SATPREP
Worksheet – Vector Equations and Planes

1 For A$(-1, 2, 3)$, B$(2, 0, -1)$ and C$(-3, 2, -4)$ find:
- **a** the equation of the plane defined by A, B and C
- **b** the measure of angle CAB
- **c** r, given that D$(r, 1, -r)$ is a point such that angle BDC is a right angle.

2
- **a** Find where the line through L$(1, 0, 1)$ and M$(-1, 2, -1)$ meets the plane with equation $x - 2y - 3z = 14$.
- **b** Find the shortest distance from L to the plane.

3 Given A$(-1, 2, 3)$, B$(1, 0, -1)$ and C$(1, 3, 0)$, find:
- **a** the normal vector to the plane containing A, B and C
- **b** D, the fourth vertex of parallelogram ACBD
- **c** the coordinates of the foot of the perpendicular from C to the line AB.

4 Show that the line $x - 1 = \dfrac{y+2}{2} = \dfrac{z-3}{4}$ is parallel to the plane $6x + 7y - 5z = 8$ and find the distance between them.

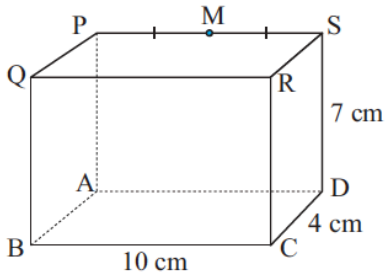
5 Consider the lines with equations $\dfrac{x-3}{2} = y - 4 = \dfrac{z+1}{-2}$ and $x = -1 + 3t$, $y = 2 + 2t$, $z = 3 - t$.
- **a** Are the lines parallel, intersecting or skew? Justify each answer.
- **b** Determine the cosine of the acute angle between the lines.

6 For A$(2, -1, 3)$ and B$(0, 1, -1)$, find:
- **a** the vector equation of the line through A and B, and hence
- **b** the coordinates of C on AB which is 2 units from A.

7 Find the equation of the plane through A$(-1, 2, 3)$, B$(1, 0, -1)$ and C$(0, -1, 5)$. If X is $(3, 2, 4)$, find the angle that AX makes with this plane.

8
- **a** Find all vectors of length 3 units which are normal to the plane $x - y + z = 6$.
- **b** Find a unit vector parallel to $\mathbf{i} + r\mathbf{j} + 3\mathbf{k}$ and perpendicular to $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.
- **c** The distance from A$(-1, 2, 3)$ to the plane with equation $2x - y + 2z = k$ is 3 units. Find k.

9



Use vector methods to determine the measure of angle QDM given that M is the midpoint of PS of the rectangular prism.

10 P$(-1, 2, 3)$ and Q$(4, 0, -1)$ are two points in space. Find:
- **a** $\overrightarrow{PQ}$
- **b** the angle that $\overrightarrow{PQ}$ makes with the X-axis.

Answer :

1 **a** $14x + 29y - 4z = 32$ **b** $\div 55.86^o$ **c** $r = \dfrac{2 \pm \sqrt{10}}{2}$

2 **a** They do not meet, the line is parallel to the plane.

b $\frac{16}{\sqrt{14}}$ units

3 **a** $\mathbf{n} = [5, -1, 3]$ **b** D$(-1, -1, 2)$ **c** $(\frac{1}{6}, \frac{5}{6}, \frac{2}{3})$

4 $\frac{31}{\sqrt{110}}$ units

5 **a** intersecting **b** $\frac{10}{3\sqrt{14}}$ units

6 **a** $[x, y, z] = [2, -1, 3] + t[-2, 2, -4], \ t \in \mathcal{R}$

b $\left(2 - \frac{2}{\sqrt{6}}, \ -1 + \frac{2}{\sqrt{6}}, \ 3 - \frac{4}{\sqrt{6}}\right)$ and

$\left(2 + \frac{2}{\sqrt{6}}, \ -1 - \frac{2}{\sqrt{6}}, \ 3 + \frac{4}{\sqrt{6}}\right)$

7 $4x + 2y + z = 3, \ \div 64.12^o$

8 **a** $[\sqrt{3}, -\sqrt{3}, \sqrt{3}]$ and $[-\sqrt{3}, \sqrt{3}, -\sqrt{3}]$

b $\frac{1}{\sqrt{74}}\mathbf{i} + \frac{8}{\sqrt{74}}\mathbf{j} + \frac{3}{\sqrt{74}}\mathbf{k}$ or $-\frac{1}{\sqrt{74}}\mathbf{i} - \frac{8}{\sqrt{74}}\mathbf{j} - \frac{3}{\sqrt{74}}\mathbf{k}$

c $k = -7$ or 11

9 $\div 26.4^o$

10 **a** $\overrightarrow{\text{PQ}} = [5, -2, -4]$ **b** $\div 41.8^o$

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