A-Level

Topic Sampling and Estimation

May 2013-May 2023

Answers

Question 1

(i)	$\bar{x} = 930/15 = (62)$ $z = 1.751$ '62' $\pm z \times \frac{12}{\sqrt{15}}$ = 56.6 to 67.4 (3 sf)	B1 B1 M1 A1	4	Any z Must be an interval
(ii)	92 % of such intervals will contain μ	B1	1	Accept P(This interval contains μ) = 0.92
(iii)	Each possible sample of this size is equally likely	B1	1	Each member of pop equally likely to be chosen
			[Total: 6]	

Question 2

(i)	est(μ) = 9750/150 = (65)	B1	
	est(σ^2) = $\frac{1}{149}(647500 - \frac{9750^2}{150})$	M1	Correct subst. in correct formula
	= 92.3 (3 s.f.)	A1 [3]	
(ii)	$z = 2.326$	B1	
	'65' $\pm z \times \frac{\sqrt{'92.28188'}}{\sqrt{150}}$	M1	Any z
	= 63.2 to 66.8 (3 s.f.)	A1 [3]	(Use of 'biased' can still score here)
(iii)	0.02^2	M1	Allow M1 for 0.02 seen
	= 0.0004 o.e.	A1 [2]	
		[Total: 8]	

Question 3

(i)	$\text{est}(\mu) = 2005/200 = (10.025)$ $\text{est}(\sigma^2) = \frac{1}{99}\left(20175 - \frac{2005^2}{200}\right)$ $= 0.376$ (3 sf)	B1 M1 A1	[3]	Correct subst in correct formula
(ii)	$\frac{10-'10.025'}{\sqrt{\frac{'0.376256'}{50}}}$ (= –0.288) $1 - \Phi('0.288')$ = 0.387 (3 sf)	M1 M1 A1	[3]	Allow without √, but ÷√50 essential (Use of 'biased' variance can still score fully in (ii))

Question 4

Est(μ) = 1.8775 or 1.88 (3 sf)	**B1**		Accept 751/400 (not 150.2/80)
$\text{Est}(\sigma^2) = \frac{80}{79}\left(\frac{820.24}{80} - "1.8775"^2\right)$	**M1**		Correct subt'n in correct formula 1/79 (820.24 – 150.2^2/80)
= 6.81316 or 6.81 (3 sf)	**A1**		
$z = 1.96$	**B1**		Seen
$"1.8775" \pm z \times \sqrt{\frac{"6.81316"}{80}}$	**M1**		
= 1.31 to 2.45 (3 sf)	**A1**	**6**	Must be an interval. NB use of biased var can still score A1.

Question 5

(i)	est (μ) = 2866 or 2870 (3 s.f.) $\text{est}(\sigma^2) = \frac{1}{49}\left(410900000 - \frac{143300^2}{50}\right)$ (= 4126.53) = 4130 (3 sf)	B1 M1 A1	[3]	Accept 143300/50 o.e. Correct subst in correct formula
(ii)	H_0: Pop mean (or μ) = 2850 H_1: Pop mean (or μ) ≠ 2850 $\frac{\frac{143300}{50} - 2850}{\frac{\sqrt{'4126.53'}}{\sqrt{50}}}$ = 1.761 '1.761' < 1.96 No evidence mean distance changed	B1 M1 A1 M1 A1f	[5]	Both. Not just 'mean' Allow '4126.53' without $\sqrt{\ }$, but must have all $\sqrt{50}$ Or correct c.v. (2867.81) for alt method For valid comparison of z values, areas or c.v. Dep 1.96; ft their 1.761 If H_1: $\mu > 2850$ and c.f. 1.645, max B0M1A1M1A0 (c.v. for 1 tail test 2864.94)

Question 6

$z = 2.576$	B1		Seen (accept 2.574 to 2.579)
$2 \times z \times \frac{0.17}{\sqrt{n}} = 0.2$ oe	M1		Allow without '2 ×' OR with incorrect z
$n = (\frac{2 \times 0.17 \times 2.576}{0.2})^2$ oe (= 19.2)	M1		Attempt to arrange equ of correct form (with correct z and '2×' into the form n= or $\sqrt{n}$ =
Smallest n is 20	A1	[4]	

Question 7

$p = 0.56$	B1		Used
'0.56' $\pm z \times \sqrt{\frac{0.56 \times 0.44}{100}}$	M1		Equation of correct form condone just +ve or –ve Must be z
$z = 2.17$, or 2.169 or 2.171	B1		
0.452 to 0.668 (3 s.f.)	A1	[4]	Seen Must be an interval

Question 8

(i)	$p = \frac{184}{400}$ or 0.46	B1		Used
	$z = 1.96$	B1		Seen
	"0.46" $\pm z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}}$	M1		Using expression of correct form
	= 0.411 to 0.509	A1	[4]	Must be an interval
(ii)	0.5 within CI			
	Claim not supported or not justified	B1√	[1]	Both needed. No contradictions. ft their **(i)**
(iii)	$z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.05$	M1		Allow M1 for $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.1$
	$z = 2.006$	A1		
	Φ('2.006') = 0.9775			
	α = '0.9775' – (1 – '0.9775')	M1		or 1 – 2(1 – '0.9775')
	= 95.5%	A1	[4]	

Question 9

$\frac{\Sigma x}{8} = \frac{2006}{8} = 250.75$ or 251 (3 s.f.)	B1		Any equivalent form
$(\Sigma x^2 = 503274)$			
$\frac{8}{7}\left(\frac{"503274"}{8} - "250.75"^2\right)$	M1		For use of formula of correct form
$= 38.5$ o.e. (accept 6.204^2)	A1	[3]	cao (as final answer)

Question 10

(i)	$0.25(1 + 4 + 9) - 1.5^2$ (=1.25 **AG**)	B1	[1]	
(ii)	$\frac{1.4-1.5}{\sqrt{\frac{5}{4}\div 300}}$ (= –1.549)	M1		$\frac{1.4-\frac{1}{600}-1.5}{\sqrt{\frac{5}{4}\div 300}}$ (= –1.523)
	$\Phi("-1.549") = 1 - \Phi("1.549")$ = 0.0607 (3 sf)	M1 A1	[3]	$\Phi("-1.523") = 1 - \Phi("1.523")$ = 0.0639 (3 sf)
(iii)	Large sample or large n ($\bar{X}$ (approx) normally distr) or Central Limit Theorem	B1	[1]	
		[Total: 5]		

Question 11

$\frac{70}{69} \times 2.70 = 2.73913$	M1A1		
$3.61 \pm z\sqrt{\frac{"2.73913"}{70}}$	M1		or $3.61 \pm z\sqrt{\frac{2.70}{69}}$ M2A1(implied)
			without $\frac{70}{69}$: $3.61 \pm z\sqrt{\frac{2.70}{70}}$ M0A0M1
$z = 1.96$	B1		$z = 1.96$ B1
3.22 to 4.00 (3 sf)	A1	[5]	3.23 to 3.99(4.00) (3 sf) A1
			Answer must be an interval
	[Total: 5]		

Question 12

	N(483.2, 537.92) or N(483.2, 23.2^2)	B1		or $\frac{8.2}{\sqrt{8}}$ or $\frac{8.2^2}{8}$ seen or implied
	$\frac{436-483.2}{\sqrt{537.92}}$ or $\frac{436-483.2}{23.2}$ (= –2.035)	M1		or $\frac{\frac{436}{8}-60.4}{8.2/\sqrt{8}}$ standardising (no mixed methods)
	Φ("–2.035") = 1 – Φ("2.035") = 0.021 or 2.1%	M1 A1	[4]	Correct area consistent with their working
		[Total: 4]		

Question 13

(i)	$\text{Est}(\mu) = \frac{7220}{80}$ or 90.25	**B1**		Accept 90.3
	$\text{Est}(\sigma^2) = \frac{80}{79}\left(\frac{656060}{80} - \left(\frac{7220}{80}\right)^2\right)$	**M1**		$\frac{1}{79}\left(656060 - \frac{7220^2}{80}\right)$
	= 56.3924 or $\frac{4455}{79}$	**A1**		
	$z = 2.17$	**B1**		Accept 56.4
	$\frac{7220}{80} \pm z \times \sqrt{\frac{'56.3924'}{80}}$	**M1**		Expression of correct form
	= 88.4 to 92.1 (3 sf)	**A1**	**[6]**	Must be an interval (N.B. biased var gives 88.4 to 92.1 scores possible B1M0A0B1M1A1)
(ii)	Pop normal	**B1**		*X* normal or full definition of pop normal
	No	**B1dep**	**[2]**	SR B1 for "no" and relevant reference to normal

Question 14

(i)	$\text{Var}(P_s) = \dfrac{\frac{33}{150}\times\frac{150-33}{150}}{150}$ (= 0.001144)	M1		
	$z = 2.576$	B1		Seen. Accept 2.574 to 2.579
	$\dfrac{33}{150} \pm z\sqrt{\text{`}0.001144\text{'}}$	M1		Expression of correct form. Any z
	= 0.133 to 0.307 (3 sf)	A1	4	Must be an interval
(ii)	$\dfrac{19035}{150}$ (= 126.9 =127(3sf))	B1		
	$\dfrac{150}{149}\left(\dfrac{4054716}{150} - \left(\dfrac{19035}{150}\right)^2\right)$ o.e.	M1		For use of a correct formula
	= 11001.17 or 11000(3 sf)	A1	3	
(iii)	4-digit nos. each digit 0-9	B1		Some valid way of generating 4 digit random nos
	Ignore nos > 9526	B1		from valid method
	Ignore repeats	B1	3	from valid method
				SR If zero score, full explanation of method for drawing numbers out of a hat can score B1.
				NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point.

Question 15

(i)	$\left(\dfrac{1508}{50}\right) = 30.16\,(30.2)$	B1		Allow any form
	$\dfrac{50}{49}\left(\dfrac{51\,825}{50} - \left(\text{'}30.16\text{'}^2\right)\right)$	M1		(129.46367)
	= 129 (3 sf) Or 130	A1	[3]	
(ii)	(1.5 × '30.16' + 10) = 55.24	B1ft		ft their 30.16
	$(1.5^2 \times$ '129….')	M1		$1.5^2 \times$ their(129) with nothing added at any stage
	= 291 (3 sf)	A1ft	[3]	Allow 290
		Total 6		

Q

Question 16

(i)	$\text{Var}(p_s) = \dfrac{0.22 \times (1-0.22)}{100}$ $\left(= \dfrac{429}{250\,000} \text{ or } 0.001716\right)$	M1		pq/100
	$0.22 \pm z\sqrt{'\dfrac{429}{250000}'}$	M1		Expression of correct form with their variance Any z (must be a z value) accept one side only
	$z = 2.17$ or 2.168/9 or 2.171	B1		Seen
	0.13(0) to 0.31(0) (2 sf)	A1	[4]	Must be an interval
(ii)	'2' × (1 – 0.97) × 0.97 = 0.0582	M1 A1	[2]	
		Total 6		

Question 17

(i)	14800/50 or 296 $\dfrac{50}{49}\left(\dfrac{4390000}{50} - '296'^2\right)$ (= 187.755) = 188 (3 sf)	B1 M1 A1	3	Oe
(ii)	$2 \times z \times \sqrt{\dfrac{'187.755'}{50}} = 5.45$ oe $z = 1.406$ or 1.405 Φ('1.406') (= 0.92 or 0.9199) $\alpha = 84$ (2 sf) allow 83.98	M1 A1 M1 A1	4	If '2 ×' omitted: $z \times \sqrt{\dfrac{'187.755'}{50}} = 5.45$ M1 $z = 2.812$ or 2.810 A0 Φ('2.812') (= 0.9975) $\alpha = 99.5$ or 99 or 100 M1 A0 For complete method to find α SR use of biased var(184) scores M1A1(1.4205) A=84.5 M1A1
(iii)	0.96^4 = 0.849 (3 sf)	M1 A1	2	

Question 18

(i)	4200/80 (=52.5) $= \dfrac{80}{79}\left(\dfrac{229\,000}{80} - '52.5'^2\right)$ (= 107.595) = 108 (3 sf)	B1 M1 A1	[3]	
(ii)	$'52.5' \pm z\sqrt{\frac{'107.595'}{80}}$ $z = 2.326$ 49.8 to 55.2	M1 B1 A1f	[3]	Correct form – must be z-value – allow one side only Seen ft their 52.5 and 107.595. Must be an interval
(iii)	49	B1	[1]	
		[Total: 7]		

Question 19

(i)	34 $2.2^2 + 1.3^2 + 2.6^2$ (=13.29)	B1 B1	[2]	Accept 13.3 or 3.65^2 Allow at early stage
(ii)	$\frac{33-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= –2.295)			
		M1		correct standardisation method for either
	$\frac{35-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= 2.295)			
		M1		For attempt to use tables to find the probability between two z values ,may be implied by next line
	Φ('2.295') – Φ('–2.295')			
		M1		For a correct method to find the area between their two z values
	= Φ('2.295') – (1 – Φ('2.295')) oe	A1	[4]	
	= 0.978 (3 sf)			
		[Total: 6]		

Question 20

(i)	est μ = 2.087	**B1**		allow 2.09
	est $\sigma^2 = \frac{100}{99}\left(\frac{435.57}{100} - 2.087^2\right)$	**M1**		1/99 (435.57 – 208.7²/100)
	= 0.000132(3232) or 131/990000	**A1**	[3]	without $\frac{100}{99}$: 0.000131 M0A0
(ii)	E(*Y* – *X*) = 2.12 – 2.087 (= 0.033)	**B1**		or 2.12 – 2.087 – 0.01 for Y – X – 0.01 < 0 allow 2.09 for 2.087
	Var(*Y* – *X*) = 0.000144 + '0.00013232'	**M1**		or √(0.012² + '0.00013232') M1
	= 0.000276(32)	**A1**		= 0.016623 A1
	$\frac{0.01-'0.033'}{\sqrt{'0.00027632'}}$ (= –1.384)	**M1**		√their E(*Y* – *X*) & Var(*Y* – *X*) var must be a combination of the two vars
	Φ('–1.384') = 1 – Φ('1.384')	**M1**		correct area/prob consistent with their working
	= 0.0832	**A1**	[6]	SR use of biased var (0.000131) in (i) and (ii) scores in (ii) B1M1 A1 for 0.000275 and M1M1 A1 for 0.0827 (6/6 available)

Question 21

(i)	0.4 or 2/5 or 26/65	**B1**	[1]	no recovery in (ii) for the B mark
(ii)	" 0.4 " $+ z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.516$ oe	**M1**		or " 0.4 " $- z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.284$ or $z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.116$ oe
	$z = \left(0.116 \times \sqrt{\frac{65}{0.4 \times 0.6}}\right) = 1.909$	**A1**		
	(Φ('1.909') = 0.97(18)) 2 ('0.97' – 1)	**M1**		for fully correct method to find α from their z
	$\alpha = 94$	**A1**	[4]	allow 94.36 or 94.4 or 94.374

Question 22

(i)	N(352, …) Variance = 2.9	**B1** **B1**	[2]	no recovery in (ii) for each B mark accept sd = $\sqrt{2.9}$ = 1.70(29) stated
(ii)	$\frac{354 - 352}{\sqrt{2.9}}$ (= 1.174)	**M1**		with their mean and var Or $\frac{354.05 - 352}{\sqrt{2.9}}$ or correct restart (= 1.204)
	1 – Φ('1.174')	**M1**		(accept sd/var mix)1 – Φ('1.204') = 0.114 (3 sf)
	= 0.120 (3 sf)	**A1**	[3]	Incorrect cc can score M1M1A0

Question 23

(i)	$\frac{3420}{60}(= 57)$	**B1**		
	$\frac{60}{59}\left(\frac{195200}{60} - '57'^2\right)$ (= 4.40678) = 4.41 (3 sf)	**M1**		Oe
		A1	[3]	As final answer
(ii)	$'57' \pm z\sqrt{\frac{'4.40678'}{60}}$	**M1**		
	$z = 2.326$	**B1**		2.326 – 2.329 (accept 2.33 if no better seen)
	[56.4 to 57.6] (3 sf)	**A1**	[3]	NB: use of biased variance in (ii) can score in full

Question 24

(i)	$\frac{6}{\sqrt{120}}$ oe seen	**B1**		Or $6^2/120$ oe seen
	$\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)}$ (= 1.826)	**M1**		± Allow without √120. No sd/var mix
	P(z > '1.826') = 1 – Φ('1.826') = 0.034 (2 sf)	**M1** **A1**	[4]	Correct tail consistent with their working 0.0339
(ii)	No n is large (⩾30)	B1		1st B1 for either comment
	Sample mean is (appr) normally distrib or The CLT applies oe	B1	[2]	2nd B1 for'No'with 2nd comment (No mark for 'No' alone)

Question 25

(i)	Est(μ) = $\frac{14\,910}{150}$ (= 99.4)	**B1**		
	Est(σ^2) = $\frac{150}{149}(\frac{1525000}{150} - \text{"99.4"}^2)$	**M1**		Allow **M1** if $\frac{150}{149}$ omitted
	= 288.228	**A1**		
	z = 2.576	**B1**		Accept 2.574–2.579
	"99.4" ± z × $\sqrt{288.228 \div 150}$	**M1**		Any z
	CI = 95.8 to 103 (3 sf)	**A1**	[6]	(NB Use of biased Var can score 5/6 max)
(ii)	100 lies within this CI Hence yes	**B1**√	[1]	Both needed, ft their CI
(iii)	To avoid bias or Necessary to enable statistical inference	**B1**	[1]	Or any equivalent

Question 26

(i)	$\lambda = 3.9$ $e^{-3.9} \times \frac{3.9^4}{4!}$ $= 0.195$	**B1** **M1** **A1**	[3]	M1 allow any λ SR Combination method B1 for $\lambda = 1.6$ AND $\lambda = 2.3$ used in combination method (at least 3 combinations) M1 All correctly combined and added
(ii)	$\bar{X} \sim N(1.6, \frac{1.6}{75})$	**B1** **B1**	[2]	B1 for N(1.6, ….)stated B1 for Var = $\frac{1.6}{75}$ stated SR, not stated but all implied in (iii): B1
(iii)	$\frac{1.7-1.6}{\sqrt{\frac{1.6}{75}}} (= 0.685)$ $1 - \Phi(\text{“}0.685\text{”})$ $= 0.247$ (3 sf)	**M1** **M1** **A1**	[3]	For standardising (using their values or correct values .Ignore cc Correct area consistent with their working Accept use of 1/2n correction leading to 0.233. NB Use of Poisson sum Po(120) and N(120,120) with $\mu = 127.5$ leads to 0.247, or 0.233 with cc
(iv)	X not normally distr. So CLT needed	**B1**	[1]	Not “it”

Question 27

$192.4 \pm z\sqrt{\frac{43.6}{150}}$	**M1**		Allow $\frac{43.6}{\sqrt{150}}$ Allow one side for M1
$z = 2.326$ to 2.329	**B1**		Condone $\sqrt{(43.6/149)}$ oe
191 to 194 (3 sf)	**A1**	[3]	CWO

Question 28

(i)	Pop too big or takes too long oe or testing destroys articles oe	**B1**	[1]	or too expensive oe or pop inaccessible oe
(ii) **(a)**	$z = 1.96$	**B1**		seen
	$65.7 \pm z \times \frac{\sqrt{15}}{10}$	**M1**		Expression of correct form (must be ‘z’ must be 65.7)
	$= 64.9$ to 66.5 (3 sf)	**A1**	[3]	Must be an interval
(b)	CI does not include 64.7 Probably has affected (or increased) mean bounce ht.	**B1**√	[1]	allow 64.7 not within CI both needed. ft their CI ft 65.7/64.7 mix

Question 29

$\frac{6.2}{\sqrt{50}}$ or $\frac{6.2^2}{50}$	**B1**		seen or implied
$\frac{51-53}{6.2\div\sqrt{50}}$ (= –2.281)	**M1**		allow without $\div\sqrt{50}$
P($z >$ '–2.281') = ϕ('2.281')	**M1**		for finding correct area consistent with working
= 0.989 (3 sf)	**A1**	[4]	as final answer

Question 30

(i)	((0.5672 + 0.6528) ÷ 2) = 0.61	**B1**	[1]	
(ii)	'0.61' + $z\sqrt{\frac{\text{'0.61'}\times(1-\text{'0.61'})}{350}}$ = 0.6528	**M1**		oe
	z = 0.0428 × $\sqrt{\frac{700}{\text{'0.61'}\times(1-\text{'0.61'})}}$ oe	**M1**		correct rearrangement of correct equn, ft '0.61'
	= 2.321	**A1**		
	98% confidence	**A1 ft**	[4]	ft their z (dep on both Ms)

Question 31

(i)	Each employee has an equal chance of being chosen	**B1**	[1]	oe
(ii)	Est (μ) = 4	**B1**		
	Est (σ^2) = $\frac{10}{9}(\frac{199.22}{10} - \text{'4'}^2)$	**M1**		sub in correct formula attempted
	= 4.36 (3 sf)	**A1**	[3]	working may not be seen
(iii)	Distances travelled by all employees at the firm	**B1**	[1]	oe

Question 32

(i)	est(μ) = 3.4 est(σ^2)= $\frac{100}{99}(\frac{1356}{100} - '3.4'^2)$ = 2.02(0202) z = 1.96 $3.4 \pm z \times \sqrt{\frac{'2.020202'}{100}}$ = 3.12 to 3.68 (3 sf)	**B1** **M1** **A1** **B1** **M1** **A1**	[6]	1 / 99 (1356 – $340^2/100$) or 200/99 correct working only allow from unbiased or biased variance
(ii)	Mean should be 3 CI does not include 3 Machine probably not working properly	**B1*** **DB1**√	[2]	stated or implied √ their CI or evidence that….

Question 33

$\frac{11.8-11}{1.6\div\sqrt{n}}$ = 1.645 $\frac{11.8-11}{1.6\div\sqrt{n}}$ = 1.96	**M1**		M1 for $\frac{11.8-11}{1.6\div\sqrt{n}}$ = any z allow var / sd mix for 1.6 but need $\sqrt{n}$
n = 10.8 (allow 11) n = 15.4 (allow 15)	**B1** **B1**		B1 for each correct z
	A1		for both
Possible values are 11, 12, 13, 14, 15	**A1**	[5]	not for just $11 \leqslant n \leqslant 15$ oe

Question 34

(a)	$63 \pm z \times \frac{9}{\sqrt{100}}$	**M1**	B1	Expression of correct form, any z
	z = 1.645	**B1**		Seen
	61.5 to 64.5 (3 sf)	**A1**	[3]	Must be an interval
(b) (i)	$z = \frac{1.96}{2}$ (= 0.98) Φ("0.98") (= 0.8365) "0.8365" – (1 – "0.8365") (= 0.673) α = 67.3 (3 sf)	**M1** **M1** **A1**	[3]	Allow $\frac{\text{any } z}{2}$ Allow 67 from correct working
(ii)	$4=(2x'z'x'\sigma')/\sqrt{n}$ n = 200	**M1** **A1**	[2]	Attempt to solve equ of correct form SR B1 for n = 100

Question 35

$\left(\frac{508}{8}\right) = 63.5$	**B1**		
$(\Sigma x^2 = 32360.12)$			
$\frac{8}{7}\left(\frac{'32360.12'}{8} - '63.5'^2\right)$	**M1**		oe
$= 14.6$ (3 sf) or $2553/175$	**A1**	[3]	From correct working

Question 36

(i)	$\frac{53-52}{6.1\div\sqrt{75}}$ (= 1.420)	**M1**		
	$\frac{51-52}{6.1\div\sqrt{75}}$ (= –1.420)	**M1**		or –"1.420" seen
	$\Phi(\text{“1.420”}) - \Phi(\text{“–1.420”})$	**M1**		
	= 0.844 (3 sfs)	**A1**		
	Total:	**4**		
(ii)	Need to assume $\bar{X}$ (approx.) normally distributed	**B1**		or X not stated to be normally distributed

Question 37

$\text{Var}(Ps) = \frac{0.3(1-0.3)}{120}$ (= 0.00175)	**M1**	Attempt correct values in correct formula
$0.3 \pm z\sqrt{"0.00175"}$	**M1**	must be a z-value, not a prob
$z = 1.645$	**B1**	
CI = 0.231 to 0.369 (3 sf)	**A1**	

Question 38

(i)	$\bar{x} = 6.7/200$ (= 67/2000 = 0.0335)	**B1**		
	$s^2 = \frac{200}{199}\times\left(\frac{0.2312}{200} - "0.0335"^2\right)$	**M1**	$s^2 = \frac{0.2312}{200} - 0.0335^2$	**M0**
	= 0.0000339(2) = 27/796000	**A1**	= 0.00003375	**A0**
	Total:	**3**		
(ii)	H_0: Pop mean level = 0.034 H_1: Pop mean level ≠ 0.034	**B1**	not just "mean", but allow just “μ”	
	$\frac{"030335" - 0.034}{\frac{\sqrt{"0.00003392"}}{\sqrt{200}}}$	**M1**	must have $\sqrt{200}$ $\frac{0.0335 - 0.034}{\frac{\sqrt{"0.00003375"}}{\sqrt{200}}}$	**M1**
	= –1.21(4) (3 sfs) (–1.22 ↔ –1.21)	**A1**	= –1.217 (3 sfs)	**A1**
	Comp with $z = -1.645$ (or 0.1124>0.05)	**M1**	0.112 > 0.05 valid comparison z or areas	
	No evidence that (mean) pollutant level has changed, accept H_0 (if correctly defined)	**A1FT**	correct conclusion no contradictions SR: One tail test: **B0**, **M1A1** as normal, **M1** (comparison with 1.282 consistent signs) **A0**	

Question 39

	10×0.46^2 (= 2.116) or $\frac{0.46}{\sqrt{10}}$	**B1**	SOI
	Total mass of ore ~ N(70, 2.116) or $\sim N\left(7, \left(\frac{0.46}{\sqrt{10}}\right)^2\right)$	**B1**	
	$\pm\frac{71-"70"}{\sqrt{"2.116"}}$ or $\pm\frac{7.1-"7.0"}{0.46/\sqrt{10}}$ (= 0.687)	**M1**	correct, using their sd or √(their var) e.g. allow $\frac{71-"70"}{4.6}$ for **M1**
	$1 - \Phi("0.687")$	**M1**	for correct area consistent with their working
	= 0.246 (3 sf)	**A1**	

Question 40

2(i)	$z = 1.751$	**B1**	
	$\frac{103}{200} \pm z\sqrt{\frac{\frac{103}{200}\times(1-\frac{103}{200})}{200}}$ oe	**M1**	all correct except for recognisable value of z, allow for on side only
	= 0.453 to 0.577 (3 sf) as final answer	**A1**	must be an interval
	Total:	**3**	
2(ii)	0.08 oe 8%, 8/100	**B1**	

Question 41

	573, 43 (or 043), 289	**B1B1B1**	Ignore incorrect numbers. But allow other correct use of table (i.e. 573, 650, 431)

Question 42

3(i)	Est (μ) = 923/400 or 2.3075 or 2.31 (3 sf)	**B1**	
	$\text{Est}(\sigma^2) = \frac{400}{399}\left(\frac{3170}{400} - "2.3075"^2\right)$ OE	**M1**	
	= 2.60696 or 2.61 (3 sf)	**A1**	(Note: Biased Var= 2.600 scores **M0**)
	Total:	**3**	
3(ii)	H_0: Pop mean (or μ) = "2.31" or "2310" H_1: Pop mean (or μ) > "2.31" or "2310"	**B1 FT**	
	$\pm\frac{2.6-"2.310"}{\sqrt{2.60696\div 50}} = 1.27$	**M1 A1**	Standardising using their values, Accept 1.28
	Comp 1.645 (OE)	**M1**	Valid comparison z values or areas
	No evidence that incomes in the region greater	**A1 FT**	OE FT their z. No contradictions (No FT for 2 tail test – max score **B0 M1 A1 M1** for comp 1.96 **A0**) **Note:** Accept alternative CV method

Question 43

$\frac{0.801\times(1-0.801)}{2000}$ (= 0.0000797)	**M1**	
$0.801\pm z\times\sqrt{"0.0000797"}$	**M1**	Allow any z-value
$z = 1.96$	**B1**	
0.784 to 0.818 (3 sf)	**A1**	As final answer. Must be an interval Allow 0.783 to 0.819

Question 44

(a)	$7.1\pm z\times\sqrt{\frac{2.6}{75}}$	**M1**	Expression of correct form must be z (note MR var = 2.6^2 can score **M1**) seen
	$z = 1.751$	**B1**	
	6.77 to 7.43 (3 sfs)	**A1**	Must be an interval
	Total:	**3**	
(b)	0.04^3	**M1**	Allow 0.08^3 for **M1**
	$= 0.000064$	**A1**	
	Total:	**2**	
(c)	e.g. Particular day or time of day	**B1**	Allow "Not random"

Question 45

(i)	$300.1\pm z\times\frac{0.9}{\sqrt{75}}$	**M1**	allow any value of z
	$z = 2.576$	**B1**	allow 2.574 to 2.579
	299.83 to 300.37 (2 dps)	**A1**	answer must be seen to 2 dps need an interval
		3	
(ii)	CI includes 300 so claim supported or justified or probably true	**B1 FT**	or equivalent FT from CI in **(i)**

Question 46

$\frac{153}{200}+z\times\sqrt{\frac{\frac{153}{200}\times\frac{200-153}{200}}{200}}=0.835$ $(\text{Var}(P_s) = 0.000898875)$ (s.d. 0.02998)	**M1**	
$z = 2.335$	**A1**	allow 2.33 or 2.34
$2\Phi(z)-1$	**M1**	or equivalent method indep
$\alpha = 98$	**A1**	allow 98.0 but not e.g. 98.04

Question 47

(i)	$4820 \pm z \times \frac{1420}{\sqrt{125}}$	**M1**	Must be a z value
	$z = 2.326$	**B1**	Accept 2.326 - 2.329
	4524/4525 to 5115/5116 or 4520 to 5120 (3 sf)	**A1**	Must be an interval
(ii)	$\bar{x} = 4840$	**B1**	or width = 280 or half width = 140
	$4840 + 1.96 \times \frac{1420}{\sqrt{n}} = 4980$ OE	**M1**	or $140 = 1.96 \times \frac{1420}{\sqrt{n}}$ OE
	$n = 395$	**A1**	CAO must be an integer

Question 48

(i)	$\bar{x} = 11.83$	**B1**	
	$11.83 \pm z \frac{0.1}{\sqrt{10}}$	**M1**	any z
	$z = 2.576$	**B1**	accept 2.574 to 2.579
	[11.75 to 11.91]	**A1**	or equiv. Accept 11.7 to 11.9
		4	
(ii)	No because pop normal (so $\bar{X}$ normally distr)	**B1**	
		1	
(iii)	11.7 not within CI	**B1FT**	
		1	
(iv)	No because 95% CI is narrower than 99% CI	**B1**	OE
		1	
(v)	Σx^2 (= 1399.67)	**M1**	attempted
	$\text{Est}(\sigma^2) = \frac{10}{9}\left(\frac{"1399.67"}{10} - \left(\frac{"118.3"}{10}\right)^2\right)$ OE	**M1**	correct sub of their Σs into correct formula
	= 0.0201 (3 sf) or 181/9000	**A1**	
		3	

Question 49

$\frac{5-4.9}{\frac{2.21}{\sqrt{75}}}$ (= 0.392)	**M1**	Correct stand'n. Must have √75
1 – Φ("0.392")	**M1**	Correct area consistent with working
= 0.348 (3 sfs)	**A1**	

Question 50

$\frac{\frac{8}{64}\times(1-\frac{8}{64})}{64}$ (= $\frac{7}{4096}$ or 0.00171)	**M1**	OE, e.g. $\frac{\frac{1}{8}\times\frac{7}{8}}{64}$
$2\times z\sqrt{"\frac{7}{4096}"} = 0.130$	**M1**	Correct equation using their variance
$z = 1.572$	**A1**	
ϕ("1.572") (= 0.942) (0.942 – (1 – 0.942) = 0.884)	**M1**	2ϕ(their z) -1
$\alpha = 88$	**A1**	CAO
	5	

Question 51

(i)	213, 165, 73, 196 Allow 073	**B1**	For 3-digit no, < 265, consisting of three consecutive integers from given digits, backwards or forward. (73 or 073 counts as a 3-digit no.)
		B1	For another three such. Other answers may be valid. If other method used, method must be clear
		2	
(ii)	$\frac{510}{25} = \frac{102}{5}$ or 20.4	**B1**	
	$\frac{25}{24}\left[\frac{13225}{25} - \left(\frac{102}{5}\right)^2\right]$	**M1**	$\frac{1}{24}\left(13225 - \frac{510^2}{25}\right)$
	118 (3 sf) or $\frac{2821}{24}$	**A1**	
		3	
(iii)	(Average) weekly earnings of all students in Amy's year	**B1**	Not 'All students in Amy's year'
		1	

Question 52

(i)	Est(μ) = 495.9	**B1**	Accept 496
	Est(σ^2) = $\frac{10}{9}(\frac{2459283}{10} - "495.9"^2)$	**M1**	Attempt Σx^2 and subst in correct formula (1/9("2459283" – "4959"^{2}/10)). May be implied by correct answer
	= 12.8 (3 sf) or 383/30	**A1**	(Note: Biased var "11.49" scores M0 A0)
		3	
(ii)	H_0: $\mu = 505$ H_1: $\mu < 505$ $\frac{75660 - 505}{3.6 \div \sqrt{150}}$	**B1**	Allow 'Pop mean' but not just 'mean'
	= –2.04	**M1**	Correct stand'n; must have √150. No sd/var mixes. Condone sample SD (3.58/3.39) Accept standardisation of totals ((75660-75750)/44.091) Accept CV method
		A1	Accept +2.04 (Note: if valid area comparison done 0.0207/0.0206 or 0.979 needed for A1)
	comp z = –2.054	**M1**	Valid comparison of z's or area (0.0207/6>0.02; 0.979(3)<0.98)
	No evidence (at 2%) that machine pkts mean mass < 505	**A1ft**	oe No contradictions. SC Two tail test can score B0 M1 A1 M1 for comparison with 2.326 A0 (max 3/5)
(iii)	Large sample, so sample mean approx normally distr'd	**B1**	Allow just 'Sample is large' or 'n is large' n>30

Question 53

(i)	$52 \pm z \times \frac{6.5}{\sqrt{15}}$	**M1**	Expression of the correct form. Any z
	z = 1.96	**B1**	Seen or used
	48.7 to 55.3 (3 sf)	**A1**	Must be an interval
		3	
(ii)	Narrower because more information or because $\frac{\sigma}{\sqrt{n}}$ smaller	**B1**	oe Accept 'sample size is larger' 'more employees' 'width inversely proportional to sq root of n' 'if n increases width decreases' '95% CI is 49.7 to 54.3' or similar. No contradictions
		1	

Question 54

(i)	$\frac{20}{100} \pm z \times \sqrt{\frac{0.2 \times (1-0.2)}{100}}$	**M1**	Any z
	z = 1.881 or 1.882	**B1**	
	= 0.125 to 0.275	**A1**	
		3	
(ii)	$\frac{1}{6}$ is within this range No evidence of bias concerning 2	**B1ft**	Both statements needed

Question 55

est(μ) (= 153.2 ÷ 75) = 2.04 (3 sf)	**B1**	
est(σ^2) = $\frac{75}{74}(\frac{340.24}{75} - "2.04267"^2)$ oe	**M1**	
= 0.369 (3 sf)	**A1**	Accept 0.368
	3	

Question 56

$\frac{12.2-12}{2.5/\sqrt{n}}$	**M1**	Standardisation. Allow cc. need correct form incl sqrt
(=) 1.96	**B1**	Correct z
$\sqrt{n}$ = 1.96 × 2.5 ÷ 0.2	**M1**	Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only
n = 600	**A1**	accept 601 SR whole number ans from 595 to 605 can score full marks if fully justified
	4	

Question 57

(i)	$176 \pm z \times \frac{7.2}{\sqrt{200}}$	**M1**	need correct form must be z
	$z = 2.24$	**B1**	allow 2.241 and 2.242
	175 to 177	**A1**	cwo
		3	
(ii)	Sample random	**B1**	oe. both words essential
		1	

Question 58

(i)	$z = 1.96$	**B1**	seen
	$330.1 \pm z \times \frac{4.8}{\sqrt{180}}$	**M1**	Must be of correct form. Any z
	= 329.4 to 330.8 (1 dp)	**A1**	Must be to 1 dp. Must be an interval.
		3	
(ii)	Yes, because vol of all cans not stated to be normal	**B1**	Or Yes, population not stated to be normal
		1	

Question 59

(i)	$\bar{x} = \frac{420}{50} = 8.4$	**B1**	
	$s^2 = \frac{50}{49}\left(\frac{27530}{50} - \left(\frac{420}{50}\right)^2\right)$	**M1**	Or $1/49(27530 - (420)^2/50)$
	= 489.8(36....)	**A1**	Must see $\geqslant$ 4 sf
		3	
(ii)	$\Phi^{-1}(0.9377) = 1.536$	**B1**	
	$\frac{5 - '8.4'}{\sqrt{\frac{490}{n}}} = -1.536$	**M1**	Attempting to standardise – must have correct form
	$n = \left(\frac{1.536}{3.4}\right)^2 \times 490$ (= 100.0048)	**M1**	Attempting numerical expression for n or $\sqrt{n}$ (must have used a 'z' value) may be implied by answer
	$n = 100$	**A1**	No errors seen. Must be whole number
		4	

Question 60

(i)	$z = 2.326$	**B1**	
	$62.3 \pm z\frac{13.2}{\sqrt{200}}$	**M1**	Any z. Expression of correct form. Must be a 'z'
	60.1 to 64.5 (3 sfs)	**A1**	Must be an interval
		3	
(ii)	Yes, because pop not (given to be) normal, or pop distribution unknown	**B1**	No contradictions
		1	

Question 61

$\frac{50}{49}\left(\frac{4361}{50}-\bar{x}^2\right)=9.62$	**M1**	or $\left(\frac{4361}{49}-\frac{(\Sigma x)^2}{50\times 49}\right)=9.62$ BOD regarding symbols used
$\bar{x}^2=\frac{4361}{50}-9.62\times\frac{49}{50}=77.7924$	**A1**	$(\Sigma x)^2=4361\times 50-9.62\times 50\times 49=194481$or $\Sigma x=441$ (Σx) or $(\bar{x})$ must be correctly identified
$\bar{x}=8.82$ (3 sf)	**A1**	SC use of 'biased' leading to 8.81 B1
	3	

Question 62

$0.6\pm z\sqrt{\frac{0.4\times 0.6}{100}}$	**M1**	Recognisable value of z
$z=2.326$	**B1**	2.326 to 2.329
0.486 to 0.714 (3 sf)	**A1**	Must be an interval
	3	

Question 63

(i)	Est(μ) = 1.85	**B1**	
	$\text{Est}(\sigma^2)=\frac{50}{49}\left(\frac{175.25}{50}-'1.85'^2\right)$	**M1**	Allow $\sqrt{\frac{50}{49}\left(\frac{175.25}{150}-'1.85'^2\right)}$ or 0.0290 for M1
	= 0.0842 (3 sf) or $\frac{33}{392}$	**A1**	Cao If $\frac{50}{49}$ omitted (giving var = 0.0825 or sd = 0.287) M0A0
		3	
(ii)	H_0: Pop mean time = 1.9 (h) H_1: Pop mean time < 1.9 (h)	**B1**	Allow 'μ' but not just 'mean'
	$\pm\frac{1.85-1.9}{\sqrt{\frac{'0.0842'}{50}}}$	**M1**	$\pm\frac{1.85-1.9}{\frac{'0.290'}{\sqrt{50}}}$ Accept totals method (92.5–95) / $\sqrt{4.21}$
	= –1.22	**A1**	= –1.22
	comp z = –1.645	**M1**	Or other valid comparison 0.888 or 0.889 < 0.95 OR 0.111 or 0.112>0.05
	No evidence that mean time < 1.9 h	**A1**	**FT** their z. Correct conclusion. No contradictions If $\frac{50}{49}$ not used in (1): var = 0.8225, sd = 0.907, cr = 1.17 can score all marks in **(ii)** Note- 2 tail test can score B0 M1 A1 M1 (comparison with 1.96) A0 (no ft) max3/5
		5	

Question 64

(i)	Normal with mean 372	**B1**	
	sd = $\frac{54}{\sqrt{36}}$	**M1**	or variance = $\frac{54^2}{36}$ M1
	(= 9)	**A1**	(= 81) A1
		3	
(ii)	Pop normal	**B1**	Allow X is normal
		1	

Question 65

(i)	Biased towards people who like tennis Excludes people who don't like tennis	**B1**	or other sensible
		1	
(ii)	Obtain a list of all people in the town	**B1**	
	Use random numbers	**B1**	or, e.g. pick numbers from a hat or other sensible
		2	
(iii)	$\text{Var}(p) = \frac{\frac{47}{350}(1-\frac{47}{350})}{350}$ (= 0.000332152)	**M1**	
	$z = 1.645$	**B1**	
	$\frac{47}{350} \pm z\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$	**M1**	Must be a z value
	0.104 to 0.164 (3 sf)	**A1**	Must be an interval
		4	
(iv)	1.25×1.645 (= 2.056)	**M1**	or 1.25 × their width ÷ 2 ÷ their $\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$ (Complete method)
	Φ('2.056') (= 0.980)	**M1**	Attempt Φ(their z)
	$x = 96$ (2 sf)	**A1**	Allow 0.96 (2 sf) CWO
		3	

Question 66

(i)	Assume population is normally distributed	**B1**	
	$\overline{x} = 25.9$	**B1**	Allow $\frac{259}{10}$
	$z = 2.17$	**B1**	
	$'25.9' \pm z \times \frac{3}{\sqrt{10}}$	**M1**	Must have correct form and z.
	23.8 to 28.0 (3 sf)	**A1**	CWO
		5	
(ii)	0.03^2 (=0.0009)	**B1**	
		1	

Question 67

(i)	$\text{est}(\mu) = \frac{25110}{50}$ (= 502.2)	**B1**	
	$\text{est}(\sigma^2) = \frac{50}{49}\left(\frac{12610300}{50} - \frac{25110}{50}\right)^2 \left(= \frac{50}{49} \times \frac{58}{50} = 1.1836\right)$	**M1**	OE
	1.18 (3 sf) or $\frac{58}{49}$	**A1**	Accept SD = 1.0879
	z = 2.054 or 2.055	**B1**	
	$502.2 \pm z \times \frac{\sqrt{\text{'1.1836'}}}{\sqrt{50}}$	**M1**	Must be of correct form.
	501.9 to 502.5 (1dp)	**A1**	CWO. Must be in interval. **SC** accept use of biased variance (1.16) for M1 A1
		6	
(ii)	More confident **or** z would be greater, Hence wider.	**B1**	OE Reason needed
		1	

Question 68

$2 \times z \times \frac{3.2}{10} = 1.25$	**M1**	OE Allow without '2 ×'
z = 1.953	**A1**	SOI
Φ('*their* 1.953') (= 0.9746)	**M1**	
= 1 – 2(1 – '0.9746') = 0.9492	**M1**	OE
α = 94.9 or 95	**A1**	CWO
	5	

Question 69

(a)	$p = \frac{70}{500}$ or 0.14	**B1**
	$z = 2.576$	**B1**
	$"0.14" \pm z \times \sqrt{\frac{"0.14"(1-"0.14")}{500}}$	**M1**
	0.100 to 0.180	**A1**
		4
(b)	0.1666... is within confidence interval Belief supported or justified	**B1**
		1
(c)	$z \times \sqrt{\frac{"0.14"(1-"0.14")}{500}} = 0.02$	**M1**
	$z = 1.289$	**A1**
	$\Phi('1.289') = 0.9013$	**M1**
	$\alpha = '0.9013' - (1 - '0.9013')$	**M1**
	80.3% (3 sf)	**A1**
		5

Question 70

Est $\mu = 15.56$	**B1**
Est $\sigma^2 = \frac{100}{99}\left(\frac{29004}{100} - "15.56"^2\right)$ or $= \frac{1}{99}\left(29004 - \frac{1556^2}{100}\right)$	**M1**
$48.4105 = 48.4$ (3 sf)	**A1**
	3

Question 71

(a)	$\frac{\Sigma x}{7} = \frac{34.7}{7} = 4.9571$ or 4.96 (3 sf) ($\Sigma x^2 = 175.15$)	**B1**
	$\frac{7}{6}\left(\frac{"175.15"}{7} - "4.9571"^2\right)$	**M1**
	0.523 (3 sf)	**A1**
		3
(b)	$'4.96' \pm z \times \sqrt{\frac{0.523}{7}}$ (**FT** *their* mean and standard deviation)	**M1**
	$z = 1.96$	**B1**
	4.42 to 5.49 (3 sf)	**A1**
		3

Question 72

(a)	$\frac{\frac{102}{250}\times\frac{250-102}{250}}{250}$ (= 0.000966144) $\frac{102}{250}\pm z\sqrt{'0.00096614'}$	
	$z = 1.645$	
	Confident Interval is 0.357 to 0.459 (3 sf)	
(b)	Estimate of mean $\left(\frac{50460}{250}\right)$ = \$201.84	
	$\frac{250}{249}\left(\frac{19854200}{250}-\left(\frac{50460}{250}\right)^2\right)$ or $\frac{1}{249}\left(19854200-\frac{50460^2}{250}\right)$	
	Estimate of variance = 38 832.75 dollars^2 or 38 800 (3 sf)	
(c)	e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected.	

Question 73

(a)	$\frac{56}{300} \pm z \times \sqrt{\frac{\frac{56}{300} \times \frac{244}{300}}{300}}$	**M1**
	$z = 2.054$ or 2.055	**B1**
	0.14(0) to 0.233 (3sf) or 0.141 to 0.233 (3sf)	**A1**
		3
(b)	$\frac{1}{6}$ $(= 0.167)$ This is within confidence interval, so no reason to believe die is biased.	**B1 FT**
		1

Question 74

(a)	$\text{Est}(\mu) = \frac{4820}{60}$ or $\frac{241}{3}$ or 80.3 (3 sf)	**B1**	
	$\text{Est}(\sigma^2) = \frac{60}{59}(\frac{392\,050}{60} - (\frac{4820}{60})^2)$	**M1**	Use of biased (80.72) score M0 A0.
	$82.0904\left(\frac{14530}{177}\right)$ to 82.635 or SD = 9.0604 to 9.0904 (3sf)	**A1**	
	$z = 2.326$	**B1**	
	$\frac{4820}{60} \pm z \times \sqrt{\frac{'82.0904'}{60}}$	**M1**	Expression of the correct form – must be z value.
	77.6 to 83.1 (3 sf)	**A1**	CWO Use of biased 77.6 to 83.0(3) can score B1M1A1 (max 4/6).
		6	
(b)	Population distribution of times unknown	**B1**	Accept 'not normal'.
		1	

Question 75

(a)	$\frac{4509}{90}$ [= 50.1]	**B1**	
	$\frac{90}{89}(\frac{225950}{90} - '50.1'^2)$ or $\frac{1}{89}\left(225950 - \frac{4509^2}{90}\right)$	**M1**	Attempted. Use of biased = 0.5455 scores M0A0
	$\frac{491}{890}$ or 0.552 (3 sf)	**A1**	
		3	
(b)	'50.1' $\pm z\sqrt{\frac{'\frac{491}{890}'}{90}}$	**M1**	Expression of the correct form, allow any z-value but must be a z-value
	$z = 2.326$	**B1**	Accept 2.326 to 2.329
	49.9 to 50.3 (3 sf)	**A1**	FT from biased variance. Must be an interval.
		3	
(c)	Population of masses is unknown	**B1**	Accept population of masses is not normal
		1	
(d)	1 – 0.98	**M1**	0.02 seen
	0.02 ÷ 2 = 0.01	**A1**	As final answer
		2	

Question 76

(a)	est $(\mu) = \frac{7570}{100}$ (= 75.7)	B1	
	est$(\sigma^2) = \frac{100}{99}\left(\frac{\sum h^2}{100} - '75.7'^2\right)$ or $\frac{1}{99}\left(588050 - \frac{7570^2}{100}\right)$ $= \frac{100}{99}\left(\frac{588050}{100} - '75.7'^2\right)$ [= 151.525]	M1	Attempted (**Note:** Biased variance (150.01) scores M0)
	= 152 (3 sf)	A1	Or $\frac{15001}{99}$
		3	
(b)	'75.7' $\pm z\sqrt{\frac{'151.525'}{100}}$	M1	For expression of correct form. Must be a z value. Condone just + or just -.
	$z = 2.576$	B1	Accept 2.574 to 2.579
	72.5 to 78.9	A1 FT	FT biased variance only Must be an interval
		3	
(c)	0.99^4	B1	
	0.961 (3 sf)	B1	
		2	

$\frac{3820}{100}$ [= 38.2]	B1	
$\frac{100}{99}\left(\frac{182200}{100}-'38.2'^2\right)$ or $\frac{1}{99}\left(182200-\frac{3820^2}{100}\right)$	M1	Use of biased (362.76) scores M0
$=\frac{12092}{33}$ or 366.424 or 366 (3 sf)	A1	Accept SD=19.1422 or 19.1(3sf)
'38.2' $\pm z\times\sqrt{\frac{'366.424'}{100}}$	M1	Expression of the correct form must be a z-value.
z = 1.881 or 1.882	B1	Seen.
34.6 to 41.8 (3 sf)	A1	Allow use of biased giving (34.6,41.8) Must be an interval.
	6	

Question 78

(a)	$\pm\frac{123-125}{\frac{6}{\sqrt{40}}}$ [= –2.108...]	M1	Must have √40 No standard deviation/variance mix. Ignore any continuity correction attempts for this mark.
	P(z < '–2.108') = 1 – Φ('2.108')	M1	For correct probability area consistent with *their* working.
	= 0.0175 or 0.0176 (3 sf)	A1	
		3	
(b)	No, population is normal	B1	Need both.
		1	

Question 79

(a)	est μ = 14 accept $\frac{560}{40}$	**B1**	
	est $\sigma^2=\frac{40}{39}\left(\frac{7850}{40}-14^2\right)$ or $\frac{1}{39}\left(7850-\frac{560^2}{40}\right)$	**M1**	
	0.25641 or 0.256 (3sf)	**A1**	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est σ^2 = 0.25 **M0 A0**.
		3	

Question 80

est(p) = 0.2 accept $\frac{15}{75}$	**B1**	SOI
$2 \times z \times \sqrt{\frac{0.2\times0.8}{75}} = 0.162$	**M1**	Expression of the correct form. Condone missing $2x$.
$z \left[= 0.081 \times \sqrt{\frac{75}{0.2\times0.8}} \right] = 1.754$	**A1**	Correct z. Condone 3sf accuracy.
Φ('1.754') = 0.96[03] '0.96' – (1 – '0.96')	**M1**	OE. Using *their* z to find alpha.
$\alpha = 92$	**A1**	Following correct working.
	5	

Question 81

(a)	N(12.5, ….)	**B1**	
	Variance = 0.4096	**B1**	Accept 0.410 (3sf), condone $\frac{10.24}{25}$
		2	
(b)	$\frac{13 - '12.5'}{\sqrt{'0.4096'}}$ [= 0.781]	**M1**	For standardising with *their* values. Accept standardising with 12.
	ϕ('0.781') – (1 – Φ('0.781'))	**M1**	For attempting to find *their* central area.
	0.565 (3sf)	**A1**	
		3	

Question 82

(a)	$0.25 \pm z\sqrt{\frac{0.25\times0.75}{140}}$	**M1**	Expression of correct form (allow **M1** for just one side stated). Must be a z-value.
	$z = 2.054$ or 2.055	**B1**	
	0.175 to 0.325 (3sf)	**A1**	Must be an interval.
		3	
(b)	0.90 × 0.95 × 0.01 + 0.90 × 0.05 × 0.99 + 0.10 × 0.95 × 0.99	**M1 M1**	**M1** for one correct triple product. **M1** for all correct and added.
	0.147	**A1**	SC If zero scored award **B1** for a 2 or 3 term expression of the form 0.90 × 0.95 [×c] OE. ($0 < c \leqslant 1$)
		3	

Question 83

(a)	E.g. Bias towards students who play instruments or only music students or e.g. the six will possibly be friends/have similar music preferences	**B1**	OE Or any reason that some are excluded e.g. because it is lunchtime or because the music building is chosen or any suggestion that opinions may not be independent. Note: 'not representative of all students' needs qualifying
		1	
(b)	28, 119, 207	**B1**	**B1** for 28, 119 (condone 028).
		B1	**B1** for 207 and only 3 values stated.
		2	

Question 84

(a)	$\frac{20.5}{40} = 0.5125$	**B1**	Accept 0.513 or $\frac{41}{80}$. Condone $\frac{20.5}{40}$.
	$\frac{40}{39}\left(\frac{10.728}{40} - \left('0.5125'^2\right)\right)$ or $\frac{1}{39}\left(10.728 - \frac{20.50^2}{40}\right)$	**M1**	Biased variance (0.005544 or $\frac{887}{160\,000}$) scores **M0 A0.**
	0.0056859 or 0.00569 (3 sf) or $\frac{887}{156\,000}$	**A1**	CAO
		3	
(b)	[11 × '0.5125' + 0.5]) = 6.1375 or $\frac{491}{80}$ or 6.14 (3sf)	**B1 FT**	FT *their* 0.5125
	11^2 × '0.0056859'	**M1**	With nothing added. Using *their* variance in **(a)** (no sd/var confusion)
	0.688 (3sf)	**A1**	CAO
		3	

Question 85

est $\mu = 14$ accept $\frac{560}{40}$	**B1**	
est $\sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right)$ or $\frac{1}{39}\left(7850 - \frac{560^2}{40}\right)$	**M1**	
0.25641 or 0.256 (3sf)	**A1**	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ **M0 A0**.
	3	

Question 86

est(p) = 0.2 accept $\frac{15}{75}$	**B1**	SOI
$2 \times z \times \sqrt{\frac{0.2 \times 0.8}{75}} = 0.162$	**M1**	Expression of the correct form. Condone missing $2x$.
$z \left[= 0.081 \times \sqrt{\frac{75}{0.2 \times 0.8}} \right] = 1.754$	**A1**	Correct z. Condone 3sf accuracy.
Φ('1.754') = 0.96[03] '0.96' – (1 – '0.96')	**M1**	OE. Using *their z* to find alpha.
$\alpha = 92$	**A1**	Following correct working.
	5	

Question 87

est(p) = 0.4	**B1**	
$'0.4' + z\sqrt{\frac{'0.4' \times (1 - '0.4')}{500}} [= 0.445]$	**M1**	OE Use of *their* 0.4 in a correct expression
$z \left[= 0.045 \div \sqrt{\frac{'0.4' \times (1 - '0.4')}{500}} \right] = 2.054$	**A1**	Condone 2.053 and 2.05
0.98 – (1 – 0.98)	**M1**	
96% confidence	**A1**	CWO, must be integer
	5	

Question 88

(a)	Est (μ) = $\frac{1199}{6}$ or 199.833 or 200 or $\frac{2398}{12}$ [mm]	**B1**	Accept in any form
	Est (σ^2) = $\frac{12}{11}\left(\frac{479226}{12} - \frac{'1199'^2}{6}\right)$ or $\frac{1}{11}\left('479226' - \frac{'2398'^2}{6}\right)$	**M1**	Use of their values in correct formula (may be implied)
	= 2.33 (3 sf) [mm^2]	**A1**	Accept $\frac{7}{3}$
		3	
(b)	Small sample	**B1**	Accept not 'not representative' unless qualified.
		1	

Question 89

(a)	$\frac{13+a}{5}$	**B1**	Accept $\frac{2+3+3+5+a}{5}$. Do not ignore subsequent working
		1	
(b)	$\frac{5}{4}\left(\frac{47+a^2}{5}-\left(\frac{13+a}{5}\right)^2\right)=4$ or $\frac{1}{4}\left(47+a^2-\frac{(13+a)^2}{5}\right)=4$	**M1**	Use of correct formula using *their* value from **(a)**, in terms of a, and equate to 4
	$2a^2-13a-7=0$	**A1**	Any correct three-term quadratic equation rearranged to a form ready to solve
	$a=7$	**A1**	Condone the other value of a $(-\frac{1}{2})$
		3	

Question 90

$\frac{1300+\frac{1}{200}-1250}{\frac{480}{10}}$ or $\frac{1300-1250}{\frac{480}{10}}$ [= 1.042]	**M1**	Allow with incorrect or omitted continuity correction Must have 10 Accept totals method
1 – Φ('1.042')	**M1**	For area consistent with *their* values
0.149 (3 s.f.)	**A1**	
	3	

Question 91

(a)	$72.3 \pm z\sqrt{\frac{64.3}{50}}$	**M1**	Expression of correct form (allow only one side for M1). Must be a z value
	$z = 1.751$	**B1**	Accept 1.75 if nothing better seen
	CI is 70.3 to 74.3 metres (3 s.f.)	**A1**	Allow without units Must be an interval
		3	
(b)	Not random sample	**B1**	Need 'random' or 'not representative/biased because…' OE
		1	

Question 92

(a)	Est(μ) = $\frac{2520}{200}$ [= 12.6]	**B1**	OE
	Est(σ^2) = $\frac{200}{199}\left(\frac{31582}{200} - \text{'12.6'}^2\right)$ or $\frac{1}{199}\left(31852 - \frac{2520^2}{200}\right)$	**M1**	Allow M1 if $\frac{200}{199}$ omitted
	= 0.5025 or 0.503 or $\frac{100}{199}$	**A1**	CWO or σ = 0.7088 or 0.709
	z = 1.96	**B1**	
	'12.6' $\pm z \times \sqrt{\text{'0.5025'} \div 200}$	**M1**	For expression of correct form Any z but must be z
	CI = 12.5 to 12.7 (3 sf)	**A1**	CWO Must be an interval Note: Use of biased can score maximum B1 M1 A0 B1 M1 A0
		6	
(b)	0.95 × 40 [= 38]	**B1**	Give at early stage
		1	

Question 93

$\frac{62.1}{10} = 6.21$	**B1**	OE
[Σx^2 = 387.05] $\frac{10}{9}\left(\frac{\textit{their '387.05'}}{10} - \left(\textit{their '6.21'}\right)^2\right)$ **or** $\frac{1}{9}\left(\frac{\textit{their '387.05'}}{10} - \frac{\left(\textit{their '6.21'}\right)^2}{10}\right)$	**M1**	Can be implied. Accept alternative methods (e.g. working mean of 6). Biased 0.1409 M0.
= 0.157 (3 sf) or $\frac{1409}{9000}$	**A1**	
	3	

Question 94

(a)	$45 \pm z \times \frac{6}{\sqrt{200}}$	**M1**	For expression of correct form, any z. Accept one side of interval for M1.
	z = 1.96	**B1**	Must be seen.
	44.2 to 45.8 (3 sf)	**A1**	Must be an interval.
		3	
(b)	$z \times \frac{11}{\sqrt{200}} = 2$	**M1**	Or … = 4 for M1
	z = 2.571	**A1**	Accept 3sf if nothing better seen.
	Φ(*their* '2.571') = 0.9949 **and** *their* '0.9949' – (1 – *their* '0.9949') [= 0.9898]	**M1**	OE For area consistent with their values. Must be seen.
	α = 99.0 (3 sf)	**A1**	Allow 99. cwo Final answer of 0.99 scores A0.
		4	

Question 95

(a)	Est $\mu = 25.6$ or $\frac{2048}{80}$ or $\frac{128}{5}$	**B1**	
	Est $\sigma^2 = \frac{80}{79}\left(\frac{52760}{80} - \left(\frac{2048}{80}\right)^2\right)$ or $\frac{1}{79}\left(52760 - \frac{2048^2}{80}\right)$	**M1**	Substitution into a correct formula. Biased 4.14 scores M0.
	= 4.19 (3 sf) or $\frac{1656}{395}$	**A1**	
		3	
(b)	'25.6' $+ z\sqrt{\frac{'4.19'}{80}} = 26.0$	**M1**	Use of correct equation with their values.
	$z = 1.748$ or 1.747	**A1**	Accept 3sf. FT Biased z = 1.758.
	(Φ('1.748') = 0.960) '0.960' – (1 – '0.960')	**M1**	Correct area using their values.
	$\alpha = 92.0$ or 91.9	**A1**	Allow 92 . FT Biased 92.1. A final answer of 0.92 or 0.919 scores A0.
		4	

Question 96

(a)	For X, $\mu = 2$ $\sigma^2 = 1.6$		
	Mean = 2	**B1**	
	Variance $= \frac{1.6}{160}$ or $\frac{1}{100}$ or 0.01	**B1**	Accept Var = 0.1^2 (accept sd=0.1 if clearly identified).
	Normal	**B1**	
		3	
(b)	$\pm\frac{1.8 - \frac{1}{320} - '2'}{\sqrt{'0.01'}}$ or $\pm\frac{1.8 - '2'}{\sqrt{'0.01'}}$ [= –2.03 or –2] or $\pm$ (287.5 – '320') / $\sqrt{'256'}$ or $\pm$ (288 – '320') / $\sqrt{'256'}$ [= –2.03 or –2]	**M1**	Allow with wrong continuity correction. M1 can be implied by correct final answer or for –2.03 / –2.0 or 0.9788 / 0.9772 seen.
	Φ('–2.03') = 1 – Φ('2.03')	**M1**	Correct area consistent with their values. M1 can be implied by correct final answer.
	= 0.0212 or 0.0228 (3 sf)	**A1**	
		3	

Question 97

(a)	$45 \pm z \times \frac{6}{\sqrt{200}}$	**M1**	For expression of correct form, any z. Accept one side of interval for M1.
	$z = 1.96$	**B1**	Must be seen.
	44.2 to 45.8 (3 sf)	**A1**	Must be an interval.
		3	
(b)	$z \times \frac{11}{\sqrt{200}} = 2$	**M1**	Or … = 4 for M1
	$z = 2.571$	**A1**	Accept 3sf if nothing better seen.
	Φ(*their* '2.571') = 0.9949 **and** *their* '0.9949' – (1 – *their* '0.9949') [= 0.9898]	**M1**	OE For area consistent with their values. Must be seen.
	$\alpha = 99.0$ (3 sf)	**A1**	Allow 99. cwo Final answer of 0.99 scores A0.
		4	

Question 98

$\frac{62.1}{10} = 6.21$	**B1**	OE
$[\Sigma x^2 = 387.05]$ $\frac{10}{9}\left(\frac{\textit{their } '387.05'}{10} - (\textit{their } '6.21')^2\right)$ **or** $\frac{1}{9}\left(\frac{\textit{their } '387.05'}{10} - \frac{(\textit{their } '6.21')^2}{10}\right)$	**M1**	Can be implied. Accept alternative methods (e.g. working mean of 6). Biased 0.1409 M0.
= 0.157 (3 sf) or $\frac{1409}{9000}$	**A1**	
	3	

Question 99

(a)	$\left[\frac{49}{140} = 0.35\right]$		
	$0.35 \pm z\sqrt{\frac{0.35(1-0.35)}{140}}$	**M1**	Use of formula of correct form, ft *their* $\frac{49}{140}$, any z (not a probability).
	$z = 2.326$	**B1**	Accept 2.326 to 2.329 .
	Confidence interval = 0.256 to 0.444 (3 sf)	**A1**	Must be an interval.
		3	
(b)	Find a smaller **percentage** confidence interval/ lower **level** of confidence	**B1**	ISW if 2 reasons given. Just saying 'use smaller z' oe B0. Accept a correct example e.g. 90% (even if not qualified with statement).
		1	

Question 100

$\bar{x} = 1700/50 = 34$	**B1**	
$\text{Est}(\sigma^2) = \frac{50}{49}\left(\frac{59050}{50} - 34^2\right)$ or $\frac{1}{49}\left(59050 - \frac{1700^2}{50}\right)$	**M1**	$\text{Est}(\sigma^2) = \frac{59050}{50} - 34^2$ biased scores M0.
$= 25.5$ (3 sf) or $\frac{1250}{49}$	**A1**	= 25 scores A0.
	3	

Question 101

(a)	$z = 1.645$	**B1**	
	$z \times \frac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$	**M1**	OE. Equation of correct form. Accept $p = x/100$. Any z. Allow missing factor of 2.
	$[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$ $x^2 - 100x + 2304 = 0$	**A1**	Any correct (likely scalar multiple) three-term quadratic equation in x or p with simplified coefficients. Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$.
	$x = 36$ or 64	**A1**	
		4	
(b)	$0.1^2 = 0.01$	**B1**	Accept either.
		1	

Question 102

(a)	180, 227	**B1**	One correct. Ignore incorrect numbers.
		B1	Both correct and no extra numbers seen. (Allow other correct use of list of digits).
		2	
(b)	These numbers are not independent of the previous numbers OR Only a finite number of digits used	**B1**	Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason).
		1	

Question 103

Est (μ) = 3.25 = 13/4 or 1625/500	**B1**	
$\text{Est}(\sigma^2) = \frac{500}{499}\left(\frac{5663.5}{500} - \text{"3.25"}^2\right)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$	**M1**	Expression of correct form.
= 0.766 (3 sf) or 1529/1996	**A1**	Biased variance of 0.7645 scores M0A0.
	3	

Question 104

$0.23 \pm z \times \sqrt{\frac{0.23 \times (1 - 0.23)}{200}}$	**M1**	Expression of correct form. Any z, but $z = 0.8328$ scores B0M0.
$z = 1.811$ or 1.812	**B1**	
0.176 to 0.284 (3 sf)	**A1**	Must be an interval.
	3	

Question 105

$\frac{5}{4}\left(\frac{1+2^2+6^2+1+a^2}{5}-\left(\frac{1+2+6+1+a}{5}\right)^2\right)=\frac{11}{2}$ or $\frac{1}{4}\left((42+a^2)-\frac{(10+a^2)}{5}\right)=\frac{11}{2}$	**M1***	OE attempted or e.g., $\frac{42+a^2}{5}-\left(\frac{10+a}{5}\right)^2=\frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.
$4a^2-20a+0=0$ or $a^2-5a+0=0$	**DM1**	Two- or three-term quadratic equation in a, with at least two terms correct.
$a=5$	**A1**	Ignore $a=0$, if seen.
	3	

Question 106

(a)	$z\times\sqrt{\frac{11.2}{n}}=1.4076\div 2$	**M1**	Any z, but must be a z.
	$z=1.881$ or 1.882	**B1**	
	$[n=\left(\frac{1.881}{0.7038}\right)^2\times 11.2]$ $n=80$	**A1**	Must be a whole number.
		3	
(b)	Jan, Feb and March not typical of whole year.	**B1**	Or, e.g., weather is different at different times of year.
		1	
(c)	$0.94^3\times 0.06\times 4$	**M1**	
	$=0.199$ (3 sf)	**A1**	
		2	