **SC** $k = 4$ unsupported scores B1 only, but see full Poisson expressions for P(4) and P(5) and 0.134 scores M1A1
		2	

Question 60

(a)	$e^{-4.2} \times \dfrac{4.2^4}{4!}$	M1	P(4), allow any λ
	0.194 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
(b)	$1 - e^{-8.4}\left(1 + 8.4 + \dfrac{8.4^2}{2} + \dfrac{8.4^3}{3!}\right)$	M1	Allow M1 with incorrect λ. Accept one end error.
	0.968 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
(c)	N(50.4, 50.4)	M1	SOI
	$\dfrac{39.5 - 50.4}{\sqrt{50.4}}$ [= −1.535]	M1	Allow wrong or no continuity correction. Must have $\sqrt{}$
	$\Phi('-1.535') = 1 - \Phi('1.535')$	M1	For correct probability area consistent with *their* working.
	0.0624 (3 sf) or 0.0623	A1	
		4	

Question 61

(a)	$Po\left(\dfrac{2}{15}\right)$	M1	SOI. Allow Po(0.133).
	$P(X \geqslant 1) = 1 - e^{\frac{2}{15}}$	M1	Allow incorrect λ allow one end error
	$= 0.125$ (3 sf)	A1	**SC** Partially unsupported final answer: $Po\left(\dfrac{2}{15}\right)$ stated B1 then unsupported 0.125 B1 **SC** Use of Binomial (0.1248) B1 only Use of Normal scores M0
		3	
(b)	$\lambda = \dfrac{n}{75000}$	B1	
	$e^{-\frac{n}{75000}} > 0.9$	M1	Allow '=' Allow incorrect λ
	$-\dfrac{n}{75000} > \ln 0.9$ [$n < 7902.04$]	M1	Attempt ln both sides
	Largest value of n is 7902	A1	CWO. Must be an integer.

Question 62

$\lambda = (3.1 + 1.7) \times 2$	M1	Attempt combined mean. Allow $3.1 + 1.7$ for M1
$= 9.6$	A1	Correct mean
$1 - e^{-9.6} \left(1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!}\right)$	M1	Allow incorrect mean. Allow one end error.
$= 0.986$ (3 sf)	A1	**SC** If 9.6 seen and unsupported 0.986 M1A1B1. **SC** Unsupported correct answer of 0.986 only if 9.6 also not seen scores B2 only.
	4	

Question 63

(i)	Po(0.025)	B1	For Poisson and correct parameter.
	$n = 2500 > 50$, $np = 0.025 < 5$	B1	Must show 2500 and 0.025. Accept p $= \frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$.
		2	
(ii)	$1 - e^{-0.025}$	M1	Allow any λ. FT *their* **(a)(i)** if normal; must have continuity correction.
	0.0247 (3sf)	A1	Must be from Poisson. Unsupported correct answer scores **B1** instead of **M1 A1**.
		2	

Question 64

N(45.2, 45.2)	B1	SOI
$\dfrac{60.5 - 45.2}{\sqrt{45.2}}$ [= 2.276]	M1	Allow with wrong or no continuity correction.
$1 - \phi('2.276')$	M1	
0.0114	A1	
	4	

Question 65

(a)	$e^{-2}(1 + 2 + \frac{2^2}{2!})$	M1	P($X < 3$) any λ. Allow one end error.
	0.677 (3sf)	A1	Unsupported correct answer scores SC **B1** only.
		2	
(b)	N(40, 40)	M1	SOI
	$\frac{50.5 - 40}{\sqrt{40}}$ [= 1.660]	M1	For standardising with *their* values. Allow with wrong or no cc must have square root.
	P($z >$ '1.660') $= 1 - \Phi$('1.660')	M1	Correct area consistent with *their* working.
	0.0485 or 0.0484 (3sf)	A1	
		4	
(c)	$\lambda = 10$	B1	Condone mean = 10.
	$e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!} + \frac{10^{10}}{10!} + \frac{10^{11}}{11!}\right)$	M1	Allow any λ (allow one end error).
	0.477 (3sf)	A1	Unsupported correct answer scores SC **B2** only.
		3	

Question 66

(a)(i)	Po(0.025)	B1	For Poisson and correct parameter.
	$n = 2500 > 50$, $np = 0.025 < 5$	B1	Must show 2500 and 0.025. Accept p = $\frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$.
		2	
(a)(ii)	$1 - e^{-0.025}$	M1	Allow any λ. FT *their* **(a)(i)** if normal; must have continuity correction.
	0.0247 (3sf)	A1	Must be from Poisson. Unsupported correct answer scores **B1** instead of **M1 A1**.
		2	
(b)	H$_0$: $p = 0.3$ H$_1$: $p < 0.3$	B1	
	$0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$	M1	Use of B(28, 0.3). Addition of terms must be intended. Allow one term wrong or omitted or extra.
	0.0474	A1	Unsupported correct answer scores **B1** instead of **M1 A1**.
	0.0474 > 0.02 [Not reject H$_0$]	M1	Valid comparison.
	No evidence that suspicion is true.	A1 ft	Not definite e.g. not 'Suspicion is not true', in context, no contradictions. SC use of N(8.4, 5.88) leading to 0.054 > 0.2 OE can score **B1** only for comparison and correct conclusion. Correct hypotheses with p will also score B1.
		5	

Question 67

N(45.2, 45.2)	**B1**	SOI
$\dfrac{60.5 - 45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**	Allow with wrong or no continuity correction.
$1 - \phi('2.276')$	**M1**	
0.0114	**A1**	
	4	

Question 68

(a)(i)	0.024×50 [= 1.2] and 0.018×60 [= 1.08]	**B1**	
	$(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.08}(1 + 1.08))$	**M1**	For $(1 - e^{-\lambda}(1+\lambda)) \times (1 - e^{-\mu}(1+\mu))$ any λ, μ $(\lambda \neq \mu)$ Allow one end error on either or both terms
	= 0.0991 (3 sf)	**A1**	Unsupported answer scores maximum **SC B1 B1** **SC** Use of binomial 0.0994 scores B1 only
		3	
(a)(ii)	$\lambda = 0.024 \times 50 + 0.018 \times 60$	**M1**	or *their* 1.2 + 1.08 (NB 0.024+0.018 is M0)
	$1 - e^{-2.28} \times \left(1 + 2.28 + \dfrac{2.28^2}{2!} + \dfrac{2.28^3}{3!}\right)$	**M1**	any λ and allow one end error
	= 0.197 (3 sf)	**A1**	Unsupported answer scores maximum **SC B2**
		3	
(b)	$e^{-\lambda} = \left[e^{-\mu}\right]^2 = e^{-2\mu}$	**M1**	
	$e^{-\lambda} \times \dfrac{\lambda^2}{2} = k\left[e^{-\mu} \times \mu\right]^2$	**M1**	
	$e^{-2\mu} \times 2\mu^2 = k \times e^{-2\mu} \times \mu^2$	**M1**	OE. Use of $\lambda = 2\mu$ to find equation in μ and k only (or λ and k only)
	$k = 2$	**A1**	
		4	

Question 69

(a)	$\lambda = 6.6$	B1	
	$e^{-6.6} \times \dfrac{6.6^6}{6!}$	M1	Any λ
	0.156 (3 s.f.)	A1	If M0 awarded SC B1 for 0.156
		3	
(b)	$1 - e^{-2.2}(1 + 2.2 + \dfrac{2.2^2}{2} + \dfrac{2.2^3}{3!} + \dfrac{2.2^4}{4!})$	M1	Allow one end error. Need $1 - \ldots$ Any λ
	0.0725 (3 s.f.)	A1	If M0 awarded SC B1 for 0.0725
		2	
(c)	N(26.4, 26.4)	B1	Give at early stage 2.2×12
	$\dfrac{19.5 - '26.4'}{\sqrt{'26.4'}}$ [$= -1.343$]	M1	Standardising with *their* values. Allow wrong or no continuity correction
	$\phi('-1.343') = 1 - \phi('1.343')$	M1	Area consistent with *their* working
	0.0897 or 0.0896 (3 s.f.)	A1	
		4	

Question 70

$\bar{X} \sim N(2.9, \dfrac{2.9}{100})$ OR Totals method N(290, 290)	B1	B1 for N(2.9, …) OR N(290, …)
	B1	B1 for Var $= \dfrac{2.9}{100}$ OR for var $= 290$ SOI
$\dfrac{2.88 - 2.90}{\sqrt{\dfrac{2.9}{100}}}$ [$= -0.1174$] OR $\dfrac{288 - 290}{\sqrt{290}}$	M1	Standardising with *their* values Allow without $-$ve sign AND/OR with incorrect continuity correction No mixed methods
$1 - \Phi('0.1174')$	M1	For area consistent with *their* values
0.453 (3 sf)	A1	As final answer

Question 71

(a)	Poisson	B1	SOI
	Mean = 3.6	B1	Can be awarded for N(3.6, …)
	$e^{-3.6}(1 + 3.6 + \frac{3.6^2}{2})$	M1	Allow any λ Allow one end error Expression must be seen
	0.303 (3 s.f.)	A1	If M0 awarded allow SC B1 for 0.303 SC Use of binomial: B1 for answer 0.300 (3 sf)
		4	
(b)	[Binomial with] 200 > 50	B1	
	[200 × 0.018 =] 3.6 < 5 or [p =] 0.018 < 0.1	B1	If B0 B0 then SC n large, p small: B1 or n large $np < 5$: B1 or $n > 50$ and either $np < 5$ or $p < 0.1$: B1
		2	

Question 72

(a)	$\lambda = 4.5$	B1	
	$1 - e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!})$	M1	Allow one end error Allow any λ. Poisson expressions must be seen
	= 0.468 (3 sf)	A1	If M0 awarded allow SC B1 for 0.468
		3	
(b)	$\lambda = 162$ $(X \sim Po(162) \Rightarrow X \sim N(162, 162))$	B1	
	$\frac{149.5 - '162'}{\sqrt{'162'}}$ and $\frac{160.5 - '162'}{\sqrt{'162'}}$ (= −0.982 and −0.118)	M1	One of these; allow with incorrect or no continuity correction
	$\Phi('0.982') - \phi('0.118')$ oe	M1	Area consistent with *their* values (both standardisations must be seen)
	= 0.290 (3 sf)	A1	Allow 0.29
		4	
(c)	$\lambda = \frac{13.5}{6} + 3.6 \times \frac{2}{3}$ OE or 4.65	M1	Attempt to find λ
	$e^{-4.65}(\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!})$	M1	Allow any λ Allow one end error Poisson terms not be seen
	0.494 (3 sf)	A1	If M0 allow SC B1 for 0.494
		3	

Question 73

(a)	$\lambda = 5.2 \div 2$ [= 2.6]	B1	
	$1 - e^{-2.6}(1 + 2.6 + \frac{2.6^2}{2})$ or $1 - e^{-2.6}(1 + 2.6 + 3.38)$ **or** 1- (0.07427 + 0.1931 + 0.2510)	M1	Allow any λ. Allow one end error. Must see expression.
	= 0.482 (3 sf)	B1	
		3	
(b)	N(120×5.2, 120×5.2)	B1	Stated or implied. Give at early stage.
	$\dfrac{649.5 - their\ '624'}{\sqrt{their\ '624'}}$ [= 1.021]	M1	Allow with no or wrong continuity correction.
	$1 - \Phi(their\ '1.021')$	M1	For area consistent with *their* working.
	= 0.154 (3 sf)	A1	
		4	

Question 74

(a)	Use of Poisson. mean = 3.2	B1 B1	
	$1 - e^{-3.2}\left(1 + 3.2 + \dfrac{3.2^2}{2} + \dfrac{3.2^3}{3!}\right)$ or $1 - e^{-3.2}(1 + 3.2 + 5.12 + 5.46133)$ or $1 - (0.04076 + 0.1304 + 0.2087 + 0.2226)$	M1	Allow any λ. Allow one end error.
	= 0.397 or 0.398	A1	**SC** Use of binomial: B1 for answer 0.398 (3 sf). 0.397 or 0.398 with no working scores **SC B1**.
		4	
(b)	[Binomial with] [n =] 200 > 50	B1	
	[np =][200 × 0.016 =] 3.2 < 5 or [p =]0.016 < 0.1	B1	If B0 B0 SC *n* large (or n > 50), **and** *p* small or p < 0.1 or np <5 : B1.
		2	

Question 75

(a)	$\lambda = 5.2 \div 2$ [= 2.6]	B1	
	$1 - e^{-2.6}(1 + 2.6 + \frac{2.6^2}{2})$ or $1 - e^{-2.6}(1 + 2.6 + 3.38)$ **or** 1- (0.07427 + 0.1931 + 0.2510)	M1	Allow any λ. Allow one end error. Must see expression.
	= 0.482 (3 sf)	B1	
		3	
(b)	N(120×5.2, 120×5.2)	B1	Stated or implied. Give at early stage.
	$\dfrac{649.5 - their\ '624'}{\sqrt{their\ '624'}}$ [= 1.021]	M1	Allow with no or wrong continuity correction.
	$1 - \Phi(their\ '1.021')$	M1	For area consistent with *their* working.
	= 0.154 (3 sf)	A1	
		4	

Question 76

(a)	Orders arrive at constant mean rate (must say mean or rate) Orders arrive at random Orders arrive independently Orders arrive singly		Must be in context (accept 25.2 as context).
		B1	Any one reason correctly stated.
		B1	A second reason correctly stated.
			SC B1: both correct, not in context.
		2	

(b)(i)	$\lambda = \frac{3}{8} \times 25.2 \ [= 9.45]$	**B1**	
	$e^{-9.45}(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!})$ **or** $e^{-9.45}(140.65 + 332.29 + 628.03)$ **or** $0.01107 + 0.02615 + 0.04942$	**M1**	Allow any λ. Allow end errors. Expression must be seen.
	$= 0.0866$ (3 sf)	**A1**	If M0 allow **SC B1** for 0.0866 no working seen.
		3	

(b)(ii)	$e^{-3.15} \times 3.15$ **or** $(1 - e^{-3.15}(1 + 3.15))$ **or** 0.135 or 0.822 (3 sf)	**B1**	
	$e^{-3.15} \times 3.15 \times (1 - e^{-3.15}(1 + 3.15))$	**M1**	M1 for product of two Poisson probabilities $P(1) \times (1 - P(0,1))$ (no end errors accepted). Accept any λ.
	$\times 2$ or 0.111×2	**M1**	M1 for *their* product of two Poisson probabilities (accept end errors) $\times 2$. Accept any λ
	0.222 (3 sf)	**A1**	
		4	

(c)	N(113.4, 113.4)	**B1**	SOI
	$\frac{120.5 - 113.4}{\sqrt{113.4}} \ [= 0.667]$	**M1**	Standardise with *their* values. Allow wrong or no cc. Must have $\sqrt{\ }$.
	$1 - \phi(\text{their } '0.667')$	**M1**	For probability area consistent with *their* values.
	$= 0.252$ (3 sf)	**A1**	
		4	

Question 77

(a)	$X \sim \text{Po}(2.5)$	**B1**	SOI.
	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ. Allow one end error.
	$= 0.758$ (3 sf)	**A1**	**SC** use of binomial B1 for 0.758. **SC** when no working is shown, $X \sim \text{Po}(2.5)$ seen scores B1, 0.758 seen also scores B1.
		3	

(b)	$E(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995	***B1**	Just an answer of 2.5 for the variance is not sufficient. However, 2.4995 is sufficient.
	These are almost equal	**DB1**	Condone 'equal'.
		2	

Question 78

(a)	Books received independently or singly or randomly.	**B1**	OE. Must be in context. If more than one condition given, ignore extras.
		1	
(b)	$e^{-15.3} \times \dfrac{15.3^{10}}{10!}$	**M1**	Allow incorrect λ.
	$= 0.0439$ (3sf)	**A1**	**SC** No working shown but correct answer seen scores **B1**.
		2	
(c)	N(153, 153)	**B1**	Seen or implied.
	$\dfrac{180.5-153}{\sqrt{153}}$ [= 2.223]	**M1**	For standardising with their values (can be implied). Allow with wrong or missing continuity correction.
	$1 - \phi(\text{`2.223'})$	**M1**	For correct probability area consistent with their values.
	$= 0.0131$ (3sf)	**A1**	
		4	
(d)	$(\lambda =)\ 5.1 + 2.5$ [= 7.6]	**B1**	Give at early stage (seen or implied).
	$1 - e^{-7.6}(1+7.6+\dfrac{7.6^2}{2}+\dfrac{7.6^3}{3!}) = 1 - e^{-7.6}(1+7.6+28.88+73.16)$ $= 1 - (0.0005005+0.003803+0.01445+0.03661)$	**M1**	Allow incorrect λ. Allow one end error. Must see an expression (accept correct sigma notation).
	$= 0.945$ (3sf)	**A1**	**SC** No working, 0.945 B1(could be implied) **SC B1**.
		3	

Question 79

(a)	$E(W) = Var(W)$.	**B1**	Allow 'they are the same' OE. Must be $=$ not $\approx$ (and not both $=$ and $\approx$). Condone $E(W) = \lambda$ and $Var(W) = \lambda$.
		1	
(b)	$np \approx np(1-p)$, hence $1-p$ must be close to 1	**B1**	OE. Must see formulae and $q = 1-p$ must be seen or implied and conclusion made.
		1	
(c)	$\lambda = 1.4$	**B1**	Seen.
	$1 - e^{-1.4}(1 + 1.4 + \dfrac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or $1 - (0.2466+0.3452+0.2417)$	**M1**	Allow any λ; allow one end error. Expression must be seen (accept correct sigma notation).
	$= 0.167$ (3 sf) or 0.166	**A1**	Use of Binomial scores SCB1 for 0.167 or 0.166 . No working: 0.167 [or 0.166] **SC B1**. Note: λ=1.4 and 0.167 with no working seen scores **SC B1B1**. Use of Normal scores B0M0.
		3	

Question 80

(a)	$20540/6012300 = 0.0034163$	**B1**	
	$[1000 \times 0.0034163 = 3.4163]$		
	Po(3.4163)	**B1**	Could be implied by expression seen.
	$e^{-their\ '3.4163'}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-their\ '3.4163'}(1 + 3.4163 + 5.8356 + 6.6453)$ or $0.03283 + 0.1122 + 0.1916 + 0.21819)$	**M1**	Allow any λ. Allow with one end error. Must see expression.
	$= 0.555$ (3sf)	**A1**	CAO **SC** No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4). **SC** Binomial: B1 B0 B1 correct answer (max 2/4).
		4	
(b)	$n = 1000 > 50$	**B1**	Must show comparison with 50.
	$np = 3.4163 < 5$	**B1**	Must show comparison with 5.
		2	**SC B1**: $n > 50$ (or n large), $np < 5$. **SC B1**: n large, p small.

Question 81

(a)(i)	$\lambda = 3$	**B1**	For mean = 3.
	$1 - e^{-3}(1 + 3 + \frac{3^2}{2} + \frac{3^3}{3!})$ or $1 - e^{-3}(1 + 3 + 4.5 + 4.5)$ or $1 - (0.04979 + 0.14936 + 0.22404 + 0.22404)$	**M1**	Any λ. Allow one end error.
	$= 0.353$ (3 sf)	**A1**	No working scores B1.
		3	
(a)(ii)	N(54, 54)	**M1**	soi
	$\frac{39.5 - 54}{\sqrt{54}}$ $(= -1.973)$	**M1**	Allow with wrong or no continuity correction. For standardising with their mean and variance.
	$1 - \phi\ ('1.973')$	**M1**	For area consistent with their working.
	$= 0.0242$ (3 sf)	**A1**	Special case: if no working seen, 0.0242 scores **SC B3**, 0.0284 scores **SC B2**.
		4	
(b)(i)	'Mean not constant' or 'number of hits per minute not constant' or 'not a constant rate'	**B1**	
		1	
(b)(ii)	$2p + p = 2 \times 0.3$ $[\Rightarrow p = 0.2]$ [where p is the rate per minute for night time]	**M1**	May be implied by answer.
	[During day-time]: Po(0.4). [During night-time]: Po(0.2)	**A1**	Accept Po(24) [per daytime hour], Po(12) [per night time hour]. Accept Po(288) [per day time shift], Po(144)[per night time shift]. Note: Po(432), Po(216) scores M0A0.
		2	

Question 82

$\frac{405 - 410}{\frac{20}{6}}$ $[= -1.5]$	**M1**	For standardising, must have $\sqrt{36}$. Allow totals method $\frac{14580 - 14760}{\sqrt{14400}}$. No mixed methods.
$\phi('-1.5') = 1 - \phi('1.5')$	**M1**	For area consistent with their working.
$= 0.0668$	**A1**	
	3	

Question 83

(a)	$\left(e^{-2.4}\left(\dfrac{2.4^2}{2!} + \dfrac{2.4^3}{3!} \right) \right) = e^{-2.4}\left(2.88 + 2.304 \right) = 0.2613 + 0.2090$	**M1**	Allow M1 for $e^{-2.4}\left(\dfrac{2.4^2}{2!} + \dfrac{2.4^3}{3!} + \dfrac{2.4^4}{4!} \right)$. Expression must be seen.
	$= 0.47(0)$	**A1**	**SC B1** 0.47(0) with no working.
		2	
(b)	$1 - e^{-2.4}(1 + 2.4)$ [$= 0.691558$]	**M1**	Allow one end error. Allow any λ.
	$(1 - e^{-2.4}(1 + 2.4))^2$	**M1**	Squaring their probability ($\lambda \neq 4.8$).
	$= 0.478$ (3 sf)	**A1**	**SC B2** 0.478 with no working.
		3	
(c)(i)	$e^{-2.4} \times \dfrac{2.4^r}{r!} < e^{-2.4} \times \dfrac{2.4^{r+1}}{(r+1)!}$	**M1**	For both expressions seen.
	$r + 1 < 2.4$	**A1***	Or $r < 1.4$ (must have correct inequality).
	Set is $r = 0, 1$	**DA1**	
		3	For trial and error solutions. M1 for substituting one value into correct expression (can be implied by correct values). A1* for [P(0) = 0.0907], P(1) = 0.218, P(2) = 0.261, P(3) = 0.209 (accept 2sf accuracy). **DA1** Set is $r = 0,1$.
(c)(ii)	$r = 2$	**B1**	
		1	

Question 84

(a)	$X \sim N(25, 25)$	**B1**	soi
	$\dfrac{30.5 - 25}{\sqrt{25}}$ [$= 1.1$]	**M1**	Standardising with their values. Allow with missing or incorrect continuity correction.
	$1 - \phi('1.1')$	**M1**	For area consistent with their working.
	$= 0.136$ (3 sf)	**A1**	
		4	
(b)	$e^{-100p}\left(1 + 100p + \dfrac{(100p)^2}{2!} \right)$	**M1**	For $P_o(100p)$ expression. Accept un-simplified terms (e.g. $p^0/0!$ For M1). Allow one end error (e.g. for correct with extra term $e^{-100p} \times \dfrac{(100p)^3}{3!}$ oe), or brackets omitted.
	$e^{-100p}\left(1 + 100p + \dfrac{(100p)^2}{2!} \right)$ or $e^{-100p} + e^{-100p} \times 100p + e^{-100p} \times \dfrac{(100p)^2}{2!}$ or $e^{-100p}(1 + 100p + 5000p^2)$ oe	**A1**	Must have brackets. Allow with or without ! sign (but not 0! or p^0). ISW once a fully correct answer seen.
		2	

Question 85

(a)	$[\lambda =]\ 0.6$		**B1**	Mean = 0.6 seen.
	$1 - e^{-0.6}\left(1 + 0.6 + \dfrac{0.6^2}{2}\right)$ or $1 - e^{-0.6}(1 + 0.6 + 0.18)$ or $1 - (0.5488 + 0.3293 + 0.09879)$		**M1**	Any λ Allow one end error. Must see expression. Accept correct Σ notation.
	$= 0.0231$		**A1**	**SC** 0.0231 and no working scores B1 (could be implied). **SC** use of binomial scores M1A1 for 0.0231.
			3	
(b)	$6000 > 50$ and either $np = 0.6 < 5$ or $\dfrac{1}{10000} < 0.1$		**B1**	Must state values of n and either np or p. Note: 'n large, p small' is insufficient.
			1	

Question 86

(a)	$\lambda = 1.9 + 2.2\ [= 4.1]$		**B1**	
	$e^{-4.1}\left(1 + 4.1 + \frac{4.1^2}{2!} + \frac{4.1^3}{3!}\right)$ or $e^{-4.1}(1 + 4.1 + 8.405 + 11.487)$ or $0.01657 + 0.06795 + 0.13929 + 0.19037$		**M1**	Allow any λ. Allow one end error. Must see expression.
	$= 0.414\ (3\text{sf})$		**A1**	**SC:** unsupported answer 0.414 scores **B1 B1**.
			3	
(b)	$P(X + Y < 4 \text{ and } X = 2) = P(2, 0 \text{ or } 2, 1)$		**M1**	Stated or implied.
	$= e^{-1.9} \times \frac{1.9^2}{2}\left(e^{-2.2} + e^{-2.2} \times 2.2\right)$ $\qquad[= 0.0957]$		**M1**	
	$P(X = 2 \mid X + Y < 4) = \frac{'0.0957'}{'0.414'}$		**M1**	Attempt $\frac{P(X+Y<4 \text{ and } X=2)}{P(X+Y<4)}$. Prob for denominator can be found in **(a)**.
	$0.231\ (3\text{sf})$		**A1**	
			4	
(c)	$E(X + Y) = 4.1\ \text{Var}(X + Y) = 4.1$ or Po(246)		**B1**	SOI
	Normal and var $= \frac{4.1}{60}$ Or normal and var $= 246$		**M1**	
	$\frac{4.0 - 4.1}{\sqrt{4.1 \div 60}}$ or totals method $\frac{240 - 246}{\sqrt{246}}$ or use of continuity correction		**M1**	No mixed methods. Or continuity correction: $\frac{4.0 - \frac{1}{120} - 4.1}{\sqrt{(4.1 \div 60)}}$ or $\frac{239.5 - 246}{\sqrt{246}}$. Condone incorrect continuity correction for M1.
	$= -0.383\ (3\text{sf})$		**A1**	$= -0.414$
	$\Phi('-0.383') = 1 - \Phi('0.383')$		**M1**	$\Phi('-0.414') = 1 - \Phi('0.414')$
	$= 0.351\ (3\text{sf})$		**A1**	$= 0.340$ or 0.339
			6	

Question 87

(a)	$[\lambda =] 4$	**B1**	
	$e^{-4}(\frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!})$ or $e^{-4}(8 + 10.67 + 10.67)$ or $0.1465 + 0.19537 + 0.19537$	**M1**	Allow one end error. Any λ. Expression must be seen.
	$= 0.537$ (3sf)	**A1**	**SC B1 B1** for unsupported correct answer. **SC B2** for use of Binomial leading to 0.537. Note: use of normal could score B1 only for mean = 4.
		3	
(b)	$n = 4000 > 50$ and either $np = 4 < 5$ or $p = 0.001 < 0.1$	**B1**	Explicit values seen.
		1	

Question 88

(a)	$e^{-3.1}(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!})$ or $e^{-3.1}(1 + 3.1 + 4.805 + 4.965)$ or $0.0450 + 0.1397 + 0.2165 + 0.22368$	**M1**	Condone one end error. Any λ. Accept fully correct Σ notation. Expression must be seen.
	$= 0.625$ (3sf)	**A1**	Correct answer with no working scores **SC B1**.
		2	
(b)	$[\lambda]= 5.5$	**B1**	SOI
	$1 - e^{-5.5}(1 + 5.5 + \frac{5.5^2}{2!} + \frac{5.5^3}{3!} + \frac{5.5^4}{4!})$ or $1 - e^{-5.5}(1 + 5.5 + 15.125 + 27.7292 + 38.1276)$ or $1 - e^{-5.5}(0.004087 + 0.0224772 + 0.061812 + 0.113323 + 0.155819)$	**M1**	Condone one end error. Any λ. Accept fully correct Σ notation. Expression must be seen.
	$= 0.642$ or 0.643 (3sf)	**A1**	Correct answer with no working scores **SC B1 B1**.
		3	
(c)	$[P(X = 3) \times P(Y = 2) =] = e^{-3.1} \times \frac{3.1^3}{3!} \times e^{-2.4} \times \frac{2.4^2}{2!}$ or $0.223676 \times 0.261267 [= 0.05844]$	**M1**	Find P(3 in first half AND 2 in second half). Must see expression.
	$[P(\text{total } 5) =]\ e^{-5.5} \times \frac{5.5^5}{5!}$ or 0.17140	**M1**	Use of 5.5 to find P(5).
	$P(P(\text{exactly 3 in 1}^{st}\text{ half given total 5}) = \frac{P(\text{exactly 3 in 1}^{st}\text{half and total 5})}{P(\text{total 5})}$	**M1**	Attempt at conditional probability; numerator = *their* 0.05844 and denominator = P(total 5) Note: $(\frac{3.1^3}{3!} \times \frac{2.4^2}{2!}) \div (\frac{5.5^5}{5!})$ scores M1 M1 M1.
	$[= \frac{'0.05844'}{'0.17140'}] = 0.341$ (3sf)	**A1**	
		4	

Question 89

(a)	N(145, 145)	**B1**	Stated or implied.
	$\pm \frac{150.5 - 145}{\sqrt{145}}$ $\qquad [= \pm 0.457]$	**M1**	Condone incorrect or omitted continuity correction.
	$\Phi('0.457')$	**M1**	For area consistent with their working.
	$= 0.676$ (3sf)	**A1**	**SC**: Unsupported answer of 0.676 scores **B3**. Unsupported answer of 0.646 or 0.661 scores **B2**. Unsupported answer of 0.6799 scores **B1**.
		4	
(b)	$145 > 15$	**B1**	Explicit. $\lambda > 15$ B0 if $\lambda = 145$ not stated. Accept $\geqslant$ Accept mean for λ.
		1	

Question 90

(a)	Constant average rate	**B1**	OE Accept constant rate. Allow without context.	
		1		
(b)	$\lambda = 14.4$	**B1**		
	$e^{-14.4}\left(\frac{14.4^{13}}{13!} + \frac{14.4^{14}}{14!} + \frac{14.4^{15}}{15!}\right)$ or $e^{-14.4}(183837 + 189089 + 181526)$ or $(0.102469 + 0.105396 + 0.101181)$	**M1**	Poisson P(13, 14, 15). Expression must be seen. Allow one end error; allow any λ. Allow fully correct sigma notation.	
	$= 0.309$ (3sf)	**A1**	**SC:** 0.309 with no working scores **B1 B1**.	
		3		
(c)	N(140×1.2, 140×1.2) or N(168, 168)	**B1**	Stated or implied.	
	$\dfrac{149.5 - 168}{\sqrt{168}}$ [= −1.427]	**M1**	Standardising using their mean and variance. Allow with wrong or no continuity correction.	
	$\Phi(\text{“}-1.427\text{”}) = 1 - \Phi(\text{“}1.427\text{”})$	**M1**	For area consistent with their working.	
	$= 0.0768$ or 0.0767 (3sf)	**A1**		
		4		

Question 91

$\lambda = 7.2$	**B1**		
$P(X > 3) = 1 - e^{-7.2}\left(1 + 7.2 + \frac{7.2^2}{2!} + \frac{7.2^3}{3!}\right)$ or $1 - e^{-7.2}(1 + 7.2 + 25.92 + 62.21)$ or $1 - (0.0007466 + 0.005375 + 0.01935 + 0.04644)$	**M1**	Allow any λ. Allow one end error. Must see expression. Allow fully correct sigma notation.
$= 0.928$ (3sf)	**A1**	**SC** 0.928 with no working seen scores **B1 B1**.
	3	

Question 92

(a)	$[\lambda = 2.7]$ $1 - e^{-2.7}(1 + 2.7 + \frac{2.7^2}{2})$ or $1 - e^{-2.7}(1 + 2.7 + 3.645)$ or $1 - (0.06721 + 0.1815 + 0.2450)$	**M1**	Any λ. Allow one end error. Must see expression.
	$= 0.506$ (3 sf)	**A1**	SC unsupported answer 0.506 scores **B1**.
		2	
(b)	$\lambda = 1.95$	**B1**	
	$e^{-1.95}(1 + 1.95 + \frac{1.95^2}{2} + \frac{1.95^3}{3!})$ or $e^{-1.95}(1 + 1.95 + 1.90125 + 1.2358)$ or $0.1423 + 0.2774 + 0.2705 + 0.1758$	**M1**	Any λ. Allow one end error. Must see expression.
	$= 0.866$	**A1**	SC unsupported answer 0.866 scores **B1B1**.
		3	
(c)	$1 - e^{-2.1x} \geqslant 0.90$ or $1 - e^{-\lambda} \geqslant 0.90$	**M1**	OE Condone use of '=' throughout.
	$[e^{-2.1x} \leq 0.1]$ or $e^{-\lambda} \leq 0.1$ $-2.1x \leq \ln 0.1$ or $-\lambda \leq \ln 0.1$ $[\lambda \geq 2.3026, 2.3026/2.1]$	**M1**	Rearrange and attempt take logs of relevant form.
	1.096 or 10.96 accept 1.097 or 10.97	***A1**	Seen.
	She must wait for at least 11 minutes	**A1 dep**	
			SC Use of trial and improvement. Use of $1 - e^{-\lambda}$ any numerical λ (not 2.1) ie one trial **M1**. Use of enough trials to give an answer of 0.90 (2sf) **M1**. $\lambda = 2.30$ i.e. 3sf accuracy AND 1.09… or 10.9 … **A1**. Then 11 **A1** dep.
		4	

Question 93

$\lambda = 4.5$	**B1**	
$1 - e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2} + \frac{4.5^3}{3!}) = 1 - e^{-4.5}(1 + 4.5 + 10.125 + 15.1875)$ $= 1 - (0.011109 + 0.049999 + 0.11248 + 0.16872)$	**M1**	Expression must be seen or implied by correct figures. Any λ. Allow one end error. Accept fully correct Σ notation.
0.658 (3 sf)	**A1**	SC unsupported 0.658 scores **B1 B1**.
	3	

Question 94

(a)	$1 - e^{-1.5}(1 + 1.5 + \frac{1.5^2}{2}) = 1 - e^{-1.5}(1 + 1.5 + 1.125) = 1 - (0.22313 + 0.334695 + 0.25102)$	M1	Allow one end error. Accept fully correct sigma notation.
	$= 0.191$	A1	SC unsupported correct answer scores B1.
		2	
(b)	$\lambda = 4.5$	B1	
	$e^{-4.5}(\frac{4.5^3}{3!} + \frac{4.5^4}{4!} + \frac{4.5^5}{5!}) = e^{-4.5}(15.1875 + 17.0859 + 15.3773)$ $= 0.1687 + 0.1898 + 0.1708$	M1	Any λ. Allow one end error. Accept fully correct sigma notation.
	$= 0.529$	A1	SC. Unsupported correct answer scores B1 B1.
		3	
(c)	$T \sim N(1.5n, 1.5n)$	B1	May be implied.
	$\Phi^{-1}(1 - 0.0391)\ [= 1.761]$	M1	Attempted.
	$\frac{330.5 - 1.5n}{\sqrt{1.5n}} = 1.761$	M1*	Attempt to standardise and $= \Phi^{-1}(1 - 0.0391)$. Allow no or incorrect cc instead of 330.5. Note: $(330.5 - 1.5n)/\sqrt{1.5}$ scores either B1M0* or B0M1*.
	$[1.5n + 1.761\sqrt{1.5n} - 330.5 = 0$ or $1.5n + 2.1568\sqrt{n} - 330.5 = 0$ or $2.25n^2 - 996.1516n + 109230.25 = 0]$	M1dep	Correctly forming and valid attempt to solve a quadratic equation in $\sqrt{n}$ or n or $\sqrt{(1.5n)}$ OE. SOI by correct answer.
	$[\sqrt{n} = 14.14\ [\text{or } -15.58]\ ] \Rightarrow n = 200$ (only)	A1	**Dep 330.5 used.**
		5	

Question 95

(a)	The probabilities are not equal	B1	OE. SOI.
	e.g. $P(n=0) = \frac{1}{4}$ and $P(n=1) = \frac{1}{2}$ or $P(n=1) = 2 \times P(n=0)$ or similar	B1	Numerical justification or there is one way to get 0 but two ways to get 1, or similar **B1B1**.
		2	
(b)	Choose PQ or QR or RP soi or reject R or P or Q soi	B1	Identifying the three correct cases to choose or reject and correctly identifying choosing or rejecting.
	eg 1 or 2 [choose]PQ. 3 or 4 [choose] QR. 5 or 6 [choose]RP		OE.
	or 1 or 2 [reject] R. 3 or 4[reject] P. 5 or 6 [reject] Q	B1	OE. Using the score on the dice to give each of the three cases equal probability. More than one throw of the dice scores **B0B0**.
		2	

Question 96

(a)	$\lambda = \dfrac{23.4}{60}$ or 0.39	**B1**	First **B1** only scored if see 0.39.
	$1 - e^{-0.39}(1 + 0.39) = 1 - (0.67706 + 0.26405)$	**M1**	Any λ. Allow one end error.
	$= 0.0589$ (3 sf)	**A1**	**SC** unsupported answer score **B1** instead of **M1A1**.
		3	
(b)(i)	N(23.4, 23.4)	**B1B1**	**B1** for N(23.4, ….) **B1** for Var(X) = 23.4. Note: marks for **(i)** cannot be recovered from **(ii)**.
		2	
(b)(ii)	$\dfrac{20.5 - 23.4}{\sqrt{23.4}}$ [= –0.5995] $\dfrac{29.5 - 23.4}{\sqrt{23.4}}$ [= 1.2610]	**M1**	Attempt to standardise both with their values. Must have square roots. Allow no (or incorrect) continuity correction(s).
	$\Phi('1.2610') - \Phi('-0.5995')$ (0.8964 – 0.2743)	**M1**	For attempt at area between 20 and 30 consistent with their working.
	$= 0.622$ (3 sf)	**A1**	**SC** no working seen can score **B2** for 0.622.
		3	

Question 97

(a)(i)	$(\lambda) = 3.5$	**B1**	
	$1 - e^{-3.5}(1 + 3.5 + \dfrac{3.5^2}{2!} + \dfrac{3.5^3}{3!}) = 1 - e^{-3.5}(1 + 3.5 + 6.125 + 7.1458)$ $= 1 - (0.030197 + 0.105691 + 0.1849589 + 0.215785)$	**M1**	Any λ. Expression or terms must be seen. Allow one end error. Accept fully correct Σ notation.
	$= 0.463$ (3 sf)	**A1**	**SC** Unsupported correct answer scores **B1B1** **SC** Use of Binomial scores **B1** for 0.464. Note: Use of Normal can score **B1** for mean =3.5.
		3	
(a)(ii)	$(\lambda =) 7$	**B1**	Seen.
	$e^{-7}(1 + 7 + \dfrac{7^2}{2!}) = e^{-7}(1 + 7 + 24.5) = 0.0009119 + 0.006383 + 0.0223411$	**M1**	Any λ. Expression or terms must be seen. Allow one end error Accept fully correct Σ notation. Accept combination method for Poisson (6 combinations) allow 6 correct combinations identified **B1**, attempt to calculate and combine at least four correct combinations **M1**.
	$= 0.0296$ (3 sf)	**A1**	**SC** Unsupported correct answer scores **B1B1**. **SC** Use of Bin **B1** for 0.0294. Note: Use of Normal can score **B1** for mean =7.
		3	
(b)	N(200, 200)	**M1**	SOI.
	$\dfrac{205.5 - 200}{\sqrt{200}}$ [= 0.38891]	**M1**	Allow with omitted or incorrect cc.
	$1 - \Phi("0.38891")$	**M1**	For finding area consistent with their values.
	$= 0.349$ (3 sf)	**A1**	
		4	

Question 98

(a)	$e^{-15} \times \dfrac{15^{12}}{12!}$	**B1**	Seen.
		1	
(b)	$e^{-15} \times \dfrac{15^{n}}{n!} = e^{-15} \times \dfrac{15^{n+1}}{(n+1)!}$	**B1**	If brackets (n+1)! missing allow benefit of doubt.
	$1 = \dfrac{15}{n+1}$	**M1**	OE. Attempt to legitimately remove powers and factorials i.e. powers reduced to $15/\lambda$ seen, and factorials reduced to n + 1 seen.
	$n = 14$	**A1**	Note: Trial and error solutions: **B1B2** for 14.
		3	

Question 99

$(\lambda) = 4$		**B1**	SOI.
$e^{-4}\left(1+4+\dfrac{4^2}{2!}+\dfrac{4^3}{3!}\right) = e^{-4}\ (1 + 4 + 8 + 10.6666)$ $= 0.018316 + 0.073263 + 0.146525 + 0.19537$		**M1**	Allow any λ, allow one end error.
$= 0.433$		**A1**	**SC₁** Unsupported answer of 0.433 scores **B1B1**. **SC₂** Use of Normal mean = 4 **B1**. **SC₃** Use Binomial 0.432 or 0.433 scores **B1**.
		3	