

A-Level

Topic Sampling and Estimation

May 2013-May 2025

Answers

Question 1

(i)	$\bar{x} = 930/15 = (62)$ $z = 1.751$ $'62' \pm z \times \frac{12}{\sqrt{15}}$ $= 56.6 \text{ to } 67.4 \text{ (3 sf)}$	B1 B1 M1 A1	4	Any z Must be an interval
(ii)	92% of such intervals will contain μ	B1	1	Accept $P(\text{This interval contains } \mu) = 0.92$
(iii)	Each possible sample of this size is equally likely	B1	1	Each member of pop equally likely to be chosen
[Total: 6]				

Question 2

(i)	$\text{est}(\mu) = 9750/150 = (65)$ $\text{est}(\sigma^2) = \frac{1}{149} \left(647500 - \frac{9750^2}{150} \right)$ $= 92.3 \text{ (3 s.f.)}$	B1 M1 A1 [3]	Correct subst. in correct formula
(ii)	$z = 2.326$ $'65' \pm z \times \frac{\sqrt{92.28188}}{\sqrt{150}}$ $= 63.2 \text{ to } 66.8 \text{ (3 s.f.)}$	B1 M1 A1 [3]	Any z (Use of 'biased' can still score here)
(iii)	0.02^2 $= 0.0004 \text{ o.e.}$	M1 A1 [2]	Allow M1 for 0.02 seen
[Total: 8]			

Question 3

(i)	$\text{est}(\mu) = 2005/200 = (10.025)$ $\text{est}(\sigma^2) = \frac{1}{99} 20175 - \frac{2005^2}{200}$ $= 0.376 \text{ (3 sf)}$	B1	Correct subst in correct formula
		M1	
		A1	
		[3]	
(ii)	$\frac{10 - '10.025'}{\sqrt{'0.376256'}}$ $\sqrt{50}$ $1 - \Phi('0.288')$ $= 0.387 \text{ (3 sf)}$	(= -0.288)	Allow without $\sqrt{\quad}$, but $\div\sqrt{50}$ essential (Use of 'biased' variance can still score fully in (ii))
		M1	
		M1	
		A1	
		[3]	

Question 4

$\text{Est}(\mu) = 1.8775 \text{ or } 1.88 \text{ (3 sf)}$ $\text{Est}(\sigma^2) = \frac{80}{79} \left(\frac{820.24}{80} - '1.8775'^2 \right)$ $= 6.81316 \text{ or } 6.81 \text{ (3 sf)}$ $z = 1.96$ $'1.8775' \pm z \times \sqrt{\frac{'6.81316''}{80}}$ $= 1.31 \text{ to } 2.45 \text{ (3 sf)}$	B1	6	Accept 751/400 (not 150.2/80) Correct subst'n in correct formula 1/79 (820.24 - 150.2 ² /80) Seen Must be an interval. NB use of biased var can still score A1.
	M1		
	A1		
	B1		
	M1		
	A1		

Question 5

(i)	$\text{est}(\mu) = 2866 \text{ or } 2870 \text{ (3 s.f.)}$ $\text{est}(\sigma^2) = \frac{1}{49} (410900000 - \frac{143300^2}{50})$ $(= 4126.53)$ $= 4130 \text{ (3 sf)}$	B1	Accept 143300/50 o.e. Correct subst in correct formula
		M1	
		A1	
		[3]	
(ii)	$H_0: \text{Pop mean (or } \mu) = 2850$ $H_1: \text{Pop mean (or } \mu) \neq 2850$ $\frac{\frac{143300}{50} - 2850}{\frac{\sqrt{'4126.53'}}{\sqrt{50}}}$ $= 1.761$ $'1.761' < 1.96$ $\text{No evidence mean distance changed}$	B1	Both. Not just 'mean' Allow '4126.53' without $\sqrt{\quad}$, but must have all $\sqrt{50}$ Or correct c.v. (2867.81) for alt method For valid comparison of z values, areas or c.v. Dep 1.96; ft their 1.761 If $H_1: \mu > 2850$ and c.f. 1.645, max B0M1A1M1A0 (c.v. for 1 tail test 2864.94)
		M1	
		A1	
		M1	
		A1f	

Question 6

$$z = 2.576$$

$$2 \times z \times \frac{0.17}{\sqrt{n}} = 0.2 \text{ oe}$$

$$n = \left(\frac{2 \times 0.17 \times 2.576}{0.2}\right)^2 \text{ oe } (= 19.2)$$

Smallest n is 20

B1	Seen (accept 2.574 to 2.579)
M1	Allow without '2 ×' OR with incorrect z
M1	Attempt to arrange equ of correct form (with correct z and '2×' into the form $n =$ or $\sqrt{n} =$
A1 [4]	

Question 7

$$p = 0.56$$

$$'0.56' \pm z \times \sqrt{\frac{0.56 \times 0.44}{100}}$$

$$z = 2.17, \text{ or } 2.169 \text{ or } 2.171$$

$$0.452 \text{ to } 0.668 \text{ (3 s.f.)}$$

B1	Used
M1	Equation of correct form condone just +ve or -ve Must be z
B1	
A1 [4]	Seen Must be an interval

Question 8

(i) $p = \frac{184}{400}$ or 0.46

$$z = 1.96$$

$$"0.46" \pm z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}}$$

$$= 0.411 \text{ to } 0.509$$

B1	Used
B1	Seen
M1	Using expression of correct form
A1 [4]	Must be an interval

(ii) 0.5 within CI
Claim not supported or not justified

B1✓ [1]	Both needed. No contradictions. ft their (i)
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(iii) $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.05$

$$z = 2.006$$

$$\Phi('2.006') = 0.9775$$

$$\alpha = '0.9775' - (1 - '0.9775')$$

$$= 95.5\%$$

M1	Allow M1 for $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.1$
A1	
M1	or $1 - 2(1 - '0.9775')$
A1 [4]	

Question 9

$\frac{\Sigma x}{8} = \frac{2006}{8} = 250.75$ or 251 (3 s.f.)	B1	Any equivalent form
$(\Sigma x^2 = 503274)$		
$\frac{8}{7} \left(\frac{"503274"}{8} - "250.75"{}^2 \right)$	M1	For use of formula of correct form
$= 38.5$ o.e. (accept 6.204 ²)	A1 [3]	cao (as final answer)

Question 10

(i)	$0.25(1 + 4 + 9) - 1.5^2$ (=1.25 AG)	B1 [1]
(ii)	$\frac{1.4-1.5}{\sqrt{\frac{5}{4} \div 300}}$ (= -1.549)	M1
	$\Phi(" -1.549 ") = 1 - \Phi(" 1.549 ")$ = 0.0607 (3 sf)	M1 A1 [3]
(iii)	Large sample or large n (\bar{X} (approx) normally distr) or Central Limit Theorem	B1 [1]
		[Total: 5]

Question 11

$\frac{70}{69} \times 2.70 = 2.73913$	M1A1	
$3.61 \pm z \sqrt{\frac{"2.73913"}{70}}$	M1	or $3.61 \pm z \sqrt{\frac{2.70}{69}}$ M2A1(implied)
$z = 1.96$ 3.22 to 4.00 (3 sf)	B1 A1 [5]	without $\frac{70}{69}$: $3.61 \pm z \sqrt{\frac{2.70}{70}}$ M0A0M1 $z = 1.96$ 3.23 to 3.99(4.00) (3 sf) B1 A1 Answer must be an interval
		[Total: 5]

Question 12

$N(483.2, 537.92)$ or $N(483.2, 23.2^2)$ $\frac{436-483.2}{\sqrt{537.92}}$ or $\frac{436-483.2}{23.2}$ (= -2.035) $\Phi(-2.035) = 1 - \Phi(2.035)$ $= 0.021$ or 2.1%	B1 M1 M1 A1 [4]	or $\frac{8.2}{\sqrt{8}}$ or $\frac{8.2^2}{8}$ seen or implied or $\frac{436-60.4}{8.2/\sqrt{8}}$ standardising (no mixed methods) Correct area consistent with their working
[Total: 4]		

Question 13

<p>(i)</p> $\text{Est}(\mu) = \frac{7220}{80} \text{ or } 90.25$ $\text{Est}(\sigma^2) = \frac{80}{79} \left(\frac{656060}{80} - \left(\frac{7220}{80} \right)^2 \right)$ $= 56.3924 \text{ or } \frac{4455}{79}$ $z = 2.17$ $\frac{7220}{80} \pm z \times \sqrt{\frac{56.3924}{80}}$ $= 88.4 \text{ to } 92.1 \text{ (3 sf)}$	B1 M1 A1 B1 M1 A1 [6]	Accept 90.3 $\frac{1}{79} \left(656060 - \frac{7220^2}{80} \right)$ Accept 56.4 Expression of correct form Must be an interval (N.B. biased var gives 88.4 to 92.1 scores possible B1M0A0B1M1A1)
<p>(ii)</p> Pop normal No	B1 B1dep [2]	X normal or full definition of pop normal SR B1 for “no” <u>and</u> relevant reference to normal

Question 14

(i)	$\text{Var}(P_s) = \frac{\frac{33}{150} \times \frac{150-33}{150}}{150} \quad (= 0.001144)$ $z = 2.576$ $\frac{33}{150} \pm z\sqrt{0.001144}$ $= 0.133 \text{ to } 0.307 \text{ (3 sf)}$	M1	4	<p>Seen. Accept 2.574 to 2.579</p> <p>Expression of correct form. Any z</p> <p>Must be an interval</p>
		B1		
		M1		
		A1		
(ii)	$\frac{19035}{150} \quad (= 126.9 = 127(3\text{sf}))$ $\frac{150 \left(\frac{4054716}{150} - \left(\frac{19035}{150} \right)^2 \right)}{149} \quad \text{o.e.}$ $= 11001.17 \text{ or } 11000(3 \text{ sf})$	B1	3	<p>For use of a correct formula</p>
		M1		
		A1		
(iii)	<p>4-digit nos. each digit 0-9</p> <p>Ignore nos > 9526</p> <p>Ignore repeats</p>	B1	3	<p>Some valid way of generating 4 digit random nos from valid method</p> <p>SR If zero score, full explanation of method for drawing numbers out of a hat can score B1.</p> <p>NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point.</p>
		B1		
		B1		

Question 15

(i)	$\left(\frac{1508}{50} \right) = 30.16 \text{ (30.2)}$ $\frac{50 \left(\frac{51825}{50} - (30.16)^2 \right)}{49}$ $= 129 \text{ (3 sf) Or } 130$	B1	[3]	<p>Allow any form</p> <p>(129.46367)</p>
		M1		
		A1		
(ii)	$(1.5 \times '30.16' + 10)$ $= 55.24$ $(1.5^2 \times '129....')$ $= 291 \text{ (3 sf)}$	B1ft	[3]	<p>ft their 30.16</p> <p>$1.5^2 \times \text{their}(129)$ with nothing added at any stage</p> <p>Allow 290</p>
		M1		
		A1ft		
		Total 6		

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Question 16

(i)	$\text{Var}(p_s) = \frac{0.22 \times (1 - 0.22)}{100}$ $\left(= \frac{429}{250\,000} \text{ or } 0.001716 \right)$ $0.22 \pm z \sqrt{\frac{429}{250\,000}}$ $z = 2.17 \text{ or } 2.168/9 \text{ or } 2.171$ $0.13(0) \text{ to } 0.31(0) \text{ (2 sf)}$	M1 M1 B1 A1	pq/100 Expression of correct form with their variance Any z (must be a z value) accept one side only Seen Must be an interval
(ii)	$'2' \times (1 - 0.97) \times 0.97$ $= 0.0582$	M1 A1	[2]
Total 6			

Question 17

(i)	$14800/50 \text{ or } 296$ $\frac{50}{49} \left(\frac{4390000}{50} - '296'^2 \right) (= 187.755)$ $= 188 \text{ (3 sf)}$	B1 M1 A1	Oe 3
(ii)	$2 \times z \times \sqrt{\frac{'187.755'}{50}} = 5.45 \quad \text{oe}$ $z = 1.406 \text{ or } 1.405$ $\Phi('1.406') (= 0.92 \text{ or } 0.9199)$ $\alpha = 84 \text{ (2 sf)} \quad \text{allow } 83.98$	M1 A1 M1 A1	If '2 ×' omitted: $z \times \sqrt{\frac{'187.755'}{50}} = 5.45$ M1 $z = 2.812 \text{ or } 2.810$ A0 $\Phi('2.812') (= 0.9975)$ $\alpha = 99.5 \text{ or } 99 \text{ or } 100$ M1 A0 For complete method to find α SR use of biased var(184) scores M1A1(1.4205) A=84.5 M1A1
(iii)	0.96^4 $= 0.849 \text{ (3 sf)}$	M1 A1	2

Question 18

(i)	$4200/80 (=52.5)$ $= \frac{80}{79} \left(\frac{229\,000}{80} - '52.5'^2 \right) (= 107.595)$ $= 108 \text{ (3 sf)}$	B1 M1 A1	[3]
(ii)	$'52.5' \pm z \sqrt{\frac{'107.595'}{80}}$ $z = 2.326$ $49.8 \text{ to } 55.2$	M1 B1 A1f	[3] Correct form – must be z-value – allow one side only Seen ft their 52.5 and 107.595. Must be an interval
(iii)	49	B1	[1]
[Total: 7]			

Question 19

<p>(i) 34 $2.2^2 + 1.3^2 + 2.6^2 (=13.29)$</p>	<p>B1 B1 [2]</p>	<p>Accept 13.3 or 3.65^2 Allow at early stage</p>
<p>(ii) $\frac{33-34'}{\sqrt{\frac{13.29'}{70}}}$ (= -2.295)</p>	<p>M1</p>	<p>correct standardisation method for either</p>
<p>$\frac{35-34'}{\sqrt{\frac{13.29'}{70}}}$ (= 2.295)</p>	<p>M1</p>	<p>For attempt to use tables to find the probability between two z values ,may be implied by next line</p>
<p>$\Phi('2.295') - \Phi(' -2.295')$</p>	<p>M1</p>	<p>For a correct method to find the area between their two z values</p>
<p>= $\Phi('2.295') - (1 - \Phi('2.295'))$ oe</p>	<p>A1 [4]</p>	
<p>= 0.978 (3 sf)</p>		
<p>[Total: 6]</p>		

Question 20

<p>(i) est $\mu = 2.087$ est $\sigma^2 = \frac{100}{99} \left(\frac{435.57}{100} - 2.087^2 \right)$ = 0.000132(3232) or 131/990000</p>	<p>B1 M1 A1 [3]</p>	<p>allow 2.09 $1/99 (435.57 - 208.7^2/100)$ without $\frac{100}{99}$: 0.000131 M0A0</p>
<p>(ii) $E(Y - X) = 2.12 - 2.087 (= 0.033)$ $\text{Var}(Y - X) = 0.000144 + '0.00013232'$ = 0.000276(32) $\frac{0.01 - '0.033'}{\sqrt{0.00027632}}$ (= -1.384) $\Phi(' -1.384') = 1 - \Phi('1.384')$ = 0.0832</p>	<p>B1 M1 A1 M1 M1 A1 [6]</p>	<p>or $2.12 - 2.087 - 0.01$ for $Y - X - 0.01 < 0$ allow 2.09 for 2.087 or $\sqrt{(0.01^2 + '0.00013232')}$ M1 = 0.016623 A1 \checkmarktheir $E(Y - X)$ & $\text{Var}(Y - X)$ var must be a combination of the two vars correct area/prob consistent with their working SR use of biased var (0.000131) in (i) and (ii) scores in (ii) B1M1 A1 for 0.000275 and M1M1 A1 for 0.0827 (6/6 available)</p>

Question 21

(i)	0.4 or 2/5 or 26/65	B1	[1]	no recovery in (ii) for the B mark
(ii)	$\text{“0.4”} + z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.516 \text{ oe}$ $z = \left(\frac{0.116 \times \sqrt{65}}{\sqrt{0.4 \times 0.6}} \right) = 1.909$ $(\Phi(1.909) = 0.97(18))$ $2(0.97 - 1)$ $\alpha = 94$	M1 A1 M1 A1	[4]	or $\text{“0.4”} - z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.284$ or $z \times \sqrt{\frac{0.4 \times 0.6}{65}} = 0.116 \text{ oe}$ for fully correct method to find α from their z allow 94.36 or 94.4 or 94.374

Question 22

(i)	$N(352, \dots)$ Variance = 2.9	B1 B1	[2]	no recovery in (ii) for each B mark accept $sd = \sqrt{2.9} = 1.70(29)$ stated
(ii)	$\frac{354 - 352}{\sqrt{2.9}} \quad (= 1.174)$ $1 - \Phi(1.174)$ $= 0.120 \text{ (3 sf)}$	M1 M1 A1	[3]	with their mean and var Or $\frac{354.05 - 352}{\sqrt{2.9}}$ or correct restart (= 1.204) (accept sd/var mix)1 $-\Phi(1.204)$ $= 0.114 \text{ (3 sf)}$ Incorrect cc can score M1M1A0

Question 23

(i)	$\frac{3420}{60} (= 57)$ $\frac{60 \left(\frac{195200}{60} - 57^2 \right)}{59} \quad (= 4.40678)$ $= 4.41 \text{ (3 sf)}$	B1 M1 A1	[3]	Oe As final answer
(ii)	$57 \pm z \sqrt{\frac{4.40678}{60}}$ $z = 2.326$ $[56.4 \text{ to } 57.6] \text{ (3 sf)}$	M1 B1 A1	[3]	2.326 – 2.329 (accept 2.33 if no better seen) NB: use of biased variance in (ii) can score in full

Question 24

(i)	$\frac{6}{\sqrt{120}} \quad \text{oe seen}$ $\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)} \quad (= 1.826)$ $P(z > '1.826') = 1 - \Phi('1.826')$ $= 0.034 \text{ (2 sf)}$	B1 M1 M1 A1 [4]	Or $6^2/120$ oe seen \pm Allow without $\sqrt{120}$. No sd/var mix Correct tail consistent with their working 0.0339
(ii)	No n is large (≥ 30) Sample mean is (appr) normally distrib or The CLT applies oe	B1 B1 [2]	1 st B1 for either comment 2 nd B1 for 'No' with 2 nd comment (No mark for 'No' alone)

Question 25

(i)	$\text{Est}(\mu) = \frac{14910}{150} \quad (= 99.4)$ $\text{Est}(\sigma^2) = \frac{150}{149} \left(\frac{1525000}{150} - "99.4"{}^2 \right)$ $= 288.228$ $z = 2.576$ $"99.4" \pm z \times \sqrt{288.228 \div 150}$ $\text{CI} = 95.8 \text{ to } 103 \text{ (3 sf)}$	B1 M1 A1 B1 M1 A1 [6]	Allow M1 if $\frac{150}{149}$ omitted Accept 2.574–2.579 Any z (NB Use of biased Var can score 5/6 max)
(ii)	100 lies within this CI Hence yes	B1 ✓ [1]	Both needed, ft their CI
(iii)	To avoid bias or Necessary to enable statistical inference	B1 [1]	Or any equivalent

Question 26

<p>(i) $\lambda = 3.9$ $e^{-3.9} \times \frac{3.9^4}{4!}$ $= 0.195$</p>	<p>B1 M1 A1 [3]</p>	<p>M1 allow any λ SR Combination method B1 for $\lambda = 1.6$ AND $\lambda = 2.3$ used in combination method (at least 3 combinations) M1 All correctly combined and added</p>
<p>(ii) $\bar{X} \sim N(1.6, \frac{1.6}{75})$</p>	<p>B1 B1 [2]</p>	<p>B1 for $N(1.6, \dots)$ stated B1 for $\text{Var} = \frac{1.6}{75}$ stated SR, not stated but all implied in (iii): B1</p>
<p>(iii) $\frac{1.7-1.6}{\sqrt{\frac{1.6}{75}}} (= 0.685)$ $1 - \Phi("0.685")$ $= 0.247$ (3 sf)</p>	<p>M1 M1 A1 [3]</p>	<p>For standardising (using their values or correct values). Ignore cc Correct area consistent with their working Accept use of $1/2n$ correction leading to 0.233. NB Use of Poisson sum $Po(120)$ and $N(120, 120)$ with $\mu = 127.5$ leads to 0.247, or 0.233 with cc</p>
<p>(iv) X not normally distr. So CLT needed</p>	<p>B1 [1]</p>	<p>Not "it"</p>

Question 27

<p>$192.4 \pm z \sqrt{\frac{43.6}{150}}$ $z = 2.326$ to 2.329 191 to 194 (3 sf)</p>	<p>M1 B1 A1 [3]</p>	<p>Allow $\frac{43.6}{\sqrt{150}}$ Allow one side for M1 Condone $\sqrt{(43.6/149)}$ oe CWO</p>
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Question 28

<p>(i) Pop too big or takes too long oe or testing destroys articles oe</p>	<p>B1 [1]</p>	<p>or too expensive oe or pop inaccessible oe</p>
<p>(ii) (a) $z = 1.96$ $65.7 \pm z \times \frac{\sqrt{15}}{10}$ $= 64.9$ to 66.5 (3 sf)</p>	<p>B1 M1 A1 [3]</p>	<p>seen Expression of correct form (must be 'z' must be 65.7) Must be an interval</p>
<p>(b) CI does not include 64.7 Probably has affected (or increased) mean bounce ht.</p>	<p>B1 [1]</p>	<p>allow 64.7 not within CI both needed. ft their CI ft 65.7/64.7 mix</p>

Question 29

$\frac{6.2}{\sqrt{50}} \text{ or } \frac{6.2^2}{50}$ $\frac{51-53}{6.2\sqrt{50}} (= -2.281)$ $P(z > '-2.281') = \phi('2.281')$ $= 0.989 \text{ (3 sf)}$	B1	seen or implied
	M1	allow without $\div\sqrt{50}$
	M1	for finding correct area consistent with
	A1 [4]	working as final answer

Question 30

(i)	$((0.5672 + 0.6528) \div 2)$ $= 0.61$	B1 [1]	
(ii)	$'0.61' + z\sqrt{\frac{0.61 \times (1-0.61)}{350}} = 0.6528$ $z = 0.0428 \times \sqrt{\frac{700}{0.61 \times (1-0.61)}} \text{ oe}$ $= 2.321$ <p>98% confidence</p>	M1 M1 A1 A1 ft [4]	oe correct rearrangement of correct equn, ft '0.61' ft their z (dep on both Ms)

Question 31

(i)	Each employee has an equal chance of being chosen	B1 [1]	oe
(ii)	$\text{Est } (\mu) = 4$ $\text{Est } (\sigma^2) = \frac{10}{9} \left(\frac{199.22}{10} - 4^2 \right)$ $= 4.36 \text{ (3 sf)}$	B1 M1 A1 [3]	sub in correct formula attempted working may not be seen
(iii)	Distances travelled by all employees at the firm	B1 [1]	oe

Question 32

<p>(i)</p> $\text{est}(\mu) = 3.4$ $\text{est}(\sigma^2) = \frac{100}{99} \left(\frac{1356}{100} - '3.4'^2 \right)$ $= 2.02(0202)$ $z = 1.96$ $3.4 \pm z \times \sqrt{\frac{'2.020202'}{100}}$ $= 3.12 \text{ to } 3.68 \text{ (3 sf)}$	<p>B1 M1 A1</p> <p>B1</p> <p>M1 A1</p>	<p>1 / 99 (1356 – 340²/100) or 200/99</p> <p>correct working only allow from unbiased or biased variance</p> <p>[6]</p>
<p>(ii)</p> <p>Mean should be 3</p> <p>CI does not include 3 Machine probably not working properly</p>	<p>B1*</p> <p>DB1✓</p>	<p>stated or implied</p> <p>✓ their CI or evidence that....</p> <p>[2]</p>

Question 33

$\frac{11.8-11}{1.6+\sqrt{n}} = 1.645$ $\frac{11.8-11}{1.6+\sqrt{n}} = 1.96$ $n = 10.8 \quad (\text{allow } 11)$ $n = 15.4 \quad (\text{allow } 15)$ <p>Possible values are 11, 12, 13, 14, 15</p>	<p>M1</p> <p>B1 B1</p> <p>A1</p> <p>A1</p>	<p>M1 for $\frac{11.8-11}{1.6+\sqrt{n}} = \text{any } z$ allow var / sd mix for 1.6 but need \sqrt{n}</p> <p>B1 for each correct z</p> <p>for both</p> <p>not for just $11 \leq n \leq 15$ oe</p> <p>[5]</p>
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Question 34

<p>(a)</p>	$63 \pm z \times \frac{9}{\sqrt{100}}$ $z = 1.645$ $61.5 \text{ to } 64.5 \text{ (3 sf)}$	<p>M1</p> <p>B1</p> <p>A1</p>	<p>B1 Expression of correct form, any z</p> <p>Seen</p> <p>[3] Must be an interval</p>
<p>(b) (i)</p>	$z = \frac{1.96}{2} \quad (= 0.98)$ $\Phi("0.98") \quad (= 0.8365)$ $"0.8365" - (1 - "0.8365")$ $\quad \quad \quad (= 0.673)$ $\alpha = 67.3 \text{ (3 sf)}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Allow $\frac{\text{any } z}{2}$</p> <p>[3] Allow 67 from correct working</p>
<p>(ii)</p>	$4 = (2x^2 z^2 x^2 \sigma^2) / \sqrt{n}$ $n = 200$	<p>M1</p> <p>A1</p>	<p>Attempt to solve equ of correct form</p> <p>[2] SR B1 for $n = 100$</p>

Question 35

$\left(\frac{508}{8}\right) = 63.5$	B1	
$(\sum x^2 = 32360.12)$		
$\frac{8}{7} \left(\frac{32360.12}{8} - 63.5^2 \right)$	M1	oe
$= 14.6$ (3 sf) or 2553/175	A1	[3] From correct working

Question 36

(i)	$\frac{53-52}{6.1+\sqrt{75}}$ (= 1.420)	M1	
	$\frac{51-52}{6.1+\sqrt{75}}$ (= -1.420)	M1	or -"1.420" seen
	$\Phi("1.420") - \Phi("-1.420")$	M1	
	= 0.844 (3 sfs)	A1	
	Total:	4	
(ii)	Need to assume \bar{X} (approx.) normally distributed	B1	or X not stated to be normally distributed

Question 37

$\text{Var}(Ps) = \frac{0.3(1-0.3)}{120}$ (= 0.00175)	M1	Attempt correct values in correct formula
$0.3 \pm z\sqrt{0.00175}$	M1	must be a z-value, not a prob
$z = 1.645$	B1	
CI = 0.231 to 0.369 (3 sf)	A1	

Question 38

(i)	$\bar{x} = 6.7/200$ (= 67/2000 = 0.0335)	B1	
	$s^2 = \frac{200}{199} \times \left(\frac{0.2312}{200} - 0.0335^2 \right)$	M1	$s^2 = \frac{0.2312}{200} - 0.0335^2$ M0
	= 0.0000339(2) = 27/796000	A1	= 0.00003375 A0
	Total:	3	
(ii)	H_0 : Pop mean level = 0.034 H_1 : Pop mean level \neq 0.034	B1	not just "mean", but allow just " μ "
	$\frac{0.030335 - 0.034}{\frac{\sqrt{0.00003392}}{\sqrt{200}}}$	M1	must have $\sqrt{200}$ $\frac{0.0335 - 0.034}{\frac{\sqrt{0.00003375}}{\sqrt{200}}}$ M1
	= -1.21(4) (3 sfs) (-1.22 \leftrightarrow -1.21)	A1	= -1.217 (3 sfs) A1
	Comp with $z = -1.645$ (or 0.1124 > 0.05)	M1	0.112 > 0.05 valid comparison z or areas
	No evidence that (mean) pollutant level has changed, accept H_0 (if correctly defined)	A1FT	correct conclusion no contradictions SR: One tail test: B0, M1A1 as normal, M1 (comparison with 1.282 consistent signs) A0

Question 39

$10 \times 0.46^2 (= 2.116)$ or $\frac{0.46}{\sqrt{10}}$	B1	SOI
Total mass of ore $\sim N(70, 2.116)$ or $\sim N\left(7, \left(\frac{0.46}{\sqrt{10}}\right)^2\right)$	B1	
$\pm \frac{71 - "70"}{\sqrt{2.116}}$ or $\pm \frac{7.1 - "7.0"}{0.46/\sqrt{10}}$ ($= 0.687$)	M1	correct, using their sd or $\sqrt{(\text{their var})}$ e.g. allow $\frac{71 - "70"}{4.6}$ for M1
$1 - \Phi("0.687")$	M1	for correct area consistent with their working
$= 0.246$ (3 sf)	A1	

Question 40

(i)	$z = 1.751$	B1	
	$\frac{103}{200} \pm z \sqrt{\frac{103 \times (1 - \frac{103}{200})}{200}}$ oe	M1	all correct except for recognisable value of z , allow for on side only
	$= 0.453$ to 0.577 (3 sf) as final answer	A1	must be an interval
	Total:	3	
(ii)	0.08 oe 8%, 8/100	B1	

Question 41

573, 43 (or 043), 289	B1B1B1	Ignore incorrect numbers. But allow other correct use of table (i.e. 573, 650, 431)
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Question 42

(i)	Est (μ) = $923/400$ or 2.3075 or 2.31 (3 sf)	B1	
	Est(σ^2) = $\frac{400}{399} \left(\frac{3170}{400} - "2.3075"{}^2 \right)$ OE	M1	
	$= 2.60696$ or 2.61 (3 sf)	A1	(Note: Biased Var = 2.600 scores M0)
	Total:	3	
(ii)	H_0 : Pop mean (or μ) = "2.31" or "2310" H_1 : Pop mean (or μ) > "2.31" or "2310"	B1 FT	
	$\pm \frac{2.6 - "2.310"}{\sqrt{2.60696 - 50}} = 1.27$	M1 A1	Standardising using their values, Accept 1.28
	Comp 1.645 (OE)	M1	Valid comparison z values or areas
	No evidence that incomes in the region greater	A1 FT	OE FT their z . No contradictions (No FT for 2 tail test – max score B0 M1 A1 M1 for comp 1.96 A0) Note: Accept alternative CV method

Question 43

$\frac{0.801 \times (1 - 0.801)}{2000}$ (= 0.0000797)	M1	
$0.801 \pm z \times \sqrt{0.0000797}$	M1	Allow any z-value
$z = 1.96$	B1	
0.784 to 0.818 (3 sf)	A1	As final answer. Must be an interval Allow 0.783 to 0.819

Question 44

(a)	$7.1 \pm z \times \sqrt{\frac{2.6}{75}}$	M1	Expression of correct form must be z (note MR var = 2.6 ² can score M1) seen
	$z = 1.751$	B1	
	6.77 to 7.43 (3 sfs)	A1	Must be an interval
	Total:	3	
(b)	0.04^3	M1	Allow 0.08 ³ for M1
	= 0.000064	A1	
	Total:	2	
(c)	e.g. Particular day or time of day	B1	Allow "Not random"

Question 45

(i)	$300.1 \pm z \times \frac{0.9}{\sqrt{75}}$	M1	allow any value of z
	$z = 2.576$	B1	allow 2.574 to 2.579
	299.83 to 300.37 (2 dps)	A1	answer must be seen to 2 dps need an interval
	Total:	3	
(ii)	CI includes 300 so claim supported or justified or probably true	B1 FT	or equivalent FT from CI in (i)

Question 46

$\frac{153}{200} + z \times \sqrt{\frac{\frac{153}{200} \times \frac{200-153}{200}}{200}} = 0.835$ (Var(P_s) = 0.000898875) (s.d. 0.02998)	M1	
$z = 2.335$	A1	allow 2.33 or 2.34
$2\Phi(z) - 1$	M1	or equivalent method indep
$\alpha = 98$	A1	allow 98.0 but not e.g. 98.04

Question 47

(i)	$4820 \pm z \times \frac{1420}{\sqrt{125}}$	M1	Must be a z value
	$z = 2.326$	B1	Accept 2.326 - 2.329
	4524/4525 to 5115/5116 or 4520 to 5120 (3 sf)	A1	Must be an interval
(ii)	$\bar{x} = 4840$	B1	or width = 280 or half width = 140
	$4840 + 1.96 \times \frac{1420}{\sqrt{n}} = 4980$ OE	M1	or $140 = 1.96 \times \frac{1420}{\sqrt{n}}$ OE
	$n = 395$	A1	CAO must be an integer

Question 48

(i)	$\bar{x} = 11.83$	B1	
	$11.83 \pm z \frac{0.1}{\sqrt{10}}$	M1	any z
	$z = 2.576$	B1	accept 2.574 to 2.579
	[11.75 to 11.91]	A1	or equiv. Accept 11.7 to 11.9
		4	
(ii)	No because pop normal (so \bar{X} normally distr)	B1	
		1	
(iii)	11.7 not within CI	B1FT	
		1	
(iv)	No because 95% CI is narrower than 99% CI	B1	OE
		1	
(v)	Σx^2 (= 1399.67)	M1	attempted
	$\text{Est}(\sigma^2) = \frac{10}{9} \left(\frac{1399.67}{10} - \left(\frac{118.3}{10} \right)^2 \right)$ OE	M1	correct sub of their Σ s into correct formula
	= 0.0201 (3 sf) or 181/9000	A1	
		3	

Question 49

$\frac{5 - 4.9}{\frac{2.21}{\sqrt{75}}}$	(= 0.392)	M1	Correct stand'n. Must have $\sqrt{75}$
$1 - \Phi(“0.392”)$		M1	Correct area consistent with working
$= 0.348$ (3 sfs)		A1	

Question 50

$\frac{\frac{8}{64} \times (1 - \frac{8}{64})}{64}$	(= $\frac{7}{4096}$ or 0.00171)	M1	OE, e.g. $\frac{1 \times 7}{8 \times 8}$
$2 \times z \sqrt{\frac{7}{4096}} = 0.130$		M1	Correct equation using their variance
$z = 1.572$		A1	
$\Phi(“1.572”) (= 0.942)$ $(0.942 - (1 - 0.942) = 0.884)$		M1	$2\phi(\text{their } z) - 1$
$\alpha = 88$		A1	CAO
		5	

Question 51

(i)	213, 165, 73, 196 Allow 073	B1	For 3-digit no, < 265, consisting of three consecutive integers from given digits, backwards or forward. (73 or 073 counts as a 3-digit no.)
		B1	For another three such. Other answers may be valid. If other method used, method must be clear
(ii)	$\frac{510}{25} = \frac{102}{5}$ or 20.4	2 B1	
	$\frac{25}{24} \left[\frac{13225}{25} - \left(\frac{102}{5} \right)^2 \right]$	M1	$\frac{1}{24} \left(13225 - \frac{510^2}{25} \right)$
	118 (3 sf) or $\frac{2821}{24}$	A1	
		3	
(iii)	(Average) weekly earnings of all students in Amy's year	B1	Not 'All students in Amy's year'
		1	

Question 52

(i)	$\text{Est}(\mu) = 495.9$	B1	Accept 496
	$\text{Est}(\sigma^2) = \frac{10}{9} \left(\frac{2459283}{10} - "495.9"{}^2 \right)$	M1	Attempt Σx^2 and subst in correct formula ($1/9("2459283" - "4959"{}^2/10)$). May be implied by correct answer
	$= 12.8$ (3 sf) or 383/30	A1	(Note: Biased var "11.49" scores M0 A0)
		3	
(ii)	$H_0: \mu = 505$ $H_1: \mu < 505$ $\frac{75660 - 505}{150}$ $3.6 \div \sqrt{150}$	B1	Allow 'Pop mean' but not just 'mean'
	$= -2.04$	M1	Correct stand'n; must have $\sqrt{150}$. No sd/var mixes. Condone sample SD (3.58/3.39) Accept standardisation of totals ((75660-75750)/44.091) Accept CV method
		A1	Accept +2.04 (Note: if valid area comparison done 0.0207/0.0206 or 0.979 needed for A1)
	comp $z = -2.054$	M1	Valid comparison of z's or area (0.0207/6 > 0.02; 0.979(3) < 0.98)
	No evidence (at 2%) that machine pkts mean mass < 505	A1ft	oe No contradictions. SC Two tail test can score B0 M1 A1 M1 for comparison with 2.326 A0 (max 3/5)
(iii)	Large sample, so sample mean approx normally distr'd	B1	Allow just 'Sample is large' or 'n is large' n > 30

Question 53

(i)	$52 \pm z \times \frac{6.5}{\sqrt{15}}$	M1	Expression of the correct form. Any z
	$z = 1.96$	B1	Seen or used
	48.7 to 55.3 (3 sf)	A1	Must be an interval
		3	
(ii)	Narrower because more information or because $\frac{\sigma}{\sqrt{n}}$ smaller	B1	oe Accept 'sample size is larger' 'more employees' 'width inversely proportional to sq root of n' 'if n increases width decreases' '95% CI is 49.7 to 54.3' or similar. No contradictions
		1	

Question 54

(i)	$\frac{20}{100} \pm z \times \sqrt{\frac{0.2 \times (1-0.2)}{100}}$	M1	Any z
	$z = 1.881$ or 1.882	B1	
	$= 0.125$ to 0.275	A1	
		3	
(ii)	$\frac{1}{6}$ is within this range No evidence of bias concerning 2	B1ft	Both statements needed

Question 55

$\text{est}(\mu) (= 153.2 \div 75) = 2.04$ (3 sf)	B1	
$\text{est}(\sigma^2) = \frac{75}{74} \left(\frac{340.24}{75} - 2.04267^2 \right)$ oe	M1	
$= 0.369$ (3 sf)	A1	Accept 0.368
	3	

Question 56

$\frac{12.2 - 12}{2.5 / \sqrt{n}}$	M1	Standardisation. Allow cc. need correct form incl sqrt
$(=) 1.96$	B1	Correct z
$\sqrt{n} = 1.96 \times 2.5 \div 0.2$	M1	Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only
$n = 600$	A1	accept 601 SR whole number ans from 595 to 605 can score full marks if fully justified
	4	

Question 57

(i)	$176 \pm z \times \frac{72}{\sqrt{200}}$	M1	need correct form must be z
	$z = 2.24$	B1	allow 2.241 and 2.242
	175 to 177	A1	cwo
		3	
(ii)	Sample random	B1	oe. both words essential
		1	

Question 58

(i)	$z = 1.96$	B1	seen
	$330.1 \pm z \times \frac{4.8}{\sqrt{180}}$	M1	Must be of correct form. Any z
	= 329.4 to 330.8 (1 dp)	A1	Must be to 1 dp. Must be an interval.
		3	
(ii)	Yes, because vol of all cans not stated to be normal	B1	Or Yes, population not stated to be normal
		1	

Question 59

(i)	$\bar{x} = \frac{420}{50} = 8.4$	B1	
	$s^2 = \frac{50}{49} \left(\frac{27530}{50} - \left(\frac{420}{50} \right)^2 \right)$	M1	Or $1/49(27530 - (420)^2/50)$
	= 489.8(36....)	A1	Must see ≥ 4 sf
		3	
(ii)	$\Phi^{-1}(0.9377) = 1.536$	B1	
	$\frac{5-8.4}{\sqrt{\frac{500}{n}}} = -1.536$	M1	Attempting to standardise – must have correct form
	$n = \left(\frac{1.536}{3.4} \right)^2 \times 490$ (= 100.0048)	M1	Attempting numerical expression for n or \sqrt{n} (must have used a 'z' value) may be implied by answer
	$n = 100$	A1	No errors seen. Must be whole number
		4	

Question 60

(i)	$z = 2.326$	B1	
	$62.3 \pm z \frac{13.2}{\sqrt{200}}$	M1	Any z. Expression of correct form. Must be a 'z'
	60.1 to 64.5 (3 sfs)	A1	Must be an interval
		3	
(ii)	Yes, because pop not (given to be) normal, or pop distribution unknown	B1	No contradictions
		1	

Question 61

$\frac{50}{49} \left(\frac{4361}{50} - \bar{x}^2 \right) = 9.62$	M1	or $\left(\frac{4361}{49} - \frac{(\Sigma x)^2}{50 \times 49} \right) = 9.62$ BOD regarding symbols used
$\bar{x}^2 = \frac{4361}{50} - 9.62 \times \frac{49}{50} = 77.7924$	A1	$(\Sigma x)^2 = 4361 \times 50 - 9.62 \times 50 \times 49 = 194481$ or $\Sigma x = 441$ (Σx) or (\bar{x}) must be correctly identified
$\bar{x} = 8.82$ (3 sf)	A1	SC use of 'biased' leading to 8.81 B1
	3	

Question 62

$0.6 \pm z \sqrt{\frac{0.4 \times 0.6}{100}}$	M1	Recognisable value of z
$z = 2.326$	B1	2.326 to 2.329
0.486 to 0.714 (3 sf)	A1	Must be an interval
	3	

Question 63

(i)	Est(μ) = 1.85	B1	
	Est(σ^2) = $\frac{50}{49} \left(\frac{175.25}{50} - 1.85^2 \right)$	M1	Allow $\sqrt{\frac{50}{49} \left(\frac{175.25}{150} - 1.85^2 \right)}$ or 0.0290 for M1
	= 0.0842 (3 sf) or $\frac{33}{392}$	A1	Cao If $\frac{50}{49}$ omitted (giving var = 0.0825 or sd = 0.287) M0A0
		3	
(ii)	H ₀ : Pop mean time = 1.9 (h) H ₁ : Pop mean time < 1.9 (h)	B1	Allow ' μ ' but not just 'mean'
	$\pm \frac{1.85 - 1.9}{\sqrt{\frac{0.0842}{50}}}$	M1	$\pm \frac{1.85 - 1.9}{\sqrt{0.290}}$ Accept totals method $(92.5 - 95) / \sqrt{4.21}$ $\sqrt{50}$
	= -1.22	A1	= -1.22
	comp $z = -1.645$	M1	Or other valid comparison 0.888 or 0.889 < 0.95 OR 0.111 or 0.112 > 0.05
	No evidence that mean time < 1.9 h	A1	FT their z. Correct conclusion. No contradictions If $\frac{50}{49}$ not used in (1): var = 0.8225, sd = 0.907, cr = 1.17 can score all marks in (ii) Note- 2 tail test can score B0 M1 A1 M1 (comparison with 1.96) A0 (no ft) max 3/5
		5	

Question 64

(i)	Normal with mean 372	B1	
	sd = $\frac{54}{\sqrt{36}}$	M1	or variance = $\frac{54^2}{36}$ M1
	(= 9)	A1	(= 81) A1
		3	
(ii)	Pop normal	B1	Allow X is normal
		1	

Question 65

(i)	Biased towards people who like tennis Excludes people who don't like tennis	B1	or other sensible
		1	
(ii)	Obtain a list of all people in the town	B1	
	Use random numbers	B1	or, e.g. pick numbers from a hat or other sensible
		2	
(iii)	$\text{Var}(p) = \frac{\frac{47}{350}(1-\frac{47}{350})}{350} (= 0.000332152)$	M1	
	$z = 1.645$	B1	
	$\frac{47}{350} \pm z\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$	M1	Must be a z value
	0.104 to 0.164 (3 sf)	A1	Must be an interval
		4	
(iv)	1.25×1.645 (= 2.056)	M1	or $1.25 \times \text{their width} \div 2 \div \text{their } \sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$ (Complete method)
	$\Phi(2.056)$ (= 0.980)	M1	Attempt $\Phi(\text{their } z)$
	$x = 96$ (2 sf)	A1	Allow 0.96 (2 sf) CWO
		3	

Question 66

(i)	Assume population is normally distributed	B1	
	$\bar{x} = 25.9$	B1	Allow $\frac{259}{10}$
	$z = 2.17$	B1	
	$25.9 \pm z \times \frac{3}{\sqrt{10}}$	M1	Must have correct form and z.
	23.8 to 28.0 (3 sf)	A1	CWO
		5	
(ii)	0.03^2 (=0.0009)	B1	
		1	

Question 67

(i)	$\text{est}(\mu) = \frac{25110}{50} (= 502.2)$	B1	
	$\text{est}(\sigma^2) = \frac{50}{49} \left(\frac{12610300}{50} - \frac{25110^2}{50} \right) \left(= \frac{50}{49} \times \frac{58}{50} = 1.1836 \right)$	M1	OE
	1.18 (3 sf) or $\frac{58}{49}$	A1	Accept SD = 1.0879
	$z = 2.054$ or 2.055	B1	
	$502.2 \pm z \times \frac{\sqrt{1.1836}}{\sqrt{50}}$	M1	Must be of correct form.
	501.9 to 502.5 (1dp)	A1	CWO. Must be in interval. SC accept use of biased variance (1.16) for M1 A1
		6	
(ii)	More confident or z would be greater, Hence wider.	B1	OE Reason needed
		1	

Question 68

$2 \times z \times \frac{32}{10} = 1.25$	M1	OE Allow without '2 ×'
$z = 1.953$	A1	SOI
$\Phi(\text{'their' } 1.953) (= 0.9746)$	M1	
$= 1 - 2(1 - \text{'0.9746'})$ $= 0.9492$	M1	OE
$\alpha = 94.9$ or 95	A1	CWO
	5	

Question 69

(a)	$p = \frac{70}{500}$ or 0.14	B1
	$z = 2.576$	B1
	$0.14 \pm z \times \sqrt{\frac{0.14(1-0.14)}{500}}$	M1
	0.100 to 0.180	A1
		4
(b)	0.1666... is within confidence interval Belief supported or justified	B1
		1
(c)	$z \times \sqrt{\frac{0.14(1-0.14)}{500}} = 0.02$	M1
	$z = 1.289$	A1
	$\Phi(1.289) = 0.9013$	M1
	$\alpha = 0.9013 - (1 - 0.9013)$	M1
	80.3% (3 sf)	A1
		5

Question 70

	Est $\mu = 15.56$	B1
	Est $\sigma^2 = \frac{100}{99} \left(\frac{29004}{100} - 15.56^2 \right)$ or $= \frac{1}{99} \left(29004 - \frac{1556^2}{100} \right)$	M1
	48.4105 = 48.4 (3 sf)	A1
		3

Question 71

(a)	$\frac{\Sigma x}{7} = \frac{34.7}{7} = 4.9571$ or 4.96 (3 sf) ($\Sigma x^2 = 175.15$)	B1
	$\frac{7}{6} \left(\frac{175.15}{7} - 4.9571^2 \right)$	M1
	0.523 (3 sf)	A1
		3
(b)	'4.96' $\pm z \times \sqrt{\frac{0.523}{7}}$ (FT their mean and standard deviation)	M1
	$z = 1.96$	B1
	4.42 to 5.49 (3 sf)	A1
		3

Question 72

(a)
$$\frac{\frac{102}{250} \times \frac{250 - 102}{250}}{250} (= 0.000966144)$$

$$\frac{102}{250} \pm z\sqrt{0.000966144}$$

$z = 1.645$

Confident Interval is 0.357 to 0.459 (3 sf)

(b) Estimate of mean $\left(\frac{50460}{250}\right) = \201.84

$$\frac{250}{249} \left(\frac{19854200}{250} - \left(\frac{50460}{250} \right)^2 \right) \text{ or } \frac{1}{249} \left(19854200 - \frac{50460^2}{250} \right)$$

Estimate of variance = 38 832.75 dollars² or 38 800 (3 sf)

(c) e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected.

Question 73

(a)	$\frac{56}{300} \pm z \times \sqrt{\frac{\frac{56}{300} \times \frac{244}{300}}{300}}$	M1
	$z = 2.054$ or 2.055	B1
	0.14(0) to 0.233 (3sf) or 0.141 to 0.233 (3sf)	A1
		3
(b)	$\frac{1}{6}$ (= 0.167) This is within confidence interval, so no reason to believe die is biased.	B1 FT
		1

Question 74

(a)	$\text{Est}(\mu) = \frac{4820}{60}$ or $\frac{241}{3}$ or 80.3 (3 sf)	B1
	$\text{Est}(\sigma^2) = \frac{60}{59} \left(\frac{392050}{60} - \left(\frac{4820}{60} \right)^2 \right)$	M1 Use of biased (80.72) score M0 A0.
	82.0904 $\left(\frac{14530}{177} \right)$ to 82.635 or SD = 9.0604 to 9.0904 (3sf)	A1
	$z = 2.326$	B1
	$\frac{4820}{60} \pm z \times \sqrt{\frac{82.0904}{60}}$	M1 Expression of the correct form – must be z value.
	77.6 to 83.1 (3 sf)	A1 CWO Use of biased 77.6 to 83.0(3) can score B1M1A1 (max 4/6).
		6
(b)	Population distribution of times unknown	B1 Accept 'not normal'.
		1

Question 75

(a)	$\frac{4509}{90}$ [= 50.1]	B1	
	$\frac{90}{89} \left(\frac{225950}{90} - '50.1'^2 \right)$ or $\frac{1}{89} \left(225950 - \frac{4509^2}{90} \right)$	M1	Attempted. Use of biased = 0.5455 scores M0A0
	$\frac{491}{890}$ or 0.552 (3 sf)	A1	
		3	
(b)	$'50.1' \pm z \sqrt{\frac{491}{890}}$	M1	Expression of the correct form, allow any z-value but must be a z-value
	$z = 2.326$	B1	Accept 2.326 to 2.329
	49.9 to 50.3 (3 sf)	A1	FT from biased variance. Must be an interval.
		3	
(c)	Population of masses is unknown	B1	Accept population of masses is not normal
		1	
(d)	$1 - 0.98$	M1	0.02 seen
	$0.02 \div 2 = 0.01$	A1	As final answer
		2	

Question 76

i(a)	$\text{est}(\mu) = \frac{7570}{100}$ (= 75.7)	B1	
	$\text{est}(\sigma^2) = \frac{100}{99} \left(\frac{\sum h^2}{100} - '75.7'^2 \right)$ or $\frac{1}{99} \left(588050 - \frac{7570^2}{100} \right)$ = $\frac{100}{99} \left(\frac{588050}{100} - '75.7'^2 \right)$ [= 151.525]	M1	Attempted (Note: Biased variance (150.01) scores M0)
	= 152 (3 sf)	A1	Or $\frac{15001}{99}$
		3	
i(b)	$'75.7' \pm z \sqrt{\frac{151.525}{100}}$	M1	For expression of correct form. Must be a z value. Condone just + or just -.
	$z = 2.576$	B1	Accept 2.574 to 2.579
	72.5 to 78.9	A1 FT	FT biased variance only Must be an interval
		3	
i(c)	0.99^4	B1	
	0.961 (3 sf)	B1	
		2	

$\frac{3820}{100} [= 38.2]$	B1	
$\frac{100}{99} \left(\frac{182200}{100} - '38.2'^2 \right)$ or $\frac{1}{99} \left(182200 - \frac{3820^2}{100} \right)$	M1	Use of biased (362.76) scores M0
$= \frac{12092}{33}$ or 366.424 or 366 (3 sf)	A1	Accept SD=19.1422 or 19.1(3sf)
$'38.2' \pm z \times \sqrt{\frac{366.424}{100}}$	M1	Expression of the correct form must be a z-value.
$z = 1.881$ or 1.882	B1	Seen.
34.6 to 41.8 (3 sf)	A1	Allow use of biased giving (34.6,41.8) Must be an interval.
	6	

Question 78

(a)	$\pm \frac{123-125}{\frac{6}{\sqrt{40}}} [= -2.108...]$	M1	Must have $\sqrt{40}$ No standard deviation/variance mix. Ignore any continuity correction attempts for this mark.
	$P(z < '-2.108') = 1 - \Phi('2.108')$	M1	For correct probability area consistent with <i>their</i> working.
	$= 0.0175$ or 0.0176 (3 sf)	A1	
		3	
(b)	No, population is normal	B1	Need both.
		1	

Question 79

(a)	est $\mu = 14$ accept $\frac{560}{40}$	B1	
	est $\sigma^2 = \frac{40}{39} \left(\frac{7850}{40} - 14^2 \right)$ or $\frac{1}{39} \left(7850 - \frac{560^2}{40} \right)$	M1	
	0.25641 or 0.256 (3sf)	A1	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ M0 A0.
		3	

Question 80

$\text{est}(p) = 0.2 \quad \text{accept } \frac{15}{75}$	B1	SOI
$2 \times z \times \sqrt{\frac{0.2 \times 0.8}{75}} = 0.162$	M1	Expression of the correct form. Condone missing $2x$.
$z \left[= 0.081 \times \sqrt{\frac{75}{0.2 \times 0.8}} \right] = 1.754$	A1	Correct z . Condone 3sf accuracy.
$\Phi('1.754') = 0.96[03]$ '0.96' - (1 - '0.96')	M1	OE. Using <i>their</i> z to find alpha.
$\alpha = 92$	A1	Following correct working.
	5	

Question 81

(a)	N(12.5, ...)	B1	
	Variance = 0.4096	B1	Accept 0.410 (3sf), condone $\frac{10.24}{25}$
		2	
(b)	$\frac{13 - '12.5'}{\sqrt{0.4096}} [= 0.781]$	M1	For standardising with <i>their</i> values. Accept standardising with 12.
	$\Phi('0.781') - (1 - \Phi('0.781'))$	M1	For attempting to find <i>their</i> central area.
	0.565 (3sf)	A1	
		3	

Question 82

(a)	$0.25 \pm z \sqrt{\frac{0.25 \times 0.75}{140}}$	M1	Expression of correct form (allow M1 for just one side stated). Must be a z -value.
	$z = 2.054$ or 2.055	B1	
	0.175 to 0.325 (3sf)	A1	Must be an interval.
		3	
(b)	$0.90 \times 0.95 \times 0.01$ $+ 0.90 \times 0.05 \times 0.99$ $+ 0.10 \times 0.95 \times 0.99$	M1 M1	M1 for one correct triple product. M1 for all correct and added.
	0.147	A1	SC If zero scored award B1 for a 2 or 3 term expression of the form $0.90 \times 0.95 [\times c]$ OE. ($0 < c \leq 1$)
		3	

Question 83

(a)	E.g. Bias towards students who play instruments or only music students or e.g. the six will possibly be friends/have similar music preferences	B1	OE Or any reason that some are excluded e.g. because it is lunchtime or because the music building is chosen or any suggestion that opinions may not be independent. Note: 'not representative of all students' needs qualifying
		1	
(b)	28, 119, 207	B1	B1 for 28, 119 (condone 028).
		B1	B1 for 207 and only 3 values stated.
		2	

Question 84

(a)	$\frac{20.5}{40} = 0.5125$	B1	Accept 0.513 or $\frac{41}{80}$. Condone $\frac{20.5}{40}$.
	$\frac{40}{39} \left(\frac{10.728}{40} - (0.5125)^2 \right)$ or $\frac{1}{39} \left(10.728 - \frac{20.50^2}{40} \right)$	M1	Biased variance (0.005544 or $\frac{887}{160\,000}$) scores M0 A0 .
	0.0056859 or 0.00569 (3 sf) or $\frac{887}{156\,000}$	A1	CAO
		3	
(b)	$[11 \times '0.5125' + 0.5] = 6.1375$ or $\frac{491}{80}$ or 6.14 (3sf)	B1 FT	FT <i>their</i> 0.5125
	$11^2 \times '0.0056859'$	M1	With nothing added. Using <i>their</i> variance in (a) (no sd/var confusion)
	0.688 (3sf)	A1	CAO
		3	

Question 85

est $\mu = 14$ accept $\frac{560}{40}$	B1	
est $\sigma^2 = \frac{40}{39} \left(\frac{7850}{40} - 14^2 \right)$ or $\frac{1}{39} \left(7850 - \frac{560^2}{40} \right)$	M1	
0.25641 or 0.256 (3sf)	A1	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ M0 A0 .
	3	

Question 86

$\text{est}(p) = 0.2$ accept $\frac{15}{75}$	B1	SOI
$2 \times z \times \sqrt{\frac{0.2 \times 0.8}{75}} = 0.162$	M1	Expression of the correct form. Condone missing 2x.
$z \left[= 0.081 \times \sqrt{\frac{75}{0.2 \times 0.8}} \right] = 1.754$	A1	Correct z. Condone 3sf accuracy.
$\Phi('1.754') = 0.96[03]$ '0.96' - (1 - '0.96')	M1	OE. Using <i>their</i> z to find alpha.
$\alpha = 92$	A1	Following correct working.
	5	

Question 87

$\text{est}(p) = 0.4$	B1	
$'0.4' + z \sqrt{\frac{'0.4 \times (1 - '0.4)'}{500}} [= 0.445]$	M1	OE Use of <i>their</i> 0.4 in a correct expression
$z \left[= 0.045 \div \sqrt{\frac{'0.4 \times (1 - '0.4)'}{500}} \right] = 2.054$	A1	Condone 2.053 and 2.05
$0.98 - (1 - 0.98)$	M1	
96% confidence	A1	CWO, must be integer
	5	

Question 88

(a)	$\text{Est}(\mu) = \frac{1199}{6}$ or 199.833 or 200 or $\frac{2398}{12}$ [mm]	B1	Accept in any form
	$\text{Est}(\sigma^2) = \frac{12}{11} \left(\frac{479226}{12} - \frac{'1199^2}{6} \right)$ or $\frac{1}{11} \left('479226' - \frac{'2398^2}{6} \right)$	M1	Use of their values in correct formula (may be implied)
	$= 2.33$ (3 sf) [mm ²]	A1	Accept $\frac{7}{3}$
		3	
(b)	Small sample	B1	Accept not 'not representative' unless qualified.
		1	

Question 89

(a)	$\frac{13+a}{5}$	B1	Accept $\frac{2+3+3+5+a}{5}$. Do not ignore subsequent working
		1	
(b)	$\frac{5}{4}\left(\frac{47+a^2}{5} - \left(\frac{13+a}{5}\right)^2\right) = 4$ or $\frac{1}{4}\left(47+a^2 - \frac{(13+a)^2}{5}\right) = 4$	M1	Use of correct formula using <i>their</i> value from (a), in terms of a , and equate to 4
	$2a^2 - 13a - 7 = 0$	A1	Any correct three-term quadratic equation rearranged to a form ready to solve
	$a = 7$	A1	Condone the other value of a ($-\frac{1}{2}$)
		3	

Question 90

	$\frac{1300 + \frac{1}{200} - 1250}{\frac{480}{10}}$ or $\frac{1300 - 1250}{\frac{480}{10}}$ [= 1.042]	M1	Allow with incorrect or omitted continuity correction Must have 10 Accept totals method
	$1 - \Phi(1.042)$	M1	For area consistent with <i>their</i> values
	0.149 (3 s.f.)	A1	
		3	

Question 91

(a)	$72.3 \pm z\sqrt{\frac{64.3}{50}}$	M1	Expression of correct form (allow only one side for M1). Must be a z value
	$z = 1.751$	B1	Accept 1.75 if nothing better seen
	CI is 70.3 to 74.3 metres (3 s.f.)	A1	Allow without units Must be an interval
		3	
(b)	Not random sample	B1	Need 'random' or 'not representative/biased because...' OE
		1	

Question 92

(a)	$\text{Est}(\mu) = \frac{2520}{200} [= 12.6]$	B1	OE
	$\text{Est}(\sigma^2) = \frac{200}{199} \left(\frac{31582}{200} - '12.6'^2 \right)$ or $\frac{1}{199} \left(31852 - \frac{2520^2}{200} \right)$	M1	Allow M1 if $\frac{200}{199}$ omitted
	$= 0.5025$ or 0.503 or $\frac{100}{199}$	A1	CWO or $\sigma = 0.7088$ or 0.709
	$z = 1.96$	B1	
	$'12.6' \pm z \times \sqrt{0.5025 \div 200}$	M1	For expression of correct form Any z but must be z
	CI = 12.5 to 12.7 (3 sf)	A1	CWO Must be an interval Note: Use of biased can score maximum B1 M1 A0 B1 M1 A0
		6	
(b)	$0.95 \times 40 [= 38]$	B1	Give at early stage
		1	

Question 93

	$\frac{62.1}{10} = 6.21$	B1	OE
	$[\Sigma x^2 = 387.05]$	M1	Can be implied. Accept alternative methods (e.g. working mean of 6). Biased 0.1409 M0.
	$\frac{10}{9} \left(\frac{\text{their '387.05'}}{10} - (\text{their '6.21'})^2 \right)$ or $\frac{1}{9} \left(\frac{\text{their '387.05'}}{10} - \frac{(\text{their '6.21'})^2}{10} \right)$		
	$= 0.157$ (3 sf) or $\frac{1409}{9000}$	A1	
		3	

Question 94

(a)	$45 \pm z \times \frac{6}{\sqrt{200}}$	M1	For expression of correct form, any z . Accept one side of interval for M1.
	$z = 1.96$	B1	Must be seen.
	44.2 to 45.8 (3 sf)	A1	Must be an interval.
		3	
(b)	$z \times \frac{11}{\sqrt{200}} = 2$	M1	Or ... = 4 for M1
	$z = 2.571$	A1	Accept 3sf if nothing better seen.
	$\phi(\text{their '2.571'}) = 0.9949$ and $\text{their '0.9949'} - (1 - \text{their '0.9949'}) [= 0.9898]$	M1	OE For area consistent with their values. Must be seen.
	$\alpha = 99.0$ (3 sf)	A1	Allow 99. cwo Final answer of 0.99 scores A0.
		4	

Question 95

(a)	Est $\mu = 25.6$ or $\frac{2048}{80}$ or $\frac{128}{5}$	B1	
	Est $\sigma^2 = \frac{80}{79} \left(\frac{52760}{80} - \left(\frac{2048}{80} \right)^2 \right)$ or $\frac{1}{79} \left(52760 - \frac{2048^2}{80} \right)$	M1	Substitution into a correct formula. Biased 4.14 scores M0.
	= 4.19 (3 sf) or $\frac{1656}{395}$	A1	
		3	
(b)	'25.6' + $z \sqrt{\frac{4.19}{80}} = 26.0$	M1	Use of correct equation with their values.
	$z = 1.748$ or 1.747	A1	Accept 3sf. FT Biased $z = 1.758$.
	$(\Phi(1.748) = 0.960)$ '0.960' - $(1 - '0.960')$	M1	Correct area using their values.
	$\alpha = 92.0$ or 91.9	A1	Allow 92. FT Biased 92.1. A final answer of 0.92 or 0.919 scores A0.
		4	

Question 96

(a)	For X , $\mu = 2$ $\sigma^2 = 1.6$		
	Mean = 2	B1	
	Variance = $\frac{1.6}{160}$ or $\frac{1}{100}$ or 0.01	B1	Accept Var = 0.1 ² (accept sd=0.1 if clearly identified).
	Normal	B1	
		3	
(b)	$\pm \frac{1.8 - \frac{1}{320} - '2'}{\sqrt{0.01}}$ or $\pm \frac{1.8 - '2'}{\sqrt{0.01}}$ [= -2.03 or -2] or $\pm (287.5 - '320') / \sqrt{256}$ or $\pm (288 - '320') / \sqrt{256}$ [= -2.03 or -2]	M1	Allow with wrong continuity correction. M1 can be implied by correct final answer or for -2.03 / -2.0 or 0.9788 / 0.9772 seen.
	$\Phi(-2.03) = 1 - \Phi(2.03)$	M1	Correct area consistent with their values. M1 can be implied by correct final answer.
	= 0.0212 or 0.0228 (3 sf)	A1	
		3	

Question 97

(a)	$45 \pm z \times \frac{6}{\sqrt{200}}$	M1	For expression of correct form, any z . Accept one side of interval for M1.
	$z = 1.96$	B1	Must be seen.
	44.2 to 45.8 (3 sf)	A1	Must be an interval.
		3	
(b)	$z \times \frac{11}{\sqrt{200}} = 2$	M1	Or ... = 4 for M1
	$z = 2.571$	A1	Accept 3sf if nothing better seen.
	$\Phi(\text{their '2.571'}) = 0.9949$ and $\text{their '0.9949'} - (1 - \text{their '0.9949'}) [= 0.9898]$	M1	OE For area consistent with their values. Must be seen.
	$\alpha = 99.0$ (3 sf)	A1	Allow 99. cwo Final answer of 0.99 scores A0.
		4	

Question 98

	$\frac{62.1}{10} = 6.21$	B1	OE
	$[\Sigma x^2 = 387.05]$	M1	Can be implied. Accept alternative methods (e.g. working mean of 6). Biased 0.1409 M0.
	$\frac{10}{9} \left(\frac{\text{their '387.05'}}{10} - (\text{their '6.21'})^2 \right)$ or $\frac{1}{9} \left(\frac{\text{their '387.05'}}{10} - \frac{(\text{their '6.21'})^2}{10} \right)$		
	$= 0.157$ (3 sf) or $\frac{1409}{9000}$	A1	
		3	

Question 99

(a)	$\left[\frac{49}{140} = 0.35 \right]$		
	$0.35 \pm z \sqrt{\frac{0.35(1-0.35)}{140}}$	M1	Use of formula of correct form, fit $\text{their } \frac{49}{140}$, any z (not a probability).
	$z = 2.326$	B1	Accept 2.326 to 2.329.
	Confidence interval = 0.256 to 0.444 (3 sf)	A1	Must be an interval.
		3	
(b)	Find a smaller percentage confidence interval/ lower level of confidence	B1	ISW if 2 reasons given. Just saying 'use smaller z ' oe B0. Accept a correct example e.g. 90% (even if not qualified with statement).
		1	

Question 100

$\bar{x} = 1700/50 = 34$	B1	
$\text{Est}(\sigma^2) = \frac{50}{49} \left(\frac{59050}{50} - 34^2 \right)$ or $\frac{1}{49} \left(59050 - \frac{1700^2}{50} \right)$	M1	$\text{Est}(\sigma^2) = \frac{59050}{50} - 34^2$ biased scores M0.
$= 25.5$ (3 sf) or $\frac{1250}{49}$	A1	= 25 scores A0.
	3	

Question 101

(a)	$z = 1.645$	B1	
	$z \times \frac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$	M1	OE. Equation of correct form. Accept $p = x/100$. Any z . Allow missing factor of 2.
	$[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$ $x^2 - 100x + 2304 = 0$	A1	Any correct (likely scalar multiple) three-term quadratic equation in x or p with simplified coefficients. Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$.
	$x = 36$ or 64	A1	
		4	
(b)	$0.1^2 = 0.01$	B1	Accept either.
		1	

Question 102

(a)	180, 227	B1	One correct. Ignore incorrect numbers.
		B1	Both correct and no extra numbers seen. (Allow other correct use of list of digits).
		2	
(b)	These numbers are not independent of the previous numbers OR Only a finite number of digits used	B1	Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason).
		1	

Question 103

$\text{Est}(\mu) = 3.25 = 13/4$ or $1625/500$	B1	
$\text{Est}(\sigma^2) = \frac{500}{499} \left(\frac{5663.5}{500} - 3.25^2 \right)$ or $\frac{1}{499} \left(5663.5 - \frac{1625^2}{500} \right)$	M1	Expression of correct form.
$= 0.766$ (3 sf) or $1529/1996$	A1	Biased variance of 0.7645 scores M0A0.
	3	

Question 104

$0.23 \pm z \times \sqrt{\frac{0.23 \times (1 - 0.23)}{200}}$	M1	Expression of correct form. Any z , but $z = 0.8328$ scores B0M0.
$z = 1.811$ or 1.812	B1	
0.176 to 0.284 (3 sf)	A1	Must be an interval.
	3	

Question 105

$$\frac{5}{4} \left(\frac{1+2^2+6^2+1+a^2}{5} - \left(\frac{1+2+6+1+a}{5} \right)^2 \right) = \frac{11}{2}$$

or $\frac{1}{4} \left((42+a^2) - \frac{(10+a)^2}{5} \right) = \frac{11}{2}$

$$4a^2 - 20a + 0 = 0 \text{ or } a^2 - 5a + 0 = 0$$

$$a = 5$$

M1* OE attempted or e.g., $\frac{42+a^2}{5} - \left(\frac{10+a}{5}\right)^2 = \frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.

DM1 Two- or three-term quadratic equation in a , with at least two terms correct.

A1 Ignore $a = 0$, if seen.

3

Question 106

(a) $z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$

$$z = 1.881 \text{ or } 1.882$$

$$\left[n = \left(\frac{1.881}{0.7038} \right)^2 \times 11.2 \right]$$

$$n = 80$$

M1 Any z , but must be a z .

B1

A1 Must be a whole number.

3

(b) Jan, Feb and March not typical of whole year.

B1 Or, e.g., weather is different at different times of year.

1

(c) $0.94^3 \times 0.06 \times 4$

$$= 0.199 \text{ (3 sf)}$$

M1

A1

2

Question 107

$$\frac{134}{300} + z \sqrt{\frac{\frac{134}{300} \times \frac{166}{300}}{\frac{300}{300}}} = 0.487$$

$$z = 1.405$$

$$\Phi^{-1}(1.405) = 0.9199 \text{ or } 0.92; 1 - 2(1 - 0.92)$$

$$\alpha = 84$$

M1 For expression of the correct form.

A1 Accept 1.404, or anything that rounds to 1.39 to 1.41.

M1 Attempt area above or below their 1.405 and convert to a confidence level.

A1 Allow $\alpha = 84\%$.
cwo
Note: final answer 0.84 scores A0.

4

Question 108

(a) $\frac{4590}{85} \pm z \times \frac{8.3}{\sqrt{85}}$

$$z = 1.96$$

$$52.2 \text{ to } 55.8 \text{ (3 sf)}$$

M1 For expression of correct form. Any z (but not $\Phi(z)$).

B1

A1 Must be an interval.

3

(b) $1.96 \times \frac{s}{\sqrt{100}} = 1.4$ or $2 \times 1.96 \times \frac{s}{\sqrt{100}} = 2.8$

$$s = 7.14 \text{ (3 sf)} \text{ or } \frac{50}{7}$$

M1 Equation of correct form (any z).
Allow factor of 2 error (i.e. first equation = 2.8).

A1

2

Question 109

(a)	$\frac{78}{250} \pm z \times \sqrt{\frac{\frac{78}{250} \times (1 - \frac{78}{250})}{250}}$	M1	Use of a correct formula (any z).
	$z = 2.326$	B1	
	$= 0.244 \text{ to } 0.38[0] \text{ (3 sf)}$	A1	Must be an interval.
		3	
(b)	Unlikely to be true because confidence interval does not contain 0.4.	B1 ft	FT their confidence interval. Must include this reason and 'unlikely', oe. Allow "not true because 0.4 is not in the confidence interval." But "Confidence interval only goes up to 0.38 so not true" and "'it' lies outside the confidence interval" both score B0.
		1	

Question 110

(a)	$\text{est}(\mu) = 0.368 = \frac{46}{125}$	B1	Oe.
	$\text{est}(\sigma^2) = \frac{100}{99} \left(\frac{17.34}{100} - \text{their } 0.368^2 \right)$ or $\frac{1}{99} \left(17.34 - \frac{36.8^2}{100} \right)$	M1	For use of a correct formula (ft <i>their</i> μ).
	$= 0.0384 \text{ (3 sf)}$	A1	
		3	
(b)	Must be a random sample	B1	E.g. <ul style="list-style-type: none"> • Values must have been randomly selected. • Sample should be representative of the population. • All values should have equal chance of being selected. • It should be an unbiased sample. • Independent sample/insect lengths are independent of one another. ISW
		1	

Question 111

(a)	$\hat{\mu} = \frac{307}{200}$ or 1.535	B1	Accept 3sf if nothing better seen (1.53 or 1.54).
	$\Sigma x^2 f = 627, \text{ Est}(\sigma^2) = \frac{200}{199} \left(\frac{627}{200} - 1.535^2 \right)$ or $\frac{1}{199} \left(627 - \frac{307^2}{200} \right)$	M1	Use of a correct formula with <i>their</i> values.
	$= 0.783$	A1	AG Correctly obtained with no errors seen.
		3	
(b)	$H_0: \mu = 1.65$ $H_1: \mu < 1.65$	B1	Accept 'population mean' but not just 'mean'.

(c)	$\frac{1.535-1.65}{\sqrt{0.783+200}}$	M1	Standardising with <i>their</i> mean.
	= -1.838 or -1.84	A1*	
	$\Phi(0.05)$ and $\Phi(0.01)$ attempted	M1	Or $P(z < -1.838')$ attempted. SC: Condone $\Phi(0.025) = 2.807$ and $\Phi(0.005) = 3.291$ following two-tailed test in (b).
	-1.645 > -1.838 > -2.326 [Hence significant at 5% but not 1% level]	DA1	AG = 0.033 and $0.05 > 0.033 > 0.01$ SC: use of 1.54 or 1.53 for the mean leading to $-1.645 > -1.758 > -2.326$ or $-1.645 > -1.918 > -2.326$ or $0.95 < 0.9606$ or $0.9724 < 0.99$ scores M1 M1 A1 . Accept use of critical value method 1.535 < 1.547 or accept $1.65 > 1.638$.
		4	
(d)	At the 1% level H_0 is not rejected Or a Type I error can only occur if H_0 is rejected.	B1	OE
		1	

Question 112

(a)	$3.12 + z \times \frac{\sigma}{\sqrt{150}} = 3.23$	M1	OE – correct expression. Any z , but must be a z value.
	$z = 1.96$	B1	
	$\sigma = 0.687$ (3sf) [cm]	A1	
		3	
(b)	Yes, because population [of widths] not given to be normally distributed	B1	Or ‘underlying distribution’ instead of population. Allow ‘yes, because population distribution not known’. Need both statements.
		1	

Question 113

$\frac{18}{50} - z \times \sqrt{\frac{\frac{18}{50} \times (1 - \frac{18}{50})}{50}} = 0.244$	M1	Use of correct equation.
$z = 1.709$ or 1.708	A1	Accept 1.71 if nothing better seen.
$\Phi^{-1}(1.709) = 0.956$; $1 - 2(1 - '0.956')$ [= 0.912]	M1	Attempt area above or below their 1.709 and use correct method to find α .
$\alpha = 91$	A1	Allow $\alpha = 91\%$ 0.91 or 91.2 score A0.
	4	

Question 114

(a)	[567, 109], 665, 21	B2	B1 for each. Allow 021. If more than 2 answers given, count first two and ISW.
		2	
(b)	$\text{Est}(\mu) = \frac{610}{30}$ or $\frac{61}{3}$	B1	OE or 20.3.
	$\text{Est}(\sigma^2) = \frac{30}{29} \left(\frac{12405}{30} - \left(\frac{610}{30} \right)^2 \right)$ or $\frac{1}{29} \left(12405 - \frac{610^2}{30} \right)$	M1	Use of correct formula.
	= 0.0575 (3sf)	A1	Accept $\frac{5}{87}$.
		3	
(c)	Variance is [unrealistically] small so Henri has [probably] made a mistake/claim is [probably] correct	B1 FT	Need both parts. Need 'small' OE, not just < 0.1. FT <i>their</i> < 0.1 variance value (not -ve), e.g. 0.0556 (if omit $\frac{30}{29}$). Accept 's.d. = 0.24 is small, so Henri has probably made a mistake'. Note: 'mean is large/small' scores B0, but 'mean large compared to variance so Henri prob made a mistake' scores B1.
		1	

Question 115

(a)	$\text{Est}(\mu) = \frac{2044}{8}$ [=255.5]	B1	Accept 3sf if nothing better seen.
	$\text{Est}(\sigma^2) = \frac{8}{7} \left(\frac{522348}{8} - "255.5"{}^2 \right)$ or $\frac{1}{7} \left("522348" - \frac{2044^2}{8} \right)$	M1	Attempt to find Σx^2 and substitute in correct formula. May be implied by correct answer. Biased 13.25 scores M0.
	= 15.1 (3 sf) or $\frac{106}{7}$	A1	OE
		3	
b)(i)	$H_0: \mu = 253$ $H_1: \mu > 253$	B1	Allow 'Population mean' but not just 'mean'.
	$\frac{25360 - 253}{100}$ $3.5 \div \sqrt{100}$	M1	Standardising must have $\sqrt{100}$.
	= 1.714	A1	
	$1.714 > 1.645$ or $0.0432 < 0.05$	M1	OE
	[Reject H_0] There is sufficient evidence (at 5% level) to suggest [mean] mass is greater than 253	A1FT	OE FT <i>their</i> '1.714' in context, not definite, no contradictions. Accept critical value method of $253.57 < 253.60$ or $253.02 > 253$. Use of a two-tailed test scores B0 M1 A1 M1 A0 (comp with $\frac{0.025}{1.96}$).
		5	
(b)(ii)	Not true. Large sample, [so sample mean is approx normally distributed].	B1	OE Allow 'Not true. Large sample' or 'Not true. n is large' or 'Not true. CLT used'.
		1	

Question 116

(a)	$81.5 \pm z \times \frac{5.8}{\sqrt{20}}$	M1	For a correct expression (accept if only one side of the interval calculated). Any z (must be a z).
	$z = 2.326$	B1	
	78.5 to 84.5 (3sf)	A1	Must be an interval.
		3	
(b)	Not true. C. I. is for mean time, not individual times.	B1	OE Both comments needed.
		1	
(c)	$\frac{0.98^2}{1-0.02^2}$	M1	Attempt $\frac{P(\text{both contain } \mu)}{P(\text{at least one contains } \mu)}$ with numerator attempt 0.98^2 and denominator attempt involving 0.02 . Must see their quotient.
	$= 0.961$ (3sf)	A1	NB: $[0.98^2 =] 0.9604$ scores M0 A0.
		2	

Question 117

(a)	$\bar{r} = \frac{230}{75} [= 3.0666\dots \text{ or } 3.07 \text{ (3 sf)}] [\text{ Or } 46/15]$	B1	
	$s^2 = \frac{75}{74} \left(\frac{930}{75} - \left(\frac{230}{75} \right)^2 \right)$ or $1/74(930 - 230^2/75)$	M1	Use of correct formula.
	$= 3.0360\dots \text{ or } 3.04 \text{ (3 sf) or } = 337/111$	A1	
		3	
(b)	$[\Phi^{-1}(1 - 0.234)] = 0.726$	B1	
	$\pm \frac{a - 3.0667'}{\sqrt{3.04/75}} = \pm '0.726'$	M1	Ft their 0.726 but must be a z value. Note using 0.766 is M0. Must have sqrt 75.
	$a = 3.21$ (3 sf)	A1	CWO
		3	

Question 118

(a)	$z = 2.054$ or 2.055	B1	Accept 3 sf if nothing better seen (2.05 or 2.06).
	$1.42 \pm z \frac{0.35}{\sqrt{150}}$	M1	Must be a z value.
	1.36 to 1.48 [m] (3 sf)	A1	Correct working only. Must be an interval.
		3	
(b)	No. CI is about mean, not individual values.	B1	Or similar. Need both.
		1	

Question 119

(a)	$\frac{36-35}{8.1+\sqrt{140}} [= 1.461]$	M1	Ignore inclusion of cc for M1 . Must have $\sqrt{140}$.
	$1 - \Phi(1.461)$	M1	For area consistent with their values.
	$= 0.0720$ (3 sf)	A1	Allow 0.072.
		3	
(b)	$[\Phi^{-1}(0.986)] = 2.197$ to 2.198	B1	Seen. Note: 2.2 and nothing better seen scores B0
	$\pm \frac{a-35}{8.1+\sqrt{140}} = \pm '2.198'$	M1	Must be a z value.
	$a = 36.5$ (3 sf)	A1	CWO Note: use of 2.2 scores A1 so 2/3. But e.g. 2.196 gives 36.5 but scores B0 M1 A0 so 1/3
		3	

Question 120

(a)	254 [m]	B1	
		1	
(b)	$263 = '254' + 1.96 \times \frac{\sigma}{\sqrt{50}}$ oe or $2 \times 1.96 \times \frac{\sigma}{\sqrt{50}} = 18$	M1	ft their '254' accept 1.96 or 1.645 for M1 .
	$[\sigma = \frac{9\sqrt{50}}{1.96} =]$. s.d. = 32.5 [m] (3 sf)	A1	
		2	
(c)	No Because the sample mean is approximately normally distributed [for large n]	B1	Both needed. Or because of the Central Limit theorem. Or because n is large [accept ≥ 30 condone ≥ 50].
		1	

Question 121

(a)	$\bar{t} = \frac{230}{75} [= 3.0666\dots \text{ or } 3.07 \text{ (3 sf)}] [\text{ Or } 46/15]$	B1	
	$s^2 = \frac{75}{74} \left(\frac{930}{75} - \left(\frac{230}{75} \right)^2 \right)$ or $1/74(930 - 230^2/75)$	M1	Use of correct formula.
	$= 3.0360\dots \text{ or } 3.04 \text{ (3 sf) or } = 337/111$	A1	
		3	
(b)	$[\Phi^{-1}(1 - 0.234)] = 0.726$	B1	
	$\pm \frac{a - 3.0667}{\sqrt{3.04/75}} = \pm '0.726'$	M1	Ft their 0.726 but must be a z value. Note using 0.766 is M0. Must have sqrt 75.
	$a = 3.21$ (3 sf)	A1	CWO
		3	

Question 122

(a)	e.g. No. The views of students in sports may be different from other students.	B1	No and any sensible reason for disagree.
			Allow just No, because biased (or not random) or not representative.
		1	
(b)	$\frac{45}{60} \pm z \sqrt{\frac{\frac{45}{60} \times \frac{15}{60}}{60}}$	M1	Any z.
	$z = 1.96$	B1	
	0.640 to 0.860 (3 sf) or 0.64 to 0.86	A1	Must be an interval. Mark at the most accurate.
		3	
(c)	$\Phi^{-1}(0.995) [= 2.574 \text{ to } 2.579]$	M1	Allow $\Phi^{-1}(0.99)$.
	$\Phi('2.576' \div 2) [= \Phi('1.288') = 0.901 \text{ to } 0.9015]$	M1	FT their 2.576.
	'0.9012' - (1 - '0.9012')	M1	OE.
	$[= 0.802 \text{ to } 0.803] \Rightarrow x = 80.2 \text{ to } 80.3 \text{ or } x = 80$	A1	Allow $x = 80\%$.
		4	

Question 123

Assume the men on the bridge are a random sample	B1	OE. Allow: assume that all the men on the bridge are independent of one another.
$\frac{1500 - 70.3n}{5.9\sqrt{n}} = 2.326$	M1	Must have square root n. Allow wrong z but must be a z value.
$70.3n + 13.7234\sqrt{n} - 1500 = 0$ Or $4942.09 n^2 - 211088.33 n + 2250000 = 0$	A1	Correct quadratic equation or inequality in square root n. OE.
$[\sqrt{n} = 4.5226 \text{ (3 sf)}]$ Max value of n is 20	A1	CWO.
	4	

Question 124

(a)(i)	$\text{Est}(\mu) = \frac{46350}{150} \text{ or } 309$	B1	
	$\text{Est}(\sigma^2) = \frac{150}{149} \left[\frac{14410800}{150} - \left(\frac{46350}{150} \right)^2 \right] \left[= \frac{150}{149} \times 591 \right]$	M1	Use of correct formula. $\frac{1}{149} \left(14410800 - \frac{46350^2}{150} \right)$. OE.
	$= 595 \text{ (3 sf) or } 88650/149$	A1	
		3	
(a)(ii)	$z = 1.96$	B1	Seen.
	$309 \pm z \times \sqrt{\frac{595}{150}}$	M1	Any z. Use of correct formula.
	$= 305 \text{ to } 313 \text{ (3 sf)}$	A1	Use of biased (591) gives same interval but scores B1M1A0 . Unsupported correct answer scores max B1B1 .
		3	

(b)	Either Smaller sample means more uncertainty about the value of μ .		OE.
	or $n < 120$ means that $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{150}}$	B1*	OE.
	Therefore Wider	B1dep	Dep on previous B1 .
		2	

Question 125

$$2 \times 1.645 \times \sqrt{\frac{a(1-a)}{200}} = 0.1066$$

$$z = 1.645$$

$$a^2 - a + 0.20997 = 0$$

$$a = 0.3(00) \text{ 3sf or } 0.7(00) \text{ 3sf}$$

M1 Allow any z and/or omit ' $2 \times$ '.
 Condone confusion between a and $\frac{a}{200}$.
 E.g. $2 \times 1.645 \times \sqrt{\frac{\frac{a}{200} \left(1 - \frac{a}{200}\right)}{200}} = 0.1066$

B1

M1 Or for valid attempt to reach quadratic in a by confusing between a and $\left(\frac{a}{200}\right)$.
 E.g. $a^2 - 200a + 8398.7 = 0$

A1

Note: Do not ISW $a = 0.3 \times 200$ or 0.7×200 score **A0**.

4

Question 126

$$15.6 = \frac{100}{99} \left(\frac{\Sigma h^2}{100} - 80.2^2 \right) \text{ OR } 15.6 = 1/99 (\Sigma h^2 - 8020^2/100)$$

$$\Sigma h^2 = 644748.4 \text{ or } 645000 \text{ (3 sf) or } 3223742/5$$

M1 For attempt biased or unbiased = 15.6.

A1

For correct expression = 15.6.

A1

3

Question 127

$$\text{Est}(\mu) = \frac{2750}{60} \text{ or } \frac{275}{6} \text{ or } 45.8 \text{ (3 sf)}$$

$$\text{Est}(\sigma^2) = \frac{60}{59} \left(\frac{127000}{60} - \left(\frac{275}{6} \right)^2 \right)$$

$$= 16.2 \text{ (3 sf) or } \frac{2875}{177}$$

B1

M1 Or $\frac{1}{59} \left(127000 - \frac{2750^2}{60} \right)$

Note: σ (4.03) can score **M1** for correct expression
 Use of biased (15.97) **M0**.

A1

3

Question 128

(a)	N(6, ...)	B1	Normal used with a mean of 6.
	$\sigma^2 = \frac{3}{2} = 1.5$ (or $\sigma = \sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$ or 1.22 (3sf))	M1	Var of 1.5 (or sd found).
	$\frac{6.2 - 6}{\sqrt{\frac{1.5}{100}}}$	M1	For standardising. Must have $\sqrt{100}$. Ignore incorrect continuity correction (correct cc if used is $1/2n$).
	= 1.633	A1	Or accept 1.674 with correct cc.
	$P(\bar{X} > 6.2) = 1 - \Phi(1.633)$	M1	
	= 0.0512 or 0.0513 (3sf)	A1	Or accept 0.0471 with correct cc (scores full marks).
		6	
(b)	Population (X) is not normally distributed / Population (X) is Binomial	B1	OE. Accept underlying distribution.
		1	

