A-Level

Topic : Sampling and Estimation

May 2013-May 2025

Questions

Question 1

Each of a random sample of 15 students was asked how long they spent revising for an exam. The results, in minutes, were as follows.

50 70 80 60 65 110 10 70 75 60 65 45 50 70 50

Assume that the times for all students are normally distributed with mean μ minutes and standard deviation 12 minutes.

(i) Calculate a 92% confidence interval for μ. [4]

(ii) Explain what is meant by a 92% confidence interval for μ. [1]

(iii) Explain what is meant by saying that a sample is 'random'. [1]

Question 2

The masses, in grams, of a certain type of plum are normally distributed with mean μ and variance σ^2. The masses, m grams, of a random sample of 150 plums of this type were found and the results are summarised by $\Sigma m = 9750$ and $\Sigma m^2 = 647\,500$.

(i) Calculate unbiased estimates of μ and σ^2. [3]

(ii) Calculate a 98% confidence interval for μ. [3]

Two more random samples of plums of this type are taken and a 98% confidence interval for μ is calculated from each sample.

(iii) Find the probability that neither of these two intervals contains μ. [2]

Question 3

The lengths, x m, of a random sample of 200 balls of string are found and the results are summarised by $\Sigma x = 2005$ and $\Sigma x^2 = 20\,175$.

(i) Calculate unbiased estimates of the population mean and variance of the lengths. [3]

(ii) Use the values from part **(i)** to estimate the probability that the mean length of a random sample of 50 balls of string is less than 10 m. [3]

(iii) Explain whether or not it was necessary to use the Central Limit theorem in your calculation in part **(ii)**. [2]

Question 4

A random sample of 80 values of a variable X is taken and these values are summarised below.

$$n = 80 \qquad \Sigma x = 150.2 \qquad \Sigma x^2 = 820.24$$

Calculate unbiased estimates of the population mean and variance of X and hence find a 95% confidence interval for the population mean of X. [6]

Question 5

Following a change in flight schedules, an airline pilot wished to test whether the mean distance that he flies in a week has changed. He noted the distances, x km, that he flew in 50 randomly chosen weeks and summarised the results as follows.

$$n = 50 \qquad \Sigma x = 143\,300 \qquad \Sigma x^2 = 410\,900\,000$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) In the past, the mean distance that he flew in a week was 2850 km. Test, at the 5% significance level, whether the mean distance has changed. [5]

Question 6

Heights of a certain species of animal are known to be normally distributed with standard deviation 0.17 m. A conservationist wishes to obtain a 99% confidence interval for the population mean, with total width less than 0.2 m. Find the smallest sample size required. [4]

Question 7

A die is thrown 100 times and shows an odd number on 56 throws. Calculate an approximate 97% confidence interval for the probability that the die shows an odd number on one throw. [4]

Question 8

Mahmoud throws a coin 400 times and finds that it shows heads 184 times. The probability that the coin shows heads on any throw is denoted by p.

(i) Calculate an approximate 95% confidence interval for p. [4]

(ii) Mahmoud claims that the coin is not fair. Use your answer to part **(i)** to comment on this claim. [1]

(iii) Mahmoud's result of 184 heads in 400 throws gives an α% confidence interval for p with width 0.1. Calculate the value of α. [4]

Question 9

The weights, in grams, of a random sample of 8 packets of cereal are as follows.

250 248 255 244 259 250 242 258

Calculate unbiased estimates of the population mean and variance. [3]

Question 10

The score on one throw of a 4-sided die is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3
$P(X = x)$	0.25	0.25	0.25	0.25

(i) Show that $\text{Var}(X) = 1.25$. [1]

The die is thrown 300 times. The score on each throw is noted and the mean, $\overline{X}$, of the 300 scores is found.

(ii) Use a normal distribution to find $P(\overline{X} < 1.4)$. [3]

(iii) Justify the use of the normal distribution in part **(ii)**. [1]

Question 11

A die is biased. The mean and variance of a random sample of 70 scores on this die are found to be 3.61 and 2.70 respectively. Calculate a 95% confidence interval for the population mean score. [5]

Question 12

The masses, in grams, of apples of a certain type are normally distributed with mean 60.4 and standard deviation 8.2. The apples are packed in bags, with each bag containing 8 randomly chosen apples. The bags are checked by Quality Control and any bag containing apples with a total mass of less than 436 g is rejected. Find the proportion of bags that are rejected. [4]

Question 13

The times, in minutes, taken by people to complete a walk are normally distributed with mean μ. The times, t minutes, for a random sample of 80 people were summarised as follows.

$$\Sigma t = 7220 \qquad \Sigma t^2 = 656\,060$$

(i) Calculate a 97% confidence interval for μ. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

Question 14

In a survey a random sample of 150 households in Nantville were asked to fill in a questionnaire about household budgeting.

(i) The results showed that 33 households owned more than one car. Find an approximate 99% confidence interval for the proportion of all households in Nantville with more than one car. [4]

(ii) The results also included the weekly expenditure on food, x dollars, of the households. These were summarised as follows.

$$n = 150 \qquad \Sigma x = 19\,035 \qquad \Sigma x^2 = 4\,054\,716$$

Find unbiased estimates of the mean and variance of the weekly expenditure on food of all households in Nantville. [3]

(iii) The government has a list of all the households in Nantville numbered from 1 to 9526. Describe briefly how to use random numbers to select a sample of 150 households from this list. [3]

Question 15

The marks, x, of a random sample of 50 students in a test were summarised as follows.

$$n = 50 \qquad \Sigma x = 1508 \qquad \Sigma x^2 = 51\,825$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Each student's mark is scaled using the formula $y = 1.5x + 10$. Find estimates of the population mean and variance of the scaled marks, y. [3]

Question 16

A die is biased so that the probability that it shows a six on any throw is p.

(i) In an experiment, the die shows a six on 22 out of 100 throws. Find an approximate 97% confidence interval for p. [4]

(ii) The experiment is repeated and another 97% confidence interval is found. Find the probability that exactly one of the two confidence intervals includes the true value of p. [2]

Question 17

The volumes, v millilitres, of juice in a random sample of 50 bottles of Cooljoos are measured and summarised as follows.

$$n = 50 \qquad \Sigma v = 14\,800 \qquad \Sigma v^2 = 4\,390\,000$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) An α% confidence interval for the population mean, based on this sample, is found to have a width of 5.45 millilitres. Find α. [4]

Four random samples of size 10 are taken and a 96% confidence interval for the population mean is found from each sample.

(iii) Find the probability that these 4 confidence intervals all include the true value of the population mean. [2]

Question 18

The masses, m grams, of a random sample of 80 strawberries of a certain type were measured and summarised as follows.

$$n = 80 \qquad \Sigma m = 4200 \qquad \Sigma m^2 = 229\,000$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 98% confidence interval for the population mean. [3]

50 random samples of size 80 were taken and a 98% confidence interval for the population mean, μ, was found from each sample.

(iii) Find the number of these 50 confidence intervals that would be expected to include the true value of μ. [1]

Question 19

The daily times, in minutes, that Yu Ming takes showering, getting dressed and having breakfast are independent and have the distributions $N(9, 2.2^2)$, $N(8, 1.3^2)$ and $N(17, 2.6^2)$ respectively. The total daily time that Yu Ming takes for all three activities is denoted by T minutes.

(i) Find the mean and variance of T. [2]

(ii) Yu Ming notes the value of T on each day in a random sample of 70 days and calculates the sample mean. Find the probability that the sample mean is between 33 and 35. [4]

Question 20

The diameter, in cm, of pistons made in a certain factory is denoted by X, where X is normally distributed with mean μ and variance σ^2. The diameters of a random sample of 100 pistons were measured, with the following results.

$$n = 100 \qquad \Sigma x = 208.7 \qquad \Sigma x^2 = 435.57$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

The pistons are designed to fit into cylinders. The internal diameter, in cm, of the cylinders is denoted by Y, where Y has an independent normal distribution with mean 2.12 and variance 0.000 144. A piston will not fit into a cylinder if $Y - X < 0.01$.

(ii) Using your answers to part **(i)**, find the probability that a randomly chosen piston will not fit into a randomly chosen cylinder. [6]

Question 21

From a random sample of 65 people in a certain town, the proportion who own a bicycle was noted. From this result an $\alpha\%$ confidence interval for the proportion, p, of all people in the town who own a bicycle was calculated to be $0.284 < p < 0.516$.

(i) Find the proportion of people in the sample who own a bicycle. [1]

(ii) Calculate the value of α correct to 2 significant figures. [4]

Question 22

It is known that the number, N, of words contained in the leading article each day in a certain newspaper can be modelled by a normal distribution with mean 352 and variance 29. A researcher takes a random sample of 10 leading articles and finds the sample mean, $\overline{N}$, of N.

(i) State the distribution of $\overline{N}$, giving the values of any parameters. [2]

(ii) Find $P(\overline{N} > 354)$. [3]

Question 23

Jagdeesh measured the lengths, x minutes, of 60 randomly chosen lectures. His results are summarised below.

$$n = 60 \qquad \Sigma x = 3420 \qquad \Sigma x^2 = 195\,200$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 98% confidence interval for the population mean. [3]

Question 24

The mean and standard deviation of the time spent by people in a certain library are 29 minutes and 6 minutes respectively.

(i) Find the probability that the mean time spent in the library by a random sample of 120 people is more than 30 minutes. [4]

(ii) Explain whether it was necessary to assume that the time spent by people in the library is normally distributed in the solution to part **(i)**. [2]

Question 25

The 150 oranges in a random sample from a certain supplier were weighed and the masses, X grams, were recorded. The results are summarised below.

$$n = 150 \qquad \Sigma x = 14\,910 \qquad \Sigma x^2 = 1\,525\,000$$

(i) Calculate a 99% confidence interval for the population mean of X. [6]

(ii) The supplier claims that the mean mass of his oranges is 100 grams. Use your answer to part **(i)** to explain whether this claim should be accepted. [1]

(iii) State briefly why the sample should be random. [1]

Question 26

X and Y are independent random variables with distributions Po(1.6) and Po(2.3) respectively.

(i) Find $P(X + Y = 4)$. [3]

A random sample of 75 values of X is taken.

(ii) State the approximate distribution of the sample mean, $\overline{X}$, including the values of the parameters. [2]

(iii) Hence find the probability that the sample mean is more than 1.7. [3]

(iv) Explain whether the Central Limit theorem was needed to answer part **(ii)**. [1]

Question 27

The time taken for a particular type of paint to dry was measured for a sample of 150 randomly chosen points on a wall. The sample mean was 192.4 minutes and an unbiased estimate of the population variance was 43.6 minutes^2. Find a 98% confidence interval for the mean drying time. [3]

Question 28

(i) Give a reason for using a sample rather than the whole population in carrying out a statistical investigation. [1]

(ii) Tennis balls of a certain brand are known to have a mean height of bounce of 64.7 cm, when dropped from a height of 100 cm. A change is made in the manufacturing process and it is required to test whether this change has affected the mean height of bounce. 100 new tennis balls are tested and it is found that their mean height of bounce when dropped from a height of 100 cm is 65.7 cm and the unbiased estimate of the population variance is 15 cm^2.

(a) Calculate a 95% confidence interval for the population mean. [3]

(b) Use your answer to part **(ii)(a)** to explain what conclusion can be drawn about whether the change has affected the mean height of bounce. [1]

Question 29

The length of time, in minutes, taken by people to complete a task has mean 53.0 and standard deviation 6.2. Find the probability that the mean time taken to complete the task by a random sample of 50 people is more than 51 minutes. [4]

Question 30

Based on a random sample of 700 people living in a certain area, a confidence interval for the proportion, p, of all people living in that area who had travelled abroad was found to be $0.5672 < p < 0.6528$.

(i) Find the proportion of people in the sample who had travelled abroad. [1]

(ii) Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. [4]

Question 31

A researcher is investigating the lengths, in kilometres, of the journeys to work of the employees at a certain firm. She takes a random sample of 10 employees.

(i) State what is meant by 'random' in this context. [1]

The results of her sample are as follows.

1.5 2.0 3.6 5.9 4.8 8.7 3.5 2.9 4.1 3.0

(ii) Find unbiased estimates of the population mean and variance. [3]

(iii) State what is meant by 'population' in this context. [1]

Question 32

A variable X takes values 1, 2, 3, 4, 5, and these values are generated at random by a machine. Each value is supposed to be equally likely, but it is suspected that the machine is not working properly. A random sample of 100 values of X, generated by the machine, gives the following results.

$$n = 100 \qquad \Sigma x = 340 \qquad \Sigma x^2 = 1356$$

(i) Find a 95% confidence interval for the population mean of the values generated by the machine. [6]

(ii) Use your answer to part **(i)** to comment on whether the machine may be working properly. [2]

Question 33

The manufacturer of a tablet computer claims that the mean battery life is 11 hours. A consumer organisation wished to test whether the mean is actually greater than 11 hours. They invited a random sample of members to report the battery life of their tablets. They then calculated the sample mean. Unfortunately a fire destroyed the records of this test except for the following partial document.

Test of the mean batter the table	
Sample size, n	
Sample mean (hours)	11.8
Is the result significant at the 5% level?	Yes
Is the result significant at the 2.5% level?	No

Given that the population of battery lives is normally distributed with standard deviation 1.6 hours, find the set of possible values of the sample size, n. [5]

Question 34

(a) The masses, in grams, of certain tomatoes are normally distributed with standard deviation 9 grams. A random sample of 100 tomatoes has a sample mean of 63 grams. Find a 90% confidence interval for the population mean mass of these tomatoes. [3]

Question 35

The weights, in kilograms, of a random sample of eight 16-year old males are given below.

58.9 63.5 62.7 59.4 66.9 68.0 60.4 68.2

Find unbiased estimates of the population mean and variance of the weights of all 16-year old males. [3]

Question 36

The length, in centimetres, of a certain type of snake is modelled by the random variable X with mean 52 and standard deviation 6.1. A random sample of 75 snakes is selected, and the sample mean, $\overline{X}$, is found.

(i) Find $P(51 < \overline{X} < 53)$. [4]

(ii) Explain why it was necessary to use the Central Limit theorem in the solution to part **(i)**. [1]

Question 37

In a survey, 36 out of 120 randomly selected voters in Hungton said that if there were an election next week they would vote for the Alpha party. Calculate an approximate 90% confidence interval for the proportion of voters in Hungton who would vote for the Alpha party. [4]

Question 38

Last year the mean level of a certain pollutant in a river was found to be 0.034 grams per millilitre. This year the levels of pollutant, X grams per millilitre, were measured at a random sample of 200 locations in the river. The results are summarised below.

$$n = 200 \qquad \Sigma x = 6.7 \qquad \Sigma x^2 = 0.2312$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test, at the 10% significance level, whether the mean level of pollutant has changed. [5]

Question 39

The mass, in tonnes, of iron ore produced per day at a mine is normally distributed with mean 7.0 and standard deviation 0.46. Find the probability that the total amount of iron ore produced in 10 randomly chosen days is more than 71 tonnes. [5]

Question 40

In a random sample of 200 shareholders of a company, 103 said that they wanted a change in the management.

(i) Find an approximate 92% confidence interval for the proportion, p, of all shareholders who want a change in the management. [3]

(ii) State the probability that a 92% confidence interval does not contain p. [1]

Question 41

A residents' association has 654 members, numbered from 1 to 654. The secretary wishes to send a questionnaire to a random sample of members. In order to choose the members for the sample she uses a table of random numbers. The first line in the table is as follows.

1096 4357 3765 0431 0928 9264

The numbers of the first two members in the sample are 109 and 643. Find the numbers of the next three members in the sample. [3]

Question 42

Household incomes, in thousands of dollars, in a certain country are represented by the random variable X with mean μ and standard deviation σ. The incomes of a random sample of 400 households are found and the results are summarised below.

$$n = 400 \qquad \Sigma x = 923 \qquad \Sigma x^2 = 3170$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

(ii) A random sample of 50 households in one particular region of the country is taken and the sample mean income, in thousands of dollars, is found to be 2.6. Using your values from part **(i)**, test at the 5% significance level whether household incomes in this region are greater, on average, than in the country as a whole. [5]

Question 43

In a survey of 2000 randomly chosen adults, 1602 said that they owned a smartphone. Calculate an approximate 95% confidence interval for the proportion of adults in the whole population who own a smartphone. [4]

Question 44

(a) The waiting time at a certain bus stop has variance 2.6 minutes^2. For a random sample of 75 people, the mean waiting time was 7.1 minutes. Calculate a 92% confidence interval for the population mean waiting time. [3]

(b) A researcher used 3 random samples to calculate 3 independent 92% confidence intervals. Find the probability that all 3 of these confidence intervals contain only values that are greater than the actual population mean. [2]

(c) Another researcher surveyed the first 75 people who waited at a bus stop on a Monday morning. Give a reason why this sample is unsuitable for use in finding a confidence interval for the mean waiting time. [1]

Question 45

The lengths, in millimetres, of rods produced by a machine are normally distributed with mean μ and standard deviation 0.9. A random sample of 75 rods produced by the machine has mean length 300.1 mm.

(i) Find a 99% confidence interval for μ, giving your answer correct to 2 decimal places. [3]

The manufacturer claims that the machine produces rods with mean length 300 mm.

(ii) Use the confidence interval found in part **(i)** to comment on this claim. [1]

Question 46

After an election 153 adults, from a random sample of 200 adults, said that they had voted. Using this information, an α% confidence interval for the proportion of all adults who voted in the election was found to be 0.695 to 0.835, both correct to 3 significant figures. Find the value of α, correct to the nearest integer. [4]

Question 47

The number of words in History essays by students at a certain college has mean μ and standard deviation 1420.

(i) The mean number of words in a random sample of 125 History essays was found to be 4820. Calculate a 98% confidence interval for μ. [3]

(ii) Another random sample of n History essays was taken. Using this sample, a 95% confidence interval for μ was found to be 4700 to 4980, both correct to the nearest integer. Find the value of n. [3]

Question 48

A nutritionist wishes to investigate the mean sugar content in some cereal bars. He takes a random sample of 10 of the bars and measures the mass, in grams, of sugar in each bar. His results are shown below.

11.9 11.7 11.8 11.9 11.6 12.1 11.7 11.9 11.8 11.9

Assume that the mass, in grams, of sugar in bars of this type has the distribution N(μ, 0.01).

(i) Calculate a 99% confidence interval for μ. [4]

(ii) Explain whether it was necessary to use the Central Limit theorem in the calculation in part **(i)**. [1]

(iii) The manufacturer claims that the mean mass of sugar in bars of this type is 11.7 g. Explain why your answer to part **(i)** does not support this claim. [1]

(iv) The manufacturer suggests that a 95% confidence interval would be more likely to support his claim than a 99% confidence interval. **Without doing a calculation**, explain whether this suggestion is correct. [1]

Question 49

X is a random variable with mean 4.9 and standard deviation 2.21. A random sample of 75 values of X is taken. Find the probability that the sample mean is greater than 5.0. [3]

Question 50

A researcher wishes to estimate the proportion, p, of houses in London Road that have only one occupant. He takes a random sample of 64 houses in London Road and finds that 8 houses in the sample have only one occupant. Using this sample, he calculates that an approximate α% confidence interval for p has width 0.130. Find α correct to the nearest integer. [5]

Question 51

Amy has to choose a random sample from the 265 students in her year at college. She numbers the students from 1 to 265 and then uses random numbers generated by her calculator. The first two random numbers produced by her calculator are 0.213 165 448 and 0.073 165 196.

(i) Use these figures to find the numbers of the first four students in her sample. [2]

There were 25 students in Amy's sample. She asked each of them how much money, \$$x$, they earned in a week, on average. Her results are summarised below.

Question 52

The mean mass of packets of sugar is supposed to be 505 g. A random sample of 10 packets filled by a certain machine was taken and the masses, in grams, were found to be as follows.

500 499 496 495 498 490 492 501 494 494

(i) Find unbiased estimates of the population mean and variance. [3]

The mean mass of packets produced by this machine was found to be less than 505 g, so the machine was adjusted. Following the adjustment, the masses of a random sample of 150 packets from the machine were measured and the total mass was found to be 75 660 g.

(ii) Given that the population standard deviation is 3.6 g, test at the 2% significance level whether the machine is still producing packets with mean mass less than 505 g. [5]

(iii) Explain why the use of the normal distribution is justified in carrying out the test in part **(ii)**. [1]

Question 53

The management of a factory wished to find a range within which the time taken to complete a particular task generally lies. It is given that the times, in minutes, have a normal distribution with mean μ and standard deviation 6.5. A random sample of 15 employees was chosen and the mean time taken by these employees was found to be 52 minutes.

(i) Calculate a 95% confidence interval for μ. [3]

Later another 95% confidence interval for μ was found, based on a random sample of 30 employees.

(ii) State, with a reason, whether the width of this confidence interval was less than, equal to or greater than the width of the previous interval. [1]

Question 54

A six-sided die is suspected of bias. The die is thrown 100 times and it is found that the score is 2 on 20 throws. It is given that the probability of obtaining a score of 2 on any throw is p.

(i) Find an approximate 94% confidence interval for p. [3]

(ii) Use your answer to part **(i)** to comment on whether the die may be biased. [1]

Question 55

A random sample of 75 values of a variable X gave the following results.

$$n = 75 \qquad \Sigma x = 153.2 \qquad \Sigma x^2 = 340.24$$

Find unbiased estimates for the population mean and variance of X. [3]

Question 56

A population has mean 12 and standard deviation 2.5. A large random sample of size n is chosen from this population and the sample mean is denoted by $\overline{X}$. Given that $P(\overline{X} < 12.2) = 0.975$, correct to 3 significant figures, find the value of n. [4]

Question 57

The standard deviation of the heights of adult males is 7.2 cm. The mean height of a sample of 200 adult males is found to be 176 cm.

(i) Calculate a 97.5% confidence interval for the mean height of adult males. [3]

(ii) State a necessary condition for the calculation in part **(i)** to be valid. [1]

Question 58

The standard deviation of the volume of drink in cans of Koola is 4.8 centilitres. A random sample of 180 cans is taken and the mean volume of drink in these 180 cans is found to be 330.1 centilitres.

(i) Calculate a 95% confidence interval for the mean volume of drink in all cans of Koola. Give the end-points of your interval correct to 1 decimal place. [3]

(ii) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

Question 59

The lifetimes, X hours, of a random sample of 50 batteries of a certain kind were found. The results are summarised by $\Sigma x = 420$ and $\Sigma x^2 = 27\,530$.

(i) Calculate an unbiased estimate of the population mean of X and show that an unbiased estimate of the population variance is 490, correct to 3 significant figures. [3]

(ii) The lifetimes of a further large sample of n batteries of this kind were noted, and the sample mean, $\overline{X}$, was found. Use your estimates from part **(i)** to find the value of n such that $P(\overline{X} > 5) = 0.9377$. [4]

Question 60

The masses of a certain variety of plums are known to have standard deviation 13.2 g. A random sample of 200 of these plums is taken and the mean mass of the plums in the sample is found to be 62.3 g.

(i) Calculate a 98% confidence interval for the population mean mass. [3]

(ii) State with a reason whether it was necessary to use the Central Limit theorem in the calculation in part **(i)**. [1]

Question 61

The length of worms is denoted by X cm. The lengths of a random sample of 50 worms were measured. Some of the results were lost, but the following results are available.

- $\Sigma x^2 = 4361$
- An unbiased estimate of the population variance of X is 9.62.

Calculate the mean length of the 50 worms. [3]

Question 62

A coin is thrown 100 times and it shows heads 60 times. Calculate an approximate 98% confidence interval for the probability, p, that the coin shows heads on any throw. [3]

Question 63

It is claimed that, on average, a particular train journey takes less than 1.9 hours. The times, t hours, taken for this journey on a random sample of 50 days were recorded. The results are summarised below.

$$n = 50 \qquad \Sigma t = 92.5 \qquad \Sigma t^2 = 175.25$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test the claim at the 5% significance level. [5]

Question 64

The random variable X has mean 372 and standard deviation 54.

(i) Describe fully the distribution of the mean of a random sample of 36 values of X. [3]

(ii) The distribution in part **(i)** might be either exact or approximate. State a condition under which the distribution is exact. [1]

Question 65

Ramesh plans to carry out a survey in order to find out what adults in his town think about local sports facilities. He chooses a random sample from the adult members of a tennis club and gives each of them a questionnaire.

(i) Give a reason why this will not result in Ramesh having a random sample of adults who live in the town. [1]

(ii) Describe briefly a valid method that Ramesh could use to choose a random sample of adults in the town. [2]

Ramesh now uses a valid method to choose a random sample of 350 adults from the town. He finds that 47 adults think that the local sports facilities are good.

(iii) Calculate an approximate 90% confidence interval for the proportion of all adults in the town who think that the local sports facilities are good. [4]

(iv) Ramesh calculates a confidence interval whose width is 1.25 times the width of this 90% confidence interval. Ramesh's new interval is an x% confidence interval. Find the value of x. [3]

Question 66

The times, in minutes, taken by competitors to complete a puzzle have mean μ and standard deviation 3. The times taken by a random sample of 10 competitors are noted and the results are given below.

25.2 26.8 18.5 25.5 30.1 28.9 27.0 26.1 26.0 24.9

(i) Stating a necessary assumption, calculate a 97% confidence interval for μ. [5]

(ii) Two more random samples, each of 10 competitors, are taken. Their times are used to calculate two more 97% confidence intervals for μ. Find the probability that neither of these intervals contains the true value of μ. [1]

Question 67

The masses, in grams, of bags of flour are normally distributed with mean μ. The masses, m grams, of a random sample of 50 bags are summarised by $\Sigma m = 25\,110$ and $\Sigma m^2 = 12\,610\,300$.

(i) Calculate a 96% confidence interval for μ, giving the end-points correct to 1 decimal place. [6]

Another random sample of 50 bags of flour is taken and a 99% confidence interval for μ is calculated.

(ii) Without calculation, state whether this confidence interval will be wider or narrower than the confidence interval found in part **(i)**. Give a reason for your answer. [1]

Question 68

Lengths of a certain species of lizard are known to be normally distributed with standard deviation 3.2 cm. A naturalist measures the lengths of a random sample of 100 lizards of this species and obtains an α% confidence interval for the population mean. He finds that the total width of this interval is 1.25 cm.

Find α. [5]

Question 69

Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p. Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p. [4]

(b) Sunita believes that the die is fair. Use your answer to part **(a)** to comment on her belief. [1]

(c) Sunita uses the result of her 500 throws to calculate an α% confidence interval for p. This interval has width 0.04.

Find the value of α. [5]

Question 70

A random sample of 100 values of a variable X is taken. These values are summarised below.

$$n = 100 \qquad \Sigma x = 1556 \qquad \Sigma x^2 = 29\,004$$

Calculate unbiased estimates of the population mean and variance of X. [3]

Question 71

The lengths, X centimetres, of a random sample of 7 leaves from a certain variety of tree are as follows.

5.2 4.8 5.5 6.1 4.8 3.9 4.4

(a) Calculate unbiased estimates of the population mean and variance of X. [3]

It is now given that the true value of the population variance of X is 0.55, and that X has a normal

Question 72

In a survey, a random sample of 250 adults in Fromleigh were asked to fill in a questionnaire about their travel.

(a) It was found that 102 adults in the sample travel by bus. Find an approximate 90% confidence interval for the proportion of all the adults in Fromleigh who travel by bus. [3]

(b) The survey included a question about the amount, x dollars, spent on travel per year. The results are summarised as follows.

$$n = 250 \qquad \Sigma x = 50\,460 \qquad \Sigma x^2 = 19\,854\,200$$

Find unbiased estimates of the population mean and variance of the amount spent per year on travel. [3]

A councillor wanted to select a random sample of houses in Fromleigh. He planned to select the first house on each of the 143 streets in Fromleigh.

(c) Explain why this would not provide a random sample. [1]

Question 73

A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws.

(a) Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3]

(b) Maroulla claims that the die is biased.

Use your answer to part **(a)** to comment on this claim. [1]

Question 74

A construction company notes the time, t days, that it takes to build each house of a certain design. The results for a random sample of 60 such houses are summarised as follows.

$$\Sigma t = 4820 \qquad \Sigma t^2 = 392\,050$$

(a) Calculate a 98% confidence interval for the population mean time. [6]

Question 75

The masses, m kilograms, of flour in a random sample of 90 sacks of flour are summarised as follows.

$$n = 90 \qquad \Sigma m = 4509 \qquad \Sigma m^2 = 225\,950$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Calculate a 98% confidence interval for the population mean. [3]

(c) Explain why it was necessary to use the Central Limit theorem in answering part **(b)**. [1]

Question 76

The heights, h centimetres, of a random sample of 100 fully grown animals of a certain species were measured. The results are summarised below.

$$n = 100 \qquad \Sigma h = 7570 \qquad \Sigma h^2 = 588\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Calculate a 99% confidence interval for the mean height of animals of this species. [3]

Four random samples were taken and a 99% confidence interval for the population mean, μ, was found from each sample.

(c) Find the probability that all four of these confidence intervals contain the true value of μ. [2]

Question 77

100 randomly chosen adults each throw a ball once. The length, l metres, of each throw is recorded. The results are summarised below.

$$n = 100 \qquad \Sigma l = 3820 \qquad \Sigma l^2 = 182\,200$$

Calculate a 94% confidence interval for the population mean length of throws by adults. [6]

Question 78

The time, in minutes, taken by students to complete a test has the distribution N(125, 36).

(a) Find the probability that the mean time taken to complete the test by a random sample of 40 students is less than 123 minutes. [3]

(b) Explain whether it was necessary to use the Central Limit theorem in the solution to part **(a)**. [1]

Question 79

The random variable T denotes the time, in seconds, for 100 m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the following results.

$$n = 40 \qquad \Sigma t = 560 \qquad \Sigma t^2 = 7850$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

Question 80

A random sample of 75 students at a large college was selected for a survey. 15 of these students said that they owned a car. From this result an approximate α% confidence interval for the proportion of all students at the college who own a car was calculated. The width of this interval was found to be 0.162.

Calculate the value of α correct to 2 significant figures. [5]

Question 81

It is known that the height H, in metres, of trees of a certain kind has the distribution N(12.5, 10.24). A scientist takes a random sample of 25 trees of this kind and finds the sample mean, $\overline{H}$, of the heights.

(a) State the distribution of $\overline{H}$, giving the values of any parameters. [2]

(b) Find $P(12 < \overline{H} < 13)$. [3]

Question 82

The probability that a certain spinner lands on red on any spin is p. The spinner is spun 140 times and it lands on red 35 times.

(a) Find an approximate 96% confidence interval for p. [3]

From three further experiments, Jack finds a 90% confidence interval, a 95% confidence interval and a 99% confidence interval for p.

(b) Find the probability that exactly two of these confidence intervals contain the true value of p. [3]

Question 83

Andy and Jessica are doing a survey about musical preferences. They plan to choose a representative sample of six students from the 256 students at their college.

(a) Andy suggests that they go to the music building during the lunch hour and choose six students at random from the students who are there.

Give a reason why this method is unsatisfactory. [1]

(b) Jessica decides to use another method. She numbers all the students in the college from 1 to 256. Then she uses her calculator and generates the following random numbers.

204393 162007 204028 587119 207395

From these numbers, she obtains six student numbers. The first three of her student numbers are 204, 162 and 7.

Continue Jessica's method to obtain the next three student numbers. [2]

Question 84

The mass, in kilograms, of a block of cheese sold in a supermarket is denoted by the random variable M. The masses of a random sample of 40 blocks are summarised as follows.

$$n = 40 \qquad \Sigma m = 20.50 \qquad \Sigma m^2 = 10.7280$$

(a) Calculate unbiased estimates of the population mean and variance of M. [3]

(b) The price, $\$P$, of a block of cheese of mass M kg is found using the formula $P = 11M + 0.50$.

Find estimates of the population mean and variance of P. [3]

Question 85

The random variable T denotes the time, in seconds, for 100 m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the following results.

$$n = 40 \qquad \Sigma t = 560 \qquad \Sigma t^2 = 7850$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

Question 86

A random sample of 75 students at a large college was selected for a survey. 15 of these students said that they owned a car. From this result an approximate $\alpha\%$ confidence interval for the proportion of all students at the college who own a car was calculated. The width of this interval was found to be 0.162.

Calculate the value of α correct to 2 significant figures. [5]

Question 87

A random sample of 500 households in a certain town was chosen. Using this sample, a confidence interval for the proportion, p, of all households in that town that owned two or more cars was found to be $0.355 < p < 0.445$.

Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. [5]

Question 88

The lengths, in millimetres, of a random sample of 12 rods made by a certain machine are as follows.

200 201 198 202 200 199 199 201 197 202 200 199

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Give a statistical reason why these estimates may not be reliable. [1]

Question 89

A random sample of 5 values of a variable X is given below.

$$2 \qquad 3 \qquad 3 \qquad 5 \qquad a$$

(a) Find an expression, in terms of a, for the mean of these values. [1]

It is given that an unbiased estimate of the population variance of X, using these values, is 4. It is also given that a is positive.

(b) Find and simplify a quadratic equation in terms of a and hence find the value of a. [3]

Question 90

The number of characters in emails sent by a particular company is modelled by the distribution $N(1250, 480^2)$.

Find the probability that the mean number of characters in a random sample of 100 emails sent by the company is more than 1300. [3]

Question 91

(a) A javelin thrower noted the lengths of a random sample of 50 of her throws. The sample mean was 72.3 m and an unbiased estimate of the population variance was 64.3 m^2.

Find a 92% confidence interval for the population mean length of throws by this athlete. [3]

(b) A discus thrower wishes to calculate a 92% confidence interval for the population mean length of his throws. He bases his calculation on his first 50 throws in a week.

Comment on this method. [1]

Question 92

The diameters, x millimetres, of a random sample of 200 discs made by a certain machine were recorded. The results are summarised below.

$$n = 200 \qquad \Sigma x = 2520 \qquad \Sigma x^2 = 31852$$

(a) Calculate a 95% confidence interval for the population mean diameter. [6]

(b) Jean chose 40 random samples and used each sample to calculate a 95% confidence interval for the population mean diameter.

How many of these 40 confidence intervals would be expected to include the true value of the population mean diameter? [1]

Question 93

The heights, in metres, of a random sample of 10 mature trees of a certain variety are given below.

5.9 6.5 6.7 5.9 6.9 6.0 6.4 6.2 5.8 5.8

Find unbiased estimates of the population mean and variance of the heights of all mature trees of this variety. [3]

Question 94

A builders' merchant sells stones of different sizes.

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams.

Find a 95% confidence interval for the population mean mass of size A stones. [3]

(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an α% confidence interval for the population mean mass is found to have width 4 grams.

Find α. [4]

Question 95

Each of a random sample of 80 adults gave an estimate, h metres, of the height of a particular building. The results were summarised as follows.

$$n = 80 \qquad \Sigma h = 2048 \qquad \Sigma h^2 = 52\,760$$

(a) Calculate unbiased estimates of the population mean and variance. [3]

(b) Using this sample, the upper boundary of an $\alpha\%$ confidence interval for the population mean is 26.0.

Find the value of α. [4]

Question 96

X is a random variable with distribution B(10, 0.2). A random sample of 160 values of X is taken.

(a) Find the approximate distribution of the sample mean, including the values of the parameters. [3]

(b) Hence find the probability that the sample mean is less than 1.8. [3]

Question 97

A builders' merchant sells stones of different sizes.

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams.

Find a 95% confidence interval for the population mean mass of size A stones. [3]

(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an $\alpha\%$ confidence interval for the population mean mass is found to have width 4 grams.

Find α. [4]

Question 98

The heights, in metres, of a random sample of 10 mature trees of a certain variety are given below.

5.9 6.5 6.7 5.9 6.9 6.0 6.4 6.2 5.8 5.8

Find unbiased estimates of the population mean and variance of the heights of all mature trees of this variety. [3]

Question 99

Anita carried out a survey of 140 randomly selected students at her college. She found that 49 of these students watched a TV programme called *Bunch*.

(a) Calculate an approximate 98% confidence interval for the proportion, p, of students at Anita's college who watch *Bunch*. [3]

Carlos says that the confidence interval found in **(a)** is not useful because it is too wide.

(b) Without calculation, explain briefly how Carlos can use the results of Anita's survey to find a narrower confidence interval for p. [1]

Question 100

Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

Find unbiased estimates of the population mean and variance. [3]

Question 101

In a random sample of 100 students at Luciana's college, x students said that they liked exams. Luciana used this result to find an approximate 90% confidence interval for the proportion, p, of all students at her college who liked exams. Her confidence interval had width 0.157 92.

(a) Find the two possible values of x. [4]

Suzma independently took another random sample and found another approximate 90% confidence interval for p.

(b) Find the probability that neither of the two confidence intervals contains the true value of p. [1]

Question 102

A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows.

10612 11801 21473 22759

(a) The numbers of the first two members in the sample are 106 and 121.

Write down the numbers of the next two members in the sample. [2]

(b) To obtain the numbers for members after the 4th member, Donash starts with the second random digit, 0, and obtains the numbers 061 and 211.

Explain why this method will not produce a random sample. [1]

Question 103

The masses, in kilograms, of newborn babies in country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

Calculate unbiased estimates of μ and σ^2. [3]

Question 104

In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they owned a car.

Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. [3]

Question 105

A sample of 5 randomly selected values of a variable X is as follows:

$$1 \qquad 2 \qquad 6 \qquad 1 \qquad a$$

where $a > 0$.

Given that an unbiased estimate of the variance of X calculated from this sample is $\frac{11}{2}$, find the value of a. [3]

Question 106

A certain train journey takes place every day throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

(a) The mean time for a random sample of n of these journeys was found. A 94% confidence interval for the population mean time was calculated and was found to have a width of 1.4076 minutes, correct to 4 decimal places.

Find the value of n. [3]

(b) A passenger noted the times for 50 randomly chosen journeys in January, February and March.

Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time. [1]

(c) A researcher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.

Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean. [2]

Question 107

In a survey of 300 randomly chosen adults in Rickton, 134 said that they exercised regularly. This information was used to calculate an $\alpha\%$ confidence interval for the proportion of adults in Rickton who exercise regularly. The upper bound of the confidence interval was found to be 0.487, correct to 3 significant figures.

Find the value of α correct to the nearest integer. [4]

Question 108

The length, in minutes, of mathematics lectures at a certain college has mean μ and standard deviation 8.3.

(a) The total length of a random sample of 85 lectures was 4590 minutes.

Calculate a 95% confidence interval for μ. [3]

The length, in minutes, of history lectures at the college has mean m and standard deviation s.

(b) Using a random sample of 100 history lectures, a 95% confidence interval for m was found to have width 2.8 minutes.

Find the value of s. [2]

Question 109

A random sample of 250 people living in Barapet was chosen. It was found that 78 of these people owned a BETEC phone.

(a) Calculate an approximate 98% confidence interval for the proportion of people living in Barapet who own a BETEC phone. [3]

(b) Manjit claims that more than 40% of the people living in Barapet own a BETEC phone.

Use your answer to part **(a)** to comment on this claim. [1]

Question 110

The lengths, Xcm, of a sample of 100 insects of a certain type were summarised as follows.

$$n = 100 \qquad \sum x = 36.8 \qquad \sum x^2 = 17.34$$

(a) Calculate unbiased estimates for the population mean and variance of X. [3]

(b) State a necessary condition for the estimates found in part **(a)** to be reliable. [1]

Question 111

The numbers of green sweets in 200 randomly chosen packets of Frutos are summarised in the table.

Number of green sweets	0	1	2	3	> 3
Number of packets	32	50	97	21	0

(a) Calculate an unbiased estimate for the population mean of the number of green sweets in a packet of Frutos, and show that an unbiased estimate of the population variance is 0.783 correct to 3 significant figures. [3]

The manufacturers of Frutos claim that the mean number of green sweets in a packet is 1.65 .

Anji believes that the true value of the mean, μ, is less than 1.65 . She uses the results from the 200 randomly chosen packets to test the manufacturers' claim.

(b) State suitable null and alternative hypotheses for the test. [1]

(c) Show that the result of Anji's test is significant at the 5% level but not at the 1% level. [4]

(d) It is given that Anji made a Type I error.

Explain how this shows that the significance level that Anji used in her test was not 1%. [1]

Question 112

The widths, w cm, of a random sample of 150 leaves of a certain kind were measured. The sample mean of w was found to be 3.12 cm.

Using this sample, an approximate 95% confidence interval for the population mean of the widths in centimetres was found to be [3.01, 3.23].

(a) Calculate an estimate of the population standard deviation. [3]

(b) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(a)**. [1]

Question 113

A student wishes to estimate the proportion, p, of students at her college who have exactly one brother. She surveys a random sample of 50 students at her college and finds that 18 of them have exactly one brother. She calculates an approximate α% confidence interval for p and finds that the lower limit of the confidence interval is 0.244 correct to 3 significant figures.

Find α correct to the nearest integer. [4]

Question 114

Henri wants to choose a random sample from the 804 students at his college. He numbers the students from 1 to 804 and then uses random numbers generated by his calculator. The first 20 random digits produced by his calculator are as follows.

5 6 7 1 0 9 8 4 3 1 0 9 6 6 5 0 2 1 7 6

Henri's first two student numbers are 567 and 109.

(a) Use Henri's digits to find the numbers of the next two students in the sample. [2]

There were 30 students in Henri's sample. He asked each of them how much time, X hours, they spent on social media each week, on average. He summarised the results as follows.

$$n = 30 \qquad \Sigma x = 610 \qquad \Sigma x^2 = 12405$$

(b) Use this information to calculate an unbiased estimate of the mean of X and show that an unbiased estimate of the variance of X is less than 0.1 . [3]

(c) Henri's friend claims that Henri has probably made a mistake in his calculation of Σx or Σx^2.

Use your answer to part **(b)** to comment on this claim. [1]

Question 115

(a) A random sample of 8 boxes of cereal from a certain supplier was taken. Each box was weighed and the masses in grams were as follows.

261 249 259 252 255 256 258 254

Find unbiased estimates of the population mean and variance. [3]

(b) The supplier claims that the mean mass of boxes of cereal is 253 g. A quality control officer suspects that the mean mass is actually more than 253 g. In order to test this claim, he weighs a random sample of 100 boxes of cereal and finds that the total mass is 25 360 g.

(i) Given that the population standard deviation of the masses is 3.5 g, test at the 5% significance level whether the population mean mass is more than 253 g. [5]

An employee says, 'This test is invalid because it uses the normal distribution, but we do not know whether the masses of the boxes are normally distributed.'

(ii) Explain briefly whether this statement is true or not. [1]

Question 116

The time taken in minutes for a certain daily train journey has a normal distribution with standard deviation 5.8 . For a random sample of 20 days the journey times were noted and the mean journey time was found to be 81.5 minutes.

(a) Calculate a 98% confidence interval for the population mean journey time. [3]

A student was asked for the meaning of this confidence interval. The student replied as follows.

'The times for 98% of these journeys are likely to be within the confidence interval.'

(b) Explain briefly whether this statement is true or not. [1]

Two independent 98% confidence intervals are found.

(c) Given that at least one of these intervals contains the population mean, find the probability that both intervals contain the population mean. [2]

Question 117

The times, T minutes, taken by a random sample of 75 students to complete a test were noted. The results were summarised by $\Sigma t = 230$ and $\Sigma t^2 = 930$.

(a) Calculate unbiased estimates of the population mean and variance of T. [3]

You should now assume that your estimates from part **(a)** are the true values of the population mean and variance of T.

(b) The times taken by another random sample of 75 students were noted, and the sample mean, $\overline{T}$, was found.

Find the value of a such that $P(\overline{T} > a) = 0.234$. [3]

Question 118

The heights of a certain species of deer are known to have standard deviation 0.35 m. A zoologist takes a random sample of 150 of these deer and finds that the mean height of the deer in the sample is 1.42 m.

(a) Calculate a 96% confidence interval for the population mean height. [3]

(b) Bubay says that 96% of deer of this species are likely to have heights that are within this confidence interval.

Explain briefly whether Bubay is correct. [1]

Question 119

A population is normally distributed with mean 35 and standard deviation 8.1 . A random sample of size 140 is chosen from this population and the sample mean is denoted by $\overline{X}$.

(a) Find $P(\overline{X} > 36)$. [3]

(b) It is given that $P(\overline{X} < a) = 0.986$. Find the value of a. [3]

Question 120

The lengths of a random sample of 50 roads in a certain region were measured. Using the results, a 95% confidence interval for the mean length, in metres, of all roads in this region was found to be $[245, 263]$.

(a) Find the mean length of the 50 roads in the sample. [1]

(b) Calculate an estimate of the standard deviation of the lengths of roads in this region. [2]

(c) It is now given that the lengths of roads in this region are normally distributed.

State, with a reason, whether this fact would make any difference to your calculation in part **(b)**. [1]

Question 121

The times, T minutes, taken by a random sample of 75 students to complete a test were noted. The results were summarised by $\Sigma t = 230$ and $\Sigma t^2 = 930$.

(a) Calculate unbiased estimates of the population mean and variance of T. [3]

You should now assume that your estimates from part **(a)** are the true values of the population mean and variance of T.

(b) The times taken by another random sample of 75 students were noted, and the sample mean, $\overline{T}$, was found.

Find the value of a such that $P(\overline{T} > a) = 0.234$. [3]

Question 122

Nikki is investigating the views of students at her school about the school sports facilities. She plans to give a survey to a sample of students.

Nikki's friend says, "This survey is about sports facilities, so you should choose a sample of students from the school sports teams."

(a) State, with a reason, whether you agree with Nikki's friend. [1]

Nikki chooses an appropriate random sample of 60 students. She finds that 45 of these students think that the sports facilities are good.

(b) Calculate an approximate 95% confidence interval for the proportion of students who think that the sports facilities are good. [3]

For a different investigation, Nikki uses another large random sample to calculate a 99% confidence interval and an x% confidence interval.

The width of the 99% confidence interval is double the width of the x% confidence interval.

(c) Calculate the value of x. [4]

Question 123

In Urberia, the masses, in kilograms, of men have the distribution $N(70.3, 5.9^2)$. A certain footbridge in Urberia can take a maximum safe load of 1500 kg. When n men stand on the bridge, the probability that the bridge is unsafe is less than 0.01.

Stating a necessary assumption, find the maximum value of n. [4]

Question 124

A machine dispenses coffee into cups. The volume, $V\text{cm}^3$, of coffee in a cup was measured for a random sample of 150 cups. The results were summarised as follows.

$$\Sigma v = 46\,350 \qquad \Sigma v^2 = 14\,410\,800$$

(a) **(i)** Calculate unbiased estimates of the population mean, μ, and population variance, σ^2. [3]

(ii) Calculate a 95% confidence interval for μ. [3]

Another random sample of n cups of coffee is taken, where $100 < n < 120$. A 95% confidence interval for μ is calculated using this sample. You may assume that, for large samples, unbiased estimates of σ^2 are very similar.

(b) Without calculation, state whether this confidence interval would be wider or narrower than the confidence interval found in part **(a)(ii)**. Give a reason for your answer. [2]

Question 125

A biased spinner has four sides. Each side is of a different colour: yellow, red, green or black. The probability, p, that the spinner will land on red is unknown. The spinner was spun 200 times, and the proportion, a, of times that it landed on red was noted. This proportion was used to calculate an approximate 90% confidence interval for p. The width of this confidence interval was 0.1066 correct to 4 significant figures.

Find the two possible values of a. [4]

Question 126

The height of a certain species of plant is denoted by Hcm. The heights of a random sample of 100 plants were measured, and the following results were found.

- The mean, $\overline{h}$, for the sample was 80.2.
- An unbiased estimate of the population variance of H was 15.6.

Calculate the value of Σh^2. [3]

Question 127

The time, T minutes, for a certain daily bus journey is normally distributed. The bus company claims that the mean of T is 45. A passenger believes that the mean of T is actually greater than 45. She notes the times taken for this journey on a random sample of 60 days. The results are summarised below.

$$n = 60 \qquad \Sigma t = 2750 \qquad \Sigma t^2 = 127000$$

Calculate unbiased estimates of the population mean and variance. [3]

Question 128

The random variable X has the distribution $\mathrm{B}\left(8, \frac{3}{4}\right)$. A random sample of 100 values of X is chosen, and the sample mean, $\overline{X}$, is found.

(a) Find $\mathrm{P}(\overline{X} > 6.2)$. You are **not** expected to use a continuity correction. [6]

(b) State why the Central Limit Theorem was needed in the calculation in part **(a)**. [1]