AS-Level

Topic : Permutation and Combination

May 2013-May 2023

Answers

Question 1

7 (i)			
S(10) R(14) P(6)	M1		Summing 2 or more 3-factor options perms or combs
1 2 4 = 10C1×14C2×6C4= 13650			
1 3 3 = 10C1×14C3×6C3= 72800	M1		Mult 3 combs or 4 combs with Σr=7
2 2 3 = 10C2×14C2×6C3= 81900	B1		2 options correct, unsimplified
Total = 168350 or 168000	A1	**[4]**	Correct answer
(ii) 2! × 2! × 5!	M1		2! × 2! oe, seen mult by an integer ≥1, no division
	M1		Mult by 5!, or 5! alone, seen mult by an integer ≥ 1 no division
= 480	A1	**[3]**	Correct answer
If M0 earned $\frac{2!\times 2!}{2!\times 2!}$ or $\frac{5!}{3!}$ or both, seen mult by an integer ≥ 1 Or 2!×2!×5! divided by a value	SCM1		
(iii) spaniels and retrievers in 4! ways	M1		4! seen multiplied by an integer >1
gaps in 5P3 or 5 × 4 × 3 ways	M1		Mult by 5P3 oe
= 1440	A1	**[3]**	Correct answer
If M0 earned $\frac{4!}{2!\times 2!}$ or $\frac{{}_5P_3}{3!}$ or both, seen multiplied by an integer > 1	SCM1		${}_5C_3$ oe
or			
7! – 5! × 3!	M1		oe
– {(4! × 2 × 4 × 3!) + (4! × 3 × 4 × 3!)}	M1		oe, e.g. 6 × 5 × 4 × 4!
= 1440	A1		
If M0 earned 3! × 2! × 2! used as a denominator in all 4 terms	SCM1		Marks cannot be earned from both methods.

Question 2

(i)	H J O 1. 28 2 = 4C2×9C8×2C2 = 54 3 7 2 = 4C3×9C7×2C2 = 144 4 6 2 = 4C4×9C6×2C2 = 84	M1 M1 A1		Mult 3 combs, 2C2 may be implied 4Cx×9Cy×2Cz Summing 2 or 3 three-factor options 2 options correct unsimplified
	Total = 282 ways	A1	**[4]**	Correct answer
(ii)	4! × 6! × 2! × 3!	M1 M1		4! × 6! × 2! oe seen multiplied by int ≥ 1 3! seen mult by int ≥ 1
	= 207360 (207000)	A1	**[3]**	Correct answer
(iii)	8 J and O trees in 8! = 40320 ways 9 gaps × 8 × 7 × 6	B1 M1		8! seen mult by int ≥ 1 no division 9P4 oe or 7P4 or 8P4 seen mult by int ≥ 1 no division
	= 121,927,680 (122,000,000)	A1	**[3]**	Correct answer
(i)	SR 4C2×9C2×2C2×9C6	M1		
(ii)	SR $\frac{4!\times6!\times2!}{4!\times6!\times2!}$ or 3! or both M1	M1		
(iii)	SR1 12! – 9! 4! SR2 $\frac{9P4}{4!}$ or $\frac{8!}{6!\,2!}$ or both	M1 M1		

Question 3

(i)	4! × 3! × 5! × 2! × 4! = 829440	B1 B1 B1	 **[3]**	4!, 3!, 5!, 2 seen multiplied 1, not in denominator Mult by 4! Correct answer
(ii)	8! × 9 × 8 × 7 × 6 × 5 × 4 = 2438553600 (2.44×10^9)	B1 B1 B1	 **[3]**	8! seen multiplied 1 Mult by ${}_9P_6$ Correct answer
(iii)	8C3 × 5C3 × 2C2 = 560	B1 B1 B1	 **[3]**	8C3 seen mult 5C3 seen mult Correct answer

Question 4

(i)	$\frac{8!}{3!2!2!}$	M1		8! Divided by at least one of 3!2!2! oe
	= 1680	A1	**2**	Correct answer
(ii)	5!	M1		5! Seen (not added, may be divided/multipled)
	= 120	A1	**2**	Correct answer
(iii)	$\frac{5!4!}{3!2!2!}$	B1		5! Or 4! Seen in sum or product in numerator (denominator may by 1)
		M1		$\frac{k5!4!}{3!2!2!}$ in a numerical expression
	= 120	A1	**3**	Correct final answer
(iv)	GG with AA, AE, EE, RA, RE, RT, TA, TE, = 8 ways	M1		Summing 2 options (could be lists)
	GGG with A, E, R, T = 4 ways	A1		1 correct option
	Total = 12 ways	A1	**3**	Correct answer

Question 5

(i)	1663200	B1	**[1]**	
(ii)	M xxxxxxxxx M	M1		9! or 9P9 seen
	Number of ways = $\frac{9!}{3!2!}$ = 30240	A1	**[2]**	Correct answer
(iii)	4 vowels together = 8! × 4/2!2!	M1		8!/2!2! seen mult by something
	= 40320	M1		4 oe 4!/3! or 4C1 etc. seen mult by something
	1663200 – 40320 = 1622880	B1	**[3]**	Correct answer SC 7!/2!2! × 8P4 or 7! × 8P4/3! Or 7!/2!2! × 8P4/3! M1
(iv)	Exactly 2 Es 4C2 = 6	M1		Summing 2 options
	Exactly 3 Es 4C1 = 4	B1		One option correct
	Total = 10 ways	A1	**[3]**	Correct answer
	OR 5C2	M2		M1 for k5C2
	= 10	A1		Correct ans

Question 6

(i) M R O 3 1 2 = 7C3 × 5C1 × 8C2 = 4900	M1		Summing more than one 3term option involving combs (can be added)
3 2 1 = 7C3 × 5C2 × 8C1 = 2800	M1		Mult 3 combs only (indep)
2 2 2 = 7C2 × 5C2 × 8C2 = 5880	A1		1 option correct unsimplified
Total = 13580	A1	**4**	Correct answer
(ii) 4 groups in 4! ways 3 mountain in 3! ways	M1		4! seen mult by something
2 ordinary in 2! ways	M1		Mult by 3! for racing or 2! for ordinary
4! × 3! × 2 = 288	A1	**3**	Correct answer
(iii) e.g. s O x x x x O s s s Ordinary in 2!	M1		2! or 4! seen mult
Rest of bikes in 4! Bikes and spaces 5 groups in 5 ways	M1		Mult by 5 (ssssb)
2! × 4! × 5 = 240	A1	**3**	Correct answer

Question 7

(i) **(a)** 6!	M1		Seen in a single term expression as numerator
(×) 4! **OR** (×) 4 × 3	M1		Seen in a single term expression as numerator (denominator may be 1)
÷ 2!2!3! **OR** ÷ 2!3!	M1		Seen in a single term expression as denominator
Total 720 ways	A1	**4**	Correct ans
(i) **(b)** 1*******3 = $\frac{7!}{3!2!}$ = 420	B1		$\frac{7!}{3!2!}$ seen oe
3*******1= 420 3*******3= 420	M1		Attempting to evaluate and sum at least 2 of 1***3, 3***1, 3***3
Total = 1260 ways	A1	**3**	Correct ans
(ii) **(a)** 5 × 4 × 3 = 60 ways ($^5P_3$)	M1 A1	 **2**	$^5P_3$ or $^5C_3$ ×3! (can be implied) Correct ans
(ii) **(b)** 2** in 212, 213, 214, 216, 221, 223, 224, 226, 231, 232, 233, 234, 236, 241, 242, 243, 246 261, 262, 263, 264, 266	M1		Listing attempt starting with 2, at least 10 correct entries
Total = 22 ways	A1	**2**	Correct ans
Alternative Methods: $3 \times {}^4C_1 + 2 \times {}^5C_1$	M1		$p \times {}^4C_1 + q \times {}^5C_1$, oe $p + q > 2$
OR $^5P_2 + {}^2C_1$	**OR** M1		 $^5P_2$ seen
OR $^4P_2 + 2 \times {}^4P_1 + {}^2C_1$	**OR** M1		 Any 2 terms added

Question 8

(i)	$5! \times 3!$ or $6!$	B1		5! or 3! or 6! oe seen mult or alone
	$= 720$	B1	**2**	Correct final answer
(ii)	3**4, 3**8, 4**8	M1		considering at least 2 types of 4-figure options ending with 4 or 8 and starting with 3 or 4
		B1		One option correct unsimplified can be implied
	$= 5 \times 4 + 5 \times 4 + 5 \times 4 = 60$	A1	**3**	Correct final answer
(iii)	5, *5, **5,	M1		Appreciating that the number must end in 5 (can be implied)
	$= 1 + 7 + 7^2$	M1		summing numbers ending in 5 with at least 2 different numbers of digits
	$= 57$	A1	**3**	Correct final answer

Question 9

Y1(7) Y2(2)Y3(2) 1 2 2 $= 7 \times 1 \times 1 = 7$	B1		One unsimplified correct 3-factor product of combinations
2 1 2 $= {}^7C_2 \times {}^2C_1 \times 1 = 42$ 2 2 1 $= {}^7C_2 \times 1 \times {}^2C_1 = 42$	B1		A second unsimplified correct 3-factor product of combinations
3 1 1 $= {}^7C_3 \times {}^2C_1 \times {}^2C_1 = 140$	M1		Summing 3 or 4 options allow perms, wrong combs but second numbers must sum to 5 etc.
Total = 231	A1	**4**	Correct answer

Question 10

(i)	$\frac{6!}{2!} = 360$	B1		6! Seen alone
		B1	**2**	Dividing by 2! only
(ii)	$\frac{4!}{2!} \times \frac{4!}{3!}$	B1		4! seen mult
		B1		Dividing by 2! or 3! (Mult by 4 implied B1B1)
	$= 48$	B1	**3**	Correct answer
(iii)	1N and 1A: N A xx in ${}^3C_2$	M1		${}^3C_x$ or ${}^xC_2$ seen alone
	= 3 ways	A1	**2**	Correct answer
(iv)	0 A : Nxxx = 1 way	M1		Finding ways with 0 or 2 or 3 As
	2 As: NAAx in ${}^3C_1$ = 3 ways	M1		Summing 3 or 4 options
	3 As: NAAA in 1 way			
	Total = 8 ways	A1	**3**	Correct answer

Question 11

(a) 1*****3 or 3*****1 or 2*****2	M1		Mult by 6^5 (for middle 5 dice outcomes)
$= 6^5 \times 3$	M1		Mult by 3 or summing 3 different combinations (for end dice outcomes)
$= 23328$	A1	**3**	Correct answer accept 23 300
(b) W J H			
1 1 $7 = {}^9C_1 \times {}^8C_1 \times 1 = 72$	M1		Multiplying 3 combinations (may be implied)
1 7 $1 = {}^9C_1 \times {}^8C_7 \times 1 = 72$	A1		1 unsimplified correct answer (72, 504, 1680, 216 or 3024)
7 1 $1 = {}^9C_7 \times {}^2C_1 \times 1 = 72$			
1 3 $5 = {}^9C_1 \times {}^8C_3 \times 1 = 504$ mult by 3!	A1		A 2nd unsimplified different correct answer
3 3 $3 = {}^9C_3 \times {}^6C_3 \times 1 = 1680$	M1		Summing options for 1,1.7 or 1,3,5 oe (mult by 3 or 3!)
	M1		Summing at least 2 different options of the 3
Total 4920	A1	**6**	Correct ans
If no marks gained			If games replaced M1M1M1 max available
Listing all 10 different outcomes	SCM1		If factorials used M0M1M1 max available

Question 12

(i) $6! \times 5!$	B1		6! oe seen multiplied by integer $k \geqslant 1$
	B1		5! oe seen multiplied by integer $k \geqslant 1$
$= 86400$	B1	**3**	Correct final answer
(ii) $6! \times 7 \times 6 \times 5 \times 4$	B1		6! seen mult by integer $k \geqslant 1$
	B1		Mult by ${}^7P_4$ oe
$= 604800$	B1	**3**	Correct final answer

Question 13

${}^{48}C_{43}$	B1		48 seen in a single term combination oe
	B1		43 or 5 seen in a single term combination oe
			Both can be mult by integer $k \geqslant 1$
$= 1712304$ (1710000)	B1	**3**	Correct final answer

Question 14

(i) W(8) M(5)			
4 $2 = {}^8C_4 \times {}^5C_2 = 700$	M1		Mult 2 combs, ${}^8C_x \times {}^5C_y$
5 $1 = {}^8C_5 \times {}^5C_1 = 280$	M1		Summing 2 or 3 options
6 $0 = {}^8C_6 \times {}^5C_0 = 28$	A1		2 correct options unsimplified
Total = 1008	A1	**4**	Correct answer
(ii) M1 and MMWWW $= {}^3C_2 \times {}^8C_3 = 168$	M1		Summing 3 options
M2 and MMWWW $= {}^3C_2 \times {}^8C_3 = 168$			
Neither and MMMWWW $= {}^3C_1 \times {}^8C_3 = 56$	B1		One correct option
Total = 392	A1	**3**	Correct answer
OR total, no restrictions $= {}^5C_3 \times {}^8C_3 = 560$	M1		Subt 2 men together from no restrictions
M1M2 and MWWW $= {}^3C_1 \times {}^8C_3 = 168$	B1		One correct of 560 or 168
$560 - 168 = 392$	A1		Correct answer
(iii) e.g. WWMWWW	M1		5! Seen mult by integer $\geqslant 1$
$= 5!$ (women) $\times 4 = 480$	M1		Mult by 4
	A1	**3**	Correct answer

Question 15

(i)	W S D			
	1 1 3 = $6\times4\times{}^3C_3 = 24$	M1		Listing at least 4 different options
	1 3 1 = $6\times{}^4C_3\times3 = 72$	M1		Mult 3 (combs) together assume 6 = ${}^6C_1$, $\Sigma r=5$
	3 1 1 = ${}^6C_3\times4\times3 = 240$	M1		Summing at least 4 different evaluated/unsimplified options >1
	1 2 2 = $6\times{}^4C_2\times{}^3C_2 = 108$			
	2 1 2 = ${}^6C_2\times4\times{}^3C_2 = 180$			
	2 2 1 = ${}^6C_2\times{}^4C_2\times3 = 270$	B1		At least 3 correct unsimplified options
	Total = 894	A1	**[5]**	Correct answer
(ii)	${}^3P_2 \times {}^{10}P_8$	B1		${}^3P_2$ oe seen multiplied either here or in (iii)
		B1		$k\,{}^{10}P_x$ seen or $k\,{}^yP_8$ with no addition, $k\geqslant1$, $y>8$, $x<10$
	= 10886400	B1	**[3]**	Correct answer, nfww
(iii)	DSWSWSWSWD or DWSWSWSWSD			If ${}^3P_2$ has not gained credit in (ii) may be awarded
	D in ${}^3P_2$ ways = 6	B1		${}^4P_4$ or ${}^6P_4$ oe seen multiplied or common in all terms (no division)
	S in ${}^4P_4$ ways = 24			
	W in ${}^6P_4$ = 360			
	Swap SW in 2 ways	B1		Mult by 2 (condone 2!)
	Total = 103680 ways	B1	**[3]**	Correct answer, 3sf or better, nfww

Question 16

(a) (i)	N*****B			
	Number of ways = $\frac{5!}{3!}$	B1		5! Seen in num oe or alone mult by $k \geqslant 1$
		B1		3! Seen in denom can be mult by $k \geqslant 1$
	= 20	B1	3	Correct final answer
(ii)	B(AAA)NNS			
	Number of ways = $\frac{5!}{2!}$ or ${}^5P_3$	M1		5! seen as a num can be mult by $k \geqslant 1$
		M1		Dividing by 2!
	= 60	A1	3	Correct final answer
(b)	${}^{14}C_9$ total options = 2002	M1		${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt
	T and M both in ${}^{12}C_7 = 792$	B1		${}^{12}C_7$ (792) seen
	Ans 2002 – 792 = 1210	A1	3	Correct final answer
	OR			
	Neither in ${}^{12}C_9 = 220$	M1		Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only
	One in ${}^{12}C_8 = 495$			
	Other in ${}^{12}C_8 = 495$	B1		Second correct option seen accept another 495 or if M1 not awarded, any correct option

Question 17

(a) **(i)**	$\frac{9!}{2!2!3!}$ = 15120 ways	B1 B1 **[2]**	Dividing by 2!2!3! Correct answer
(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways	B1	Correct ways end in 3
	********7 in $\frac{8!}{2!3!}$ = 3360 ways	B1	Correct ways end in 7
	Total even = 15120 – 1680 – 3360	M1	Finding odd and subt from 15120 or their **(i)**
	= 10080 ways	A1 **[4]**	Correct answer
	OR		
	********2 in 8!/2!3! = 3360 ways	B1	One correct way end in even
	********6 in 8!/2!2!3! = 1680 ways	B1	correct way end in another even
	********8 in 8!/2!2!2! = 5040ways	M1	Summing 2 or 3 ways
	Total = 10080 ways	A1	Correct answer
	OR		
	"15120" ×6/9 = 10080	M2 A2	Mult their **(i)** by 2/3 oe Correct answer
(b)	T(3) S(6) G(14)		
	1 1 3 in $3\times6\times{}^{14}C_3$ = 6552	M1	Mult 3 (combinations) together assume 6 = ${}^6C_1$etc
	1 3 1 in $3\times{}^6C_3\times14$ = 840		
	3 1 1 in 1×6×14 = 84	M1	Listing at least 4 different options
	2 2 1 in ${}^3C_2\times{}^6C_2\times14$ = 630	M1	Summing at least 4 different options
	2 1 2 in ${}^3C_2\times6\times{}^{14}C_2$ = 1638		
	1 2 2 in $3\times{}^6C_2\times{}^{14}C_2$ = 4095	B1	At least 3 correct numerical options
	Total ways = 13839 (13800)	A1 **[5]**	Correct answer

Question 18

(a)	e.g. **(AAOOOI)*****	**B1**		8! (8 × 7!) or 6! seen anywhere, either alone or in numerator)
	$\frac{8!}{2!2!}\times\frac{6!}{2!3!}=604800$	**M1**		Dividing by at least 3 of 2!2!2!3! (may be fractions added)
		A1	3	Correct answer
(b)	C(7) E(6) A(4)	**M1**		Mult 3 appropriate combinations together assume $6={}^6C_1$, $1={}^4C_0$ etc., $\sum r=4$, C&E both present
	1 1 2 $= 7\times 6\times {}^4C_2 = 252$			
	1 2 1 $= 7\times {}^6C_2\times 4 = 420$			
	1 3 0 $= 7\times {}^6C_3\times 1 = 140$			
	2 1 1 $= {}^7C_2\times 6\times 4 = 504$	**A1**		At least 3 correct unsimplified products
	2 2 0 $= {}^7C_2\times {}^6C_2\times 1 = 315$			
	3 1 0 $= {}^7C_3\times 6\times 1 = 210$	**M1***		Listing at least 4 different correct options
		DM1		Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$
	Total = 1841	**A1**	5	Correct answer
				SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1\times{}^6C_1\times$(sum of at least 4 outcomes)

Question 19

(i)	Two in same taxi:	**M1**		${}^6C_4$ or ${}^6C_2$ oe seen anywhere
	${}^6C_2\times{}^4C_4\times 2$ or ${}^6C_2+{}^6C_4$	**M1**		'something' ×2 only or adding 2 equal terms
	= 30	**A1**	3	Correct final answer
(ii)	MJS in taxi	**M1**		${}^5P_1$, ${}^5C_1$ or 5 seen anywhere
	$({}^5C_1\times2\times2)\times{}^4P_4$	**M1**		Mult by 2 or 4 oe
		M1		Mult by ${}^4P_4$ oe eg 4! or $4\times{}^3P_3$ or can be part of 5!
	= 480	**A1**	4	Correct final answer

Question 20

P(no men) $\frac{{}^9C_6}{{}^{16}C_6}=\frac{84}{8008}=\frac{21}{2002}=\frac{3}{286}$	**B1**		${}^9C_6$ seen anywhere
$= 0.0105$	**B1**		${}^{16}C_6$ seen as denom of fraction oe
	B1	3	Correct final answer
OR $\frac{9}{16}\times\frac{8}{15}\times\frac{7}{14}\times\frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}=0.0105$	**B1**		
	B1		(9 × 8 × 7 × 6 × 5 × 4) seen anywhere
	B1		Correct unsimplified denom
			Correct final answer

Question 21

(i)	**5 (i)** eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!}$ = 180	**M1** **M1** **A1** [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*******T or T*******S Number of ways = $\frac{7!}{4!2!} \times 2$ = 210	**M1** **M1** **A1** [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in ${}^6C_3$ ways = 20	**M1** **M1** **A1** [3]	${}^6C_x$ as a single answer ${}^xC_3$ as a single answer correct answer

Question 22

(i)	${}^{15}P_5$ = 360360	**M1** **A1**	 2	oe, can be implied Not ${}^{15}C_5$ Correct answer
(ii)	5 × 10 × 4 × 9 × 3 = 5400	**M1** **A1**	 2	Mult 5 numbers Correct answer
(iii)	M(5) F(10) 3 2 = ${}^5C_3 \times {}^{10}C_2$ = 450 ways 4 1 = ${}^5C_4 \times {}^{10}C_1$ = 50 5 0 = ${}^5C_5 \times {}^{10}C_0$ = 1 Total = 501 ways	 **M1** **M1** **A1**	 3	 Mult 2 combs, ${}^5C_x \times {}^{10}C_y$ Summing 2 or 3 two-factor options, $x + y = 5$ Correct answer
(iv)	(Couple) M(4) F(9) ManWife + 3 0 = ${}^4C_3 \times {}^9C_0$ = 4 ManWife + 2 1 = ${}^4C_2 \times {}^9C_1$ = 54 Total = 58	 **M1** **M1** **A1**	 3	 Mult 2 combs ${}^4C_x$ and ${}^9C_y$ Summing both options $x + y = 3$, gender correct Correct answer

Question 23

(i)	7560 ways	B1 [1]	
(ii)	RxxxxxxG in $\frac{7!}{4!}$	B1	7! alone seen in num or 4! alone in denom. Must be in a fraction. $\frac{7!\times 2}{4!\times 2}$ gets full marks
	= 210 ways	B1 [2]	
(iii)	eg EEEExxxxx in $\frac{6!}{2!}$	B1	6! or 5! × 6 seen in numerator or on own. Can be 6! × *k* but not 6! ± *k*
	= 360 ways	B1 [2]	
(iv)	1 R eg RVG or RVN or RGN = 3	B1 [1]	
(v)	no Rs eg VGN or 3C3 ways = 1 2 Rs eg RRV or 3C1 ways = 3	M1	Summing at least 2 options for R
		A1	Correct outcome for no Rs or 2 Rs – evaluated
	Total = 7	A1 [3]	

Question 24

(a) (i)	$\frac{10!}{2!3!} = 302400$	B1 [1]	Exact value only, isw rounding
(ii)	e.g. *W******W*, **W******W, W******W**	M1	8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!}$ × 3(for the Ws)	M1	Dividing by 3! (removing repeated L's)
		M1	Mult by 3 (different W positions) may be sum of 3 terms
	= 20160	A1 [4]	
(b)	S(5) A(7) C(4) 1 3 2 : $5\times{}^7C_3\times{}^4C_2 = 1050$ 1 4 1 : $5\times{}^7C_4\times 4 = 700$ 2 3 1 : ${}^5C_2\times{}^7C_3\times 4 = 1400$ 3 2 1 : ${}^5C_3\times{}^7C_2\times 4 = 840$ (Outcomes : Options)	M1	Mult 3 combinations, ${}^5C_x$, ${}^7C_y$, ${}^4C_z$ (not 5 x 7 x 4)
		A1	2 correct options unsimplified
		M1	Summing only 3 or 4 correct outcomes involving combs or perms
	Total = 3990	A1 [4]	

Question 25

(a)	**(i)**	$9 \times 9 \times 8$	**M1 M1**	Logical listing attempt
		$= 648$	**A1** [3]	
		OR $900 - 28 \times 9 = 648$		
	(ii)	(7….in $1 \times 8 \times 4 = 32$ ways	**M1**	Listing #s starting with 7 or 9 and ending odd
		8 …in $1 \times 8 \times 5 = 40$	**M1**	
		9… in $1 \times 8 \times 4 = 32$	**M1**	
		Total 104 ways	**A1** [4]	
(b)		R(6) T(5) D(4)		
		2 2 3 = $^6C_2 \times {}^5C_2 \times {}^4C_3 = 600$	**M1**	Mult 3 combs, $^6C_x \times {}^5C_y \times {}^4C_z$
		2 3 2 = $^6C_2 \times {}^5C_3 \times {}^4C_2 = 900$	**M1**	Summing 2 or 3 three-factor outcomes can be perms, + instead of ×
		3 2 2 = $^6C_3 \times {}^5C_2 \times {}^4C_2 = 1200$		
			A1	2 options correct unsimplified
		Total = 2700	**A1** [4]	

Question 26

(i)	e.g. **5 in $^3P_2$ ways = 6	**M1**		Recognising ends in 5 or 7, can be implied
	**7 in $^3P_2 = 6$	**M1**		Summing ends in 5 + ends in 7 oe
	Total 12 AG	**A1**	[3]	Correct answer following legit working
	OR listing 457, 547, 467, 647, 567, 657, 475, 745 465, 645, 675, 765	**M1**		Listing at least 5 different numbers ending in 5
		M1		Listing at least 5 different numbers ending in 7
	Total 12 AG	**A1**		
(ii)	1 digit in 2 ways 2 digits in *5 or *7 = $^3P_1 \times 2 = 6$	**M1**		Consider at least 3 options with different number of digits. If no working, must be 3 or 4 from 2, 6, 12, 12
	4 digits in ***5 or ***7 = $^3P_3 \times 2 = 12$	**A1**		One option correct from 1, 2 or 4 digits
	Total ways = 32	**A1**	[3]	

Question 27

total ways $^{10}C_5$=252	**M1**		$^{10}C_5 - \ldots$ or $252 - \ldots$
MW together e.g. (MW)*** in $^8C_3$ ways = 56			
MW not together = 252 – 56	**B1**		252 and 56 seen, may be unsimplified
= 196 ways	**A1**	[3]	
OR 1			
$2\,{}^8C_4 + {}^8C_5$	**M1**		$2\,{}^nC_4 + {}^nC_5$
$2\,{}^8C_4 = 2 \times 70 = 140$; $^8C_5 = 56$	**B1**		140 and 56 seen may be unsimplified
$2\,{}^8C_4 + {}^8C_5 = 196$	**A1**		
OR 2			
$2\,{}^9C_5 - {}^8C_5$	**M1**		$2\,{}^9C_5 - ..$
$2\,{}^9C_5 = 2 \times 126 = 252$; $^8C_5 = 56$	**B1**		252 and 56 seen, may be unsimplified
$2\,{}^9C_5 - {}^8C_5 = 196$	**A1**		

Question 28

(i)	e.g. (OAEE)(CPNHGN) or cv $\frac{4!}{2!}\times\frac{6!}{2!}\times 2 = 8640$	**M1** **M1** **A1**	[3]	4!/2! or 6!/2! seen anywhere All multiplied by 2 oe
(ii)	First Method Total ways = 10!/2!2! = 907200 EE together in 9!/2! ways = 181440 EE not together = 907200 – 181440 = 725760	**B1** **M1** **M1** **A1**	[4]	Total ways together correct EE together attempt alone Considering total – EE together
	OR Second Method C ↑ P N H G N O A in 8!/2! ways	**B1**		8!/2! Seen
	Insert E in 9 ways	**M1**		Interspersing an E, x n where n=7,8,9. Condone additional factors.
	Insert 2nd E in 8 ways, ÷ 2 Total = 8!/2!×9×8 ÷ 2 = 725760	**M1** **A1**		Mult by 9×8(÷2), $^9C_2$ or $^9P_2$ only oe
(iii)	First Method EN** in $^6C_2$ ways	**M1** **M1**		^6C_x or $^yC_2$ seen alone or mult by $k > 1$, x<6, y>2 (1x1x) $^6C_2$ seen strictly alone or added to their EENN only
	= 15 different ways	**A1**		
	EENN in 1 way Total 16 ways	**B1** **A1**	[5]	
	OR Second Method Listing with at least 8 different correct options Listing all correct options Total = 15 different ways EENN in 1 way Total 16 ways	**M1** **M1** **A1** **B1** **A1**		Value stated or implied by final answer correct value stated Award 16 SRB2 if no method is present

Question 29

5 (a)	e.g. P*N*P*P*L	M1		Mult by 5! in num
	$= \frac{5!}{3!} \times \frac{^6P_4}{2!}$	M1		Dividing by 3! or 2!
		M1		Mult by $^6P_4$ oe
	= 3600	A1	[4]	
(b) (i)	$^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$	M1		Mult 4 combs of which three are correct
	= 420	A1	[2]	
(ii)	both in team	M1		Evaluating both in team and subtracting from (i)
	$^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$	M1		240 seen can be unsimplified ft their 420, their 240
	420 – 240 = 180 ways	A1		
	OR			
	Bat in bowl out + bowl in bat out + both out	M1		summing 2 or 3 options not both in team
	$= {}^6C_4 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_4 \times 2 \times 2$	A1		2 or 3 options correct unsimplified
	= 60 + 96 + 24 = 180 ways	A1		Correct ans from correct working
	OR			
	Bat in bowl out + bat out	M1		As above, or bowl in bat out + bowl out
	$= 60 + {}^6C_5 \times {}^5C_4 \times 2 \times 2 = 60 + 120 = 180$ ways	A1 A1	[3]	

Question 30

5(i)	$^{12}C_1 + {}^{12}C_3 + {}^{12}C_5 + {}^{12}C_7 + {}^{12}C_9 + {}^{12}C_{11}$	M1	Summing at least 4 $^{12}C_x$ combinations with x = odd numbers
		A1	Correct unsimplified answer (can be implied by final answer)
	= 2048	A1	Correct answer
	Total:	3	
(ii)	$7! \times {}^8P_4$	B1	7! seen alone or multiplied only (cupcakes ordered)
		M1	multiplying by $^8P_4$ o.e (placing brownies)
	= 8467200	A1	correct answer
	Total:	3	
(iii)	$9! / (6! \times 2!)$	B1	9! oe seen alone or as numerator
		M1	dividing by at least one of 6!,2! (removing repeated shortbread or gingerbread biscuits) ignore 4! if present
	= 252	A1	correct answer
	Total:	3	

Question 31

5(a)(i)	First digit in 2 ways. 2 × 4 × 3 × 2 or 2 × 4P3	**M1**	1, 2 or 3 × 4P3 OE as final answer
	Total = 48 ways	**A1**	
	Total:	**2**	
5(a)(ii)	2 × 5 × 5 × 3	**M1** **M1**	Seeing 5^2 mult; this mark is for correctly considering the middle two digits with replacement Mult by 6; this mark is for correctly considering the first and last digits
	= 150 ways	**A1**	
	Totals:	**3**	

5(b)(i)	OO**** in ${}^{18}C_4$ ways	**M1**	${}^{18}C_x$ or the sum of five 2-factor products with n = 14 and 4, may be × by 2C2: 4C0 × 14C4 + 4C1 × 14C3 + 4C2 × 14C2 + 4C3 × 14C1 + 4C4 (× 14C0)
	= 3060	**A1**	
	Totals:	**2**	
(b)(ii)	Choc / Not Choc 0 / 6= 1 × ${}^{16}C_6$ = 8008 0.2066 1 / 5= ${}^{4}C_1$ × ${}^{16}C_5$ = 17472 0.4508 2 / 4= ${}^{4}C_2$ × ${}^{16}C_4$ = 10920 0.2817 OR Choc / Oats / Ginger 0 / 0 / 6 0 / 1 / 5 0 / 2 / 4 1 / 0 / 5 1 / 1 / 4 1 / 2 / 3 2 / 0 / 4 2 / 1 / 3 2 / 2 / 2	**B1**	The correct number of ways with one of 0, 1 or 2 chocs , unsimplified **or** any three correct number of ways of combining choc/oat/ginger, unsimplified
	Total = 36400 ways	**M1**	sum the number of ways with 0, 1 and 2 chocs and two must be totally correct, unsimplified OR sum the nine combinations of choc, ginger, oats, six must be totally correct, unsimplified
	Probability = 36400/ ${}^{20}C_6$	**M1**	dividing by ${}^{20}C_6$ (38760) oe
	= 0.939 (910/969)	**A1**	
	Totals:	**4**	

Question 32

5(i)	*EITHER*: Route 1 *A*********A* in 9! / 2!2!5! = 756 ways	(*M1	*Considering AA and BB options* with values
	*B*********B* in 9! / 4!5! = 126 ways	A1	Any one option correct
	756 + 126	DM1	*Summing their AA and BB outcomes* only
	Total = 882 ways	A1)	
	OR1: Route 2 *A*********A* in ${}^9C_5 \times {}^4C_2 = 756$ ways	(M1	*Considering AA and BB options* with values
	*B*********B* in ${}^9C_4 \times {}^5C_5 = 126$ ways	A1	Any one option correct
	756 + 126	DM1	*Summing their AA and BB outcomes* only
	Total = 882	A1)	
	Total:	**4**	
5(ii)	*EITHER:* (The subtraction method) *A*s together, no restrictions 8! / 2!5! = 168	(*M1	*Considering all As together* – 8! seen alone or as numerator – condone × 4! for thinking A's not identical
	*A*s together and *B*s together 7! / 5! = 42	M1	*Considering all As together and all Bs together* – 7! seen alone or numerator
		M1	*Removing repeated Bs or Cs* – Dividing by 5! either expression or 2! 1st expression only – OE
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR1: *A*s together, no restrictions ${}^8C_5 \times {}^3C_1 = 168$	(*M1	${}^8C_5$ seen alone or multiplied
		M1	${}^7C_5$ seen alone or multiplied
	*A*s together and *B*s together ${}^7C_5 \times {}^2C_1 = 42$	M1	First expression x ${}^3C_1$ or second expression x ${}^2C_1$
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR2: (The intersperse method)	(M1	*Considering all "As together" with Cs* – Mult by 6!
	(AAAA)CCCCC then intersperse *B* and another *B*	M1	*Removing repeated Cs* – Dividing by 5!– [Mult by 6 implies M2]
		*M1	*Considering positions for Bs* – Mult by 7P2 oe –

Question 33

(a)	*EITHER:* e.g. xxxxx =5! for the other children	(B1	5! OE seen alone or mult by integer k ⩾ 1, no addition
	Put y in 6 ways, then 5 then 4 for the youngest children	B1	Mult by 6P3 OE
	Answer 5! × 6P3 = 14400	B1)	Correct answer
	OR: total – 3 tog – 2 tog = 8! – 6!3! – 6! × 2 × 5 × 3 = 14400	(B1	8! – 6! × k ⩾ 1seen
		B1	6!3! or 6! × 2 × 5 × 3 seen subtracted
		B1)	Correct answer
	Total:	**3**	
(b)	D W M 2 2 1 = 6C2 × 4C2 × 1 = 90	B1	One correct unsimplified option
	3 1 1 = 6C3 × 4 × 1 = 80	M1	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1 3 1 = 6 × 4C3 × 1 = 24	M1	Summing the correct 3 unsimplified outcomes only
	Total=194 ways	A1	
	Total:	**4**	

(c)	C D S 2 1 1 = $^{26}C_2 \times 9 \times 5 \times 4!$ = 351 000	M1	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1 2 1 = $26 \times {}^{9}C_2 \times 5 \times 4!$ = 112 320	M1	4 relevant products seen excluding 4! e.g. 26 × 9 × 8 × 5 or $26 \times {}^{9}P_2 \times 5$ for 2nd outcome, condone 26 × 9 × 5 × 37 as being relevant
	1 1 2 = $26 \times 9 \times {}^{5}C_2 \times 4!$ = 56 160	M1	mult all terms by 4! or 4!/2!
	Total = 519 480	A1	
	Total:	**4**	

Question 34

(i)	$^{18}P_5$	M1	$^{18}P_x$ or $^yP_5$ OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geqslant 1$
	= 1 028 160	A1	
		2	
(ii)	*EITHER:* e.g. ***(CCCCC)********** in 5!×14 ways	(B1	5! OE mult by k ⩾ 1, considering the arrangements of cars next to each other
	= 1680	B1	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = 1680/1 028 160	M1	Dividing by **(i)** for probability
	P(not next to each other) = 1 – 1680/1 028 160	M1	Subtracting prob from 1 (or their ‘ 5! × 14 ’ from **(i)**)
	$= 0.998\left(\frac{611}{612}\right)$ OE	A1)	
	OR1: $\frac{5! \times 14!}{18!} = 0.001634$	(B1	5! OE mult by k ⩾ 1 (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by 14!, (or 14! on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by 18!, total number of ways of arranging spaces
	1 – 0.001634	M1	Subtracting prob from 1 (or ‘5! × 14!’ from 18!)
	= 0.998(366)	A1)	
	OR2: 4 together – $2\times5!\times14C12 = 21\,840$ 3, 1, 1 – $3\times5!\times14C11 = 131\,040$ 3, 2 – $2\times5!\times14C12 = 21\,840$ 2,2,1 – $3\times5!\times14C11 = 131\,040$ 2,1,1,1 – $4\times5!\times14C10 = 480\,480$ 1,1,1,1,1 – $5!\times14C9\ or\ 14P5 = 240\,240$	(M1	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
		M1	Summing total of the six scenarios, at least 2 correct unsimplified
	Total = 1 026 480	A1	Total of 1 026 480
		M1	Dividing their 1 026 480 by their **6(i)**
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	A1)	
		5	
(iii)	R(5) W(4) B(3) Scenarios — No. of ways 1 1 1 = 5 × 4 × 3 = 60 0 1 2 = 4 × $^3C_2$ = 12 0 2 1 = $^4C_2$ × 3 = 18 1 0 2 = 5 × $^3C_2$ = 15 2 0 1 = $^5C_2$ × 3 = 30 1 2 0 = 5 × $^4C_2$ = 30 2 1 0 = $^5C_2$ × 4 = 40	B1	$5C1\times4C1\times3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of $^5C_2$ or $^4C_2$ or $^3C_2$ seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified ‘no. of ways’ other than 5C1 × 4C1 × 3C1
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	A1	
	OR		
	$^{12}C_3$ –	M1	Seeing ‘$^{12}C_3$ –’, considering all selections of 3 cars
	– $^5C_3$	M1	Subt $^5C_3$ OE, removing only red selections
	– $^4C_3$	M1	Subt $^4C_3$ OE, removing only white selections
	– $^3C_3$	M1	Subt $^3C_3$ OE, removing only black selections
	= 205	A1	Correct answer
		5	

Question 35

5(a)(i)	*EITHER:* 3**, 4**, 6**, 8**	(M1	$^5P_2$ or $^5C_2 \times 2!$ or 5×4 OE (considering final 2 digits)
	options $4 \times 5 \times \mathbf{4} = 80$	M1	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	OR: Total number of values: $6 \times 5 \times 4 = 120$	(M1	Calculating total number of values (with subtraction seen)
	Number of values less than 300: $2 \times 5 \times 4 = 40$	M1	Calculating number of unwanted values
	Number of evens = $120 - 40 = 80$	A1)	Correct final answer
		3	
5(a)(ii)	3**, 4**, 6**, 8** *EITHER:* options $4 \times 6 \times 4$ (last)	(M1	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4×4 (condone $\times 16$) (No additional figures present for both M's to be awarded)
	= 96	A1)	
	OR: Total number of values $4 \times 6 \times 6 = 144$	(M1	Calculating total number of values (with subtraction seen)
	Number of odd values $4 \times 6 \times 2 = 48$	M1	Calculating number of unwanted values
	Number of evens = $144 - 48 = 96$	A1)	
		3	
5(b)(i)	252	B1	
		1	
5(b)(ii)	B (6)G(4) 5 0 in $^6C_5 (\times {}^4C_0) = 6 \times 1 = 6$ 4 1 in $^6C_4 \times {}^4C_1 = 15 \times 4 = 60$ 3 2 in $^6C_3 \times {}^4C_2 = 20 \times 6 = 120$	M1	Multiplying 2 combinations $^6C_q \times {}^4C_r$, $q + r = 5$, or $^6C_5$ seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	A1	
		3	

Question 36

(a)(i)	$^{40}P_5$	**M1**	$^{40}P_x$ or $^{y}P_5$ oe seen, can be mult by $k \geqslant 1$
	$= 78\ 960\ 960$	**A1**	
		2	
(a)(ii)	not front row e.g. WEJ** in $3\times 3! = 18$ ways	**B1**	3! seen mult by $k \geqslant 1$
	7 rows in $7 \times 18 = 126$ ways	**B1**	mult by 7
	front row: e.g. *MA** in $4 \times 2 = 8$ ways	**M1**	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total $126 \times 8 = 1008$	**A1**	
		4	
6(b)	*EITHER:* e.g. *R** in $^8C_3$ ways = 56 ways *L** in $^8C_3 = 56$ ways	**(M1**	Considering either R or L only in team
	**** in $^8C_4 = 70$ ways	**M1***	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	**A1)**	
	OR1: No restrictions $^{10}C_4 = 210$ ways	**(M1**	$^{10}C_4$ – , Considering no restrictions with subtraction
	RL = $^8C_2 = 28$	**M1***	Considering both in team
	$210 - 28$	**DM1**	subt
	= 182 ways	**A1)**	
(b)	*OR2:* R out in $^9C_4 = 126$ ways L out in $^9C_4 = 126$ ways	**(M1**	Considering either R out or L out
	Both out in $^8C_4 = 70$	**M1***	Considering both out
		DM1	Summing 2 scenarios and subtracting 1 scenario
	$126 + 126 - 70 = 182$ ways.	**A1)**	
		4	

Question 38

(i)	1 L: $^6C_2 = 15$	**B1**	
		1	
(ii)	No L: $^6C_3 = 20$ (1 L: $^6C_2 = 15$)	**M1**	Either 0L or 2L correct unsimplified
	2 L: $^6C_1 = 6$	**M1**	Summing the 3 correct scenarios
	Total = 41	**A1**	
		3	

Question 39

(i)	****E**** Other letters arranged in $\frac{8!}{2!3!}$ = 3360 ways	**M1**	Mult by 8! or $^8P_8$ oe (arrangements ignoring repeats)
		A1	Correct final answer www
	OR $\frac{8\times7\times6\times5\times4\times4\times3\times2\times1}{4!2!} = 3360$ ways	**M1**	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	**2**	
(ii)	* * * * * ↑ Arrangements other letters × ways Es inserted $= \frac{5!}{2!} \times {}^6C_4 \left(\frac{5!}{2!}\times\frac{^6P_4}{4!}\right)$	**M1**	k mult by $^6C_4$ or $^6P_4$ oe (ways to insert Es ignoring repeats), k can = 1 or k mult by $\frac{5!}{2!}$
		M1	Correct unsimplified expression or $\frac{5!}{2!} \times {}^6P_4$
	= 900 ways	**A1**	Correct answer
	OR Total no of ways – no of ways with Es touching 9!/(4! × 2!) – … or 7 560 – … $\frac{6!}{2!} + {}^6P_2\times\frac{5!}{2!} + \frac{^6P_2}{2!}\times\frac{5!}{2!} + \frac{^6P_3}{2!\times\frac{5!}{2!}}$ = 360 + 1800 + 900 + 3600 = 6660	**M1**	7560 unsimplified – k
		M1	Attempting to find four ways of Es touching (4 Es, 3Es and a single, 2 lots of 2 Es, 2 Es and 2 singles)
	7 560 – 6 660 = 900	**A1**	Correct answer
(ii)	OR Adding the number of ways with the first E in the 1st (E_1), 2nd (E_2) or 3rd (E_3) position. $\frac{5!}{2!}(E_1 + E_2 + E_3)$ where $E_1 = 10$, $E_2 = 4$, $E_3 = 1$ $\frac{5!}{2!}(E_1 + E_2 + E_3)$	**M1**	For any values for E_1, E_2 and E_3
		M1	For any two correct values of E_1, E_2 and E_3
	600 + 240 + 60 = 900	**A1**	Correct answer
	Total:	**3**	
(iii)	EENN* in 3 ways	**B1**	Numerical value must be stated
	Total:	**1**	

(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from **(iii)**	**M1**	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	**A1**	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	**A1**	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	**M1**	At least 1 option listed for each of EE^^^, EEE^^, EEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	**4**	

Question 40

(a)(i)	(AAAIU) * * * * Arrangements of vowels/repeats × arrangements of (consonants & vowel group) =	**M1**	$k \times 5!$ (k is an integer, $k \geqslant 1$)
	$\frac{5! \times 5!}{3!}$	**M1**	$\frac{m}{3}!$ (m is an integer, $m \geqslant 1$) **Both** Ms can **only** be awarded if expression is **fully correct**
	= 2400	**A1**	Correct answer
		3	
(a)(ii)	E.g. R * * * T * * * L . Arrangements of consonants RL, RS, SL = $^3P_2 = 6$ Arrangements of remaining letters = $\frac{6!}{3!} = 120$	**M1**	$k \times \frac{6!}{3!}$ **or** $k \times {}^3P_2$ **or** $k \times {}^3C_2$ **or** $k \times 3!$ **or** $k \times 3 \times 2$ (k is an integer, $k \geqslant 1$), no irrelevant addition
	Total 120 × 6	**M1**	Correct unsimplified expression or $\frac{6!}{3!} \times {}^3C_2$
	= 720 ways	**A1**	Correct answer
		3	
(b)	**Method 1** N(2) R(8) Br(4) 1 2 1 $= 2 \times {}^8C_2 \times 4 = 224$	**M1**	Multiply 3 combinations, ${}^2C_x \times {}^8C_y \times {}^4C_z$. Accept ${}^2C_1 = 2$ etc.
	2 1 1 $= 1 \times {}^8C_1 \times 4 = 32$ 1 1 2 $= 2 \times 8 \times {}^4C_2 = 96$	**A1**	3 or more options correct unsimplified
	2 0 2 $= 1 \times 1 \times {}^4C_2 = 6$ 1 0 3 $= 2 \times 1 \times 4 = 8$	**M1**	Summing *their* values of 4 or 5 legitimate scenarios (no extra scenarios)
	Total = 366 ways	**A1**	Correct answer
	Method 2 ${}^{14}C_4$ – (2N2R or 1N3R or 4R or 3R1B or 2R2B or 1R3B or 4B)	**M1**	'${}^{14}C_4 - k$' seen, k an integer from an expression containing ${}^8C_x$
	$1001 - (1 \times {}^8C_2 + 2 \times {}^8C_3 + {}^8C_4 + {}^8C_3 \times 4 + {}^8C_2 \times {}^4C_2 + 8 \times 4 + 1)$	**A1**	4 or more 'subtraction' options correct unsimplified, may be in a list
	1001 – (28 + 112 + 70 + 224 + 168 + 32 + 1)	**M1**	*Their* ${}^{14}C_4$ – [*their* values of 6 or more legitimate scenarios] (no extra scenarios, condone omission of final bracket)
	= 366	**A1**	Correct answer
		4	

Question 41

(i)	$\frac{9!}{2!2!} = 90720$	B1	Must see 90720
		1	
(ii)	**Method 1** ↑ * * * * * A	B1	5! seen multiplied (arrangement of consonants allowing repeats)
	No. arrangements of consonants × ways of inserting vowels =	B1	$^6P_4$ oe (i.e. 6 × 5 × 4 × 3, $^6C_4$ × 4!) seen mult (allowing repeats) no extra terms
	$\frac{5!}{2!}$ $\times \frac{^6P_4}{2!}$	B1	Dividing by at least one 2! (removing at least one set of repeats)
	Answer $\frac{^6P_4}{2!} \times \frac{5}{2} = 10\,800$	B1	Correct final answer
		4	
(iii)	$^5C_3 = 10$	M1	^5C_x or ^5P_x seen alone, $x = 2$ or 3
		A1	Correct final answer not from $^5C_2$
		2	
(iv)	**Method 1** Considering separate groups	M1	Considering two scenarios of MME or EEM or MMEE with attempt, may be probs or perms
	MME** = $^5C_2$ = 10 MEE** = $^5C_2$ = 10 MMEE* = $^5C_1$ = 5	M1	Summing three appropriate scenarios from the four need ^5C_x seen in all of them
	ME*** = $^5C_3$ = 10 see **(iii)** Total = 35	A1	Correct final answer
	Method 2 Considering criteria are met if ME are chosen	M1	^7C_x only seen, no other terms
		M1	$^xC_3$ only seen, no other terms
	ME *** = $^7C_3$ = 35	A1	Correct final answer
		3	

Question 42

Method 1		
... M … M … M … M … M …	M1	k × 5! (120) or k × 6P2 (30), k is an integer ⩾ 1,
No. ways men placed × No. ways women placed in gaps = 5! × $^6P_2$	M1	Correct unsimplified expression
= 3600	A1	Correct answer
Method 2		
Number with women together = 6! × 2 (1440) Total number of arrangements = 7! (5040)	M1	6! × 2 or 7! – k seen, k is an integer ⩾ 1
Number with women not together = 7! – 6! × 2	M1	Correct unsimplified expression
= 3600	A1	Correct answer
	3	

Question 43

(i)	Total number of selections = ${}^{12}C_7 = 792$	**B1**	Seen as denominator of fraction
	Selections with boy included = ${}^{11}C_6$ or ${}^{12}C_7 - {}^{11}C_7 = 462$	**M1**	Correct unsimplified expression for selections with boy included seen as numerator of fraction
	Probability = 462/792 = 7/12 (0.583)	**A1**	Correct answer
(ii)	**Method 1**		
	Scenarios are: 2G + 5B: ${}^4C_2 \times {}^8C_5 = 336$	**B1**	One unsimplified product correct
	3G + 4B: ${}^4C_3 \times {}^8C_4 = 280$ 4G + 3B: ${}^4C_4 \times {}^8C_3 = 56$	**M1**	No of selections (products of ${}^nC_r$ and ${}^nP_r$) added for 2, 3 and 4 girls with no of girls and no of boys summing to 7
	Total = 672	**A1**	Correct total
	Probability = 672/792 (28/33) (0.848)	**A1ft**	Correct answer – 'total'/('total no of selections' from i)
	Method 2		
	0G + 7B ${}^4C_0 \times {}^8C_7 = 8$	**B1**	One unsimplified no of selections correct
	1G + 6B ${}^4C_1 \times {}^8C_6 = 112$ Total = 8 + 112 = 120	**M1**	No of selections (products of ${}^nC_r$ and ${}^nP_r$) added for 0 and 1 girls with no of girls and no of boys summing to 7
	$({}^{12}C_7 - 120)/792$ or $1 - 120/792$	**A1**	792 – 120 = 672 or 1 – 120/792
	Probability = 672/792 (28/33) (0.848)	**A1ft**	'672' over '792' from i

Question 44

(i)	$\frac{11!}{4!4!2!}$	**M1**	$\frac{11!}{4!\times k}$ or $\frac{11!}{2!\times k}$, k a positive integer
	= 34650	**A1**	Correct final answer
		2	
(ii)	**Method 1**		
	$P(SS) = \frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911)	**B1**	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	$P(PP) = \frac{2}{11}\times\frac{1}{10}=\frac{2}{110}$ (= 0.01818) $P(II) = \frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911) $\frac{4}{11}\times\frac{3}{10}$	**M1**	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\frac{4}{11}\times\frac{a}{b}+\frac{2}{11}\times\frac{c}{b}+\frac{4}{11}\times\frac{a}{b}$ where $a = 4$ *or* 3, $b = 11$ *or* 10, $c = 2$ *or* 1)
	Total = $\frac{26}{110}=\frac{13}{55}$ oe (0.236)	**A1**	Correct final answer
	Method 2		
	Total number of selections = ${}^{11}C_2 = 55$ Selections with 2 Ps = 1	**B1**	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = ${}^4C_2 = 6$ Selections with 2 Is = ${}^4C_2 = 6$,	**M1**	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\frac{13}{55}$ oe (0.236)	**A1**	Correct final answer, without use of permutations
		3	

Question 45

(i)	$5! \times 6! \times 2$	B1	$k\times5!$ or $m\times6!$ (k,m integer, $k,m \geqslant 1$), no inappropriate addition
		B1	$n \times 5! \times 6!$ (n integer, $n \geqslant 1$), no inappropriate addition
	$= 172800$	B1	Correct final answer, isw rounding (www scores B3) All marks based on their **final** answer
		3	
(ii)	… G … G … G … G … G … G … No. ways girls placed × No. ways boys placed in gaps =	M1	$k \times 6!$ or $k \times {}^7P_5$ (k is an integer, $k \geqslant 1$) no inappropriate add. (${}^7P_5 = 7 \times 6 \times 5 \times 4 \times 3$ or ${}^7C_5 \times 5!$)
	$6! \times {}^7P_5$	M1	Correct unsimplified expression
	$= 1814400$	A1	Correct exact final answer (ignore subsequent rounding)
		3	

Question 46

${}^9C_4 \times {}^5C_3 \times {}^2C_2$	B1	${}^9C_4$ or ${}^9C_3$ or ${}^9C_2$ seen *(1st group)*
$=126 \times 10 \times 1$	B1	${}^{5 \text{ or } 7}C_3$ or ${}^{6 \text{ or } 7}C_4$ or ${}^{6 \text{ or } 5}C_2$ times an integer *(2nd group)*
$=1260$	B1	Correct answer
	3	

Question 47

(i)	Scenarios are: 4V + 1C + 1DB: ${}^{11}C_4 \times {}^5C_1 \times {}^4C_1$	M1	${}^{11}C_a \times {}^5C_b \times {}^4C_c$, $a+b+c=6$,
	4V + 2C: ${}^{11}C_4 \times {}^5C_2$ 5V + 1C: ${}^{11}C_5 \times {}^5C_1$	B1	2 correct unsimplified options
	$6600 + 3300 + 2310$	M1	Add 2 or 3 correct scenarios only
	$= 12210$	A1	Correct answer
		4	
(ii)	$4! \times 3!$	M1	k multiplied by 3! or 4!, k an integer $\geqslant 1$
		A1	Correct unsimplified expression
	$= 144$	A1	Correct answer
		3	

Question 48

(i)	$\frac{9!}{2!3!}$	M1	9! alone on numerator, 2! and/or 3! on denominator
	$= 30240$	A1	Exact value, final answer
		2	
(ii)	A ^ ^ ^ A ^ ^ ^ A Arrangements $= \frac{6!}{2!} = 360$	B1	Final answer
		1	
(iii)	M ^ M ^ ^ ^ ^ ^ ^ $= \frac{7!}{3!} \times 7$	M1	7! in numerator, (considering letters not M)
		M1	Division by 3! only (removing repeated As)
		M1	Multiply by 7 (positions of M-M)
	$= 5880$	A1	Exact value, final answer
	Method 2 (choosing letter between Ms)		
	$1 \times \frac{6!}{2!} \times 7 + 4 \times \frac{6!}{3!} \times 7$	M1	6! in sum of 2 expressions $a6! + b6!$
		M1	Multiply by 7 in both expressions (positions of M-M)
	$= 2520 + 3360$	M1	$\frac{c}{2!} + \frac{d}{3!}$ seen (removing repeated As)
	$= 5880$	A1	Exact value
(iii)	**Method 3**		
	(MAM) ^ ^ ^ ^ ^ ^ = 7!/2! = 2520	M1	7! in numerator (considering 6 letters + block)
	(MA'M) ^ ^ ^ ^ ^ ^ = 7!/3! × 4 = 840 × 4 = 3360	M1	Division by 2! and 3! seen in different terms
	Total = 2520 + 3360	M1	Summing 5 correct scenarios only
	= 5880	A1	Exact value
		4	
(iv)	M A ^ = $^4C_1 = 4$	B1	Final answer
		1	
(v)	M ^ ^ : $^4C_2$ = 6 M M ^ : $^4C_1$ = 4	M1	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M M A : = 1 M A A : = 1 (M A _ : $^4C_1$ = 4)	M1	Add 4 or 5 correct scenarios only
	Total = 16	A1	Value must be clearly stated
	Method 2		
	M M ^ = $^5C_1$ = 5	M1	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M ^ ^ = $^5C_2$ = 10	M1	Adding 2 or 3 correct scenarios only
	M A A = = 1 Total = 16	A1	Value must be clearly stated
		3	

Question 49

4(i)	M(8) W(4) 4 2 in $^8C_4 \times {^4C_2} = 420$ ways 5 1 in $^8C_5 \times {^4C_1} = 224$ ways 6 0 in $^8C_6 \times {^4C_0} = 28$ ways	B1	One unsimplified product correct
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios
	Total 672 ways	A1	Correct answer
		3	
(ii)	Total number of selections = $^{12}C_6 = 924$ (A)	M1	$^{12}C_x$ – (subtraction seen), accept unsimplified
	Selections with males together = $^{10}C_4 = 210$ (B)	A1	Correct unsimplified expression
	Total = (A) – (B) = 714	A1	Correct answer
	Alternative method for question 4(ii)		
	No males + Only male 1 + Only male 2 $= {^{10}C_6} + {^{10}C_5} + {^{10}C_5}$	M1	$^{10}C_x + 2 \times {^{10}C_y}$, $x \neq y$ seen, accept unsimplified
	= 210 + 252 + 252	A1	Correct unsimplified expression
	= 714	A1	Correct answer
	Alternative method for question 4(ii)		
	Pool without male 1 + Pool without male 2 – Pool without either male	M1	$2 \times {^{11}C_x} - {^{10}C_x}$
	$= {^{11}C_6} + {^{11}C_6} - {^{10}C_6}$ = 462 + 462 – 210	A1	Correct unsimplified expression
	= 714	A1	Correct answer
		3	

Question 50

7(a)	$^6C_3 \times {^3C_2} \times {^1C_1}$	M1	$^6C_a \times {^{6-a}C_b} \times {^{6-a-b}C_{6-a-b}}$ seen oe $^{6-a-b}C_{6-a-b}$ can be implied by 1 or omission, condone use of permutations,
	= 20 × 3	A1	Any correct method seen no addition/additional scenarios
	= 60	A1	Correct answer
	Alternative method for question 7(a)		
	$\frac{^6P_6}{^3P_3 \times {^2P_2} \times {^1P_1}} = \frac{6!}{3! \times 2!}$	M1	$^6P_6 / ({^nP_n} \times k)$ with $3 \geqslant n > 1$ and $6 \geqslant k$ an integer $\geqslant 1$, not 6!/1
		A1	Correct method with no additional terms
	= 60	A1	Correct answer
		3	
7(b)(i)	$\frac{4!}{3!} \times \frac{3!}{2!} \times 2$	M1	A single expression with either $4!/3! \times k$ or $3!/2! \times k$, k a positive integer seen oe (condone 2 identical expressions being added)
		M1	Correctly multiplying *their* single expression by 2 or 2 identical expressions being added.
	= 24	A1	Correct answer
		3	

(b)(ii)	Total no of arrangements = $\frac{7!}{2!3!}$ = 420 (A)	B1	Accept unsimplified
	No with 2s together = $\frac{6!}{3!}$ = 120 (B)	B1	Accept unsimplified
	With 2s not together: *their* (A) – *their* (B)	M1	Subtraction indicated, possibly by *their* answer, no additional terms present
	= 300 ways	A1	Exact value www
	Alternative method for question 7(b)(ii)		
	3 _ 7 _ 7 _ 7 _ 8 _		
	$\frac{5!}{3!} \times \frac{6 \times 5}{2}$	B1	*k* x 5! in numerator, *k* a positive integer
		B1	*m* x 3! In denominator, *m* a positive integer
		M1	*Their* 5!/3! multiplied by $^6C_2$ only (no additional terms)
	= 300 ways	A1	Exact value www
		4	

Question 51

(i)	($^9C_4$ =) 126	B1	
		1	
(ii)	$^7C_2$	B1	^7C_x or $^yC_2$ (implied by correct answer) or ^7P_x or ^7P_y, seen alone
	= 21	B1	correct answer
		2	
(iii)	_ C_1 (B_1 B_2 B_3) C_2 _ C_3 _ C_4 _ C_5 _ C_6	B1	3! or 6! seen alone or multiplied by k > 1 need not be an integer
	3! × 6! × 7	B1	3! and 6! seen multiplied by k > 1, integer, no division
	= 30240	B1	Exact value
	Alternative method for question 8(iii)		
	C_1 (B_1 B_2 B_3) C_2 C_3 C_4 C_5 C_6	B1	3! or 7! seen alone or multiplied by k > 1 need not be an integer
	3! × 7!	B1	3! and 7! seen multiplied by k > or = 1, no division
	= 30240	B1	Exact value
		3	
(iv)	C_1 _ C_2 _ C_3 _ C_4 _ C_5 _C_6	B1	6! or 4! X 6P2 seen alone or multiplied by k > 1, no division (arrangements of cars)
	6! × 5P3 *or* 6! × 5 × 4 × 3 or 6! x 3! x10	B1	Multiply by 5P3 oe i.e. putting Bs in between 4 of the Cs OR multiply by 3! x n where n = 7, 8, 9, 10 (number of options)
	= 43200	B1	Correct answer
		3	

Question 52

(i)	$\frac{9!}{2!3!} = 30240$	B1	9! Divided by at least one of 2! or 3!
		B1	Exact value
		2	
(ii)	D _ _ _ _ _ _ _ R: $\frac{7!}{2!2!} = 1260$ D _ _ _ _ _ _ _ O: $\frac{7!}{3!} = 840$	B1	7! Seen alone or as numerator in a term, can be multiplied not + or –
		B1	One term correct, unsimplified
	Total = 2100	B1	Final answer
		3	

Question 53

(i)	$3A\ 2D\ 2M: {}^6C_3 \times {}^5C_2 \times {}^4C_2$ (= 1200) $4A\ 2D\ 1M: {}^6C_4 \times {}^5C_2 \times {}^4C_1$ (= 600) $3A\ 3D\ 1M: {}^6C_3 \times {}^5C_3 \times {}^4C_1$ (= 800)	M1	${}^6C_x \times {}^5C_y \times {}^4C_z,\ x + y + z = 7$
		A1	2 correct products, allow unsimplified
		M1	Summing their totals for 3 correct scenarios only
	Total = 2600	A1	Correct answer SC1 ${}^6C_3 \times {}^5C_2 \times {}^4C_1 \times {}^9C_1 = 7200$
		4	
(ii)	${}^7C_4 \times 1$	B1	${}^7C_3$ or ${}^7C_4$ seen anywhere
	35	B1	
		2	

Question 54

(i)	$6! = 720$	B1	Evaluated
		1	
(ii)	Total no of arrangements: $\frac{9!}{2!3!} = 30240$	B1	Accept unevaluated
	No with Ts together $= \frac{8!}{3!} = 6720$	B1	Accept unevaluated
	With Ts not together: 30 240 – 6720	M1	correct or $\frac{9!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ts together
	23 520	A1	CAO
	Alternative method for question 7(ii)		
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	B1	$7! \times (k > 0)$ in numerator, cannot be implied by $^7P_2$, etc.
		B1	$3! \times (k > 0)$ in denominator
		M1	$\frac{\textit{their } 7!}{\textit{their } 3!} \times {}^8C_2$ or $^8P_2$
	23 520	A1	CAO
		4	

'(iii)	Number of arrangements $= \frac{7!}{3!}$ Probability $= \frac{\textit{their} \frac{7!}{3!}}{\textit{their} \frac{9!}{3!2!}} = \frac{840}{30240}$	M1	$\frac{\textit{their} \text{ identified number of arrangements with T at ends}}{\textit{their} \text{ identified total number of arrangements}}$ or $\frac{\frac{7!}{m}}{\frac{9!}{n}}$ m, n integers > 1
	$\frac{1}{36}$ or 0.0278	A1	Final answer
		2	
'(iv)	OOT_ _ $^4C_2 = 6$ OOTT_ $^4C_1 = 4$ OOOT_ $^4C_1 = 4$ OOOTT $= 1$	M1	^4C_x seen alone or ^4C_x x $k \geq 1$, k an integer, $0 < x < 4$
		A1	$^4C_2$ x k, $k = 1$ oe or $^4C_1$ x m, $m = 1$ oe alone
		M1	Add 3 or 4 identified correct scenarios only, accept unsimplified
	(Total) = 15	A1	CAO, WWW Only dependent on 2nd M mark
		4	

Question 55

(i)	$\frac{9!}{2!} = 181\,440$	B1	Exact value
		1	
(ii)	Total no of ways $= \frac{12!}{2!4!} = 9\,979\,200$ (A)	B1	Accept unevaluated
	With Ss together $= \frac{11!}{4!} = 1\,663\,200$ (B)	B1	Accept unevaluated
	With Ss not together = (B) – (A)	M1	Correct or $\frac{12!}{m} - \frac{8!}{n}$, m, n integers > 1 or *their* identified total – *their* identified Ss together
	8 316 000	A1	Exact value
	Alternative method for question 6(ii)		
	_ T _ E _ E _ P _ L _ E _ C _ H _ A _ E _	B1	$10! \times k$ in numerator k integer $\geqslant 1$
	$\frac{10!}{4!} \times \frac{11 \times 10}{2!}$	B1	$4! \times k$ in numerator k integer $\geqslant 1$
	$\frac{their\,10!}{their\,4!} \times {}^{11}C_2$ or ${}^{11}P_2$	M1	OE
	8 316 000	A1	Exact value
		4	
(iii)	S E E E : 1	M1	${}^6C_x$ seen alone or times $K > 1$
	S E E _ : ${}^6C_1 = 6$ S E _ _ : ${}^6C_2 = 15$ S _ _ _ : ${}^6C_3 = 20$	B1	${}^6C_3$ or ${}^6C_2$ or ${}^6C_1$ alone
	Add 3 or 4 correct scenarios	M1	No extras
	Total = 42	A1	
		4	

Question 56

${}^{38}C_r$ or ${}^{n}C_{34}$	M1	Either expression seen OE, no other terms, condone x1
${}^{38}C_{34}$	A1	Correct unsimplified OE
73815	A1	If M0, **SCB1** ${}^{38}C_{34}$ x k, k an integer
	3	

Question 57

(a)	R ^^^^^^^^^^ R $\frac{9!}{3!6!}$	M1	9! Alone on numerator, $3! \times k$ or $6! \times k$ on denominator
	= 84	A1	
		2	
(b)	^ (B B B) ^^^^^	M1	$\frac{7!}{6!} \times k$ or $7k$ seen, k an integer > 0
	$\frac{7!}{6!} \times \frac{8 \times 7}{2}$	M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n=7, 8 or 9, m an integer > 0
		M1	n = 8 used in above expression
	= 196	A1	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!} - \frac{8!}{6!}$	M1	9! Seen alone or as numerator with subtraction
	= [252 – 56]	M1	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x k, k an integer
	= 196	A1	
		4	

Question 58

(a)	$\frac{9!}{2!2!} = 90\,720$	B1
		1
(b)	$\frac{6!}{2!}$	M1
	360	A1
		2

(c)	2 Es together $= \frac{8!}{2!}(=20160)$	**M1**
	Es not together $= 90720 - 20160 = 70560$	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
	Alternative method for question 7(c)	
	_^_^_^_^_^_^_^_^_ $\frac{7!}{2!} \times \frac{8 \times 7}{2} = 70560$	
	$7! \times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	**M1**
	Multiplying by ${}^8C_2$ OE	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
		4
(d)	Scenarios are: E L _ _ _ ${}^5C_3$ 10 E E L _ _ ${}^5C_2$ 10 E _ _ _ _ ${}^5C_4$ 5 E E _ _ _ ${}^5C_3$ 10	**M1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total $= 35$	**A1**
		3

Question 59

(a)	$\frac{8!}{3!}$	**M1**
	6720	**A1**
		2

(b)	Total number $= \frac{10!}{2!3!}$ (302400) (A)	B1
	With Es together $= \frac{9!}{3!}$ (60480) (B)	B1
	Es not together = *their* (A) – *their* (B)	M1
	241920	A1
	Alternative method for question 6(b)	
	_^_^_^_^_^_^_^_^_ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	$8! \times k$ in numerator, k integer ≥ 1, denominator ≥ 1	B1
	$3! \times m$ in denominator, m integer ≥ 1	B1
	Their $\frac{8!}{3!}$ Multiplied by ${}^9C_2$ (OE) only (no additional terms)	M1
	241920	A1
		4
(c)	Scenarios: E M M M ${}^5C_0 = 1$ E M M _ ${}^5C_1 = 5$ E M _ _ ${}^5C_2 = 10$	M1
	Summing the number of ways for 2 or 3 correct scenarios	M1
	Total = 16	A1
		3

Question 60

(a)	6!	M1
	720	A1
		2
(b)	Total number: $\frac{9!}{3!2!}$(30240)	M1
	Number with Ls together $= \frac{8!}{3!}$(6720)	M1
	Number with Ls not together $= \frac{9!}{3!2!} - \frac{8!}{3!}$ $= 30\,240 - 6720$	M1
	23 520	A1
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	$7! \times k$ in numerator, k integer ≥ 1	M1
	$8 \times 7 \times m$ in numerator or 8C2 $\times m$, m integer ≥ 1	M1
	3! in denominator	M1
	23 520	A1
		4

Question 61

Scenarios: 2P 3V 2G $^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for $^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
Two unsimplified products correct	**B1**
Summing the number of ways for 3 or 4 correct scenarios	**M1**
Total: 13 720	**A1**
	4

Question 62

(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	**B1**	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2! \times 2!}$ (= 180)	**B1**	$\frac{6!}{2! \times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No $\times, \pm$
	Probability = $\frac{180}{3360}$ $\left(= \frac{3}{56}\right)$ or 0.0536	**B1 FT**	Final answer from *their* $\frac{6!}{2! \times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8} \times \frac{3}{7}$	**M1**	$\frac{a}{8} \times \frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c} \times \frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	**A1**	
		3	
(b)	Rs together and Es together: 5! (120)	**B1**	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no $\times, +, -$
	Es together: $\frac{6!}{2!} (= 360)$	**B1**	Alone or as denominator of probability to represent the number of ways with Es together, no $\times$, + or –
	Probability = $\frac{5!}{\frac{6!}{2!}}$	**M1**	$\frac{their\ 5!}{their\ \frac{6!}{2!}}$ seen
	$\frac{1}{3}$	**A1**	OE

Question 63

(a)	Scenarios: 6W 0M $^9C_6 = 84$ 5W 1M $^9C_5 \times {}^5C_1 = 126 \times 5 = 630$ 4W 2M $^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$	**M1**	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	**A1**	
		3	
(b)	Total number of ways = $^{14}C_6$ (3003) Number with sister and brother = $^{12}C_4$ (495) Number required = $^{14}C_6$ –	**M1**	$^{14}C_6$ – a value
	$^{12}C_4 = 3003 - 495$	**M1**	$^{12}C_x$ or $^nC_4$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	**A1**	

Question 64

(a)	${}^{9}C_6$ (× ${}^{3}C_3$)	**M1**	${}^{9}C_k \times n$, $k = 6, 3$, $n = 1,2$ oe Condone ${}^{9}C_6 + {}^{3}C_3$, ${}^{9}P_6 \times {}^{3}P_3$
	84	**A1**	Accept unevaluated.
		2	
(b)	Number with 3 Baker children = ${}^{6}C_2$ or 15	**B1**	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^{9}C_5$ or 126 Probability = $\dfrac{\text{number of selections with3 Baker children}}{\text{total number of selections}}$	**M1**	Seen as denominator of fraction
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
	Alternative method for question 6(b)		
	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times {}^{5}C_3$	**B1**	${}^{5}C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times k$, $1 \leqslant k$, k integer
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
		3	
(c)	[Total no of arrangements = 9!] [Arrangements with men together = 8! × 2] Not together: 9! –	**M1**	9! – k or 362880 – k, k an integer<362 880
	8! × 2	**B1**	8! × 2(!) or 80 640 seen anywhere
	282 240	**A1**	Exact value
	Alternative method for question 6(c)		
	7! × 8 × 7	**B1**	7! × k, k positive integer > 1
		M1	$m \times 8 \times 7$, $m \times {}^{8}P_2$, $m \times {}^{8}C_2$ m positive integer > 1
	282 240	**A1**	Exact value
		3	
(d)	7! × 2 × 7	**M1**	7! × k, k positive integer > 1 If 7! not seen, condone 7 × 6 × 5 × 4 × 3 × 2 × (1) × k *or* 7 × 6! × k only
		M1	$m \times 2 \times 7$, m positive integer > 1
	70 560	**A1**	
		3	

Question 65

(a)	$\frac{8!}{2!}$	**M1**	$\frac{8!}{k} \equiv \frac{7! \times 8}{k}$, where $k \in \mathbb{N}$, $\frac{a!}{2(!)}$, where $a \in \mathbb{N}$
	20160	**A1**	
		2	

(b)	Total number of ways: $\frac{10!}{2!3!}$ $(= 302\,400)$ (A)	**B1**	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ (= 60 480) (B)	**B1**	Accept unsimplified
(c)	Probability = $\frac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\frac{\frac{8!}{2!}}{\frac{10!}{2! \times 3!}} = \frac{20160}{302\,400}$	**M1**	$\frac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1 \leqslant k, l \in \mathbb{N} \leqslant 3$, FT denominator from **7(b)** or correct
	$\frac{1}{15}$, 0·0667	**A1**	
(d)	Scenarios: P E E E $\quad {}^5C_0 = 1$ P E E _ $\quad {}^5C_1 = 5$ P E _ _ $\quad {}^5C_2 = 10$ P _ _ _ $\quad {}^5C_3 = 10$	**M1**	${}^5C_x$ seen alone, $1 \leqslant x \leqslant 4$
		M1	Summing the number of ways for 3 or 4 correct scenarios (can be unsimplified), no incorrect scenarios
	Total = 26	**A1**	
		3	

Question 66

(a)	$\frac{11!}{2!2!2!}$	**M1**	11! alone as numerator. $2! \times m! \times n!$ on denominator, $m = 1, 2$, $n = 1, 2$. no additional terms, no additional operations.
	4989600	**A1**	Exact answer only.
		2	

Question	Answer	Marks	Guidance
(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$	**B1**	$\frac{7!}{2!}\times k$ seen, k an integer > 1.
	Number of ways of placing As in non-adjacent places = ${}^{8}C_2$ $\frac{7!}{2!}\times {}^{8}C_2$	**M1**	$m\times n(n-1)$ or $m\times {}^{n}C_2$ or $m\times {}^{n}P_2$, n = 7, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!}\times {}^{8}C_2$ or $\frac{7!}{p!}\times {}^{8}P_2$, p integer ⩾ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$	**M1**	$\frac{9!}{2!m!}-k$, 90720 > k integer > 1, m = 1, 2.
	Arrangements with R at ends and As together = $\frac{8!}{2!}$	**B1**	$s-\frac{8!}{2!}$, s an integer >1
	With As not together = $\frac{9!}{2!2!}-\frac{8!}{2!}$	**M1**	$\frac{9!}{p}-\frac{8!}{q}$, p, q integers ⩾ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	
(c)	**Method 1**		
	R R A L _ _ ${}^{5}C_2$ = 10 R R A L L _ ${}^{5}C_1$ = 5 R R A A L _ ${}^{5}C_1$ = 5 R R A A L L = 1	**M1**	${}^{5}C_x$ seen alone or ${}^{5}C_x\times k$, 2⩾ k ⩾ 1, k an integer, $0<x<5$ linked to an appropriate scenario.
		A1	${}^{5}C_2\times k$, k = 1 oe or ${}^{5}C_1\times m$, m = 1,2 oe alone. **SC** if ${}^{5}C_x$ not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. ${}^{2}C_w\times {}^{2}C_x\times {}^{2}C_y\times {}^{5}C_z$, $w+x+y+z$=6 identifies w Rs, x As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: ${}^{5}C_2+{}^{5}C_1+{}^{5}C_1+1$ = 21 is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = ${}^{7}C_2$	**M1**	${}^{7}C_x$ seen alone or ${}^{7}C_x\times k$, 2⩾ k⩾ 1, k an integer, $0<x<7$. Condone ${}^{7}P_x$ or ${}^{7}P_x\times k$, 2⩾ k⩾ 1, k an integer, $0<x<7$.
		M1	${}^{7}C_2\times k$, 2⩾ k⩾ 1oe
		A1	${}^{7}C_2\times k$, k = 1oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

Question 67

	Answer	Marks	Guidance
(a)	$\frac{11!}{2!3!}$	M1	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3$, 1 can be implied but cannot both = 1. No additional terms
	3326400	A1	Exact value only
		2	
(b)	8! = 40320	B1	Evaluate, exact value only
		1	
(c)	$\frac{9!}{3!}\times 7$	M1	$\frac{9!}{3!}\times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7!\ (\times\frac{3!}{3!})$	M1	$9C3\times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	A1	Exact value only but there must be evidence of $\times\frac{3!}{3!}$
(c) cont'd	**Alternative method for Question 6(c)**		
	$3\times 7\times\frac{8!}{2!}$	M1	$3\times\frac{8!}{2!}\times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	$7\times\frac{2}{11}\times\frac{9}{10}\times\frac{8}{9}\times\frac{7}{8}\times\frac{1}{7}\times$ total no. of arrangements	M1	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs $-\frac{{}^6C_3\times 3!\times 7!}{3!} = 100800$ 1E between the Rs $-\frac{{}^6C_2\times 3!\times 7!}{2!} = 226800$ 2Es between the Rs $-{}^6C_1\times 3!\times 7! = 90720$ 3Es between the Rs $- 7! = 5040$	M1	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified.
		M1	Adding the number of ways for 3 or 4 correct scenarios
	$[\text{Total} = 7!\times(20+45+18+1) = 7!\times 84 =]\,423360$	A1	CAO
		3	

(d)	E E R _ _ $^6C_2 = 15$ E E R R _ $^6C_1 = 6$ E E E R _ $^6C_1 = 6$ E E E R R $^6C_0 = 1$	**M1**	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. $^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	**A1**	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = $^8C_2$	**M1**	^8C_x seen alone or $^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ^8P_x or $^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	$^8C_2 \times k$, $k = 1$ or 2 OE
		M1	$^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	**A1**	Value stated
		4	

Question 68

(a)	$\frac{8!}{2!3!}$	M1	$\frac{8!}{k! \times m!}$ $k = 1$ or 2, $m = 1$ or 3, not $k = m = 1$ no additional terms
	3360	A1	
		2	
(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together =] $\frac{6!}{3!} - 4!$	M1	$\frac{6!}{k!} - m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2\times\left(\frac{6!}{k!}\right) - m$.
		M1	$w - 4!$ or $w - 24$, w an integer Condone $w - 2 \times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	$^4C_3 \times 3! + {}^4C_2 \times 2 \times 3!$	M1	$^4C_3 \times 3! + r$ or $4 \times 3! + r$ or $^4P_3 \times 3! + r$, r an integer. Condone $2 \times {}^4C_3 \times 3! + r$. $2 \times 4 \times 3! + r$ or $2 \times {}^4P_3 \times 3! + r$.
		M1	$q + {}^4C_2 \times 3! \times k$ or $q + {}^4P_2 \times 3! \times k$, $k = 1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
(c)	**Method 1** Identified scenarios		
	OORR $^3C_2 \times {}^2C_2 \times \left[{}^3C_0\right] = 3\times1 = 3$ ORR_ $^3C_1 \times {}^2C_2 \times {}^3C_1 = 3\times1\times3 = 9$ OOR_ $^3C_2 \times {}^2C_1 \times {}^3C_1 = 3\times2\times3 = 18$ OR_ _ $^3C_1 \times {}^2C_1 \times {}^3C_2 = 3\times2\times3 = 18$ OOOR $^3C_3 \times {}^2C_1 \times \left[{}^3C_0\right] = 1\times2 = 2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{^8C_4}$	M1	$\frac{their\ '50'}{^8C_4}$, accept numerator unevaluated

(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM $^3C_1 \times {}^2C_1 = 6$ ORTW $^3C_1 \times {}^2C_1 = 6$ ORMW $^3C_1 \times {}^2C_1 = 6$ ORRM $^3C_1 \times {}^2C_2 = 3$ ORRW $^3C_1 \times {}^2C_2 = 3$ ORRT $^3C_1 \times {}^2C_2 = 3$ OROR $^3C_2 \times {}^2C_2 = 3$ OROT $^3C_2 \times {}^2C_1 = 6$ OROM $^3C_2 \times {}^2C_1 = 6$ OROW $^3C_2 \times {}^2C_1 = 6$ OROO $^3C_3 \times {}^2C_1 = 2$	B1 M1	Outcomes for 5 identifiable scenarios correct, accept unsimplified. Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{^8C_4}$	M1	$\frac{their\ '50'}{^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

Question 69

(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\frac{their\ 5040}{their\ 6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7!\times 2}{3!}$ [= 5040]	*M1 *M1	*Their* 6720 – k, k a positive integer $(m-)\frac{7!\times k}{3!}, k = 1, 2$

Question 70

RRRRB $^8C_4 \times {}^4C_1 = 280$ BBBBR $^8C_1 \times {}^4C_4 = 8$ RRRRR $^8C_5 = 56$	M1	$^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone $^8C_1 \times 1$
	A1	Two correct outcomes evaluated
	M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
[Total =] 344	A1	WWW, only dependent on 2nd M mark
	4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

Question 71

$^{23}C_{17}$	**M1**	$^{23}C_x$ or $^yC_{17}$ or $^zC_6$, x, y or z are integers no $+$, $-$, $\times$ or $\div$.
100947	**A1**	CAO
	2	

Question 72

(a)	$^5P_2 \times {}^7P_4$ **or** $5 \times 4 \times 7 \times 6 \times 5 \times 4$	**M1**	$^5P_x \times {}^7P_y$, $1 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 6$
	16 800	**A1**	
		2	
(b)	**Method 1** [Identify scenarios]		
	With A and no 5: $8 \times {}^6P_4$ or $(1 \times 4 \times 6 \times 5 \times 4 \times 3) \times 2$ or $4C1 \times 2! \times 6P4 = 2880$ With 5 and no A: $^4P_2 \times 4 \times {}^6P_3$ or $(4 \times 3 \times 1 \times 6 \times 5 \times 4) \times 4$ or $4P2 \times 6C3 \times 4! = 5760$ With A and 5: $8 \times 4 \times {}^6P_3$ or $(4 \times 1 \times 1 \times 6 \times 5 \times 4) \times 8$ or $4C1 \times 2! \times 6C3 \times 4! = 3840$	**M1**	One number of ways correct, accept unsimplified.
		M1	Add 2 or 3 identified correct scenarios only, accept unsimplified.
	[Total =] 12 480	**A1**	CAO
	Method 2 [total number of codes – number of codes with no A or 5]		
	No A or 5 : $(4 \times 3) \times (6 \times 5 \times 4 \times 3) = 4320$	**M1**	$^4P_2 \times {}^6P_4$ or $^4C_2 \times {}^6C_4$ seen, accept unsimplified.
	Required number = *their* **(a)** – *their* 4320	**M1**	*Their* **5(a)** (or correct) – *their* (No A or 5) value.
	12 480	**A1**	
	Method 3 [subtracting **double counting**]		
	With A $^4P_1 \times {}^7P_4 \times 2$ or $^4C_1 \times 2 \times {}^7C_4 \times 4! = 6720$ With 5 $^5P_2 \times {}^6P_3 \times 4$ or $^5C_2 \times 2 \times {}^6C_3 \times 4! = 9600$ With A and 5 = $^4P_1 \times {}^6P_3 \times 8$or $4C1 \times 2! \times 6C3 \times 4! \times 8 = 3840$	**M1**	One outcome correct, accept unsimplified.
	Required number = 6720 + 9600 – 3840	**M1**	Adding 'with a' to 'with 5' and subtracting 'A and 5'.
	12 480	**A1**	CAO
		3	
(c)	**Method 1** – number of successful codes divided by total		
	$(1 \times)\ 3 \times {}^5P_2$	**M1**	$3 \times {}^5P_n$, $n = 2, 3$. Condone $3 \times {}^5C_2$, no + or –.
	Probability = $\dfrac{their\ 3 \times 5P2}{their\ 16\,800}$	**M1**	Probability = $\dfrac{their\ 60}{their\ 16\,800}$.
	$\dfrac{1}{280}$, 0.00357	**A1**	
	Method 2 – product of probabilities of each part of code		
	$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{7} \times \frac{3}{6}\left(\times \frac{5}{5} \times \frac{4}{4}\right)$ or $\frac{1}{5} \times \frac{1}{4} \times \frac{3 \times 5P2}{7P4}$	**M1**	$\frac{1}{5} \times \frac{1}{4} \times k$ where $0 < k < 1$ for considering letters.
		M1	$t \times \frac{1}{7} \times \frac{3}{6}$ or $t \times \frac{3 \times 5P2}{7P4}$ where $0 < t < 1$.
	$\dfrac{1}{280}$	**A1**	CAO
		3	

Question 73

(a)	$\frac{9!}{3!}$	**M1**	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	**A1**	
		2	
(b)	$\frac{7!}{3!}\times 2\times 6$	**M1**	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times 2\times q$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!}\times p\times 6$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t \times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	**A1**	Exact value.

Question 74

(a)	$^{11}C_5 \times {}^4C_1$	**M1**	$^{11}C_5 \times {}^4C_1$ condone $^{11}P_5 \times {}^4P_1$ no +, –, × or ÷.
	1848	**A1**	CAO as exact.
		2	
(b)	**Method 1** [Identifying scenarios]		
	[Neither selected =] $^{13}C_6$ [= 1716] [Only Jane selected =] $^{13}C_5$ [= 1287] [Only Kate selected =] $^{13}C_5$ [= 1287]	**M1**	Either $^{13}C_6$ seen alone or $^{13}C_5$ seen alone or × 2 (condone $^{13}P_n$, n = 5,6).
	[Total =] 1716 + 1287 + 1287	**M1**	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	**A1**	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	$^{15}C_6$ - $^{13}C_4$ [= 5005 – 715]	**M1**	$^{15}C_6 - k$, k a positive integer < 5005, condone $^{15}P_6$.
		M1	$m - {}^{13}C_4$, m integer > 715, condone $n - {}^{13}P_4$, n > 17 160.
	4290	**A1**	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 $^4C_1 \times {}^9C_5 + {}^4C_1 \times {}^9C_4 \times 2$ Method 2 $^4C_1 \times {}^{11}C_5 - {}^4C_1 \times {}^9C_3$

Question 75

(a)	[8! =] 40 320	B1	Evaluated, exact value only.
		1	
(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7! \times {}^8C_2 \times 2$	M1	$7! \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 7, 8 or 9, m an integer > 1.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	$9! - 8! \times 2$	M1	$9! - k$, k an integer < 362 880 .
		M1	$m - 8! \times n$, m an integer > 40 320, n = 1,2.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
(c)	${}^9C_5$ [$\times {}^4C_4$]	M1	${}^9C_x$ [$\times {}^{9-x}C_{9-x}$,] x = 4, 5. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	A1	WWW
		2	
(d)	[Number of ways with Raman and Sanjay together on back row =] ${}^7C_3$ [Number of ways with Raman and Sanjay together on front row =] ${}^7C_2$	M1	${}^7C_x$ seen, x = 3 or 2.
	[Total =] 35 + 21	M1	Summing two correct scenarios.
	56	A1	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability = $\frac{their\ 56}{their\ (c)} = \frac{56}{126}, \frac{4}{9}$, 0.444	B1 FT	FT *their* 56 from adding 2 or more scenarios in numerator and *their* **(c)** or correct as denominator.
		4	

Question 76

(a)	${}^5C_1 \times {}^7C_4$	M1	${}^7C_4 \times k$, k integer ⩾ 1 Condone ${}^5P_1$ for M1 only
	175	A1	
		2	
(b)	2B 1G 2A $\quad {}^3C_2 \times {}^4C_1 \times {}^5C_2$ = 120 2B 2G 1A $\quad {}^3C_2 \times {}^4C_2 \times {}^5C_1$ = 90 2B 3G $\quad {}^3C_2 \times {}^4C_3$ = 12 3B 1G 1A $\quad {}^3C_3 \times {}^4C_1 \times {}^5C_1$ = 20 3B 2G $\quad {}^3C_3 \times {}^4C_2$ = 6	M1	${}^3C_x \times {}^4C_y \times {}^5C_z$, $x + y + z = 5$, x,y,z integers ⩾1 Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	A1	Note: Only dependent upon M marks
		4	
(c)	$8! \times 3! \times {}^5P_2$	M1	$8! \times m$, m an integer ⩾ 1 Accept $8 \times 7!$ for 8!
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {}^5P_2$, $p \times {}^5C_2 \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	A1	Exact value required
		4	

Question 77

(a)	$^{12}C_5 \times {}^{7}C_4$ [$\times {}^{3}C_3$]	M1	$^{12}C_r \times q$, r = 3, 4, 5 q a positive integer > 1, no + or − .
		M1	$^{12}C_s \times {}^{12-s}C_t$ [$\times {}^{12-s-t}C_u$] s = 3, 4, 5; t = 3, 4, 5 $\neq s$; u = 3, 4, 5 $\neq s, t$
	Alternative method for question 7(a)		
	$\frac{12!}{5! \times 3! \times 4!}$	M1	12! ÷ by a product of three factorials.
		M1	$\frac{n!}{5! \times 3! \times 4!}$
	[792 × 35 =] 27 720	A1	CAO
		3	
(b)	4! (Lizo) × 6! (Kenny) × 2! (Martin) × 2! (Nantes)	M1	Product involving at least 3 of 4!, 6!, 2!, 2!
	× 3! (orders of K, M and N)	M1	w × 3! , w integer > 1.
	414 720	A1	WWW CAO
		3	
(c)	$^{7}C_4$ (adults) × $^{4}C_1 \times {}^{3}C_1$	M1	$^{7}C_4 \times b$, b integer > 1 no + or − .
	420	A1	
		2	
(d)	K not L $^{5}C_3 \times {}^{8}C_3$ = 560 L not K $^{5}C_3 \times {}^{8}C_3$ = 560 L and K $^{5}C_2 \times {}^{8}C_3$ = 560	M1	$^{8}C_3$(or $^{8}P_3$) × c for one of the products or $^{5}C_3$ (or $^{5}P_3$)× c, positive integer >1 for first 2 products only.
		M1	Add 2 or 3 correct scenarios only values, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified.
	[Total or Difference=] 1680	A1	
	Alternative method for question 7(d)		
	Total no of ways – neither L nor K Total = $^{7}C_4 \times {}^{8}C_3$ = 1960 Neither K nor L = $^{5}C_4 \times {}^{8}C_3$ =280	M1	$^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting the number of ways with neither from their total number of ways.
	[Total or Difference=] 1680	A1	
7(d)	**Alternative method for question 7(d)**		
	Subtracting K and L from sum of K and L K $^{6}C_3 \times {}^{8}C_3$ = 1120 L $^{6}C_3 \times {}^{8}C_3$ = 1120 L and K $^{5}C_2 \times {}^{8}C_3$= 560 1120 + 1120 – 560 = 1680	M1	$^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting number of ways with both from sum of number of ways with K and number of ways with L.
	[Total or Difference=] 1680	A1	
		3	

Question 78

(a)	$\left[\frac{9!}{2!2!} =\right]$ 90 720	B1	
		1	
(b)	**Method 1** Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!} - 6!$ [= 2520 – 720]	M1	$\frac{w!}{2!} - y$, w = 6, 7 y an integer. Condone $2 \times \left(\frac{w!}{2!}\right) - y$.
		M1	a – 6! or a – 720, a an integer resulting in a positive answer.
	1800	A1	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5! \times \frac{6 \times 5}{2!}$ =	M1	5! × b, b integer >1.
		M1	$c \times \left(\frac{6 \times 5}{2!} \text{ or } {}^{6}C_2 \text{ or } \frac{^{6}P_2}{2!} \text{ or } 15\right)$, c integer > 1.
	1800	A1	
		3	

(c)	CCO _ $^5C_1 = 5$ CC _ _ $^5C_2 = 10$ OOC _ $^5C_1 = 5$ OO _ _ $^5C_2 = 10$ C _ _ _ $^5C_3 = 10$ O _ _ _ $^5C_3 = 10$	**B1**	Correct outcome/value for 1 identified scenario. Accept unsimplified. WWW
		M1	Add 5 or 6 values of appropriate scenarios only, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified. Condone use of permutations.
	[Total =] 50	**A1**	
		3	
(d)	Both Os in group with a C $^5C_2 = 10$ Both Os in group without a C $^5C_2 \times {}^3C_2 = 30$ One O in a C group, one not $^5C_1 \times {}^4C_2 = 30$ One O with each C $({}^5C_1 \times {}^4C_1) \div 2! = 10$	**B1**	A correct scenario calculated accurately. Accept unsimplified.
		M1	Add 3 or 4 correct scenario values, no incorrect scenarios, accept repeated scenarios. Accept unsimplified.
	[Total =] 80	**A1**	
	Alternative method for question 6(d)		
	CCO O^^ ^^^ = $^5C_2 = 10$ CC^ O^^ O^^ = $^5C_1 \times {}^4C_2 = 30$ CC^ OO^ ^^^ = $^5C_1 \times {}^4C_1 = 20$	**B1**	A correct scenario calculated accurately. Accept unsimplified.
	Total ways of making three groups $\frac{{}^9C_6 \times {}^6C_3}{2 \times 2 \times 3} = 140$ 140 – (*their* 10+ *their* 30+ *their* 20)	**M1**	Total subtract 2 or 3 correct scenario values, no incorrect scenarios. Accept unsimplified.
	80	**A1**	
		3	

Question 79

(a)	$^{12}C_4 \times 2$	**M1**	$^gC_4 \times h$ $g = 12, 13$, $h = 1,2$
	990	**A1**	
	Alternative method for question 2(a)		
	[total – both on – neither on] $^{14}C_5 - ({}^{12}C_3 + {}^{12}C_5)$ = [2002 – 220 – 792]	**M1**	$^kC_5 - ({}^aC_3 + {}^aC_5)$ $a = 12$, 13 and $k = 13$, 14
	990	**A1**	
		2	
(b)	[Mrs Lan plus] 2W 2M $^7C_2 \times {}^6C_2$ = 315 3W 1M $^7C_3 \times {}^6C_1$ = 210 4W $^7C_4$ = 35	**M1**	$^7C_r \times {}^6C_{4-r}$ for $r = 2$, 3 or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing $^7C_r \times {}^6C_s$ $r + s = 4$ to imply r women and s men for both **B & M** marks only
	[Total =] 560	**A1**	
		4	

Question 80

(a)	5!	**M1**	$k!$ where $k = 5$, 6 or 7 Condone × 1 OE
	120	**A1**	
		2	
(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	**M1**	$\frac{8!}{a!b!}$, $a = 1,2$ $b = 1,3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	**M1**	$\frac{6!}{c!}$, $c = 1,2$ seen in an addition/subtraction
	[With 3Es not together] = 3360 – 360	**M1**	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1,2$ & $e = 1,3$
	3000	**A1**	
		4	

Question 81

(a)	$\frac{9!}{2!2!}$	M1	$\frac{h!}{2!\times j!}$, $h = 7, 8, 9; j = 1, 2$
	90720	A1	
		2	
(b)	**Arrangements with 5 letters between As + Arrangements with 6 letters between As + Arrangements with 7 letters between As**		
	With gap of 5: $\frac{7!}{2!}\times 3$ [= 7560] With gap of 6: $\frac{7!}{2!}\times 2$ [= 5040] With gap of 7: $\frac{7!}{2!}\times 1$ [= 2520]	M1	$\frac{7!}{2!}\times k$, k positive integer 1< k < 7
		M1	Add their no of ways for 3 identified correct scenarios, no additional incorrect scenarios, accept unsimplified.
	[Total no = $\frac{7!}{2!}\times 6$ =] 15120	A1	
		3	

(c)	**Method 1: Summing number of ways**		
	AT _ _ _ $2\times2\times{}^5C_3$ 40 A _ _ _ _ $2\times{}^5C_4$ 10 AATT _ ${}^5C_1$ 5 AAT _ _ $2\times{}^5C_2$ 20 AA _ _ _ ${}^5C_3$ 10 _ _ _ _ _ ${}^5C_5$ 1	B1	Correct no of ways for 4 correctly identified scenarios, accept unsimplified.
		M1	Add no of ways for 5 or 6 identified correct scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total no of ways not containing more Ts than As =] = 40+10+5+20+10+1 [=86]	A1	All correct and added
	Probability = $\frac{86}{{}^9C_5}$	M1	$\frac{\textit{their } 86}{9C5 \textit{ or their identified total}}$ accept numerator unevaluated
	$\frac{86}{126}, \frac{43}{63}$, 0.683	A1	
	Method 2: Subtracting no of ways with more Ts from total		
	T _ _ _ _ $2\times{}^5C_4$ 10 TTA _ _ $2\times{}^5C_2$ 20 TT _ _ _ ${}^5C_3$ 10	B1	Correct no of ways for 2 correctly identified scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations
		M1	Add no of ways for 2 or 3 correct scenarios and subtract from their total no of ways All correct and subtracted
	Total no of ways with more Ts than As =40 ${}^9C_5 - 40 = 86$	A1	
	Probability = $\frac{86}{{}^9C_5}$	M1	$\frac{\textit{their } 86}{9C5 \textit{ or their identified total}}$ accept numerator unevaluated

(c)	$\frac{43}{63}$, 0.683	A1	
		5	

Question 82

(a)	7!	M1	$\frac{7!}{b! \times c!}$ $b,c = 1,2$ $7! \times \frac{2!}{2!} \times \frac{2!}{2!}$ oe, no further terms present.
	5040	A1	
		2	

(b)	**Method 1 for first 3 marks: Arrangements of 6 letters including Ls between As**		
	$5! \times 5 \times 2$	M1	$5! \times d$, d integer > 1
		M1	$e! \times f \times g$, $e = 5, 6, 7$; $f = 1, 5$; $g = 1, 2$; $f \neq g$, 1 can be implicit.
	1200	A1	
	Method 2 for first 3 marks: Number of arrangements of LL^^^^^ – number of arrangements with the Ls split by an A		
	$6! \times 2 - 5! \times 2$	M1	$6! \times 2 - h$ h an integer $1 < h < 1440$
		M1	$k - 5! \times 2$ k an integer $k > 240$
	1200	A1	
	Method 3 for first 3 marks: Alternative approaches to Method 1		
	^A ^ ^ ^ ^ ^ A $\quad {}^5P_1 \times {}^1P_1 \times {}^5P_5 \times {}^1P_1 = 600$	M1	LL treated as a single unit.
		M1	
	1200	A1	

'(b)	**Final 2 marks of Question 7(b)**		
	[Total number of arrangements =] $\left[\frac{9!}{2!2!} = \right] 90\,720$	B1	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{1200}{90\,720}, \frac{5}{378}$, 0.0132	B1 FT	$\frac{their\ 1200}{their\ 90\,720}$ unsimplified B1 FT if *their* 1200 and *their* 90 720 supported by work in this part.
		5	

Question 83

(a)	5M0W ${}^8C_5$ [$\times {}^7C_0$] = 56 4M1W ${}^8C_4 \times {}^7C_1 = 490$ 3M2W ${}^8C_3 \times {}^7C_2 = 1176$	M1	${}^8C_x \times {}^7C_{5-x}$ for $x = 1, 2, 3, 4$, or 5
		B1	Outcome for 4M1W **or** 3M2W correct and identified, accept unsimplified.
		M1	Add 3 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified. Addition may be implied by final answer.
	[Total =] 1722	A1	Value stated WWW.
	Alternative method for Question 6(a)		
	2M3W ${}^8C_2 \times {}^7C_3 = 980$ 1M4W ${}^8C_1 \times {}^7C_4 = 280$ 0M5W ${}^8C_0 \times {}^7C_5 = 21$	M1	${}^8C_x \times {}^7C_{5-x}$ for $x = 1, 2, 3, 4$, or 5
		B1	Outcome for 2M3W **or** 1M4W correct and identified, accept unsimplified.
	[Total = ${}^{15}C_5$ – (980 + 280 + 21)] 3003 – (980 + 280 + 21)	M1	Subtract 3 values of appropriate scenarios from *their* identified total or correct, no incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total =] 1722	A1	Value stated WWW.
		4	
(b)	${}^{15}C_3 \times {}^{12}C_5$ [$\times {}^7C_7$] [= 455 × 792]	M1	${}^{15}C_r \times q$, $r = 3, 5, 7$; q a positive integer >1
		M1	${}^{15}C_s \times {}^{15-s}C_t$ [$\times {}^{15-s-t}C_u$] $s = 3,5,7$; $t = 3,5,7 \neq s$; $u = 3,5,7 \neq s,t$
	360360	A1	Final answer. If A0 awarded **SC B1** for final answer 360360.
		3	

(c)

Method 1: Total number of arrangements with AB together – Arrangements with AB and FG together		
$6! \times 2 - 5! \times 2 \times 2$ [= 1440 – 480]	M1	$a! \times 2! \times b$, $a = 5, 6$; $b = 1,2$ seen.
	M1	Either $6! \times 2 - c$, $1 < c < 1440$ or $d - 5! \times 2 \times 2$, $1440 < d$
960	A1	
Method 2: arrangements with AB together with F and G not together.		
$2 \times 4! \times 5 \times 4$	M1	$2 \times 4! \times e$, e positive integer >1
	M1	$f \times 5 \times 4$, f positive integer >1 condone $f \times 20$, $f \times {}^5C_2$, f positive integer >1
960	A1	
	3	

Question 84

(a)

Method 1: Arrangements with 3 Es together – arrangements with 3 Es together and 2 Ds together		
$\frac{7!}{2!} - 6!$	B1	$\frac{7!}{2!} - e$, e a positive integer (including 0).
	M1	$f - 6!$, $f > 6!$
	M1	$\frac{7!}{a!b!} - \frac{6!}{c!d!}$, $a,c = 1, 2$ and $b,d = 1, 3$.
1800	A1	
Method 2: Identified scenarios ^ EEE ^ ^ ^		
$5! \times \frac{6 \times 5}{2}$	B1	$5! \times j$, j a positive integer ($j = 1$ may be implied).
	M1	$\frac{k!}{m!} \times \frac{6 \times 5}{2}$, $\frac{k!}{m!} \times {}^6C_2$, $\frac{k!}{m!} \times \frac{{}^6P_2}{2}$ or $k! \times \frac{7 \times 6}{n}$, k a positive integer ($k = 1$ may be implied), $m = 1, 2$ $n = 1, 2, 3$.
	M1	$k! \times \frac{m \times (m-1)}{n}$ k a positive integer > 1, $m = 10, 9, 8, 7, 6$ and $n = 1, 2$.
1800	A1	
	4	

(b)

First 2 marks: Method 1 – Number of arrangements with 2 Ds in one position with 4 letters in between – repeats allowed		
$7! \times 4 \times 2$	M1	$7! \times s$, s = positive integer > 1.
	M1	$t! \times 4 \times 2$, $t = 8, 7, 6$. Condone $t! \times 8$.
First 2 marks: Method 2 – Picking 2Ds, arranging 4 letters from remaining letters between and then arranging terms		
${}^7P_4 \times 4! \times 2!$	M1	${}^7P_4 \times a! \times b!$, $1 \leqslant a \leqslant 6$ and $b = 1, 2, 3$.
	M1	${}^7P_c \times 4! \times 2!$, $c = 3, 4, 5$.
First 2 marks: Method 3 – Identified scenarios involving Es between Ds		
D ^ ^ ^ ^ D E E E = ${}^4C_4 \times 4! \times 4! \times 2! = 1152$ D E ^ ^ ^ D E E ^ = ${}^4C_3 \times 4! \times 4! \times 3 \times 2! = 13824$ D E E ^ ^ D E ^ ^ = ${}^4C_2 \times 4! \times 4! \times 3 \times 2! = 20736$ D E E E ^ D ^ ^ ^ = ${}^4C_1 \times 4! \times 4! \times 2! = 4608$	M1	1 identified scenario value correct.
	M1	4 appropriate scenarios added, no incorrect.

'(b)	**Final 3 marks for Methods 1, 2 and 3**		
	40320	**A1**	If A0 scored, **SC B1** for 40320 WWW.
	[Total number of arrangements =] $[9!=]$ 362880	**B1**	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{40320}{362880}=\frac{1}{9}$	**B1FT**	$\frac{\textit{their}\ 40320}{\textit{their}\ 362880}$, accept unsimplified. B1FT if *their* 40320 and *their* 362880 supported by work in this part. Condone *their* 362880 supported by calculation in **7(a)**.
		5	
'(c)	Scenarios D E _ _ _ $^4C_3$ 4 D E E _ _ $^4C_2$ 6 D E E E _ $^4C_1$ 4 D D E _ _ $^4C_2$ 6 D D E E _ $^4C_1$ 4 D D E E E [$^4C_0$] 1	**B1**	1 correct unsimplified outcome/value for one identified scenario excluding DDEEE. Note: $^4C_1$ cannot be used for $^4C_3$.
		M1	Add values of 6 appropriate scenarios, no additional, incorrect or repeated scenarios. Accept unsimplified.
	[Total =] 25	**A1**	
		3	

Question 85

(a)	**Method 1: Total number of arrangements – number of arrangements with Cs together**		
	$\frac{10!}{2!4!}-\frac{9!}{4!}$ [75600-15120]	**M1**	$\frac{10!}{a!b!}-c$, $a\neq b$, a = 1, 2, b = 1, 4, with c being a positive integer.
		M1	$d-\frac{e!}{4!}$, e = 8, 9, 10, with d being a positive integer.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
	Method 2: Arrangements ^ ^ C ^ C ^ ^ ^ ^ ^		
	$\frac{8!}{4!}\times\frac{9\times 8}{2}$	**M1**	$\frac{8!}{4!}\times f$ seen, with f being a positive integer.
		M1	$g\times\frac{9\times 8}{h}$, with g being a positive integer, h = 1, 2. $g\times {}^9C_2$ and $g\times {}^9P_2$ are acceptable.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
		3	
(b)	AC^^^C^^^A $\frac{6!}{2!}\times 4$	**M1**	$\frac{6!}{2!}\times s$, with s being a positive integer.
		M1	$\frac{t!}{r!}\times 4$, r = 1, 2, 3 and t = 8, 7, 6.
	1440	**A1**	
	Alternative Method for Question 7(b)		
	$\frac{4\times {}^6P_3\times 3!}{2!}$	**M1**	$\frac{{}^6P_3}{2!}\times k$, with k being a positive integer.
		M1	$4\times 3!\times\frac{{}^6P_m}{n!}$, m = 2, 3 and n = 1, 2, 3.
	1440	**A1**	
		3	

(c)	Scenarios AA _ _ _ $^5C_3 = 10$ AAA _ _ $^5C_2 = 10$ AAAA _ $^5C_1 = 5$	**B1**	Correct number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www.
		M1	Add 3 values for 2, 3 and 4 As, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
	Alternative Method 2 for Question 7(c)		
	Scenarios: AAC _ _ $^4C_2 = 6$ AA _ _ _ $^4C_3 = 4$ AAAC _ $^4C_1 = 4$ AAA _ _ $^4C_2 = 6$ AAAAC 1 AAAA _ 4	**B1**	Correct total number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www (e.g., both values for AAC^^ and AA^^^ shown would be fine for 2As).
		M1	Add 6 values of appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
		3	

Question 86

(a)	S + 4C + 2R $^6C_1 \times {}^8C_4 \times {}^{11}C_2$ [$= 6 \times 70 \times 55$] = 23 100 S + 5C + 1R $^6C_1 \times {}^8C_5 \times {}^{11}C_1$ [$= 6 \times 56 \times 11$] = 3696 S + 6C [+ 0R] $^6C_1 \times {}^8C_6$ [$\times {}^{11}C_0$][$= 6 \times 28$] = 168	**M1**	$^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ seen.
		B1	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 3 correct scenarios. No incorrect scenarios, no repeated scenarios. Condone $^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ to identify S, C, R.
	[Total =] 26964	**A1**	cao
		4	
(b)	$2! \times 3! \times 4! \times 6$	**M1**	$2! \times 3! \times 4! \times k$, k an integer > 0. 1 can be implied.
	=1728	**A1**	If A0 scored **SC B1** for 1728 www.
		2	
(c)	**Method 1**		
	$6! \times 7 \times 6 \times 5$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 7 \times n \times r$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant n, r \leqslant 6$, $n \neq r$.
		M1	$\frac{m!}{a! \times b!} \times 7 \times 6 \times 5$; $6 \leqslant m \leqslant 9$; $a =1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 2		
	$6! \times {}^7P_3$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_q$, or $\frac{m!}{a! \times b!} \times {}^7C_q \times q!$; $6 \leq m \leq 9$; $a =1, 2$; $b = 1, 4$; $1 \leq q \leq 6$.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_3$, or $\frac{m!}{a! \times b!} \times {}^7C_3 \times 3!$; $6 \leq m \leq 9$; $a =1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.

(c)	**Method 3**		
	$6! \times 35 \times 3!$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 35 \times q!$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant q \leqslant 3$.
		M1	$\frac{m!}{a! \times b!} \times 35 \times 6$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 4		
	$9! - 7!3! - {}^3P_2 \times 6! \times 7 \times 6$ Or $9! - 7!3! - 3! \times 7! \times 6$ [= 362 880 – 30 240 – 181 440]	**M1**	$9! - 7!r! - q$, r an integer > 1, q an integer $\leqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - q$; $s = 8, 9$; $a = 1, 2$; $b = 1, 3$; $c = 1, 4$; q an integer $\geqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - {}^3P_2 \times 6! \times 6 \times 7$, $6 \leqslant s \leqslant 9$, **or** $\frac{s!}{a! \times b! \times c!} - 7!3! - 3! \times 7! \times 6$, $6 \leqslant s \leqslant 9$. $a = 1, 2$ $b = 1, 3$ $c = 1, 4$. 1 may be implied.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
		4	

Question 87

(a)	${}^6C_3 \times {}^8C_3$	**M1**	${}^6C_3 \times b$ or $c \times {}^8C_3$ seen. b, c integers $\geqslant 1$ (1 may be implied).
	1120	**A1**	
		2	
(b)	**Method 1**		
	0 brothers [${}^3C_0$] $\times {}^{11}C_6$ 462 1 brother ${}^3C_1 \times {}^{11}C_5$ 1386 2 brothers ${}^3C_2 \times {}^{11}C_4$ 990	**B1**	${}^3C_x \times {}^{11}C_{6-x}$, with $x = 1$ or 2 seen.
		M1	Add values of 3 correct scenarios, (may be identified by the appropriate calculations) no incorrect/repeated scenarios, condone use of permutations.
	2838	**A1**	Only dependent on the M mark. **SC B1** for the correct calculation or 2838 seen WWW.
	Method 2		
	${}^{14}C_6 - {}^{11}C_3$ 3003 – 165	**B1**	${}^{14}C_6 - d$, where d a positive integer.
		M1	$e - {}^{11}C_3$, where e is a positive integer >165.
	= 2838	**A1**	
		3	

Question 88

(a)	$\left[\frac{8!}{2!3!} =\right]$ 3360	**B1**	
		1	
(b)	$\frac{6!}{2!2!}$	**M1**	$\frac{6!}{2!f!}$; $f = 1, 2, 3$.
	180	**A1**	
		2	
(c)	$\left[P(OOO \mid CC) = \frac{P(OOO \cap CC)}{P(CC)} = \right]$ $\frac{5!}{\frac{7!}{3!}}$	**M1**	$\frac{5!}{g}$ g a positive integer, $g \neq 3360$, 1. Condone numerator of $\frac{5!}{3360g}$.
		M1	$\frac{h}{\frac{7!}{3!}}$ or $\frac{h}{\frac{8!}{3!}}$, where h is a positive integer. Condone division by 3360 in denominator.
	$= \frac{120}{840}, \frac{1}{7}, 0.143$	**A1**	0.1428571… to at least 3SF. If M0 scored **SC B1** for $\frac{1}{7}$ WWW.
		3	