AS-Level

Topic : Binomial & Geometric Distribution

May :2013- May : 2023

Answers

Question 1

(i)	$(0.8)^n < 0.001$	M1		Eqn or inequ involving 0.8^n or 0.2^n and 0.001 or 0.999
	$n > 30.9$	M1		Trial and error or logs (can be implied)
	$n = 31$	A1	**[3]**	Correct answer **MR** 0.01, max available M1M1A0
(ii)	$\mu = 120 \times 0.2 = 24$	B1		24 and 19.2 or $\sqrt{19.2}$ seen
	$\sigma^2 = 120 \times 0.2 \times 0.8 = 19.2$	M1		Standardising with or without cc, must have sq rt in denom
	$P(x < 33) = P \times \left(z < \frac{32.5 - 24}{\sqrt{19.2}} \right)$	M1		Continuity correction 32.5 or 33.5
	$= P(z < 1.9398)$ $= 0.974$	A1	**[4]**	Correct answer

Question 2

(i)	$p = 4/9$ or $5/9$	B1		Binomial term ${}_5C_x p^x (1-p)^{5-x}$ seen
	P(at least 2) = 1 – P(0, 1) $= 1 - (5/9)^5 - (4/9)(5/9)^4\ {}_5C_1$	M1		
	$= 0.735$	A1	**[3]**	Correct answer
(ii)	$np = 96$ $npq = 32$ $p = P(\leq k)$	M1		Using $np = 96$ $npq = 32$ to obtain eqn in 1 variable
	$p = 2/3$ $q = 1/3$ $n = 144$	A1		1/3 or 2/3 seen or implied
	$k = 6$	A1ft		Correct k ft $k = 9p$
	$n = 144$	A1	**[4]**	correct n

Question 3

(i)	$X \sim \text{Bin}(12, 0.2)$	B1 B1 B1	**[3]**	Bin or B 12 0.2 or 1/5
(ii)	$P(X = 3, 4, 5) = 0.2^3 0.8^9\ {}_{12}C_3 + 0.2^4 0.8^8\ {}_{12}C_4 + 0.2^5 0.8^7\ {}_{12}C_5$	M1		Bin exprerssion with any p
	$= 0.23622 + 0.13287 + 0.05315$	A1ft		Correct unsimplified expression, their p
	$= 0.422$	A1	**[3]**	Correct answer
(iii)	$P(X = 0) < 0.01$	M1	**[3]**	Statement involving P(X = 0) and 0.01 can be implied
	$0.8^n < 0.01$	M1		Equn involving ‘0.8’, 0.01 or 0.99
	$n = 21$	A1		Correct answer

Question 4

(i)	$(p=)0.85$ $P(<12) = 1 - P(12, 13, 14)$ $= 1 - [(0.85)^{12}(0.15)^2{}_{14}C_{12} + (0.85)^{13}(0.15){}_{14}C_{13} + (0.85)^{14}]$ $= 1 - 0.6479$ $= 0.352$	B1 M1 A1	 **3**	$(p=)0.85$ oe seen anywhere Summing 2 or 3 consistent bin probs, any $p<1$, $n=14$ (or summing 12 or 13 consistent bin probs) Correct answer
(ii)	$(0.85)^n \geqslant 0.1$ $n \leqslant 14.2$ $n = 14$	M1 M1 A1	 **3**	Eqn or inequality in 0.85(or 0.15), n, 0.1, n as a power Attempt to solve (can be implied) if n a power Correct answer – must be equals, not approx. MR allowed for 0.01, M1M1A0 max.

Question 5

$X \sim B(19, 0.12)$ $P(X<4) = P(0, 1, 2, 3)$ $= (0.88)^{19} + {}^{19}C_1(0.12)^1(0.88)^{18} + {}^{19}C_2(0.12)^2(0.88)^{17} + {}^{19}C_3(0.12)^3(0.88)^{16}$ $= 0.813$	M1 M1 M1 A1	 **4**	Any binomial term ${}^{19}C_x\, p^x(1-p)^{19-x}$, $0<p<1$ Any binomial term ${}^nCx(0.12 \text{ or } 0.88)^x(0.88 \text{ or } 0.12)^{n-x}$ P(0, 1, 2, 3) binomial expr with at least 2 consistent terms Correct answer

Question 6

(i)	constant / given p, independent trials, fixed / given no. of trials, only two outcomes	B1 B1	 **2**	Any one correct Any 3 correct
(ii)	$P(x \geqslant 3) = 1 - P(0, 1, 2)$ $= 1 - [(0.85)^{18} + (0.85)^{17}(0.15) \times 18 + (0.85)^{16}(0.15)^2 \times {}^{18}C_2]$ $= 0.520$	M1 M1 A1	 **3**	Any binomial expression $p^r(1-p)^{18-r}\,{}^{18}C_r$ seen $1 - P(0, 1, 2)$, any n,p,q Correct answer

Question 7

(i)	max = 12 $P(12) = (0.7)^{12} = 0.0138$	B1 B1	 **2**	(Implied by P(12) with power 12) Accept 0.014
(ii)	P(fewer than 10) = 1 – P (10, 11, 12) $= 1 - {}^{12}C_{10} \times (0.7)^{10}(0.3)^2 - 12 \times (0.7)^{11}(0.3) - (0.7)^{12}$ $= 1 - 0.2528$ $= 0.747$	M1 A1 A1	 **3**	Binomial term ${}^{12}C_r(0.7)^r(0.3)^{12-r}$ or ${}^{12}C_r(p)^r(q)^{12-r}$, $0.99 \leqslant p+q \leqslant 1.00$ Correct unsimplified expression oe Correct answer

Question 8

(i)	1 1 1 2 or 1 1 2 1 or 1 2 1 1 or 2 1 1 1	M1	One of 1 1 1 2 seen
	Prob = $\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times 4$	M1	Mult a prob by 4 or $(\frac{1}{6})^4 \times$ integer $k \geqslant 1$ seen
	$= \frac{1}{324}$ (0.00309)	A1 **3**	Correct answer
(ii)	$P(1,2) = {}^7C_1 \times (1/324)\,(323/324)^6 + {}^7C_2(1/324)^2(323/324)^5$	M1	Bin term ${}^7C_x p^x (q)^{7-x}$, $0.99 \leqslant p + q \leqslant 1$
		M1	Using their p from **(i)** in a bin term
		M1	Correct unsimplified answer
	= 0.0214	A1 **4**	Correct answer

Question 9

(i)	$1.2 = 15p \quad p = 0.08$	M1	Attempt to find p using $1.2 = 15p$
	Var = $npq = 15 \times 0.08 \times 0.92 = 1.104$ **AG**	A1 **2**	Correct answer
(ii)	$P(0, 1, 2) = (0.92)^{15} + {}^{15}C_1(0.08)(0.92)^{14} + {}^{15}C_2(0.08)^2(0.92)^{13}$	M1	Binomial expression ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$
		M1	Correct unsimplified expression for P(0, 1, 2)
	= 0.887	A1 **3**	Correct answer
(iii)	P(at least 1 faulty screw) = 1 – P(0) = 1 – $(0.92)^{15}$	M1	Attempt at P(0) or 1 – P(0)
	= 0.7137…	A1	Rounding to 0.71
	P(at least 1 faulty screw in 7 packets) = ${}^8C_7(0.713\ldots)^7(0.2863\ldots)$	M1	Binomial expression ${}^8C_7 p^7(1-p)$ $0 < p < 1$
	= 0.216	A1 **4**	Correct answer

Question 10

P(3, 4, 5) =	M1	Bin expression of form ${}^{10}C_x(p)^x(1-p)^{10-x}$ any x any p
${}^{10}C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7 + {}^{10}C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6 + {}^{10}C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^5$	A1	Correct unsimplified answer accept (0.17, 0.83), (0.16, 0.84), (0.16, 0.83), (0.17, 0.84) or more accurate
= 0.222	A1 **3**	Correct answer

Question 11

$p = 0.76$		
P(fewer than 10) = 1 – P(10, 11)	**M1**	Any binomial term ${}^{11}C_x p^x(1-p)^{11-x}$, $0 < p < 1$
$= 1 - (0.76)^{10}(0.24)\,{}^{11}C_{10} - (0.76)^{11}$	**M1**	Any binomial term ${}^nC_x(0.76)^x(0.24)^{n-x}$
= 1 – 0.219	**M1**	1 – P(10, 11) oe binomial expression
= 0.781	**A1** [4]	Correct answer

Question 12

(i)	$p = 0.66$ $X \sim B(15, 0.66)$ P(at least 14) = P(14, 15) = ${}^{15}C_{14}\,(0.66)^{14}(0.34) + (0.66)^{15}$ = 0.0171	**M1** **M1** **A1** [3]	Bin term ${}^{15}C_x p^x(1-p)^{15-x}$ seen any p Unsimplified correct expression for P(14,15)
(ii)	$(0.87)^n < 0.04$ $n = 24$	**M1** **M1** **A1** [3]	Eqn involving 0.87, power of n, 0.04 only Solving by logs or trial and error(can be implied). Must be exponential equation

Question 13

P (throwing a 4) = (1 – 0.4) / 4 = 0.15 P(at most 1) = P(0, 1) or 1 – P(2, 3) $= (0.85)^3 + {}^3C_1\,(0.15)\,(0.85)^2$ = 0.939	**M1** **A1** **M1** **M1** **A1** [5]	Sensible attempt to find P(1) Correct answer A binomial term with ${}^3C_n$ oe any p Binomial expression with ${}^3C_n$ P(0, 1) or 1 – P(2, 3) $p = 0.15$ or 0.85

Question 14

(i)	$p = 1/3$ $P(\geqslant 2) = 1 - P(0, 1) = 1 - (2/3)^4 - {}^4C_1(1/3)(2/3)^3$ or $P(2,3,4) = {}^4C_2(1/3)^2(2/3)^2 + {}^4C_3(1/3)^3(2/3) + (1/3)^4$ $= \frac{11}{27}$, 0.407	 **M1** **M1** **A1**	 [3]	 Bin term ${}^4C_x p^x(1-p)^{4-x}$ $0 < p < 1$ Correct unsimplified answer
(ii)	P(sum is 5) = P(1, 1, 1, 2) ×4 $= (1/3)^4 \times 4$ $= \frac{4}{81}$, 0.0494	**M1** **M1** **A1**	 [3]	1, 1, 1, 2 seen or 4 options Mult by $(1/3)^4$

Question 15

(i)	0.9 × 0.95 × 0.85 × 0.1= 0.0727	**B1**	[1]	
(ii)	P(0, 1, 2) $= (0.9)^{12} + {}^{12}C_1\,(0.1)(0.9)^{11} + {}^{12}C_2\,(0.1)^2(0.9)^{10}$ = 0.889	**M1** **M1** **A1**	 [3]	Bin term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$ $p < 1, x \neq 0$ Bin expression $p = 0.1$ or 0.9, $n = 12$, 2 or 3 terms
(iii)	$X \sim B(50, 0.85)$ Expectation = 50 × 0.85 (= 42.5) Var = 50 × 0.85 × 0.15 (= 6.375)	**M1** **A1**	 [2]	50 × 0.85 seen oe can be implied Correct unsimplified mean and var

Question 16

(i)	constant probability (of completing)	**B1**	Any one condition of these two
	independent trials/events	**B1**	The other condition
	Totals:	**2**	
(ii)	$P(5, 6, 7) = {}^7C_5(0.7)^5(0.3)^2 + {}^7C_6(0.7)^6(0.3)^1 + (0.7)^7$	**M1** **A1**	Bin term ${}^7C_x(0.7)^x(0.3)^{7-x}$, $x \neq 0, 7$ Correct unsimplified answer (sum) OE
	= 0.647	**A1**	
	Total:	**3**	
(iii)	P(0, 1, 2, 3, 4) = 1 – their '0.6471' = 0.3529	**M1**	Find P($\leqslant 4$) either by subtracting their (ii) from 1 or from adding Probs of 0,1,2,3,4 with n=7 (or 10) and p = 0.7
	$P(3) = {}^{10}C_3(0.3529)^3(0.6471)^7$	**M1**	${}^{10}C_3$ (their $0.353)^3(1 -$ their $0.353)^7$ on its own
	= 0.251	**A1**	

Question 17

(i)	$p = 0.07$	**B1**	
	$P(2) = {}^{20}C_2(0.07)^2(0.93)^{18}$	**M1**	Bin term ${}^{20}C_x p^x(1-p)^{20-x}$ their p
	= 0.252	**A1**	
	Total:	**3**	
(ii)	P(at least 1 cracked egg)=$1-(0.93)^{20}$=1–0.2342	**M1**	Attempt to find P(at least1 cracked egg) with their p from (i) allow 1 – P(0, 1) OE
	= 0.766	**A1**	Rounding to 0.766
	Total:	**2**	
(iii)	$(0.7658)^n<0.01$	**M1**	Eqn or inequal containing (their $0.766)^n$ or (their $0.234)^n$, together with 0.01 or 0.99
	n = 18	**A1**	
	Total:	**2**	

Question 18

(i)

x	–3	0	5	32
Prob	1/6	1/2	1/6	1/6

B1	At least 3 different correct values of X (can be unsimplified)
B1	Four correct probabilities in a Probability Distribution table
B1	Correct probs with correct values of X
3	

Question 19

3(i)	$p = 0.207$	B1	
		1	
3(ii)	Var = 30 × 0.207 × 0.793 = 4.92	B1	
		1	
(iii)	P(⩾ 2) = 1 – P(0, 1)	M1	
	$= 1 - (0.793)15 - \binom{15}{1}(0.207)(0.793)14$	M1	1 – P(0, 1) seen n =15 p = any prob
	= 0.848	A1	
		3	

Question 20

(i)	$P(4) + P(5) = {}^5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$	M1	One binomial term, with $p < 1$, n=5, $p + q$=1
	= 0.014648.. + 0.00097656..	M1	Add 2 correct unsimplified binomial terms
	= 0.0156 or $\frac{1}{64}$	A1	
		3	
(ii)	$1 - P(0) > 0.995$: $0.75^n < 0.005$	M1	Equation or inequality involving 0.75^n and 0.005 or 0.25^n and 0.995
	$n\log 0.75 < \log 0.005$ $n > 18.4$:	M1	Attempt to solve *their* exponential equation using logs, or trial and error May be implied by their answer
	$n = 19$	A1	
		3	
(iii)	$p = 0.25$, $n = 160$: mean = 160 x 0.25 (= 40) variance = 160 x 0.25 x 0.75 (=30)	B1	Correct unsimplified mean and variance
	$P(X < 50) = P\left(Z < \frac{49.5 - 40}{\sqrt{30}}\right)$	M1	Use standardisation formulae must include square root.
		M1	Use continuity correction ±0.5 (49.5 or 50.5)
	$= P(Z < 1.734) = 0.959$	A1	Correct final answer
		4	

Question 21

(i)	$z = 0.674$	B1	z value ± 0.674
	$0.674 = \frac{0 - -3}{\sigma}$	M1	$\pm$Standardising with 0 and equating to a z-value
	$\sigma = 4.45$	A1	Correct answer www ie not ignoring a minus sign
	Total:	**3**	
(ii)	P(0, 1)	M1	Any bin of form ${}^8C_x(0.75)^x(0.25)^{8-x}$any x
	$= (0.75)^8 + {}^8C_1(0.25)(0.75)^7$	M1	Correct unsimplified answer, may be implied by numerical values
	$0.1001 + 0.2670 \quad = 0.367$	A1	Correct answer
	Method 2 $1 - P(8,7,6,5,4,3,2) = 1 - (0.25)^8 - {}^8C_1(0.75)(0.25)^7 - \ldots$	M1	Any bin of form ${}^8C_x(0.75)^x(0.25)^{8-x}$any x
	$- {}^8C_2(0.75)^6(0.25)^2$	M1	Correct unsimplified answer
	$= 0.367$	A1	Correct answer
	Total:	**3**	

Question 22

(i)	**Method 1** $P(< 11) = 1 - P(11, 12, 13)$	M1	Binomial expression of form ${}^{13}C_x\ (p)^x(1-p)^{13-x}$, $0 < x < 13$, $0 < p < 1$
	$= 1 - {}^{13}C_{11}(0.6)^{11}(0.4)^2 - {}^{13}C_{12}(0.6)^{12}(0.4) - (0.6)^{13}$	M1	Correct unsimplified answer
	$= 0.942$	A1	CAO
	Method 2 $P(< 11) = P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$	M1	Binomial expression of form ${}^{13}C_x\ (p)^x(1-p)^{13-x}$ $0 < x < 13$, $0 < p < 1$
	$= (0.4)^{13} + {}^{13}C_1(0.4)^{12}(0.6) + \ldots + {}^{13}C_{10}(0.4)^3(0.6)^{10}$	M1	Correct unsimplified answer
	$= 0.942$	A1	CAO
		3	
(ii)	$\mu = 130 \times 0.35 = 45.5 \quad \text{var} = 130 \times 0.35 \times 0.65 = 29.575$	B1	Correct unsimplified mean and var (condone $\sigma^2 = 29.6$, $\sigma = 5.438$)
	$P(\geqslant 50) = P\left(z > \frac{49.5 - 45.5}{\sqrt{29.575}}\right) = P(z > 0.7355)$	M1	Standardising, using $\pm\left(\frac{x - \textit{their}\text{ mean}}{\textit{their}\ \sigma}\right)$, x = value to standardise 49.5 or 50.5 seen in $\pm$ standardisation equation
	$= 1 - \Phi(0.7355)$	M1	Correct final area
	$= 1 - 0.7691$	M1	
	$= 0.231$	A1	Correct final answer
		5	

Question 23

(i)	P(4, 5, 6) = $^{15}C_4(0.22)^4(0.78)^{11} + {}^{15}C_5(0.22)^5(0.78)^{10}$ +	M1	One binomial term $^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$
	$^{15}C_6(0.22)^6(0.78)^9$	A1	Correct unsimplified expression
	= 0.398	A1	Correct answer
		3	
(ii)	$\mu = 145 \times 0.22 = 31.9$ $\quad \sigma^2 = 145 \times 0.22 \times 0.78 = 24.882$	B1	Correct unsimplified mean and variance
	$P(x > 26) = P\left(z > \frac{26.5 - 31.9}{\sqrt{24.882}}\right) = P(z > -1.08255)$	M1	Standardising must have sq rt
		M1	25.5 or 26.5 seen as a cc
	$= \Phi(1.08255)$	M1	Correct area Φ, must agree with their μ
	= 0.861	A1	Correct final answer accept 0.861, or 0.860 from 0.8604 not from 0.8599
		5	

Question 24

(i)	$z_1 = \pm\frac{90-120}{24} = -\frac{5}{4}$, $z_2 = \pm\frac{140-120}{24} = \frac{5}{6}$	M1	At least one standardisation, no cc, no sq rt, no sq using 120 and 24 and either 90 or 140
	$= \Phi\left(\frac{20}{24}\right) - \Phi\left(-\frac{30}{24}\right)$	A1	–5/4 and 5/6 unsimplified
	$= \Phi(0.8333) - (1 - \Phi(1.25))$ = 0.7975 – (1 – 0.8944) or 0.8944 – 0.2025 = 0.6919	M1	Correct area Φ – Φ legitimately obtained and evaluated from phi(their z_2) – phi (their z_1)
	= 0.692 AG	A1	Correct answer obtained from 0.7975 and 0.1056 oe to 4sf or 0.6919 seen www
		4	

(ii)	**Method 1**		
	Probability = P(2, 3, 4) $= 0.692^2(1 - 0.692)^2 \times {}^4C_2 + 0.692^3(1 - 0.692) \times {}^4C_3 + 0.692^4$	M1	Any binomial term of form $4C_x p^x(1-p)^{4-x}, x \neq 0$ or 4
		B1	One correct bin term with $n = 4$ and $p = 0.692$,
	= 0.27256 + 0.40825 + 0.22931	M1	Correct unsimplified expression using 0.692 or better
	= 0.910	A1	Correct answer
	Method 2:		
	1 – P(0, 1) =	M1	Any binomial term of form $4C_x p^x(1-p)^{4-x}, x \neq 0$ or 4
	$1 - 0.692^0(1 - 0.692)^4 \times {}^4C_0 - 0.692^1(1 - 0.692)^3 \times {}^4C_1$	B1	One correct bin term with $n = 4$ and $p = 0.692$
	= 1 – 0.00899 – 0.0808757	M1	Correct unsimplified expression using 0.692 or better
	= 0.910	A1	Correct answer
		4	

Question 25

3(i)	**Method 1**		
	$P(3)+P(4)+P(5) = {}^5C_3\ 0.75^3 \times 0.25^2 +$	M1	One binomial term ${}^5C_x p^x (1-p)^{5-x}$, $x \neq 0$ or 5, any p
	${}^5C_4\ 0.75^4 \times 0.25^1 + {}^5C_5\ 0.75^5 \times 0.25^0$	M1	Correct unsimplified expression
	$= 0.26367 + 0.39551 + 0.23730$ $= 0.896\ (459/512)$	A1	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
	Method 2		
	$1 - P(0) - P(1) - P(2) = 1 - {}^5C_0\ 0.75^0 \times 0.25^5$	M1	One binomial term ${}^5C_x p^x (1-p)^{5-x}$, $x \neq 0$ or 5, any p
	$-\ {}^5C_1\ 0.75^1 \times 0.25^4 -\ {}^5C_2\ 0.75^2 \times 0.25^3$	M1	Correct simplified expression
	$= 1 - 0.00097656 - 0.014648 - 0.087891$ $= 0.896\ (459/512)$	A1	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
		3	
(ii)	**Method 1**		
	P(C,C) + P(C,C′) + P(C′,C) 0.8×0.9	B1	Unsimplified prob completed on both days
	$0.8 \times 0.1 + 0.2 \times 0.6$	M1	Unsimplified prob $0.8 \times a + 0.2 \times b$, $a = 0.1$or 0.4, $b = 0.6$ or 0.9
	= 0.92 oe	A1	Correct final answer
	Method 2		
	$1 - P(C', C') = 1 - 0.2 \times 0.4$	B1	Unsimplified prob completed on no days
		M1	$1 - 0.2 \times a$, a=0.1or 0.4 allow unsimplified
	= 0.92	A1	Correct final answer
		3	

Question 26

(i)	$1 - (P(7) + P(8) + P(9))$ $= 1 - ({}^9C_7\ 0.8^7 \times 0.2^2 + {}^9C_8\ 0.8^8 \times 0.2^1 + {}^9C_9\ 0.8^9 \times 0.2^0)$	M1	Any binomial term of form ${}^9C_x p^x (1-p)^{9-x}$, $x \neq 0$
		M1	Correct unsimplified expression
	$= 1 - (0.3019899 + 0.3019899 + 0.1342177)$ $= 0.262$	A1	Correct answer
		3	
(ii)	Mean = 200 × 0.8 = 160: var = 200 × 0.8 × 0.2 = 32	B1	Both unsimplified
	$P(X > 166) = P(Z > \frac{166.5 - 160}{\sqrt{32}})$	M1	Standardise, $z = \pm \frac{x - their\,160}{\sqrt{their\,32}}$ with square root
		M1	166.5 or 165.5 seen in attempted standardisation expression
	$= P(Z > 1.149) = 1 - 0.8747$	M1	1 – a Φ -value, correct area expression, linked to final answer
	$= 0.125$	A1	Correct final answer
		5	
(iii)	$np = 160$, $nq = 40$: both > 5 (so normal approx. holds)	B1	Both parts required
		1	

Question 27

(i)	$P(4, 5, 6) = {}^6C_4 0.35^4 0.65^2 + {}^6C_5 0.35^5 0.65^1 + 0.35^6$	M1	Binomial term of form ${}^6C_x p^x(1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
		A1	Correct unsimplified answer
	$= 0.117$	A1	
		3	
(ii)	$1 - 0.65^n > 0.95$ $0.65^n < 0.05$	M1	Equation or inequality involving '0.65^n or 0.35^n' and '0.95 or 0.05'
	$n > \frac{\log 0.05}{\log 0.65} = 6.95$	M1	Attempt to solve *their* exponential equation using logs or Trial and Error.
	$n = 7$	A1	CAO
		3	
(iii)	Mean $= 0.35 \times 100 = 35$ Variance $= 0.35 \times 0.65 \times 100 = 22.75$	B1	Correct unsimplified np and npq,
	$P\left(z > \frac{39.5 - 35}{\sqrt{22.75}}\right) = P(z > 0.943)$	M1	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'39.5'*.
		M1	Using continuity correction 39.5 or 40.5
	$= 1 - 0.8272$	M1	Appropriate area Φ from standardisation formula P(z>….) in final solution, (>0.5 if z is -ve, <0.5 if z is +ve)
	$= 0.173$	A1	Final answer
		5	

Question 28

(i)	$P(0, 1, 2) = (0.66)^{14} + {}^{14}C_1(0.34)(0.66)^{13} + {}^{14}C_2(0.34)^2(0.66)^{12}$	M1	Binomial term of form ${}^{14}C_x p^x(1-p)^{14-x}$ $0 < p < 1$ any p, $x \neq 14,0$
	$= 0.0029758 + 0.02146239 + 0.071866$	A1	Correct unsimplified answer
	$= 0.0963$	A1	Correct answer
		3	
(ii)	Mean $= 600 \times 0.34 = 204$, Var $= 600 \times 0.34 \times 0.66 = 134.64$	B1	Correct unsimplified np and npq (or sd = 11.603 or Variance = 3366/25)
	$P(< 190) = P\left(z < \frac{189.5 - 204}{\sqrt{134.64}}\right) = P(z < -1.2496)$	M1	Substituting *their* μ and σ, (no σ^2 or $\sqrt{\sigma}$) into the Standardisation Formula with a numerical value for *'189.5'*. Condone ± standardisation formula
		M1	Using continuity correction 189.5 or 190.5 within a Standardisation formula
	$= 1 - \Phi(1.2496)$	M1	Appropriate area Φ from standardisation formula P(z<….) in final solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	$= 1 - 0.8944 = 0.106$	A1	Correct final answer
		5	

Question 29

(i)	P(at most 7) = 1 – P(8, 9, 10) $= 1 - {}^{10}C_8(0.35)^8(0.65)^2 - {}^{10}C_9(0.35)^9(0.65)^1 - (0.35)^{10}$	M1	Use of normal approximation M0 Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	[= 1 – 0.004281 – 0.0005123 – 0.00002759]	A1	Correct unsimplified (or individual terms evaluated) answer seen Condone 1 – A + B + C leading to correct solution
	= 0.995	B1	B1 **not** dependent on previous marks.
	Alternative method for question 3(i)		
	P(at most 7) = P(0,1,2,3,4,5,6,7)	M1	Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	$= (0.65)^{10} + {}^{10}C_1(0.35)^1(0.65)^9 + \ldots + {}^{10}C_7(0.35)^7(0.65)^3$	A1	Correct unsimplified answer or individual terms evaluated seen
	= 0.995	B1	
		3	
(ii)	$1 - (0.65)^n > 0.99$ $0.01 > (0.65)^n$	M1	Equation or inequality with $(0.65)^n$ **and** 0.01 or $(0.35)^n$ **and** 0.99 only (Note 1 – 0.99 is equivalent to 0.01 etc.)
	$n > 10.69$	M1	Solving their $a^n = c$, $0 < a,c < 1$ using logs **or** Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark
	smallest $n = 11$	A1	CAO
		3	

Question 30

(i)	(P > 12) = P(13, 14, 15)	M1	Binomial term of form ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$ any p, $x \neq 15,0$
	$= {}^{15}C_{13}(0.65)^{13}(0.35)^2 + {}^{15}C_{14}(0.65)^{14}(0.35)^1 + (0.65)^{15}$	A1	Correct unsimplified answer
	= 0.0617	A1	SC if use np and npq with justification give (12.5 – 9.75)/√3.41 M1 1–F(1.489) A1 0.0681 A0
		3	
(ii)	mean = 250 × 0.65 = 162.5 variance = 250 × 0.65 × 0.35 = 56.875	B1	Correct unsimplified *np* and *npq*
	$P(< 179) = P\left(z < \frac{178.5 - 162.5}{\sqrt{56.875}}\right) = P(z < 2.122)$	M1	Substituting *their* μ and σ (condone σ^2) into the Standardisation Formula with a numerical value for *'178.5'*. Continuity correct not required for this M1. Condone ± standardisation formula
	Using continuity correction 178.5 or 179.5	M1	
	= 0.983	A1	Correct final answer
		4	

Question 31

(a)	$P(0, 1, 2) = {}^6C_0\, 0.3^0\, 0.7^6 + {}^6C_1\, 0.3^1\, 0.7^5 + {}^6C_2\, 0.3^2\, 0.7^4$	M1	Binomial term of form ${}^6C_x p^x(1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
	0.1176 ... + 0.3025 ... + 0.3241 ...	A1	Correct unsimplified answer
	0.744	A1	Correct final answer
		3	
(b)	P(support neither choir) = 1 – (0.3 + 0.45) = 0.25	M1	0.25^n seen alone, $1 < n \leqslant 6$
	P(6 support neither choir) = 0.25^6 $= 0.000244$ or $\frac{1}{4096}$	A1	Correct final answer
		2	

Question 32

(i)	P(8, 9, 10) = $^{10}C_8\, 0.66^8\, 0.34^2 + {}^{10}C_9\, 0.66^9\, 0.34^1 + 0.66^{10}$	**M1**	Correct binomial term, $^{10}C_a\, 0.66^a(1–0.66)^b$ $a+b = 10,\ 0 < a,b < 10$
		A1	Correct unsimplified expression
	0.284	**B1**	CAO
		3	
(ii)	$np = 0.66 \times 150 = 99$ $npq = 0.66 \times (1 - 0.66) \times 150 = 33.66$	**B1**	Accept evaluated or unsimplified μ, σ^2 numerical expressions, condone $\sigma = \sqrt{33.66} = 5.8017$ or 5.802 CAO
	$P(X > 84) = P\left(Z > \frac{84.5 - 99}{\sqrt{33.66}}\right)$	**M1**	± Standardise, $\frac{x - their\, 99}{\sqrt{their\, 33.66}}$, condone σ^2, x a value
		M1	84.5 or 83.5 used in *their* standardisation formula
	$(= P(Z > -2.499))$	**M1**	Correct final area
	0.994	**A1**	Final answer (accept 0.9938) SC if no standardisation formula seen, B2 $P(Z > -2.499) = 0.994$
		5	

Question 33

(i)	$1 - ({}^{10}C_2\, 0.42^8\, 0.58^2 + {}^{10}C_9\, 0.42^9\, 0.58^1 + 0.42^{10})$	**M1**	Binomial term of form $^{10}C_a p^a (1-p)^b$ $0 < p < 1$ any p, $0 \leqslant a,b \leqslant 10$
		A1	Correct unsimplified expression
	0.983	**A1**	
		3	
(ii)	$1 - P(0) > 0.995 \quad 0.58^n < 0.005$	**M1**	Equation or inequality involving 0.58^n or 0.42^n and 0.995 or 0.005
	$n > \frac{\log 0.005}{\log 0.58}$ $n > 9.727$	**M1**	Attempt to solve using logs or Trial and Error. May be implied by their answer (rounded or truncated)
	$n = 10$	**A1**	CAO
		3	

Question 34

(a)	$1 - P(6, 7, 8)$ $= 1 - ({}^8C_6\, 0.7^6 0.3^2 + {}^8C_7\, 0.7^7 0.3^1 + 0.7^8)$	M1	One term ${}^8C_x\, p^x (1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
	$= 1 - 0.55177$	A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
	Alternative method for question 5(a)		
	$P(0, 1, 2, 3, 4, 5)$ $= 0.3^8 + {}^8C_1\, 0.7^1 0.3^7 + {}^8C_2\, 0.7^2 0.3^6 + {}^8C_3\, 0.7^3 0.3^5 +$ ${}^8C_4 0.7^4 0.3^4 + {}^8C_5\, 0.7^5 0.3^3$	M1	One term ${}^8C_x\, p^x (1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
		A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
		3	
(b)	Mean $= 120 \times 0.7 = 84$ Var $= 120 \times 0.7 \times 0.3 = 25.2$	B1	Correct mean and variance, allow unsimplified
	P(more than 75) $= P\left(z > \frac{75.5 - 84}{\sqrt{25.2}}\right)$	M1	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
		M1	Using continuity correction 75.5 or 74.5
	$P(z > -1.693)$	M1	Appropriate area Φ, from final process, must be a probability
	$= 0.955$	A1	Allow $0.9545 < p \leqslant 0.955$
		5	

Question 35

(a)	$0.22^3 = 0.0106$	B1
		1
(b)	$P(2, 3, 4) = {}^{16}C_2\, 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	M1
	$0.179205 + 0.235877 + 0.216221$	A1
	0.631	A1
		3

Question 36

(a)	$1 - P(10, 11, 12)$ $= 1 - [{}^{12}C_{10} 0.72^{10} 0.28^2 + {}^{12}C_{11}\, 0.72^{11} 0.28^1 + 0.72^{12}]$	M1
	$1 - (0.19372 + 0.09057 + 0.01941)$	A1
	0.696	A1
		3
(b)	$0.28^3 \times 0.72 = 0.0158$	B1
		1

Question 37

$1 - P(8, 9, 10) = 1 - \left[{}^{10}C_8\, 0.64^8 0.36^2 + {}^{10}C_9\, 0.64^9 0.36^1 + 0.64^{10}\right]$	M
$1 - (0.164156 + 0.064852 + 0.11529)$	M
0.759	A

Question 37

(a)	$0.65^7 + {}^7C_1\ 0.65^6\, 0.35^1 + {}^7C_2\ 0.65^5\, 0.35^2$	**M1**	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1$, any p, $x \neq 0, 7$
	$0.049022 + 0.184776 + 0.29848$	**A1**	Correct unsimplified answer
	0.532	**A1**	
		3	
(b)	Mean $= 142 \times 0.35 = 49.7$ Variance $= 142 \times 0.35 \times 0.65 = 32.305$	**B1**	Correct unsimplified *np* and *npq* (condone $\sigma = 5.684$ evaluated)
	$P(X > 40) = P(z > \frac{40.5 - 49.7}{\sqrt{32.305}})$	**M1**	Substituting *their* μ *and* σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z > -1.619)$	**M1**	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z > \ldots)$ in final solution, must be probability
	0.947	**A1**	Correct final answer
		5	

Question 38

(a)	$\left(\frac{5}{6}\right)^8$	**M1**	p^8, $0 < p < 1$, no x, + or -
	0.233	**A1**	
		2	
(b)	36	**B1**	
		1	
(c)	$P(X=10) + P(X=11) = \left(\frac{35}{36}\right)^9 \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \frac{1}{36}$	**M1**	OE, unsimplified expression in form $p^9 q + p^{10} q$, $p + q = 1$, no ×
	0.0425	**A1**	
		2	

Question 39

(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$
	$0{\cdot}598, \frac{4651}{7776}$	**A1**	
		2	
(b)	$(1 - P(0, 1, 2))$ $1-\left(\left(\frac{5}{6}\right)^{10}+{}^{10}C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9+{}^{10}C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	**M1**	${}^{10}C_x\, p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0,10$
	$1 - (0{\cdot}1615056 + 0{\cdot}3230111 + 0{\cdot}290710)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket
	$0{\cdot}225$	**A1**	$0{\cdot}2247 < p \leq 0{\cdot}225$, WWW
		3	

Question 40

(a)	$P(X > 6) = 0.75^6$	**M1**	p^n, $n = 6, 7$ $0 < p < 1$
	$0.178, \frac{729}{4096}$	**A1**	$0{\cdot}17797\ldots$
		2	
(b)	$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\, 0.25^1\, 0.75^9 + {}^{10}C_2\, 0.25^2\, 0.75^8)$	**M1**	Binomial term of form ${}^{10}C_x\, p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$
	$1 - (0{\cdot}0563135 + 0{\cdot}1877117 + 0{\cdot}2815676)$	**A1**	Correct unsimplified expression
	$0{\cdot}474$	**A1**	$0{\cdot}474 \leqslant p \leqslant 0{\cdot}4744$
		3	

Question 41

(a)	$\left[\left(\frac{4}{5}\right)^7\frac{1}{5}=\right]$ $\frac{16384}{390625}$ or $0{\cdot}0419[43\ldots]$	**B1**	Evaluated, final answer.
		1	
(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$ Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or $0{\cdot}672[32]$	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5+{}^5C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)+{}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2+{}^5C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3+{}^5C_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5 + {}^5C_4(p)^4(q) + {}^5C_3(p)^3(q)^2 + {}^5C_2(p)^2(q)^3 + {}^5C_1(p)(q)^4$ or $(p)^6 + {}^6C_5(p)^5(q) + {}^6C_4(p)^4(q)^2 + {}^6C_3(p)^3(q)^3 + {}^6C_2(p)^2(q)^4 + {}^6C_1(p)(q)^5$, $0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or $0{\cdot}672[32]$	**A1**	Final answer.
		2	

Question 42

(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability = $\left(\frac{1}{6}\right)^3 \times 3$	**M1**	$\left(\frac{1}{6}\right)^3 \times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	**A1**	Accept $\frac{3}{216}$ or $0.013\dot{8}$ or 0.0139
		3	
(b)	$P(18)=\left(\frac{1}{6}\right)^3\left[=\frac{1}{216}\right]$	**B1**	
	$P(18 \text{ on 5th throw})=\left(\frac{215}{216}\right)^4\times\frac{1}{216}$	**M1**	$(1-p)^4p$, $0 < their\ p < 1$
	0.00454	**A1**	
		3	

Question 43

(a)	$[(0.7)^3 =]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3 + {}^3C_1(0.15)^2(0.55) + {}^3C_2(0.15)(0.55)^2 + (0.55)^3 =]$ 0.343	B1	Evaluated WWW
		1	
(b)	$1 - (0.85^9 + {}^9C_1\,0.15^1\,0.85^8 + {}^9C_2\,0.15^2\,0.85^7)$ [1 – (0.231617 + 0.367862 + 0.259667)]	M1	One term: ${}^9C_x\,p^x(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\,0.15^3\,0.85^6 + {}^9C_4\,0.15^4\,0.85^5 + {}^9C_5\,0.15^5\,0.85^4 + {}^9C_6\,0.15^6\,0.85^3 + {}^9C_7\,0.15^7\,0.85^2 + {}^9C_8\,0.15^8\,0.85 + 0.15^9$	M1	One term: ${}^9C_x\,p^x(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	
5(c)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765 \le \sigma \le 2.77$ imply correct variance)
	$[(X\ge12)=]P\left(Z>\frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

Question 44

(a)	6	B1	WWW
		1	
(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p)+p^4(1-p)+p^5(1-p)+p^6(1-p)$, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3-\left(\frac{5}{6}\right)^7$	M1	p^3-p^7, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
(c)	$1-\left(\frac{5}{6}\right)^9$	M1	$1-p^n$, $0<p<1$, $n=9, 10$
	0.806	A1	

Question 45

(a)	$1-P(10, 11, 12) = 1-({}^{12}C_{10}\,0.6^{10}\,0.4^2+{}^{12}C_{11}\,0.6^{11}\,0.4^1+{}^{12}C_{12}\,0.6^{12}\,0.4^0)$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x p^x(1-p)^{12-x}$ for $0<x<12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	$P(0,1,2,3,4,5,6,7,8,9) = {}^{12}C_0 0.6^0\,0.4^{12}+{}^{12}C_1 0.6^1\,0.4^{11}+\ldots\ldots\ldots\ldots{}^{12}C_9\,0.6^9\,0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x p^x(1-p)^{12-x}$ for $0<x<12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	
(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6\times150\times0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X<81)=P\left(Z<\frac{80.5-90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833)=1-0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
(c)	$np=90$, $nq=60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

Question 46

(a)	[P(0, 1, 2) =] ${}^{10}C_0\,0.16^0\,0.84^{10}+{}^{10}C_1\,0.16^1\,0.84^9+{}^{10}C_2\,0.16^2\,0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	**M1**	One term: ${}^{10}C_x p^x(1-p)^{10-x}$ for $0<x<10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	**A1**	$0.7935<p\leqslant0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
(b)	$(0.84)^7\,0.16$	**M1**	$(1-p)^7p$, $0<p<1$
	0.0472	**A1**	0.0472144 to at least 3sf.
		2	
(c)	$4\times0.0472\times(1-0.0472)^3$	**M1**	$4\times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	**A1**	$0.163\leqslant p\leqslant0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

Question 47

(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	M1	$(1-p)^6 p$, $0 < p < 1$
	0.0445, $\frac{729}{16384}$	A1	
		2	
(b)	$\left(\frac{3}{4}\right)^9$	M1	$\left(\frac{3}{4}\right)^n$ *or* p^n, $0 < p < 1, n = 8,9,10$
	0.0751, $\frac{19683}{262144}$	A1	
		2	

Question 48

(a)	$\left[\left(\frac{5}{6}\right)^7 \times \frac{1}{6} = \right] 0.0465, \frac{78125}{1679616}$	B1	$0.0465 \leqslant p < 0.04652$
		1	
(b)	$P(X < 6) = 1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	M1	$1-p^n$, $0 < p < 1$, $n = 4, 5, 6$ or sum of 4, 5 or 6 terms $p \times (1-p)^n$ for $n = 0,1,2,3,4(5)$.
	0.598, $\frac{4651}{7776}$	A1	
		2	
(c)	[Probability of total less than 4 is] $\frac{3}{36}$ or $\frac{1}{12}$	B1	SOI
	[1 – P(0, 1, 2)] $=1-({}^{10}C_0\left(\frac{1}{12}\right)^0\left(\frac{11}{12}\right)^{10} + {}^{10}C_1\left(\frac{1}{12}\right)^1\left(\frac{11}{12}\right)^9 + {}^{10}C_2\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)^8)$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$.
	1 – (0.418904 + 0.380822 + 0.155791)	A1 FT	Correct expression. Accept unsimplified.
	0.0445	A1	$0.04448 \leqslant p \leqslant 0.0445$
		4	

Question 49

(a)	[P(10, 11, 12) =] ${}^{12}C_{10}0.72^{10}0.28^2 + {}^{12}C_{11}0.72^{11}0.28^1 + {}^{12}C_{12}0.72^{12}0.28^0$	M1	One term ${}^{12}C_x\,p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	= 0.193725 + 0.0905726 + 0.0194084	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	[1 – P(0,1,2,3,4,5,6,7,8,9) =] $1-({}^{12}C_0 0.72^0 0.28^{12} + {}^{12}C_1 0.72^1 0.28^{11} + {}^{12}C_2 0.72^2 0.28^{10} + {}^{12}C_3 0.72^3 0.28^9 + {}^{12}C_4 0.72^4 0.28^8 + {}^{12}C_5 0.72^5 0.28^7 + {}^{12}C_6 0.72^6 0.28^6 + {}^{12}C_7 0.72^7 0.28^5 + {}^{12}C_8 0.72^8 0.28^4 + {}^{12}C_9 0.72^9 0.28^3)$	M1	One term ${}^{12}C_x\,p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
		3	
(b)	Mean = $[0.52 \times 90 =] 46.8$, var $= [0.52 \times 0.48 \times 90] = 22.464$	B1	46.8 and 22.464 or 22.46 seen, allow unsimplified, ($4.739 < \sigma \leqslant 4.740$ imply correct variance).
	$[P(X < 40) =]\ P\left(z < \frac{39.5-46.8}{\sqrt{22.464}}\right)$	M1	Substituting *their* mean and *their* variance into ±standardisation formula (any number for 39.5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	$= [P(Z < -1.540)] = 1 - 0.9382$	M1	Appropriate area Φ, from final process, must be probability.
	0.0618	A1	$0.06175 \leqslant p \leqslant 0.0618$
		5	

Question 50

(a)	$a = \text{P(1 head)} = 0.7\times(0.5)^3 + 0.3\times(0.5)^3\times3 = \frac{1}{5}$	B1	Clear statement of unevaluated correct calculation $= \frac{1}{5}$. AG
	$b = 0.7\times0.5^3\times3+0.3\times0.5^3\times3 = \frac{3}{8}$ $c = 0.7\times0.5^3\times3+0.3\times0.5^3 = \frac{3}{10}$ $\left[or\ c = \frac{27}{40} - b\right]$	M1	Clear statement of unevaluated calculation for either b or c
		A1	For either b or c correct
		B1 FT	$their\ b + their\ c = \frac{27}{40}$
		4	
(b)	$\left[E(X) = \frac{3\times0+16\times1+30\times2+24\times3+7\times4}{80} = \right] \frac{176}{80}$ or 2.2	B1 FT	Correct or accept unsimplified calculation using *their* values for b and c seen (sum of probabilities = 1)
		1	
(c)	[P(0, 1, 2) =]$^{10}C_0\ 0.2^0\ 0.8^{10} + {}^{10}C_1\ 0.2^1\ 0.8^9 + {}^{10}C_2\ 0.2^2\ 0.8^8$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
	0.107374 + 0.268435 + 0.301989	A1	Correct expression, accept unsimplified leading to final answer
	0.678	B1	$0.677 < p \leqslant 0.678$
	Alternative method for question 4(c)		
	$1 - [{}^{10}C_{10}\ 0{\cdot}2^{10}0.8^0 + {}^{10}C_9\ 0{\cdot}2^9 0.8^1 + {}^{10}C_8\ 0{\cdot}2^8 0.8^2 + {}^{10}C_7\ 0{\cdot}2^7 0.8^3 + {}^{10}C_6\ 0{\cdot}2^6 0.8^4 + {}^{10}C_5\ 0{\cdot}2^5 0.8^5 + {}^{10}C_4\ 0{\cdot}2^4 0.8^6 + {}^{10}C_3\ 0{\cdot}2^3 0.8^7]$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	B1	$0.677 < p \leq 0.678$
		4	
(d)	$0.8^6\times0.2 + 0.8^7\times0.2 = 0.0524288 + 0.041943$	M1	$p^l\times(1-p) + p^m\times(1-p)$, $l = 6, 7$ $m = l + 1$, $0 < p < 1$
	0.0944	A1	$0.09437 \leqslant p \leqslant 0.0944$
		2	

Question 51

[P(0, 1, 2) =] $^{10}C_0\ 0.1^0\ 0.9^{10} + {}^{10}C_1\ 0.1^1\ 0.9^9 + {}^{10}C_2\ 0.1^2\ 0.9^8$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, $0<p<1, x\neq0$
= 0.348678+0.38742+0.19371	A1	Correct expression, accept unsimplified.
0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
Alternative method for Question 5(a)		
[1 – P(3, 4, 5, 6, 7, 8, 9, 10) = $1 - ({}^{10}C_3\ 0.9^7\ 0.1^3 + {}^{10}C_4\ 0.9^6\ 0.1^4 + {}^{10}C_5\ 0.9^5\ 0.1^5 + {}^{10}C_6\ 0.9^4\ 0.1^6 + {}^{10}C_7\ 0.9^3\ 0.1^7 + {}^{10}C_8\ 0.9^2\ 0.1^8 + {}^{10}C_9\ 0.9^1\ 0.1^9 + {}^{10}C_{10}\ 0.9^0\ 0.1^{10})$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, $0<p<1, x\neq0$
	A1	Correct expression, accept unsimplified.
0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
	3	

Question 52

[1 – P(10, 11, 12) =] $1-({}^{12}C_{10}\ 0.9^{10}\ 0.1^2 + {}^{12}C_{11}\ 0.9^{11}\ 0.1^1 + {}^{12}C_{12}\ 0.9^{12}\ 0.1^0)$ = 1 – (0.230128 + 0.376573 + 0.282430)	M1	One term $^{12}C_x\ p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$
	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
0.111	B1	Mark the final answer at the most accurate value, $0.1108 < p \leqslant 0.111$ WWW.

Question 53

[P(3, 4, …7) = 1 – P(0, 1, 2, 8)] $= 1-({}^8C_0\ 0.48^0\ 0.52^8 + {}^8C_1\ 0.48^1\ 0.52^7 + {}^8C_2\ 0.48^2\ 0.52^6 + {}^8C_8\ 0.48^8\ 0.52^0)$	M1	One term $^8C_x\ p^x(1-p)^{8-x}$, for $0 < x < 8$, $0 < p < 1$
= 1 – (0.00534597 + 0.039478 + 0.127544 + 0.0028179)	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
0.825	B1	Mark the final answer at the most accurate value. $0.8248 < p \leqslant 0.825$ WWW.

Question 54

(a)	**Method 1 for Question 3(a)**		
	[P(X > 17) = P(18, 19, 20) =] $^{20}C_{18}\,(0.8)^{18}(0.2)^2 + {}^{20}C_{19}\,(0.8)^{19}(0.2)^1 + {}^{20}C_{20}\,(0.8)^{20}$ = 0.13691 + 0.05765 + 0.01153	**M1**	One term $^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.206	**B1**	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$.
	Method 2 for Question 3(a)		
	[P(X > 17) = 1 – P(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) =] $1-({}^{20}C_0\,(0.8)^0(0.2)^{20} + {}^{20}C_1\,(0.8)^1(0.2)^{19} + {}^{20}C_2\,(0.8)^2(0.2)^{18} + \ldots + {}^{20}C_{16}\,(0.8)^{16}(0.2)^4 + {}^{20}C_{17}\,(0.8)^{17}(0.2)^3)$ $= 1-(1.048\times10^{-14} + 8.389\times10^{-13} + 3.188\times10^{11} + \ldots + 0.2182 + 0.2054)$	**M1**	One term $^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer. If answer correct, condone omission of any 15 of the 16 middle terms.
	0.206	**B1**	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$. Condone omission of brackets.
		3	
(b)	$\left[(0.8)^4(0.2) = \right]0.08192, \frac{256}{3125}$	**B1**	Accept $\frac{8192}{100000}$ OE.
		1	

Question 55

(a)	**Method 1**		
	[P(X < 6) = P($X \leqslant$ 5) =] $1-0.8^5$	**M1**	$1-0.8^r$, r = 5, 6.
	= 0.672	**A1**	
	Method 2		
	[P(X < 6) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) =] $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	Condone an extra term $(\frac{4}{5})^5\times\frac{1}{5}$. First, last and one of the 3 middle terms implies M1.
	= 0.672	**A1**	
		2	
(b)	**Method 1**		
	[1 – P(0, 1, 2)] $= 1-({}^{12}C_0\,(0.8)^{12} + {}^{12}C_1\,(0.2)(0.8)^{11} + {}^{12}C_2\,(0.2)^2\,(0.8)^{10})$ [= 1 – (0.06872 + 0.20615 + 0.28347)]	**M1**	One term $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$, $x\neq0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. Correct unsimplified expression or better.
	= 0.442	**B1**	$0.411 < p \leqslant 0.442$ WWW.
	Method 2		
	[P(3,4,5,6,7,8,9,10,11,12) =] $^{12}C_3\,(0.2)^3\,(0.8)^9 + {}^{12}C_4\,(0.2)^4\,(0.8)^8 + \ldots + {}^{12}C_{11}\,(0.2)^{11}\,(0.8)^1 + {}^{12}C_{12}\,(0.2)^{12}$ [= 0.23622 + 0.13288 + … + 1.966×10^{-7} + 4.096×10^{-9}]	**M1**	One term $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$, $x\neq0, 1, 2$.
		A1	Correct expression, accept unsimplified, leading to final answer. Accept first, last and 8 of the middle terms.
	=0.442	**B1**	$0.411 < p \leqslant 0.442$.
		3	

(c)	$(0.2)^5 \times 5!$	M1	$(0.2)^5 \times s$, s a positive integer. 1 may be implied.
		M1	$t \times 5!$ where $0 < t < 1$.
	$= 0.0384, \frac{24}{625}$	A1	
	Alternative Method for Question 7(c)		
	$\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]}{({}^5C_1)^5}$	M1	$({}^5C_1)^5$ or 5^5 as denominator.
		M1	${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]$ or 5! as numerator.
	$= 0.0384, \frac{24}{625}$	A1	
		3	