AS-Level

Discrete Random Variable

May : 2013 - May 2023

Questions

Question 1

Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

Question 2

Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) = $\frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Question 3

James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable X is defined as follows.

- If the coin shows a **head** then X is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then X is the score on the **first throw** of the die only.

(i) Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

(ii) Show that $P(X = 3) = \frac{3}{16}$. [4]

(iii) Copy and complete the probability distribution table for X. [3]

x	1	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{8}$		$\frac{1}{16}$	$\frac{1}{32}$

Question 4

A pet shop has 6 rabbits and 3 hamsters. 5 of these pets are chosen at random. The random variable X represents the number of hamsters chosen.

(i) Show that the probability that exactly 2 hamsters are chosen is $\frac{10}{21}$. [2]

(ii) Draw up the probability distribution table for X. [4]

Question 5

Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

(i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

(ii) Draw up the probability distribution table for the number of heads obtained. [4]

(iii) Find the expectation of the number of heads obtained. [2]

Question 6

A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable X represents the number of paperback books she chooses.

(i) Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

(ii) Draw up the probability distribution table for X. [3]

(iii) You are given that $E(X) = 3$. Find $Var(X)$. [2]

Question 7

The number of phone calls, X, received per day by Sarah has the following probability distribution.

x	0	1	2	3	4	$\geqslant 5$
$P(X = x)$	0.24	0.35	$2k$	k	0.05	0

(i) Find the value of k. [2]

(ii) Find the mode of X. [1]

(iii) Find the probability that the number of phone calls received by Sarah on any particular day is more than the mean number of phone calls received per day. [3]

Question 8

A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

Question 9

A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered -3, -2, -1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	-3	-2			
	-2			1	
	-1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find $\text{Var}(X)$. [3]

(iv) Find the probability that X is even, given that X is positive. [2]

Question 10

Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

Question 11

A flower shop has 5 yellow roses, 3 red roses and 2 white roses. Martin chooses 3 roses at random. Draw up the probability distribution table for the number of white roses Martin chooses. [4]

Question 12

Two ordinary fair dice are thrown. The resulting score is found as follows.

- If the two dice show different numbers, the score is the smaller of the two numbers.
- If the two dice show equal numbers, the score is 0.

(i) Draw up the probability distribution table for the score. [4]

(ii) Calculate the expected score. [2]

Question 13

A particular type of bird lays 1, 2, 3 or 4 eggs in a nest each year. The probability of x eggs is equal to kx, where k is a constant.

(i) Draw up a probability distribution table, in terms of k, for the number of eggs laid in a year and find the value of k. [3]

(ii) Find the mean and variance of the number of eggs laid in a year by this type of bird. [3]

Question 14

A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [6]

Question 15

Noor has 3 T-shirts, 4 blouses and 5 jumpers. She chooses 3 items at random. The random variable X is the number of T-shirts chosen.

(i) Show that the probability that Noor chooses exactly one T-shirt is $\frac{27}{55}$. [3]

(ii) Draw up the probability distribution table for X. [4]

Question 16

Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

Question 17

In a probability distribution the random variable X takes the value x with probability kx^2, where k is a constant and x takes values $-2, -1, 2, 4$ only.

(i) Show that $\mathrm{P}(X = -2)$ has the same value as $\mathrm{P}(X = 2)$. [1]

(ii) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

(iii) Find $\mathrm{E}(X)$. [2]

Question 18

A fair die with faces numbered 1, 2, 2, 2, 3, 6 is thrown. The score, X, is found by squaring the number on the face the die shows and then subtracting 4.

(i) Draw up a table to show the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

Question 19

A box contains 6 identical-sized discs, of which 4 are blue and 2 are red. Discs are taken at random from the box in turn and not replaced. Let X be the number of discs taken, up to and including the first blue one.

(i) Show that $P(X = 3) = \frac{1}{15}$. [2]

(ii) Draw up the probability distribution table for X. [3]

Question 20

The discrete random variable X has the following probability distribution.

x	1	2	3	6
$P(X = x)$	0.15	p	0.4	q

Given that $E(X) = 3.05$, find the values of p and q. [4]

Question 21

The discrete random variable X has the following probability distribution.

x	-2	0	1	3	4
$P(X = x)$	0.2	0.1	p	0.1	q

(i) Given that $E(X) = 1.7$, find the values of p and q. [4]

(ii) Find $\text{Var}(X)$. [2]

Question 22

A game is played with 3 coins, A, B and C. Coins A and B are biased so that the probability of obtaining a head is 0.4 for coin A and 0.75 for coin B. Coin C is not biased. The 3 coins are thrown once.

(i) Draw up the probability distribution table for the number of heads obtained. [5]

(ii) Hence calculate the mean and variance of the number of heads obtained. [3]

Question 23

Mrs Rupal chooses 3 animals at random from 5 dogs and 2 cats. The random variable X is the number of cats chosen.

(i) Draw up the probability distribution table for X. [4]

(ii) You are given that $E(X) = \frac{6}{7}$. Find the value of $\text{Var}(X)$. [2]

Question 24

Andy has 4 red socks and 8 black socks in his drawer. He takes 2 socks at random from his drawer.

(i) Find the probability that the socks taken are of different colours. [2]

The random variable X is the number of red socks taken.

(ii) Draw up the probability distribution table for X. [3]

(iii) Find $E(X)$. [1]

Question 25

A fair 6-sided die has the numbers -1, -1, 0, 0, 1, 2 on its faces. A fair 3-sided spinner has edges numbered -1, 0, 1. The die is thrown and the spinner is spun. The number on the uppermost face of the die and the number on the edge on which the spinner comes to rest are noted. The sum of these two numbers is denoted by X.

(i) Draw up a table showing the probability distribution of X. [3]

(ii) Find $\text{Var}(X)$. [3]

Question 26

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 3 sides, numbered 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [3]

(ii) Find $\text{Var}(X)$. [3]

(iii) Find the probability that X is equal to 1, given that X is non-zero. [3]

Question 27

A random variable X has the probability distribution shown in the following table, where p is a constant.

x	-1	0	1	2	4
$P(X = x)$	p	p	$2p$	$2p$	0.1

(i) Find the value of p. [1]

(ii) Given that $E(X) = 1.15$, find $\text{Var}(X)$. [2]

Question 28

The random variable X takes the values -1, 1, 2, 3 only. The probability that X takes the value x is kx^2, where k is a constant.

(i) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $E(X)$ and $\text{Var}(X)$. [3]

Question 29

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

(b) Find Var(X). [3]

Question 30

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Find E(X) and Var(X). [3]

Question 31

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

Question 32

Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

(c) Given that $E(X) = \frac{32}{15}$, find Var(X). [2]

Question 33

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Given that $E(X) = \frac{15}{8}$, find Var(X). [2]

Question 34

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

(b) Find the probability that $Y = 2$ given that Y is even. [2]

Question 35

The probability that a student at a large music college plays in the band is 0.6. For a student who plays in the band, the probability that she also sings in the choir is 0.3. For a student who does not play in the band, the probability that she sings in the choir is x. The probability that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. [3]

Two students from the college are chosen at random.

(b) Find the probability that both students play in the band and both sing in the choir. [2]

Question 36

The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5 - x)$, where k is a constant.

(a) Draw up the probability distribution table for X, in terms of k. [2]

(b) Show that $\text{Var}(X) = 1.05$. [4]

Question 37

The random variable X can take only the values $-2, -1, 0, 1, 2$. The probability distribution of X is given in the following table.

x	-2	-1	0	1	2
$P(X = x)$	p	p	0.1	q	q

Given that $P(X \geqslant 0) = 3P(X < 0)$, find the values of p and q. [4]

Question 38

A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered -2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $\text{Var}(X)$. [3]

Question 39

Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X = 3) = \frac{6}{35}$. [2]

(b) Draw up the probability distribution table for X. [4]

(c) Find $\text{Var}(X)$. [3]

Question 40

In a game, Jim throws three darts at a board. This is called a 'turn'. The centre of the board is called the bull's-eye.

The random variable X is the number of darts in a turn that hit the bull's-eye. The probability distribution of X is given in the following table.

x	0	1	2	3
$P(X = x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$.

(a) Find the values of p and q. [4]

(b) Find $\text{Var}(X)$. [2]

Jim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the probability that $X = 1$ in at least 3 of 12 randomly chosen turns. [3]

(d) Find the probability that Jim first succeeds in hitting the bull's-eye with all three darts on his 9th turn. [1]

Question 41

A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$. [3]

The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$. [1]

Question 42

A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered -1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find Var(X). [3]

Question 43

A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered $-3, -2, -1, -1$. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

(b) Given that E(X) = 0.25, find the value of Var(X). [2]

Question 44

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find E(X) and Var(X). [3]

Question 45

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

(b) Find E(X) and Var(X). [3]

Question 46

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find E(X). [1]

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

Question 47

Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X = 2) = \frac{7}{64}$. [3]

(b) Complete the probability distribution table for X. [2]

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$. [1]

(d) Find $P(Y > 4)$. [2]

Question 48

Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw. [3]

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws. [2]

Question 49

The random variable X is the number of heads obtained when Eric throws the three coins.

Draw up the probability distribution table for X. [3]

Question 50

The probability distribution table for a random variable X is shown below.

x	−2	−1	0.5	1	2
$P(X = x)$	0.12	p	q	0.16	0.3

Given that $E(X) = 0.28$, find the value of p and the value of q. [4]

Question 51

Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

(c) Given that $E(X) = 2.1$, find the value of $\text{Var}(X)$. [2]

Question 52

Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X.

(a) State the value of E(X). [1]

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

Question 53

The random variable X takes the values 1, 2, 3, 4. It is given that $P(X = x) = kx(x + a)$, where k and a are constants.

(a) Given that $P(X = 4) = 3P(X = 2)$, find the value of a and the value of k. [4]

(b) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [1]

(c) Given that $E(X) = 3.2$, find Var(X). [2]

Question 54

The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find E(X) and Var(X). [3]

Question 55

A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

(c) Find the probability that it rains on exactly one of the 4 days of the event. [3]

Question 56

A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X = 4)$. [1]

(b) Find $P(X < 6)$. [2]

Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

The two spinners are spun at the same time repeatedly .

(d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

Question 57

Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$			$\frac{3}{64}$	$\frac{1}{256}$

(c) Find $E(X)$. [2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]