AS-Level

Probability

May : 2013- May : 2023

Questions

Question 1

(a) John plays two games of squash. The probability that he wins his first game is 0.3. If he wins his first game, the probability that he wins his second game is 0.6. If he loses his first game, the probability that he wins his second game is 0.15. Given that he wins his second game, find the probability that he won his first game. [4]

(b) Jack has a pack of 15 cards. 10 cards have a picture of a robot on them and 5 cards have a picture of an aeroplane on them. Emma has a pack of cards. 7 cards have a picture of a robot on them and $x - 3$ cards have a picture of an aeroplane on them. One card is taken at random from Jack's pack and one card is taken at random from Emma's pack. The probability that both cards have pictures of robots on them is $\frac{7}{18}$. Write down an equation in terms of x and hence find the value of x. [4]

Question 2

The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

Question 3

Q is the event 'Nicola throws two fair dice and gets a total of 5'. S is the event 'Nicola throws two fair dice and gets one low score (1, 2 or 3) and one high score (4, 5 or 6)'. Are events Q and S independent? Justify your answer. [4]

Question 4

Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

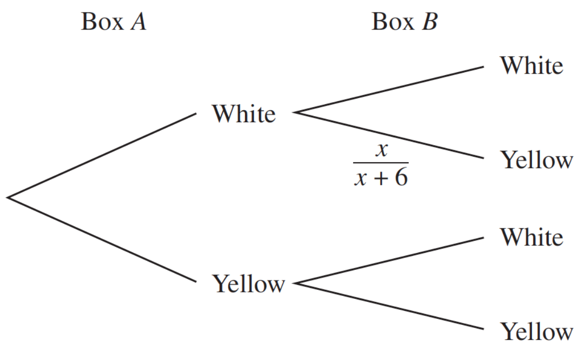
(i) Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag. [2]

(ii) Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate. [4]

(iii) The random variable X is the number of times a chocolate is taken. State the possible values of X and draw up a table to show the probability distribution of X. [5]

Question 5

Box A contains 8 white balls and 2 yellow balls. Box B contains 5 white balls and x yellow balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.



(i) Justify the probability $\frac{x}{x+6}$ on the tree diagram. [1]

(ii) Copy and complete the tree diagram. [4]

(iii) If the ball chosen from box A is white then the probability that the ball chosen from box B is also white is $\frac{1}{3}$. Show that the value of x is 12. [2]

(iv) Given that the ball chosen from box B is yellow, find the conditional probability that the ball chosen from box A was yellow. [4]

Question 6

Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

(i) Find the probability that the two digits chosen are equal. [3]

(ii) Find the probability that one digit is a 5 and one digit is not a 5. [3]

(iii) Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

(iv) The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

Question 7

Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) $= \frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event R is 'the sum of the numbers on the three cards is 11'. Event S is 'the number on the first card taken is a 3'.

(iii) Determine whether events R and S are independent. Justify your answer. [3]

(iv) Determine whether events R and S are exclusive. Justify your answer. [1]

Question 8

On Saturday afternoons Mohit goes shopping with probability 0.25, or goes to the cinema with probability 0.35 or stays at home. If he goes shopping the probability that he spends more than \$50 is 0.7. If he goes to the cinema the probability that he spends more than \$50 is 0.8. If he stays at home he spends \$10 on a pizza.

(i) Find the probability that Mohit will go to the cinema and spend less than \$50. [1]

(ii) Given that he spends less than \$50, find the probability that he went to the cinema. [4]

Question 9

James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable X is defined as follows.

- If the coin shows a **head** then X is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then X is the score on the **first throw** of the die only.

(i) Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

(ii) Show that $P(X = 3) = \frac{3}{16}$. [4]

(iii) Copy and complete the probability distribution table for X. [3]

x	1	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{8}$		$\frac{1}{16}$	$\frac{1}{32}$

Event Q is 'James throws a tail'. Event R is 'the value of X is 7'.

(iv) Determine whether events Q and R are exclusive. Justify your answer. [2]

Question 10

The people living in two towns, Mumbok and Bagville, are classified by age. The numbers in thousands living in each town are shown in the table below.

	Mumbok	Bagville
Under 18 years	15	35
18 to 60 years	55	95
Over 60 years	20	30

One of the towns is chosen. The probability of choosing Mumbok is 0.6 and the probability of choosing Bagville is 0.4. Then a person is chosen at random from that town. Given that the person chosen is between 18 and 60 years old, find the probability that the town chosen was Mumbok. [5]

Question 11

Tom and Ben play a game repeatedly. The probability that Tom wins any game is 0.3. Each game is won by either Tom or Ben. Tom and Ben stop playing when one of them (to be called the champion) has won two games.

(i) Find the probability that Ben becomes the champion after playing exactly 2 games. [1]

(ii) Find the probability that Ben becomes the champion. [3]

(iii) Given that Tom becomes the champion, find the probability that he won the 2nd game. [4]

Question 12

Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.6. For sets after the first, the probability that Roger wins the set is 0.7 if he won the previous set, and is 0.25 if he lost the previous set. No set is drawn.

(i) Find the probability that there is a winner of the match after exactly two sets. [3]

(ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets. [2]

Question 13

Playground equipment consists of swings (S), roundabouts (R), climbing frames (C) and play-houses (P). The numbers of pieces of equipment in each of 3 playgrounds are as follows.

Playground X	Playground Y	Playground Z
$3S$, $2R$, $4P$	$6S$, $3R$, $1C$, $2P$	$8S$, $3R$, $4C$, $1P$

Each day Nur takes her child to one of the playgrounds. The probability that she chooses playground X is $\frac{1}{4}$. The probability that she chooses playground Y is $\frac{1}{4}$. The probability that she chooses playground Z is $\frac{1}{2}$. When she arrives at the playground, she chooses one piece of equipment at random.

(i) Find the probability that Nur chooses a play-house. [4]

(ii) Given that Nur chooses a climbing frame, find the probability that she chose playground Y. [4]

Question 14

A box contains 2 green apples and 2 red apples. Apples are taken from the box, one at a time, without replacement. When both red apples have been taken, the process stops. The random variable X is the number of apples which have been taken when the process stops.

(i) Show that $P(X = 3) = \frac{1}{3}$. [3]

(ii) Draw up the probability distribution table for X. [3]

Another box contains 2 yellow peppers and 5 orange peppers. Three peppers are taken at random from the box without replacement.

(iii) Given that at least 2 of the peppers taken from the box are orange, find the probability that all 3 peppers are orange. [5]

Question 15

Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find $\mathrm{E}(X)$. [4]

Question 16

Jodie tosses a biased coin and throws two fair tetrahedral dice. The probability that the coin shows a head is $\frac{1}{3}$. Each of the dice has four faces, numbered 1, 2, 3 and 4. Jodie's score is calculated from the numbers on the faces that the dice land on, as follows:

- if the coin shows a head, the two numbers from the dice are added together;
- if the coin shows a tail, the two numbers from the dice are multiplied together.

Find the probability that the coin shows a head given that Jodie's score is 8. [5]

Question 17

When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

Question 18

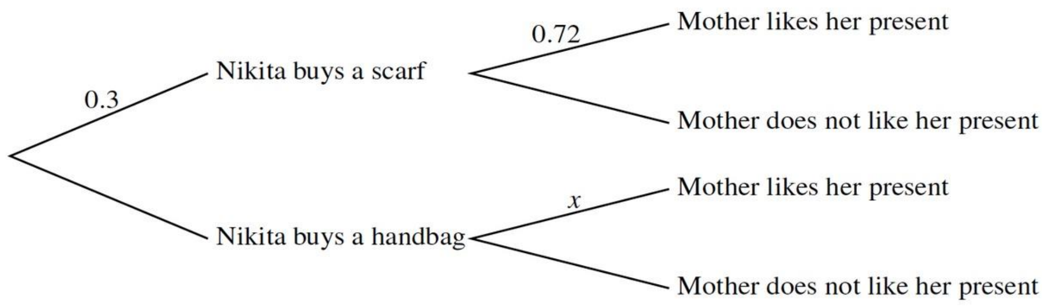
A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

Question 19

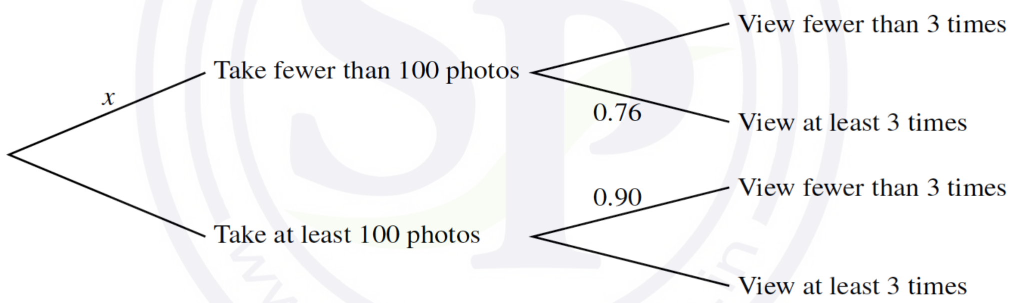


Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x. This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x. [3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf. [4]

Question 20



A survey is undertaken to investigate how many photos people take on a one-week holiday and also how many times they view past photos. For a randomly chosen person, the probability of taking fewer than 100 photos is x. The probability that these people view past photos at least 3 times is 0.76. For those who take at least 100 photos, the probability that they view past photos fewer than 3 times is 0.90. This information is shown in the tree diagram. The probability that a randomly chosen person views past photos fewer than 3 times is 0.801.

(i) Find x. [3]

(ii) Given that a person views past photos at least 3 times, find the probability that this person takes at least 100 photos. [4]

Question 21

Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

(i) Determine whether events A and B are independent, showing your working. [5]

(ii) Are events A and B mutually exclusive? Justify your answer. [1]

Question 22

Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is 'the sum of the two numbers is 4'. Event T is 'the product of the two numbers is an odd number'.

(i) Determine whether events S and T are independent, showing your working. [5]

(ii) Are events S and T exclusive? Justify your answer. [1]

Question 23

In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

(i) Find the probability that the person chosen is from country X. [1]

(ii) Find the probability that the person chosen has fair hair. [2]

(iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]

Question 24

One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

Question 25

In a certain town, 35% of the people take a holiday abroad and 65% take a holiday in their own country. Of those going abroad 80% go to the seaside, 15% go camping and 5% take a city break. Of those taking a holiday in their own country, 20% go to the seaside and the rest are divided equally between camping and a city break.

(i) A person is chosen at random. Given that the person chosen goes camping, find the probability that the person goes abroad. [5]

(ii) A group of n people is chosen randomly. The probability of all the people in the group taking a holiday in their own country is less than 0.002. Find the smallest possible value of n. [3]

Question 26

A fair eight-sided die has faces marked 1, 2, 3, 4, 5, 6, 7, 8. The score when the die is thrown is the number on the face the die lands on. The die is thrown twice.

- Event R is 'one of the scores is exactly 3 greater than the other score'.
- Event S is 'the product of the scores is more than 19'.

(i) Find the probability of R. [2]

(ii) Find the probability of S. [2]

(iii) Determine whether events R and S are independent. Justify your answer. [3]

Question 27

In a group of 30 adults, 25 are right-handed and 8 wear spectacles. The number who are right-handed and do not wear spectacles is 19.

(i) Copy and complete the following table to show the number of adults in each category. [2]

	Wears spectacles	Does not wear spectacles	Total
Right-handed			
Not right-handed			
Total			30

An adult is chosen at random from the group. Event X is 'the adult chosen is right-handed'; event Y is 'the adult chosen wears spectacles'.

(ii) Determine whether X and Y are independent events, justifying your answer. [3]

Question 28

Ayman's breakfast drink is tea, coffee or hot chocolate with probabilities 0.65, 0.28, 0.07 respectively. When he drinks tea, the probability that he has milk in it is 0.8. When he drinks coffee, the probability that he has milk in it is 0.5. When he drinks hot chocolate he always has milk in it.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Find the probability that Ayman's breakfast drink is coffee, given that his drink has milk in it. [3]

Question 29

The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

Question 30

For a group of 250 cars the numbers, classified by colour and country of manufacture, are shown in the table.

	Germany	Japan	Korea
Silver	40	26	34
White	32	22	26
Red	28	12	30

One car is selected at random from this group. Find the probability that the selected car is

(i) a red or silver car manufactured in Korea, [1]

(ii) not manufactured in Japan. [1]

X is the event that the selected car is white. Y is the event that the selected car is manufactured in Germany.

(iii) By using appropriate probabilities, determine whether events X and Y are independent. [5]

Question 31

When Anya goes to school, the probability that she walks is 0.3 and the probability that she cycles is 0.65; if she does not walk or cycle she takes the bus. When Anya walks the probability that she is late is 0.15. When she cycles the probability that she is late is 0.1 and when she takes the bus the probability that she is late is 0.6. Given that Anya is late, find the probability that she cycles. [5]

Question 32

Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

(i) Find the probability that, when Deeti carries out operation T, she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T.

(ii) Find $P(X = 1)$. [3]

(iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

Question 33

Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(i) Show that $P(X = 2) = \frac{2}{15}$. [2]

(ii) Draw up the probability distribution table for X. [4]

(iii) Given that $X = 3$, find the probability that the card chosen from pack A is a 1. [3]

Question 34

A bag contains 10 pink balloons, 9 yellow balloons, 12 green balloons and 9 white balloons. 7 balloons are selected at random without replacement. Find the probability that exactly 3 of them are green. [3]

Question 35

A shop sells two makes of coffee, Café Premium and Café Standard. Both coffees come in two sizes, large jars and small jars. Of the jars on sale, 65% are Café Premium and 35% are Café Standard. Of the Café Premium, 40% of the jars are large and of the Café Standard, 25% of the jars are large. A jar is chosen at random.

(i) Find the probability that the jar is small. [2]

(ii) Find the probability that the jar is Café Standard given that it is large. [3]

Question 36

A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

Question 37

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

(i) Find the probability that Khalid hurts himself on any particular day. [2]

(ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]

(iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]

(iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

Question 38

Two identical biased triangular spinners with sides marked 1, 2 and 3 are spun. For each spinner, the probabilities of landing on the sides marked 1, 2 and 3 are p, q and r respectively. The score is the sum of the numbers on the sides on which the spinners land. You are given that P(score is 6) = $\frac{1}{36}$ and P(score is 5) = $\frac{1}{9}$. Find the values of p, q and r. [6]

Question 39

Rani and Diksha go shopping for clothes.

(i) Rani buys 4 identical vests, 3 identical sweaters and 1 coat. Each vest costs \$5.50 and the coat costs \$90. The mean cost of Rani's 8 items is \$29. Find the cost of a sweater. [3]

(ii) Diksha buys 1 hat and 4 identical shirts. The mean cost of Diksha's 5 items is \$26 and the standard deviation is \$0. Explain how you can tell that Diksha spends \$104 on shirts. [2]

Question 40

Redbury United soccer team play a match every week. Each match can be won, drawn or lost. At the beginning of the soccer season the probability that Redbury United win their first match is $\frac{3}{5}$, with equal probabilities of losing or drawing. If they win the first match, the probability that they win the second match is $\frac{7}{10}$ and the probability that they lose the second match is $\frac{1}{10}$. If they draw the first match they are equally likely to win, draw or lose the second match. If they lose the first match, the probability that they win the second match is $\frac{3}{10}$ and the probability that they draw the second match is $\frac{1}{20}$.

(i) Draw a fully labelled tree diagram to represent the first two matches played by Redbury United in the soccer season. [2]

(ii) Given that Redbury United win the second match, find the probability that they lose the first match. [4]

Question 41

Ashfaq throws two fair dice and notes the numbers obtained. R is the event 'The product of the two numbers is 12'. T is the event 'One of the numbers is odd and one of the numbers is even'. By finding appropriate probabilities, determine whether events R and T are independent. [5]

Question 42

At the end of a revision course in mathematics, students have to pass a test to gain a certificate. The probability of any student passing the test at the first attempt is 0.85. Those students who fail are allowed to retake the test once, and the probability of any student passing the retake test is 0.65.

(i) Draw a fully labelled tree diagram to show all the outcomes. [2]

(ii) Given that a student gains the certificate, find the probability that this student fails the test on the first attempt. [4]

Question 43

A fair tetrahedral die has faces numbered 1, 2, 3, 4. A coin is biased so that the probability of showing a head when thrown is $\frac{1}{3}$. The die is thrown once and the number n that it lands on is noted. The biased coin is then thrown n times. So, for example, if the die lands on 3, the coin is thrown 3 times.

(i) Find the probability that the die lands on 4 and the number of times the coin shows heads is 2. [3]

(ii) Find the probability that the die lands on 3 and the number of times the coin shows heads is 3. [1]

(iii) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. [3]

Question 44

Over a period of time Julian finds that on long-distance flights he flies economy class on 82% of flights. On the rest of the flights he flies first class. When he flies economy class, the probability that he gets a good night's sleep is x. When he flies first class, the probability that he gets a good night's sleep is 0.9.

(i) Draw a fully labelled tree diagram to illustrate this situation. [2]

The probability that Julian gets a good night's sleep on a randomly chosen flight is 0.285.

(ii) Find the value of x. [2]

(iii) Given that on a particular flight Julian does not get a good night's sleep, find the probability that he is flying economy class. [3]

Question 45

Last Saturday, Sarah recorded the colour and type of 160 cars in a car park. All the cars that were not red or silver in colour were grouped together as 'other'. Her results are shown in the following table.

		Type of car		
		Saloon	Hatchback	Estate
Colour of car	Red	20	40	12
	Silver	14	26	10
	Other	6	24	8

(i) Find the probability that a randomly chosen car in the car park is a silver estate car. [1]

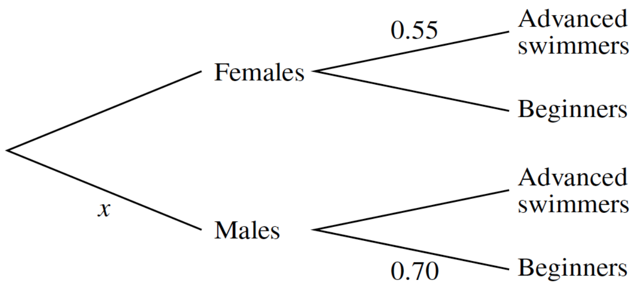
(ii) Find the probability that a randomly chosen car in the car park is a hatchback car. [1]

(iii) Find the probability that a randomly chosen car in the car park is red, given that it is a hatchback car. [2]

(iv) One of the cars in the car park is chosen at random. Determine whether the events 'the car is a hatchback car' and 'the car is red' are independent, justifying your answer. [2]

Question 46

The members of a swimming club are classified either as 'Advanced swimmers' or 'Beginners'. The proportion of members who are male is x, and the proportion of males who are Beginners is 0.7. The proportion of females who are Advanced swimmers is 0.55. This information is shown in the tree diagram.



For a randomly chosen member, the probability of being an Advanced swimmer is the same as the probability of being a Beginner.

(i) Find x. [3]

Question 47

In a group of students, $\frac{3}{4}$ are male. The proportion of male students who like their curry hot is $\frac{3}{5}$ and the proportion of female students who like their curry hot is $\frac{4}{5}$. One student is chosen at random.

(i) Find the probability that the student chosen is either female, or likes their curry hot, or is both female and likes their curry hot. [4]

(ii) Showing your working, determine whether the events 'the student chosen is male' and 'the student chosen likes their curry hot' are independent. [2]

Question 48

Vehicles approaching a certain road junction from town A can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town A, 55% turn left, 15% turn right and 30% go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.

(i) Find the probability that, of the next three vehicles approaching the junction from town A, one goes straight on and the other two either both turn left or both turn right. [4]

(ii) Three vehicles approach the junction from town A. Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on. [4]

Question 49

Andy has 4 red socks and 8 black socks in his drawer. He takes 2 socks at random from his drawer.

(i) Find the probability that the socks taken are of different colours. [2]

The random variable X is the number of red socks taken.

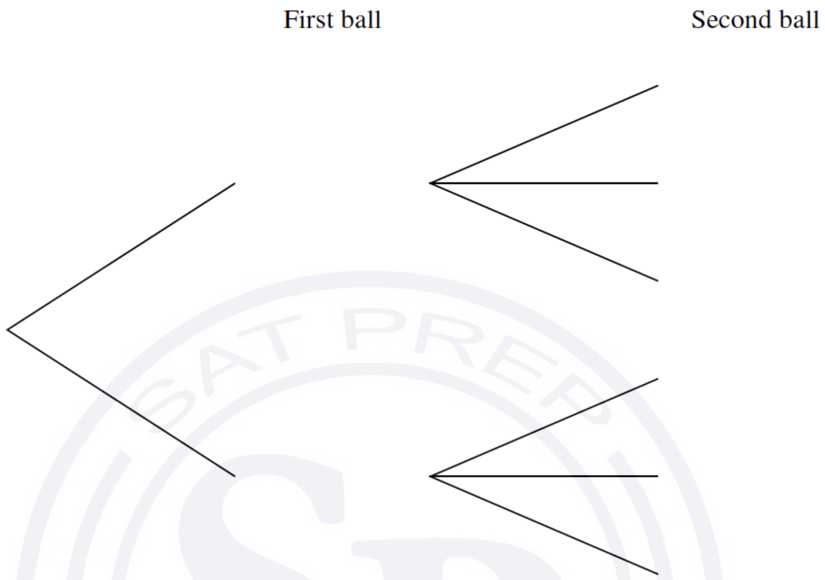
(ii) Draw up the probability distribution table for X. [3]

(iii) Find $E(X)$. [1]

Question 50

A box contains 3 red balls and 5 blue balls. One ball is taken at random from the box and not replaced. A yellow ball is then put into the box. A second ball is now taken at random from the box.

(i) Complete the tree diagram to show all the outcomes and the probability for each branch. [2]



(ii) Find the probability that the two balls taken are the same colour. [2]

(iii) Find the probability that the first ball taken is red, given that the second ball taken is blue. [3]

Question 51

Two letters are chosen at random from the 11 letters in the word MISSISSIPPI. Find the probability that these two letters are the same. [3]

Question 52

In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

	Art	Music	Drama
Boys	24	40	32
Girls	15	12	37

(i) Find the probability that a randomly chosen student is studying Music. [1]

(ii) Determine whether the events 'a randomly chosen student is a boy' and 'a randomly chosen student is studying Music' are independent, justifying your answer. [2]

(iii) Find the probability that a randomly chosen student is not studying Drama, given that the student is a girl. [2]

(iv) Three students are chosen at random. Find the probability that exactly 1 is studying Music and exactly 2 are boys. [5]

Question 53

On each day that Tamar goes to work, he wears either a blue suit with probability 0.6 or a grey suit with probability 0.4. If he wears a blue suit then the probability that he wears red socks is 0.2. If he wears a grey suit then the probability that he wears red socks is 0.32.

(i) Find the probability that Tamar wears red socks on any particular day that he is at work. [2]

(ii) Given that Tamar is not wearing red socks at work, find the probability that he is wearing a grey suit. [3]

Question 54

Megan sends messages to her friends in one of 3 different ways: text, email or social media. For each message, the probability that she uses text is 0.3 and the probability that she uses email is 0.2. She receives an immediate reply from a text message with probability 0.4, from an email with probability 0.15 and from social media with probability 0.6.

(i) Draw a fully labelled tree diagram to represent this information. [2]

(ii) Given that Megan does not receive an immediate reply to a message, find the probability that the message was an email. [4]

Question 55

Two ordinary fair dice are thrown and the numbers obtained are noted. Event S is 'The sum of the numbers is even'. Event T is 'The sum of the numbers is either less than 6 or a multiple of 4 or both'. Showing your working, determine whether the events S and T are independent. [4]

Question 56

Maryam has 7 sweets in a tin; 6 are toffees and 1 is a chocolate. She chooses one sweet at random and takes it out. Her friend adds 3 chocolates to the tin. Then Maryam takes another sweet at random out of the tin.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Draw up the probability distribution table for the number of toffees taken. [3]

(iii) Find the mean number of toffees taken. [1]

(iv) Find the probability that the first sweet taken is a chocolate, given that the second sweet taken is a toffee. [4]

Question 57

Jameel has 5 plums and 3 apricots in a box. Rosa has x plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in x and hence find x. [3]

Question 58

A fair six-sided die is thrown twice and the scores are noted. Event X is defined as 'The total of the two scores is 4'. Event Y is defined as 'The first score is 2 or 5'. Are events X and Y independent? Justify your answer. [4]

Question 59

There are 300 students at a music college. All students play exactly one of the guitar, the piano or the flute. The numbers of male and female students that play each of the instruments are given in the following table.

	Guitar	Piano	Flute
Female students	62	35	43
Male students	78	40	42

(i) Find the probability that a randomly chosen student at the college is a male who does not play the piano. [1]

(ii) Determine whether the events 'a randomly chosen student is male' and 'a randomly chosen student does not play the piano' are independent, justifying your answer. [2]

Question 60

There are 300 students at a music college. All students play exactly one of the guitar, the piano or the flute. The numbers of male and female students that play each of the instruments are given in the following table.

	Guitar	Piano	Flute
Female students	62	35	43
Male students	78	40	42

(i) Find the probability that a randomly chosen student at the college is a male who does not play the piano. [1]

(ii) Determine whether the events 'a randomly chosen student is male' and 'a randomly chosen student does not play the piano' are independent, justifying your answer. [2]

Question 61

Benju cycles to work each morning and he has two possible routes. He chooses the hilly route with probability 0.4 and the busy route with probability 0.6. If he chooses the hilly route, the probability that he will be late for work is x and if he chooses the busy route the probability that he will be late for work is $2x$. The probability that Benju is late for work on any day is 0.36.

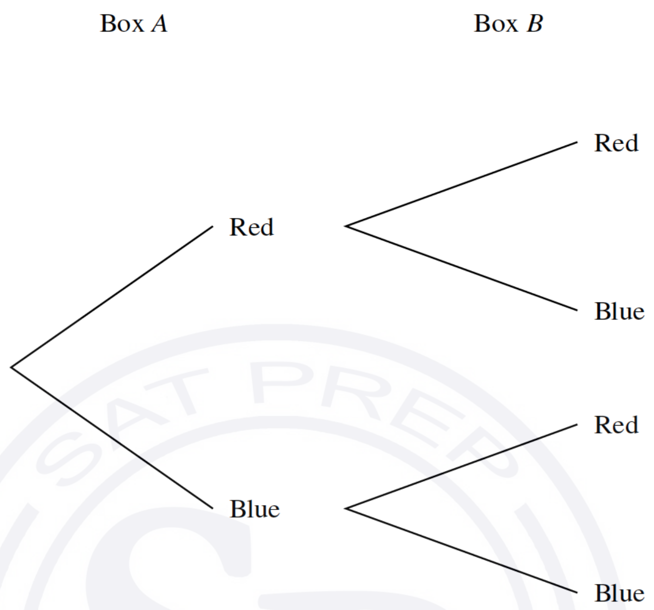
(i) Show that $x = 0.225$. [2]

(ii) Given that Benju is not late for work, find the probability that he chooses the hilly route. [3]

Question 62

Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



(b) Find the probability that the two balls chosen are not the same colour. [2]

(c) Find the probability that the ball chosen from box A is blue given that the ball chosen from box B is blue. [4]

Question 63

Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Juan goes to college by car given that he arrives early. [4]

Question 64

A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

Question 65

On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with probabilities 0.35, 0.44, 0.21 respectively. When she cooks a pizza, Rani has some fruit with probability 0.3. When she cooks a burger, she has some fruit with probability 0.8. When she cooks a curry, she never has any fruit.

(a) Draw a fully labelled tree diagram to represent this information. [2]

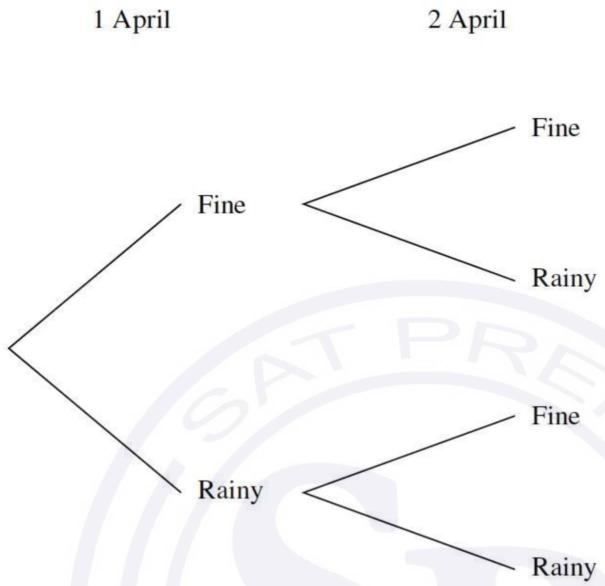
(b) Find the probability that Rani has some fruit. [2]

(c) Find the probability that Rani does not have a burger given that she does not have any fruit. [4]

Question 66

In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

Question 67

Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $P(A \cap B)$. [2]

(b) Hence determine whether or not the events A and B are independent. [2]

Question 68

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

Question 69

Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears a blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3.

(a) Find the probability that on a randomly chosen day Georgie wears a hat. [2]

(b) Find the probability that on a randomly chosen day Georgie wears a yellow scarf given that she does not wear a hat. [3]

Question 70

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

Question 71

On each day that Alexa goes to work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that she arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the probability that she arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. [3]

(b) Find the probability that Alexa travels to work by train given that she arrives late. [3]

Question 72

To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If a student fails both attempts at the written test, then they cannot attempt the practical test.

The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. [3]

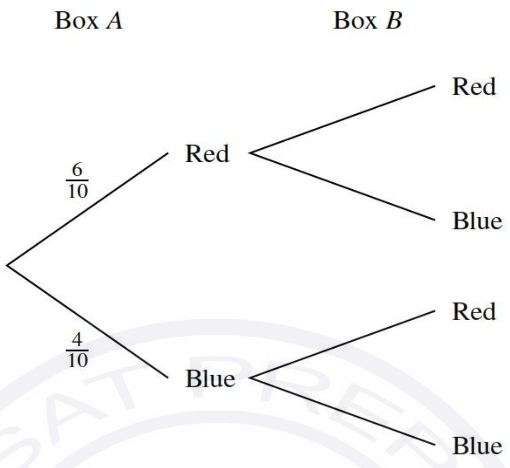
(b) Find the probability that a randomly chosen student will succeed in gaining a place at the college. [2]

(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [2]

Question 73

Box A contains 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram below, giving the remaining four probabilities in terms of x. [3]



(b) Show that the probability that both balls chosen are blue is $\frac{4}{x+10}$. [2]

It is given that the probability that both balls chosen are blue is $\frac{1}{6}$.

(c) Find the probability, correct to 3 significant figures, that the ball chosen from box A is red given that the ball chosen from box B is red. [5]

Question 74

Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table.

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar. [1]

(b) Find the probability that the student is male given that the student plays the drums. [2]

(c) Determine whether the events ‘the student plays the guitar’ and ‘the student is female’ are independent, justifying your answer. [2]

Question 75

For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]

Question 76

A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

Question 77

In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

Question 78

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

Question 79

Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

Question 80

Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

Question 81

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

(b) Find the probability that Janice finishes the game. [3]

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

Question 82

Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]

(b) Find the probability that Tom wins the game on his second turn. [4]

(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

Question 83

On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

Question 84

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

(b) Find $P(A \mid B)$. [2]

Question 85

A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2, 3, 4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two throws.

(a) Draw a fully labelled tree diagram to represent this information. [2]

Events A and B are defined as follows.

A: the player's score is 5, 6, 7, 8 or 9

B: the player has two throws

(b) Show that $P(A) = \frac{1}{3}$. [3]

(c) Determine whether or not events A and B are independent. [2]

(d) Find $P(B \mid A')$. [3]

Question 86

The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining, the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day it is not raining and Aran is not wearing a hat or a scarf is 0.36.

Find the value of x. [3]

Question 87

Marco has four boxes labelled K, L, M and N. He places them in a straight line in the order K, L, M, N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A: The white marble is in either box L or box M.

B: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent. [3]

Question 88

Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows.

A: The sum of the two numbers is less than 6.

B: The difference between the two numbers is at most 2.

(a) Determine whether or not the events A and B are independent. [4]

(b) Find $P(B \mid A')$. [3]