AS-Level

Discrete Random Variable

May : 2013- May : 2023

Answers

Question 1

x	0	1	2
$P(X = x)$	5/12	1/2	1/12

B1 Values 0, 1, 2 seen in table with at least 1 prob

$P(0) = P(\bar{5}, \bar{5}) = \frac{6}{9} \times \frac{5}{8} = 30/72$ (5/12) (0.4166) B1 Correct P(0) unsimplified

P(1) = 0.5 from part (ii)

P(2) = 6/72 (1/12) (0.0833) from part (i) B1ft **3** If x=0,1,2(,3) ft Σp = 1, no –ve values, all probabilities <1

Question 2

(i) options (3, 4, 4,) or (4, 3, 4) or (4, 4, 3) M1 Summing three 3-factor options oe
Probs (4/10 × 6/9 × 5/8) ×3C1 M1 10 × 9 × 8 seen in denom
= 360/720
= ½ AG A1 **[3]** Correct answer

OR $\frac{{}_6C_2 \times {}_4C_1}{{}_{10}C_3} = \frac{1}{2}$ AG M1 One of 6C2 or 4C1 seen in num
M1 10C3 in denom
A1 Correct answer

(ii) B1 **[4]** 9, 10, 11, 12 only seen

sum	9	10	11	12
Prob	24/720	216/720	360/720	120/720

B1 One correct prob other than P(11), with or without replacement

P(3, 3, 3) = 4/10 × 3/9 × 2/8 = 24/720 (1/30) B1 Another correct prob
P(3, 3, 4) = 4/10 × 3/9 × 6/8 × 3C1
= 216/720 (3/10)
P(4, 4, 4) = 6/10 × 5/9 × 4/8 = 120/720(1/6) B1 Σ all 4 probs = 1

Question 3

(i)	if throw H then smallest score is 2 P(T, 1) = 1/2 × 1/4 = 1/8 **AG**	B1 B1	**2**	Or equivalent
(ii)	P(3) from two dice = 2/16 seen P(H, 3) = 1/2 × 2/16 = 2/32 P(T, 3) = 1/2 × 1/4 = 1/8 So P(3) = 6/32 = 3/16 **AG**	B1 M1 A1 A1	**4**	From (1, 2) and (2, 1) Summing P(H, 3) and P(T, 3) One correct Correct answer must see clear reasoning
(iii)				

X	1	2	3	4	5	6	7	8
Prob		5/32		7/32		3/32		

B1		One correct prob
B1		A second correct prob
B1	**3**	A third correct prob

Question 4

(i)	$P(2) = {}^6C_3 \times {}^3C_2/{}^9C_5$ **OR** $\dfrac{{}^6C_3 \times {}^3C_2}{{}^6C_5+{}^6C_4\times{}^3C_1+{}^6C_3\times{}^3C_2+{}^6C_2\times{}^3C_3}$ **OR** $3/9 \times 2/8 \times 6/7 \times 5/6 \times 4/5 \times {}^5C_2 = 10/21$ $= 60/126$ **AG**	M1 **OR** M1 **OR** M1 A1	**2**	Using combinations ${}^aC_b \times {}^cC_d/{}^eC_f$ Mult 5 probs with a ${}^pC_q$ If ${}^5C_2$ replace by 10, oe must be justified Legit method, as answer given
(ii)				

x	0	1	2	3
Prob	2/42	15/42	20/42	5/42

	B1		0, 1, 2, 3 only seen in table. Condone $x = 4,5$ in table if $P(x) = 0$ or blank and values in table for $x = 0,1,2,3$
$P(0) = {}^6C_5/{}^9C_5 = 6/126$	B1		Any correct prob other than P(2)
$P(1) = {}^6C_4 \times {}^3C_1/{}^9C_5 = 45/126$	B1		Any other correct prob
$P(3) = {}^6C_2 \times {}^3C_3/126 = 15/126$	B1√	**4**	$\Sigma P(x) = 1$, $3<n(x)<6$

Question 5

(i)	A:P(H) = 2/3, P(T) = 1/3 B: P(H) = ¼, P(T) = 3/4	M1		Using some of 2/3, 1/3, ¼ or 3/4 in a calculation involving prod of 3 probs
	P(1H) = P(HTT) + P(THT) + P(TTH) = (2/3 × 1/3 × 3/4) + (1/3 × 2/3 × 3/4)	M1		Summing 3 options not all the same
	+ (1/3 × 1/3 × 1/4) = 13/36 **AG**	A1	**3**	Correct answer
(ii)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P</td><td>3/36</td><td>13/36</td><td>16/36</td><td>4/36</td></tr></table>	B1		0, 1, 2, 3 seen for table no probs needed, table not absolutely necessary if calcs shown
	P(0H) = P(TTT) = 1/3 × 1/3 × 3/4 = 1/12	B1		One prob correct other than (i) condone 0.083 for 0.0833
	P(2H) = P(HHT) + P(HTH) + P(THH) = (2/3 × 2/3 × 3/4) + (2/3 × 1/3 × 1/4) + (1/3 × 2/3 × 1/4) = 4/9 not 2/3 × 2/3	B1		A second prob correct need 3 factors can be implied
	P(3H) = P(HHH) = 2/3 × 2/3 × 1/4 = 1/9	B1√	**4**	A third prob correct ft 23/36 – Σ their 2 probs
(iii)	E(X) = 13/36 + 32/36 + 12/36	M1		Attempt to evaluate Σxp at least 3 vals of x in table
	= 57/36 (19/12) (1.58)	A1	**2**	Correct answer

Question 6

(i)	P (exactly 2) = $\frac{{}^6C_2}{{}^8C_4} = \frac{15}{70} = \frac{3}{14}$ AG OR P (2) = $\frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} \times {}^4C_2 = \frac{3}{14}$ AG	M1 A1	 **2**	${}^6Cx$ / ${}^8Cx$ seen or ${}^4C_2$ mult by 4 fractions (last 2 can be implied) Answer legit obtained
(ii)	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Prob</td><td>3/14</td><td>8/14</td><td>3/14</td></tr></table>	B1 B1 B1√	 **3**	2, 3, 4 only in top line one correct prob other than P (2) third correct prob ft Σ = 1
(iii)	Var(X) = $\frac{12}{14} + \frac{72}{14} + \frac{48}{14} - 3^2$ = $\frac{3}{7}$ (0.429)	M1 A1	 **2**	using $\Sigma x^2 p - 3^2$ (or their $\{E(X)\}^2$) must be evaluated correct answer

Question 7

(i)	$0.24 + 0.35 + 2k + k + 0.05 = 1$ $k = 0.12$	M1 A1	**2**	Summing probs = 1 Correct answer
(ii)	model number is 1	B1	**1**	
(iii)	mean = $1 \times 0.35 + 2 \times 0.24 + 3 \times 0.12 + 4 \times 0.05$ P(>1.39) = P(2, 3, 4) = 0.41	B1 M1 B1	**3**	1.39 seen Finding P(X > their mean) Correct ans following mean or mode only

Question 8

(i)	P(1 W) = 6/9×3/8 + 3/9×6/8	M1		summing 2 two-factor probs (condone replacement) not ½×½ + ½×½
	= ½ AG	A1	**[2]**	Correct answer, fully justified
	OR $\frac{{}^6C_1 \times {}^3C_1}{{}^9C_2}$	M1		Using combinations consistent, correct format
	= ½ AG	A1		Correct answer, fully justified
(ii)	$P(\overline{W},\overline{W})$ = 3/9×2/8 = 6/72 (1/12) $P(W,W)$ = 6/9×5/8 = 30/72 (5/12) x: 0, 1, 2 Prob: 1/12, 1/2, 5/12	B1 B1 B1√	**[3]**	Distribution table with 0,1,2 only $P(W,W)$ or $P(\overline{W},\overline{W})$ correct $P(W,W) + P(\overline{W},\overline{W})$ = 0.5
(iii)	E(X) = 16/12 (4/3) (1.33) isw	B1	**[1]**	Condone 1(.3) if correct working seen, nfww

Question 9

(i)

Spinner *B* \ Spinner *A*	1	2	3	3
–3	(–2)	–1	0	0
–2	–1	0	(1)	1
–1	0	1	2	2
1	2	3	4	4

B1 1

(ii)

x	–2	–1	0	1	2	3	4
prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$

M1 Their values in (i) as the top line, seen listed in (ii) or used in part (iii)

M1 Attempt at probs seen evaluated, need at least 4 correct from their table

A1 3 Correct table seen

(iii)

$E(X) = 1$

$\text{Var}(X) = ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$

$= \frac{62}{16} - 1$

$= \left(\frac{23}{8}\right)(2.875)$

OR using $\Sigma p(x - \bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$

$= \frac{46}{16} = 2.875$

M1 Attempt at $E(X)$ from their table if $\Sigma p = 1$

M1 Evaluating $\Sigma x^2 p$ – [their $E(X)$]^2 allow $\Sigma p \neq 1$ but all p's <1

A1 3 Correct answer

M1

M1

A1

(iv)

P(even given +ve)

$= \frac{5}{9}$

OR P(even given +ve) $= \frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$

$= \frac{5}{9}(0.556)$

M1 Counting their even numbers and dividing by their positive numbers

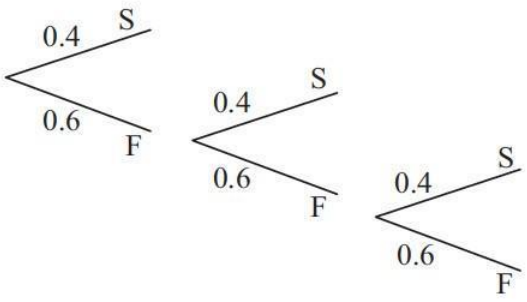
A1 2 Correct answer

M1 Using cond prob formula not P(E) × P(+ve) need fraction over fraction accept any of $\frac{5/16 or 6/16 or 9/16}{9/16 or 10/16 or 13/16}$

A1 Correct answer

Question 10

(i)



M1 3 pairs S (bank, log in, success oe) and F oe seen no extra bits.

A1 Exactly 3 pairs, must be labelled

A1 [3] Correct diagram with all probs correct

(ii)

x	0	1	2	3
Prob	0.4		0.144	0.216

B1 P(0) correct

M1 Multiplying two of more factors of 0.4 and 0.6

A1 One more correct prob

B1 [4] One more correct prob

(iii) $E(X) = 0.24 + 2\times 0.144 + 3\times 0.216$
$= 1.176\ (1.18)$

M1 Using $\Sigma p_i x_i$

A1 [2] Correct answer

Question 11

No of W	0	1	2
Prob	42/90	42/90	6/90

B1 0, 1, 2, seen in table with attempt at prob.

$P(0) = 8/10 \times 7/9 \times 6/8 = 42/90$

M1 3-factor prob seen with different denoms.

$P(1W) = P(W,NW, NW) \times 3 = 2/10 \times 8/9 \times 7/8 \times 3$
$= 42/90$

M1 Mult by 3

$P(2W) = P(W, W, NW) \times 3 = 2/10 \times 1/9 \times 8/8 \times 3$
$= 6/90$

A1 4 All correct

Question 12

(i) $P(0) = 6/36$, $P(1) = 10/36$, $P(2) = 8/36$

B1 Table oe seen with 0, 1, 2, 3, 4, 5 (6 if P(6) = 0)

B1 Any three probs correct

M1 $\Sigma p = 1$ and at least 3 outcomes

$P(3) = 6/36$, $P(4) = 4/36$, $P(5) = 2/36$

A1 [4] All probs correct

(ii) mean score $= (0\times 6 + 1\times 10 + 16 + 18 + 16 + 10)/36$

M1 Using Σxp (unsimplified) on its own – condone Σp not =1

$= 70/36\ (35/18, 1.94)$

A1 [2]

Question 13

(i)			
	x | 1 | 2 | 3 | 4 P(x) | k | $2k$ | $3k$ | $4k$	**B1**	Probability Distribution Table, either k or correct numerical values
	$10k = 1$	**M1**	Summing probs involving k to = 1, 3 or 4 terms
	$k = 1/10$	**A1** [3]	
(ii)	$E(X) = 1/10 + 4/10 + 9/10 + 16/10 = 3$ $Var(X) = 1/10 + 8/10 + 27/10 + 64/10 - 3^2$ $= 1$	**B1** **M1** **A1** [3]	Correct mean Correct method seen for var, their k and μ

Question 14

$[P(X = 0)] = P(B, B) = 5/7 \times 4/6 = 10/21$	**M1**	Attempt to find P(0) or P(1) or P(2) can be seen as P(BB) etc. or table unsimplified
$[P(X = 1)] = P(G,B) + P(B,G) = 2/7 \times 5/6 \times 2 = 10/21$	**A1**	P(1) or P(BG)+P(GB) correct
$[P(X = 2] = P(G, G) = 2/7 \times 1/6 = 1/21$	**A1**	P(0) or P(2) correct must see X value
$E(X) = 0 + 10/21 + 2/21 = 4/7$ (0.571)	**B1**√	Correct answer ft their probs P(1) and P(2)
$Var(X) = 0 + 10/21 + 4/21 - (4/7)^2$ $= 50/147$ (0.340)	**M1** **A1** [6]	Attempt at $\Sigma x^2 p - [E(X)]^2$

Question 15

(i)	P(1 T-shirt) = $\frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$	**B1**		Correct num unsimplified
		B1		Correct denom unsimplified
	= 27/55 AG	**B1**	[3]	Answer given, so process needs to be convincing
	OR $3/12 \times 9/11 \times 8/10 \times {}^3C_1$ oe	**M1**		Mult 3 probs diff denoms (not a/3 x b/4 x c/5)
		M1		Mult by ${}^3C_1$ oe
	= 27/55 AG	**A1**		Answer given, so process needs to be convincing
(ii)	X | 0 | 1 | 2 | 3 Prob | 84/220 | 27/55 | 27/220 | 1/220	**B1**		0, 1, 2, 3 only seen in top line (condone additional values if Prob stated as 0)
		B1		One correct prob, correctly placed in table
		B1		One other correct prob, correctly placed in table
		B1√	[4]	One other correct prob ft $\Sigma p = 1$, 4 values in table

Question 16

diff	0	1	2	3	4	5
prob	6/36	10/36	8/36	6/36	4/36	2/36

Expectation $= (0+10+16+18+16+10)/36$
$= 70/36$
$= 1.94$

B1		0, 1, 2, 3, 4, 5 seen in table heading or considering all different differences
M1		Attempt at finding prob of any difference
A1		1 correct prob
M1		Probs summing to 1
A1	[5]	

Question 17

(i)	$k(-2)^2$ is the same as $k(2)^2 = 4k$	**B1**	need to see $-2^2\,k$, 2^2k and $4k$, algebraically correct expressions OE
	Total:	**1**	
(ii)	x | -2 | -1 | 2 | 4 Prob | $4k$ | k | $4k$ | $16k$	**B1**	-2, -1, 2, 4 only seen in a table, together with at least one attempted probability involving k
	$4k + k + 4k + 16k = 1$	**M1**	Summing 4 probs equating to 1. Must all be positive (table not required)
	$k = 1/25$ (0.04)	**A1**	CWO
	Total:	**3**	
(iii)	$E(X) = -8k + -k + 8k + 64k = 63k$	**M1**	using Σpx unsimplified. FT their k substituted before this stage, no inappropriate dividing
	$= 63/25$ (2.52)	**A1**	
	Total:	**2**	

Question 18

(i)	x | -3 | 0 | 5 | 32 Prob | 1/6 | 1/2 | 1/6 | 1/6	**B1**	At least 3 different correct values of X (can be unsimplified)
		B1	Four correct probabilities in a Probability Distribution table
		B1	Correct probs with correct values of X
		3	
(ii)	$E(X) = -3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	**M1**	Subst their attempts at scores in correct formula as long as 'probs' sum to 1
	$Var(X) = 9/6 + 25/6 + 1024/6 - (34/6)^2$	**M1**	Subst their attempts at scores in correct var formula
	$= 144\left(\frac{1298}{9}\right)$	**A1**	Both answers correct
		3	

Question 19

(i)	*EITHER:* $P(X=3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	**(M1**	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$, accept $\times 1$ as $\frac{4}{r}$.
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR1: $P(X=3) = P(RRB) = \frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ included
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR2: $P(X=3) = P(RRB) = \frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities in order $\frac{{}^2C_1}{{}^pC_1} \times \frac{{}^1C_1}{{}^qC_1} \times \frac{{}^4C_1}{{}^rC_1}$ $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ acceptable)
	= 1/15 AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
		2	
(ii)	$P(1) = P(B) = \frac{4}{6}$ $(\frac{2}{3} = 0.667)$ $P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ $(= 0.267)$ $P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15}$ $(= 0.0667)$ x: 1, 2, 3 P: $\frac{10}{15}$, $\frac{4}{15}$, $\frac{1}{15}$	**B1**	Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leqslant p < 1$.
		B1	P(1) or P(2) correct unsimplified, or better, and identified.
		B1	All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0
		3	

Question 20

$p + q = 0.45$	**M1**	Equation involving $\Sigma P(x) = 1$
$0.15 + 2p + 1.2 + 6q = 3.05$	**M1**	Equation using $E(X) = 3.05$
$q = 0.2$	**M1**	Solving simultaneous equations to one variable
$p = 0.25$	**A1**	Both answers correct
	4	

Question 21

(i)	$\Sigma p = 1$: $0.2 + 0.1 + p + 0.1 + q = 1$: $p + q = 0.6$	**M1**	Unsimplified sum of probabilities equated to 1
	$\Sigma px = 1.7$: $-0.4 + 0 + p + 0.3 + 4q = 1.7$:	**M1**	Unsimplified Sum of px equated to 1.7
	$p + 4q = 1.8$	**M1**	Solve simult. equations to find expression in p or q
	$p = 0.2$, $q = 0.4$	**A1**	
		4	
(ii)	$Var(X) = \Sigma px^2 - 1.7^2 = 4x0.2 + 1p + 9x0.1 + 16q - 1.7^2$ $= 8.3 - 2.89$	**M1**	Use correct unsimplified expression for variance
	$= 5.41$	**A1**	
		2	

Question 22

(i)	$P(0) = 0.6 \times 0.25 \times 0.5 = 0.075$ $P(1) = 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 + 0.6 \times 0.25 \times 0.5 = 0.35$ $P(2) = 0.4 \times 0.75 \times 0.5 + 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 = 0.425$ $P(3) = 0.4 \times 0.75 \times 0.5 = 0.15$	**B1**	0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)
		M1	Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table
		M1	Summing 3 probabilities for P(1) or P(2) with or without a table
		B1	One correct probability seen.
		A1	All correct in a table
	Total:	**5**	
(ii)	$E(X) = 0.35 + 2 \times 0.425 + 3 \times 0.15 = 1.65 \left(\frac{33}{20} \text{ oe}\right)$	**M1**	Correct unsimplified expression for the mean using their table, $\sum p = 1$; can be implied by correct answer
(ii)	$\text{Var}(X) = 0.35 + 4 \times 0.425 + 9 \times 0.15 - 1.65^2$	**M1**	Correct unsimplified expression for the variance using their table and their mean^2 subtracted, $\sum p = 1$
	$= 0.678\ (0.6775) \left(\frac{271}{400} \text{ oe}\right)$	**A1**	Correct answer
	Total:	**3**	

No of heads	0	1	2	3
Prob	0.075 $\left(\frac{3}{40}\right)$	0.35 $\left(\frac{7}{20}\right)$	0.425 $\left(\frac{17}{40}\right)$	0.15 $\left(\frac{3}{20}\right)$

Question 23

(i)		**B1**	Prob distribution table drawn, top row correct with at least one probability $0 < p < 1$ entered, condone additional values with $p = 0$ stated
	$P(0) = \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} = \frac{2}{7}\ (0.2857)$	**B1**	One probability correct (need not be in table)
	$P(1) = \frac{2}{7} \times \frac{5}{6} \times \frac{4}{5} \times {}^3C_1 = \frac{4}{7}\ (0.5713)$	**B1**	Another probability correct (need not be in table).
	$P(2) = \frac{2}{7} \times \frac{1}{6} \times \frac{5}{5} \times {}^3C_2 = \frac{1}{7}\ (0.1429)$	**B1**	Values in table, all probs correct (to 3SF) or 3 probabilities summing to 1
		4	
(ii)	$\text{Var}(X) = 1 \times \frac{4}{7} + 4 \times \frac{1}{7} - (\frac{6}{7})^2$ $= \frac{8}{7} - (\frac{6}{7})^2$	**M1**	Unsimplified correct numerical expression for variance **or** *their* probabilities from **(i)** $0 < p < 1$ in **unsimplified** variance expression
	$= \frac{20}{49}$ or 0.408	**A1**	Correct answer (0.40816…) nfww Final answer does **not** imply the method mark
		2	

X	0	1	2
Prob	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

Question 24

(i)	P(RB) + P(BR) = $\frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$ oe	M1	Multiply 2 probs together and summing two 2-factor probs, unsimplified, condone replacement
	P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) (0.485) oe	A1	Correct answer
	Method 2 1 – P(BB) – P(RR) = $1 - \frac{4}{12} \times \frac{3}{11} - \frac{8}{12} \times \frac{7}{11}$	M1	Multiply 2 probs together and subtracting two 2-factor probs from 1, unsimplified, condone replacement
	P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) oe	A1	Correct answer
	Method 3 P(diff colours) = $\frac{({}^4C_1 \times {}^8C_1)}{{}^{12}C_2}$	M1	Multiply 2 combs together and dividing by a combination
	= $\frac{16}{33}$	A1	Correct answer
		2	
(ii)	<table><tr><td>Number of red socks</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>$\frac{14}{33}$</td><td>$\frac{16}{33}$</td><td>$\frac{3}{33}$</td></tr></table>	B1	Prob distribution table drawn, top row correct, condone additional values with $p = 0$ stated
		B1	P(0) or P(2) correct to 3sf (need not be in table)
		B1	All probs correct to 3sf, condone P(0) and P(2) swapped if correct
		3	
(iii)	E(X) = $1 \times \frac{16}{33} + 2 \times \frac{3}{33} = \frac{16}{33} + \frac{6}{33} = \frac{22}{33}$ ($\frac{2}{3}$)	B1ft	ft their table if 0, 1, 2 only, $0 < p < 1$
		1	

Question 25

(i)	<table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P($X = x$)</td><td>$\frac{2}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{5}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{2}{18}$</td><td>$\frac{1}{18}$</td></tr></table>	B1	−2, −1, 0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(−2), P(−1), P(0), P(1), P(2), P(3),
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	
(ii)	E(X) = $\frac{-4-4+0+4+4+3}{18} = \frac{1}{6}$	M1	Correct unsimplified expression for the mean using their table, $\Sigma p = 1$, may be implied
	Var(X) = $\frac{8+4+0+4+8+9}{18} - \left(\frac{1}{6}\right)^2$ =11/6 – 1/36 (1.8333 – 0.02778)	M1	Correct, unsimplified expression for the variance using their table, and their mean^2 subtracted. Allow $\Sigma p \neq 1$
	= 65/36, (1.81)	A1	Correct answer
		3	

Question 26

(i)	x | -2 | -1 | 0 | 1 | 2 | 3 p | $\frac{1}{12}$ | $\frac{2}{12}$ | $\frac{3}{12}$ | $\frac{3}{12}$ | $\frac{2}{12}$ | $\frac{1}{12}$	**B1**	$-2, -1, 0, 1, 2, 3$ seen as top line of a pdf table with at least 1 probability OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3) (condone additional values with p=0 stated)
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	
(ii)	$E(X) = \frac{-2\times1-1\times2+0+1\times3+2\times2+1\times3}{12} = 0.5$	**M1**	Unsimplified expression for mean using *their* pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.
	$\text{Var}(X) = \frac{(-2)^2\times1+(-1)^2\times2+1^2\times3+2^2\times2+3^2\times1}{12} - (\textit{their}\ 0.5)^2$	**M1**	Unsimplified expression for variance using *their* pdf table (or correct) with at least 2 non-zero values and *their* E(X). Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly
	26/12 −1/4 = 23/12	**A1**	Correct final answer
		3	

Question 27

(i)	$6p + 0.1 = 1$ $p = 0.15$	**B1**	Correct answer
		1	
(ii)	$\text{Var}(X) = 1\times p + 1\times 2p + 4\times 2p + 16\times 0.1 - 1.15^2$	**M1**	Correct unsimplified formula, *their p* substituted (allow 1 error)
	$0.15 + 0 + 0.3 + 1.2 + 1.6 - 1.15^2$ $= 1.9275 = 1.93$ (3sf)	**A1**	Correct answer
		2	

Question 28

(i)

x	-1	1	2	3
p	k	k	$4k$	$9k$

B1 Probability distribution table with correct values of x, no additional values unless with probability 0 stated, at least one correct probability including k

$15k = 1$, **M1** Equating $\Sigma p = 1$, may be implied by answer

$k = \frac{1}{15}$ **A1** If 0 scored, SCB2 for probability distribution table with correct numerical probabilities.

3

(ii)

Method 1

$E(X) = 8k + 27k = 35k = \frac{35}{15} = \frac{7}{3}$ **B1FT** FT if 0< *their k*<1

$\text{Var}(X) = (k + k + 16k + 81k) - (35k)^2$ **M1** Correct formula for variance, in terms of k at least – must have '– mean^2'(ft).

$= 1.16, \frac{52}{45}$ **A1**

Method 2

$E(X) = \frac{8}{15} + \frac{27}{15} = \frac{35}{15} = \frac{7}{3}$ **B1FT** FT if 0< *their k*<1

$\text{Var}(X) = \frac{1}{15} + \frac{1}{15} + \frac{16}{15} + \frac{81}{15} - \left(\frac{7}{3}\right)^2$ **M1** Subst *their* values in correct var formula – must have '– mean^2'(ft) (condone probs not summing to exactly 1)

$= 1.16 (= 52/45)$ **A1** Using their values from **(i)**

3

Question 29

(a)

-1	0	0	1
0	1	1	2
2	3	3	4

x	-1	0	1	2	3	4
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

Probability distribution table with correct scores with at least one probability **B1**

At least 4 probabilities correct **B1**

All probabilities correct **B1**

3

(b)

$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$ **B1**

$\text{Var}(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$ **M1**

$\frac{37}{18}$ $(= 2.06)$ **A1**

3

Question 30

(a)

	1	1	2	2	3
1	1	1	2	2	3
2	2	2	2	2	3
3	3	3	3	3	3

M1

$\frac{7}{15}$ **AG** **A1**

2

(b)

x	1	2	3
Probability	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{7}{15}$

B1

P(1) or P(2) correct **B1**

3rd probability correct, **FT** sum to 1 **B1**

3

(c)

$E(X) = \frac{2+12+21}{15} = \frac{35}{15} = \frac{7}{3}$ **B1**

$\text{Var}(X) = \frac{1^2\times2+2^2\times6+3^2\times7}{15} - \left(\frac{7}{3}\right)^2$ **M1**

$\frac{22}{45}(0.489)$ **A1**

3

Question 31

x	0	1	2	3
Probability	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

B1

(**B1** for probability distribution table with correct outcome values)

$P(0) = \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6} = \frac{1}{56}$

$P(1) = \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3 = \frac{15}{56}$

$P(2) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times3 = \frac{30}{56}$

$P(3) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6} = \frac{10}{56}$

(**M1** for denominator $8\times7\times6$) **M1**

Any one probability correct (with correct outcome) **A1**

All probabilities correct **A1**

4

Question 32

(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	**M1**	One 3 factor probability with 3, 3, 5 as denominators
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total = $\frac{20}{45}=\frac{4}{9}$	**A1**	AG, Total of 3 products with clear context
		3	
(b)	x: 0, 1, 2, 3 Prob.: $\frac{1}{45}$, $\frac{8}{45}$, $\frac{20}{45}$, $\frac{16}{45}$	**B1**	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
(c)	$\text{Var(X)}=\frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45}-\left(\frac{32}{15}\right)^2$ $=\frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have '– mean^2' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	**A1**	
		2	

Question 33

(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3$	**M1**	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times3$, $1\leqslant a,b,c\leqslant5$, $d, e, f\leqslant8$, a, b, c, d, e, f, k all integers. $1<k\leqslant3$,
	$\frac{15}{56}$	**A1**	AG, WWW
	Alternative method for question 2(a)		
	$\frac{{}^5C_1\times{}^3C_2}{{}^8C_3}$	**M1**	$\frac{{}^aC_1\times{}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e\left(or\ {}^aC_1\times{}^bC_2\right)}{{}^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a+b=8$, $d+e=3$
	$\frac{15}{56}$	**A1**	AG, WWW, $\frac{15}{56}$ must be seen
		2	
(b)	x: 0, 1, 2, 3 Prob.: $\frac{1}{56}$, $\frac{15}{56}$, $\frac{30}{56}=\frac{15}{28}$, $\frac{10}{56}=\frac{5}{28}$ 0·0179, 0·268, 0·536, 0·179	**B1**	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	

(c)	$\text{Var}(X) = \dfrac{(0^2\times1)+1^2\times15+2^2\times30+3^2\times10}{56} - \left(\dfrac{15}{8}\right)^2$ $= \dfrac{15}{56}+\dfrac{120}{56}+\dfrac{90}{56}-\left(\dfrac{15}{8}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have ' – mean^2' (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\dfrac{225}{448}$, 0·502	**A1**	
		2	

Question 34

(a)	y: 1, 2, 3, 4 prob: $\frac{7}{16}$, $\frac{5}{16}$, $\frac{3}{16}$, $\frac{1}{16}$	**B1**	| | 1 | 2 | 3 | 4 | | 1 | 1 | 1 | 2 | 3 | | 2 | 1 | 2 | 1 | 2 | | 3 | 2 | 1 | 3 | 1 | | 4 | 3 | 2 | 1 | 4 | Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated'
		B1	One probability (linked with correct score) correct
		B1	2 more probs (linked with correct scores) correct
		B1 FT	4th prob correct, FT sum of 3 or 4 terms = 1
		4	
(b)	$\text{P(2}\mid\text{even)} = \dfrac{\frac{5}{16}}{\frac{6}{16}}$	**M1**	$\dfrac{\textit{their}\,\text{P}(2)}{\textit{their}\,\text{P}(2)+\textit{their}\,\text{P}(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or 0·833	**A1**	
		2	

Question 35

(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1-x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1-x) = 0{\cdot}58$	**M1**	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1-x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	**M1**	Equation of form 0·6 x a + 0·4 x b = 0·42; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1-x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
		3	
(b)	$(0.6\times0.3)^2$	**M1**	$(a \times b)^2$, a = 0·6, 0·4 and b = 0·7, 0·3, x, $(1-x)$ or $0{\cdot}18^2$, alone.
	0.0324	**A1**	
		2	

Question 36

(a)	x | 1 | 2 | 3 | 4 prob | $4k$ | $6k$ | $6k$ | $4k$	**B1**	Table with × values and one correct probability expressed in terms of k. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
(b)	$[4k + 6k + 6k + 4k = 1]\ k = \frac{1}{20}\ (= 0.05)$	**B1**	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X) = 1\times\frac{4}{20} + 2\times\frac{6}{20} + 3\times\frac{6}{20} + 4\times\frac{4}{20} = \frac{4}{20} + \frac{12}{20} + \frac{18}{20} + \frac{16}{20}$ $(= 2.5)$	**M1**	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\mathrm{Var}(X) = 1^2\times\frac{4}{20} + 2^2\times\frac{6}{20} + 3^2\times\frac{6}{20} + 4^2\times\frac{4}{20} - \left(their\ 2\frac{1}{2}\right)^2$ $= (4 + 24 + 54 + 64)\times their\ 0.05 - (their\ 2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20} + (2-2.5)^2\times\frac{6}{20} + (3-2.5)^2\times\frac{6}{20} + (4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(\mathrm{E}(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	**A1**	AG, NFWW.
		4	

Question 37

$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
$0.1 + 2q = 3(2p)$	**B1**	Use given information
Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
	4	

Question 38

(a)

X	–1	0	1	2	3
$P(X)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{2}{9}$

B1	Table with correct X values and at least one probability Condone any additional X values if probability stated as 0.
B1	2 correct probabilities linked with correct outcomes, may not be in table.
B1	3 further correct probabilities linked with correct outcomes, may not be in table. **SC** if less than 2 correct probabilities seen, award **SCB1** for sum of *their* 4 or 5 probabilities in table = 1
3	

(b)

$\left[E(X) = \frac{-1\times1 + (0\times2) + 1\times1 + 2\times3 + 3\times2}{9} = \right]$ $\frac{-1+1+6+6}{9}$	M1 May be implied by use in variance, accept unsimplified expression. FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999
[Var (X) =] $\left[\frac{-1^2\times1 + (0^2\times2) + 1^2\times1 + 2^2\times3 + 3^2\times2}{9} - (\text{their } E(X))^2\right]$ $\frac{1+0+1+12+18}{9} - (\text{their } E(X))^2$	M1 Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table even if *their* 3 or more probabilities not summing to 1.
$E(X) = \frac{4}{3}$ or 1.33 and $Var(X) = \frac{16}{9}$ or 1.78	A1 Answers for E(X) and Var(X) must be identified
	3 **N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1

Question 39

(a)	$P(X=3)=\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$	M1	$\frac{m}{7}\times\frac{n}{6}\times\frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

(b)	x: 1, 2, 3, 4, 5 p: $\frac{15}{35}$, $\frac{10}{35}$, $\frac{6}{35}$, $\frac{3}{35}$, $\frac{1}{35}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X=3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X=3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
(c)	$[E(X)=1\times\frac{15}{35}+2\times\frac{10}{35}+3\times\frac{6}{35}+4\times\frac{3}{35}+5\times\frac{1}{35}]$ $E(X)=\frac{15+20+18+12+5}{35}\left[=\frac{70}{35}=2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X)=\left[\frac{1^2\times15+2^2\times10+3^2\times6+4^2\times3+5^2\times1}{35}-2^2=\right]$ $\frac{15+40+54+48+25}{35}-2^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[=\frac{182}{35}-4\right]=\frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

Question 40

(a)	$p+q+0.65=1$	**B1**	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	**B1**	Use given information.
	Solve 2 linear equations	**M1**	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3,\frac{3}{10},\quad q=0.05,\frac{1}{20}$	**A1**	CAO, both WWW If **M0** with correct answers **SC B1**.
		4	
(b)	Var (X) = *their* $0.3+4\times$*their* $0.05+9\times0.05-0.55^2$	**M1**	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475\left[\frac{259}{400}\right]$	**A1**	CAO (must be exact).
		2	
(c)	$1-P(0,1,2)=1-({}^{12}C_0\,0.3^0\,0.7^{12}+{}^{12}C_1\,0.3^1\,0.7^{11}+{}^{12}C_2\,0.3^2\,0.7^{10})$	**M1**	One correct term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0<x<12$, $0<p<1$.
	$1-(0.01384+0.07118+0.16779)$	**A1FT**	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to **FT** *their* p from **6(a)** or correct.
	0.747	**A1**	
		3	
(d)	$(0.95)^8\times0.05=0.0332$ or $0.95^8-0.95^9=0.0332$	**B1**	Evaluated.
		1	

Question 41

(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	**M1**	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability × 3, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$, (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
(b)	X: 0, 1, 2, 3 $P(X)$: $\frac{24}{504}$ $\left[=\frac{1}{21}, 0.0476\right]$; $\frac{180}{504}$ $\left[=\frac{5}{14}, 0.357\right]$; $\frac{240}{504}$ $\left[=\frac{10}{21}, 0.476\right]$; $\frac{60}{504}$ $\left[=\frac{5}{42}, 0.119\right]$	**B1**	Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or X value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
(c)	$[E(X) =]$ $\frac{840}{504}, \frac{5}{3}$, 1.67	**B1**	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

Question 42

(a)	x: -1, 0, 1, 2, 3 p: $\frac{1}{12}=0.0833$; $\frac{2}{12}=0.167$; $\frac{4}{12}=0.333$; $\frac{3}{12}=0.25$; $\frac{2}{12}=0.167$	**B1**	<table><tr><td></td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>– 1</td><td>–1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>3</td></tr></table> Table with x values and at least one probability substituted, $0 < p < 1$. Condone any additional x values if probability stated as 0.
		B1	2 correct identified probabilities.
		B1	All probabilities correct (accept to 3sf). **SC** if less than 2 correct probabilities: **SC B1** 4 or 5 probabilities summing to one.
		3	
(b)	$E(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$	**M1**	May be implied by use in Variance, accept unsimplified expression. Probabilities must sum to 1 ± 0.001.
	$\text{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$	**M1**	Appropriate variance formula using *their* $(E(X))^2$. **FT** accept probabilities not summing to 1. Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ *or* $\frac{35}{12}-\frac{25}{9}$ from correct table.
	$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48}$, 1.35	**A1**	WWW
		3	

Question 43

(a)

X	−2	−1	0	1	2
P(X)	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{16}$
	0.0625	0.1875	0.3125	0.3125	0.125

B1 Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.

B1 3 correct probabilities linked with correct outcomes, may not be in table.

B1 2 further correct probabilities linked with correct outcomes, may not be in table
No repeated X values.

SC if less than 3 correct probabilities seen, award
SCB1 Sum of *their* probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1±0·01 or better).

3

(b)

$\left[\frac{1}{16}\times -2^2+\frac{3}{16}\times -1^2\left(+\frac{5}{16}\times 0^2\right)+\frac{5}{16}\times 1^2+\frac{2}{16}\times 2^2-\left(\frac{1}{4}\right)^2\right]$

$\frac{1\times 4+3\times 1+5\times 0+5\times 1+2\times 4}{16}-0.25^2$

M1 Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT *their* table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$.
Condone 1 error providing all probabilities <1 and 0.25^2 used

$\left[=\frac{5}{4}-\frac{1}{16}=\right]\frac{19}{16}$, 1.1875

A1 Condone 1.188 or 1.19 WWW

2

Question 44

(a)

$k=\frac{1}{18}$ $(4k+k+4k+9k=18k=1)$

x	−2	1	2	3
P(X=x)	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

B1 SOI

M1 Table with correct x values and at least one probability accurate using *their k*.
Values need not be in order, lines may not be drawn, may be vertical, x and P(X=x) may be omitted.
Condone any additional X values if probability stated as 0.

A1 Remaining probabilities correct.

3

(b)

$\left[E(X)=\frac{4\times -2+1\times 1+4\times 2+9\times 3}{18}=\right]$

$\frac{-8+1+8+27}{18}$

M1 $-8k+k+8k+27k$
May be implied by use in Variance. Accept unsimplified expression.
FT *their* table if probabilities sum to 1 or 0.999.
SC B1 28k.

$\left[\text{Var}(X)=\frac{4\times(-2)^2+1\times 1^2+4\times 2^2+9\times 3^2}{18}-(\textit{their}\,E(X))^2=\right]$

$=\frac{16+1+16+81}{18}-\left(\textit{their}\,\frac{28}{18}\right)^2$

M1 $16k+k+16k+81k-(\textit{their}\ \text{mean})^2$
FT *their* table even if probabilities not summing to 1.
Note: If table is correct, $\frac{114}{18}-(\textit{their}\,E(X))^2$ M1.
SC B1 $114k-(\textit{their}\ \text{mean})^2$.

E(X) = $\frac{14}{9}$, $1\frac{5}{9}$, 1.56, Var (X) = $\frac{317}{81}$, $3\frac{74}{81}$, 3.91

A1 Answers for E(X) and Var(X) must be identified.
$3.91 \leqslant \text{Var}(X) \leqslant 3.914$

3

Question 45

(a)

x	2	3	4	5	6
p	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
	0.02778	0.1111	0.2778	0.3333	0.25

B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
3	**SC B1** for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.

(b) If method FT from *their* incorrect **(a)**, expressions for E(X) and Var(X) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.

$\left[\mathrm{E}(X)=\frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36}=\right]\ \frac{\mathbf{2+12+40+60+54}}{\mathbf{36}}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to $0.999 \leqslant$ total $\leqslant 1$ ($0 < p < 1$).
$\left[\mathrm{Var}(X)=\frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36}-(\mathit{their}\ \mathrm{E}(X))^2=\right]$ $\frac{\mathbf{1\times4+4\times9+10\times16+12\times25+9\times36}}{\mathbf{36}}-\left(\mathit{their}\ \frac{\mathbf{14}}{\mathbf{3}}\right)^2$ $\left[\frac{4+36+160+300+324}{36}-\left(\mathit{their}\ \frac{14}{3}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
$\mathrm{E}(X)=\frac{168}{36},\ \frac{14}{3},\ 4.67$ $\mathrm{Var}(X)=\frac{10}{9},\ 1\frac{1}{9},\ 1.11,\ \frac{1440}{1296}$	**A1**	Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
	3	

Question 46

(a)

$a = \mathrm{P(1\ head)} = 0.7\times(0.5)^3 + 0.3\times(0.5)^3\times3 = \frac{1}{5}$	**B1**	Clear statement of unevaluated correct calculation $=\frac{1}{5}$. AG
$b = 0.7\times0.5^3\times3 + 0.3\times0.5^3\times3 = \frac{3}{8}$	**M1**	Clear statement of unevaluated calculation for either b or c
$c = 0.7\times0.5^3\times3 + 0.3\times0.5^3 = \frac{3}{10}$	**A1**	For either b or c correct
$\left[or\ c=\frac{27}{40}-b\right]$	**B1 FT**	*their* b + *their* $c = \frac{27}{40}$
	4	

(b)

$\left[\mathrm{E}(X)=\frac{3\times0+16\times1+30\times2+24\times3+7\times4}{80}=\right]\frac{176}{80}$ or 2.2	**B1 FT**	Correct or accept unsimplified calculation using *their* values for b and c seen (sum of probabilities = 1)
	1	

(c)

[P(0, 1, 2) =]$^{10}C_0\ 0.2^0\ 0.8^{10} + {}^{10}C_1\ 0.2^1\ 0.8^9 + {}^{10}C_2\ 0.2^2\ 0.8^8$	**M1**	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
0.107374 + 0.268435 + 0.301989	**A1**	Correct expression, accept unsimplified leading to final answer
0.678	**B1**	$0.677 < p \leqslant 0.678$
Alternative method for question 4(c)		
$1 - [{}^{10}C_{10}\ 0{\cdot}2^{10}0.8^0 + {}^{10}C_9\ 0{\cdot}2^9 0.8^1 + {}^{10}C_8\ 0{\cdot}2^8 0.8^2 + {}^{10}C_7\ 0{\cdot}2^7 0.8^3 + {}^{10}C_6\ 0{\cdot}2^6 0.8^4 + {}^{10}C_5\ 0{\cdot}2^5 0.8^5 + {}^{10}C_4\ 0{\cdot}2^4 0.8^6 + {}^{10}C_3\ 0{\cdot}2^3 0.8^7]$	**M1**	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
	A1	Correct expression, accept unsimplified
0.678	**B1**	$0.677 < p \leq 0.678$
	4	

(d)

$0.8^6\times0.2 + 0.8^7\times0.2 = 0.0524288 + 0.041943$	**M1**	$p^l\times(1-p) + p^m\times(1-p)$, $l = 6, 7$ $m = l + 1$, $0 < p < 1$
0.0944	**A1**	$0.09437 \leqslant p \leqslant 0.0944$
	2	

Question 47

(a)

Method 1: Scenarios identified

[no of ways for score of 2 are] 222, 211, 212, 221, 122, 112, 121 [Total options = 64]	**B1**	7 correct scenarios identified, no incorrect.
[So P($X = 2$) =] $\frac{7}{4\times4\times4} = \frac{7}{64}$	**M1**	$\frac{a}{4\times4\times4}$, a = *their* number of correct identified scenarios > 4
	A1	Approach identified, WWW.

Method 2: P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one spinner and 1 on two spinners)

$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + {}^3C_1\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$	**B1**	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(or\ {}^3C_1\right)\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + d$, $0 < d < 1$
	M1	$\left(\frac{1}{4}\right)^3 + e\left(\frac{1}{4}\right)^3 + f\left(\frac{1}{4}\right)^3$ $1 < e < 5$ and $1 < f < 5$
[So P($X = 2$) =] $= \frac{7}{64}$	**A1**	Approach identified, WWW.

Method 3: P(1 or 2 on each spinner) – P(1 on all spinners)

$\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$	**B1**	$\left(\frac{1}{2}\right)^3 - b$ seen, $0 < b < 1$
	M1	$\left(\frac{1}{2}\right)^3 - c^3$, $0 < c < ½$
[So P($X = 2$) =] $\frac{7}{64}$	**A1**	Approach identified, WWW.
	3	

(b)

P($X = 1$) $= \frac{1}{64}$	**B1**	P($X = 1$) or P($X = 4$) correct. Condone answers not in probability distribution table if clearly identified.
P($X = 4$) $= \left[1 - \frac{1}{64} - \frac{7}{64} - \frac{19}{64} = \right] \frac{37}{64}$	**B1 FT**	All 4 probabilities summing to 1.
	2	

(c)

P($Y = 6$) $= \left[\left(\frac{3}{4}\right)^5 \times \frac{1}{4} = \right] 0.0593, \frac{243}{4096}$	**B1**	Accept 0.059326… to 4 or more SF.
	1	

(d)

$\left(\frac{3}{4}\right)^4$	**M1**	$\left(\frac{3}{4}\right)^g$, $g = 4, 5$ **or** p^4 where $0 < p < 1$
$= \frac{81}{256}, 0.316$	**A1**	Accept 0.316406…to 4 or more SF.

Alternative method for Question 4(d)

P($Y > 4$) $= 1 - $ P($Y \leqslant 4$) $= 1 - \left(\frac{1}{4} + \frac{3}{4}\times\frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4}\right)$ $\left[= 1 - \frac{175}{256}\right]$	**M1**	Correct or $1 - \left(\frac{1}{4} + \frac{3}{4}\times\frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^4\right)$ **or** $1 - \left(p + qp + q^2p + q^3p\right)$ where $0 < p < 1$ and $q = 1 - p$
$= \frac{81}{256}, 0.316$	**A1**	Accept 0.316406…to 4 or more SF.
	2	

Question 48

(a)	[P(17 or 18) =] $\frac{4}{216}=\frac{1}{54}$,0.0185(185…)	**B1**	May be seen used in calculation.
	P(X = 6) = $\left(\frac{53}{54}\right)^5.\frac{1}{54}$	**M1**	$p(1-p)^5$, $0<p<1$
	0.0169	**A1**	$0.01686 < p \leqslant 0.0169$ If A0 scored **SC B1** for $0.01686 < p \leqslant 0.0169$
		3	
(b)	[P(X < 8) =] $1-\left(\frac{53}{54}\right)^7$	**M1**	$1-\left(their\left(\frac{53}{54}\text{ or }0.98148\right)\text{or correct}\right)^r$, $r=7,8$ $0<their\ p<1$
	0.123	**A1**	$0.1225 \leqslant p \leqslant 0.123$
	Alternative method for Question 3(b)		
	[P(X < 8) =] $\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^2\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^3\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^4\left(\frac{1}{54}\right)+$ $\left(\frac{53}{54}\right)^5\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^6\left(\frac{1}{54}\right)$	**M1**	$q+pq+p^2q+p^3q+p^4q+p^5q\left[+p^6q\right]$, $p+q=1, 0<p,q<1, q=their\frac{53}{54}$
	0.123	**A1**	$0.1225 \leqslant p \leqslant 0.123$
		2	

Question 49

P(1H) = $\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}=\frac{15}{32}$

P(2H) = $\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{7}{32}$

X	0	1	2	3
$p(X)$	$\frac{9}{32}$ 0.28125	$\frac{15}{32}$ 0.46875	$\frac{7}{32}$ 0.21875	$\frac{1}{32}$ 0.03125

B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
B1	P(1) or P(2) correct, need not be in table, accept unsimplified.
B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF.
	SC B1 for 4 probabilities ($0<p<1$) sum to 1 ± 0.005 with P(1) **and** P(2) incorrect.
3	

Question 50

$0.12+p+q+0.16+0.3=1$	**B1**	Sum of probabilities = 1 $p+q$ = 0.42 OE.
$-0.24-p+0.5q+0.16+0.6=0.28$	**B1**	Form equation using E(X) = 0.28 $-p+0.5q=-0.24$ OE. Accept unsimplified.
Attempt to solve *their* two equations in p and q	**M1**	**Either** Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** Elimination method by writing both equations in the same form (usually $ap+bq=c$) and + or – to find an equation in one unknown and finding values for both unknowns.
$q=0.12,\ p=0.3$	**A1**	CAO, both WWW. If M0 awarded **SC B1** for both correct WWW.
	4	

Question 51

(a)	$0.6(0.5)^3 + 0.4(0.5)^3 \times 3$	**B1**	Either $0.6(0.5)^3 + a$ or $b + 0.4(0.5)^3 \times (3 \text{ or } {}^3C_1)$, $0 < a, b < 1$ seen.
		M1	$0.6(0.5)^3 + 0.4(0.5)^3 \times d$ seen, $d = 1, 3$. Condone $0.075 + 0.05 \times d$, $d = 1, 3$.
	$= 0.225$	**A1**	AG full supporting working required. Scenarios identified and linked to calculations.
		3	
(b)		**B1**	Either [P(2) =] 0.375, $\frac{3}{8}$ or [P(3) =] 0.275, $\frac{11}{40}$ seen. Condone not in table if identified.
		B1 FT	Both values in table. FT P(2) + P(3) = 0.650 .
		2	
(c)	Var(X) $= [1^2 \times]0.225 + 2^2 \times \textit{their}\, 0.375 + 3^2 \times \textit{their}\, 0.275 + 4^2 \times 0.075 - 2.1^2$	**M1**	Appropriate variance formula from their probability distribution table with at least 4 terms, $0 < \textit{their}\ P(x) < 1$. Condone 4.41 for 2.1^2. Condone mean clearly recalculated inaccurately. Or $0.225 + 4 \times \textit{their}\, 0.375 + 9 \times \textit{their}\, 0.275 + 16 \times 0.075 - 2.1^2$ Condone 2.1^2 for 4.41.
	$\left[5.4 - 2.1^2\right] = 0.99[0]$	**A1**	If M0 awarded **SC B1** for 0.99[0] WWW.
		2	

x	0	1	2	3	4
P($X = x$)	0.05	0.225	**0.375**	**0.275**	0.075

Question 52

(a)	$\left[P(HH) = \frac{1}{4}\right]$ [E(X) =] 4	**B1**	
		1	
(b)	$\left[P(X = 5) = \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) = \right]$ 0.0791	**B1**	$\frac{81}{1024}$
		1	
(c)	$[P(X < 7) =]1 - \left(\frac{3}{4}\right)^6$ or $\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3^2}{4} \times \frac{1}{4} + \ldots + \frac{3^5}{4} \times \frac{1}{4}$	**M1**	$1 - p^n$, $0 < p < 1$, $n = 6, 7$ or $p + p(1-p) + p(1-p)^2 + \ldots + p(1-p)^n$, where $n = 4, 5$.
	$= \frac{3367}{4096}$, 0.822	**A1**	Accept 0.82202148… to at least 3SF.
		2	

Question 53

(a)	$[P(X=4)=3P(X=2)]$ $4k(4+a)=3\times 2k(2+a)$ $16k+4ak=12k+6ak$	**M1**	Using $P(X=4)=3P(X=2)$ to form an equation in a and k.
	$a=2$	**A1**	If M0 scored, **SC B1** for $a=2$ www.
	$3k+8k+15k+24k=1$	**M1**	Using sum of probabilities = 1 to form an equation in k: $k(1+a)+2k(2+a)+3k(3+a)+4k(4+a)=1$.
	$k=\frac{1}{50}$	**A1**	If M0 scored, **SC B1** for $k=\frac{1}{50}$ www.
		4	
(b)	X: 1, 2, 3, 4 $P(X)$: $\frac{3}{50}$, 0.06 ; $\frac{8}{50}$, 0.16 ; $\frac{15}{50}$, 0.3 ; $\frac{24}{50}$, 0.48	**B1 FT**	X: 1, 2, 3, 4 $P(X)$: $k(1+a)$, $2k(2+a)$, $3k(3+a)$, $4k(4+a)$ $0<p<1$ for all outcomes, must be numerical.
		1	
(c)	$\text{Var}(X)=\frac{3}{50}\times 1+\frac{8}{50}\times 2^2+\frac{15}{50}\times 3^2+\frac{24}{50}\times 4^2-3.2^2$	**M1**	Correct formula for variance method from their probability distribution table, $0 \leqslant$ *their* $P(x) \leqslant 1$. Accept $\frac{3+32+135+384}{50}-\frac{256}{25}$.
	$[=11.08-3.2^2=]\ 0.84[0],\ \frac{21}{25}$	**A1**	If M0 score **SC B1** for 0.84 www.
		2	

Question 54

(a)	$[3k+3k+8k=1, so]\ k=\frac{1}{14}$	**B1**	
	x: –2, 2, 3 $P(x)$: $\frac{3}{14}$, 0.214 ; $\frac{3}{14}$, 0.214 ; $\frac{8}{14}$, 0.571	**B1 FT**	Table with correct values of x, and at least one correct probability linked with outcome. FT *their k*. Condone any additional X values if probability stated as 0.
		B1 FT	The outcomes in the table must be –2, 2 and 3. 2 further correct probabilities in table **or** 3 correct probabilities not in table linked to outcomes, **or** 3 correct FT probabilities in table using *their k*, **or** 3 incorrect probabilities summing to 1 in table if k not stated.
			If k not calculated, **SC B1** for the below. x: –2, 2, 3 $P(x)$: $3k$, $3k$, $8k$
		3	
(b)	$\left[E(X)=-2\times\frac{3}{14}+2\times\frac{3}{14}+3\times\frac{8}{14}=\right]$ $-\frac{6}{14}+\frac{6}{14}+\frac{24}{14}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 3 probabilities summing to $0.999 \leqslant$ *total* $\leqslant 1$ $(0<p<1)$ or in terms of k.
	$\left[\text{Var}(X)=(-2)^2\times\frac{3}{14}+2^2\times\frac{3}{14}+3^2\times\frac{8}{14}-(\textit{their}\,E(X))^2=\right]$ $4\times\frac{3}{14}+4\times\frac{3}{14}+9\times\frac{8}{14}-\left(\textit{their}\ \frac{12}{7}\right)^2$ $\left[\frac{12+12+72}{14}-\left(\textit{their}\ \frac{12}{7}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with 3 or more probabilities $(0<p<1)$ which need not sum to 1, or in terms of k with an expression no more evaluated than shown.
	$E(X)=\frac{12}{7}$, 1.71, $1\frac{5}{7}$ $\text{Var}(X)=\frac{192}{49}$, 3.92, $3\frac{45}{49}$	**A1**	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E, V, μ, σ^2, etc.). If A0 earned, **SC B1** for identified correct final answers.
		3	

Question 55

(a)	[P(no rain) = $0.6\times(0.8)^3$ =] 0.3072, $\frac{192}{625}$	**B1**	Exact value required
		1	
(b)	$0.6\times0.8\times0.2$	**M1**	$a\times b\times c$ where a, b = 0.6, 0.8, c = 0.2, 0.4, 0.7 . Condone including Wednesday with both 0.3 and 0.7 used.
	= 0.096[0], $\frac{12}{125}$	**A1**	
		2	
(c)	$P(RDDD)=0.4\times0.3\times0.8\times0.8=0.0768,\ \frac{48}{625}$ $P(DRDD)=0.6\times0.2\times0.3\times0.8=0.0288,\ \frac{18}{625}$ $P(DDRD)=0.6\times0.8\times0.2\times0.3=0.0288,\ \frac{18}{625}$ $P(DDDR)=0.6\times0.8\times0.8\times0.2=0.0768,\ \frac{48}{625}$	**B1**	Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient.
		M1	Add 4 probability values, $0<p<1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark: Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	0.2112, $\frac{132}{625}$	**A1**	Accept 0.211 If 0/3 scored **SC B1** for 0.2112, $\frac{132}{625}$.
		3	

Question 56

(a)	$[P(X=4) = (0.8)^3(0.2) =] \ 0.1024, \frac{64}{625}$	**B1**	Condone 0.102 .
		1	

(b)	$[P(X<6) =] 1-0.8^5$	**M1**	$1-0.8^d$, $d = 5, 6$.
	$= 0.672, \frac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\frac{2101}{3125}$ or 0.67232 only.
	Alternative Method for Question 4(b)		
	$[P(X<6) =] \left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	**M1**	If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$
	$= 0.672, \frac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\frac{2101}{3125}$ or 0.67232 only.
		2	

(c)	$\left[P(X>0 \mid X\neq 2) = \frac{P(X>0 \cap X\neq 2)}{P(X\neq 2)} =\right]$ $= \frac{\frac{14}{25}}{\frac{19}{25}}$ $= \frac{14}{19}, 0.737$	**M1**	$[P(X>0 \cap X\neq 2) =] \ \frac{14}{25}, 0.56[0]$ seen as numerator or denominator of conditional probability fraction.
		M1	$[P(X\neq 2) =] \ \frac{19}{25}, 0.76[0]$ seen as denominator of conditional probability fraction.
		A1	Final answer = $\frac{14}{19}$, 0.7368421… to at least 3SF. If A0, **SC B1** for correct final answer www.
	Alternative Method for Question 4(c)		
	1 2 3 4 5 1 0 1 2 3 4 2 1 0 1 2 3 3 2 1 0 1 2 4 3 2 1 0 1 5 4 3 2 1 0 $\left[P(X>0 \mid X\neq 2) = \frac{\text{Number of outcome}(X>0 \cap X\neq 2)}{\text{Number of outcomes } X\neq 2} =\right]$ $\frac{14}{19}, 0.737$	**M1**	$\left[\text{Number of outcome}(X>0 \cap X\neq 2) =\right] 14$ seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\text{Number of outcome}(X\neq 2) =\right] 19$ seen as denominator of conditional probability fraction.
		A1	Final answer = $\frac{14}{19}$, 0.7368421… to at least 3SF.
		3	

(d)	$[P(X>2) = 1 - P(0, 1, 2) \text{ with } p = \frac{6}{25}]$ $1-({}^9C_0\left(\frac{19}{25}\right)^9+{}^9C_1\left(\frac{6}{25}\right)^1\left(\frac{19}{25}\right)^8+{}^9C_2\left(\frac{6}{25}\right)^2\left(\frac{19}{25}\right)^7)$ $[1 - (0.08459 + 0.2404 + 0.3037)]$	**M1**	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0<p<1, 0<x<9$.
		A1	$1-({}^9C_0\,(1-p)^9+{}^9C_1\,(p)^1(1-p)^8$ $+{}^9C_2\,(p)^2(1-p)^7)$, $0<p<1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.
	0.371	**B1**	$0.371 \leqslant p < 0.3715$.
	Alternative Method for Question 4(d)		
	$[P(X>2) = P(3,4,5,6,7,8,9) \text{ with } p = \frac{6}{25}]$ ${}^9C_3\left(\frac{6}{25}\right)^3\left(\frac{19}{25}\right)^6+{}^9C_4\left(\frac{6}{25}\right)^4\left(\frac{19}{25}\right)^5+\ldots+{}^9C_8\left(\frac{6}{25}\right)^8\left(\frac{19}{25}\right)^1+{}^9C_9\left(\frac{6}{25}\right)^9$ $[0.2238 + 0.1060 + \ldots + 7.529\times10^{-5} + 2.642\times10^{-6}]$	**M1**	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0<p<1, 0<x<9$.
		A1	${}^9C_3\,(p)^3(1-p)^6+{}^9C_4\,(p)^4(1-p)^5+\ldots+{}^9C_8$ $(p)^8(1-p)^1+{}^9C_9\,(p)^9$, $0<p<1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer.
	0.371	**B1**	$0.371 \leqslant p < 0.3715$.
		3	

Question 57

(a)	$[\mathrm{P}(X=3)=]\ \frac{3}{4}\times\left(\frac{1}{4}\right)^3\times 4$	**M1**	$\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times q$; q a positive integer (1 may be implied).
	$=\frac{3}{64}$	**A1**	AG.
		2	
(b)	x: 0, 1, 2, 3, 4 $\mathrm{P}(X=x)$: $\frac{81}{256}$, $\frac{27}{64}$, $\frac{27}{128}$, $\frac{3}{64}$, $\frac{1}{256}$	**B1**	Either $\mathrm{P}(1)=\frac{27}{64}, 0.421875$ or $\mathrm{P}(2)=\frac{27}{128}, 0.2109375$ correct to at least 3SF. Condone not in table.
		B1 FT	Both values in table. FT $\mathrm{P}(1)+\mathrm{P}(2)=\frac{81}{128}, 0.6328125$.
		2	
(c)	$[\mathrm{E}(X)=]\ [0\times\frac{81}{256}]+1\times \textit{their}\ \frac{27}{64}+2\times \textit{their}\ \frac{27}{128}+3\times\frac{12}{256}+4\times\frac{1}{256}$	**M1**	Correct method from *their* probability distribution table with at least 4 terms, $0<$ *their* $\mathrm{P}(x)<1$, accept partially evaluated. $=0+\frac{27}{64}+\frac{54}{128}+\frac{36}{256}+\frac{4}{256}$
	$=1$	**A1**	
		2	
(d)	Mean $=96\times\frac{67}{256}=25.125$ Var $=96\times\frac{67}{256}\times\frac{189}{256}=18.549$	**B1**	25.125, $25\frac{1}{8}$ and 18.5493… to at least 3SF seen, allow unsimplified ($4.3068\leqslant\sigma\leqslant 4.307$ implies correct variance).
	$\mathrm{P}(X<20)=\mathrm{P}(Z<\frac{19.5-25.125}{\sqrt{18.549}})$	**M1**	Substituting *their* μ and σ into ± standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula. Note: $\frac{\pm 5.625}{\sqrt{18.549}}$ seen gains M2 BOD.
	$[=\mathrm{P}(Z<-1.306)=1-\Phi(1.306)=]\ 1-0.9042=$	**M1**	Appropriate area Φ, from final process. Must be a probability.
	0.0958	**A1**	$0.0957\leqslant p\leqslant 0.0958$. **SC B1** for $0.0957\leqslant p\leqslant 0.0958$ if B1M0M0M1 scored.
		5	