AS-Level
Permutation and Combination
May : 2013- May : 2023
Questions

Question 1

There are 10 spaniels, 14 retrievers and 6 poodles at a dog show. 7 dogs are selected to go through to the final.

(i) How many selections of 7 different dogs can be made if there must be at least 1 spaniel, at least 2 retrievers and at least 3 poodles? [4]

2 spaniels, 2 retrievers and 3 poodles go through to the final. They are placed in a line.

(ii) How many different arrangements of these 7 dogs are there if the spaniels stand together and the retrievers stand together? [3]

(iii) How many different arrangements of these 7 dogs are there if no poodle is next to another poodle? [3]

Question 2

A town council plans to plant 12 trees along the centre of a main road. The council buys the trees from a garden centre which has 4 different hibiscus trees, 9 different jacaranda trees and 2 different oleander trees for sale.

(i) How many different selections of 12 trees can be made if there must be at least 2 of each type of tree? [4]

The council buys 4 hibiscus trees, 6 jacaranda trees and 2 oleander trees.

(ii) How many different arrangements of these 12 trees can be made if the hibiscus trees have to be next to each other, the jacaranda trees have to be next to each other and the oleander trees have to be next to each other? [3]

(iii) How many different arrangements of these 12 trees can be made if no hibiscus tree is next to another hibiscus tree? [3]

Question 3

Four families go to a theme park together. Mr and Mrs Lin take their 2 children. Mr O'Connor takes his 2 children. Mr and Mrs Ahmed take their 3 children. Mrs Burton takes her son. The 14 people all have to go through a turnstile one at a time to enter the theme park.

(i) In how many different orders can the 14 people go through the turnstile if each family stays together? [3]

(ii) In how many different orders can the 8 children and 6 adults go through the turnstile if no two adults go consecutively? [3]

Once inside the theme park, the children go on the roller-coaster. Each roller-coaster car holds 3 people.

(iii) In how many different ways can the 8 children be divided into two groups of 3 and one group of 2 to go on the roller-coaster? [3]

Question 4

(i) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the first letter is R. [2]

(ii) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the 3 letters G are together, both letters A are together and both letters E are together. [2]

(iii) The letters G, R and T are consonants and the letters A and E are vowels. Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if consonants and vowels occur alternately. [3]

(iv) Find the number of different selections of 4 letters of the word AGGREGATE which contain exactly 2 Gs or exactly 3 Gs. [3]

Question 5

The 11 letters of the word REMEMBRANCE are arranged in a line.

(i) Find the number of different arrangements if there are no restrictions. [1]

(ii) Find the number of different arrangements which start and finish with the letter M. [2]

(iii) Find the number of different arrangements which do not have all 4 vowels (E, E, A, E) next to each other. [3]

4 letters from the letters of the word REMEMBRANCE are chosen.

(iv) Find the number of different selections which contain no Ms and no Rs and at least 2 Es. [3]

Question 6

A shop has 7 different mountain bicycles, 5 different racing bicycles and 8 different ordinary bicycles on display. A cycling club selects 6 of these 20 bicycles to buy.

(i) How many different selections can be made if there must be no more than 3 mountain bicycles and no more than 2 of each of the other types of bicycle? [4]

The cycling club buys 3 mountain bicycles, 1 racing bicycle and 2 ordinary bicycles and parks them in a cycle rack, which has a row of 10 empty spaces.

(ii) How many different arrangements are there in the cycle rack if the mountain bicycles are all together with no spaces between them, the ordinary bicycles are both together with no spaces between them and the spaces are all together? [3]

(iii) How many different arrangements are there in the cycle rack if the ordinary bicycles are at each end of the bicycles and there are no spaces between any of the bicycles? [3]

Question 7

Nine cards are numbered 1, 2, 2, 3, 3, 4, 6, 6, 6.

(i) All nine cards are placed in a line, making a 9-digit number. Find how many different 9-digit numbers can be made in this way

(a) if the even digits are all together, [4]

(b) if the first and last digits are both odd. [3]

(ii) Three of the nine cards are chosen and placed in a line, making a 3-digit number. Find how many different numbers can be made in this way

(a) if there are no repeated digits, [2]

(b) if the number is between 200 and 300. [2]

Question 8

Find how many different numbers can be made from some or all of the digits of the number 1 345 789 if

(i) all seven digits are used, the odd digits are all together and no digits are repeated, [2]

(ii) the numbers made are even numbers between 3000 and 5000, and no digits are repeated, [3]

(iii) the numbers made are multiples of 5 which are less than 1000, and digits can be repeated. [3]

Question 9

A school club has members from 3 different year-groups: Year 1, Year 2 and Year 3. There are 7 members from Year 1, 2 members from Year 2 and 2 members from Year 3. Five members of the club are selected. Find the number of possible selections that include at least one member from each year-group. [4]

Question 10

Find the number of different ways in which all 8 letters of the word TANZANIA can be arranged so that

(i) all the letters A are together, [2]

(ii) the first letter is a consonant (T, N, Z), the second letter is a vowel (A, I), the third letter is a consonant, the fourth letter is a vowel, and so on alternately. [3]

4 of the 8 letters of the word TANZANIA are selected. How many possible selections contain

(iii) exactly 1 N and 1 A, [2]

(iv) exactly 1 N? [3]

Question 11

(a) Seven fair dice each with faces marked 1, 2, 3, 4, 5, 6 are thrown and placed in a line. Find the number of possible arrangements where the sum of the numbers at each end of the line add up to 4. [3]

(b) Find the number of ways in which 9 different computer games can be shared out between Wainah, Jingyi and Hebe so that each person receives an odd number of computer games. [6]

Question 12

Find the number of different ways that 6 boys and 4 girls can stand in a line if

(i) all 6 boys stand next to each other, [3]

(ii) no girl stands next to another girl. [3]

Question 13

The 50 members of a club include both the club president and the club treasurer. All 50 members want to go on a coach tour, but the coach only has room for 45 people. In how many ways can 45 members be chosen if both the club president and the club treasurer must be included? [3]

Question 14

A committee of 6 people is to be chosen from 5 men and 8 women. In how many ways can this be done

(i) if there are more women than men on the committee, [4]

(ii) if the committee consists of 3 men and 3 women but two particular men refuse to be on the committee together? [3]

One particular committee consists of 5 women and 1 man.

(iii) In how many different ways can the committee members be arranged in a line if the man is not at either end? [3]

Question 15

Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

(i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

Rachel displays 10 of the 13 ornaments in a row on her window-sill. Find the number of different arrangements that are possible if

(ii) she has a duck at each end of the row and no ducks anywhere else, [3]

(iii) she has a duck at each end of the row and wooden animals and sea-shells are placed alternately in the positions in between. [3]

Question 16

(a) Find the number of different ways the 7 letters of the word BANANAS can be arranged

(i) if the first letter is N and the last letter is B, [3]

(ii) if all the letters A are next to each other. [3]

(b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]

Question 17

(a) Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

(i) there are no restrictions, [2]

(ii) the number made is an even number. [4]

(b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

Question 18

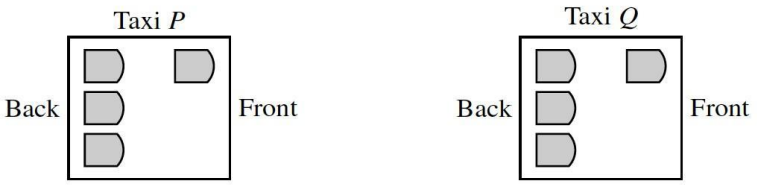
(a) Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3]

(b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

Question 19

A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

Question 20

A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

Question 21

(a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged

(i) if all the letters E are together, [3]

(ii) if the T is at one end and there is an S at the other end. [3]

(b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

Question 22

Hannah chooses 5 singers from 15 applicants to appear in a concert. She lists the 5 singers in the order in which they will perform.

(i) How many different lists can Hannah make? [2]

Of the 15 applicants, 10 are female and 5 are male.

(ii) Find the number of lists in which the first performer is male, the second is female, the third is male, the fourth is female and the fifth is male. [2]

Hannah's friend Ami would like the group of 5 performers to include more males than females. The order in which they perform is no longer relevant.

(iii) Find the number of different selections of 5 performers with more males than females. [3]

(iv) Two of the applicants are Mr and Mrs Blake. Find the number of different selections that include Mr and Mrs Blake and also fulfil Ami's requirement. [3]

Question 23

Find the number of ways all 9 letters of the word EVERGREEN can be arranged if

(i) there are no restrictions, [1]

(ii) the first letter is R and the last letter is G, [2]

(iii) the Es are all together. [2]

Three letters from the 9 letters of the word EVERGREEN are selected.

(iv) Find the number of selections which contain no Es and exactly 1 R. [1]

(v) Find the number of selections which contain no Es. [3]

Question 24

(a) Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if

(i) there are no restrictions, [1]

(ii) there are exactly six letters between the two Ws. [4]

(b) A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be at least 1 from each activity and there must be more athletes than cyclists. Find the number of different ways in which the team can be chosen. [4]

Question 25

(a) **(i)** Find how many numbers there are between 100 and 999 in which all three digits are different. [3]

(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

Question 26

Numbers are formed using some or all of the digits 4, 5, 6, 7 with no digit being used more than once.

(i) Show that, using exactly 3 of the digits, there are 12 different odd numbers that can be formed. [3]

(ii) Find how many odd numbers altogether can be formed. [3]

Question 27

A committee of 5 people is to be chosen from 4 men and 6 women. William is one of the 4 men and Mary is one of the 6 women. Find the number of different committees that can be chosen if William and Mary refuse to be on the committee together. [3]

Question 28

Find the number of ways all 10 letters of the word COPENHAGEN can be arranged so that

(i) the vowels (A, E, O) are together and the consonants (C, G, H, N, P) are together, [3]

(ii) the Es are not next to each other. [4]

Four letters are selected from the 10 letters of the word COPENHAGEN.

(iii) Find the number of different selections if the four letters must contain the same number of Es and Ns with at least one of each. [5]

Question 29

(a) Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

Question 30

(i) A plate of cakes holds 12 different cakes. Find the number of ways these cakes can be shared between Alex and James if each receives an odd number of cakes. [3]

(ii) Another plate holds 7 cup cakes, each with a different colour icing, and 4 brownies, each of a different size. Find the number of different ways these 11 cakes can be arranged in a row if no brownie is next to another brownie. [3]

(iii) A plate of biscuits holds 4 identical chocolate biscuits, 6 identical shortbread biscuits and 2 identical gingerbread biscuits. These biscuits are all placed in a row. Find how many different arrangements are possible if the chocolate biscuits are all kept together. [3]

Question 31

(a) Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,

(i) if digits are not repeated, [2]

(ii) if digits can be repeated and the number formed is odd. [3]

(b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.

(i) Find the number of different selections that include the 2 oatmeal biscuits. [2]

(ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

Question 32

A library contains 4 identical copies of book A, 2 identical copies of book B and 5 identical copies of book C. These 11 books are arranged on a shelf in the library.

(i) Calculate the number of different arrangements if the end books are either both book A or both book B. [4]

(ii) Calculate the number of different arrangements if all the books A are next to each other and none of the books B are next to each other. [5]

Question 33

(a) Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]

(b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]

(c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

Question 34

A car park has spaces for 18 cars, arranged in a line. On one day there are 5 cars, of different makes, parked in randomly chosen positions and 13 empty spaces.

(i) Find the number of possible arrangements of the 5 cars in the car park. [2]

(ii) Find the probability that the 5 cars are not all next to each other. [5]

On another day, 12 cars of different makes are parked in the car park. 5 of these cars are red, 4 are white and 3 are black. Elizabeth selects 3 of these cars.

(iii) Find the number of selections Elizabeth can make that include cars of at least 2 different colours. [5]

Question 35

(a) Find the number of different 3-digit numbers greater than 300 that can be made from the digits

(a) A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.

(i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]

(ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]

(b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

Question 36

Question 37

The digits 1, 3, 5, 6, 6, 6, 8 can be arranged to form many different 7-digit numbers.

(i) How many of the 7-digit numbers have all the even digits together and all the odd digits together? [3]

(ii) How many of the 7-digit numbers are even? [3]

Question 38

A selection of 3 letters from the 8 letters of the word COLLIDER is made.

(i) How many different selections of 3 letters can be made if there is exactly one L? [1]

(ii) How many different selections of 3 letters can be made if there are no restrictions? [3]

Question 39

Find the number of ways the 9 letters of the word SEVENTEEN can be arranged in each of the following cases.

(i) One of the letter Es is in the centre with 4 letters on either side. [2]

(ii) No E is next to another E. [3]

5 letters are chosen from the 9 letters of the word SEVENTEEN.

(iii) Find the number of possible selections which contain exactly 2 Es and exactly 2 Ns. [1]

(iv) Find the number of possible selections which contain at least 2 Es. [4]

Question 40

(a) Find the number of ways in which all 9 letters of the word AUSTRALIA can be arranged in each of the following cases.

(i) All the vowels (A, I, U are vowels) are together. [3]

(ii) The letter T is in the central position and each end position is occupied by one of the other consonants (R, S, L). [3]

Question 41

Find the number of different ways in which all 9 letters of the word MINCEMEAT can be arranged in each of the following cases.

(i) There are no restrictions. [1]

(ii) No vowel (A, E, I are vowels) is next to another vowel. [4]

5 of the 9 letters of the word MINCEMEAT are selected.

(iii) Find the number of possible selections which contain exactly 1 M and exactly 1 E. [2]

(iv) Find the number of possible selections which contain at least 1 M and at least 1 E. [3]

Question 42

A group consists of 5 men and 2 women. Find the number of different ways that the group can stand in a line if the women are not next to each other. [3]

Question 43

Out of a class of 8 boys and 4 girls, a group of 7 people is chosen at random.

(i) Find the probability that the group of 7 includes one particular boy. [3]

(ii) Find the probability that the group of 7 includes at least 2 girls. [4]

Question 44

(i) How many different arrangements are there of the 11 letters in the word MISSISSIPPI? [2]

(ii) Two letters are chosen at random from the 11 letters in the word MISSISSIPPI. Find the probability that these two letters are the same. [3]

Question 45

(i) Find the number of different ways that 5 boys and 6 girls can stand in a row if all the boys stand together and all the girls stand together. [3]

(ii) Find the number of different ways that 5 boys and 6 girls can stand in a row if no boy stands next to another boy. [3]

Question 46

9 people are to be divided into a group of 4, a group of 3 and a group of 2. In how many different ways can this be done? [3]

Question 47

In an orchestra, there are 11 violinists, 5 cellists and 4 double bass players. A small group of 6 musicians is to be selected from these 20.

(i) How many different selections of 6 musicians can be made if there must be at least 4 violinists, at least 1 cellist and no more than 1 double bass player? [4]

The small group that is selected contains 4 violinists, 1 cellist and 1 double bass player. They sit in a line to perform a concert.

(ii) How many different arrangements are there of these 6 musicians if the violinists must sit together? [3]

Question 48

Find the number of different arrangements that can be made of all 9 letters in the word CAMERAMAN in each of the following cases.

(i) There are no restrictions. [2]

(ii) The As occupy the 1st, 5th and 9th positions. [1]

(iii) There is exactly one letter between the Ms. [4]

Three letters are selected from the 9 letters of the word CAMERAMAN.

(iv) Find the number of different selections if the three letters include exactly one M and exactly one A. [1]

(v) Find the number of different selections if the three letters include at least one M. [3]

Question 49

(i) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if there must be at least twice as many men as there are women on the committee. [3]

(ii) Find the number of ways a committee of 6 people can be chosen from 8 men and 4 women if 2 particular men refuse to be on the committee together. [3]

Question 50

(a) A group of 6 teenagers go boating. There are three boats available. One boat has room for 3 people, one has room for 2 people and one has room for 1 person. Find the number of different ways the group of 6 teenagers can be divided between the three boats. [3]

(b) Find the number of different 7-digit numbers which can be formed from the seven digits 2, 2, 3, 7, 7, 7, 8 in each of the following cases.

(i) The odd digits are together and the even digits are together. [3]

(ii) The 2s are not together. [4]

Question 51

Freddie has 6 toy cars and 3 toy buses, all different. He chooses 4 toys to take on holiday with him.

(i) In how many different ways can Freddie choose 4 toys? [1]

(ii) How many of these choices will include both his favourite car and his favourite bus? [2]

Freddie arranges these 9 toys in a line.

(iii) Find the number of possible arrangements if the buses are all next to each other. [3]

(iv) Find the number of possible arrangements if there is a car at each end of the line and no buses are next to each other. [3]

Question 52

(i) How many different arrangements are there of the 9 letters in the word CORRIDORS? [2]

(ii) How many different arrangements are there of the 9 letters in the word CORRIDORS in which the first letter is D and the last letter is R or O? [3]

Question 53

A sports team of 7 people is to be chosen from 6 attackers, 5 defenders and 4 midfielders. The team must include at least 3 attackers, at least 2 defenders and at least 1 midfielder.

(i) In how many different ways can the team of 7 people be chosen? [4]

The team of 7 that is chosen travels to a match in two cars. A group of 4 travel in one car and a group of 3 travel in the other car.

(ii) In how many different ways can the team of 7 be divided into a group of 4 and a group of 3? [2]

Question 54

(i) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that all three Os are together and both Ts are together. [1]

(ii) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that the Ts are not together. [4]

(iii) Find the probability that a randomly chosen arrangement of the 9 letters of the word TOADSTOOL has a T at the beginning and a T at the end. [2]

(iv) Five letters are selected from the 9 letters of the word TOADSTOOL. Find the number of different selections if the five letters include at least 2 Os and at least 1 T. [4]

Question 55

(i) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that all four Es are together. [1]

(ii) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that the Ss are not next to each other. [4]

Four letters are selected from the 12 letters of the word STEEPLECHASE.

(iii) Find the number of different selections if the four letters include exactly one S. [4]

Question 56

The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in the coach and Saed will be in the car.

In how many ways can the members who will travel in the coach be chosen? [3]

Question 57

Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges the 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. [2]

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and the red candles are not together. [4]

Question 58

(a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

Question 59

(a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

Question 60

(a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

Question 61

In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

Question 62

The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

Question 63

A committee of 6 people is to be chosen from 9 women and 5 men.

(a) Find the number of ways in which the 6 people can be chosen if there must be more women than men on the committee. [3]

The 9 women and 5 men include a sister and brother.

(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. [3]

Question 64

Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

Question 65

(a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

Question 66

(a) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

Question 67

(a) How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

Question 68

(a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

Question 69

(a) How many different arrangements are there of the 8 letters in the word RELEASED? [1]

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

Question 70

A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

Question 71

The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.

In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [2]

Question 72

A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from {A, B, C, D, E} and the digits are chosen from {1, 2, 3, 4, 5, 6, 7}. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed? [2]

(b) Find the number of different codes that include the letter A or the digit 5 or both. [3]

A security code is formed at random.

(c) Find the probability that the code is DE followed by a number between 4500 and 5000. [3]

Question 73

(a) In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

Question 74

A group of 6 people is to be chosen from 4 men and 11 women.

(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? [2]

Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group? [3]

Question 75

Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

Question 76

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

Question 77

A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

Question 78

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

Question 79

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

Question 80

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

Question 81

(a) Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

Question 82

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

Question 83

A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

In one photograph Abel, Betty, Cally, Doug, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? [3]

Question 84

(a) Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

Question 85

(a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

Question 86

In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

Question 87

(a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

Question 88

(a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]