SATPREP

Applications of sine and cosine rule

1. Given ΔABC, with lengths shown in the diagram below, find the length of the line segment [CD].

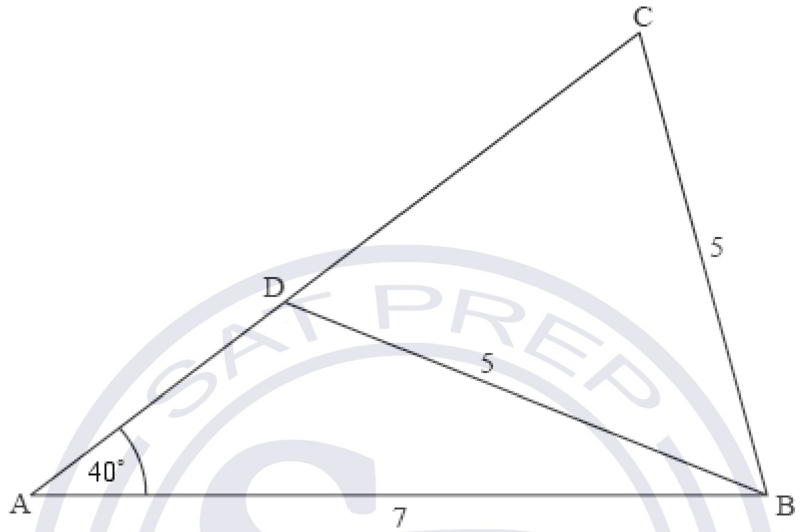


diagram not to scale

2. Triangle ABC has AB = 5cm, BC = 6 cm and area 10 cm^2.

   (a) Find $\sin \hat{B}$.

   (b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

3. A triangle has sides of length $(n^2 + n + 1)$, $(2n + 1)$ and $(n^2 - 1)$ where $n > 1$.

   (a) Explain why the side $(n^2 + n + 1)$ must be the longest side of the triangle.

   (b) Show that the largest angle, θ, of the triangle is 120°.

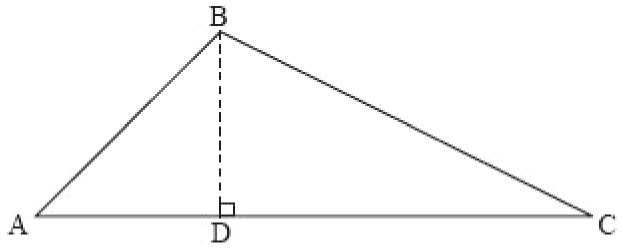
4. Consider triangle ABC with $B\hat{A}C$ = 37.8°, AB = 8.75 and BC = 6.

   Find AC.

5. In triangle ABC, AB = 9 cm, AC =12 cm, and $\hat{B}$ is twice the size of $\hat{C}$.

   Find the cosine of $\hat{C}$.

6. In triangle ABC, BC = a, AC = b, AB = c and [BD] is perpendicular to [AC].



(a) Show that CD = $b - c\cos A$.

(b) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC.

(c) If $\mathrm{A\hat{B}C} = 60°$, use the cosine rule to show that $c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$.

7. The lengths of the sides of a triangle ABC are $x - 2$, x and $x + 2$. The largest angle is 120°.

(a) Find the value of x.

(b) Show that the area of the triangle is $\frac{15\sqrt{3}}{4}$.

(c) Find $\sin A + \sin B + \sin C$ giving your answer in the form $\frac{p\sqrt{q}}{r}$ where $p, q, r \in \mathbb{Z}$.

8. A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is 60°.

(a) Calculate the length of the third side of the field.

(b) Find the area of the field in the form $p\sqrt{3}$, where p is an integer.

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length x metres.

(c) (i) Show that the area o the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

(ii) Hence, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(d) Prove that $\frac{\mathrm{BD}}{\mathrm{DC}} = \frac{5}{8}$.

Answer

1. **METHOD 1**

$$\frac{\sin C}{7} = \frac{\sin 40}{5}$$

$B\hat{C}D = 64.14...°$

CD = 2 × 5cos 64.14...

Note: A1so allow use of sine or cosine rule.

CD = 4.36

METHOD 2

let AC = x

cosine rule

$5^2 = 7^2 + x^2 - 2 \times 7 \times x \cos 40$

$x^2 - 10.7... x + 24 = 0$

$$x = \frac{10.7... \pm \sqrt{(10.7...)^2 - 4 \times 24}}{2}$$

$x = 7.54; 3.18$

CD is the difference in these two values = 4.36

Note: Other methods may be seen.

2. (a) area = $\frac{1}{2} \times BC \times AB \times \sin B$

$\left(10 = \frac{1}{2} \times 5 \times 6 \times \sin B\right)$

$\sin \hat{B} = \frac{2}{3}$

(b) $\cos B = \pm \frac{\sqrt{5}}{3}$ (= ±0.7453...) or B = 41.8... and 138.1...

$AC^2 = BC^2 + AB^2 - 2 \times BC \times AB \times \cos B$

AC = $\sqrt{5^2 + 6^2 - 2 \times 5 \times 6 \times 0.7453...}$ or $\sqrt{5^2 + 6^2 + 2 \times 5 \times 6 \times 0.7453...}$

AC = 4.03 or 10.28

3. (a) a reasonable attempt to show either that $n^2 + n + 1 > 2n + 1$ or $n^2 + n + 1 > n^2 - 1$

complete solution to each inequality

(b) $$\cos \theta = \frac{(2n+1)^2 + (n^2-1)^2 - (n^2+n+1)^2}{2(2n+1)(n^2-1)}$$

$$= \frac{-2n^3 - n^2 + 2n + 1}{2(2n+1)(n^2-1)}$$

$$= -\frac{(n-1)(n+1)(2n+1)}{2(2n+1)(n^2-1)}$$

$$= -\frac{1}{2}$$

$\theta = 120°$

4. METHOD 1

Attempting to use the cosine rule i.e.

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \ B\hat{A}C$

$6^2 = 8.75^2 + AC^2 - 2 \times 8.75 \times AC \times \cos 37.8°$ (or equivalent)

Attempting to solve the quadratic in AC e.g. graphically, numerically or with quadratic formula

Evidence from a sketch graph or their quadratic formula (AC =…)

that there are two values of AC to determine.)

AC = 9.60 or AC = 4.22

Note: Award (M1)A1M1A1(A0)A1A0 for one correct value of AC.

METHOD 2

Attempting to use the sine rule i.e. $\dfrac{\text{BC}}{\sin \text{B}\hat{\text{A}}\text{C}} = \dfrac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}}$

$\sin C = \dfrac{8.75 \sin 37.8°}{6}$ $(= 0.8938...)$

$C = 63.3576...°$

$C = 116.6423...°$ **and** $B = 78.842...°$ or $B = 25.5576...°$

EITHER

Attempting to solve $\dfrac{\text{AC}}{\sin 78.842...°} = \dfrac{6}{\sin 37.8°}$ or

$\dfrac{\text{AC}}{\sin 25.5576...°} = \dfrac{6}{\sin 37.8°}$

OR

Attempting to solve $\text{AC}^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 25.5576...°$ or

$\text{AC}^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 78.842...°$

AC = 9.60 or AC = 4.22

Note: Award (M1)(A1)A1A0M1A1A0 for one correct value of AC.

5. $\dfrac{9}{\sin C} = \dfrac{12}{\sin B}$

$\dfrac{9}{\sin C} = \dfrac{12}{\sin 2C}$

Using double angle formula $\dfrac{9}{\sin C} = \dfrac{12}{2 \sin C \cos C}$

$\Rightarrow 9(2 \sin C \cos C) = 12 \sin C$

$\Rightarrow 6 \sin C\,(3 \cos C - 2) = 0$ or equivalent

$(\sin C \neq 0)$

$\Rightarrow \cos C = \dfrac{2}{3}$

6. (a) $\text{CD} = \text{AC} - \text{AD} = b - c \cos A$

(b) **METHOD 1**

$BC^2 = BD^2 + CD^2$

$a^2 = (c \sin A)^2 + (b - c \cos A)^2$

$= c^2 \sin^2 A + b^2 - 2bc \cos A + c^2 \cos^2 A$

$= b^2 + c^2 - 2bc \cos A$

METHOD 2

$BD^2 = AB^2 - AD^2 = BC^2 - CD^2$

$\Rightarrow c^2 - c^2 \cos^2 A = a^2 - b^2 + 2bc \cos A - c^2 \cos^2 A$

$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$

(c) **METHOD 1**

$b^2 = a^2 + c^2 - 2ac \cos 60° \Rightarrow b^2 = a^2 + c^2 - ac$

$\Rightarrow c^2 - ac + a^2 - b^2 = 0$

$$\Rightarrow c = \frac{a \pm \sqrt{(-a)^2 - 4(a^2 - b^2)}}{2}$$

$$= \frac{a \pm \sqrt{4b^2 - 3a^2}}{2} = \frac{a}{2} \pm \sqrt{\frac{4b^2 - 3a^2}{4}}$$

$$= \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$$

Note: Candidates can only obtain a maximum of the first three marks if they **verify** that the answer given in the question satisfies the equation.

METHOD 2

$b^2 = a^2 + c^2 - 2ac \cos 60° \Rightarrow b^2 = a^2 + c^2 - ac$

$c^2 - ac = b^2 - a^2$

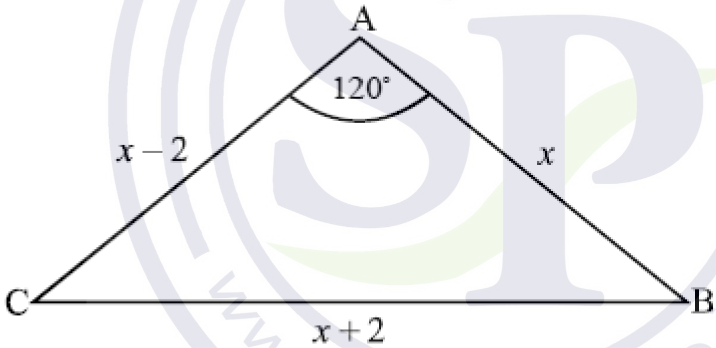
$c^2 - ac + \left(\frac{a}{2}\right)^2 = b^2 - a^2 + \left(\frac{a}{2}\right)^2$

$\left(c - \frac{a}{2}\right)^2 = b^2 - \frac{3}{4}a^2$

$c - \frac{a}{2} = \pm\sqrt{b^2 - \frac{3}{4}a^2}$

$\Rightarrow c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$

7. (a)



$(x + 2)^2 = (x - 2)^2 + x^2 - 2(x - 2)\, x\cos 120°$
$x^2 + 4x + 4 = x^2 - 4x + 4 + x^2 + x^2 - 2x$
$0 = 2x^2 - 10x$
$0 = x(x - 5)$
$x = 5$

(b) Area $= \frac{1}{2} \times 5 \times 3 \times \sin 120°$

$= \frac{1}{2} \times 15 \times \frac{\sqrt{3}}{2}$

$= \frac{15\sqrt{3}}{4}$

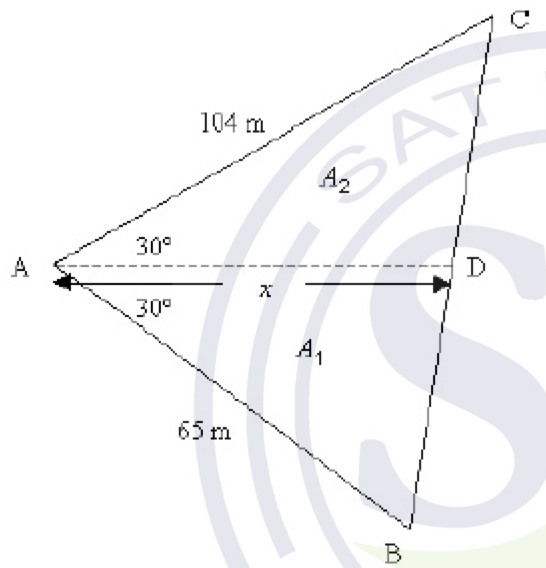
(c) $\sin A = \dfrac{\sqrt{3}}{2}$

$\dfrac{15\sqrt{3}}{4} = \dfrac{1}{2} \times 5 \times 7 \times \sin B \Rightarrow \sin B = \dfrac{3\sqrt{3}}{14}$

Similarly $\sin C = \dfrac{5\sqrt{3}}{14}$

$\sin A + \sin B + \sin C = \dfrac{15\sqrt{3}}{14}$

8.



(a) Using the cosine rule ($a^2 = b^2 + c^2 - 2bc \cos A$)
Substituting correctly
$BC^2 = 65^2 + 104^2 - 2\ (65)\ (104) \cos 60°$
$= 4225 + 10\,816 - 6760 = 8281$
$\Rightarrow$ BC = 91m

(b) Finding the area using = $\dfrac{1}{2} bc \sin A$

Substituting correctly, area = $\dfrac{1}{2}(65)\ (104) \sin 60°$

$= 1690\sqrt{3}$ (accept $p = 1690$)

(c) (i) Smaller area $A_1 = \left(\dfrac{1}{2}\right)(65)\ (x)\sin 30°$

$= \dfrac{65x}{4}$

Larger area $A_2 = \left(\dfrac{1}{2}\right)(104)\ (x) \sin 30°$

$= 26x$

(ii) Using $A_1 + A_2 = A$

Substituting $\frac{65x}{4} + 26x = 1690\sqrt{3}$

Simplifying $\frac{169x}{4} = 1690\sqrt{3}$

Solving $x = \frac{4 \times 1690\sqrt{3}}{169}$

$\Rightarrow x = 40\sqrt{3}$ (accept $q = 40$)

(d) Using sin rule in ΔADB and ΔACD

Substituting correctly $\frac{BD}{\sin 30°} = \frac{65}{\sin A\hat{D}B} \Rightarrow \frac{BD}{65} = \frac{\sin 30°}{\sin A\hat{D}B}$

and $\frac{DC}{\sin 30°} = \frac{104}{\sin A\hat{D}C} \Rightarrow \frac{DC}{104} = \frac{\sin 30°}{\sin A\hat{D}C}$

Since $A\hat{D}B + A\hat{D}C = 180°$

It follows that $\sin A\hat{D}B = \sin A\hat{D}C$

$\frac{BD}{65} = \frac{DC}{104} \Rightarrow \frac{BD}{DC} = \frac{65}{104}$

$\Rightarrow \frac{BD}{DC} = \frac{5}{8}$