

AS-Level
Discrete Random Variable
May : 2013- May : 2025
Answers

Question 1

x	0	1	2
$P(X = x)$	5/12	1/2	1/12

B1

Values 0, 1, 2 seen in table with at least 1 prob

$$P(0) = P(\bar{5}, \bar{5}) = \frac{6}{9} \times \frac{5}{8} = 30/72 \text{ (5/12)}$$

(0.4166)

B1

Correct P(0) unsimplified

$$P(1) = 0.5 \text{ from part (ii)}$$

$$P(2) = 6/72 \text{ (1/12) (0.0833) from part (i)}$$

B1 ft 3

If $x=0,1,2,(3)$ ft $\Sigma p = 1$, no -ve values, all probabilities <1

Question 2

- (i) options (3, 4, 4,) or (4, 3, 4) or (4, 4, 3)
 Probs $(4/10 \times 6/9 \times 5/8) \times 3C1$
 $= 360/720$
 $= \frac{1}{2}$ AG

M1

Summing three 3-factor options oe
 $10 \times 9 \times 8$ seen in denom

M1

$$\text{OR } \frac{{}^6C_2 \times {}^4C_1}{{}^{10}C_3} = \frac{1}{2} \text{ AG}$$

A1

[3] Correct answer

M1

One of $6C_2$ or $4C_1$ seen in num

M1

$10C_3$ in denom

A1

Correct answer

(ii)

sum	9	10	11	12
Prob	24/720	216/720	360/720	120/720

B1

[4] 9, 10, 11, 12 only seen

B1

One correct prob other than P(11), with
 or without replacement

$$P(3, 3, 3) = 4/10 \times 3/9 \times 2/8 = 24/720 \text{ (1/30)}$$

B1

Another correct prob

$$P(3, 3, 4) = 4/10 \times 3/9 \times 6/8 \times 3C1$$

$$= 216/720 \text{ (3/10)}$$

$$P(4, 4, 4) = 6/10 \times 5/9 \times 4/8 = 120/720 \text{ (1/6)}$$

B1

Σ all 4 probs = 1

Question 3

<p>(i) if throw H then smallest score is 2 $P(T, 1) = 1/2 \times 1/4 = 1/8$ AG</p>	<p>B1 B1</p>	<p>2</p>	<p>Or equivalent</p>																		
<p>(ii) P(3) from two dice = 2/16 seen</p> <p>$P(H, 3) = 1/2 \times 2/16 = 2/32$ $P(T, 3) = 1/2 \times 1/4 = 1/8$ So $P(3) = 6/32 = 3/16$ AG</p>	<p>B1 M1 A1 A1</p>	<p>4</p>	<p>From (1, 2) and (2, 1)</p> <p>Summing P(H, 3) and P(T, 3)</p> <p>One correct</p> <p>Correct answer must see clear reasoning</p>																		
<p>(iii)</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>Prob</td> <td></td> <td>5/32</td> <td></td> <td>7/32</td> <td></td> <td>3/32</td> <td></td> <td></td> </tr> </table>	X	1	2	3	4	5	6	7	8	Prob		5/32		7/32		3/32			<p>B1 B1 B1</p>	<p>3</p>	<p>One correct prob</p> <p>A second correct prob</p> <p>A third correct prob</p>
X	1	2	3	4	5	6	7	8													
Prob		5/32		7/32		3/32															

Question 4

<p>(i) $P(2) = {}^6C_3 \times {}^3C_2 / {}^9C_5$ OR</p> $\frac{{}^6C_3 \times {}^3C_2}{{}^6C_5 + {}^6C_4 \times {}^3C_1 + {}^6C_3 \times {}^3C_2 + {}^6C_2 \times {}^3C_3}$ <p>OR</p> $3/9 \times 2/8 \times 6/7 \times 5/6 \times 4/5 \times {}^5C_2 = 10/21$ <p>$= 60/126$ AG</p>	<p>M1 OR M1 OR M1 A1</p>	<p>2</p>	<p>Using combinations ${}^aC_b \times {}^cC_d / {}^eC_f$</p> <p>Mult 5 probs with a pC_q</p> <p>If 5C_2 replace by 10, oe must be justified</p> <p>Legit method, as answer given</p>										
<p>(ii)</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Prob</td> <td>2/42</td> <td>15/42</td> <td>20/42</td> <td>5/42</td> </tr> </table> <p>$P(0) = {}^6C_5 / {}^9C_5 = 6/126$ $P(1) = {}^6C_4 \times {}^3C_1 / {}^9C_5 = 45/126$ $P(3) = {}^6C_2 \times {}^3C_3 / 126 = 15/126$</p>	x	0	1	2	3	Prob	2/42	15/42	20/42	5/42	<p>B1 B1 B1[†]</p>	<p>4</p>	<p>0, 1, 2, 3 only seen in table. Condone $x = 4, 5$ in table if $P(x) = 0$ or blank and values in table for $x = 0, 1, 2, 3$</p> <p>Any correct prob other than P(2)</p> <p>Any other correct prob</p> <p>$\Sigma P(x) = 1, 3 < n(x) < 6$</p>
x	0	1	2	3									
Prob	2/42	15/42	20/42	5/42									

Question 5

(i) A: P(H) = 2/3, P(T) = 1/3 B: P(H) = 1/4, P(T) = 3/4	M1		Using some of 2/3, 1/3, 1/4 or 3/4 in a calculation involving prod of 3 probs
P(1H) = P(HTT) + P(THT) + P(TTH) = (2/3 × 1/3 × 3/4) + (1/3 × 2/3 × 3/4)	M1		Summing 3 options not all the same
+ (1/3 × 1/3 × 1/4) = 13/36 AG	A1	3	Correct answer

(ii)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>P</td> <td>3/36</td> <td>13/36</td> <td>16/36</td> <td>4/36</td> </tr> </table>	x	0	1	2	3	P	3/36	13/36	16/36	4/36	B1		0, 1, 2, 3 seen for table no probs needed, table not absolutely necessary if calcs shown
x	0	1	2	3										
P	3/36	13/36	16/36	4/36										
P(0H) = P(TTT) = 1/3 × 1/3 × 3/4 = 1/12	B1			One prob correct other than (i) condone 0.083 for 0.0833										
P(2H) = P(HHT) + P(HTH) + P(THH) = (2/3 × 2/3 × 3/4) + (2/3 × 1/3 × 1/4) + (1/3 × 2/3 × 1/4) = 4/9 not 2/3 × 2/3	B1			A second prob correct need 3 factors can be implied										
P(3H) = P(HHH) = 2/3 × 2/3 × 1/4 = 1/9	B1✓	4		A third prob correct ft 23/36 – Σ their 2 probs										
(iii) E(X) = 13/36 + 32/36 + 12/36 = 57/36 (19/12) (1.58)	M1			Attempt to evaluate Σxp at least 3 vals of x in table										
	A1	2		Correct answer										

Question 6

(i) P(exactly 2) = $\frac{{}^6C_2}{{}^8C_4} = \frac{15}{70} = \frac{3}{14}$ AG	M1			${}^6C_x / {}^8C_x$ seen or 4C_2 mult by 4 fractions (last 2 can be implied)								
OR P(2) = $\frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} \times {}^4C_2 = \frac{3}{14}$ AG	A1	2		Answer legit obtained								
(ii)	<table border="1"> <tr> <td>x</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>Prob</td> <td>3/14</td> <td>8/14</td> <td>3/14</td> </tr> </table>	x	2	3	4	Prob	3/14	8/14	3/14	B1 B1 B1✓	3	2, 3, 4 only in top line one correct prob other than P(2) third correct prob ft Σ = 1
x	2	3	4									
Prob	3/14	8/14	3/14									
(iii) Var(X) = $\frac{12}{14} + \frac{72}{14} + \frac{48}{14} - 3^2$	M1			using Σx ² p – 3 ² (or their {E(X)} ²) must be evaluated								
= $\frac{3}{7}$ (0.429)	A1	2		correct answer								

Question 7

(i)	$0.24 + 0.35 + 2k + k + 0.05 = 1$ $k = 0.12$	M1 A1	2	Summing probs = 1 Correct answer
(ii)	model number is 1	B1	1	
(iii)	mean = $1 \times 0.35 + 2 \times 0.24 + 3 \times 0.12 + 4 \times 0.05$ $P(>1.39) = P(2, 3, 4) = 0.41$	B1 M1 B1	3	1.39 seen Finding $P(X > \text{their mean})$ Correct ans following mean or mode only

Question 8

(i)	$P(1 W) = 6/9 \times 3/8 + 3/9 \times 6/8$ $= \frac{1}{2} \text{ AG}$ OR $\frac{{}^6C_1 \times {}^3C_1}{{}^9C_2}$ $= \frac{1}{2} \text{ AG}$	M1 A1	[2]	summing 2 two-factor probs (condone replacement) not $\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$ Correct answer, fully justified								
(ii)	$P(\overline{W}, \overline{W}) = 3/9 \times 2/8 = 6/72 (1/12)$ $P(W, W) = 6/9 \times 5/8 = 30/72 (5/12)$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>Prob</td> <td>1/12</td> <td>1/2</td> <td>5/12</td> </tr> </table>	x	0	1	2	Prob	1/12	1/2	5/12	B1 B1 B1	[3]	Distribution table with 0,1,2 only $P(W, W)$ or $P(\overline{W}, \overline{W})$ correct $P(W, W) + P(\overline{W}, \overline{W}) = 0.5$
x	0	1	2									
Prob	1/12	1/2	5/12									
(iii)	$E(X) = 16/12 (4/3) (1.33) \text{ isw}$	B1	[1]	Condone 1(.3) if correct working seen, nfw								

Question 9

(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2" rowspan="2"></th> <th colspan="4">Spinner A</th> </tr> <tr> <th>1</th> <th>2</th> <th>3</th> <th>3</th> </tr> </thead> <tbody> <tr> <th rowspan="4">Spinner B</th> <th>-3</th> <td>(-2)</td> <td>-1</td> <td>0</td> <td>0</td> </tr> <tr> <th>-2</th> <td>-1</td> <td>0</td> <td>(1)</td> <td>1</td> </tr> <tr> <th>-1</th> <td>0</td> <td>1</td> <td>2</td> <td>2</td> </tr> <tr> <th>1</th> <td>2</td> <td>3</td> <td>4</td> <td>4</td> </tr> </tbody> </table>			Spinner A				1	2	3	3	Spinner B	-3	(-2)	-1	0	0	-2	-1	0	(1)	1	-1	0	1	2	2	1	2	3	4	4	B1 1	
				Spinner A																														
		1	2	3	3																													
Spinner B	-3	(-2)	-1	0	0																													
	-2	-1	0	(1)	1																													
	-1	0	1	2	2																													
	1	2	3	4	4																													
(ii)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>prob</td> <td>$\frac{1}{16}$</td> <td>$\frac{2}{16}$</td> <td>$\frac{4}{16}$</td> <td>$\frac{3}{16}$</td> <td>$\frac{3}{16}$</td> <td>$\frac{1}{16}$</td> <td>$\frac{2}{16}$</td> </tr> </tbody> </table>	x	-2	-1	0	1	2	3	4	prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	M1 M1 A1 3	<p>Their values in (i) as the top line, seen listed in (ii) or used in part (iii)</p> <p>Attempt at probs seen evaluated, need at least 4 correct from their table</p> <p>Correct table seen</p>															
x	-2	-1	0	1	2	3	4																											
prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$																											
(iii)	<p>$E(X) = 1$</p> <p>$Var(X) = ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$</p> <p>$= \frac{62}{16} - 1$</p> <p>$= \left(\frac{23}{8}\right) (2.875)$</p> <p>OR using $\sum p(x - \bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$</p> <p>$= \frac{46}{16} = 2.875$</p>	M1 M1 A1 3 M1 M1 A1	<p>Attempt at $E(X)$ from their table if $\sum p = 1$</p> <p>Evaluating $\sum x^2 p - [their E(X)]^2$ allow $\sum p \neq 1$ but all p's < 1</p> <p>Correct answer</p>																															
(iv)	<p>$P(\text{even given +ve})$</p> <p>$= \frac{5}{9}$</p> <p>OR $P(\text{even given +ve}) = \frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$</p> <p>$= \frac{5}{9} (0.556)$</p>	M1 A1 2 M1 A1	<p>Counting their even numbers and dividing by their positive numbers</p> <p>Correct answer</p> <p>Using cond prob formula not $P(E) \times P(+ve)$ need fraction over fraction accept any of $\frac{5/16 \text{ or } 6/16 \text{ or } 9/16}{9/16 \text{ or } 10/16 \text{ or } 13/16}$</p> <p>Correct answer</p>																															

Question 10

(i)		<p>M1</p> <p>A1</p> <p>A1 [3]</p>	<p>3 pairs S (bank, log in, success) and F (seen no extra bits).</p> <p>Exactly 3 pairs, must be labelled</p> <p>Correct diagram with all probs correct</p>										
(ii)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Prob</td> <td>0.4</td> <td></td> <td>0.144</td> <td>0.216</td> </tr> </table>	x	0	1	2	3	Prob	0.4		0.144	0.216	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1 [4]</p>	<p>$P(0)$ correct</p> <p>Multiplying two of more factors of 0.4 and 0.6</p> <p>One more correct prob</p> <p>One more correct prob</p>
x	0	1	2	3									
Prob	0.4		0.144	0.216									
(iii)	$E(X) = 0.24 + 2 \times 0.144 + 3 \times 0.216$ $= 1.176 \text{ (1.18)}$	<p>M1</p> <p>A1 [2]</p>	<p>Using $\sum p_i x_i$</p> <p>Correct answer</p>										

Question 11

<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>No of W</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>Prob</td> <td>42/90</td> <td>42/90</td> <td>6/90</td> </tr> </table>	No of W	0	1	2	Prob	42/90	42/90	6/90	<p>B1</p>	<p>0, 1, 2, seen in table with attempt at prob.</p>
No of W	0	1	2							
Prob	42/90	42/90	6/90							
$P(0) = 8/10 \times 7/9 \times 6/8 = 42/90$	<p>M1</p>	<p>3-factor prob seen with different denoms.</p>								
$P(1W) = P(W, NW, NW) \times 3 = 2/10 \times 8/9 \times 7/8 \times 3$ $= 42/90$	<p>M1</p>	<p>Mult by 3</p>								
$P(2W) = P(W, W, NW) \times 3 = 2/10 \times 1/9 \times 8/8 \times 3$ $= 6/90$	<p>A1</p>	<p>4 All correct</p>								

Question 12

(i)	$P(0) = 6/36, P(1) = 10/36, P(2) = 8/36$ $P(3) = 6/36, P(4) = 4/36, P(5) = 2/36$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 [4]</p>	<p>Table seen with 0, 1, 2, 3, 4, 5 (6 if $P(6) = 0$)</p> <p>Any three probs correct</p> <p>$\sum p = 1$ and at least 3 outcomes</p> <p>All probs correct</p>
(ii)	$\text{mean score} = (0 \times 6 + 1 \times 10 + 16 + 18 + 16 + 10) / 36$ $= 70/36 \text{ (35/18, 1.94)}$	<p>M1</p> <p>A1 [2]</p>	<p>Using $\sum xp$ (unsimplified) on its own – condone</p> <p>$\sum p \text{ not } = 1$</p>

Question 13

(i)	x	1	2	3	4	B1	Probability Distribution Table, either k or correct numerical values
	$P(x)$	k	$2k$	$3k$	$4k$		
	$10k = 1$					M1	Summing probs involving k to = 1, 3 or 4 terms
	$k = 1/10$					A1 [3]	
(ii)	$E(X) = 1/10 + 4/10 + 9/10 + 16/10 = 3$					B1	Correct mean
	$Var(X) = 1/10 + 8/10 + 27/10 + 64/10 - 3^2 = 1$					M1	
						A1 [3]	

Question 14

$[P(X=0)] = P(B, B) = 5/7 \times 4/6 = 10/21$	M1	Attempt to find $P(0)$ or $P(1)$ or $P(2)$ can be seen as $P(BB)$ etc. or table unsimplified
$[P(X=1)] = P(G, B) + P(B, G) = 2/7 \times 5/6 \times 2 = 10/21$	A1	
$[P(X=2)] = P(G, G) = 2/7 \times 1/6 = 1/21$	A1	$P(0)$ or $P(2)$ correct must see X value
$E(X) = 0 + 10/21 + 2/21 = 4/7 (0.571)$	B1 [*]	Correct answer ft their probs $P(1)$ and $P(2)$
$Var(X) = 0 + 10/21 + 4/21 - (4/7)^2 = 50/147 (0.340)$	M1	Attempt at $\Sigma x^2 p - [E(X)]^2$
	A1 [6]	

Question 15

(i)	$P(1 \text{ T-shirt}) = \frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$	B1	Correct num unsimplified					
	$= 27/55$	B1		Correct denom unsimplified				
	AG	B1 [3]	Answer given, so process needs to be convincing					
	OR $3/12 \times 9/11 \times 8/10 \times {}^3C_1$ oe	M1	Mult 3 probs diff denoms (not $a/3 \times b/4 \times c/5$)					
	$= 27/55$	M1		Mult by 3C_1 oe				
	AG	A1	Answer given, so process needs to be convincing					
(ii)	X	0	1	2	3	B1	0, 1, 2, 3 only seen in top line (condone additional values if Prob stated as 0)	
	Prob	84/220	27/55	27/220	1/220			
						B1	One correct prob, correctly placed in table	
						B1		One other correct prob, correctly placed in table
						B1 [*] [4]		One other correct prob ft $\Sigma p = 1$, 4 values in table

Question 16

diff	0	1	2	3	4	5
prob	6/36	10/36	8/36	6/36	4/36	2/36

$$\begin{aligned} \text{Expectation} &= (0+10+16+18+16+10)/36 \\ &= 70/36 \\ &= 1.94 \end{aligned}$$

B1

M1

A1

M1

A1

[5]

0, 1, 2, 3, 4, 5 seen in table heading or considering all different differences
Attempt at finding prob of any difference
1 correct prob
Probs summing to 1

Question 17

(i)	$k(-2)^2$ is the same as $k(2)^2 = 4k$	B1	need to see $-2^2 k$, $2^2 k$ and $4k$, algebraically correct expressions OE										
	Total:	1											
(ii)	<table border="1"> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>2</td> <td>4</td> </tr> <tr> <td>Prob</td> <td>$4k$</td> <td>k</td> <td>$4k$</td> <td>$16k$</td> </tr> </table>	x	-2	-1	2	4	Prob	$4k$	k	$4k$	$16k$	B1	-2, -1, 2, 4 only seen in a table, together with at least one attempted probability involving k
x	-2	-1	2	4									
Prob	$4k$	k	$4k$	$16k$									
	$4k + k + 4k + 16k = 1$	M1	Summing 4 probs equating to 1. Must all be positive (table not required)										
	$k = 1/25$ (0.04)	A1	CWO										
	Total:	3											
(iii)	$E(X) = -8k + -k + 8k + 64k = 63k$	M1	using $\sum px$ unsimplified. FT their k substituted before this stage, no inappropriate dividing										
	$= 63/25$ (2.52)	A1											
	Total:	2											

Question 18

(i)	<table border="1"> <tr> <td>x</td> <td>-3</td> <td>0</td> <td>5</td> <td>32</td> </tr> <tr> <td>Prob</td> <td>1/6</td> <td>1/2</td> <td>1/6</td> <td>1/6</td> </tr> </table>	x	-3	0	5	32	Prob	1/6	1/2	1/6	1/6	B1	At least 3 different correct values of X (can be unsimplified)
x	-3	0	5	32									
Prob	1/6	1/2	1/6	1/6									
		B1	Four correct probabilities in a Probability Distribution table										
		B1	Correct probs with correct values of X										
		3											
(ii)	$E(X) = -3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	M1	Subst their attempts at scores in correct formula as long as 'probs' sum to 1										
	$\text{Var}(X) = 9/6 + 25/6 + 1024/6 - (34/6)^2$	M1	Subst their attempts at scores in correct var formula										
	$= 144 \left(\frac{1298}{9} \right)$	A1	Both answers correct										
		3											

Question 19

(i)	<i>EITHER:</i> $P(X=3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	(M1)	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leq 6$ and $p \geq q \geq r, r \geq 4$, accept $\times 1$ as $\frac{4}{r}$.								
	$= \frac{1}{15}$ AG	(A1)	Needs either P(RRB) OE stated or identified on tree diagram.								
	<i>OR1:</i> $P(X=3) = P(RRB) = \frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	(M1)	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ included								
	$= \frac{1}{15}$ AG	(A1)	Needs either P(RRB) OE stated or identified on tree diagram.								
	<i>OR2:</i> $P(X=3) = P(RRB) = \frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^3C_1} \times \frac{{}^4C_1}{{}^4C_1}$	(M1)	probabilities in order $\frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^3C_1} \times \frac{{}^4C_1}{{}^4C_1}$, $p, q, r \leq 6$ and $p \geq q \geq r, r \geq 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ acceptable)								
	$= 1/15$ AG	(A1)	Needs either P(RRB) OE stated or identified on tree diagram.								
(ii)	$P(1) = P(B) = \frac{4}{6} \left(\frac{2}{3} = 0.667 \right)$ $P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15} (= 0.267)$ $P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15} (= 0.0667)$	2									
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>P</td> <td>$\frac{10}{15}$</td> <td>$\frac{4}{15}$</td> <td>$\frac{1}{15}$</td> </tr> </tbody> </table>	x	1	2	3	P	$\frac{10}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	B1	Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leq p < 1$.
x	1	2	3								
P	$\frac{10}{15}$	$\frac{4}{15}$	$\frac{1}{15}$								
		B1	P(1) or P(2) correct unsimplified, or better, and identified.								
		B1	All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0								
		3									

Question 20

$p + q = 0.45$	M1	Equation involving $\sum P(x) = 1$
$0.15 + 2p + 1.2 + 6q = 3.05$	M1	Equation using $E(X) = 3.05$
$q = 0.2$	M1	Solving simultaneous equations to one variable
$p = 0.25$	A1	Both answers correct
	4	

Question 21

(i)	$\sum p = 1: 0.2 + 0.1 + p + 0.1 + q = 1: \quad p + q = 0.6$	M1	Unsimplified sum of probabilities equated to 1
	$\sum px = 1.7: -0.4 + 0 + p + 0.3 + 4q = 1.7:$	M1	Unsimplified Sum of px equated to 1.7
	$p + 4q = 1.8$	M1	Solve simult. equations to find expression in p or q
	$p = 0.2, q = 0.4$	A1	
		4	
(ii)	$\text{Var}(X) = \sum px^2 - 1.7^2 = 4 \times 0.2 + 1p + 9 \times 0.1 + 16q - 1.7^2$ $= 8.3 - 2.89$	M1	Use correct unsimplified expression for variance
	$= 5.41$	A1	
		2	

Question 22

(i)	$P(0) = 0.6 \times 0.25 \times 0.5 = 0.075$ $P(1) = 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 + 0.6 \times 0.25 \times 0.5 = 0.35$ $P(2) = 0.4 \times 0.75 \times 0.5 + 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 = 0.425$ $P(3) = 0.4 \times 0.75 \times 0.5 = 0.15$	B1	0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)										
		M1	Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table										
	<table border="1"> <tr> <td>No of heads</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Prob</td> <td>0.075 $\left(\frac{3}{40}\right)$</td> <td>0.35 $\left(\frac{7}{20}\right)$</td> <td>0.425 $\left(\frac{17}{40}\right)$</td> <td>0.15 $\left(\frac{3}{20}\right)$</td> </tr> </table>	No of heads	0	1	2	3	Prob	0.075 $\left(\frac{3}{40}\right)$	0.35 $\left(\frac{7}{20}\right)$	0.425 $\left(\frac{17}{40}\right)$	0.15 $\left(\frac{3}{20}\right)$	M1	Summing 3 probabilities for P(1) or P(2) with or without a table
	No of heads	0	1	2	3								
	Prob	0.075 $\left(\frac{3}{40}\right)$	0.35 $\left(\frac{7}{20}\right)$	0.425 $\left(\frac{17}{40}\right)$	0.15 $\left(\frac{3}{20}\right)$								
	B1	One correct probability seen.											
	A1	All correct in a table											
	Total:	5											
(ii)	$E(X) = 0.35 + 2 \times 0.425 + 3 \times 0.15 = 1.65 \left(\frac{33}{20} \text{ oe}\right)$	M1	Correct unsimplified expression for the mean using their table, $\sum p = 1$; can be implied by correct answer										
(ii)	$\text{Var}(X) = 0.35 + 4 \times 0.425 + 9 \times 0.15 - 1.65^2$	M1	Correct unsimplified expression for the variance using their table and their mean ² subtracted, $\sum p = 1$										
	$= 0.678 \text{ (0.6775)} \left(\frac{271}{400} \text{ oe}\right)$	A1	Correct answer										
	Total:	3											

Question 23

(i)	<table border="1"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>Prob</td> <td>$\frac{2}{7}$</td> <td>$\frac{4}{7}$</td> <td>$\frac{1}{7}$</td> </tr> </table>	X	0	1	2	Prob	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$	B1	Prob distribution table drawn, top row correct with at least one probability $0 < p < 1$ entered, condone additional values with $p = 0$ stated
	X	0	1	2							
	Prob	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$							
	$P(0) = \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} = \frac{2}{7} \text{ (0.2857)}$	B1	One probability correct (need not be in table)								
$P(1) = \frac{2}{7} \times \frac{5}{6} \times \frac{4}{5} \times {}^3C_1 = \frac{4}{7} \text{ (0.5713)}$	B1	Another probability correct (need not be in table).									
$P(2) = \frac{2}{7} \times \frac{1}{6} \times \frac{5}{5} \times {}^3C_2 = \frac{1}{7} \text{ (0.1429)}$	B1	Values in table, all probs correct (to 3SF) or 3 probabilities summing to 1									
		4									
(ii)	$\text{Var}(X) = 1 \times \frac{4}{7} + 4 \times \frac{1}{7} - \left(\frac{6}{7}\right)^2$ $= \frac{8}{7} - \left(\frac{6}{7}\right)^2$	M1	Unsimplified correct numerical expression for variance or their probabilities from (i) $0 < p < 1$ in unsimplified variance expression								
	$= \frac{20}{49} \text{ or } 0.408$	A1	Correct answer (0.40816...) nfw Final answer does not imply the method mark								
		2									

Question 24

(i)	$P(RB) + P(BR) = \frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$ oe	M1	Multiply 2 probs together and summing two 2-factor probs, unsimplified, condone replacement								
	$P(\text{diff colours}) = \frac{64}{132} \left(\frac{16}{33}\right)$ (0.485) oe	A1	Correct answer								
	Method 2 $1 - P(BB) - P(RR) = 1 - \frac{4}{12} \times \frac{3}{11} - \frac{8}{12} \times \frac{7}{11}$	M1	Multiply 2 probs together and subtracting two 2-factor probs from 1, unsimplified, condone replacement								
	$P(\text{diff colours}) = \frac{64}{132} \left(\frac{16}{33}\right)$ oe	A1	Correct answer								
	Method 3 $P(\text{diff colours}) = \frac{{}^4C_1 \times {}^8C_1}{{}^{12}C_2}$	M1	Multiply 2 combs together and dividing by a combination								
	$= \frac{16}{33}$	A1	Correct answer								
		2									
(ii)	<table border="1"> <thead> <tr> <th>Number of red socks</th> <th>0</th> <th>1</th> <th>2</th> </tr> </thead> <tbody> <tr> <td>Prob</td> <td>$\frac{14}{33}$</td> <td>$\frac{16}{33}$</td> <td>$\frac{3}{33}$</td> </tr> </tbody> </table>	Number of red socks	0	1	2	Prob	$\frac{14}{33}$	$\frac{16}{33}$	$\frac{3}{33}$	B1	Prob distribution table drawn, top row correct, condone additional values with $p = 0$ stated
Number of red socks	0	1	2								
Prob	$\frac{14}{33}$	$\frac{16}{33}$	$\frac{3}{33}$								
		B1	$P(0)$ or $P(2)$ correct to 3sf (need not be in table)								
		B1	All probs correct to 3sf, condone $P(0)$ and $P(2)$ swapped if correct								
		3									
(iii)	$E(X) = 1 \times \frac{16}{33} + 2 \times \frac{3}{33} = \frac{16}{33} + \frac{6}{33} = \frac{22}{33} \left(\frac{2}{3}\right)$	B1ft	fit their table if 0, 1, 2 only, $0 < p < 1$								
		1									

Question 25

(i)	<table border="1"> <thead> <tr> <th>x</th> <th>-2</th> <th>-1</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>$P(X=x)$</td> <td>$\frac{2}{18}$</td> <td>$\frac{4}{18}$</td> <td>$\frac{5}{18}$</td> <td>$\frac{4}{18}$</td> <td>$\frac{2}{18}$</td> <td>$\frac{1}{18}$</td> </tr> </tbody> </table>	x	-2	-1	0	1	2	3	$P(X=x)$	$\frac{2}{18}$	$\frac{4}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{2}{18}$	$\frac{1}{18}$	B1	-2, -1, 0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate $P(-2)$, $P(-1)$, $P(0)$, $P(1)$, $P(2)$, $P(3)$.
x	-2	-1	0	1	2	3											
$P(X=x)$	$\frac{2}{18}$	$\frac{4}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{2}{18}$	$\frac{1}{18}$											
		B1	At least 4 probs correct (need not be in table)														
		B1	All probs correct in a table														
		3															
(ii)	$E(X) = \frac{-4 - 4 + 0 + 4 + 4 + 3}{18} = \frac{1}{6}$	M1	Correct unsimplified expression for the mean using their table, $\Sigma p = 1$, may be implied														
	$\text{Var}(X) = \frac{8 + 4 + 0 + 4 + 8 + 9}{18} - \left(\frac{1}{6}\right)^2$ $= 11/6 - 1/36$ (1.8333 - 0.02778)	M1	Correct, unsimplified expression for the variance using their table, and their mean ² subtracted. Allow $\Sigma p \neq 1$														
	$= 65/36$, (1.81)	A1	Correct answer														
		3															

Question 26

(i)	<table border="1"> <tbody> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>p</td> <td>$\frac{1}{12}$</td> <td>$\frac{2}{12}$</td> <td>$\frac{3}{12}$</td> <td>$\frac{3}{12}$</td> <td>$\frac{2}{12}$</td> <td>$\frac{1}{12}$</td> </tr> </tbody> </table>	x	-2	-1	0	1	2	3	p	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	B1	-2, -1, 0, 1, 2, 3 seen as top line of a pdf table with at least 1 probability OR attempting to evaluate $P(-2)$, $P(-1)$, $P(0)$, $P(1)$, $P(2)$, $P(3)$ (condone additional values with $p=0$ stated)
x	-2	-1	0	1	2	3											
p	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{1}{12}$											
		B1	At least 4 probs correct (need not be in table)														
		B1	All probs correct in a table														
		3															
(ii)	$E(X) = \frac{-2 \times 1 - 1 \times 2 + 0 + 1 \times 3 + 2 \times 2 + 1 \times 3}{12} = 0.5$	M1	Unsimplified expression for mean using <i>their</i> pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.														
	$\text{Var}(X) = \frac{(-2)^2 \times 1 + (-1)^2 \times 2 + 1^2 \times 3 + 2^2 \times 2 + 3^2 \times 1}{12} - (\text{their } 0.5)^2$	M1	Unsimplified expression for variance using <i>their</i> pdf table (or correct) with at least 2 non-zero values and <i>their</i> $E(X)$. Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly														
	$26/12 - 1/4 = 23/12$	A1	Correct final answer														
		3															

Question 27

(i)	$6p + 0.1 = 1$ $p = 0.15$	B1	Correct answer
		1	
(ii)	$\text{Var}(X) = 1 \times p + 1 \times 2p + 4 \times 2p + 16 \times 0.1 - 1.15^2$	M1	Correct unsimplified formula, <i>their</i> p substituted (allow 1 error)
	$0.15 + 0 + 0.3 + 1.2 + 1.6 - 1.15^2$ $= 1.9275 = 1.93$ (3sf)	A1	Correct answer
		2	

Question 28

(i)	<table border="1"> <tr> <td>x</td> <td>-1</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>p</td> <td>k</td> <td>k</td> <td>$4k$</td> <td>$9k$</td> </tr> </table>	x	-1	1	2	3	p	k	k	$4k$	$9k$	B1	Probability distribution table with correct values of x , no additional values unless with probability 0 stated, at least one correct probability including k
x	-1	1	2	3									
p	k	k	$4k$	$9k$									
	$15k = 1,$	M1	Equating $\Sigma p = 1$, may be implied by answer										
	$k = \frac{1}{15}$	A1	If 0 scored, SCB2 for probability distribution table with correct numerical probabilities.										
		3											
(ii)	Method 1												
	$E(X) = 8k + 27k = 35k = \frac{35}{15} = \frac{7}{3}$	B1FT	FT if 0 < their $k < 1$										
	$\text{Var}(X) = (k + k + 16k + 81k) - (35k)^2$	M1	Correct formula for variance, in terms of k at least – must have ‘– mean ² ’(ft).										
	$= 1.16, \frac{52}{45}$	A1											
	Method 2												
	$E(X) = \frac{8}{15} + \frac{27}{15} = \frac{35}{15} = \frac{7}{3}$	B1FT	FT if 0 < their $k < 1$										
	$\text{Var}(X) = \frac{1}{15} + \frac{1}{15} + \frac{16}{15} + \frac{81}{15} - \left(\frac{7}{3}\right)^2$	M1	Subst <i>their</i> values in correct var formula – must have ‘– mean ² ’(ft) (condone probs not summing to exactly 1)										
	$= 1.16 (= 52/45)$	A1	Using their values from (i)										
		3											

Question 29

(a)	<table border="1"> <tr> <td>-1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>3</td> <td>3</td> <td>4</td> </tr> </table>	-1	0	0	1	0	1	1	2	2	3	3	4				
-1	0	0	1														
0	1	1	2														
2	3	3	4														
	<table border="1"> <tr> <td>x</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>Probability</td> <td>$\frac{1}{12}$</td> <td>$\frac{3}{12}$</td> <td>$\frac{3}{12}$</td> <td>$\frac{2}{12}$</td> <td>$\frac{2}{12}$</td> <td>$\frac{1}{12}$</td> </tr> </table>	x	-1	0	1	2	3	4	Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$		
x	-1	0	1	2	3	4											
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$											
	Probability distribution table with correct scores with at least one probability		B1														
	At least 4 probabilities correct		B1														
	All probabilities correct		B1														
			3														
(b)	$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$		B1														
	$\text{Var}(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$		M1														
	$\frac{37}{18} (= 2.06)$		A1														
			3														

Question 30

(a)	<table border="1"> <tr> <td></td> <td>1</td> <td>1</td> <td>2</td> <td>2</td> <td>3</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td>2</td> <td>3</td> </tr> <tr> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>3</td> </tr> <tr> <td>3</td> <td>3</td> <td>3</td> <td>3</td> <td>3</td> <td>3</td> </tr> </table>		1	1	2	2	3	1	1	1	2	2	3	2	2	2	2	2	3	3	3	3	3	3	3	M1
	1	1	2	2	3																					
1	1	1	2	2	3																					
2	2	2	2	2	3																					
3	3	3	3	3	3																					
	$\frac{7}{15}$ AG	A1																								
		2																								
(b)	<table border="1"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Probability</td> <td>$\frac{2}{15}$</td> <td>$\frac{6}{15}$</td> <td>$\frac{7}{15}$</td> </tr> </table>	x	1	2	3	Probability	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{7}{15}$	B1																
x	1	2	3																							
Probability	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{7}{15}$																							
	P(1) or P(2) correct	B1																								
	3 rd probability correct, FT sum to 1	B1																								
		3																								
(c)	$E(X) = \frac{2+12+21}{15} = \frac{35}{15} = \frac{7}{3}$	B1																								
	$\text{Var}(X) = \frac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 7}{15} - \left(\frac{7}{3}\right)^2$	M1																								
	$\frac{22}{45}$ (0.489)	A1																								
		3																								

Question 31

	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Probability</td> <td>$\frac{1}{56}$</td> <td>$\frac{15}{56}$</td> <td>$\frac{30}{56}$</td> <td>$\frac{10}{56}$</td> </tr> </table>	x	0	1	2	3	Probability	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$	B1
x	0	1	2	3								
Probability	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$								
	(B1 for probability distribution table with correct outcome values)											
	$P(0) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$ $P(1) = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times 3 = \frac{15}{56}$ $P(2) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times 3 = \frac{30}{56}$ $P(3) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{10}{56}$	M1										
	(M1 for denominator $8 \times 7 \times 6$)											
	Any one probability correct (with correct outcome)	A1										
	All probabilities correct	A1										
		4										

Question 32

(a)	Scenarios:	M1	One 3 factor probability with 3, 3, 5 as denominators										
	HHT: $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{4}{45}$ HTH: $\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{8}{45}$ THH: $\frac{1}{3} \times \frac{2}{3} \times \frac{4}{5} = \frac{8}{45}$	M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios										
	Total = $\frac{20}{45} = \frac{4}{9}$	A1	AG, Total of 3 products with clear context										
		3											
(b)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Prob.</td> <td>$\frac{1}{45}$</td> <td>$\frac{8}{45}$</td> <td>$\frac{20}{45}$</td> <td>$\frac{16}{45}$</td> </tr> </table>	x	0	1	2	3	Prob.	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$	B1	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		x	0	1	2	3							
		Prob.	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$							
		B1	2 of P(0), P(1) and P(3) correct										
B1 FT	3 or 4 probabilities sum to 1 with P(2) correct												
3													
(c)	$\text{Var}(X) = \frac{0^2 \times 1 + 1^2 \times 8 + 2^2 \times 20 + 3^2 \times 16}{45} - \left(\frac{32}{15}\right)^2$ $= \frac{8}{45} + \frac{80}{45} + \frac{144}{45} - \left(\frac{32}{15}\right)^2$	M1	Substitute <i>their</i> attempts at scores in correct variance formula, must have '- mean ² ' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values										
		A1	$\frac{136}{225}$ or 0.604										
	2												

Question 33

(a)	P(1 red) = $\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times 3$	M1	$\frac{a}{8} \times \frac{b}{7} \times \frac{c}{6} \times k$ or $\frac{5}{d} \times \frac{3}{e} \times \frac{2}{f} \times 3, 1 \leq a, b, c \leq 5, d, e, f \leq 8, a, b, c, d, e, f, k$ all integers. $1 < k \leq 3$,															
	$\frac{15}{56}$	A1	AG, WWW															
	Alternative method for question 2(a)																	
	$\frac{{}^5C_1 \times {}^3C_2}{{}^8C_3}$	M1	$\frac{{}^aC_1 \times {}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d \times {}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d \times {}^3C_e (or {}^aC_1 \times {}^bC_2)}{{}^5C_3 \times {}^3C_0 + {}^5C_2 \times {}^3C_1 + {}^5C_1 \times {}^3C_2 + {}^5C_0 \times {}^3C_3}$, $a + b = 8, d + e = 3$															
	$\frac{15}{56}$	A1	AG, WWW, $\frac{15}{56}$ must be seen															
		2																
(b)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Prob.</td> <td>$\frac{1}{56}$</td> <td>$\frac{15}{56}$</td> <td>$\frac{30}{56} = \frac{15}{28}$</td> <td>$\frac{10}{56} = \frac{5}{28}$</td> </tr> <tr> <td></td> <td>0.0179</td> <td>0.268</td> <td>0.536</td> <td>0.179</td> </tr> </table>	x	0	1	2	3	Prob.	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56} = \frac{15}{28}$	$\frac{10}{56} = \frac{5}{28}$		0.0179	0.268	0.536	0.179	B1	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		x	0	1	2	3												
		Prob.	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56} = \frac{15}{28}$	$\frac{10}{56} = \frac{5}{28}$												
			0.0179	0.268	0.536	0.179												
B1	2 of P(0), P(2) and P(3) correct																	
B1 FT	4 th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct																	
3																		

(c)	$\text{Var}(X) = \frac{(0^2 \times 1) + 1^2 \times 15 + 2^2 \times 30 + 3^2 \times 10}{56} - \left(\frac{15}{8}\right)^2$ $= \frac{15}{56} + \frac{120}{56} + \frac{90}{56} - \left(\frac{15}{8}\right)^2$	M1	Substitute <i>their</i> attempts at scores in correct variance formula, must have ' $- \text{mean}^2$ ', (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\frac{225}{448}, 0.502$	A1	
		2	

Question 34

(a)	<table border="1"> <tr> <td>y</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>prob</td> <td>$\frac{7}{16}$</td> <td>$\frac{5}{16}$</td> <td>$\frac{3}{16}$</td> <td>$\frac{1}{16}$</td> </tr> </table>	y	1	2	3	4	prob	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	B1	<table border="1"> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>2</td> <td>1</td> <td>2</td> <td>1</td> <td>2</td> </tr> <tr> <td>3</td> <td>2</td> <td>1</td> <td>3</td> <td>1</td> </tr> <tr> <td>4</td> <td>3</td> <td>2</td> <td>1</td> <td>4</td> </tr> </table> <p>Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated</p>		1	2	3	4	1	1	1	2	3	2	1	2	1	2	3	2	1	3	1	4	3	2	1	4
y	1	2	3	4																																		
prob	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$																																		
	1	2	3	4																																		
1	1	1	2	3																																		
2	1	2	1	2																																		
3	2	1	3	1																																		
4	3	2	1	4																																		
		B1	One probability (linked with correct score) correct																																			
		B1	2 more probs (linked with correct scores) correct																																			
		B1 FT	4 th prob correct, FT sum of 3 or 4 terms = 1																																			
		4																																				
(b)	$P(2 \text{even}) = \frac{\frac{5}{16}}{\frac{6}{16}}$	M1	$\frac{\text{their } P(2)}{\text{their } P(2) + \text{their } P(4)}$ seen or correct outcome space.																																			
	$\frac{5}{6}$ or 0.833	A1																																				
		2																																				

Question 35

(a)	$0.6 \times 0.7 + 0.4(1-x) = 0.58$ $\equiv 0.42 + 0.4(1-x) = 0.58$	M1	Equation of form $0.6 \times a + 0.4 \times b = 0.58$; $a = 0.3, 0.7, b = x, (1-x)$
		B1	Single correct product seen, condone 0.42, in an equation of appropriate form
	$x = 0.6$	A1	
Alternative method for question 2(a)			
	$0.6 \times 0.3 + 0.4x = 0.42$ $\equiv 0.18 + 0.4x = 0.42$	M1	Equation of form $0.6 \times a + 0.4 \times b = 0.42$; $a = 0.3, 0.7, b = x, (1-x)$
		B1	Single correct product seen, condone 0.18, in an equation of appropriate form
	$x = 0.6$	A1	
		3	
(b)	$(0.6 \times 0.3)^2$ 0.0324	M1	$(a \times b)^2, a = 0.6, 0.4$ and $b = 0.7, 0.3, x, (1-x)$ or 0.18^2 , alone.
		A1	
		2	

Question 36

(a)	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>prob</td> <td>$4k$</td> <td>$6k$</td> <td>$6k$</td> <td>$4k$</td> </tr> </table>	x	1	2	3	4	prob	$4k$	$6k$	$6k$	$4k$	B1	Table with \times values and one correct probability expressed in terms of k . Condone any additional \times values if probability stated as 0.
		x	1	2	3	4							
prob	$4k$	$6k$	$6k$	$4k$									
B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.												
		2											
(b)	$[4k + 6k + 6k + 4k = 1] k = \frac{1}{20} (= 0.05)$ $E(X) = 1 \times \frac{4}{20} + 2 \times \frac{6}{20} + 3 \times \frac{6}{20} + 4 \times \frac{4}{20} = \frac{4}{20} + \frac{12}{20} + \frac{18}{20} + \frac{16}{20}$ $(= 2.5)$ $\text{Var}(X) = 1^2 \times \frac{4}{20} + 2^2 \times \frac{6}{20} + 3^2 \times \frac{6}{20} + 4^2 \times \frac{4}{20} - \left(\text{their } 2\frac{1}{2} \right)^2$ $= (4 + 24 + 54 + 64) \times \text{their } 0.05 - (\text{their } 2.5)^2$ Or $(1 - 2.5)^2 \times \frac{4}{20} + (2 - 2.5)^2 \times \frac{6}{20} + (3 - 2.5)^2 \times \frac{6}{20} + (4 - 2.5)^2 \times \frac{4}{20}$	B1	Correct value for k SOI. May be calculated in 4(a). SC B1 If denominator $20k$ used throughout.										
		M1	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. Special ruling: Allow use of denominator $20k$.										
		M1	Appropriate variance formula with <i>their</i> numerical probabilities using <i>their</i> $(E(X))^2$, accept unsimplified, with <i>their</i> k substituted. Special ruling: If denominator $20k$ used throughout, accept appropriate variance formula in terms of k .										
		A1	AG, NFWW.										
		4											

Question 37

$p + p + 0.1 + q + q = 1$	B1	Sum of probabilities = 1
$0.1 + 2q = 3(2p)$	B1	Use given information
Attempt to solve two correct equations in p and q	M1	Either use of Substitution method to form a single equation in either p or q and finding values for both unknowns. Or use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or - to find an equation in one unknown and finding values for both unknowns.
$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	A1	CAO, both WWW
	4	

Question 38

(a)	<table border="1"> <tr> <td>X</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(X)$</td> <td>$\frac{1}{9}$</td> <td>$\frac{2}{9}$</td> <td>$\frac{1}{9}$</td> <td>$\frac{3}{9}$</td> <td>$\frac{2}{9}$</td> </tr> </table>	X	-1	0	1	2	3	$P(X)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	B1	Table with correct X values and at least one probability Condone any additional X values if probability stated as 0.
		X	-1	0	1	2	3								
		$P(X)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{2}{9}$								
B1	2 correct probabilities linked with correct outcomes, may not be in table.														
		B1	3 further correct probabilities linked with correct outcomes, may not be in table. SC if less than 2 correct probabilities seen, award SCB1 for sum of <i>their</i> 4 or 5 probabilities in table = 1												
		3													
(b)	$E(X) = \frac{-1 \times 1 + (0 \times 2) + 1 \times 1 + 2 \times 3 + 3 \times 2}{9} = \frac{-1 + 1 + 6 + 6}{9}$	M1	May be implied by use in variance, accept unsimplified expression. FT <i>their</i> table if <i>their</i> 3 or more probabilities sum to 1 or 0.999												
		M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table even if <i>their</i> 3 or more probabilities not summing to 1.												
	$[Var(X) = \frac{-1^2 \times 1 + (0^2 \times 2) + 1^2 \times 1 + 2^2 \times 3 + 3^2 \times 2}{9} - (their E(X))^2]$ $\frac{1 + 0 + 1 + 12 + 18}{9} - (their E(X))^2$	A1	Answers for $E(X)$ and $Var(X)$ must be identified												
	$E(X) = \frac{4}{3}$ or 1.33 and $Var(X) = \frac{16}{9}$ or 1.78	3	N.B. If method FT for M marks from <i>their</i> incorrect (b), expressions for $E(X)$ and $Var(X)$ must be seen unsimplified with all probabilities <1												

Question 39

(a)	$P(X=3) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$	M1	$\frac{m}{7} \times \frac{n}{6} \times \frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$												
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.												
			2												
(b)	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>p</td> <td>$\frac{15}{35}$</td> <td>$\frac{10}{35}$</td> <td>$\frac{6}{35}$</td> <td>$\frac{3}{35}$</td> <td>$\frac{1}{35}$</td> </tr> </table>	x	1	2	3	4	5	p	$\frac{15}{35}$	$\frac{10}{35}$	$\frac{6}{35}$	$\frac{3}{35}$	$\frac{1}{35}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
x	1	2	3	4	5										
p	$\frac{15}{35}$	$\frac{10}{35}$	$\frac{6}{35}$	$\frac{3}{35}$	$\frac{1}{35}$										
		B1	One correct probability other than $X=3$ linked to the correct outcome												
		B1	Two further correct probabilities other than $X=3$ seen linked to the correct outcome												
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1												
			4												
(c)	$E(X) = 1 \times \frac{15}{35} + 2 \times \frac{10}{35} + 3 \times \frac{6}{35} + 4 \times \frac{3}{35} + 5 \times \frac{1}{35}$ $E(X) = \frac{15+20+18+12+5}{35} = \left[\frac{70}{35} = 2 \right]$	M1	At least 4 correct terms FT <i>their</i> values in (a) with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.												
	$\text{Var}(X) = \left[\frac{1^2 \times 15 + 2^2 \times 10 + 3^2 \times 6 + 4^2 \times 3 + 5^2 \times 1}{35} - 2^2 \right]$ $\frac{15+40+54+48+25}{35} - 2^2$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$. FT <i>their</i> table accept probabilities not summing to 1.												
	$\left[\frac{182}{35} - 4 \right] = \frac{6}{5}$	A1	N.B. If method FT for M marks from <i>their</i> incorrect (b), expressions for $E(X)$ and $\text{Var}(X)$ must be seen unsimplified with all probabilities <1												
			3												

Question 40

(a)	$p+q+0.65=1$	B1	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	B1	Use given information.
	Solve 2 linear equations	M1	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3, \frac{3}{10}, q=0.05, \frac{1}{20}$	A1	CAO, both WWW If M0 with correct answers SC B1.
			4
(b)	$\text{Var}(X) = \text{their } 0.3 + 4 \times \text{their } 0.05 + 9 \times 0.05 - 0.55^2$	M1	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475 \left[\frac{259}{400} \right]$	A1	CAO (must be exact).
			2
(c)	$1 - P(0, 1, 2) = 1 - ({}^{12}C_0 0.3^0 0.7^{12} + {}^{12}C_1 0.3^1 0.7^{11} + {}^{12}C_2 0.3^2 0.7^{10})$	M1	One correct term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	$1 - (0.01384 + 0.07118 + 0.16779)$	A1FT	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to FT <i>their</i> p from 6(a) or correct.
	0.747	A1	
			3
(d)	$(0.95)^8 \times 0.05 = 0.0332$ or $0.95^8 - 0.95^9 = 0.0332$	B1	Evaluated.
			1

Question 41

<p>(a) For one yellow: YGG + GYG + GGY</p> $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times 3$ $\left[\frac{180}{504} = \right] \frac{5}{14}$	<p>M1 $\frac{a}{9} \times \frac{b}{8} \times \frac{c}{7}$, $0 < a, b, c$ integers ≤ 5, for one arrangement.</p> <p>M1 <i>Their</i> three-factor probability $\times 3$, 3C_1, 3C_2 or 3P_1, (or repeated adding) no additional terms.</p> <p>A1 AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.</p> <p>3</p>																				
<p>(b)</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>X</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>$P(X)$</td> <td>$\frac{24}{504}$</td> <td>$\frac{180}{504}$</td> <td>$\frac{240}{504}$</td> <td>$\frac{60}{504}$</td> </tr> <tr> <td></td> <td>$\left[\frac{1}{21}, \right]$</td> <td>$\left[\frac{5}{14}, \right]$</td> <td>$\left[\frac{10}{21}, \right]$</td> <td>$\left[\frac{5}{42}, \right]$</td> </tr> <tr> <td></td> <td>$[0.0476]$</td> <td>$[0.357]$</td> <td>$[0.476]$</td> <td>$[0.119]$</td> </tr> </tbody> </table>	X	0	1	2	3	$P(X)$	$\frac{24}{504}$	$\frac{180}{504}$	$\frac{240}{504}$	$\frac{60}{504}$		$\left[\frac{1}{21}, \right]$	$\left[\frac{5}{14}, \right]$	$\left[\frac{10}{21}, \right]$	$\left[\frac{5}{42}, \right]$		$[0.0476]$	$[0.357]$	$[0.476]$	$[0.119]$	<p>B1 Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.</p> <p>B1 Second identified correct probability, may not be in table.</p> <p>B1 All probabilities identified and correct . SC if less than 2 correct probabilities or X value(s) omitted: SC B1 3 or 4 probabilities summing to one.</p> <p>3</p>
X	0	1	2	3																	
$P(X)$	$\frac{24}{504}$	$\frac{180}{504}$	$\frac{240}{504}$	$\frac{60}{504}$																	
	$\left[\frac{1}{21}, \right]$	$\left[\frac{5}{14}, \right]$	$\left[\frac{10}{21}, \right]$	$\left[\frac{5}{42}, \right]$																	
	$[0.0476]$	$[0.357]$	$[0.476]$	$[0.119]$																	
<p>(c) $[E(X) =] \frac{840}{504} \frac{5}{3}, 1.67$</p>	<p>B1 OE Must be evaluated. SC B1 FT correct unsimplified expression from incorrect 3(b) using at least 3 probabilities, $0 < p < 1$.</p> <p>1</p>																				

Question 42

<p>(a)</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>x</th> <th>-1</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>p</td> <td>$\frac{1}{12} = 0.0833$</td> <td>$\frac{2}{12} = 0.167$</td> <td>$\frac{4}{12} = 0.333$</td> <td>$\frac{3}{12} = 0.25$</td> <td>$\frac{2}{12} = 0.167$</td> </tr> </tbody> </table>	x	-1	0	1	2	3	p	$\frac{1}{12} = 0.0833$	$\frac{2}{12} = 0.167$	$\frac{4}{12} = 0.333$	$\frac{3}{12} = 0.25$	$\frac{2}{12} = 0.167$	<p>B1</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>0</th> <th>1</th> <th>2</th> <th>2</th> </tr> </thead> <tbody> <tr> <th>-1</th> <td>-1</td> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <th>0</th> <td>0</td> <td>1</td> <td>2</td> <td>2</td> </tr> <tr> <th>1</th> <td>1</td> <td>2</td> <td>3</td> <td>3</td> </tr> </tbody> </table> <p>Table with x values and at least one probability substituted, $0 < p < 1$. Condone any additional x values if probability stated as 0.</p> <p>B1 2 correct identified probabilities.</p> <p>B1 All probabilities correct (accept to 3sf). SC if less than 2 correct probabilities: SC B1 4 or 5 probabilities summing to one.</p> <p>3</p>		0	1	2	2	-1	-1	0	1	1	0	0	1	2	2	1	1	2	3	3
x	-1	0	1	2	3																												
p	$\frac{1}{12} = 0.0833$	$\frac{2}{12} = 0.167$	$\frac{4}{12} = 0.333$	$\frac{3}{12} = 0.25$	$\frac{2}{12} = 0.167$																												
	0	1	2	2																													
-1	-1	0	1	1																													
0	0	1	2	2																													
1	1	2	3	3																													
<p>(b)</p> $E(X) = -\frac{1}{12} + \frac{4}{12} + \frac{6}{12} + \frac{6}{12} = \left[\frac{15}{12} \right]$ $\text{Var}(X) = \frac{1}{12} + 0 + \frac{4}{12} + \frac{12}{12} + \frac{18}{12} - \left(\frac{15}{12} \right)^2$ $\left[\frac{35}{12} - \frac{25}{16} = \right] \frac{65}{48}, 1.35$	<p>M1 May be implied by use in Variance, accept unsimplified expression. Probabilities must sum to 1 ± 0.001.</p> <p>M1 Appropriate variance formula using <i>their</i> $(E(X))^2$. FT accept probabilities not summing to 1. Condone $\frac{35}{12} - \left(\frac{15}{12} \right)^2$ or $\frac{35}{12} - \frac{25}{9}$ from correct table.</p> <p>A1 WWW</p> <p>3</p>																																

Question 43

(a)	<table border="1"> <tr> <td>X</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>$P(X)$</td> <td>$\frac{1}{16}$</td> <td>$\frac{3}{16}$</td> <td>$\frac{5}{16}$</td> <td>$\frac{5}{16}$</td> <td>$\frac{2}{16}$</td> </tr> <tr> <td></td> <td>0.0625</td> <td>0.1875</td> <td>0.3125</td> <td>0.3125</td> <td>0.125</td> </tr> </table>	X	-2	-1	0	1	2	$P(X)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{16}$		0.0625	0.1875	0.3125	0.3125	0.125	<p>B1 Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.</p> <p>B1 3 correct probabilities linked with correct outcomes, may not be in table.</p> <p>B1 2 further correct probabilities linked with correct outcomes, may not be in table No repeated X values. SC if less than 3 correct probabilities seen, award SCB1 Sum of <i>their</i> probabilities, $0 < p < 1$, of 4, 5 or 6 X values = 1 (condone summing to 1 ± 0.01 or better).</p>
	X	-2	-1	0	1	2														
$P(X)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{16}$															
	0.0625	0.1875	0.3125	0.3125	0.125															
		3																		
(b)	$\left[\frac{1}{16} \times (-2)^2 + \frac{3}{16} \times (-1)^2 + \frac{5}{16} \times 0^2 + \frac{5}{16} \times 1^2 + \frac{2}{16} \times 2^2 - \left(\frac{1}{4} \right)^2 \right]$ $\frac{1 \times 4 + 3 \times 1 + 5 \times 0 + 5 \times 1 + 2 \times 4}{16} - 0.25^2$	<p>M1 Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT <i>their</i> table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$. Condone 1 error providing all probabilities < 1 and 0.25^2 used</p>																		
	$\left[= \frac{5}{4} - \frac{1}{16} = \right] \frac{19}{16}, 1.1875$	<p>A1 Condone 1.188 or 1.19 WWW</p>																		
		2																		

Question 44

(a)	$k = \frac{1}{18} (4k + k + 4k + 9k = 18k = 1)$	<p>B1 SOI</p>									
	<table border="1"> <tr> <td>x</td> <td>-2</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{4}{18}$</td> <td>$\frac{1}{18}$</td> <td>$\frac{4}{18}$</td> <td>$\frac{9}{18}$</td> </tr> </table>	x	-2	1	2	3	$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$
x	-2	1	2	3							
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$							
		3									
(b)	$\left[E(X) = \frac{4 \times (-2) + 1 \times 1 + 4 \times 2 + 9 \times 3}{18} = \right]$ $\frac{-8 + 1 + 8 + 27}{18}$	<p>M1 $-8k + k + 8k + 27k$ May be implied by use in Variance. Accept unsimplified expression. FT <i>their</i> table if probabilities sum to 1 or 0.999. SC B1 $28k$.</p>									
	$\left[\text{Var}(X) = \frac{4 \times (-2)^2 + 1 \times 1^2 + 4 \times 2^2 + 9 \times 3^2}{18} - (\text{their } E(X))^2 = \right]$ $= \frac{16 + 1 + 16 + 81}{18} - \left(\text{their } \frac{28}{18} \right)^2$	<p>M1 $16k + k + 16k + 81k - (\text{their mean})^2$ FT <i>their</i> table even if probabilities not summing to 1. Note: If table is correct, $\frac{114}{18} - (\text{their } E(X))^2$ M1. SC B1 $114k - (\text{their mean})^2$.</p>									
	$E(X) = \frac{14}{9}, 1\frac{5}{9}, 1.56, \text{Var}(X) = \frac{317}{81}, 3\frac{74}{81}, 3.91$	<p>A1 Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $3.91 \leq \text{Var}(X) \leq 3.914$</p>									
		3									

Question 45

(a)	x	2	3	4	5	6	B1 Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.	
	p	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$		B1 3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		0.02778	0.1111	0.2778	0.3333	0.25		B1 2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
							3 SC B1 for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.	
(b)	If method FT from <i>their</i> incorrect (a), expressions for $E(X)$ and $\text{Var}(X)$ must be seen at the stage shown in bold (or less simplified) in the scheme with all probabilities < 1 .							
	$E(X) = \frac{1 \times 2 + 4 \times 3 + 10 \times 4 + 12 \times 5 + 9 \times 6}{36} = \frac{\mathbf{2 + 12 + 40 + 60 + 54}}{\mathbf{36}}$						M1 Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 4 or more probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$).	
	$\left[\frac{\text{Var}(X) = \frac{1 \times 2^2 + 4 \times 3^2 + 10 \times 4^2 + 12 \times 5^2 + 9 \times 6^2}{36} - (\text{their } E(X))^2 = \frac{\mathbf{1 \times 4 + 4 \times 9 + 10 \times 16 + 12 \times 25 + 9 \times 36}}{\mathbf{36}} - \left(\text{their } \frac{\mathbf{14}}{\mathbf{3}}\right)^2}{\left[\frac{4 + 36 + 160 + 300 + 324}{36} - \left(\text{their } \frac{\mathbf{14}}{\mathbf{3}}\right)^2 \right]}$						M1 Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in bold (or less simplified) seen.	
	$E(X) = \frac{168}{36}, \frac{14}{3}, 4.67$ $\text{Var}(X) = \frac{10}{9}, 1\frac{1}{9}, 1.11, \frac{1440}{1296}$						A1 Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned SC B1 for identified correct final answers.	
							3	

Question 46

(a)	$a = P(1 \text{ head}) = 0.7 \times (0.5)^3 + 0.3 \times (0.5)^3 \times 3 = \frac{1}{5}$	B1 Clear statement of unevaluated correct calculation $= \frac{1}{5}$. AG
	$b = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 \times 3 = \frac{3}{8}$	M1 Clear statement of unevaluated calculation for either b or c
	$c = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 = \frac{3}{10}$ $\left[\text{or } c = \frac{27}{40} - b \right]$	A1 For either b or c correct B1 FT $\text{their } b + \text{their } c = \frac{27}{40}$
		4
(b)	$E(X) = \frac{3 \times 0 + 16 \times 1 + 30 \times 2 + 24 \times 3 + 7 \times 4}{80} = \frac{176}{80} \text{ or } 2.2$	B1 FT Correct or accept unsimplified calculation using <i>their</i> values for b and c seen (sum of probabilities = 1)
		1
(c)	$[P(0, 1, 2) =]^{10}C_0 \cdot 0.2^0 \cdot 0.8^{10} + {}^{10}C_1 \cdot 0.2^1 \cdot 0.8^9 + {}^{10}C_2 \cdot 0.2^2 \cdot 0.8^8$	M1 One term ${}^{10}C_x p^x (1-p)^{10-x}$, for $0 < x < 10, 0 < p < 1$
	0.107374 + 0.268435 + 0.301989	A1 Correct expression, accept unsimplified leading to final answer
	0.678	B1 $0.677 < p \leq 0.678$
	Alternative method for question 4(c)	
	$1 - [{}^{10}C_{10} \cdot 0.2^{10} \cdot 0.8^0 + {}^{10}C_9 \cdot 0.2^9 \cdot 0.8^1 + {}^{10}C_8 \cdot 0.2^8 \cdot 0.8^2 + {}^{10}C_7 \cdot 0.2^7 \cdot 0.8^3 + {}^{10}C_6 \cdot 0.2^6 \cdot 0.8^4 + {}^{10}C_5 \cdot 0.2^5 \cdot 0.8^5 + {}^{10}C_4 \cdot 0.2^4 \cdot 0.8^6 + {}^{10}C_3 \cdot 0.2^3 \cdot 0.8^7]$	M1 One term ${}^{10}C_x p^x (1-p)^{10-x}$, for $0 < x < 10, 0 < p < 1$
		A1 Correct expression, accept unsimplified
	0.678	B1 $0.677 < p \leq 0.678$
		4
(d)	$0.8^6 \times 0.2 + 0.8^7 \times 0.2 = 0.0524288 + 0.041943$	M1 $p^l \times (1-p) + p^m \times (1-p)$, $l = 6, 7$ $m = l + 1, 0 < p < 1$
	0.0944	A1 $0.09437 \leq p \leq 0.0944$
		2

Question 47

(a)	Method 1: Scenarios identified		
	[no of ways for score of 2 are] 222, 211, 212, 221, 122, 112, 121 [Total options = 64]	B1	7 correct scenarios identified, no incorrect.
	[So $P(X=2) = \frac{7}{4 \times 4 \times 4} = \frac{7}{64}$]	M1	$\frac{a}{4 \times 4 \times 4}$, $a = \text{their number of correct identified scenarios} > 4$
		A1	Approach identified, WWW.
	Method 2: P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one spinner and 1 on two spinners)		
	$\left(\frac{1}{4}\right)^3 + {}^3C_2 \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) + {}^3C_1 \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$	B1	$\left(\frac{1}{4}\right)^3 + {}^3C_2 \left(\text{or } {}^3C_1\right) \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) + d$, $0 < d < 1$
		M1	$\left(\frac{1}{4}\right)^3 + e \left(\frac{1}{4}\right)^3 + f \left(\frac{1}{4}\right)^3$, $1 < e < 5$ and $1 < f < 5$
	[So $P(X=2) = \frac{7}{64}$]	A1	Approach identified, WWW.
	Method 3: P(1 or 2 on each spinner) – P(1 on all spinners)		
	$\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$	B1	$\left(\frac{1}{2}\right)^3 - b$ seen, $0 < b < 1$
		M1	$\left(\frac{1}{2}\right)^3 - c^3$, $0 < c < \frac{1}{2}$
	[So $P(X=2) = \frac{7}{64}$]	A1	Approach identified, WWW.
		3	
(b)	$P(X=1) = \frac{1}{64}$	B1	$P(X=1)$ or $P(X=4)$ correct. Condone answers not in probability distribution table if clearly identified.
	$P(X=4) = \left[1 - \frac{1}{64} - \frac{7}{64} - \frac{19}{64}\right] \frac{37}{64}$	B1 FT	All 4 probabilities summing to 1.
		2	
(c)	$P(Y=6) = \left[\left(\frac{3}{4}\right)^5 \times \frac{1}{4}\right] = 0.0593, \frac{243}{4096}$	B1	Accept 0.059326... to 4 or more SF.
		1	
(d)	$\left(\frac{3}{4}\right)^4$	M1	$\left(\frac{3}{4}\right)^g$, $g = 4, 5$ or p^4 where $0 < p < 1$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406... to 4 or more SF.
	Alternative method for Question 4(d)		
	$P(Y>4) = 1 - P(Y \leq 4) = 1 - \left(\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4}\right)$ $\left[1 - \frac{175}{256}\right]$	M1	Correct or $1 - \left(\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^4\right)$ or $1 - (p + qp + q^2p + q^3p)$ where $0 < p < 1$ and $q = 1-p$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406... to 4 or more SF.
		2	

Question 48

(a)	$[P(17 \text{ or } 18) =] \frac{4}{216} = \frac{1}{54}, 0.0185(185\dots)$	B1	May be seen used in calculation.
	$P(X=6) = \left(\frac{53}{54}\right)^5 \cdot \frac{1}{54}$	M1	$p(1-p)^5, 0 < p < 1$
	0.0169	A1	$0.01686 < p \leq 0.0169$ If A0 scored SC B1 for $0.01686 < p \leq 0.0169$
		3	
(b)	$[P(X < 8) =] 1 - \left(\frac{53}{54}\right)^7$	M1	$1 - \left(\text{their } \left(\frac{53}{54} \text{ or } 0.98148\right) \text{ or correct}\right)^r$, $r = 7, 8 \quad 0 < \text{their } p < 1$
	0.123	A1	$0.1225 \leq p \leq 0.123$
Alternative method for Question 3(b)			
	$[P(X < 8) =]$ $\left(\frac{1}{54}\right) + \left(\frac{53}{54}\right)\left(\frac{1}{54}\right) + \left(\frac{53}{54}\right)^2\left(\frac{1}{54}\right) + \left(\frac{53}{54}\right)^3\left(\frac{1}{54}\right) + \left(\frac{53}{54}\right)^4\left(\frac{1}{54}\right) +$ $\left(\frac{53}{54}\right)^5\left(\frac{1}{54}\right) + \left(\frac{53}{54}\right)^6\left(\frac{1}{54}\right)$	M1	$q + pq + p^2q + p^3q + p^4q + p^5q + p^6q, p + q = 1, 0 < p, q < 1, q$ $= \text{their } \frac{53}{54}$
	0.123	A1	$0.1225 \leq p \leq 0.123$
		2	

Question 49

$$P(1H) = \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{15}{32}$$

$$P(2H) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{7}{32}$$

X	0	1	2	3
$p(X)$	$\frac{9}{32}$	$\frac{15}{32}$	$\frac{7}{32}$	$\frac{1}{32}$
	0.28125	0.46875	0.21875	0.03125

	B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
	B1	$P(1)$ or $P(2)$ correct, need not be in table, accept unsimplified.
	B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF.
	SC B1	for 4 probabilities ($0 < p < 1$) sum to 1 ± 0.005 with $P(1)$ and $P(2)$ incorrect.
	3	

Question 50

$$0.12 + p + q + 0.16 + 0.3 = 1$$

$$-0.24 - p + 0.5q + 0.16 + 0.6 = 0.28$$

Attempt to solve *their* two equations in p and q

$$q = 0.12, p = 0.3$$

$0.12 + p + q + 0.16 + 0.3 = 1$	B1	Sum of probabilities = 1 $p + q = 0.42$ OE.
$-0.24 - p + 0.5q + 0.16 + 0.6 = 0.28$	B1	Form equation using $E(X) = 0.28$ $-p + 0.5q = -0.24$ OE. Accept unsimplified.
Attempt to solve <i>their</i> two equations in p and q	M1	Either Substitution method to form a single equation in either p or q and finding values for both unknowns. Or Elimination method by writing both equations in the same form (usually $ap + bq = c$) and $+$ or $-$ to find an equation in one unknown and finding values for both unknowns.
$q = 0.12, p = 0.3$	A1	CAO, both WWW. If M0 awarded SC B1 for both correct WWW.
	4	

Question 51

(a)	$0.6(0.5)^3 + 0.4(0.5)^3 \times 3$	B1	Either $0.6(0.5)^3 + a$ or $b + 0.4(0.5)^3 \times (3 \text{ or } {}^3C_1)$, $0 < a, b < 1$ seen.												
		M1	$0.6(0.5)^3 + 0.4(0.5)^3 \times d$ seen, $d = 1, 3$. Condone $0.075 + 0.05 \times d$, $d = 1, 3$.												
	$= 0.225$	A1	AG full supporting working required. Scenarios identified and linked to calculations.												
			3												
(b)	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X = x)$</td> <td>0.05</td> <td>0.225</td> <td>0.375</td> <td>0.275</td> <td>0.075</td> </tr> </table>	x	0	1	2	3	4	$P(X = x)$	0.05	0.225	0.375	0.275	0.075	B1	Either $[P(2) =] 0.375, \frac{3}{8}$ or $[P(3) =] 0.275, \frac{11}{40}$ seen. Condone not in table if identified.
		x	0	1	2	3	4								
$P(X = x)$	0.05	0.225	0.375	0.275	0.075										
B1 FT	Both values in table. FT $P(2) + P(3) = 0.650$.														
			2												
(c)	$\text{Var}(X)$ $= [1^2 \times] 0.225 + 2^2 \times \text{their } 0.375 + 3^2 \times \text{their } 0.275 + 4^2 \times 0.075 - 2.1^2$	M1	Appropriate variance formula from their probability distribution table with at least 4 terms, $0 < \text{their } P(x) < 1$. Condone 4.41 for 2.1^2 . Condone mean clearly recalculated inaccurately. Or $0.225 + 4 \times \text{their } 0.375 + 9 \times \text{their } 0.275 + 16 \times 0.075 - 2.1^2$ Condone 2.1^2 for 4.41.												
		A1	If M0 awarded SC B1 for $0.99[0]$ WWW.												
	$[5.4 - 2.1^2] = 0.99[0]$		2												

Question 52

(a)	$\left[P(HH) = \frac{1}{4} \right] [E(X) =] 4$	B1	
			1
(b)	$\left[P(X = 5) = \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) = \right] 0.0791$	B1	$\frac{81}{1024}$
			1
(c)	$[P(X < 7) =] 1 - \left(\frac{3}{4}\right)^6$ or $\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3^2}{4} \times \frac{1}{4} + \dots + \frac{3^5}{4} \times \frac{1}{4}$	M1	$1 - p^n$, $0 < p < 1$, $n = 6, 7$ or $p + p(1-p) + p(1-p)^2 + \dots + p(1-p)^n$, where $n = 4, 5$.
		A1	Accept $0.82202148\dots$ to at least 3SF.
	$= \frac{3367}{4096}, 0.822$		2

Question 53

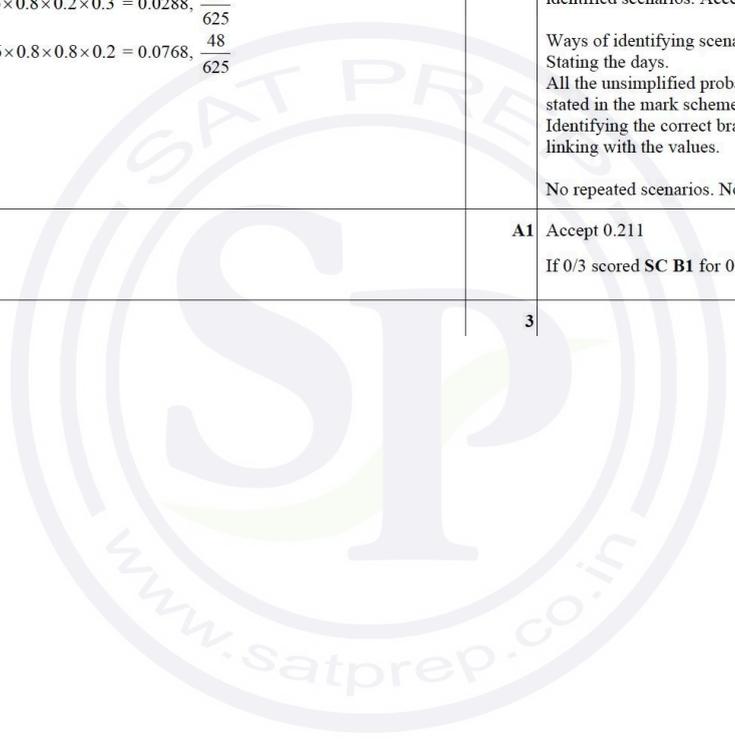
(a)	$[P(X=4) = 3P(X=2)]$ $4k(4+a) = 3 \times 2k(2+a)$ $16k + 4ak = 12k + 6ak$	M1	Using $P(X=4) = 3P(X=2)$ to form an equation in a and k .																				
	$a = 2$	A1	If M0 scored, SC B1 for $a = 2$ www.																				
	$3k + 8k + 15k + 24k = 1$	M1	Using sum of probabilities = 1 to form an equation in k : $k(1+a) + 2k(2+a) + 3k(3+a) + 4k(4+a) = 1$.																				
	$k = \frac{1}{50}$	A1	If M0 scored, SC B1 for $k = \frac{1}{50}$ www.																				
		4																					
(b)	<table border="1"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X)$</td> <td>$\frac{3}{50}, 0.06$</td> <td>$\frac{8}{50}, 0.16$</td> <td>$\frac{15}{50}, 0.3$</td> <td>$\frac{24}{50}, 0.48$</td> </tr> </table>	X	1	2	3	4	$P(X)$	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$	B1 FT	<table border="1"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X)$</td> <td>$k(1+a)$</td> <td>$2k(2+a)$</td> <td>$3k(3+a)$</td> <td>$4k(4+a)$</td> </tr> </table> <p>$0 < p < 1$ for all outcomes, must be numerical.</p>	X	1	2	3	4	$P(X)$	$k(1+a)$	$2k(2+a)$	$3k(3+a)$	$4k(4+a)$
X	1	2	3	4																			
$P(X)$	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$																			
X	1	2	3	4																			
$P(X)$	$k(1+a)$	$2k(2+a)$	$3k(3+a)$	$4k(4+a)$																			
		1																					
(c)	$\text{Var}(X) = \frac{3}{50} \times 1 + \frac{8}{50} \times 2^2 + \frac{15}{50} \times 3^2 + \frac{24}{50} \times 4^2 - 3.2^2$	M1	Correct formula for variance method from their probability distribution table, $0 \leq \text{their } P(x) \leq 1$. Accept $\frac{3+32+135+384}{50} - \frac{256}{25}$.																				
	$[= 11.08 - 3.2^2 =] 0.84[0], \frac{21}{25}$	A1	If M0 score SC B1 for 0.84 www.																				
		2																					

Question 54

(a)	$[3k + 3k + 8k = 1, \text{ so}] k = \frac{1}{14}$	B1									
	<table border="1"> <tr> <td>x</td> <td>-2</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(x)$</td> <td>$\frac{3}{14}, 0.214$</td> <td>$\frac{3}{14}, 0.214$</td> <td>$\frac{8}{14}, 0.571$</td> </tr> </table>	x	-2	2	3	$P(x)$	$\frac{3}{14}, 0.214$	$\frac{3}{14}, 0.214$	$\frac{8}{14}, 0.571$	B1 FT	Table with correct values of x , and at least one correct probability linked with outcome. FT <i>their</i> k . Condone any additional X values if probability stated as 0.
x	-2	2	3								
$P(x)$	$\frac{3}{14}, 0.214$	$\frac{3}{14}, 0.214$	$\frac{8}{14}, 0.571$								
		B1 FT	The outcomes in the table must be -2, 2 and 3. 2 further correct probabilities in table or 3 correct probabilities not in table linked to outcomes, or 3 correct FT probabilities in table using <i>their</i> k , or 3 incorrect probabilities summing to 1 in table if k not stated.								
			If k not calculated, SC B1 for the below.								
			<table border="1"> <tr> <td>x</td> <td>-2</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(x)$</td> <td>$3k$</td> <td>$3k$</td> <td>$8k$</td> </tr> </table>	x	-2	2	3	$P(x)$	$3k$	$3k$	$8k$
x	-2	2	3								
$P(x)$	$3k$	$3k$	$8k$								
		3									
(b)	$[E(X) = -2 \times \frac{3}{14} + 2 \times \frac{3}{14} + 3 \times \frac{8}{14} =]$ $\frac{-6}{14} + \frac{6}{14} + \frac{24}{14}$	M1	Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 3 probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$) or in terms of k .								
	$[\text{Var}(X) = (-2)^2 \times \frac{3}{14} + 2^2 \times \frac{3}{14} + 3^2 \times \frac{8}{14} - (\text{their } E(X))^2 =]$ $4 \times \frac{3}{14} + 4 \times \frac{3}{14} + 9 \times \frac{8}{14} - \left(\text{their } \frac{12}{7}\right)^2$ $\left[\frac{12+12+72}{14} - \left(\text{their } \frac{12}{7}\right)^2\right]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1, or in terms of k with an expression no more evaluated than shown.								
	$E(X) = \frac{12}{7}, 1.71, 1\frac{5}{7}$ $\text{Var}(X) = \frac{192}{49}, 3.92, 3\frac{45}{49}$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E, V, μ, σ^2 , etc.). If A0 earned, SC B1 for identified correct final answers.								
		3									

Question 55

(a)	$[P(\text{no rain}) = 0.6 \times (0.8)^3 =] 0.3072, \frac{192}{625}$	B1 Exact value required
		1
(b)	$0.6 \times 0.8 \times 0.2$	M1 $a \times b \times c$ where $a, b = 0.6, 0.8, c = 0.2, 0.4, 0.7$. Condone including Wednesday with both 0.3 and 0.7 used.
	$= 0.096[0], \frac{12}{125}$	A1
		2
(c)	$P(\text{RDDD}) = 0.4 \times 0.3 \times 0.8 \times 0.8 = 0.0768, \frac{48}{625}$ $P(\text{DRDD}) = 0.6 \times 0.2 \times 0.3 \times 0.8 = 0.0288, \frac{18}{625}$ $P(\text{DDRD}) = 0.6 \times 0.8 \times 0.2 \times 0.3 = 0.0288, \frac{18}{625}$ $P(\text{DDDR}) = 0.6 \times 0.8 \times 0.8 \times 0.2 = 0.0768, \frac{48}{625}$	B1 Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient. M1 Add 4 probability values, $0 < p < 1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark: Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	$0.2112, \frac{132}{625}$	A1 Accept 0.211 If 0/3 scored SC B1 for $0.2112, \frac{132}{625}$.
		3



Question 56

(a)	$[P(X=4) = (0.8)^3(0.2) =] 0.1024, \frac{64}{625}$	B1 Condone 0.102 .																																				
		1																																				
(b)	$[P(X < 6) =] 1 - 0.8^5$	M1 $1 - 0.8^d, d = 5, 6.$																																				
	$= 0.672, \frac{2101}{3125}$	A1 0.67232 to at least 3SF. If M0 awarded, SC B1 for $\frac{2101}{3125}$ or 0.67232 only.																																				
Alternative Method for Question 4(b)																																						
	$[P(X < 6) =] \left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	M1 If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$																																				
	$= 0.672, \frac{2101}{3125}$	A1 0.67232 to at least 3SF. If M0 awarded, SC B1 for $\frac{2101}{3125}$ or 0.67232 only.																																				
		2																																				
(c)	$\left[P(X > 0 X \neq 2) = \frac{P(X > 0 \cap X \neq 2)}{P(X \neq 2)} = \right]$ $= \frac{14}{25} \frac{19}{25}$ $= \frac{14}{19}, 0.737$	M1 $[P(X > 0 \cap X \neq 2) =] \frac{14}{25}, 0.56[0]$ seen as numerator or denominator of conditional probability fraction. M1 $[P(X \neq 2) =] \frac{19}{25}, 0.76[0]$ seen as denominator of conditional probability fraction. A1 Final answer = $\frac{14}{19}, 0.7368421\dots$ to at least 3SF. If A0, SC B1 for correct final answer www.																																				
Alternative Method for Question 4(c)																																						
	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td></tr> <tr><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td></tr> </table>		1	2	3	4	5	1	0	1	2	3	4	2	1	0	1	2	3	3	2	1	0	1	2	4	3	2	1	0	1	5	4	3	2	1	0	M1 [Number of outcome $(X > 0 \cap X \neq 2) =] 14$ seen as numerator or denominator of conditional probability fraction. M1 [Number of outcome $(X \neq 2) =] 19$ seen as denominator of conditional probability fraction.
	1	2	3	4	5																																	
1	0	1	2	3	4																																	
2	1	0	1	2	3																																	
3	2	1	0	1	2																																	
4	3	2	1	0	1																																	
5	4	3	2	1	0																																	
	$\left[P(X > 0 X \neq 2) = \frac{\text{Number of outcome } (X > 0 \cap X \neq 2)}{\text{Number of outcomes } X \neq 2} = \right]$ $\frac{14}{19}, 0.737$	A1 Final answer = $\frac{14}{19}, 0.7368421\dots$ to at least 3SF.																																				
		3																																				
(d)	$[P(X > 2) = 1 - P(0, 1, 2) \text{ with } p = \frac{6}{25}]$ $1 - ({}^9C_0 \left(\frac{19}{25}\right)^9 + {}^9C_1 \left(\frac{6}{25}\right)^1 \left(\frac{19}{25}\right)^8 + {}^9C_2 \left(\frac{6}{25}\right)^2 \left(\frac{19}{25}\right)^7)$ $[1 - (0.08459 + 0.2404 + 0.3037)]$	M1 One term ${}^9C_x (p)^x (1-p)^{9-x}, 0 < p < 1, 0 < x < 9.$ A1 $1 - ({}^9C_0 (1-p)^9 + {}^9C_1 (p)^1 (1-p)^8 + {}^9C_2 (p)^2 (1-p)^7), 0 < p < 1.$ Correct expression from <i>their</i> p , accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.																																				
	0.371	B1 $0.371 \leq p < 0.3715 .$																																				
Alternative Method for Question 4(d)																																						
	$[P(X > 2) = P(3,4,5,6,7,8,9) \text{ with } p = \frac{6}{25}]$ ${}^9C_3 \left(\frac{6}{25}\right)^3 \left(\frac{19}{25}\right)^6 + {}^9C_4 \left(\frac{6}{25}\right)^4 \left(\frac{19}{25}\right)^5 + \dots + {}^9C_8 \left(\frac{6}{25}\right)^8 \left(\frac{19}{25}\right)^1 + {}^9C_9 \left(\frac{6}{25}\right)^9$ $[0.2238 + 0.1060 + \dots + 7.529 \times 10^{-5} + 2.642 \times 10^{-6}]$	M1 One term ${}^9C_x (p)^x (1-p)^{9-x}, 0 < p < 1, 0 < x < 9.$ A1 ${}^9C_3 (p)^3 (1-p)^6 + {}^9C_4 (p)^4 (1-p)^5 + \dots + {}^9C_8 (p)^8 (1-p)^1 + {}^9C_9 (p)^9, 0 < p < 1.$ Correct expression from <i>their</i> p , accept unsimplified, no terms omitted leading to final answer.																																				
	0.371	B1 $0.371 \leq p < 0.3715 .$																																				
		3																																				

Question 57

(a)	$[P(X=3)] = \frac{3}{4} \times \left(\frac{1}{4}\right)^3 \times 4$	M1	$\frac{3}{4} \times \left(\frac{1}{4}\right)^3 \times q$; q a positive integer (1 may be implied).												
	$= \frac{3}{64}$	A1	AG.												
		2													
(b)	<table border="1" style="width: 100%; text-align: center;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>P(X=x)</td> <td>$\frac{81}{256}$</td> <td>$\frac{27}{64}$</td> <td>$\frac{27}{128}$</td> <td>$\frac{3}{64}$</td> <td>$\frac{1}{256}$</td> </tr> </table>	x	0	1	2	3	4	P(X=x)	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$	B1	Either $P(1) = \frac{27}{64}, 0.421875$ or $P(2) = \frac{27}{128}, 0.2109375$ correct to at least 3SF. Condone not in table.
x	0	1	2	3	4										
P(X=x)	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$										
		B1 FT	Both values in table. FT $P(1) + P(2) = \frac{81}{128}, 0.6328125$.												
		2													
(c)	$[E(X)] = \left[0 \times \frac{81}{256}\right] + 1 \times \text{their } \frac{27}{64} + 2 \times \text{their } \frac{27}{128} + 3 \times \frac{12}{256} + 4 \times \frac{1}{256}$	M1	Correct method from <i>their</i> probability distribution table with at least 4 terms, $0 < \text{their } P(x) < 1$, accept partially evaluated. $= 0 + \frac{27}{64} + \frac{54}{128} + \frac{36}{256} + \frac{4}{256}$												
	= 1	A1													
		2													
(d)	$\text{Mean} = 96 \times \frac{67}{256} = 25.125$ $\text{Var} = 96 \times \frac{67}{256} \times \frac{189}{256} = 18.549$	B1	$25.125, 25\frac{1}{8}$ and $18.5493\dots$ to at least 3SF seen, allow unsimplified ($4.3068 \leq \sigma \leq 4.307$ implies correct variance).												
	$P(X < 20) = P\left(Z < \frac{19.5 - 25.125}{\sqrt{18.549}}\right)$	M1	Substituting <i>their</i> μ and σ into \pm standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.												
		M1	Using continuity correction 19.5 or 20.5 in <i>their</i> standardisation formula. Note: $\frac{\pm 5.625}{\sqrt{18.549}}$ seen gains M2 BOD.												
	$[= P(Z < -1.306) = 1 - \Phi(1.306)] = 1 - 0.9042 =$	M1	Appropriate area Φ , from final process. Must be a probability.												
	0.0958	A1	$0.0957 \leq p \leq 0.0958$. SC B1 for $0.0957 \leq p \leq 0.0958$ if B1M0M0M1 scored.												
		5													

Question 58

(a)	$2k + 6k + 12k + 20k = 1, \left[k = \frac{1}{40} \right]$	M1	Using sum of probabilities = 1 to form an equation in k . Accept $1 \times 2 \times k + 2 \times 3 \times k + 3 \times 4 \times k + 4 \times 5 \times k = 1$.																									
	<table border="1"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X)$</td> <td>$\frac{2}{40}$</td> <td>$\frac{6}{40}$</td> <td>$\frac{12}{40}$</td> <td>$\frac{20}{40}$</td> </tr> <tr> <td></td> <td>0.05</td> <td>0.15</td> <td>0.3</td> <td>0.5</td> </tr> </table>	X	1	2	3	4	$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$		0.05	0.15	0.3	0.5	M1	<table border="1"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X)$</td> <td>$2k$</td> <td>$6k$</td> <td>$12k$</td> <td>$20k$</td> </tr> </table> <p>Two correctly linked, accurate probabilities. May be in terms of k. May not be in a table.</p>	X	1	2	3	4	$P(X)$	$2k$	$6k$	$12k$	$20k$
X	1	2	3	4																								
$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$																								
	0.05	0.15	0.3	0.5																								
X	1	2	3	4																								
$P(X)$	$2k$	$6k$	$12k$	$20k$																								
		A1	Table with correct X values and correct probabilities.																									
		3																										
(b)	$[E(X) =] [E(X) = \frac{1 \times 2 + 2 \times 6 + 3 \times 12 + 4 \times 20}{40}, \frac{2 + 12 + 36 + 80}{40}]$	M1	$[E(X) = 1 \times 2k + 2 \times 6k + 3 \times 12k + 4 \times 20k = 130k]$ Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 3 or more probabilities summing to 1 ($0 < p < 1$). If there are outcomes in the table without probabilities, condone and treat as $p = 0$.																									
	$\left[\text{Var}(X) = \frac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 12 + 4^2 \times 20}{40} - (\text{their } E(X))^2 = \right]$ $\frac{1 \times 2 + 4 \times 6 + 9 \times 12 + 16 \times 20}{40} - \left(\text{their } \frac{13}{4} \right)^2$ $\left[\frac{2 + 24 + 108 + 320}{40} - \left(\text{their } \frac{13}{4} \right)^2 \right]$	M1	$[\text{Var}(X) = 1^2 \times 2k + 2^2 \times 6k + 3^2 \times 12k + 4^2 \times 20k - (130k)^2]$ Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the highlighted calculation (or less simplified) seen. Note: if table is correct, $\frac{454}{40} \left(\text{or } \frac{227}{20} \text{ or any calculation} \right) - (\text{their } E(X))^2$ implies M1.																									
	$E(X) = \frac{13}{4}, 3\frac{1}{4}, 3.05 \quad \text{Var}(X) = \frac{63}{80}, 0.7875$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in variance. Condone E, V, μ, σ etc. If A0 earned, SC B1 for identified correct final solutions.																									
		3																										

Question 59

(a)	<table border="1"> <thead> <tr> <th colspan="4">Aces</th> <th colspan="4">Jets</th> </tr> </thead> <tbody> <tr> <td>9</td><td>8</td><td>6</td><td>4</td> <td>16</td><td>6</td><td>8</td><td></td> </tr> <tr> <td></td><td>4</td><td>3</td><td>1</td> <td>17</td><td>0</td><td>4</td><td>4 5</td> </tr> <tr> <td>2</td><td>2</td><td>1</td><td>0</td> <td>18</td><td>1</td><td>1</td><td>8 8</td> </tr> <tr> <td></td><td></td><td></td><td></td> <td>19</td><td></td><td></td><td>0</td> </tr> </tbody> </table>	Aces				Jets				9	8	6	4	16	6	8			4	3	1	17	0	4	4 5	2	2	1	0	18	1	1	8 8					19			0	B1	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e., stem values are repeated), the remaining B marks are available.
Aces				Jets																																							
9	8	6	4	16	6	8																																					
	4	3	1	17	0	4	4 5																																				
2	2	1	0	18	1	1	8 8																																				
				19			0																																				
	Key: 1 17 0 means 171 cm for the Aces and 170 cm for the Jets	B1	Correct Aces labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.																																								
		B1	Correct Jets labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.																																								
		B1	Correct key for their diagram, need both teams labelled and 'cm' stated at least once here, or in leaf headings or title.																																								
		4																																									
(b)	Median = 173 [cm]	B1	Accept Q_2 ; must be identified.																																								
	[IQR =] 181 - 168	M1	$180 \leq UQ \leq 182 - 166 \leq LQ \leq 169$ Implied if both quartile values are stated and an appropriate IQR calculated accurately.																																								
	13 [cm]	A1	www If M0 scored SC B1 for 13 www.																																								
		3																																									
(c)	Jets have a greater variety of heights. Jets have a wider range of height. Jets have a greater/larger/bigger/wider/'more' spread of heights. Aces have a smaller variety of height etc...	B1	[Jets IQR = 18 cm, Range = 24 cm Aces IQR = <i>their</i> 4(b) , Range = 18 cm] Comment about spread in context, must include height. Comparison of values does not score until a comment in context is made. If values for range or IQR are stated, they must be correct or FT from 4(b) . If more than one comment about spread, mark the final comment. Additional comments about central tendency score B0.																																								
		1																																									

Question 60

(a)	$p + r + 0.55 = 1$	M1	Using sum of probabilities = 1 to form an equation. Accept $p + r = 0.45$ oe.
	$p + 2r + 0.45 = 1.1$	M1	Use $E(X) = 1.1$ to form an equation. Accept $p + 2r = 0.65$ oe. NB: These marks can be gained in either order; the second M may have an algebraic substitution.
	$p = 0.25, r = 0.2$	A1	If both Ms not awarded, SC B1 for $p = 0.25, r = 0.2$ stated.
		3	
(b)	$[\text{Var}(X) =] [0.4 \times 0^2 +] \text{their } 0.25 [\times 1^2] + (\text{their } 0.2) \times 2^2 + 0.15 \times 3^2 - 1.1^2$ $[= [0+] 0.25 + 0.8 + 1.35 - 1.21]$	M1	Correct formula for variance method using their probability distribution table, $0 < \text{their } P(x) < 1$.
	$= 1.19, 1 \frac{19}{100}$	A1	If M0 awarded, SC B1 for 1.19 www. $\frac{119}{100}$ is A0.
		2	

Question 61

(a)	$0.28 + 6p = 1, p = 0.12$	B1	Using sum of probabilities = 1 to form an equation. Accept $0.28 + p + 2p + 3p = 1, p = 0.12$. Substitution of 0.12 into the expression scores B0.																																																		
		1																																																			
(b)	[For fair spinners (blue and green), probability of any score is 0.25 Scenarios to give total 4 or less:]	B1	Correct probability for 1 identified scenario, accept unsimplified, www.																																																		
	<table border="1"> <thead> <tr> <th>R</th> <th>B</th> <th>G</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>2</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>2</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>2</td> <td>1</td> <td>1</td> <td>$0.12 \times (0.25)^2$</td> <td>= 0.0075</td> </tr> </tbody> </table>	R	B	G			1	1	2	$0.28 \times (0.25)^2$	= 0.0175	1	2	1	$0.28 \times (0.25)^2$	= 0.0175	1	1	1	$0.28 \times (0.25)^2$	= 0.0175	2	1	1	$0.12 \times (0.25)^2$	= 0.0075	M1	Add values of 4 correct scenarios, may be implied by correct unsimplified expressions. No incorrect/repeated scenarios.																									
R	B	G																																																			
1	1	2	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	2	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
2	1	1	$0.12 \times (0.25)^2$	= 0.0075																																																	
	0.06	A1	If A0 scored, SC B1 for 0.06 www.																																																		
		3																																																			
(c)	$[P(X \text{ is odd}) = 0.28 + 2 \times 0.12 \text{ or } 0.24] = 0.52[0]$	B1	Seen alone or as the denominator of a conditional probability fraction. Accept unsimplified.																																																		
	<table border="1"> <thead> <tr> <th>R</th> <th>B</th> <th>G</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>1</td> <td>3</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>1</td> <td>4</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>2</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>2</td> <td>2</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>3</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>1</td> <td>4</td> <td>1</td> <td>$0.28 \times (0.25)^2$</td> <td>= 0.0175</td> </tr> <tr> <td>3</td> <td>1</td> <td>1</td> <td>$0.24 \times (0.25)^2$</td> <td>= 0.015</td> </tr> </tbody> </table>	R	B	G			1	1	1	$0.28 \times (0.25)^2$	= 0.0175	1	1	2	$0.28 \times (0.25)^2$	= 0.0175	1	1	3	$0.28 \times (0.25)^2$	= 0.0175	1	1	4	$0.28 \times (0.25)^2$	= 0.0175	1	2	1	$0.28 \times (0.25)^2$	= 0.0175	1	2	2	$0.28 \times (0.25)^2$	= 0.0175	1	3	1	$0.28 \times (0.25)^2$	= 0.0175	1	4	1	$0.28 \times (0.25)^2$	= 0.0175	3	1	1	$0.24 \times (0.25)^2$	= 0.015	M1	Values of at least 5 identified correct scenarios added, accept unsimplified, condone incorrect scenarios in calculation.
R	B	G																																																			
1	1	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	2	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	3	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	4	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	2	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	2	2	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	3	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	4	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
3	1	1	$0.24 \times (0.25)^2$	= 0.015																																																	
	$[P(\text{product of 3 scores} \leq 4 \cap X \text{ is odd}) =] 0.28 \times (0.25)^2 \times 8 + 0.24 \times (0.25)^2$	M1	$0.28 \times (0.25)^2 \times x + 0.24 \times (0.25)^2$, or $0.0175 \times x + 0.015$ where $x = 4, 5, 6, 7, \text{ or } 8$. Seen alone or as numerator/denominator of a conditional probability fraction.																																																		

(c)	$\left[\frac{P(\text{product of 3 scores} \leq 4 \mid X \text{ is odd})}{P(X \text{ is odd})} = \frac{P(\text{product of 3 scores} \leq 4 \cap X \text{ is odd})}{P(X \text{ is odd})} \right]$ $\frac{0.155}{0.52}$	M1 $\frac{0.28 \times (0.25)^2 \times x + 0.24 \times (0.25)^2}{0.28 + 0.24} \quad x = 4, 5, 6, 7, 8$ or <i>their identified</i> $P(\text{product of 3 scores is 4 or less and } X \text{ is odd})$ <i>their identified</i> $P(\text{odd})$
	$= 0.298, \frac{155}{520}, \frac{31}{104}$	A1 0.2980769... to at least 3SF.
		5

Question 62

(a)	$P(X = 2) = P(\text{WLL or DDL})$ $= 0.5 \times 0.2^2 \times {}^3C_1 + 0.3^2 \times 0.2 \times {}^3C_1 = [0.06 + 0.054]$	M1 $0.5 \times 0.2^2 \times {}^3C_a (\text{or } 3) + x; 0 < x < 1; a = 1, 2.$ Or $0.3^2 \times 0.2 \times {}^3C_b (\text{or } 3) + y; 0 < y < 1; b = 1, 2.$ Or $0.5 \times 0.2^2 \times a + 0.3^2 \times 0.2 \times b; a, b = 1, 2, 3.$																								
	= 0.114	A1 AG. Fully correct solutions with outcomes identified and linked to appropriate probabilities. Condone 2 = W, 1 = D, 0 = L. Probabilities alone do not identify outcomes. If individual scenarios are identified, separate calculations must correspond to the order.																								
		2																								
(b)	<table border="1"> <thead> <tr> <th>x</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>P(X=x)</td> <td>0.008</td> <td>0.036</td> <td>0.114</td> <td>0.207</td> <td>0.285</td> <td>0.225</td> <td>0.125</td> </tr> <tr> <td></td> <td>$\frac{1}{125}$</td> <td>$\frac{9}{250}$</td> <td></td> <td></td> <td></td> <td>$\frac{9}{40}$</td> <td></td> </tr> </tbody> </table>	x	0	1	2	3	4	5	6	P(X=x)	0.008	0.036	0.114	0.207	0.285	0.225	0.125		$\frac{1}{125}$	$\frac{9}{250}$				$\frac{9}{40}$		B1 One additional correct probability in table or clearly identified. B1 A second additional correct probability in table or clearly identified.
x	0	1	2	3	4	5	6																			
P(X=x)	0.008	0.036	0.114	0.207	0.285	0.225	0.125																			
	$\frac{1}{125}$	$\frac{9}{250}$				$\frac{9}{40}$																				
		B1 Final correct probability, all probabilities in table. If 0/3 scored, SC B1 for three additional probabilities in table that sum to 0.269 exactly.																								
		3																								
(c)	$[E(X) =$ $[0.008 \times 0 + 0.036 \times 1 + 0.114 \times 2 + 0.207 \times 3 + 0.285 \times 4$ $+ 0.225 \times 5 + 0.125 \times 6 =]$ $[0] + 0.036 + 0.228 + 0.621 + 1.140 + 1.125 + 0.750 [= 3.9]$ OR $E(X) = 3(0.5 \times 2 + 0.3 \times 1) [= 3.9]$	M1 Accept unsimplified expression. May be calculated in the variance, FT <i>their</i> table with probabilities, $0 < p < 1$, that sum to 1. FT acceptable at the bold partially evaluated stage.																								
	$[\text{Var}(X) =$ $[0.008 \times 0^2 + 0.036 \times 1^2 + 0.114 \times 2^2 + 0.207 \times 3^2 + 0.285 \times 4^2$ $+ 0.225 \times 5^2 + 0.125 \times 6^2 - \text{their } 3.9^2 =]$ $[0.008 \times 0 + 0.036 \times 1 + 0.114 \times 4 + 0.207 \times 9 + 0.285 \times 16$ $+ 0.225 \times 25 + 0.125 \times 36 - \text{their } 3.9^2]$	M1 Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with probabilities, $0 < p < 1$, that may not sum to 1.																								
	$= [17.04 - 3.9^2] = 1.83$	A1 Cao. Condone $\frac{183}{100}$.																								
		3																								

Question 63

(a)	<table border="1"> <tr> <td>x</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{1}{36}$</td> <td>$\frac{4}{36}$</td> <td>$\frac{10}{36}$</td> <td>$\frac{12}{36}$</td> <td>$\frac{9}{36}$</td> </tr> <tr> <td></td> <td>$\frac{1}{36}$</td> <td>$\frac{1}{9}$</td> <td>$\frac{5}{18}$</td> <td>$\frac{1}{3}$</td> <td>$\frac{1}{4}$</td> </tr> </table> <p>Decimal equivalent 3sf: 0.0278, 0.111, 0.278, 0.333, 0.25</p>	x	2	3	4	5	6	$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$		$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$	<p>B1 Table with correct X values and at least one correct probability associated with the correct X value. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.</p> <p>B1 Three other probabilities associated with correct x values, need not be in table, accept unsimplified.</p> <p>B1 Five correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. SC B1 for five probabilities summing to 1 placed in a probability distribution table with the correct x values.</p>
x	2	3	4	5	6															
$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$															
	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$															
		3																		
(b)	$E(X) = \frac{1 \times 2 + 4 \times 3 + 10 \times 4 + 12 \times 5 + 9 \times 6}{36} =$ $\frac{2}{36} + \frac{12}{36} + \frac{40}{36} + \frac{60}{36} + \frac{54}{36} \left[= \frac{14}{3} \text{ or } 4.67 \right]$ $\text{Var}(X) = \frac{1 \times 2^2 + 4 \times 3^2 + 10 \times 4^2 + 12 \times 5^2 + 9 \times 6^2}{36} - \left(\frac{14}{3} \right)^2$ $\left[= \frac{824}{36} - \frac{196}{9} = 22.89 - 21.78 \right] = \frac{10}{9}$	<p>M1 Accept unsimplified expression or sum of fractions seen. May be calculated in variance. FT <i>their</i> table with five probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$).</p> <p>M1 Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 4 or more probabilities. ($0 < p < 1$) which need not sum to 1. Note: If table is correct, then $\left(\frac{824}{36} \text{ or } \frac{206}{9} \text{ or } 22.89 \right) - \left(\frac{196}{9} \text{ or } 21.78 \text{ or } \left(\frac{14}{3} \right)^2 \right)$ implies M1.</p> <p>A1 $1\frac{1}{9}$, 1.11[1], 1.1</p>																		
		3																		
(c)	$P(X \text{ even} X > 3) = \frac{\frac{10}{36} + \frac{9}{36}}{\frac{31}{36}}$ $= \frac{19}{31}$	<p>M1 $\frac{\text{their } P(4) + \text{their } P(6)}{\text{their } P(4) + P(5) + P(6)}$, all probabilities ($0 < p < 1$). If sample space seen in any part of the question, then M1 $\frac{\text{their } 19}{\text{their } 31}$.</p> <p>A1 0.613</p>																		
		2																		

Question 64

(a)	<p>[\\$7 =] [\\$]5 + [\\$]2 [Probability =] $\frac{1}{5} \times \frac{2}{4} \times 2 = \frac{1}{5} = 0.2$ Or [Probability =] $0.2 \times 0.5 \times 2 = 0.2$</p>	<p>B1 AG Must include [\\$7], 5, 2 and link the probabilities to the appropriate value $\frac{[{}^1C_1] \times {}^2C_1}{{}^5C_2} = 0.2$. $\frac{1}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$, not $\frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4}$ unless 5 and 2 and 2 and 5 seen in solution. If all possibilities identified (e.g. outcome table), must be clearly labelled and terms fulfilling the condition identified.</p>												
		1												
(b)	<table border="1"> <tr> <td>x</td> <td>2</td> <td>3</td> <td>4</td> <td>6</td> <td>7</td> </tr> <tr> <td>$P(X=x)$</td> <td>0.1</td> <td>0.4</td> <td>0.1</td> <td>0.2</td> <td>0.2</td> </tr> </table>	x	2	3	4	6	7	$P(X=x)$	0.1	0.4	0.1	0.2	0.2	<p>B1 Table with correct x values and at least one further non-zero probability correct. Condone extra x values if probability stated as 0.</p> <p>B1 Two more correct non-zero probabilities linked with correct outcomes. Accept probabilities not in table if clearly identified.</p> <p>B1 All five probabilities correct. Accept probabilities not in table if clearly identified. SC B1 for four further non-zero probabilities adding to 0.8 if B1 max scored.</p>
x	2	3	4	6	7									
$P(X=x)$	0.1	0.4	0.1	0.2	0.2									
		3												

(c)	$[E(X) = 0.1 \times 2 + 0.4 \times 3 + 0.1 \times 4 + 0.2 \times 6 + 0.2 \times 7]$ $0.2 + 1.2 + 0.4 + 1.2 + 1.4 [= 4.4]$	M1	Accept unsimplified expression. May be calculated in the variance, FT <i>their</i> table with at least 5 probabilities, $0 < p < 1$, that sum to 1. FT acceptable at the bold partially evaluated stage.
	$[\text{Var}(X) = 0.1 \times 2^2 + 0.4 \times 3^2 + 0.1 \times 4^2 + 0.2 \times 6^2$ $+ 0.2 \times 7^2 - 4.4^2$ $0.1 \times 4 + 0.4 \times 9 + 0.1 \times 16 + 0.2 \times 36 + 0.2 \times 49 - 4.4^2$ $= 3.24$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with at least 4 probabilities, $0 < p < 1$, that may not sum to 1. FT acceptable at the bold partially evaluated stage. Note: if table is correct, $22.6 - (4.4^2 \text{ or } 19.36)$ implies this M1.
		A1	CAO $\frac{81}{25}, 3\frac{6}{25}$ scores A0. Only dependent upon previous M1 (M0 M1 A1 possible). If M0 M0 scored, SC B1 for 3.24 WWW.
		3	

Question 65

(a)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{24}{60}$</td> <td>$\frac{26}{60}$</td> <td>$\frac{9}{60}$</td> <td>$\frac{1}{60}$</td> </tr> <tr> <td></td> <td>$\frac{2}{5}$</td> <td>$\frac{13}{30}$</td> <td>$\frac{3}{20}$</td> <td>$\frac{1}{60}$</td> </tr> <tr> <td></td> <td>0.4</td> <td>0.433</td> <td>0.15</td> <td>0.0167</td> </tr> </table>	x	0	1	2	3	$P(X=x)$	$\frac{24}{60}$	$\frac{26}{60}$	$\frac{9}{60}$	$\frac{1}{60}$		$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$		0.4	0.433	0.15	0.0167	B1	Table with correct X values and at least one probability. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.
x	0	1	2	3																			
$P(X=x)$	$\frac{24}{60}$	$\frac{26}{60}$	$\frac{9}{60}$	$\frac{1}{60}$																			
	$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$																			
	0.4	0.433	0.15	0.0167																			
		B1	$P(X=1)$ or $P(X=2)$ correct and identified, need not be in table, accept unsimplified.																				
		B1	Two more correct and identified probabilities, need not be in table, accept unsimplified.																				
		B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. SC B1 for four probabilities summing to 1 placed in a probability distribution table with the correct x values.																				
		4																					
(b)	$[P(Y=0) = \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2];$ $[P(Y=5) = \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2]$	B1	Either $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2$, not $\frac{2}{5} \times (1-p)^2$; or $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2$, not $\frac{1}{60} \times p^2$.																				
	$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2 = 6 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2$ $[24(1-p)^2 = 6 \times p^2]$ $3p^2 - 8p + 4 = 0$	M1	Equating and forming a 3 term quadratic equation. <i>Their</i> $P(Y=0) = 6 \times \text{their } P(Y=5)$.																				
	$p = \frac{2}{3}$	A1	Not dependent on B1. A0 if $p = 2$ seen and not clearly rejected.																				
		SC B1	if $p = \frac{2}{3}$ obtained from a correct quadratic with more than three terms. If $p = 2$ seen and not clearly rejected, SC B0 .																				
		3																					

Question 66

(a)	<table border="1"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>6</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{6}{36}$</td> <td>$\frac{12}{36}$</td> <td>$\frac{6}{36}$</td> <td>$\frac{6}{36}$</td> <td>$\frac{6}{36}$</td> </tr> <tr> <td></td> <td>$\frac{1}{6}$</td> <td>$\frac{1}{3}$</td> <td>$\frac{1}{6}$</td> <td>$\frac{1}{6}$</td> <td>$\frac{1}{6}$</td> </tr> <tr> <td></td> <td>0.167</td> <td>0.333</td> <td>0.167</td> <td>0.167</td> <td>0.167</td> </tr> </table>	x	1	2	3	4	6	$P(X=x)$	$\frac{6}{36}$	$\frac{12}{36}$	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{6}{36}$		$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		0.167	0.333	0.167	0.167	0.167	<p>B1 Table with correct x values and at least one correct probability linked with the correct x-value. Values need not be in order, lines may not be drawn, may be vertical, x and $P(X)$ may be omitted. Condone any additional x values if probability stated as 0.</p> <p>B1 4 correct probabilities linked with the correct x-values, need not be in table, accept unsimplified.</p> <p>B1 5 correct probabilities linked with correct x-values, may not be in table. Decimals correct to at least 3 SF. SC B1 4 or 5 probabilities summing to 1 placed in a probability distribution table with 4 or 5 x-values between 1 and 6 inclusive.</p>
x	1	2	3	4	6																					
$P(X=x)$	$\frac{6}{36}$	$\frac{12}{36}$	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{6}{36}$																					
	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$																					
	0.167	0.333	0.167	0.167	0.167																					
		3																								
(b)	$[E(X) = \frac{1}{36}(6+24+18+24+36) =] 3$	<p>B1 FT FT <i>their</i> table with 4 or 5 probabilities ($0 < p < 1$) summing to 1.</p>																								
		1																								

Question 67

(a)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{3}{12}$, $\frac{1}{4}$ 0.25</td> <td>$\frac{2}{12}$, $\frac{1}{6}$ 0.167</td> <td>$\frac{2}{12}$</td> <td>$\frac{2}{12}$</td> <td>$\frac{1}{12}$ 0.0833</td> <td>$\frac{1}{12}$</td> <td>$\frac{1}{12}$</td> </tr> </table>	x	0	2	4	6	8	10	12	$P(X=x)$	$\frac{3}{12}$, $\frac{1}{4}$ 0.25	$\frac{2}{12}$, $\frac{1}{6}$ 0.167	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$ 0.0833	$\frac{1}{12}$	$\frac{1}{12}$	<p>B1 Table with correct x values and at least 2 correct probabilities. Condone any additional x values if probability stated as 0.</p>
x	0	2	4	6	8	10	12											
$P(X=x)$	$\frac{3}{12}$, $\frac{1}{4}$ 0.25	$\frac{2}{12}$, $\frac{1}{6}$ 0.167	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$ 0.0833	$\frac{1}{12}$	$\frac{1}{12}$											
		<p>B1 Four more probabilities correctly linked to the correct x value, need not be in table, accept unsimplified.</p> <p>B1 7 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF. SC B1 7 or more probabilities summing to 1 placed in a probability distribution table.</p>																
		3																
(b)	$[E(X) =] [0] + 2 \times \frac{2}{12} + 4 \times \frac{2}{12} + 6 \times \frac{2}{12} + 8 \times \frac{1}{12} + 10 \times \frac{1}{12} + 12 \times \frac{1}{12}$ $[[0] + \frac{4}{12} + \frac{8}{12} + \frac{12}{12} + \frac{8}{12} + \frac{10}{12} + \frac{12}{12} = \frac{54}{12} = 4.5]$	<p>MI Accept unsimplified expression. May be calculated in variance. Accept $\frac{1}{3} + \frac{2}{3} + 1 + \frac{2}{3} + \frac{5}{6} + 1$ OE for the M mark FT <i>their</i> table with 7 or 8 probabilities summing to $0.999 \leq total \leq 1$ ($0 < p < 1$). FT acceptable at the bold partially evaluated stage.</p>																
	$[Var(X) =]$ $[0] + 2^2 \times \frac{2}{12} + 4^2 \times \frac{2}{12} + 6^2 \times \frac{2}{12} + 8^2 \times \frac{1}{12} + 10^2 \times \frac{1}{12} + 12^2 \times \frac{1}{12} - (their 4.5)^2$ $[\frac{[0] + 2 \times 4 + 2 \times 16 + 2 \times 36 + [1 \times] 64 + [1 \times] 100 + [1 \times] 144}{12} - \frac{81}{4}]$ $[= 35 - 4.5^2]$	<p>MI Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 6 or more probabilities. ($0 < p < 1$) which need not sum to 1 or with an expression no more evaluated than shown. FT acceptable at the bold partially evaluated stage with <i>their</i> probabilities.</p>																
	$= 14.75, 14\frac{3}{4}$	<p>A1 CAO Accept $\frac{59}{4}$. If either or both M marks not awarded, SC B1 for correct answer WWW</p>																
		3																

Question 68

(a)	[Probs $6k, 3k, 2k, 6k, 11k$ so $28k = 1$,] $k = \frac{1}{28}$	B1	k must be identified												
	<table border="1"> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{6}{28}$ 0.2143</td> <td>$\frac{3}{28}$ 0.1071</td> <td>$\frac{2}{28}$ 0.07143</td> <td>$\frac{6}{28}$ 0.2143</td> <td>$\frac{11}{28}$ 0.3929</td> </tr> </table>	x	-2	-1	0	2	3	$P(X=x)$	$\frac{6}{28}$ 0.2143	$\frac{3}{28}$ 0.1071	$\frac{2}{28}$ 0.07143	$\frac{6}{28}$ 0.2143	$\frac{11}{28}$ 0.3929	M1	Table with correct outcomes and 2 correct probabilities. FT substituting <i>their</i> k correctly into formula, with $0 < p < 1$. No additional x values unless probability 0. Condone in terms of k of the form $\frac{6k}{28}$ or $6k$.
x	-2	-1	0	2	3										
$P(X=x)$	$\frac{6}{28}$ 0.2143	$\frac{3}{28}$ 0.1071	$\frac{2}{28}$ 0.07143	$\frac{6}{28}$ 0.2143	$\frac{11}{28}$ 0.3929										
		A1	Fully correct. Decimal answers to at least 3 sig figures, condone not summing exactly to 1.												
		3													
(b)	$E(X) = \left[-2 \times \frac{6}{28} + -1 \times \frac{3}{28} + \left[0 \times \frac{2}{28} \right] + 2 \times \frac{6}{28} + 3 \times \frac{11}{28} \right]$ $\frac{1}{28} (-12 - 3 + 12 + 33) = \frac{15}{14}$	M1	Accept unsimplified expression. May be calculated in the variance. FT <i>their</i> table with 5 probabilities $0 < p < 1$ that sum to 1.												
	$\text{Var}(X) = \frac{6 \times (-2)^2 + 3 \times (-1)^2 + 6 \times 2^2 + 11 \times 3^2}{28}$ $- \text{their} \left(\frac{15}{14} \right)^2$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with at least 4 probabilities $0 < p < 1$, that may not sum to 1.												
	$= 4.21, 4 \frac{41}{196}$	A1	Condone $\frac{825}{196}$. If one or both M marks not awarded, SC B1 for correct answer WWW.												
		3													

Question 69

(a)	$\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{1}{3} \times \frac{1}{4} \times \frac{3}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} = \left[\frac{9}{60} = \right] \frac{3}{20}$ AG	B1	Order of coins must be consistent with question if not identified.										
		1											
(b)	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$P(X=x)$</td> <td>$\frac{24}{60}, \frac{8}{20}, \frac{2}{5}$ 0.4</td> <td>$\frac{26}{60}, \frac{13}{30}$ 0.433</td> <td>$\frac{9}{60}, \frac{3}{20}$ 0.15</td> <td>$\frac{1}{60}$ 0.0167</td> </tr> </table>	x	0	1	2	3	$P(X=x)$	$\frac{24}{60}, \frac{8}{20}, \frac{2}{5}$ 0.4	$\frac{26}{60}, \frac{13}{30}$ 0.433	$\frac{9}{60}, \frac{3}{20}$ 0.15	$\frac{1}{60}$ 0.0167	B1	Table with correct values of x and at least two correct non-zero probabilities.
x	0	1	2	3									
$P(X=x)$	$\frac{24}{60}, \frac{8}{20}, \frac{2}{5}$ 0.4	$\frac{26}{60}, \frac{13}{30}$ 0.433	$\frac{9}{60}, \frac{3}{20}$ 0.15	$\frac{1}{60}$ 0.0167									
		B1	One more correct non-zero probability linked with correct x value, need not be in table if clearly identified, accept unsimplified (total of 3 correct probabilities).										
		B1	4 correct probabilities linked with the correct outcomes, may not be in table. Decimals correct to at least 3SF. SC1 for 4 or more probabilities summing to 1 placed in a probability distribution table.										
		3											
(c)	$[\text{Var}(X) =] \frac{(0^2 \times 24) + 1^2 \times 26 + 2^2 \times 9 + 3^2 \times 1}{60} - \left(\frac{47}{60} \right)^2$ $= \frac{1 \times 26 + 4 \times 9 + 9 \times 1}{60} - \left(\frac{47}{60} \right)^2$	M1	Appropriate variance formula using $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1, with an expression no more evaluated than in bold . FT acceptable at the bold partially evaluated stage with <i>their</i> probabilities.										
	$= \frac{2051}{3600}, 0.570$	A1	$0.5695 < \text{Var}(X) \leq 0.570$. If M0 scored, SC1 for $\frac{2051}{3600}, 0.570$ WWW Note: 0.57 without more accurate previous value penalised as 2SF.										
		2											

Question 70

x	0	1	2	3
$P(X=x)$	$\frac{1}{24}$	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{6}{24}$
	0.0416 7	0.25	0.458 3	0.25

B1 Table with correct values of x and at least one non-zero probability correct.
Condone extra x values if probability stated as 0.

B1 Two more correct non-zero probabilities linked with correct outcome (3 correct probabilities present).
Accept probabilities not in table if clearly identified.

B1 Four correct probabilities linked with the correct outcomes. Accept probabilities not in table if clearly identified.
Non-exact decimals correct to at least 3SF.
SCB1 for 4 non-zero probabilities (not all $\frac{1}{4}$) in table with correct x values adding to 1 if **B1** max scored.

3

Question 71

(a)

Method 1

$$P(X=2) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{4!}{2!2!} = \frac{3}{7}$$

M1 AG.
 $\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times k$ or $\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} \times k$ for k an integer, $k > 1$.

A1 4C_2 may be seen for $\frac{4!}{2!2!}$.

Method 2

$$\frac{{}^6C_2 \times {}^4C_2}{10C_4} = \left[\frac{15 \times 6}{210} \right] \frac{3}{7}$$

M1 AG.
 ${}^6C_2 \times {}^4C_2$ seen as the numerator of a fraction.
Condone use of permutations if used consistently.

A1

2

(b)

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{210}$, 0.00476	$\frac{4}{35}$, 0.114	$\frac{3}{7}$	$\frac{8}{21}$, 0.381	$\frac{1}{14}$, 0.0714

B1 Table with correct values of x and at least one further non-zero probability correct.
Condone extra x values if probability stated as 0.

B1 Third probability correct.
Accept probabilities not in table if clearly identified.

B1 Fourth probability correct.
Accept probabilities not in table if clearly identified.

B1 Fifth probability correct.
Accept probabilities not in table if clearly identified.

SCB1 for 4 further non-zero probabilities adding to $\frac{4}{7}$, 0.5712 if
B2 max scored.

4

(c)

$$[P(2B, 3B | 3B1R \text{ or } 2B2R \text{ or } 1B 3R)] = \frac{\frac{3}{7} + \frac{8}{21}}{\frac{4}{35} + \frac{3}{7} + \frac{8}{21}}$$

M1 $\frac{3}{7} + \text{their } \frac{8}{21}$ seen as the numerator of a fraction.

B1 FT $\text{their } \frac{4}{35} + \frac{3}{7} + \text{their } \frac{8}{21}$ seen as the denominator of a fraction.

$$= \left[\frac{17}{21} \right]_{97} = \frac{170}{194}, \frac{85}{97}, 0.876$$

A1 Accept 0.87628... to at least 3SF.

3